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IRVINE

Applications of Synthetic Microchannel and Nanopore systems

DISSERTATION

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for the degree of

DOCTOR OF PHILOSOPHY

in Physics & Astronomy

by

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DEDICATION

(Optional dedication page)
To ...

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nanoIV	https://github.com/tphinkle/nanoIV
<i>Keithley 6487 Picoammeter control GUI program. Allows measurement of current-voltage and time-series data.</i>	
pore stats	https://github.com/tphinkle/pore_stats
<i>GUI program and Python library for extracting and analyzing resistive pulse data.</i>	

ABSTRACT OF THE DISSERTATION

Applications of Synthetic Microchannel and Nanopore systems

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Dr. Zuzanna S. Siwy, Chair

The abstract of your contribution goes here.

Chapter 1

Introduction

The introduction to this dissertation is structured as follows. First, we introduce the main topic of this dissertation and of my PhD research work, the nanopore. This section serves to introduce the reader first, to the basic definition of a nanopore and then to some of their applications, in addition to miscellaneous facts about nanopores that provide context for why they are of interest to scientists. This section is entirely qualitative and does not aim to make any statements about the underlying physics needed to understand nanopore systems. Subsequently, I introduce in three stages the physics that is relevant to nanopore systems. It turns out that if we take a macroscaled system (say, a tube rather than a nanotube) and slowly shrink it in size, then as the system size shrinks new types of physics emerges and become increasingly relevant to the system. We will start by addressing the relevant physics at the milliscale and shrink in intervals of powers of three, stopping at the sub-nanoscale. Therefore, the progression looks like milli-, micro-, nano-, and finally, sub-nano-. To provide a bigger picture of the journey through different length scales, the progression looks like ant, cell, protein, molecule.

1.1 Nanopores

Unlike most physics jargon, the name nanopore is entirely self-explanatory. In fact, the following is a terse (but accurate) definition of a nanopore:

Definition 1.1. *A nanopore is a hole on the order of 1 nm in size*

As we will discover, this definition, while accurate, does not do justice to the field of nanopore research.

Nanopores generally belong to two classes, biological and synthetic. Because nanopores can be found in biology, many scientists are interested in studying them in their own right, to better understand how biological systems function. On the other hand, nanopores, both biological and synthetic, can be used in a surprisingly large variety of applications, with the total count of such applications constantly increasing. In this section we will briefly discuss the types of biological nanopores and where they are found in nature, and then move on to describing different types of synthetic nanopores. Given the loose criteria required by 1.1 for something to constitute a nanopore, listing off every type of either biological or synthetic nanopores would be impossible; instead, I seek to introduce the most relevant types, with an admitted bias towards the types of nanopores I am most familiar with and that I have worked with in my research. After discussing the various types of nanopores, I will give a summary of applications in which nanopores see use. Some of these applications, for instance resistive-pulse sensing, are a major part of this dissertation and will therefore be discussed in much greater detail later on, while others, such as DNA sequencing, are important enough to be mentioned but not discussed in significant detail later on. An extensive list of references, including those not explicitly referenced in this work, will be included should the reader wish to read more on a particular topic.

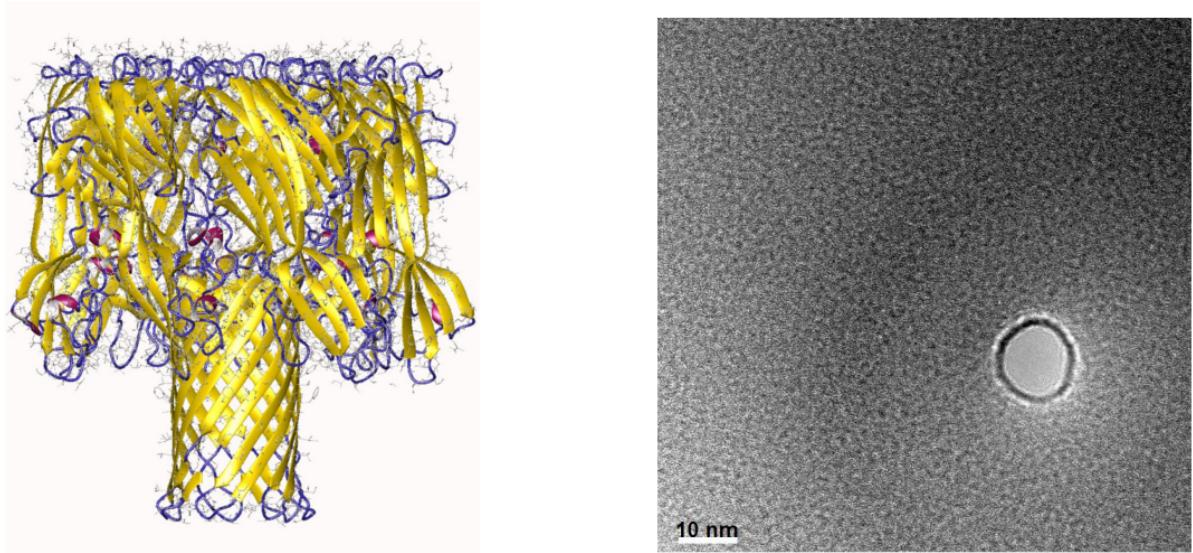


Figure 1.1: Two types of nanopores. **Left:** Alpha-hemolysin, a biological type of nanopore approximately 1 nm in diameter. **Right:** A silicon nitride nanopore drilled using a transmission electron microscope (TEM), approximately 10 nm in diameter.

1.1.1 Biological nanopores

As mentioned before, nanopores exist naturally in biology. Their most important function is to enable passage from one region to another, much like a tunnel does; however, where a tunnel permits cars, trucks, etc., the passengers in nanopore systems are water, ions, proteins, etc. Nanopores typically connect regions that are necessarily divided in order for a cell to function. Membrane-bound nanopores are structures made out of protein that are embedded in a cell's lipid bilayer that connect the outside of a cell to the inside of a cell. They serve the useful function of regulation osmotic pressure by allowing water to enter and exit the cell, and also balance voltage differences by allowing passage of important electrolytes such as sodium, potassium, and chloride. Therefore, maintaining homeostasis is one of the chief functions of these types of nanopores. These pores also enable passage of messenger molecules that enable extracellular messaging, crucial to all multicelled organisms. Like lipid bilayer pores, there are also pores in the nuclear membrane that enable transport into and out of the nucleus of a cell. These pores' chief responsibility is to allow the passage of

messenger molecules in and out of the nucleus.

The most fascinating property of nanopores is their capacity for active management of transport. For instance, some nanopores may open or close when the voltage difference across them surpasses a threshold; other types of nanopores are *ion selective*, permitting passage of some types of ions while denying passage to others. *Active* transport is absolutely crucial to biological systems; if all nanopores were passive holes that permitted anything to pass through, the system would equilibrate in a homogeneous, non-functional soup of maximum entropy—life simply could not exist. Fortunately for us, nanopores are active transport regulators and life goes on. Going back to our tunnel analogy, nanopores act more like regulated tunnels, for instance some with guards that permit small cars only and deny trucks, or permit passage in only one direction, etc.

1.1.2 Synthetic nanopores

Advances in nano- and microfabrication have led to an enormous number of different types of synthetic nanopores. Synthetic nanopores can be made from a variety of different types of materials, and often their geometry (shape and size), as well as their surface chemistry properties, can be tailored to introduce specific types of behavior. However, the type of material used to make the nanopore most often determines the types of geometries and chemistries that are allowed. The following is a list of common types of nanopores, with a description of the type of material they are made from, their permissible length scales, and other miscellaneous relevant facts about them.

Monolayer pores. The invention of graphene, a monolayer of carbon atoms, and later MoS₂, opened the possibility of creating nanopores with a length of a single atom. These types of pores are created by punching through the thin monolayer, typically with an electron or ion beam. While still a very new field of study, these pores may see future use in

desalination (removing electrolyte ions from water).

Carbon nanotubes. A carbon nanotube can be conceptually understood as a rolled up tube of one or a stack of graphene layers. Graphene is a monolayer of carbon atoms, and therefore a carbon nanotube itself is a type of pure crystal structure. The lattice arrangement of carbon atoms permits only certain tube diameters, which itself depends on the chirality of the tube (the way the tube is rolled up). Carbon nanotubes are interesting to researchers because of the exotic behaviors they exhibit that are not present in most other types of nanopores—for instance, they exhibit frictionless transport of water, hydrogen-dominant conductance (*via* the Grotthus mechanism), ion selectivity, and more. As of the submission of this dissertation, the use of CNTs as nanopore is still a new field of study, and the physical mechanism for the previously mentioned behaviors is still poorly understood. Nevertheless, the extreme confinement (1 nm and below for small CNTs) and the atomic-level precision of their structure are believed to be responsible. Another interesting aspect of the smallest CNTs is the breakdown of mean-field physics in describing their behavior. For instance, the Navier-Stokes equations which describe fluid dynamics, breaks down at this level because the water must be considered at the molecular level; the mean-field approach is no longer valid. Because they are still new, we do not yet know the exact applications that these nanopores will see use in. However, it is likely that they could be used in desalination applications (removing ions from water), or in ionic circuits as cation selective elements.

Silicon nitride. Silicon nitride is a type of crystalline semiconductor that permits engineering of nanopores through several different fabrication processes. Briefly, very thin layers of silicon nitride (e.g. 10 – 100 nm) are grown, and a hole is bored through via either electron or ion beam milling. For instance, in one project of this dissertation I describe a project conducted with silicon nitride pores that were drilled via high-energy transmission electron microscope (TEM). Depending on the type of mill used (ion or electron), the size of these nanopores ranges, but the diameter is generally in the range of 1 – 100 nm. One advantage of

silicon nitride pores is their smooth interior geometries, as well as the native silane chemistry on the surface that permits many types of chemical modifications. These types of pores are used as ionic rectifiers (after modification), and in resistive pulse applications.

Glass nanopipettes. Quartz pipettes can be heated and slowly stretched, reducing the tip diameter with the possibility of reaching the nanoscale. Unlike the previously mentioned pores which all had an approximately cylindrical geometry, these nanopipettes have a conical shape. These pores have a couple advantages. First, the stretching process itself is simple and can be used to create many pores over a short amount of time. Second, the surface chemistry of the glass or quartz is amenable towards chemical modification. These pores may be used in ionic circuits, in resistive pulse sensing, or as a surface probe, by monitoring the current through the pipette as it approaches a charged surface.

Track-etched polymer nanopores. These types of pores are perhaps the most robust, dependable types of nanopores. Pore formation is a multistep process. First, untouched polymer membranes are irradiated by a single heavy isotope of an element such as gold and xenon, that has been accelerated to high speeds in a particle accelerator. The ion rips through the membrane, uniformly dispersing some of its kinetic energy into the surrounding polymer, and breaking the bonds in the polymer surrounding its trajectory. This location of damaged polymer bonds is known as the ‘damage track’. An ion detector is placed at the exit point of the membranes so that the beam can be switched off when a single ion is detected. This detection, along with a solid mask placed in front of the beam that blocks the vast majority of the ions in the beam, ensures that films that only have a single damage track can be prepared. This step, known as irradiation, must be performed off-site at a particle accelerator. After heavy ion irradiation, the membranes are immersed in an etchant solution such as NaOH or KOH. The etchant preferentially attacks the polymer in the damage track, clearing out the polymers along the track much faster than elsewhere in the membrane. Once the track has been etched out, the rest of the membrane is isotropically etched slowly by the NaOH. Pores

prepared by the track-etch technique have a few advantages, including a customizable size and geometry. By putting etching solution on only one side of the membrane, it is possible to create conical pores of various aspect ratios. By applying a voltage across the pore during the etching process, the ionic current can be monitored and the etching process can be stopped at a particular current level, allowing the researcher customize the pore diameter. Another advantage of these pores is the carboxyl groups native to their surfaces, which permit many types of useful chemical modifications.

Hybrid biological-synthetic nanopores. Biological nanopores, such as alpha hemolysin, aerolysin, or MSPA to name a few, can be isolated from their host biological systems and inserted into synthetic systems, forming a hybrid biological-synthetic complex. In these cases, the pore is entirely biological, but the complex does not appear naturally in nature, and the pores may be used in engineering or scientific applications. To make matters even more confusing, such pores may also be genetically modified to change some of their behavior, meaning they are derived from biology but are synthesized in the lab. For instance, the company Oxford nanopore invented a genetically modified alpha hemolysin nanopore that is especially adept at differentiating between nucleotides, and is currently being used in DNA sequencing applications.

1.1.3 Applications (introductions)

So far, I have hinted at or referred to nanopore applications without getting into specifics. Just as there are too many types of nanopores to enumerate, there are too many applications of nanopores to list them all off. This is a list of some of the most important applications of nanopores.

Analyte detection and characterization with the resistive pulse technique. Surprisingly, nanopores may be used as a sort of particle characterizer using something called the

resistive pulse technique, which works as follows. A voltage is applied across the nanopore, which induces a measurable ionic current to flow through the channel. If a particle enters the nanopore, the particle will occupy a volume that otherwise would be occupied by high-conductivity ions, increasing the pore's resistance, and decreasing the measured current. It turns out that by studying the nature of the decrease in the current, we can gather some information about the transiting particle. For instance, by counting the number of pulses in the measured ionic current, we can determine the concentration of particles in the suspension. The size of the particle can be determined by relating it to the depth of the pulse—larger particles block the current to a larger degree, and therefore create deeper resistive pulses. If the particle is at least partially driven through the channel *via* electrophoresis (discussed later), then the particle's surface charge can be determined by the length of the pulse, or ‘dwell time’ of the particle.

One advantage of resistive pulse sensing is its scale-independence, which is a result of the generality of the physics involved. For this reason, resistive pulse sensing has been used in a large number of applications. The original use of the resistive pulse technique was in performing red blood cell counts (channel size approximately $\sim 100 \mu\text{m}$ which was achieved by Coulter in 1953; this is the reason why sometimes the resistive pulse technique is referred to as the Coulter counter principle, or simply the Coulter principle. Since Coulter's original design, the resistive pulse technique has been used in counting smaller specimens such as exosomes, proteins, and viruses, used to measure particle rigidity, and perhaps most importantly, in DNA sequencing. The basic idea of resistive-pulse sensing for DNA sequencing is that the four types of nucleotides have slightly different sizes, and therefore lead to four unique current blockages. DNA can be slowly threaded through a nanopore while the current is monitored, and the discrete states in the resulting time-series of the measured current yield the sequence of DNA.

Water desalination. Charged nanopores are slightly ion selective, meaning they preferen-

tially allow one polarity of charge over another to pass through. If one side of a nanopore membrane is filled with electrolyte solution and a pressure is applied, water will pass through the pore, and due to the selectivity of the membrane, fewer ions will pass through than are contained in the bulk. In this way, nanopores can be used for water desalination. This mechanism of salt rejection is electrostatic, since the finite potential in the vicinity of the pore walls is responsible for screening ions. Traditional water desalination relies on reverse osmosis membranes that are completely impermeable to salts, and therefore *sterically* reject ions. In future sections we will discuss the physics involved with water desalination.

Nanopore ionics and ionic circuits. Ion transport is fundamentally important to all cellular life. At the most basic level, single cells maintain homeostasis with their environment by maintaining a careful balance of salt ions across their membranes. This balance is regulated by a rich variety of ion channels, each with their own specific functionality. For instance, potassium can diffuse through potassium channels at nearly the bulk diffusion rate (i.e. the channel hardly impedes their motion), while sodium is completely blocked, an amazing feat considering the ions are nearly identical, save a minute difference in their hydration radii. Beyond maintaining homeostasis, ion channels are also responsible for propagating electric signals along axons, crucial for signaling in multicellular organisms.

One of the holy grails in nanopores is being able to reproduce the functionality of these biological pores, that were created by millions or billions of years of evolution, in synthetic systems. The ability to do so could create a revolution in ion conducting systems, analogous to the revolution in electronics that was created by the invention of the solid-state transistor. However, the goal of ionics is not to replace wires, capacitors, and transistors with ion channels and pores; indeed, ions travel orders of magnitude slower than electrons, and therefore ionic systems are expected to act more slowly than electronic systems. Instead, integrated ionic circuits could be used for complex signaling since each ion carries its own information, and for controlling the transport of biomolecules. One needs only to look at

biology for inspiration on the application of complex ion channels.

