

# Online Appendix to “On the Optimality of Differential Asset Taxation”

Thomas Phelan  
Federal Reserve Bank of Cleveland

August 18, 2022

This document provides a guide to the code used to produce the figures in the paper “On the Optimality of Differential Asset Taxation”. All code is written in Python 3.6.5 and is located at [https://github.com/tphelanECON/diff\\_cap\\_tax](https://github.com/tphelanECON/diff_cap_tax). If you spot errors or have questions please email me at [tom.phelan@clev.frb.org](mailto:tom.phelan@clev.frb.org).

## Preliminaries

To explain the code construction I recall some algebra from the paper. In Section 2 of the paper I defined a candidate value function

$$\bar{v} = \max_{\substack{\bar{c}, x \geq 0, x\bar{c} \leq \bar{\omega} \\ -\ln \bar{c} + x^2/2 < 1}} \frac{(Sx - 1)\bar{c}}{\rho(1 + \ln \bar{c} - x^2/2)} \quad (1)$$

where  $\bar{\omega} = \sqrt{\rho}\phi\sigma/(\rho\nu)$ . In Appendix A I defined  $\bar{x}$  and  $\bar{\bar{x}}$  to be the solutions to  $\bar{x}e^{\bar{x}^2/2} = \bar{\omega}$  and  $\bar{\bar{x}}e^{\bar{\bar{x}}^2/2-1} = \bar{\omega}$ , respectively. Following the explicit maximization in the proof of Proposition 2.3, the right-hand side of (1) may be written as

$$\bar{v} = \max_{x \in [0, \bar{\bar{x}}]} g(S, \bar{\omega}, x) h(S, \bar{\omega}, x) \quad (2)$$

where  $g$  and  $h$  are given by

$$\begin{aligned} g(S, \bar{\omega}, x) &= \frac{1}{\rho}(Sx - 1)e^{x^2/2} \\ h(S, \bar{\omega}, x) &= 1_{x < \bar{x}(\bar{\omega})} + 1_{x \geq \bar{x}(\bar{\omega})} \frac{(\bar{\omega}/x)e^{-x^2/2}}{1 + \ln(\bar{\omega}/x) - x^2/2}. \end{aligned} \quad (3)$$

## Code construction

The sole class constructor for the paper is entitled `captax_general` and is located in `classes.py`. It contains the following methods (in the following, `omegabar` is  $\bar{\omega}$ ):

- `xbar(omegabar)` and `xbarbar(omegabar)`:  $\bar{x}$  and  $\bar{\bar{x}}$ .
- `g(S,omegabar,x)` and `h(S,omegabar,x)`: the functions in (3).
- `x(S,omegabar)` and `c(S,omegabar)`:  $x(S, \bar{\omega})$  and  $\bar{c}(S, \bar{\omega})$  from the main text.
- `mu_c(S,omegabar)` and `sig_c(S,omegabar)`: mean and volatility of consumption growth, denoted  $\mu_c$  and  $\sigma_c$  in the main text.
- `omegahat(self,S,omegabar)`: the constant  $\hat{\omega}$  in the collateral constraint in the decentralization.
- `S(Pi,phi)`:  $(\Pi - \rho_S)/(\sqrt{\rho}\phi\sigma)$ .
- `f(S,omegabar,phi)`: defined as LHS - RHS of the equation in Assumption 3.1.
- `S_hat(phi)` and `Pi_hat(phi)`:  $\hat{S}$  and  $\hat{\Pi}$  from Section 3 of the text.
- `r(Pi,phi)`: interest rate in benchmark case (no private risk-sharing).
- `r_pe(Pi,phi)`: interest rate with private risk-sharing.
- `taus(Pi,phi)` and `tausW(Pi,phi)`: entrepreneur and worker taxes in benchmark case (no private risk-sharing).
- `taus_pe(Pi,phi)` and `tausW_pe(Pi,phi)`: entrepreneur and worker taxes with private risk-sharing.
- `nu_B(S,omegabar)` and `nu_K(S,omegabar,Pi)`: the wedges from the partial equilibrium setting.
- `check1(S,omegabar)` and `check2(S,omegabar)`: two checks corresponding to the assumptions in A.1 and A.2 of the appendix.

Note that figures and methods computed under the assumption of private risk-sharing have a suffix `_pe` at the end of their name for “private equity”. Figures with no suffix after them assume the most relaxed collateral constraints possible, while the figures with suffix `1.0` corresponds to the tightest collateral constraints possible.

## Figures generation

There are six scripts necessary for replication: `main.py`, `classes.py`, `parameters.py`, `tight.py`, `relaxed.py`, and `private_equity.py`.

- `main.py`: runs `tight.py`, `relaxed.py` and `private_equity.py`.
- `parameters.py`: defines the parameters used in the numerical examples.
- `classes.py`: contains the class constructor used in the paper.
- `tight.py`: produces figures pertaining to the case in which collateral constraints are tight ( $\bar{\iota} = 1.0$ ) for the benchmark decentralization. In particular, this produces Figure 1 and Figure 2 from the main text.
- `relaxed.py`: produces figures pertaining to the case in which collateral constraints are relaxed ( $\bar{\iota} \approx 0.5$ ) for the benchmark decentralization. In particular, this produces Figure 3 from the main text as well as the wedges in Figure 4 of Appendix E.
- `private_equity.py`: produces figures for taxes and interest rates for the market structure in which there is private risk-sharing. In particular, this produces Figure 5, 6 and 7 of Appendix E.