# Model Predictive Control of a Sewer System

June 14, 2018


Group 1030

Jacob Naundrup Pedersen

Thomas Holm Pilgaard

Department of Electronic Systems
Aalborg University
Denmark


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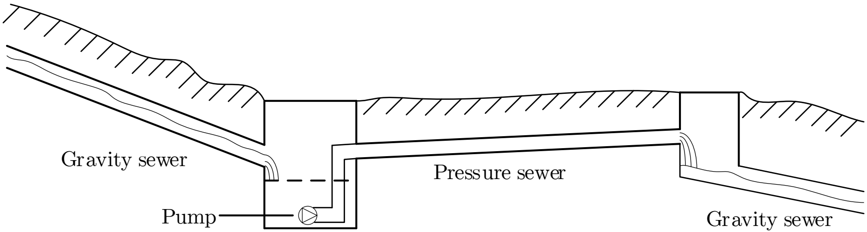
# Agenda

# Typisk opbygning af kloak ledning





# Tilstande i kloakken



- Aerob $\rightarrow O_2 \rightarrow H_2O$

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# Tilstande i kloakken



- Aerob $\rightarrow O_2 \rightarrow H_2O$
- Anaerob $\rightarrow SO_4^{-2} \rightarrow H_2S$

# Tilstande i kloakken





- Aerob $\rightarrow O_2 \rightarrow H_2O$
- Anaerob $\rightarrow SO_4^{-2} \rightarrow H_2S$
- Anoxisk $\rightarrow NO_3^{-} \rightarrow N_2$

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# Udfordringer ved spildevands rensning





- Virksomheds besøg ved Fredericia Spildevand og Energi A/S.
  - Større udledninger uden varsel

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# Udfordringer ved spildevands rensning



4

- Virksomheds besøg ved Fredericia Spildevand og Energi A/S.
  - Større udledninger uden varsel
  - Problemer for aerobe bakterier

# Udfordringer ved spildevands rensning



- Virksomheds besøg ved Fredericia Spildevand og Energi A/S.
    - Større udledninger uden varsel
    - Problemer for aerobe bakterier
    - Andre forstyrelser

# Problem formulering



How can a simulation environment be constructed, which mimic the behavior of a real sewer system, where MPC is utilized as the control scheme to obtain stable sewage output such that optimal performance can be obtained from a WWTP.

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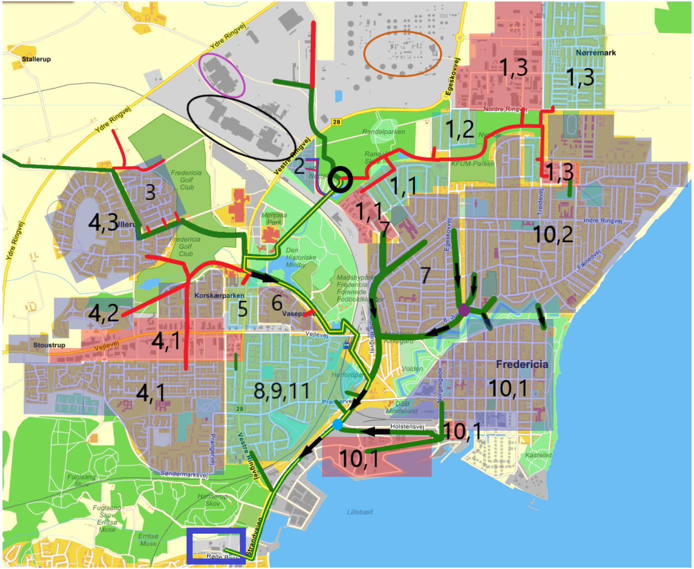
# Udgangspunkt i et virkeligt setup



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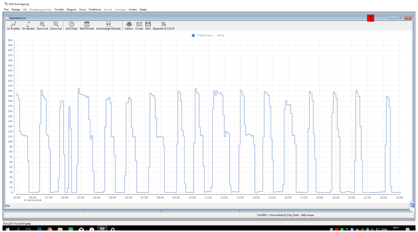
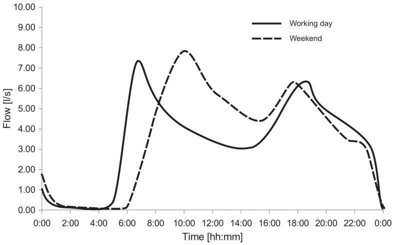
# Udgangspunkt i et virkeligt setup

- Data fra industri.
- Flow profiler af beboelse og mindre industri.



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# Valg





- Indsættelse af tank.

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# Valg

- Indsættelse af tank.
- Afgrænse simulering til enkelt kemisk component.



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# Valg



- Indsættelse af tank.
- Afgrænse simulering til enkelt kemisk component.
- Runde kloak rør.

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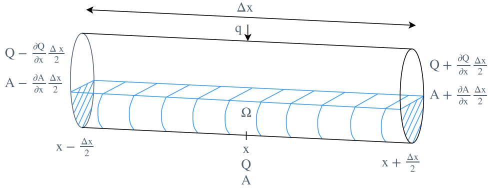
# 4 modeller



- Kloak ledning.
- Transport af concentrat i kloak ledning.
- Sammenkobling af kloakledninger.
- Tank.