1.2 Nanopore science—physics and chemistry

The actors at play in nanopore systems include the fluid solvent that fills the pore and its exterior regions, the charged ion species, the pore surfaces themselves, the electrodes, and finally, any particles (cells, biomolecules, etc.) present in the system. In the following sections we will delve into the physics that determines the interplay between these actors. As it turns out, as we shrink a system size from the macro to nanoscale, new regimes of physics become increasingly important. Therefore, not all of the physics mentioned in the upcoming sections are applicable to all length scales; for instance, the properties that make nanopores behave more interesting than just an open space only emerge close to the nanoscale, and are irrelevant in microscale systems. We can think of four distinct length scales at which the physics, and hence the system behavior, is sufficiently differentiated so as to give that scale its own identity. These scales are the milliscale, microscale, nanoscale, and subnanoscale. For example, microscale physics is sufficiently different from nanoscale physics that one may be an expert in one of the two, but not the other. While the actors at play in these two systems—water, ions, particles, etc.—are all the same, the observed behavior of the systems can still be quite different. To give a brief preview of what is to come, the most important physics involved in nanopore theory is fluid mechanics (to describe the forces on the solvent), electrostatics (because of the voltage boundary conditions imposed by electrodes and charged surfaces), electrokinetics (due to migration of charged species under electric fields), and statistical mechanics (to describe the ensemble behavior of the ions and diffusion). Before moving on, it is important to note that concepts in chemistry are also very important in nanopores. For instance, electrochemical theory explains the mechanism for how an ion current turns into an electron current at the electrode-liquid interface. As

another example, the charges present at the surfaces in almost all surfaces in contact with a fluid are due to the chemical groups, and their charged state. In the following sections, whenever chemistry is relevant, I will mention its role in creating the entity that we then go on to describe insofar as is possible with physics alone. For example, I will acknowledge the charge of a nanopore is due to chemical groups attached to its surface, but will then proceed agnostically, treating the charges as if they were a perfect model surface-charge, uniformly distributed over the surface of the pore. Chemistry will be invoked in the main chapters of this thesis as it relates to specific aspects of the projects.

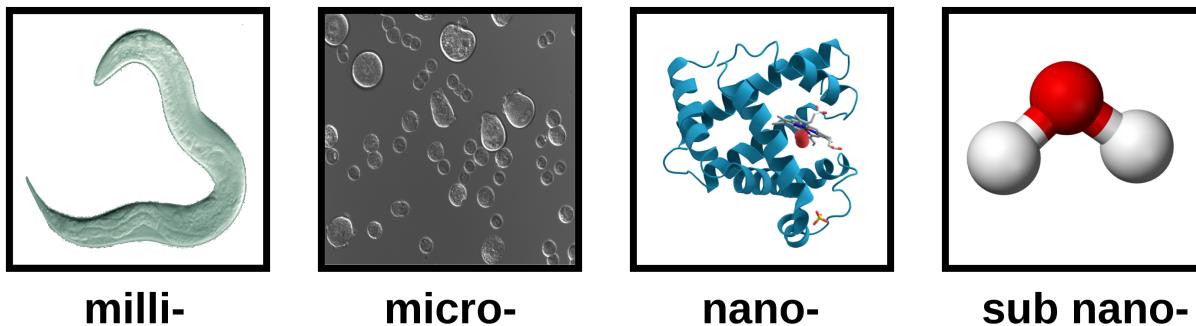


Figure 1.2: Four important length scales for fluidics and surface forces. **milli-**: *Caenorhabditis elegans* (*c. elegans*) sample, approximately 1 mm in length. Surface tension and capillary forces become significant at this length scale. Fluid dynamics is described by the complete Navier-Stokes equations. **micro-**: *HCT-116* (colorectal cancer) cells, approximately $10 \mu\text{m}$ in diameter. Surface effects become increasingly important. Gravity becomes increasingly negligible, and systems under small applied pressures are described by having low-Reynolds number. **nano-**: A protein. Fluidic systems are dominated by surface forces. Liquid within 1 nm of surfaces is entirely contained within the electrical double layer. Fluid is completely non-inertial. **sub nano-**: A water molecule. Mean-field approximations are no longer valid. Navier-Stokes breaks down. Physics is determined entirely by electrostatic and Van der Waals forces between individual atoms and molecules.

1.2.1 Pore/channel

The pore/channel itself is the primary actor of interest in our systems. The pore is created in some material, and the material properties as well as the fabrication procedure dictate the shape and geometry of the pore. As we will see, ionized chemical groups on the surface

of the pore play as important a role as geometry in determining the behavior of the pore system.

1.2.2 Electrolyte solutions

Life on Earth exists in primary a *fluid* environment, with the primary fluid being water. Water is the solvent for the chemistry of our bodies, and acts as the medium that allows electrolytes to ionize and transport between areas. In synthetic nanopore systems, we immerse our pore in an electrolyte solution that fills the channel, and under an applied voltage induces conduction of ions.

1.2.3 Electrodes

In order to create an external voltage in our system, an electronic voltage must be transformed into an ionic voltage potential. This is achieved *via* ion electrodes, which allow transferrance of electrons into the ions of the solution and vice-versa, and the ions from the electrode then become the conductors in the system. This transferrance is known as a redox reaction in chemistry. For the rest of the thesis, unless specified we turn a blind eye to the chemical conversion process at the surface of the electrodes, and pretend that our electrodes are ideal voltage sources.

1.2.4 Particles

Because so many of the applications of interest for nanopore and microchannel studies involve biomolecule manipulation, we often work with electrolyte solutions that contain a suspension of particles in addition to the ions. The particles used are tied to the application, but often

respond in the same way to forces present in the system.

1.2.5 Equations of motion for fluidic systems—Navier–Stokes equations

One of the major actors in nanopores systems is the fluid solvent, which may remain at rest or, more commonly, move in response to forces present in the system. Because the motion of the solvent couples to the motion of other actors present in the system, understanding the physical laws governing the fluid medium is crucial. For systems large compared to the size of molecules, we make the approximation that the fluid is a continuous rather than discrete medium. In reality, we know that fluids are not continuous but are made up of discrete atoms and molecules, which may make this approximation seem limiting. However, the continuum approximation has been shown to accurately describe fluids all the way down to the scale of a single nanometer, and thus are valid in most systems of interest [1]. Under the continuum approximation, the equations of motion of the fluid are described by the Navier-Stokes equation.

The Navier-Stokes equations are actually three coupled 2nd order non-linear partial differential equations that yield the fluid vector velocity \vec{u} . Despite the apparent complexity of the equations, they can be derived simply using Newton's 2nd law $\frac{d\vec{p}}{dt} = \vec{F}$ and conservation principles.

Consider an integrable quantity ϕ that is convected in a fluid field with velocity \vec{u} . For a given control volume, the continuity equation relates the time rate of change of ϕ in the volume with the flux through the boundaries due to convection in the fluid field and the sources and sinks that create or consume the field. The continuity equation is

$$\frac{d}{dt} \int_V \phi dV = - \int_A \phi \vec{u} \cdot \hat{n} dA - \int_V s dV,$$

where dV is the control volume, dA its bounding surfaces, \hat{n} the unit normal to the surface dA , and s are the sources and sinks present in the control volume, with the convention that sinks are positive and sources negative.

If we consider the quantity to be the max flux (the momentum density) $\phi = \rho \vec{u}$ and simplify the equation, we arrive at the Cauchy momentum equation:

$$\begin{aligned} \rho \frac{\partial \vec{u}}{\partial t} + \rho \vec{u} \cdot \vec{\nabla} \vec{u} &= \vec{s} \\ \rho \frac{D \vec{u}}{Dt} &= \vec{s}. \end{aligned} \tag{1.1}$$

The derivative $\frac{D}{Dt}$ in the second term is the material derivative, which describes the time rate of change of the velocity of a fluid parcel given that its change is time and position-dependent. Written in this way, we see that the equation for momentum continuity in the fluid leads us to a Newton's second law type expression. The term $\rho \vec{u} \cdot \vec{\nabla} \vec{u}$ is known as the convective term, and describes the transport of momentum due to motion of the flow-field itself.

Next, we replace the sources and sinks \vec{s} with their physical origins. Because the quantity of interest is momentum, we recognize that the sources and sinks must be forces (force densities) in the system, which can generally be broken down into body forces, such as those due to electrostatics or gravity, and surface forces. Therefore, we replace the source/sink term with $\vec{s} = \vec{\nabla} \cdot \sigma + \sum_i \vec{f}_i$, where σ is the Cauchy stress tensor and \vec{f}_i is a body force acting on the control volume. We can further break down the Cauchy stress tensor into the sum of two

separate pieces, an isotropic pressure tensor $p\mathbf{I}$ and the stress tensor τ due to *viscous* forces. For Newtonian fluids like water, there is a fundamental postulate that the stresses σ are proportional to the strain rates, with proportionality constant η , the viscosity of the fluid. In this case, $\tau = 2\eta\epsilon$, where ϵ is the strain rate tensor which describes the deformation of the fluid due to velocity gradients. When the stress tensor τ is replaced with the strain rate tensor ϵ via the constitutive relationship for Newtonian fluids, we finally arrive at the full Navier-Stokes equations:

$$\rho \frac{\partial \vec{u}}{\partial t} + \rho \vec{u} \cdot \nabla \vec{u} = -\nabla p + \eta \nabla^2 \vec{u} + \sum_i \vec{f}_i. \quad (1.2)$$

Solving the Navier-Stokes equation yields the correct fluid velocity \vec{u} at all points in the system. While it is a complex equation, in many systems it can be simplified and solved analytically. For instance, in microfluidic channels gravity is often insignificant compared to other inertial and pressure forces, and is often ignored. Additionally, in small channels and with relatively small velocities, the inertial/convective term $(\vec{u} \cdot \nabla) \vec{u}$ is often negligible and can be dropped. To quantify this notion, we introduce a dimensionless parameter called the Reynolds number:

$$Re = \frac{\rho u L}{\eta}. \quad (1.3)$$

L in the above equation is the characteristic length scale of the flow, which in pores or channels is chosen to be the diameter by convention. The Reynolds number Re is essentially a ratio of the strength of inertial/convective forces to inertial forces. For high Reynolds number, convective forces dominate and we expect turbulent and chaotic flow. At low

Reynolds number, inertial forces dominate and fluid motion is laminar and smooth. The Reynolds number is useful for knowing which two regimes we are in, and can provide intuition about the behavior of our systems. In microfluidic systems, we are seldom in the turbulent regime, and are often at medium to low Reynolds numbers. Nanofluidic systems are almost certainly non-inertial for reasonable external pressures.

1.2.6 Motion of ions in a solvent—Poisson-Nernst-Planck equation

While the Navier-Stokes equations of motion describe the motion of the fluid solvent, the Poisson-Nernst-Planck, or PNP equations describe the motion (flux) of ions in the solution. The flux of ions is due to the superposition of diffusion, advection in the fluid solvent (known as ‘streaming’ current), and migration in electric fields (e.g. due to external voltage sources). Summing the individual contributions, the PNP equation for the flux of an ion species is

$$\vec{J}_i = - \left[D_i \nabla c_i - \vec{u} c_i + \frac{D_i z_i e}{k_B T} c_i \nabla \phi \right], \quad (1.4)$$

where \vec{J}_i is the ion flux of species i , D is the diffusion coefficient, c is the ion concentration, z is the ion valence, e is the elementary charge, k_B is the Boltzmann constant, T is the temperature, nad ϕ is the electric potential. The first term in eq. 1.4 is the diffusion term, which essentially states that there is a flux of ions in the opposite direction of the gradient of the ion concentration. The second term is the advective term, which accounts for the ions moving with the fluid solvent. The final term is the electric migration term, which describes the motion of the ions in an electric field gradient.

1.2.7 The diffuse layer—Poisson-Boltzmann equation

The diffuse layer is of paramount importance in nanopore studies; without it, nanopores would simply act as passive conductors with no interesting conductance properties. When any type of material is submitted in solution, charges tend to appear at the surface. Some of these charges are due to specific adsorption or immobilization of ions or molecules onto the surface, which give the surface some charge. The other charge type which is usually more significant is due to the ionization of chemical groups native to the surface of the pore. The type of chemical group present on the pore depends on the material out of which the pore is made. For instance, polymer pores prepared via the track-etch technique are formed when an etchant solution cleaves the monomer units in half, exposing a carboxyl group, which is then deprotonated at basic solution pHs. On average there are approximately 1 of these surface groups per 4 nm^2 , yielding a net surface charge density of 0.25 e/nm^2 . In any case, the total charge that mobile ions ‘feel’ in the solution is the sum of the adsorbed charges (Stern layer), plus the chemical groups attached to the surface, plus the immobilized ions within the slip layer. The electric potential immediately outside the Stern layer due to these superimposed charged sources is known as the ζ potential.

Because the ion solution is conductive, mobile ions outside the Stern layer will redistribute themselves near the charged surface to counter or ‘screen’ its ζ -potential. This screening layer is known as the Debye layer, and is characterized by an enriched number of counterions (ions of opposite charge polarity to the wall) and a depleted number of coions. As we will see, the electrical double layer has a characteristic length scale known as the Debye length λ_D , which ranges from $0.1 - 10\text{ nm}$ for most solution concentrations. Figure 1.3 gives a view of the electrical double layer under the combined Gouy-Chapman and Stern models.

Nanopore’s interesting behavior emerges when a non-negligible amount of the total volume is within a distance of λ_D of the pore wall. Assuming $\lambda_D = 1$, figure 1.4 shows the fractional

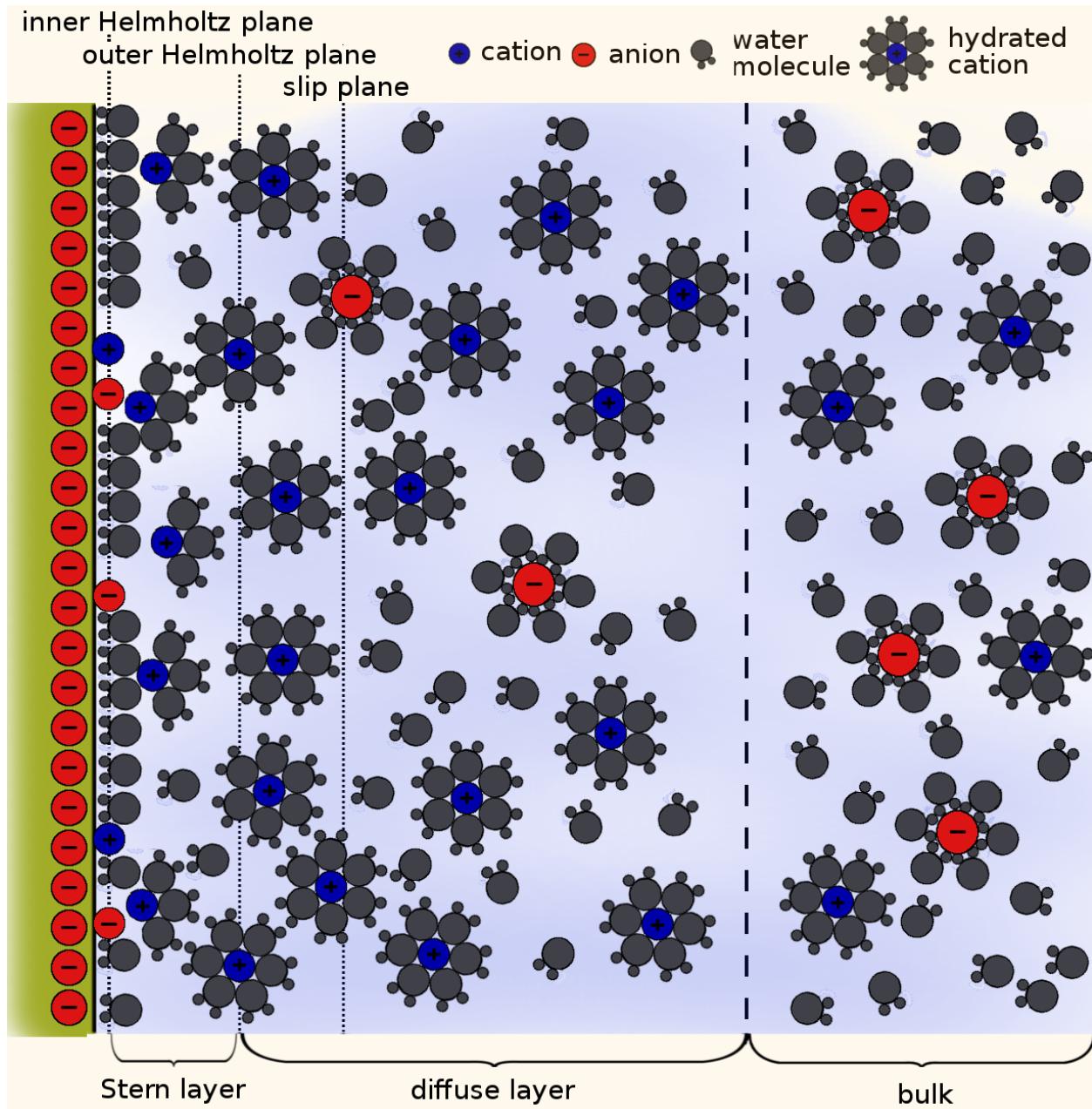


Figure 1.3: Scheme of the electrical double layer under the Gouy-Chapman and Stern model. There is an abundance of counterions near the pore surface and a reduced number of coions. At the bulk both ion species' concentrations are equal.

