Revisiting Unbiased Implicit Variational Inference

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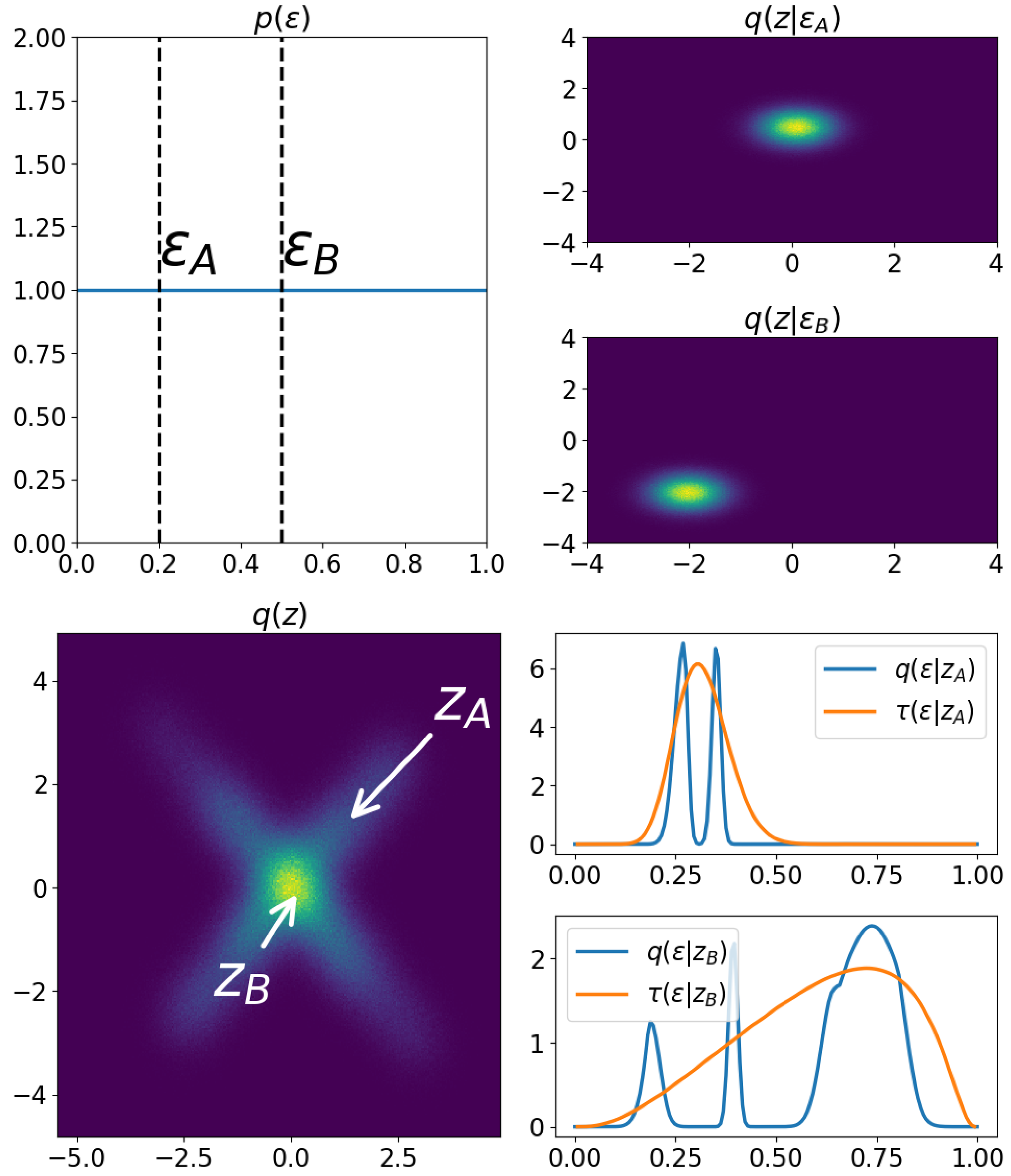
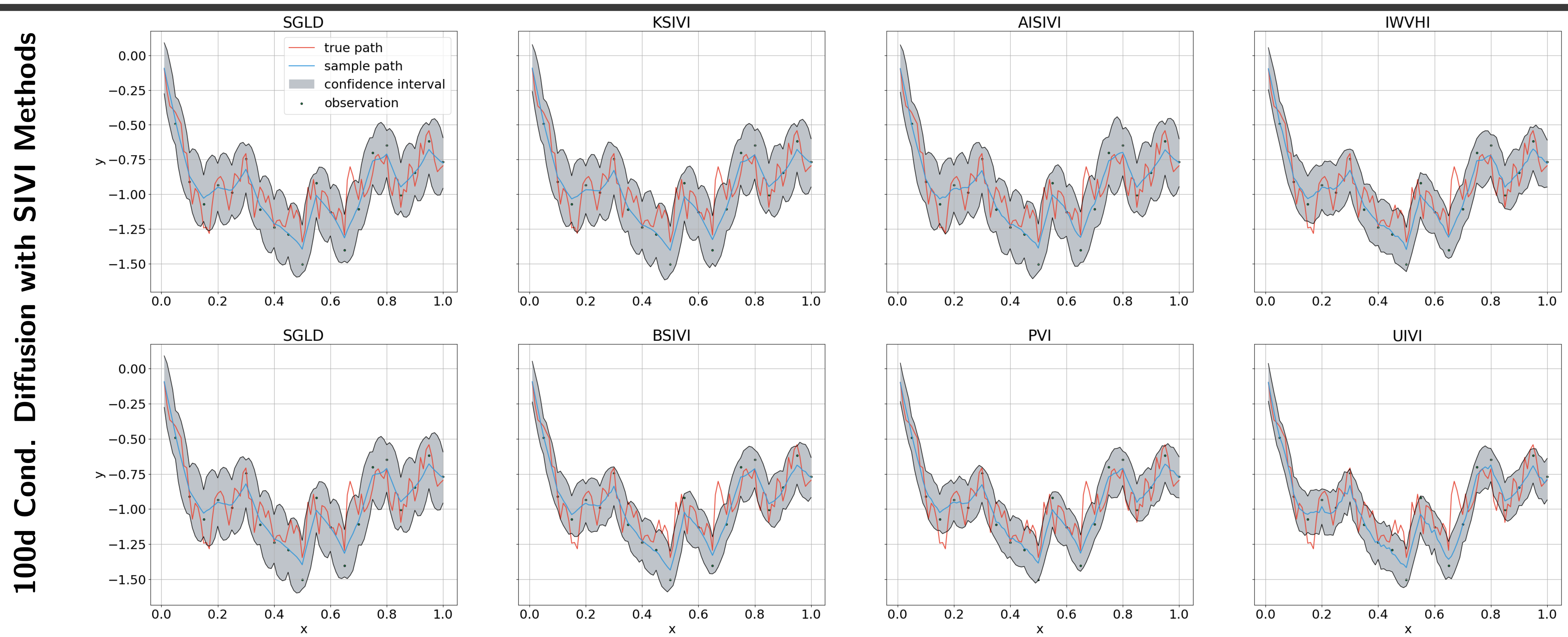
Challenges in Semi-Implicit Variational Inference (SIVI)