- $$\frac{\partial A(x,t)}{\partial t} + \frac{\partial Q(x,t)}{\partial x} = 0$$

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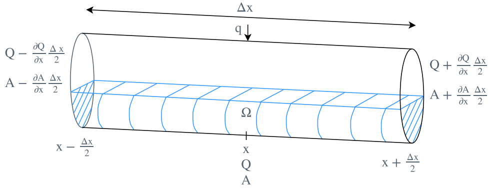
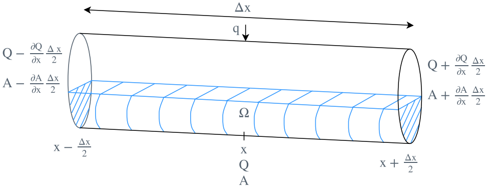
# Kloak ledning

## Saint-Venant ligningerne

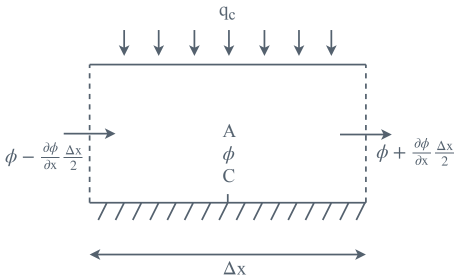
- $$\frac{\partial A(x,t)}{\partial t} + \frac{\partial Q(x,t)}{\partial x} = 0$$
- $$\frac{1}{gA}\frac{\partial Q}{\partial t} + \frac{1}{gA}\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + \frac{\partial h}{\partial x} + S_f - S_b = 0$$



- $$\frac{\partial A(x,t)}{\partial t} + \frac{\partial Q(x,t)}{\partial x} = 0$$
- $$\frac{1}{gA}\frac{\partial Q}{\partial t} + \frac{1}{gA}\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + \frac{\partial h}{\partial x} + S_f - S_b = 0$$
- Approksimationer af momentum ligningen.

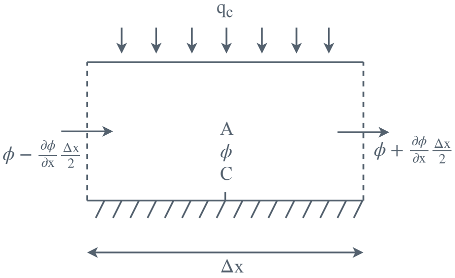


- $A \cdot \frac{\partial C}{\partial t} + Q \cdot \frac{\partial C}{\partial x} = 0$

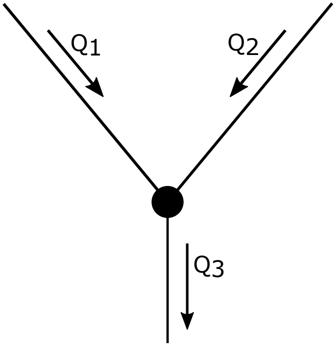


# Transport af koncentrat

- $A \cdot \frac{\partial C}{\partial t} + Q \cdot \frac{\partial C}{\partial x} = 0$
- Afhænger af kendt $A$ og $Q$.

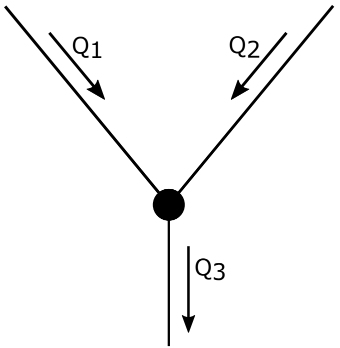


- $Q_3 = Q_1 + Q_2$



# Sammenkobling af kloak ledninger

- $Q_3 = Q_1 + Q_2$
- $C_3 = \dfrac{C_1 \cdot Q_1 + C_2 \cdot Q_2}{Q_1 + Q_2}$

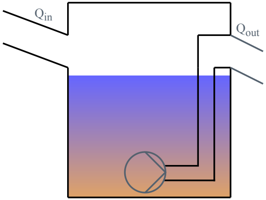


# Tank





- $$\frac{dh(t)}{dt} = \frac{1}{A}\left(Q_{in}(t) - u(t) \cdot \overline{Q}\right)$$

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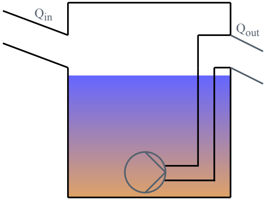
# Tank

- $$\frac{dh(t)}{dt} = \frac{1}{A}\left(Q_{in}(t) - u(t) \cdot \overline{Q}\right)$$
- $$\frac{dC_{tank}(t)}{dt} = \frac{1}{A}\left(C_{in}(t) \cdot \frac{Q_{in}(t)}{h(t)} - C_{tank}(t) \cdot \frac{Q_{out}(t)}{h(t)}\right)$$



# Tre dele



- **Intialisering**
- Opsætning af komponenter.

# Tre dele

- **Intialisering**
- Opsætning af komponenter.
- System i steady state.



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# Tre dele



- **Intialisering**
- Opsætning af komponenter.
- System i steady state.
- **Simulering**



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# Tre dele





- **Intialisering**
- Opsætning af komponenter.
- System i steady state.
- **Simulering**
- Iterativ beregning af komponenterne