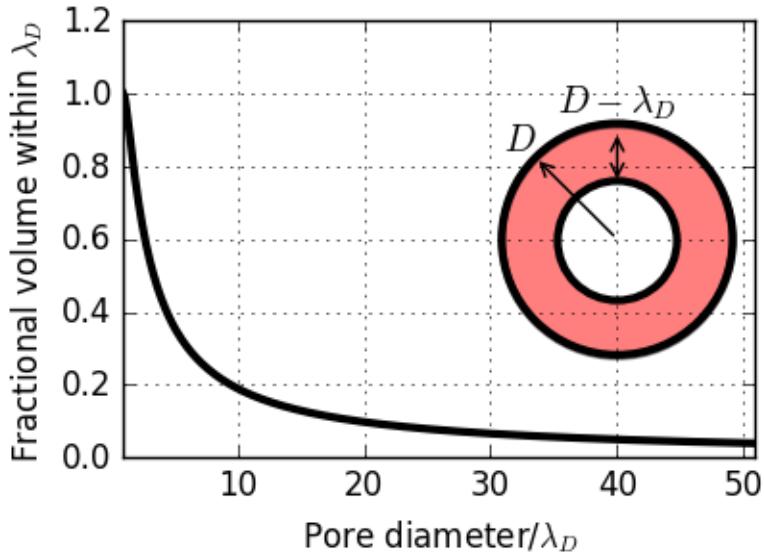


Figure 1.4: Plot of fraction of volume in a nanopore that is within the EDL versus pore diameter. The plot reveals a large super linear increase of total solution within the EDL as diameter decreases, starting at approximately $D \approx 10\lambda_D$.

area of pore contained within λ_D of the pore wall as a function of the pore diameter D .

For pores with diameter $D \gtrsim 10\lambda_D$, the total volume within the electrical double layer is negligible; however, below approximately $10\lambda_D$ the total volume within the EDL quickly increases. This explains why the ‘nano’ prefix in nanopore is important; as we approach the scale of the EDL, the structure of the solution very quickly changes, in a non-linear fashion, within the vicinity of a few nm of the pore wall.

In order to derive the appearance of the EDL, we consider the equations governing the distribution of ions in solution. In the solution, the ion species i have an electrostatic potential energy $U_i = z_i e \phi$, and have the following distribution in accordance with Boltzmann statistics

$$c_i = c_{0,i} e^{-\frac{z_i e \phi}{k_B T}}. \quad (1.5)$$

The electric potential itself is given by the Poisson equation

$$\begin{aligned}\nabla^2\phi &= -\frac{\rho}{\epsilon} \\ \nabla^2\phi &= -\frac{1}{\epsilon} \sum_i z_i e c_i(y).\end{aligned}\tag{1.6}$$

Plugging the concentrations given by the Boltzmann equation into the Poisson equation, we arrive at the Poisson-Boltzmann equation

$$\nabla^2\phi = -\frac{1}{\epsilon} \sum_i z_i e c_{0,i} e^{-\frac{z_i e \phi}{k_B T}}.\tag{1.7}$$

The Poisson-Boltzmann equation 1.7 is a second order non-linear differential equation for the electrostatic potential ϕ ; after solving for ϕ , it can be plugged back into the Poisson equation 1.6 to obtain the ion concentrations c_i . The Poisson-Boltzmann equation is generally solved numerically, but analytic solutions exist for simplified assumptions about the geometry and magnitude of the electrostatic potential in the solution.

Consider a planar surface with unit normal pointing in the \hat{y} - direction and uniform surface charge density σ immersed in a solution of water and symmetric ions with molar concentration c_{\pm} , valency $z_{\pm} = z$, and identical diffusion coefficients D_i ; this ion configuration is a close approximation of a KCl electrolyte solution, a common-used electrolyte in nanopore experiments due to the near-perfect symmetry of the ions. The ions have electrostatic potential energy given by $U_i = z_i e \phi(y)$ where the electric potential ϕ is only a function of distance from the wall due to the system's symmetry. Under these assumptions, the Poisson-Boltzmann equation becomes

$$\frac{d^2\phi}{dy^2} = -\frac{zec_0}{\epsilon}.$$

Though the solution can be solved analytically, a simple solution is obtained when we linearize the right hand side of the equation under the assumption $zec_0 \ll k_B T$ and apply the correct boundary conditions, an approximation known as the Debye-Hückel approximation. The solution is

$$\begin{aligned}\phi &= \phi_0 e^{-\frac{y}{\lambda_D}} \\ c_i &= c_{i,0} e^{-\frac{z_i e \phi_0}{k_B T} e^{-y/\lambda_D}} \\ \lambda_D &= \sqrt{\frac{\epsilon k_B T}{2c_0}} z,\end{aligned}\tag{1.8}$$

where we introduced a new parameter, the Debye length λ_D . The solution to the 1-D linearized Poisson-Boltzmann equation reveals that the electrostatic potential exponentially approaches 0 from the wall, that the counterion species are enriched at the surface and diminish towards the bulk, and the coions are diminished at the surface, but recover their concentration towards the bulk. The Debye length is perhaps the most useful parameter in nanopore systems since it gives a characteristic length scale for the region within the pore wall that departs from its bulk, ordinary behavior. The Debye length increases with dilute ion concentrations and decreases as the ion concentration is increased, and is typically somewhere in the range of 0.1–10 nm for common solution preparations. Note that although we discussed the electrical double layer here in the context of a pore surface, nearly every body immersed in solution will have some surface charge and therefore its own electrical double layer. The electrical double layers of free-bodies i.e. particles are integral in explaining

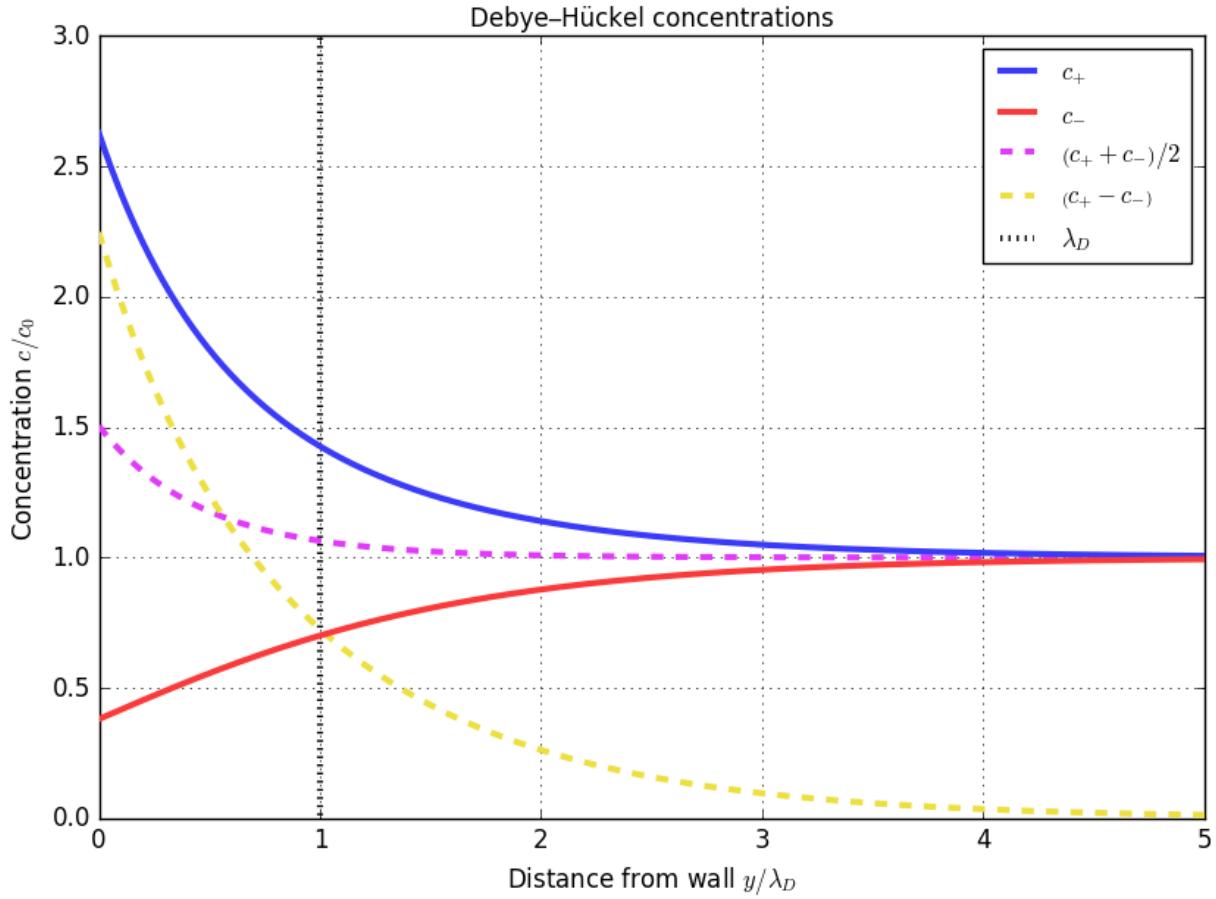


Figure 1.5: **Ion concentrations adjacent to a negatively charged surface according to the Debye-Hückel theory.** The plot reveals that the counterion concentration (blue) is enhanced near the wall, and decays approximately exponentially as distance from the wall increases. Oppositely, coions (red) are diminished near the wall, and rise to their bulk value with distance from the wall. The *total* ion concentration at the pore surface is slightly higher than the total ion concentration in the bulk. The space charge density $\rho_E \propto [c_+(y) - c_-(y)]$ is non-zero near the wall, and decays to zero in the bulk.

their motion in applied electrical fields, also known as electrophoresis.

The importance of the EDL in bestowing nanopores with their novel conductance properties cannot be overstated, and accordingly there are many phenomenon that nanopores that exhibit that are a direct consequence of the presence of the EDL. In the next few sections we will review some of these phenomena.

1.2.8 Electroosmosis

Ions migrate in externally applied electric fields, and ions of opposite polarity travel in opposite directions. When ions move, they tend to drag fluid along with them. In the bulk, this fact results in no net motion of fluid because for every anion that drags fluid towards the cathode, there is a cation that drags fluid towards the anode, and the two motions cancel each other out so that there is no net motion of fluid. However, as we've just learned there *is* a net concentration of one ion polarity in the EDL, so we expect that an electric field applied parallel to a charged surface will induce motion of the fluid. This intuition turns out to be correct, and the effect is known as electroosmosis. Interestingly, although the effect originates due to the presence of the $\sim 1\text{ nm}$ -thick EDL, the induced fluid flow propagates well into the bulk, and electroosmosis is observable in micro- and even milli-sized systems. This result, while surprising, is due to the simple viscous coupling of the fluid at the EDL surface to the rest of the fluid.

Because electroosmosis induces a general fluid flow, the effect couples with the motion of all unbound actors in the system, namely the ions species and any particles suspended in the solution. In typical nanopores, convection in the electroosmotic flow tends to be less pronounced than migration in the electric field. However, some lightly charged particles for instance will only experience a small force from the electric field (electrophoresis) compared to the drag force from the fluid moving electroosmotically, and their motion is therefore

determined primarily by electroosmosis. For the case of ions, usually the electroosmotic convective velocity adds or subtracts only a small offset to their net total motion, which remains dominated by electrophoresis. However, recently it was shown that some types of channels exhibit frictionless flow of water, and in these cases the electroosmotic velocities can become very large. In these channels it is actually expected that electroosmotic offers a large, if not dominant contribution to the motion of ions in the system. One example of these special types of pores, carbon nanotube pores, will be discussed later in this dissertation.

To understand electroosmosis, we look to the Navier-Stokes equations which describe the motion of fluids, and the Poisson-Boltzmann equation which yields the distribution of ions in the vicinity of the EDL. As an example, consider again an infinite charged plane with unit normal pointing in the \hat{y} - direction, but this time with a constant electric field \vec{E} applied parallel to its surface. The Navier-Stokes equation (eq 1.2) simplifies greatly in this case, and simply becomes

$$0 = \eta \frac{\partial^2 u}{\partial y^2} + [c_+(y) - c_-(y)] z e E_{ext}.$$

The equation shows that there is a momentum flux source term from the electric field acting on the charged solution in the EDL, which is proportional to the net charge in the solution (proportional to $c_+ - c_-$) times the electric field \vec{E} . The other term that balances the driving force is due to viscous forces between neighboring layers that have different y - velocities u , i.e. due to velocity gradients perpendicular to the wall. In order to solve this equation, we can replace the charge density $\rho_E = z e (c_+ - c_-)$ with the Laplacian of the scalar potential ϕ , integrate twice, and apply the correct boundary conditions, that the velocity at the pore wall is 0 (the so-called ‘non-slip’ boundary condition) and that the velocity gradients are 0 infinitely far from the wall. Performing these steps, we find the fluid velocity equals

$$u = \frac{\epsilon E_{ext}}{\eta} (\phi - \zeta),$$

where ϕ_0 is the electric potential immediately outside the surface charges on the pore wall. Interestingly, according to this equation the fluid velocity is proportional to the local electric potential in the screening layer, which is determined by solving the Poisson-Boltzmann equation (Eq. 1.7). Under the Debye-Hückel approximation of small surface potentials, the fluid velocity is zero at the walls, and exponentially approaches its bulk value as distance from the wall increases. This type of fluid flow profile is known as ‘plug flow’, since the velocity is approximately constant everywhere except in the thin EDL where there are sharp velocity gradients. Since the potential in the bulk is equal to $\phi = 0$, we find the bulk velocity to be equal to

$$u = -\frac{\epsilon \zeta}{\eta} E_{ext}$$

$$u = \mu_{EO} E_{ext}, \quad (1.9)$$

a result known as the Helmholtz-Smoluchowski equation. The parameter $\mu_{EO} \equiv -\frac{\epsilon \zeta}{\eta}$ introduced is known as the electroosmotic mobility, and is the proportionality constant between the applied electrical field and the resulting fluid velocity. Typical values of the mobility are $\mu_{EO} = 10^{-8} [m^2/Vs]$, which for an electric field of $1 V/\mu m^2$ yields a fluid velocity of approximately 1 cm/s .

Figure 1.6 shows the fluid flow profile in a pore with a finite slip length u_{slip} ; however, it must be noted that in the vast majority of experimental systems, $u_{\text{slip}} = 0$.

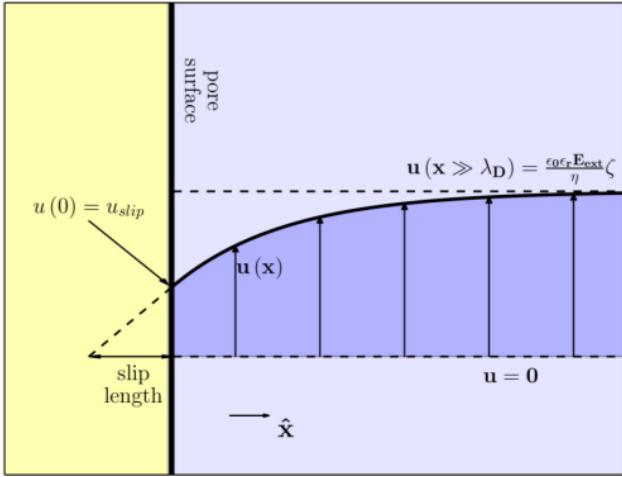


Figure 1.6: Plot of electroosmotic flow along an infinite plane with finite slip length. A net ion charge at the surface experiences a force in an electric field which drags fluid along with it. The force is only present in the electrical double layer, which viscously couples to the rest of the fluid. This fluid flow profile is known as plug flow, and extends far beyond the EDL where it originates.

It is possible to experimentally observe the fluid flow velocity, for instance by tracking the motion of small particles that move in the fluid flow, and therefore Eq. 1.9 provides a means for directly measuring the ζ -potential.

