# Online Appendix to "On the Optimality of Differential Asset Taxation"

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August 3, 2022

This document provides a guide to the code used to produce the figures in the paper "On the Optimality of Differential Asset Taxation". All code is written in Python 3.6.5 and is located at https://github.com/tphelanECON/diff_cap_tax. If you spot errors or have questions please email me at tom.phelan@clev.frb.org.

### Preliminaries

To explain the code construction I recall some algebra from the paper. In Section 2 of the paper I defined a candidate value function

$$\overline{v} = \max_{\substack{\overline{c},x\geq 0, x\overline{c}\leq\overline{\omega} \\ -\ln\overline{c}+x^2/2<1}} \frac{(Sx-1)\overline{c}}{\rho(1+\ln\overline{c}-x^2/2)} \tag{1}$$

where $\overline{\omega} = \sqrt{\rho}\phi\sigma/(\rho\iota)$. In Appendix A I defined $\overline{x}$ and $\overline{\overline{x}}$ to be the solutions to $\overline{x}e^{\overline{x}^2/2} = \overline{\omega}$ and $\overline{\overline{x}}e^{\overline{\overline{x}}^2/2-1} = \overline{\omega}$, respectively. Following the explicit maximization in the proof of Proposition 2.3, the right-hand side of (1) may be written as

$$\overline{v} = \max_{x\in[0,\overline{\overline{x}}]} g(S,\overline{\omega},x)h(S,\overline{\omega},x) \tag{2}$$

where $g$ and $h$ are given by

$$\begin{aligned} g(S,\overline{\omega},x) &= \frac{1}{\rho}(Sx-1)e^{x^2/2} \\ h(S,\overline{\omega},x) &= 1_{x<\overline{x}(\overline{\omega})} + 1_{x\geq\overline{x}(\overline{\omega})}\frac{(\overline{\omega}/x)e^{-x^2/2}}{1+\ln(\overline{\omega}/x)-x^2/2}. \end{aligned} \tag{3}$$

## Code construction

The sole class constructor for the paper is entitled captax_general and is located in classes.py. It contains the following methods (in the following, omegabar is $\overline{\omega}$):

- xbar(omegabar) and xbarbar(omegabar): $\overline{x}$ and $\overline{\overline{x}}$.
- g(S,omegabar,x) and h(S,omegabar,x): the functions in (3).
- x(S,omegabar) and c(S,omegabar): $x(S,\overline{\omega})$ and $\overline{c}(S,\overline{\omega})$ from the main text.
- mu_c(S,omegabar) and sig_c(S,omegabar): mean and volatility of consumption growth, denoted $\mu_c$ and $\sigma_c$ in the main text.
- omegahat(self,S,omegabar): the constant $\hat{\omega}$ in the collateral constraint in the decentralization.
- f(S,omegabar,phi,sigma): defined as LHS - RHS of the equation in Assumption 3.1.
- S_hat(phi,sigma) and Pi_hat(phi,sigma): $\hat{S}$ and $\hat{\Pi}$ from Section 3 of the text.
- r(Pi,phi,sigma): interest rate in benchmark case (no private risk-sharing).
- r_pe(Pi,phi,sigma): interest rate with private risk-sharing.
- taus(Pi,phi,sigma) and tausW(Pi,phi,sigma): entrepreneur and worker taxes in benchmark case (no private risk-sharing).
- taus_pe(Pi,phi,sigma) and tausW_pe(Pi,phi,sigma): entrepreneur and worker taxes with private risk-sharing.
- S(Pi,phi,sigma): $(\Pi - \rho_S)/(\sqrt{\rho}\phi\sigma)$.
- check1(S,omegabar) and check2(S,omegabar): two checks corresponding to the assumptions in A.1 and A.2 of the appendix.

Note that figures and methods computed under the assumption of private risk-sharing have a suffix _pe at the end of their name for "private equity".

### Figures generation

the script parameters.py lists the parameters used in the numerical examples and classes.py contains the above class constructor. There are three scripts that produce the figures:

- benchmark.py: produces Figure 1 as well as a number of miscellaneous figures (interest rate, revenue from profits tax, consumption risk) that were ultimately omitted from the main text.
- collateral.py: produces Figure 2, which documents taxes for tighter levels of the collateral constraint than the benchmark.
- private_equity.py: produces figures for taxes and interest rates when risk-sharing occurs in the private sector.

The script main.py runs all three of the above.