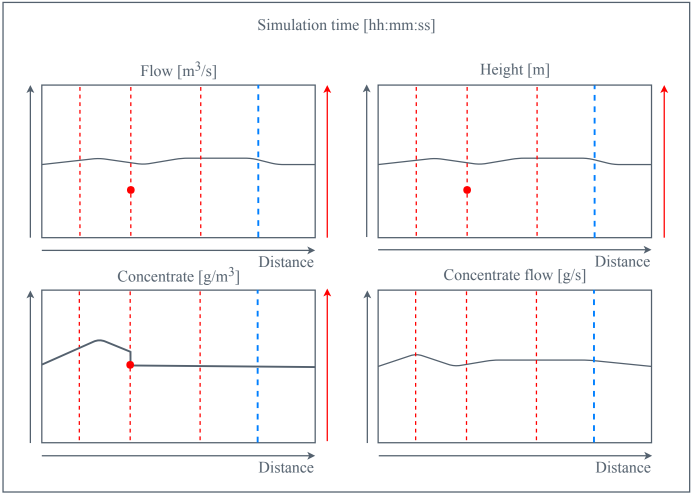
# Tre dele

- **Intialisering**
- Opsætning af komponenter.
- System i steady state.
- **Simulering**
- Iterativ beregning af komponenterne
- **Gennemgang af resultat**

# Playback funktion





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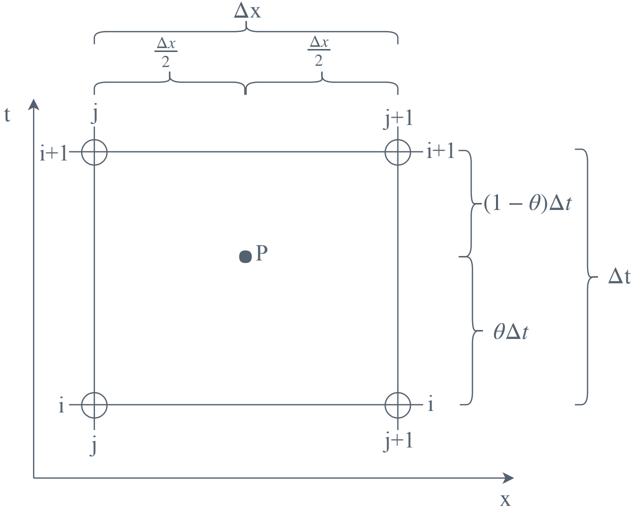
# Preissmann basic

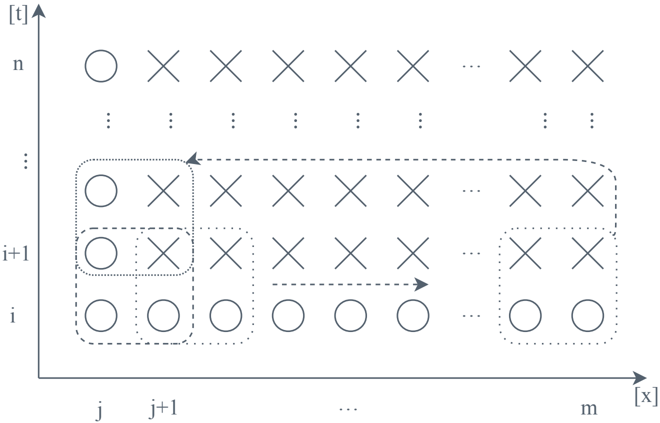


- Kinematisk bølge aproksimering.
- Fyldningsgrad kurve for rør.