1.2.9 Surface conductance and conductance saturation

Ignoring electrical double layer effects, the conductance of an ion channel is proportional to the ion concentration in the solution; doubling the ion concentration exactly doubles the measured current. However, the presence of the electrical double layer breaks this perfect linear conductance relationship for two reasons. First, the integrated *excess* ion count $\int_0^{EDL} [(c_+ + c_-) - 2c_0] dy$ in the EDL layer is not guaranteed to be the same as the total ion concentration in the bulk; in fact, in most cases it is slightly higher, meaning the electrophoretic current in the EDL is slightly higher than in the bulk. Another important contribution to the measured current is the current from convection due to electroosmosis; because there is a net charge density in the fluid, electroosmosis will create a measurable

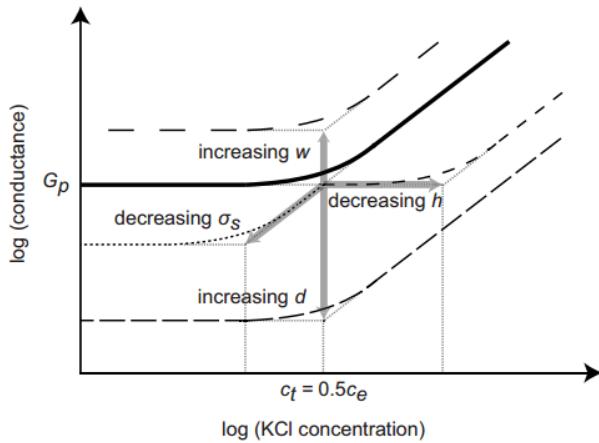


Figure 1.7: Reprinted (abstract/excerpt/figure) with permission from [(FULL REFERENCE CITATION) as follows: Author's Names, APS Journal Title, Volume Number, Page Number and Year of Publication.] Copyright (YEAR) by the American Physical Society.

ion current! In total, there are three terms that contribute to the total current measured in a channel, migration of ions in the bulk, migration of the integrated ion *excess* in the EDL, and net convection of charge in the EDL due to electroosmosis. Without applying any equations, we can use what we learned about the EDL and some general logic to reason about what the current behavior should be in the limiting cases of very large ion concentration and very small ion concentration. The equation for the Debye-length is $\lambda_D \propto c_0^{-1/2}$; for very large ion concentration, the EDL should be negligible and therefore neither the effects of electroosmosis nor electrophoretic ion excess should be significant. At the other end of the scale, in the limit of nearly zero ions in the bulk we still must have charge neutrality in the pore, meaning a finite concentration of ions despite the bulk concentration being zero. In this case, the EDL fills the entire pore and the result is that the conductance is entirely due to the integrated excess and convective terms. This finite conductance at zero concentration is known as the *saturation current*. Therefore, as we increase the length of the EDL we expect the conductance discrepancy, the difference in bulk conductance versus conductance due to the two surface effects, to increase. The end result of all this is that the conductance is a non-linear function of ion concentration at sufficiently low ion concentration; removing ions past some point does not further reduce the conductance of the channel! Consider a

nanopore immersed in solution of two symmetric ions denoted + and – (symmetric means their mobilities, valencies, and bulk concentrations are equal) with electrophoretic mobility μ_{EP} and electroosmotic mobility μ_{EO} . The following expression breaks down the total measured current into the three components described above:

$$I = I_{\text{bulk}} + I_{\text{excess}} + I_{\text{EO}}$$

$$I = \int 2c_0\mu_{EP}EezdA + \int [c_+(z) + c_-(z) - 2c_0]\mu_{EP}EezdA + \int [c_+(z) - c_-(z)]u_{\text{EO}}(z)ezdA. \quad (1.10)$$

Figure 1.7 shows a plot of the conductance versus solution concentration for a nanopore; the plot reveals a saturation at low solution concentration as expected from the preceding arguments. The saturation current is equal to the surface conductance term in the above equation.

1.2.10 Ion selectivity

A phenomenon related to surface conductance is *ion selectivity*. Since the EDL has a higher concentration of counterions rather than coions, the total flux of ions through the pore is slightly skewed towards counterions. The ion selectivity is defined as

$$S = \frac{|I_+ - I_-|}{I}. \quad (1.11)$$

1.2.11 Ionic current rectification

An ion channel is said to be rectifying if its conductance for one voltage polarity is greater than the opposite voltage polarity. The presence of an EDL alone cannot account for this, since the system has symmetry with respect to switching the electrodes, and therefore no rectification will occur. However, by introducing asymmetry into a system, along with having the pore be partially ion selective, we can create rectifying pores. There are many ways to create asymmetric nanopores, but two obvious solutions are geometry and surface charge pattern based.

If the shape of a nanopore is asymmetric, for instance as in a cone, the ionic current will be rectified. A simple explanation for this is that at the tip-side of the pore (in the case of conical nanopores), there will be an excess of counterions in the vicinity of the entrance EDL, while the other side will more closely resemble the bulk. Because these ions are at the entrance of the pore, they will more easily enter the pore than when at the other side. The result is that for one voltage polarity the ionic current is greater than at the opposite polarity.

By similar reasoning, charge patterning asymmetry can also lead to rectification. By placing two charges of opposite polarity adjacent to one another inside the pore, for one voltage polarity a depletion zone is created in analogy with the depletion zone formed between the n- and p-doped regions in a semiconductor diode. The ionic current is rectified, and because the diode junction so severely limits the ionic current under one voltage polarity, the device is said to be equivalent to an ionic diode.[6]

Figure 1.8 shows a conical nanopore with a bipolar charge distribution. The combination of the geometrical and charge asymmetry gives the pore a very large rectification value, and the near-zero current for one polarity of voltage is enough to consider the pore a diode.

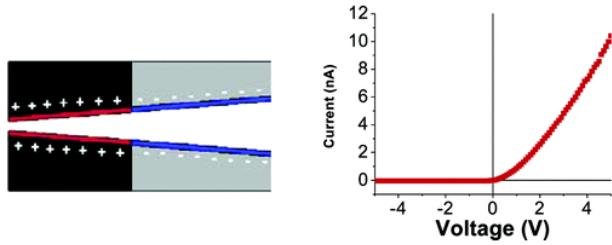


Figure 1.8: An ionic diode formed in a conical nanopore with bipolar surface charge patterning. The region between negative and positive charges is the depletion zone, and provides a large resistance to ion flow through the pore for one voltage polarity. Reprinted with permission from I. Vlassiouk and Z. S. Siwy. Nanofluidic diode. *Nano Lett.*, 7(3):552–556, 2007. PMID: 17311462. Copyright 2007 American Chemical Society.

Particle electrophoresis

Electrophoresis is the transport of a charged particle in the presence of an electric field. The effect of electrophoresis can be derived much in the way electroosmosis was derived, but instead of considering the fluid flow velocity at the surface of the particle to be zero, we set it equal to the electrophoretic velocity. For particles with thin electrical double layers (Debye-Hückel approximation) and laminar flow (Stoke's flow), the particle velocity is

$$\vec{u} = \frac{\epsilon \vec{E}_0}{\eta} \phi_0 \quad (1.12)$$

$$\vec{u} = \mu_{EP} = \frac{\epsilon \phi_0}{\eta}.$$

In equation 1.12, we've again introduced a proportionality factor between the applied electric field E_0 and the induced velocity \vec{u} ; this factor is known as the electrophoretic mobility. However, these equations are derived under the assumption that the applied electric field does not perturb the distribution of ions in the electrical double layer, which is only true for small surface potentials. If this assumption is invalid, additional effects related to the redistribution of ions must be taken into account; the net effect is a retarding force on the particle that slows its velocity.

Electrophoresis is an important effect in many applications involving particles. One useful application is that particle electrophoresis through a nanopore can be used to measure the zeta potential of a particle, related to its surface chemistry and chemical adsorption. An electric field is applied across a nanopore which is immersed in electrolyte solution containing ions and charged particles. Particles travel through the pore under electrophoresis, and the dwell time can be recorded using resistive pulse sensing or optical techniques. If the geometry of the pore is known, the dwell time can be related with the particle velocity, and the electric field strength can be determined as well. In this case, equation 1.12 can be solved for the surface potential ϕ_0 , which is the same as its zeta-potential.

Particle electrophoresis + electroosmosis

Transport of particles is in general a superposition of electrophoretic and electroosmotic effects. Combining the two, the velocity of a charged particle translocating through a charged pore is

$$\begin{aligned} v &= (\mu_{EP} + \mu_{EO}) E, \\ v &= \frac{\epsilon}{\eta} (\zeta_{\text{particle}} - \zeta_{\text{pore}}) E \end{aligned} \tag{1.13}$$

In general the net velocity can be positive, negative, or zero depending on the relative sizes and signs of the electrophoretic mobility.

Chapter 2

Development and testing of a single carbon nanotube based pore

2.1 Background

This chapter describes my research performed on measuring the ion, water, and particle transport properties of single carbon nanotubes. A carbon nanotube (CNT) is a cylindrically-shaped monolayer of carbon atoms, and because of its shape it can effectively act as a nanopore. When used as nanopores, CNTs are thought to exhibit behavior that is significantly different than traditional nanopores, including large fluxes of water, large fluxes of some ion species, steric rejection of other ion species, ultra-high proton mobilities, and an unusual sub-linear power law in the conductance-concentration relationship. These effects are poorly understood, and since it was discovered that they could be used as single nanopores many labs around the world have worked on understanding them. Besides being poorly understood, there is even a lack of consensus in the field about whether some of these properties are even real! The objective of this experiment was to test a new CNT platform

that could help point the field in the right direction; by observing transport phenomena in our CNT platform, hopefully we nudged the field in the direction of a concensus.

In the following sections, I will explain in greater depth a functional CNT nanopore platform, and the physical reasons underlying their novel transport characteristics.

Chapter 3

Experimental study of ion conductance property of hybrid conductor-insulator nanopores

3.1 Background

In the introduction of this thesis we discussed the electrical double layer (EDL) which is a screening layer of ions that forms in the proximity of a charged surface. The vast majority of pores studied are non-conductive insulators, and their surface charge properties come from the functionalization of chemical groups at their surface. For instance, the base of the pore studied in this section is Si₃N₄ (silicon nitride), whose native silane chemistry means there are exposed alcohols at the surface that are either negative or neutral depending on pH. However, charge on a surface can arise *via* other means. When a metal is exposed to an electric field, electrons in the metal rapidly redistribute themselves on the surface to cancel the external applied field. If we assume the metal was originally overall neutral, the

charge pattern will consist of an abundance of negatively-charged electrons at one end and an abundance of positively-charged holes at the other. A natural question is whether these *induced* charges have associated EDLs in the same way that static charges do.

Previous studies suggest they do. For instance, Squires *et al.* and Pascall *et al.* performed experiments with applied external electric fields on various metal surfaces and discovered that electroosmotic flow occurred. They reasoned that electroosmosis was a result of polarization of charges according to the model discussed above [5, 3]. However, these types of induced-charges have been seldom discussed in the context of nanopore transport, especially in the context of a realizable experimental platform. In order to understand the effect of induced charges on nanopore transport, we considered a hybrid metal-insulator pore created by evaporating a thin layer of gold onto the pore surface. If the pore was only the insulator, its IV curve would be symmetrical due to the lack of any symmetry breaking in the system. The addition of the metallic layer breaks this symmetry, and we expect to see ionic current rectification in the pore (see chapter 1.) However, ionic current rectification alone is insufficient for proving that induced surface charges contribute to the pore's conductance; this is because essentially *any* symmetry breaking in a pore, including a difference in static surface charges between two regions, a difference in geometrical shape between two regions, or induced surface charges, will lead to ionic current rectification.

In order to provide evidence that induced charges directly contributed to a pore's conductance properties in this system, we need to devise a model for the manner of the induced rectification properties, and to check if the model accurately describes experimental data. Figure 3.1 is a scheme of the induced-charge model, which is effectively explained by the surface charges present. First, the insulating part of the pore has an approximately uniform surface charge due to ionization of the chemical groups on its surface. On the gold, the charge patterning depends on the applied voltage. When a negative bias is applied on the gold side of the pore, electrons are repelled from the electrode while holes are attracted. The

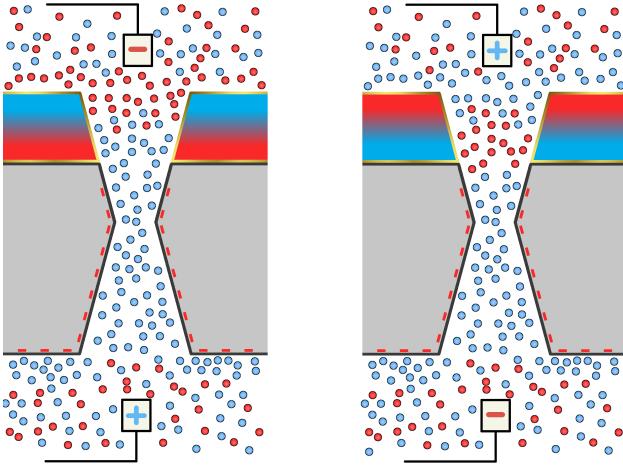


Figure 3.1: Model for the induced-charge and EDL charges in the hybrid conductor-insulator nanopore. **Left:** Negative bias applied to gold side. **Right:** Positive bias applied to gold side. For the positive bias, a depletion zone forms at the junction between the gold and SiN, which is expected to severely limit the conductance. For the negative bias, no depletion zone forms and therefore the conductance is expected to be unhindered. The double conical geometry reflects the expected geometry of SiN pores drilled *via* a transmission electron microscope electron beam.

charge pattern in the solution from top to bottom is then $- + +$. On the other hand, when a positive bias is applied on the gold-side of the pore, holes flee to the inside of the pore while electrons are drawn towards the electrode. Therefore, from the positive electrode to the negative electrode the surface charge pattern is $+ - +$ (right hand side of figure 3.1). The primary difference in the charge pattern as it relates to the influence on the nanopore between the two voltage polarity cases is the charge junction at the conductor-insulator interface; in the latter case (positive voltage applied on the metallic side), the $+ -$ junction indicates the presence of a bipolar junction that leads to a depletion zone in the EDL, as discussed in the section on ionic current rectification in chapter 1. This depletion zone is characterized by having a very large resistance, and therefore limits the total current through the pore. For the opposite polarity, the depletion zone is not present and therefore we do not expect any significant limiting of the current.

This model, known as the induced-charge model for hybrid conductor-insulator nanopores, was tested experimentally in this work.

3.2 Experimental setup

In order to test the induced-charge model, we devised a platform consisting of thin nanopores in silicon nitride (SiN) with gold (Au) deposited on top. The crucial elements of this setup consisted of the silicon nitride (SiN) membrane, evaporated gold (Au) layer, transmission electron microscope drilling of the pore, and the current-voltage characterization measurements.

3.2.1 Silicon nitride

Silicon nitride was used as the substrate through which to drill the nanopore. The effects of polarization of the gold should be apparent in truly nano-scale systems, so it was important to choose a material through which a small pore could be fabricated. As of the writing of this thesis, silicon nitride (chemical formula Si_3N_4) is a popular choice for creating small nanopores given its mechanical stability and well-understood surface chemistry. The method of pore formation, TEM drilling (discussed below) can enable pores ranging from 1 – 20 nm in diameter, and is capable of drilling through substrate lengths of up to 100 nm. It is predicted that pores with very low aspect ratios, i.e. short pores, will have severely diminished rectification properties. In order to ensure large rectification, the solution must be subjected to a sufficient length of EDL. In order to strike a compromise between TEM's capabilities and the undesirable low-aspect ratio effects, a substrate thickness of 50 nm was chosen. The silicon nitride substrate is commonly used in TEM microscopy, and therefore is available from commercial vendors. The substrates were ordered from the SPI company. The substrate itself is grown onto a thicker layer of silicon, through which a window is drilled on the backside to expose the silicon nitride. The final result is a 50 nm thick layer of SiN supported by a silicon chip. After pore fabrication, silanol groups native to the pore surface act as surface charges if immersed in solution with pH beyond their pKa value of

3.2.2 Gold deposition

Although the model of induced-charge rectification described above is independent of the type of conductor used, we chose to use gold (Au) because of its mechanical stability after deposition and its resistance to corrosion and oxidation, which could create a surface chemistry effect that could potentially obscure the relationship with the induced surface charges. With the conductor's material chosen, we think about the desired deposition thickness. In the model of induced-charge rectification described above, it is necessary to have a bipolar junction far enough inside the pore that a depletion layer can form. For this reason, we aimed to deposit a 30 nm layer of gold onto the surface of the SiN. There are a number of means of depositing metals onto surfaces, but given the thinness of the desired layer we chose to deposit *via* an e-beam evaporation. During e-beam evaporation, a chunk of metal is pumped down in vacuum and bombarded with high-energy electrons. The combination of low-pressure and large temperature caused by the bombardment with the electrons causes individual metal atoms to evaporate from the surface of the chunk along straight-line or ray paths. The atoms then hit the surface of the substrate, which is suspended above it, and stick. Although Au *can* be deposited directly onto the surface of SiN, a thin layer (3 nm) of chromium (Cr) was first evaporated, which facilitates adhesion of the Au.