Semi-implicit variational distributions allow flexible representations of complex densities - but estimating their marginal likelihood is non-trivial in high dimensions

- SIVI defines $q_z(z) = \mathbb{E}_{\epsilon\sim p(\epsilon)}[q(z|\epsilon)]$ - a powerful yet implicit representation
- The marginal density $q_z(z)$ cannot be evaluated directly, complicating optimization of divergence objectives like KL
- Unbiased Implicit Variational Inference (UIVI) requires expensive inner-loop MCMC
- **Our proposal:** AISIVI – a path-gradient-compatible training scheme with **low bias, low variance** score estimation via importance sampling and learned reverse conditionals





Training SIVI with Path Gradients

The path gradient estimator enables stable and efficient optimization - but relies on estimating the intractable score $\nabla_z \log q(z)$

Objective: Minimize reverse KL $D_{\mathrm{KL}}(q(z)\|p(z))$

Reparametrization: $z = h_\phi(\epsilon, \eta)$ with $\epsilon \sim p(\epsilon), \eta \sim p(\eta)$

Path gradient:

$$\nabla_\phi D_{\mathrm{KL}}(q\|p) = \mathbb{E}_{\epsilon,\eta}\left[\nabla_z\left(\log q(z) - \log p(z)\right)\cdot \nabla_\phi h_\phi(\epsilon,\eta)\right]$$

Challenge: No tractable form of $\nabla_z \log q(z)$

Importance Sampling and Unbiasedness

We prove: If the proposal $\tau(\epsilon|z)$ matches the reverse conditional $q(\epsilon \mid z)$, our importance-weighted score estimate becomes unbiased

Key idea: Estimate score via importance sampling, for $\tilde{z} = z$:

$$\nabla_z \log q(z) = \nabla_z \log \mathbb{E}_{\epsilon\sim p(\epsilon)}[q(z|\epsilon)] = \nabla_z \log \mathbb{E}_{\epsilon\sim\tau(\cdot|\tilde{z})}\left[\frac{p(\epsilon)q(z|\epsilon)}{\tau(\epsilon|\tilde{z})}\right]$$

Theoretical guarantee: Let $\tilde{z} \sim q(z)$ and $\tau(\epsilon|\tilde{z}) = q(\epsilon|\tilde{z})$, then:

$$\mathbb{E}_{\epsilon_i\sim\tau(\epsilon|\tilde{z})}\left[\nabla_z \log\left(\frac{1}{k}\sum_{i=1}^{k}\frac{p(\epsilon_i)q(z|\epsilon_i)}{\tau(\epsilon_i|\tilde{z})}\right)\right] = \nabla_z \log q(\tilde{z})$$

$\Rightarrow$ Low bias and variance when $\tau(\epsilon|z) \approx q(\epsilon|z)$

- Even if $\tau(\epsilon|z) \neq q(\epsilon|z)$, the estimator remains **consistent** for: supp$(\tau(\cdot|z)) \supseteq$ supp$(q(\cdot|z))$
- **Contrast:** UIVI requires **exact** samples from $q(\epsilon|z)$

Learning the Proposal $\tau(\epsilon|z)$

We minimize the forward KL between the true reverse conditional and our proposal - with a tractable gradient estimator derived in closed form

Objective: $\min_\theta \mathbb{E}_{z\sim q(z)}\left[D_{\mathrm{KL}}(q(\epsilon|z)\|\tau_\theta(\epsilon|z))\right]$

We prove that $\nabla_\theta \mathbb{E}_{z\sim q(z)}\left[D_{\mathrm{KL}}(q(\epsilon|z)\|\tau_\theta(\epsilon|z))\right]$

$$= -\mathbb{E}_{(z,\epsilon)\sim q(z,\epsilon)}\left[\nabla_\theta \log \tau_\theta(\epsilon|z)\right]$$

- We can optimize τ directly using samples from $q(\epsilon, z)$
- No need to evaluate or sample from $q(\epsilon|z)$ explicitly
- We use a conditional normalizing flow for $\tau(\cdot|z)$

Contact

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