Δx
Δx/2
Δx/2
t
j
j+1
i+1
i+1
(1 − θ)Δt
P
Δt
θΔt
i
i
j
j+1
x

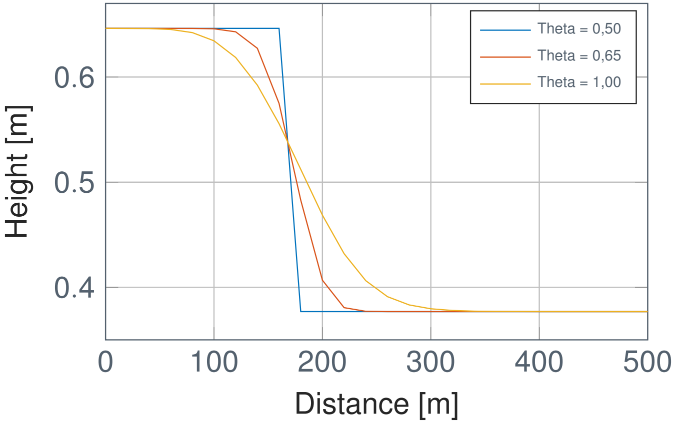
# Preissmann iteration





# Preissmann stabilitet

- Ubetinget stabilitet

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# Courant's tal

- Indikation af præcision

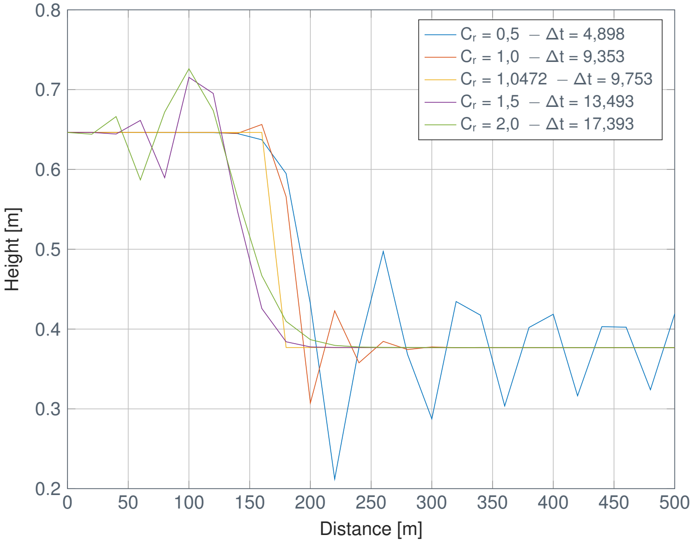
# Courant's tal



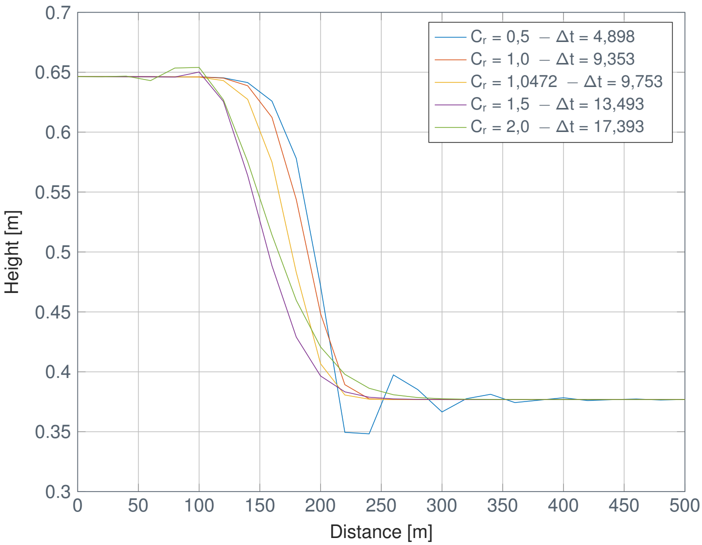
- Indikation af præcision
- $C_r = \dfrac{\sqrt{g \cdot \overline{\mathrm{H}}} \cdot \Delta t}{\Delta x}$

# Courant's tal





# Courant's tal



- Implementation
- Kontrol
- Results
- Discussion
- Conclusion

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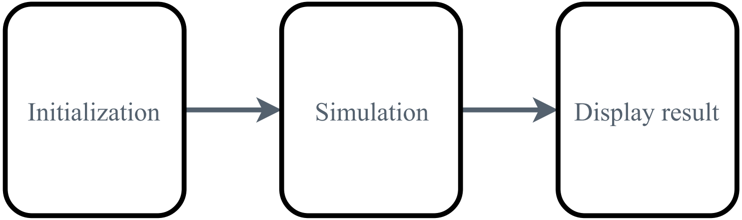


Figure: Chosen structure of simulation environment.

1. Pipe
   - Length [m]
   - Sections (Number of sections the pipe should be split in to)
   - $S_b$ (Slope) [‰]
   - $\Delta x$ = Length/Sections [m]
   - Diameter [meter]
   - Theta (parameter used in Preissmann scheme)
   - $Q_f [m^3/s]$
   - Side/lateral inflow present
   - Section location in data
2. Tank
   - Size [$m^3$]
   - Height [m]
   - Area = Size / Height [$m^2$]
   - Maximum outflow [$m^3/s$]
   - Section location in data

Table: List of parameters for pipe and tank.

| Fields | length | sections | Dx | Sb | d | Theta | Qf | side_inflow | data_location |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 700 | 35 | 20 | 0.0030 | 0.9000 | 0.6500 | 0.9730 | 0 | 1 |
| 2 | 303 | 15 | 20.2000 | 0.0030 | 0.9000 | 0.6500 | 0.9730 | 0 | 3 |
| 3 | 27 | 2 | 13.5000 | 0.0030 | 1 | 0.6500 | 1.2843 | 1 | 4 |
| 4 | 155 | 8 | 19.3750 | 0.0041 | 1 | 0.6500 | 1.5014 | 0 | 5 |
| 5 | 295 | 14 | 21.0714 | 0.0122 | 0.8000 | 0.6500 | 1.4386 | 0 | 6 |
| 6 | 318 | 15 | 21.2000 | 0.0053 | 0.9000 | 0.6500 | 1.2932 | 1 | 7 |
| 7 | 110 | 5 | 22 | 0.0036 | 0.9000 | 0.6500 | 1.0658 | 1 | 8 |
| 8 | 38 | 2 | 19 | 0.0024 | 1 | 0.6500 | 1.1487 | 1 | 9 |
| 9 | 665 | 30 | 22.1667 | 0.0030 | 1 | 0.6500 | 1.2843 | 1 | 10 |
| 10 | 155 | 7 | 22.1429 | 8.0000e-04 | 1 | 0.6500 | 0.6632 | 0 | 11 |
| 11 | 955 | 47 | 20.3191 | 0.0029 | 1.2000 | 0.6500 | 2.0415 | 1 | 12 |
| 12 | 304 | 15 | 20.2667 | 0.0030 | 1.2000 | 0.6500 | 2.0764 | 0 | 13 |
| 13 | 116 | 5 | 23.2000 | 0.0021 | 1.2000 | 0.6500 | 1.7373 | 1 | 14 |
| 14 | 283 | 12 | 23.5833 | 0.0017 | 1.4000 | 0.6500 | 2.3463 | 1 | 15 |
| 15 | 31 | 2 | 15.5000 | 0.0019 | 1.4000 | 0.6500 | 2.4805 | 1 | 16 |
| 16 | 125 | 6 | 20.8333 | 0.0021 | 1.6000 | 0.6500 | 3.7075 | 0 | 17 |
| 17 | 94 | 4 | 23.5000 | 0.0013 | 1.5000 | 0.6500 | 2.4609 | 0 | 18 |
| 18 | 360 | 18 | 20 | 0.0046 | 1.6000 | 0.6500 | 5.4872 | 1 | 19 |
| 19 | 736 | 38 | 19.3684 | 0.0012 | 1.6000 | 0.6500 | 2.8026 | 0 | 20 |

Figure: Setup in MATLAB of pipe specification of the main line in Fredericia.

| Field | Value |
|---|---|
| size | 90 |
| height | 10 |
| area | 9 |
| Q_out_max | 0.9730 |
| data_location | 2 |

Figure: Setup in MATLAB of tank specifications.

| Fields | type | component | sections |
|---|---|---|---|
| 1 | 'Pipe' | 1 | 35 |
| 2 | 'Tank' | 1 | 1 |
| 3 | 'Pipe' | 18 | 245 |
| 4 | 'Total' | 20 | 281 |

Figure: Display of structure showing system setup information in MATLAB.