3.2.3 Transmission electron microscope drilling

The transmission electron microscope (TEM) is a super resolution imaging technique that is typically used to image objects below the optical diffraction limit of 200 nm. However, if the electron beam is sufficiently energized and focused, the electron beam can act as a drill for forming nanopores instead of for imaging. This technique was used to drill pores through the SiN-Au hybrid device described in the above two steps. A highly focused beam in the TEM's scanning (S) mode is applied to the surface. The energetic electrons strike the

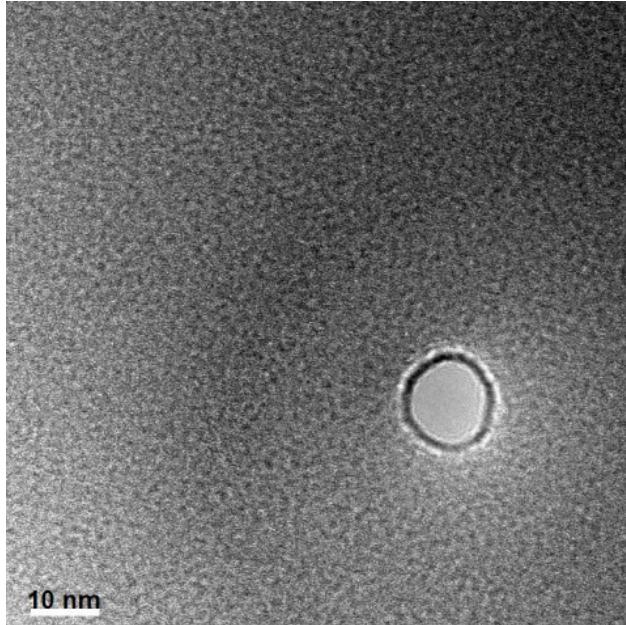


Figure 3.2: A ~ 15 nm SiN pore drilled *via* TEM.

surface and eject atoms, while simultaneously the entire structure melts. The final result is a pore with the same thickness as the substrate, and with a double conical geometry which is a result of the simultaneous ballistic ejection and melting. Because the drilling is performed inside the TEM, the resulting pore can also be immediately imaged. Figure 3.2 shows an example of a ~ 15 nm TEM-drilled pore.

3.2.4 Pore conductance characterization

In order to test the induced-charge model, we measured the current-voltage (IV) response of each of the fabricated nanopores in 100 and 500 mM KF and KCl salt solutions. The total surface charge of a pore ‘felt’ by the ions in the solution is given by a combination of the ionized chemical groups on its surface and any chemical species that are adsorbed to the surface. Cl is known to adsorb to the surface of gold, and so we expect that the gold in KCl solutions will have net charge given by the adsorbed chloride ions and the induced-charges. For this reason, measurements were performed primarily with KF solution. However, it

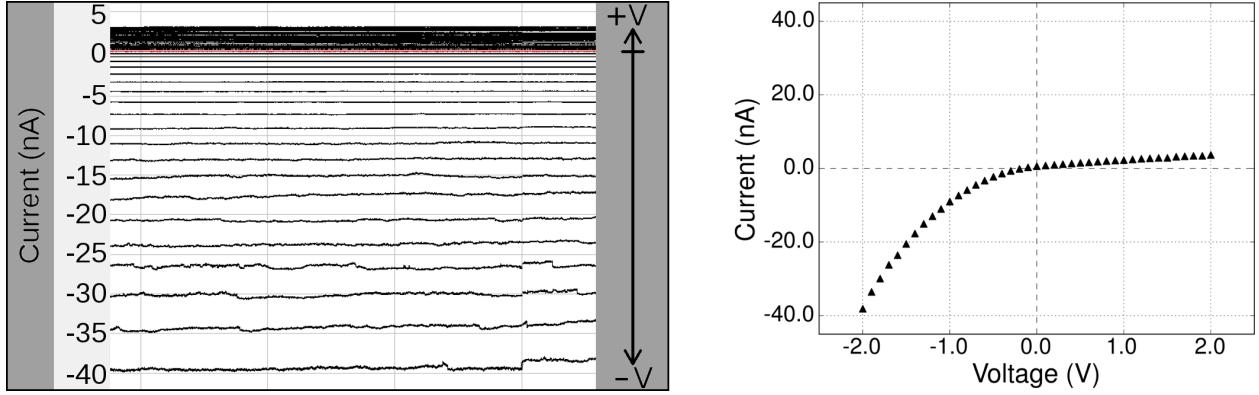


Figure 3.3: A current-voltage time series alongside the current-voltage curve produced from it. **Left:** Current-voltage time series. Each continuous line corresponds to one voltage setting; once a voltage value is set, the current does not significantly vary. **Right:** Traditional pore I-V curve. The asymmetry in the current with respect to voltage (present in both signals) is the hallmark indicator of ionic current rectification.

was predicted that adsorption of chloride on the Au surface would diminish the effects of the induced-charge, and this model was tested by performing measurements with KCl on chips that had already been measured in KF, to see if their conductance behavior changed according to this prescription.

Due to the small sizes of the pore and the low surface charge density of the silanols on the SiN surface, the interior of the pore is difficult to wet, meaning any vapor present inside the pore can be stable and difficult to remove. If this is the case, ions cannot pass through. Often times these vapor phases are quasi-stable and the pore will spontaneously wet and dewet. In order to reduce these hydrophobic effects, we use an even 50/50 mixture of water and ethanol solvent; the combination of both solvents promotes wetting of the pore.

In principle, during IV measurements one needs only to sweep the voltage along the desired range, stopping to record the current at each voltage value. Usually one delays ~ 1 second after changing the voltage before a current measurement is made to allow for the system to equilibrate, e.g. due to system capacitance charging/discharging. However, in systems in which the system conductance can stochastically change, e.g. due to spontaneous wetting/dewetting transitions, it is advantageous to record a time series of the current voltage

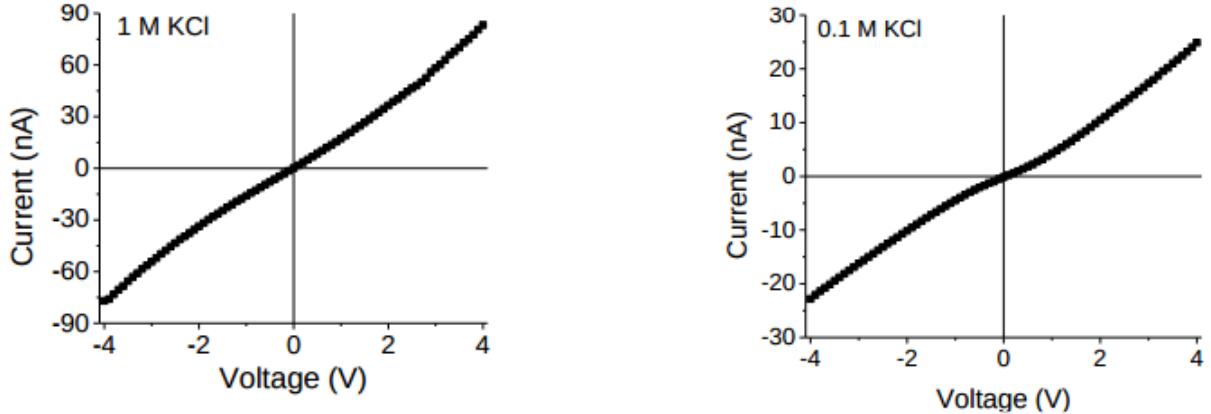


Figure 3.4: **IV curves taken of bare SiN pores.** The symmetry in the IV curve reveals no inherent rectification of the TEM drilled pores.

sweeps rather than measure single data points. If a transition occurs, it can be observed in the time series of the signal and appropriately dealt with afterwards. The protocol for measuring IV curves is then to apply a fixed voltage, record the current for a fixed amount of time, and repeat for all of the desired voltages. Then, an IV curve is produced by averaging the current time-series for a fixed interval of time (averaging is performed to smooth over the small amount of noise in the signal). In this study, voltages were applied for a total of 20 s and the current values in the last 0.5 s were averaged to create the current measurement value. Figure 3.3 shows an example current-voltage time series and the IV curve produced from it for one of the pores used in the study measured in KF solution. On the left is a current-voltage time series plot that shows the current value time series for different voltage settings, while the right hand side of the figure shows a traditional I-V curve of the pore.

3.3 Experimental results

A number of conductor-insulator pores were tested, along with plain SiN pores to test of the pores intrinsically rectify the current (e.g. due to a geometrical asymmetry).

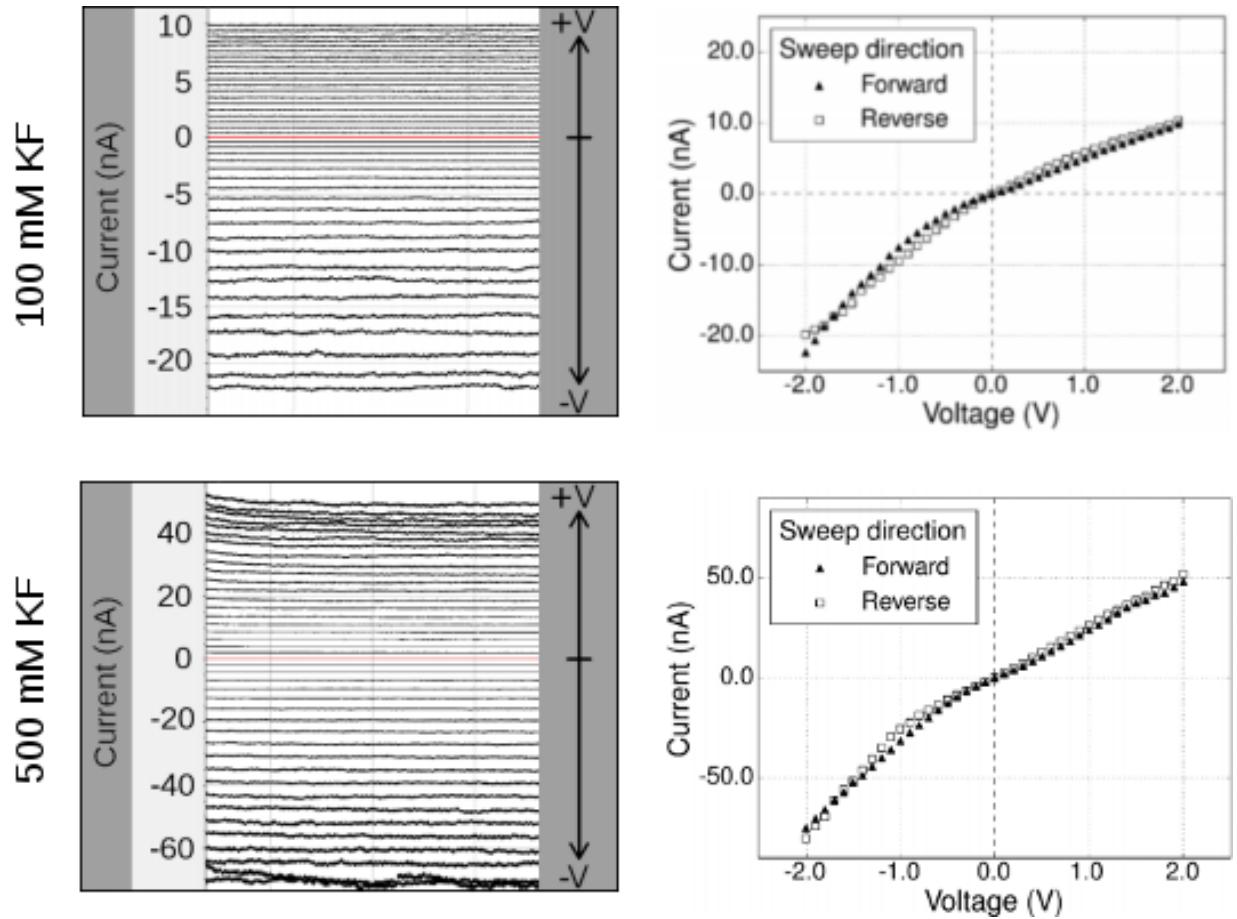


Figure 3.5: **IV curves taken of Au-SiN pore in 100 mM and 500 mM KF.** The symmetry in the IV curve reveals no inherent rectification of the TEM drilled pores. The ion rectification ratio R is lower in the higher concentration solution, as is expected from basic considerations of EDL thickness.

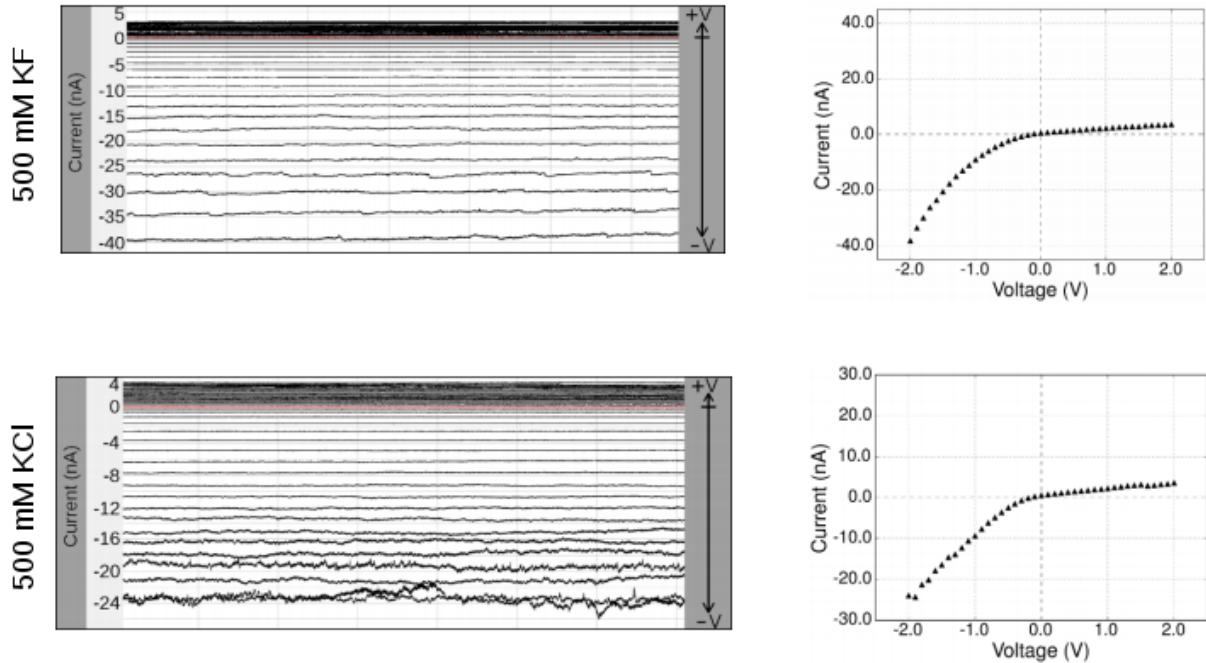


Figure 3.6: **IV curves taken of Au-SiN pore in 500 mM KCl and KF.** The rectification ratio R is lower in the KCl solution than in the KF solution, which is expected due to the additional contribution of the static charge from adsorbed Cl^- ions on the gold surface.

Figure 3.4 shows two IV curves gathered for the SiN-only (insulator pore only). The plot shows approximately symmetric IV curves, indicating a symmetric nanopore.

On the other hand, figure 3.5 shows two IV curves collected for a single nanopore approximately 9 nm in diameter. The IV curves taken in KF show significant rectification, with a ratio of approximately $R = 0.5$ for the 100 mM concentration and $R = 0.7$ for the 500 mM KF concentration. The increase in rectification R towards $R = 1$ with increasing salt concentration is consistent with the decrease in thickness of the EDL. In the model we developed for the induced-charge rectification, a depletion region is expected to form in the interior of the pore. Typically pores that have depletion zones experience much greater rectifications due to the ‘off-state’ voltage polarity having nearly zero conductance. While this pore’s rectification is not small enough (e.g. not close to zero) to describe it as an ionic diode, the rectification direction is nevertheless consistent with the proposed model. We note that this direction of rectification is inconsistent with models having only static charges on the SiN

and Au, barring the case where the *Au* has positive charges adsorbed onto its surface, which is unexpected for the ion species used in this study.

