Guide to code for "The Art of Temporal Approximation: An Investigation into Numerical Solutions to Discrete & Continuous-Time Problems in Economics"*

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1 Introduction

This document provides some notes to the python code used in the paper. If you have any questions or comments on the code or this guide please email me at tom.phelan@clev.frb.org.

I first describe the class constructors used throughout the paper and then turn to the scripts used to produce run times and figures. As we emphasize in the paper, both the continuous-time and discrete-time frameworks lead to a discrete-time Bellman equation, and so all of the class constructors below contain methods for VFI, MPFI and PFI. The frameworks differ in the matrix construction (which involves non-local transitions in the discrete-time case) and in the policy updating step (which requires either brute force or the EGM in the discrete-time case, together with an interpolation step).

Note. The scripts stat_time_accuracy.py and nonstat_time_accuracy.py take several hours to run, because they first compute the "true" values on a very fine grid and then solve the test problems many times on different grids. A reader interested in adapting the code for their own purposes ought to simply run stat_figures.py, nonstat_figures.py and mortality.py, which produces the figures used in the paper for the stationary, non-stationary, and endogenous mortality examples, respectively.

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2 File organization

2.1 Class constructors

All class constructors are contained in either classes.py in the main code folder or mortality.py within the "nonstationary" folder. The classes.py script contains three class constructors:

(i) DT_IFP: discrete-time income fluctuation problem (stationary and age-dependent).

(ii) CT_stat_IFP: continuous-time stationary income fluctuation problem.

(iii) CT_nonstat_IFP: continuous-time age-dependent income fluctuation problem.

The mortality.py script contains one class constructor:

(i) CT_nonstat_IFP_mortality: continuous-time age-dependent income fluctuation problem with endogenous mortality and labor-leisure choice.

2.2 Scripts

I now briefly describe each script appearing in the replication files.

- In the main folder there are two scripts:
  - (i) classes.py: contains the three class constructors DT_IFP, CT_stat_IFP, and CT_nonstat_IFP described in Section 2.1.
  - (ii) parameters.py: records the parameters used throughout the paper together with some miscellaneous conventions such as column lists and plotting functions.
- In the folder entitled "stationary" there are three scripts:
  - (i) stat_figures.py: produces the figures used in the stationary section of the paper. This scripts also tests the possible values of the timestep for which convergence fails in the example, and so motivates the choice of timestep in the run times for the continuous-time problem.
  - (ii) stat_time_accuracy.py: produces run times and accuracy for the stationary section of the paper.
  - (iii) true_stat.py: creates folder for "true" policy functions and value functions and computes these for the stationary setting if they do not already exist.
- In the folder entitled "nonstationary" there are five scripts:

(i) nonstat_figures.py: produces the figures used in the nonstationary section of the paper (without mortality risk).

(ii) nonstat_time_accuracy.py: produces run times and accuracy for the nonstationary section of the paper.

(iii) true_nonstat.py: creates folder for "true" policy functions and value functions and computes these for the nonstationary setting if they do not already exist.

(iv) mortality.py: solves the extension at the end of the paper with endogenous mortality risk and labor-leisure choice.

(v) naive_seq_accuracy.py: computes the policy functions and value functions for the benchmark parameters using both (naive) PFI and sequential PFI and produces a table documenting the differences.