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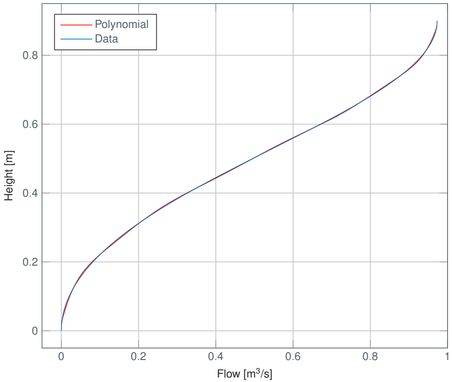


Figure: Comparison between data obtained by equation **??** and the same data curve fitted to a ninth order polynomial.

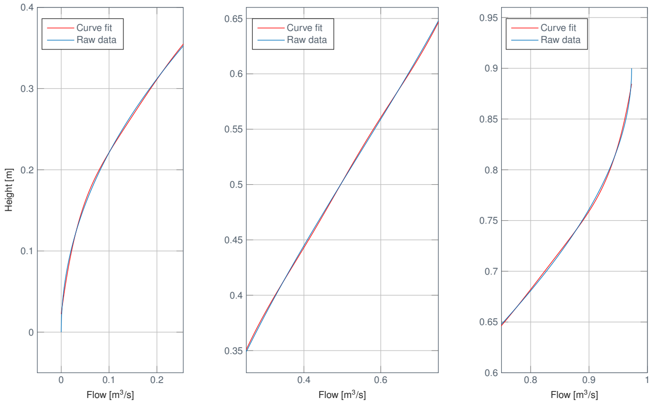


Figure: Comparison between data obtained by equation **??** and the same data curve fitted to a ninth order polynomial.

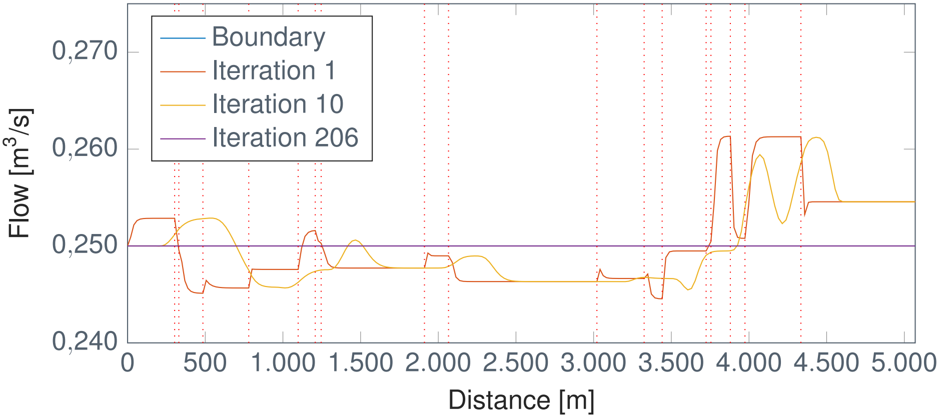


Figure: Height and flow of pipe setup from part of Fredericia where boundary conditions is found by fitted polynomial. Various amount of iterations, with constant flow input of 0,25 $m^3/s$, is performed. The dotted line indicates pipe intersections.

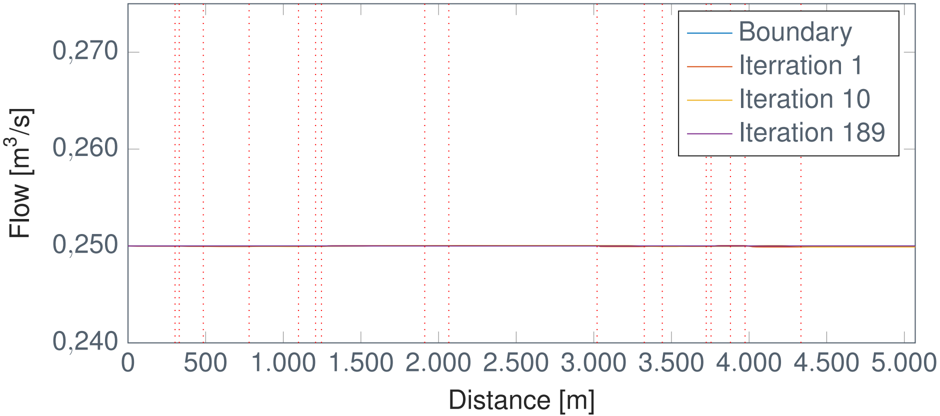


Figure: Height and flow of pipe setup from part of Fredericia where boundary conditions is found by lookup table. Various amount of iterations, with constant flow input of 0,25 $m^3/s$, is performed. The dotted line indicates pipe intersections.