In order to understand the effect of a static charge superimposed on top of the induced-charge, we next performed experiments in KCl in addition to KF. Figure 3.6 shows a 12 nm pore with IV curves measured in 500 mM concentration of both types of salts. Note that both salts give the same type of rectification as in the previous example. This result is not entirely unexpected, as the presence of adsorbed chloride will bring the two charges in the depletion zone closer together in magnitude, while still maintaining a finite, albeit reduced rectification. If the Cl^- ions adsorbed to the surface of the gold to the extent that they dominated both the induced-charge on the gold and the surface charge on the SiN, then it is possible the rectification direction could even flip direction such that $R > 1$. However, we did not observe such an effect.

3.4 Modelling results

In order to provide an additional line of evidence pointing to induced-charge based ion current rectification found in experiments, we performed finite-element analysis simulations using the COMSOL Multiphysics software package. The models were designed to emulate the experimental set up, e.g. the channel geometries and salt concentrations used were similar. However, rather than try to reproduce the double conical geometry expected for TEM drilled pores, we worked with a cylindrical geometry which greatly facilitated convergence of the solutions with the very minor penalty of not adhering exactly to the experimental conditions. Because the cone angle is expected to be shallow and because the COMSOL results are meant to be interpreted semi-quantitatively, and are primarily used to draw conclusions about the rectification direction and mechanism, replicating the exact geometry of the pore was deemed unnecessary. Details of the COMSOL simulation can be found in the appendix.

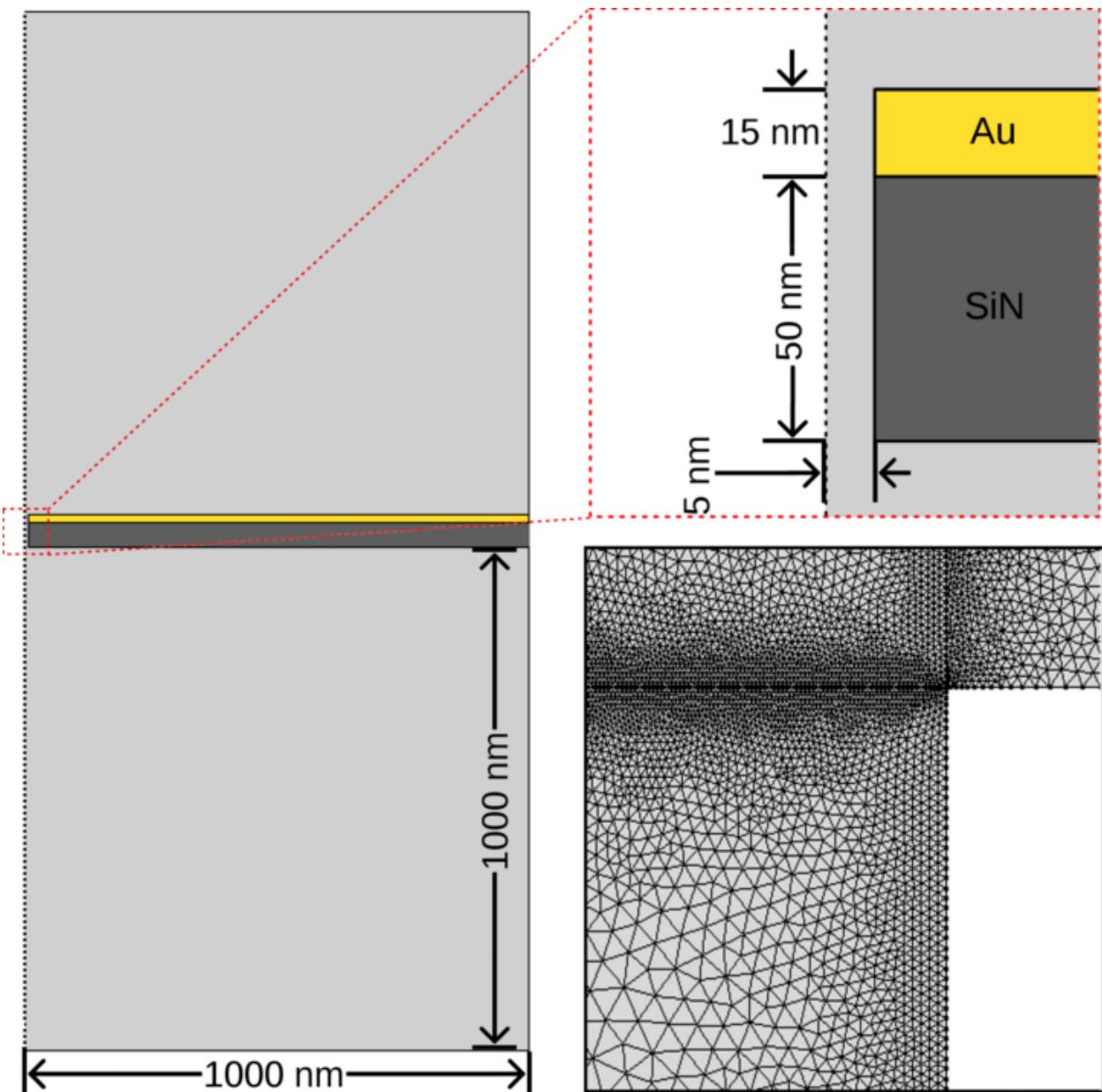


Figure 3.7: COMSOL simulations.

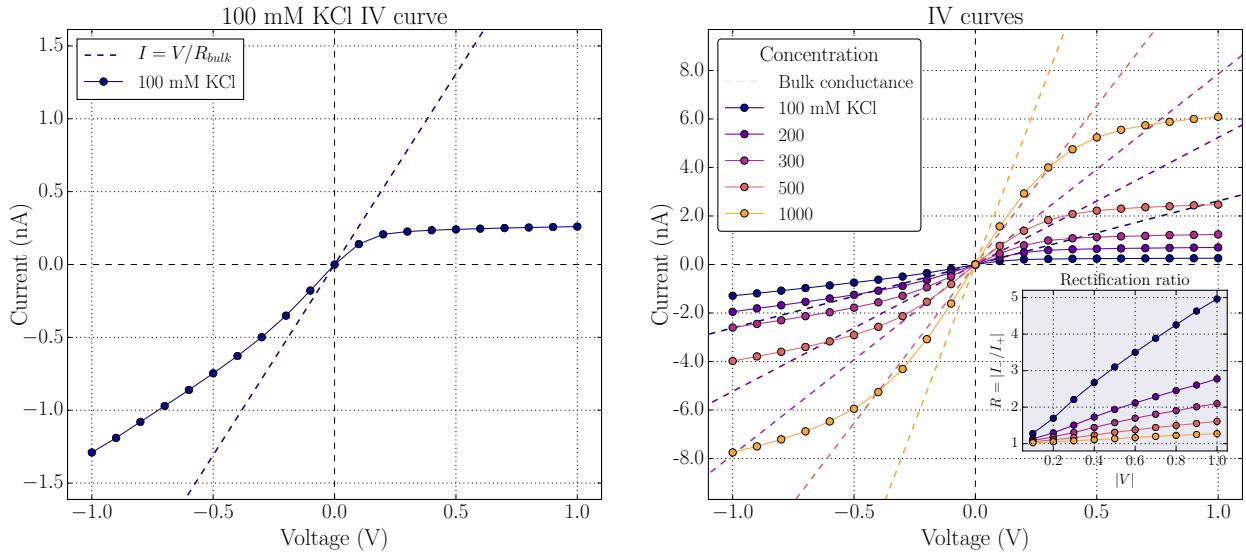


Figure 3.8: **COMSOL simulations.**

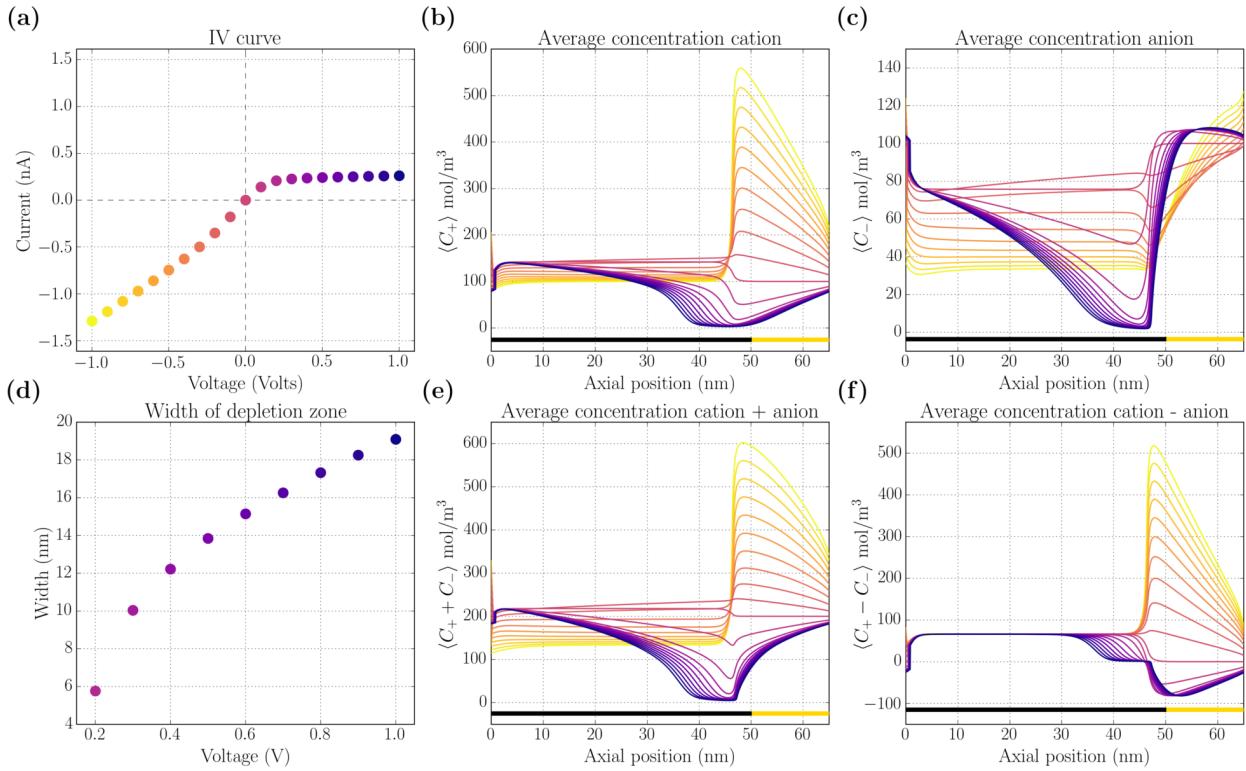


Figure 3.9: **COMSOL simulations.**

The results of the simulations are found in figures 3.8 and 3.9. We simulated IV curves for the model device and calculated the average concentrations of the cations, anions, and combinations at various axial positions inside the pore, and used these calculations to calculate the width of the depletion zone that forms for positive voltages. Figure 3.8 shows IV curves of the simulated pore at various concentrations. Notice htat at low concentrations the pore rectifies in the negative direction i.e. $R = |I_-/I_+| > 1$ and the rectification ratio R decreases with increasing electrolyte concentration, both results are consistent with the experimental data. Figure 3.9 shows the average concentrations of C_K , C_{Cl} , $C_K + C_{Cl}$ and $C_K - C_{Cl}$ at various axial positions. The concentrations of C_K and C_{Cl} along the length of the gold surface are consistent with the polarization model of gold: when a positive potential is applied on the side of the metal, electrons are drawn to the entrance of the pore, inducing formation of an EDL more positive than at the opposite terminal, and the opposite occurs for the opposite voltage. Along the length of the SiN far from the junction, the concentration of potassium is enhanced due to the negative charge of the surface, and fluoride is reduced. In the plot of $C_K + C_{Cl}$ which reflects the total ion concentration (bottom center), at positive voltages there is a drop in the concentration in the vicinity of the junction. In this region the total concentration is far below its bulk value, while elsewhere it is much closer to the bulk. This severely reduced charge density reflects the formation of the depletion zone, the zone created when ions near the diode junction are pulled apart. The resistivity $\rho \propto (C_K + C_{Cl})^{-1}$, so the depletion zone has a very large resistance and therefore limits the current through the pore as a whole. Depletion zone regions like the one that appears in the simulation severeliy limit current, which can be seen in the near 0 slope of the current for positive voltages in the IV curve. The current is limited by the depletion zone, and we identify this type of pore as a diode. The bottom-left plot of figure 3.9 shows the width of the depletion zone as a function of voltage. The depletion zone was defined as the width along the pore axis such that the total ion concentration is below half of the total ion concentration (in this case, below 100 mM, since $C_K = 100$ mM and $C_{Cl} = 100$ mM. While increasing the voltage

increases the current, the depletion zone is also widened with increasing voltage, resulting in a net sub-linear relationship between voltage and current at positive voltages.

The COMSOL results show that the model of induced-charge based electrical double layers is possible, and that the magnitude of the effect is enough to greatly influence the pore. Furthermore, the results show that the model system results in rectification ratio $R > 1$, in the same direction of the experiments' IV curves.

3.5 Conclusion & Future Work

The work presented in this chapter demonstrates the capabilities of conducting layers in affecting the ionic conductance properties of nanopore systems. We demonstrated, to our knowledge, the first instance of a conducting layer that actively modulates the current in a nanopore by depositing a thin layer of gold on top of a TEM-drilled SiN pore. In our model for our device, a dynamically induced charge acts in concert with the native, static ionized silane groups native to the SiN surface to create a diode-like rectification behavior. This type of rectification and its direction cannot be explained with static charges alone, and therefore we conclude that the induced charges on the gold layer were significant in contributing towards the total rectification of the device. Furthermore, numerical simulations using COMSOL multiphysics provided another line of evidence that the induced-charges contributed to the device's conductance. In the simulations, for the 'off-state' of the device at positive voltages there was a clear depletion zone in the channel, in agreement with our a priori expectations based on the physics of the device.

The ability to fabricate single nanopores with novel ion conductance properties is crucial in fabricating integrated ionic circuit devices. These non-trivial conductance properties are usually achieved *via* chemical modification or *via* or modification with biomolecules such as

DNA. However, this work has revealed another potential way of creating diode-like nanopores with the addition of permanent embedded metallic layers.

Chapter 4

Resistive pulse studies of aspherical mesoparticles

4.1 Background & Theory

In the introduction of this dissertation we introduced and explained the theory behind resistive pulse sensing, a particle characterization technique that works by monitoring the change in conductance of a nanopore as small particles pass through it. Probably the most important application of RP sensing is in measuring the sizes of particles. An analytic model was devised by DeBlois *et al.* to relate the size of a particle to its resistive pulse amplitude. The solution is essentially an electrostatics boundary-value problem: given an insulating particle inside a pore and a known externally applied voltage, calculate the electric field distribution inside the pore, and from the electric field distribution calculate the pore's expected resistance with the particle inside it. When this calculation is performed for spherical particles travelling along the axis of long cylindrical pores, the change in resistance or current is found

to be

$$\frac{\Delta R}{R_0} = \frac{\Delta I}{I_p} = \frac{4\rho D^3}{\pi d^4} \left[1 - 0.8 \left(\frac{d}{D} \right)^3 \right]. \quad (4.1)$$

Equation 4.1 is useful for relating the change in current—or resistance—to the diameter of spherical particles. However, it is very often the case that particles of interest are not spherical. For instance, many biomolecules such as viruses and proteins are poorly approximated as spheres, and are more rod-like in shape. In this case, the resistive pulse amplitude of on-axis translocations through a cylindrical pore is given by

$$\frac{\Delta R}{R_0} = [f_{\perp} + (f_{\parallel} - f_{\perp} \cos^2 \alpha)] \frac{v}{V}, \quad (4.2)$$

where f_{\perp} and f_{\parallel} are so called ‘shape-factors’, that depend on the particle’s major and minor axis lengths, α is the orientation of the particle with $\alpha = 0$ being axially aligned, and v and V are the volume of the particle and of the pore, respectively. Figure 4.1 shows a plot of the relative $\Delta I/I_p$ for various ellipsoids of the same volume but different axial lengths.