- Preissmann scheme




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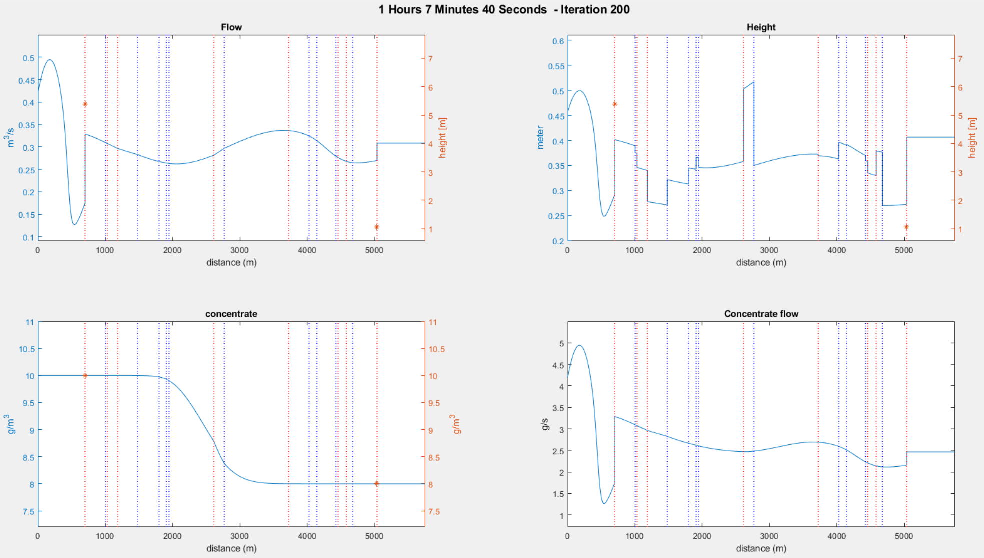

1 Hours 7 Minutes 40 Seconds - Iteration 200
Flow
Height
concentrate
Concentrate flow
distance (m)
height [m]
meter
g/s


Figure

- Linearisering af ulinear model
- Opstilles på state space form

$$\frac{\partial A(x,t)}{\partial t} + \frac{\partial Q(x,t)}{\partial x} = 0 \tag{1}$$

$$\frac{\partial A(h)}{\partial h}\frac{\partial h(x,t)}{\partial t} + \frac{\partial Q(h)}{\partial h}\frac{\partial h(x,t)}{\partial x} = 0 \tag{2}$$

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- Priessmann scheme

$$\frac{\partial A(h)}{\partial h}\left(\frac{1}{2}\frac{h_{j+1}^{i+1}-h_{j+1}^{i}}{\Delta t}+\frac{1}{2}\frac{h_{j}^{i+1}-h_{j}^{i}}{\Delta t}\right)+ \frac{\partial Q(h)}{\partial h}\left(\theta\frac{h_{j+1}^{i+1}-h_{j}^{i+1}}{\Delta x}+(1-\theta)\frac{h_{j+1}^{i}-h_{j}^{i}}{\Delta x}\right)=0 \quad (3)$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & b_1 & 0 & \cdots & 0 \\ 0 & a_1 & b_2 & \cdots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & a_{m-1} & b_m \end{bmatrix}}_{\xi} \underbrace{\begin{bmatrix} h_0^{i+1} \\ h_1^{i+1} \\ h_2^{i+1} \\ \vdots \\ h_m^{i+1} \end{bmatrix}}_{x(k+1)} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ c_0 & d_1 & 0 & \cdots & 0 \\ 0 & c_1 & d_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & c_{m-1} & d_m \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} h_0^{i} \\ h_1^{i} \\ h_2^{i} \\ \vdots \\ h_m^{i} \end{bmatrix}}_{x(k)} + \underbrace{\begin{bmatrix} 1 \\ -a_0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}}_{B} h_0^{i+1} + \underbrace{\begin{bmatrix} \frac{dh}{dQ} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}}_{B_d} d_0^{i+1} \tag{4}$$

$$
\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & b_{1,1} & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{1,1} & b_{1,2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{2,1} & b_{2,2} & 0 \\ 0 & 0 & 0 & 0 & 0 & a_{2,2} & b_{2,3} \end{bmatrix}}_{\xi} \underbrace{\begin{bmatrix} h_{1,0}^{i+1} \\ h_{1,1}^{i+1} \\ h_{1,2}^{i+1} \\ h_{tank}^{i+1} \\ h_{2,0}^{i+1} \\ h_{2,1}^{i+1} \\ h_{2,2}^{i+1} \end{bmatrix}}_{x(k+1)}
= \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ c_{1,0} & d_{1,1} & 0 & 0 & 0 & 0 & 0 \\ 0 & c_{1,1} & d_{1,2} & 0 & 0 & 0 & 0 \\ 0 & 0 & e & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{2,0} & d_{2,1} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{2,1} & d_{2,2} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} h_{1,0}^{i} \\ h_{1,1}^{i} \\ h_{1,2}^{i} \\ h_{tank}^{i} \\ h_{2,0}^{i} \\ h_{2,1}^{i} \\ h_{2,2}^{i} \end{bmatrix}}_{x(k)} + \underbrace{\begin{bmatrix} 1 & 0 \\ -a_0 & 0 \\ 0 & 0 \\ 0 & -f \\ 0 & g \\ 0 & 0 \\ 0 & 0 \end{bmatrix}}_{B} \begin{bmatrix} h_0^{i+1} \\ u_{tank} \end{bmatrix} \tag{5}
$$

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- Sinus input

| Type | Components | Sections |
|---|---|---|
| Pipe | 1 | 35 |
| Tank | 1 | 1 |
| Pipe | 18 | 227 |
| Total | 20 | 263 |

Table: System setup.



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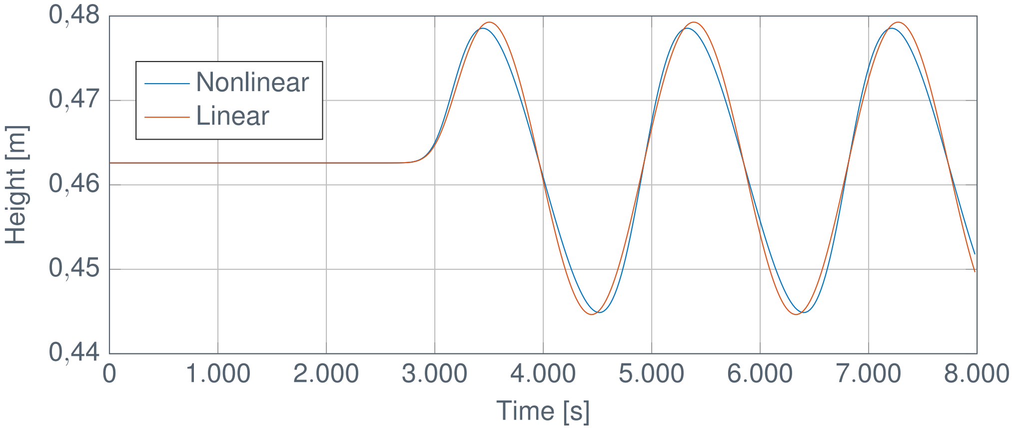


Figure: Comparison of the nonlinear and linear model at the last pipe in the setup.

- Cost function
  - Begrænset til minimiere af output
- Constraints
  - Højde
  - Kontrol input
- Linear model

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# Kontrol
## MPC





- Bestemmelse af Prediction horizon
  - Flow profiler
  - Industri
- Begrænsning af Prediction horizon

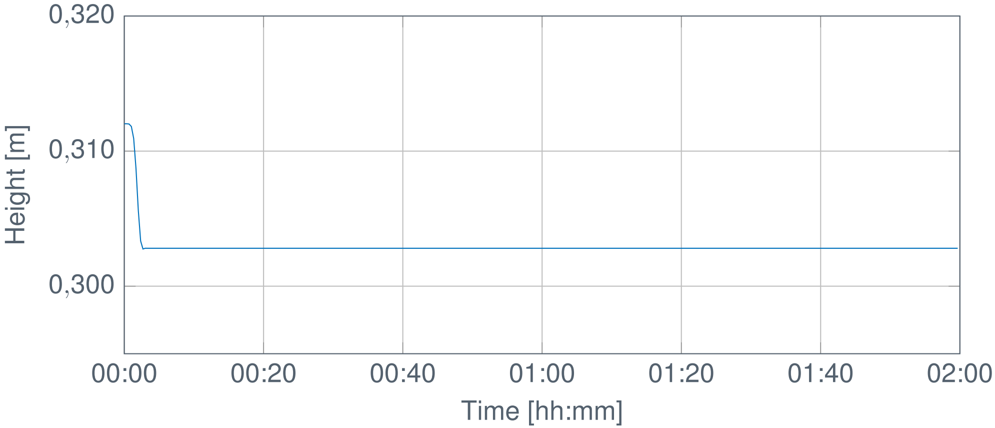


Figure: Output of the last pipe.

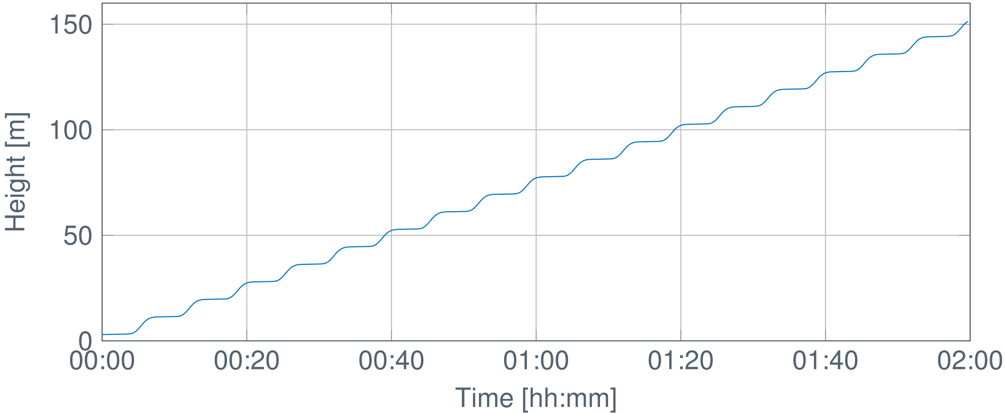


Figure: Height in the tank.

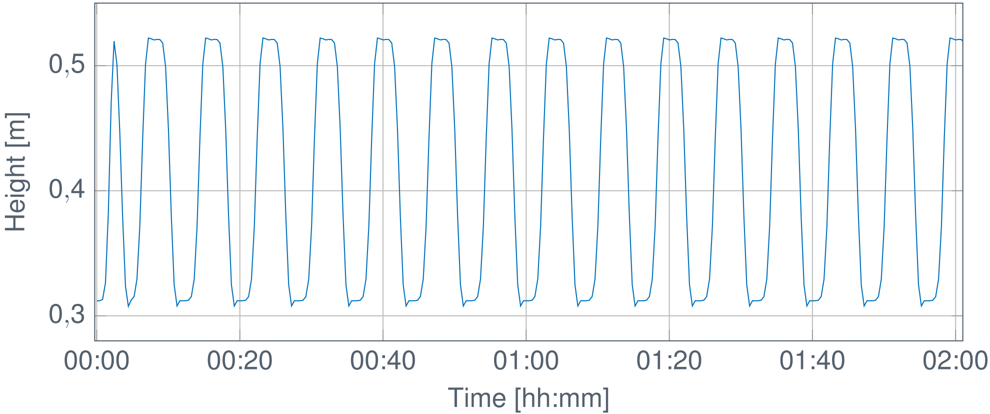


Figure: Output of the last pipe in the second simulation run.