While equation 4.2 is useful for determining the volume of spheroidal particles from their resistive pulse amplitudes, it would be useful to devise a method for measuring the length of particles in addition to measuring their volumes as an additional physical marker for their identification in a sample.

Consider a pore with a rough, non-uniform interior. Due to the irregular interior, the channel cross sections will have a range of resistance values; for instance, a large cavity in the channel will have a smaller resistance than a more narrow constriction. If a very small particle travels

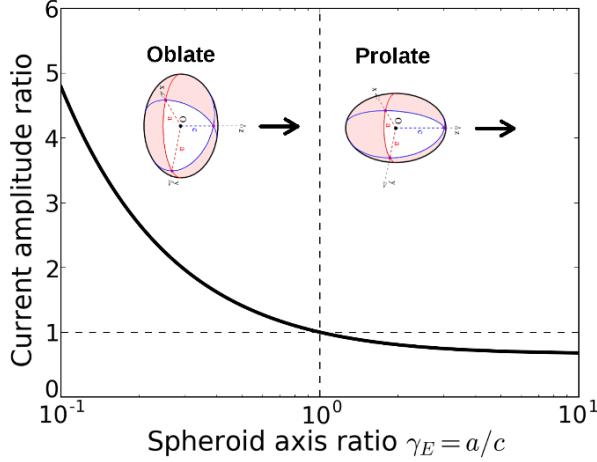


Figure 4.1: adsf

through the pore, its resistive pulse time-series is determined by the interior topology of the pore. For instance, when it happens to occupy a narrower region of the pore, the resistance in the region is larger relative to the rest of the pore, and therefore the particle will also block a relatively larger portion of the current. In this way, the particle is able to resolve the interior features of the pore. In such cases, the instantaneous resistive pulse amplitude is given by

$$\Delta R = R_p - R_0 = \frac{4\rho d^3}{\pi D^4} \left[1 - 0.8 \left(\frac{d}{D} \right)^3 \right]^{-1}. \quad (4.3)$$

$$\Delta R = \Delta R' = \int_{z=z'}^{z=z'+l} \frac{d\Delta R(z)}{dz} dz = \int_{z=z'}^{z=z'+l} \frac{4\rho d^3}{\pi D^4(z)} \left[1 - 0.8 \left(\frac{d}{D(z)} \right)^3 \right]^{-1} dz \quad (4.4)$$

Oppositely, if we consider a pore whose length extends along a significant amount of the pore's axis, it may be occupying several such regions at a time. Instead of a single particle,

we may consider the particle to be composed of many smaller segments of particles in series, and therefore the resistance values of each segment are additive. Therefore, the signals of longer particles can be seen as the convolution of the signal of an infinitesimally small particle over the long particle's length.

This fact can be used to devise an experiment that can measure the length of long particles. First, we drive a suspension of small ‘tracer’ spheres through a pore with rough interior, whose diameter is smaller than the characteristic length of the irregularities in the pore. These particles will ‘map’ the interior geometry of the pore as explained above, with resistive pulse shape given by equation 4.3. Then, we repeat the experiment with long particles for which we wish to know the length. Then we perform a convolution or moving average transformation of the signals of the tracer particles for a variety of lengths, given by the following equations:

$$I_i \rightarrow I_i^{l'} = \frac{1}{N} \sum_{j=i-\frac{N}{2}}^{j=i+\frac{N}{2}} I_j, \text{ and} \quad N = \frac{fl'}{v}, \quad (4.5)$$

where N is the total number of points in the moving average, I_i is the i th current measurement in the signal of the unprocessed short particle, $I_i^{l'}$ is the transformed data point for a moving average over length l' , f is the sampling frequency of the current acquisition instrument, and v is the *average* velocity of the particle during its translocation through the pore. However, equation 4.5 slightly oversamples particle positions in the middle of the moving average compared to particle positions at the very ends, along a distance equal to half the length of the small particle. For this reason, the moving average estimate of the correct transformation slightly overestimates the length of long particles. However, when $l \ll l'$, i.e. the tracer particle's displacement length is much less than the longer rod's displacement length, the effect is negligibly small.

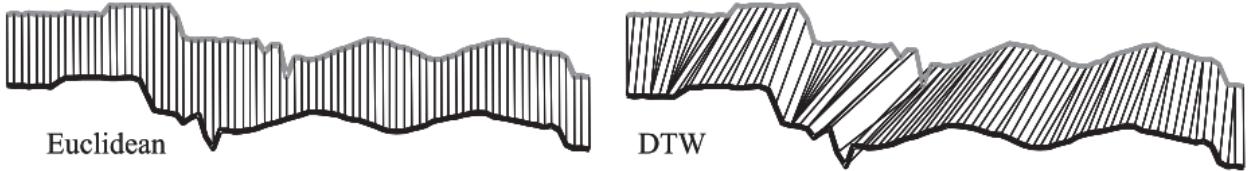


Figure 4.2: Comparison of standard Euclidean distance metric (left) and distance metric as determined by dynamic time warping (right). Notice that the Euclidean distance does not take into account small displacements of the two signals in time, despite their obvious similarity. On the other hand, dynamic time warping is able to account for small mismatches in time by finding the connections between points that minimizes the total cost.

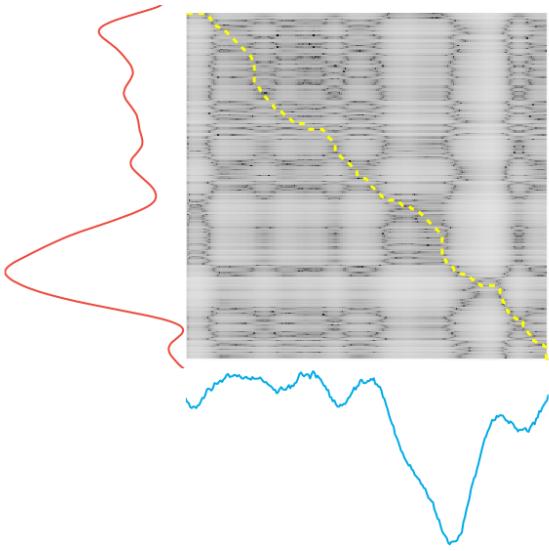
After computing the moving averages of the tracer particles over an array of lengths, a similarity measure is calculated between every pair of convoluted-raw signals for the tracer particle and the long particle, and the length with the greatest similarity measure is chosen as the correct length of the particle.

Probably the simplest similarity measure of two time-series is the point-wise summed Euclidean distance between the two signals given by the following formula:

$$C(S, S') = \sum_{i=0}^{N-1} (S_i - S'_i)^2. \quad (4.6)$$

However, the problem with this similarity measure is it is not robust against small variations in the phases of the two signals, i.e. if the two signals evolve in time noisily such that they are not instantaneously exactly in phase. For instance, Fig. 4.2A shows two resistive pulse time-series taken during an experiment and overlapped exactly in time, with lines connecting the points that would be summed in the Euclidean distance metric[2]. Notice that the two sequences are very similar, but due to a non-perfect alignment of identical features in time, the aligned Euclidean distance metric will greatly penalize the similarity of the two signals. One method of measuring the similarity of two time-series that accounts for time offsets is

Distance matrix



Cost matrix

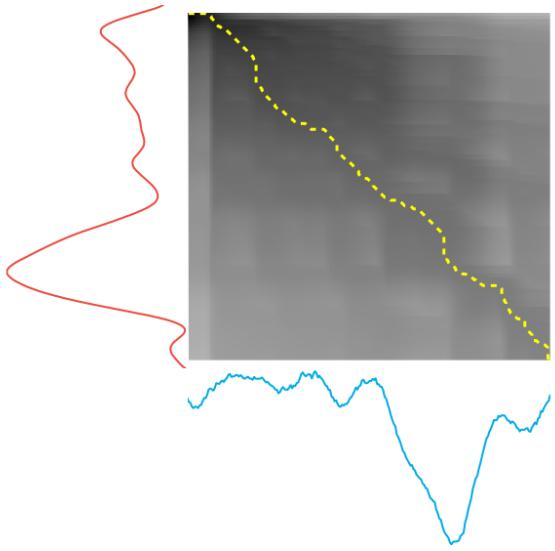


Figure 4.3: **Distance (left) and cost matrices (right) of two resistive pulse signals as determined by dynamic time warping.** The element (i, j) in the distance matrix represents the Euclidean distance between the two signals, $(S_i - S'_j)^2$. Every element in the cost matrix represents the summed Euclidean distance along the minimum path that arrives at that point. Therefore, the point at the lower right-most point in the distance matrix is the summed cost to completely connect the two signals. The path that yields the cost of that element in the matrix is represented by the yellow dashed line.

dynamic time warping.

4.1.1 Dynamic time warping

Figure 4.2B shows two time-series signals that have been aligned after dynamic time warping. The image shows that the algorithm is capable of matching points that are not perfectly aligned in time, but that reflect the actual progression of the two signals in synchronization. The algorithm works as follows.

First, a matrix $D_{i,j}$ of the point-wise Euclidean distance between all two points in the two signals is created, as shown in Fig. 4.3 (left). For instance, point $D_{3,4}$ is the Euclidean distance between point 3 and 4 in signals S and S' , respectively. The objective is to traverse through the distance matrix along the path that minimizes the total accumulated distance, or the cost C , subject to certain kinematic constraints, described below. Each step in the trajectory is defined by the indices from the two respective signals (i, j) . The non-warped Euclidean distance metric given by Eq. 4.6 would correspond to a straight diagonal trajectory through the matrix, e.g. no warping at all. Although there are several formulations of dynamic time warping, the original method only allows for three types of motion such that $(i, j) \rightarrow (i + 1, j)$ (progressive vertical motion), $(i, j) \rightarrow (i + 1, j + 1)$ (progressive diagonal motion), or $(i, j + 1) \rightarrow (i, j)$ (progressive horizontal motion). Regressive motion, which would correspond to taking a step backwards in the path back towards the upper left cell in the matrix, is forbidden. The key to dynamic time warping is the means by which we determine the optimal path that minimizes the total accumulated distance. The problem is combinatorically large and scales like $2^{\sqrt{NN'}}$, where N, N' are the number of data points in each of the two signals. Due to this scaling, random searching of paths through the matrix is not a feasible means of determining the optimum path for large NN' . As the name implies, dynamic time warping implies a dynamic computational algorithm for determining the minimum path, described as follows. First, we must recognize that, due to the rules for stepping through the matrix, the minimum path to arrive at point (i, j) must pass through one of points $(i - 1, j)$, $(i - 1, j - 1)$, or $(i, j - 1)$. Therefore, the cost C of point (i, j) is given by

$$C_{i,j} = D_{i,j} + \min [C_{i-1,j}, C_{i-1,j-1}, C_{i,j-1}] . \quad (4.7)$$

With this in mind, we can build up the complete cost matrix $C_{i,j}$ by starting from the top-left

corner $C_{0,0}$ and propagating through the matrix with Eq. 4.7. We repeat this process until we arrive at point $C_{N-1,N'-1}$, which is the minimum cost of a complete matching up of the two signals. At this point we have arrived at the answer we seeked in the beginning, the total distance metric between the two signals. However, it is often informative to calculate the actual trajectory that the dynamic time warping algorithm has determined, so as to verify that the point matching system is meaningful. In order to do so, we traverse through the matrix starting at point $(N - 1, N' - 1)$ in reverse, according to the same kinematic rules (but in reverse). At each step, we move to the nearest adjacent point with the minimum distance. This algorithm is guaranteed to return us to the starting point $(0, 0)$ along the complete minimum path between start and finish. This trajectory is shown by the yellow dashed lines in figure 4.3, and the pointwise connections are shown in the right hand side of figure 4.2.

4.2 Experiment

In order to test the method described above for determining the lengths of particles, we performed resistive pulse experiments with single mesopores ranging from 800 – 1000 nm in diameter. The materials used were polyethylene terephthalate (PET), a polymer which is known to have highly irregularly shaped interiors when pores are prepared *via* the track-etch technique, and polycarbonate (PC), another polymer but with no axial inhomogeneities which will act as a control for the method. Figure 4.4 shows images of both of these types of pores. For the particles, 280 nm and 410 nm in diameter polystyrene beads ('spheres') were used as the tracer particle, and rods of length 590 nm, diameter 210 nm ('short rods') and 1920 nm, diameter 240 nm ('long rods') were used to test the length measurement protocol; the particles are shown in figure 4.5. Particles were suspended in 100 mM KCl solution with 0.5% Tween 80, a surfactant which prevents particle aggregation. The solution was then

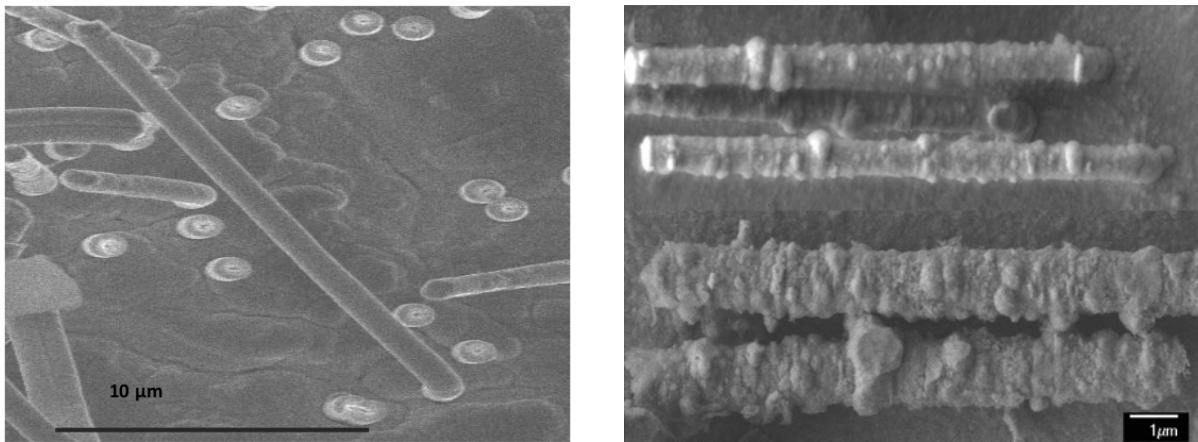


Figure 4.4: Scanning electron microscope images of metal replica of track-etched PC (left) and PET (right) pores. The pores were filled with metal and the polymer was completely etched away, leaving a metal structure that is the inverse image of the pore. The metal was imaged in a scanning electron microscope. While the PC pores are nearly perfectly cylindrical in shape, the PET pores are characterized by rough inhomogeneities across their entire surface, punctuated with large local bumps. The large bumps in the images of the metal replicas correspond to equally large cavities in the PET pore.

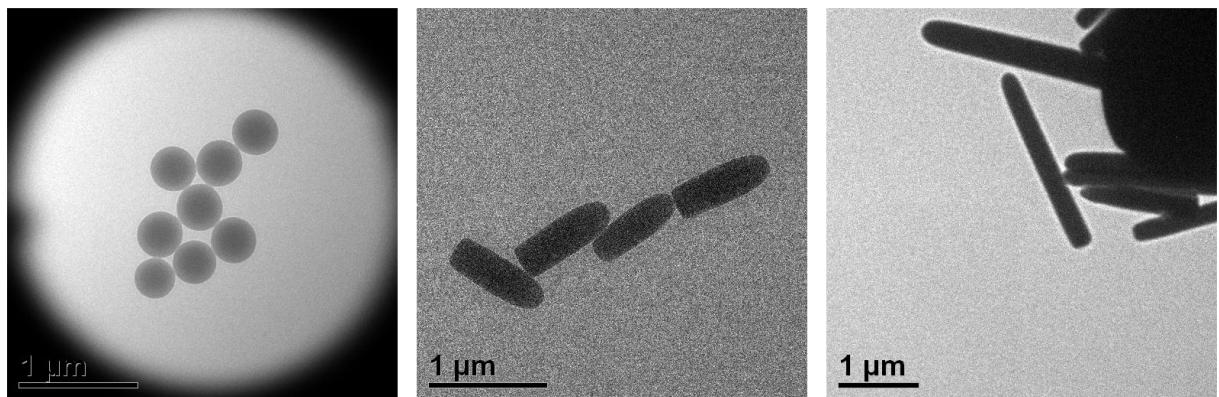


Figure 4.5: Transmission electron microscope images of polystyrene beads and silica nanorods. While the polystyrene beads are nearly perfectly spherical, the silica nanorods are approximately ‘bullet shaped’. For the purposes of this work, we approximated their shapes as ellipsoids in order to apply equation 4.2.