# Results

## First test



- System setup
- Flow profiles

| Type | Component | Sections |
|---|---|---|
| Pipe | 1 | 35 |
| Tank | 1 | 1 |
| Pipe | 17 | 207 |
| Tank | 1 | 1 |
| Pipe | 1 | 38 |
| Total | 21 | 282 |

Table: System setup.

# Results
## First test



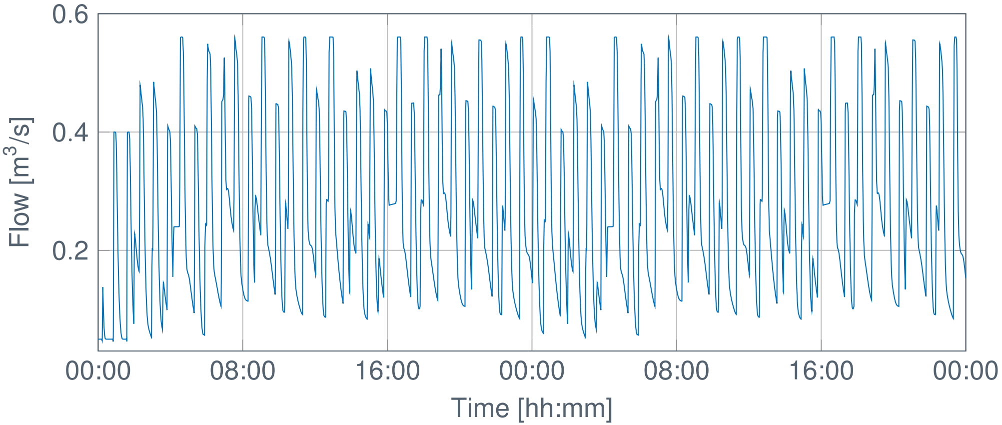


**Figure:** Output of the last pipe into the WWTP.

# Results
## First test

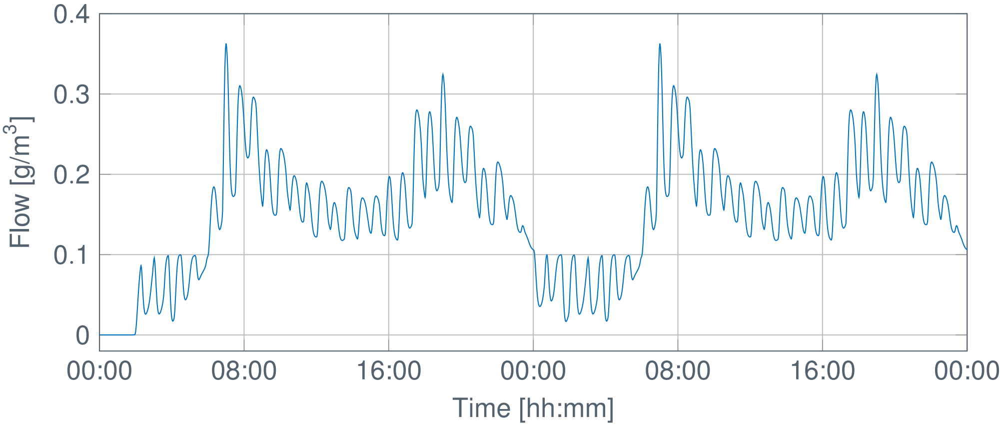


Figure: Simulation of COD output of the last pipe into the WWTP.

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# Results
## Second test

- Over dimensioneret tank
- Konstant output af tank

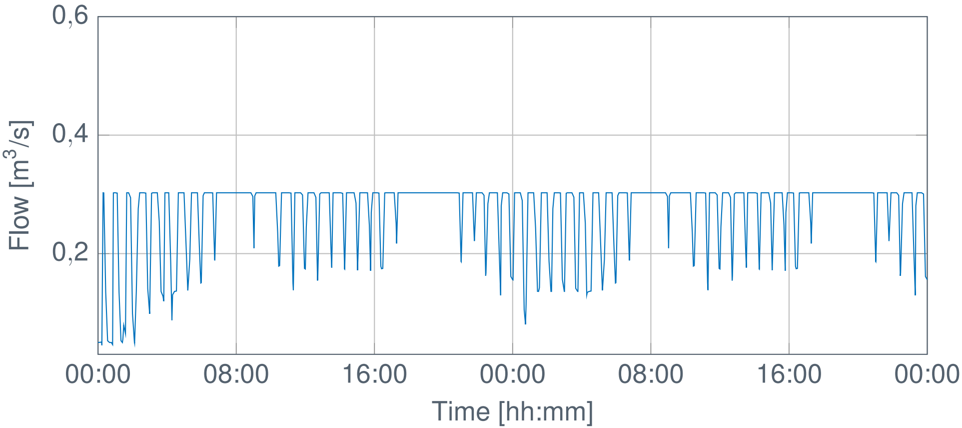


Figure: Output of the last pipe in to the WWTP, where a tank has been placed in front to reduce variation in flow into WWTP.

# Discussion



- Courant's number
- Model reduction

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# Conclusion



- Simulation
- MPC