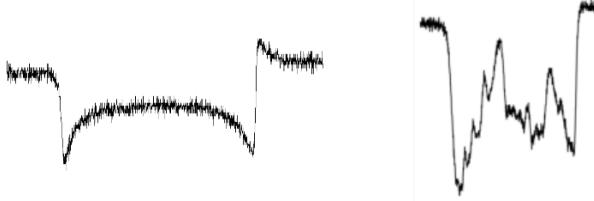


Figure 4.6: Resistive pulse events through PC (left) and PET (right) pores. As expected, the PET events are highly irregular and reflect the interior irregularities of the PET pore itself. On the other hand, the PC pores show no such irregularity. The events show that PC pores have large resistances at the beginning and end, reflecting the narrow, ‘pinched’ diameters at the openings of the pores.

injected into both sides of a conductivity cell, and a voltage was applied across the pore. The resulting ionic current was sampled at 10 kHz and recorded. The polystyrene spheres used had a larger ζ – potential than the pore, and therefore they translocated through the pore electrophoretically. On the other hand, the rods had a lower magnitude ζ – potential than the pore, and therefore travelled through the pore under electroosmotic convective forces.

4.3 Analysis & Discussion

Events were extracted from the current time series and studied separately; figure 4.6 shows two events, one from a PC pore and one from a PET pore that are representative of the translocations through each type of pore. In the initial analysis, we looked at the average $\Delta I/I_p$ of the beads and rods and compared with the theoretical predictions of Eqs. 4.1 and 4.2. Figure ?? shows the results for two pores, one 770 nm and another 1200 nm in diameter. For the two spheres studied, the measured $\Delta I/I_p$ was very close to the theoretical prediction (Eq. 4.1). The theoretical predictions were also very close to the measured values for the long rods. However, for both pores the theoretical prediction of the resistive pulse amplitude $\Delta I/I_p$ is far off for the short rods: the data shows a nearly 100% discrepancy, i.e. the measured $\Delta I/I_p$ is nearly twice its expected value. This significant of a discrepancy is highly

unusual, and one of the goals in the paper was to determine its cause. The discrepancy is made even stranger by the fact that the short and long rods are made of the same material, so any material contribution to the discrepancy e.g. due to large ζ – potentials is unlikely. Instead, we hypothesize that this discrepancy is due to their rotational dynamics. Equation 4.2 predicts that a prolate spheroid (like the rods) will have a larger $\Delta I/I_p$ when they are oriented with an off-axis component, but only considers static positioning. A very large rotational speed of the rods could couple to the measured $\Delta I/I_p$ amplitude in ways not captured by the equation, for instance by stirring the local solution in such a manner to reduce the local ion mobility. Furthermore, this hypothesis is consistent with the non-observance of the increased $\Delta I/I_p$ for the long rods, since at their length of $\sim 1920\text{ nm}$ they are too large to rotate in even the larger of the two pores (1200 nm in diameter). In any case, the net result is a measured volume for these particles that is approximately $2\times$ larger than their actual volume, which was confirmed with a combination of transmission electron microscopy images and dynamic light scattering.

The question, however, is how such a large rotation could occur. To answer this question, forces that cause rotation must be considered. In terms of translation (transport), the contributing forces are convection due to electroosmosis, and electrophoresis, as well as passive transport due to diffusion. Similarly, these forces could also contribute to rotational motion of the particles. The estimated rotational diffusion contribution of these rod particles is estimated as 13 and 0.8 rad/s for the short and long rods, respectively. In order for fluid motion to cause rotation, it is necessary to have a non-constant fluid velocity profile on the pore's length. However, because the fluid velocity is due to electroosmosis there will be a nearly constant fluid velocity profile in the pore, which is unlikely to cause significant rotation. On the other hand, the rough undulations in the pore could create axial variations in the electric field; such variations could also lead to a net torque on the particle that causes rotation. In order to estimate the potential magnitude of the rotational velocities of the rods, we considered a simple model where the pore's diameter discontinuously changes

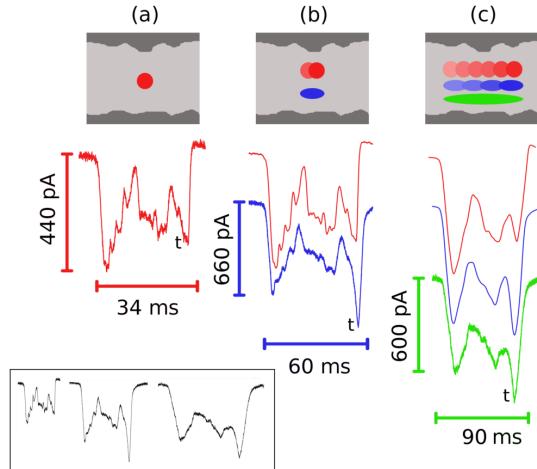


Figure 4.7: **Resistive pulse events of a 410 nm diameter sphere (red), and 590 nm (blue) and 1920 nm (green) in length rods.** The top row shows a cartoon scheme of the pore with the particles inside its interior. The bottom resistive pulse event in each column corresponds to the raw recorded data; events above this are smoothed *via* the moving average transformation. The bottom left figure shows the raw events on the same scale.

from 1000 nm to 1100 nm, a reasonable approximation for the geometries present in our system based on the images of the pores (see Fig. ??). Under this model, we calculate the electric field amplitude to differ by a factor of 1.2 between the two regions. Using the rotational diffusion coefficient of the rods and integrating the electric field differences along the length of the pore, we calculate the angular rotation rate ω to be $\omega \sim 10^4$ rad/s. Due to the preceding arguments, it is clear that the largest contribution to the rotation of the rods is from the electric forces. We believe that such a large rotation could make the rods behave as if they are much larger particles.

Figure 4.7 shows example recorded events for the three types of particles used in this study. A 410 nm sphere translocation is shown on the left column in red. Notice the roughness in the event; this roughness has been shown to correspond to the interior irregularities in the pore. Local $\Delta I/I_p(t)$ are large at relatively constricted points, and small at larger cavitated points. The green event in the right column corresponds to the raw resistive pulse signal of a long rod. Notice the difference in structure of the two events; the long rod's event shows some structure, but is missing many of the local minima and maxima that are present in

the signal of the short rod. This observation is in agreement with our previous arguments for the smoothing of the pore interior that takes place for long particles. Without making any quantitative arguments, we are able to conclude that the particle corresponding to the green event is longer than the particle corresponding to the red event. In between those two extremes are the short rods in red. The short rods at length 590 nm are slightly longer than the spheres which have diameter 410 nm, but due to their rotation it is not entirely obvious whether they should image the pore with the same resolution as the spheres. Looking at the results, the signals of the short rod and sphere are much more similar than the long rod and the sphere, but some local structure is still unresolved for the short rods.

In order to test the hypothesis that long particles behave as a moving average of shorter particles, we calculated moving averages of the spheres and the short rods. The time over which the moving average was calculated corresponded to the duration in time over which the particle travelled a distance equal to the length of the longer particle with which it is being compared. We determined the average velocity of a particle from the total translocation distance (the known length of the pore, $L' = L - D$) and the translocation duration measured from the RP signal, ΔT . The velocity is then $v = L'/\Delta t$. The moving average window then has length (in time) of $\Delta t = v/l$, which corresponds to a window of $N = f \times \Delta t$ data points in length. The transformation then from the raw signal I_i to the averaged signal I'_i is

$$I'_i = \frac{1}{N} \sum_{j=-N/2}^{N/2} I_j. \quad (4.8)$$

This moving average process was performed on the spheres and on the short rods. The red signal in the middle column corresponds to the moving average of the spheres over the length of the short rods, and the red signal in the right column corresponds to the moving average of the spheres over the length of the long rods. Finally, the moving average of the short

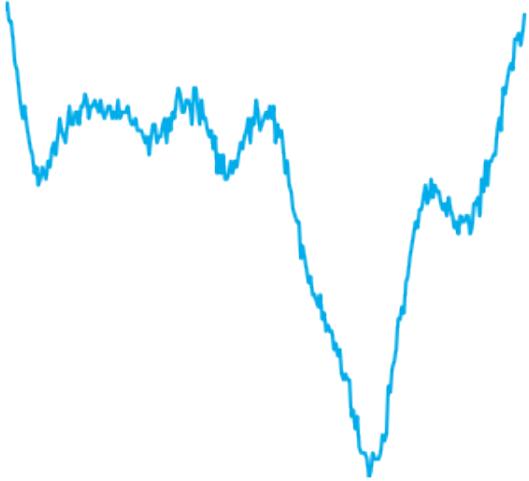


Figure 4.8: Resistive pulse signal of a single long rod passing through a PET pore. The variation in the amplitude of the current signals that the pore has an inhomogeneous interior shape.

rods over the length of the long rods is the blue signal in the right column. Qualitatively, we see that the moving average process, which is physically motivated by the equations for the resistance of small particles in series, recovers the appearance of the signals of longer particles.

In order to measure the lengths of the long rods, we performed the dynamic time warping technique described in the preceding section. Figure 4.8 shows a single instance of a long rod passing through a PET pore. Figure 4.9 shows a shorter rod particle averaged over many different length intervals. As the short rod is averaged over a greater length interval, it continuously transforms into a shape that bears a stronger resemblance to the signal of the long rods. Of all the fits calculated in Fig. 4.9, the correct length is determined by the fit with the smallest cost as determined by dynamic time warping. An example resistive pulse signal of a long rod and a shorter rod that has been transformed *via* the moving average equation is shown in Fig. 4.10. Finally, after performing the moving average and dynamic time warping minimization approach described above, we were able to determine the expected lengths of the long rods according to this algorithm. Figure 4.11 shows a plot

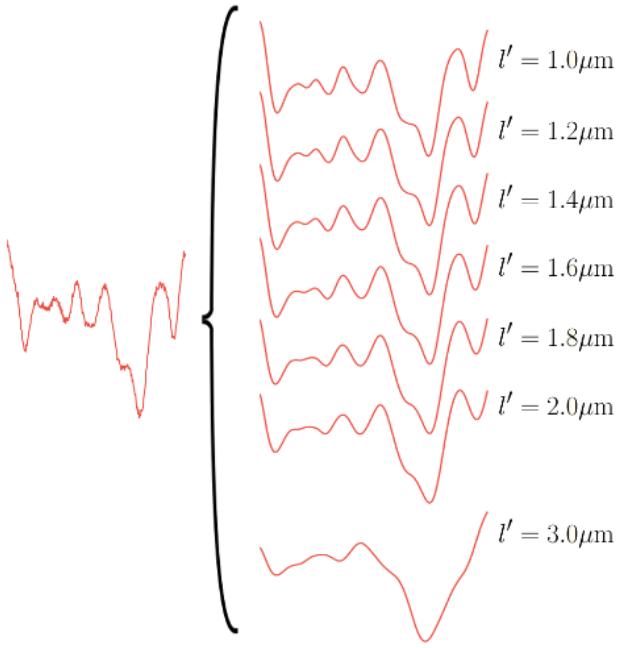


Figure 4.9: Resistive pulse signal of a single short rod passing through a PET pore alongside many instances of the moving average transformation performed on the signal for different length intervals. As the length interval is increased, the finer structure of the resistive pulse signal is smoothed over. For the greatest length measured, many of the features prominent in the raw signal have disappeared..

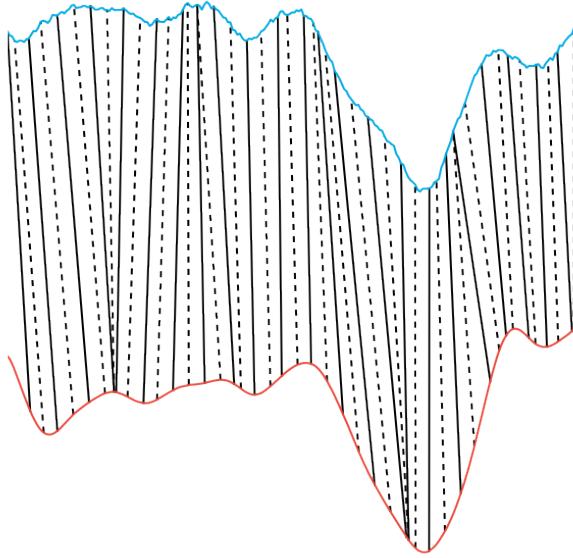


Figure 4.10: Raw resistive pulse signal of a single long rod matched with the signal of a short rod that has been transformed via the moving average process. The aligned points were determined by the dynamic time warping algorithm, and the particular transformation shown was the one that minimized the difference between the two signals, also determined by the dynamic time warping algorithm.

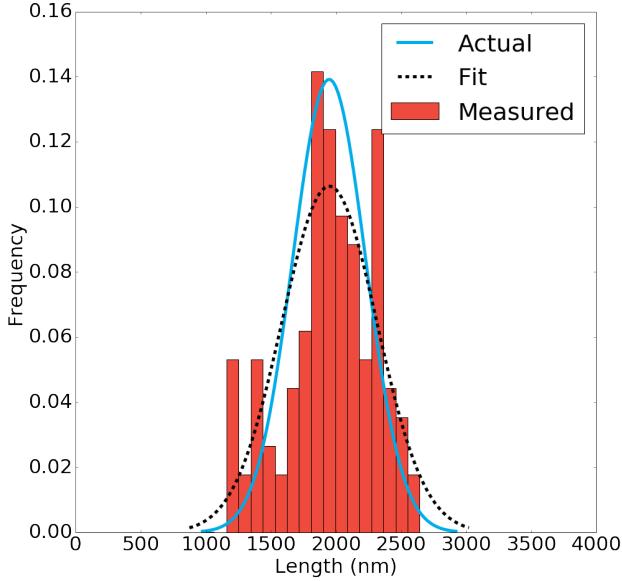


Figure 4.11: **Histogram of particle length as determined by dynamic time warping along with the actual distribution of sizes determined by electron microscopy imaging.** A normal distribution is fit to the histogram of particle lengths as determined by dynamic time warping and shown to be in good agreement with the expected distribution.

of histograms of the lengths as determined by this approach, alongside a Gaussian fit to the histogram, and a Gaussian distribution that shows the actual distribution of hte lengths of the long particles as determined by electron microscopy images. The figure reveals that, in this case, the dynamic time warping algorithm is able to successfully recover the real distribution of the known lengths relatively accurately.

4.4 Conclusions

In summary, we presented resistive pulse experiments performed with spherical and non-spherical particles in two types of pores, those with smooth interiors (PC) and rough interiors (PET). We devised a protocol for measuring the lengths of long particles by taking advantage of the rough interiors of PET pores, which reflect in the measured resistive pulse signals of all the particles. Longer particles displace a greater distance along the pore's length, and therefore their signals are a more smoothed version of the signals of shorter particles. This

smoothing attribute can be recovered by taking the moving averages of shorter particles. By performing moving averages for many different particle lengths and measuring the similarity between the raw signal of the long particles with each of the transformed signals of the shorter particles, we were able to determine the length distribution of the long particles used in experiments. Dynamic time warping was used as a means of effectively comparing the similarity of the signals of the two particles. While the general premise of the ‘smoothing’ of the resistive pulse signals by long particles compared to short particles is convincingly established, more research must be performed on the hybrid moving average-dynamic time warping method. The protocol was only tested for a small number of pores and only two types of particles, short and long rods. In the future, an ideal experiment could be performed with small spherical beads and rods of various unknown lengths. The experiments and analysis would be performed in a blind fashion, and only after the analysis is complete would the true length distribution of the long rods be revealed. If the length measurement protocol survives this blind study, it would prove to be a very effective means of determining the lengths of small particles.

Additionally, a likely unrelated phenomena was discovered where short, aspherical particles created far greater (up to 2x) resistive pulse amplitudes than predicted by the well-established classical equations. The exact reason for this effect is still unclear, however we laid out arguments that explain rotational dynamics as a likely culprit. We reasoned from our studies that an extremely rapid rate of rotation could effectively increase the radius of the particle, possibly by disturbing the local solution and decreasing its conductivity. Such rotations would not be possible for the longer rods due to their prohibitively long lengths compared to the diameter of the pore. However, more investigation must be performed in order to convincingly show that not only do short rod like objects deviate strongly from the classical equations, but also that rotational dynamics is the culprit. Additional experiments must be performed with a larger variety of short rods to show that the effect is general and not limited to the rods used in this study. Furthermore, experiments could be designed in such

a way as to retard the rods' rotations, perhaps by using smaller pores or solution with an increased viscosity.

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Appendix A

Appendix Title

A.1 Lorem Ipsum