

①

$$x(k+1) = Ax(k) + B_u u(k) + B_d u_d(k)$$

$$y(k) = Cx(k)$$

$$x(k) = Ax(k|k) + \beta_u u(k) + \beta_d u_d(k)$$

$$X(k) = \begin{bmatrix} x(k+1|k) \\ \vdots \\ x(k+H_p|k) \end{bmatrix} \quad A = \begin{bmatrix} A \\ \vdots \\ A^{H_p} \end{bmatrix} \quad \beta_u = \begin{bmatrix} B_u & 0 & \dots & 0 \\ AB_u & B_u & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A^{H_p-1} B_u & A^{H_p-2} B_u & \dots & B_u \end{bmatrix}$$

$$\beta_d = \begin{bmatrix} B_d & 0 & \dots & 0 \\ AB_d & B_d & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A^{H_p-1} B_d & A^{H_p-2} B_d & \dots & B_d \end{bmatrix}$$

$$u(k) = \begin{bmatrix} u(k) \\ \vdots \\ u(k+H_p-1) \end{bmatrix}$$

$$u_d(k) = \begin{bmatrix} u_d(k) \\ \vdots \\ u_d(k+H_p-1) \end{bmatrix}$$

$$C = \text{diag}(C)$$

$$Y(k) = \begin{bmatrix} y(k+1) \\ \vdots \\ y(k+H_p) \end{bmatrix} = Cx(k+1) = C X(k)$$

$$\Psi = CA \quad \Upsilon = C\beta_u \quad \Theta = C\beta_d$$

$$Y(k) = \Psi X(k|k) + \Upsilon u(k) + \Theta u_d(k)$$

$$J = \sum_{i=0}^{H_p} \|y(k+i) - y(k+i-1)\|_{Q(i)}^2$$

S.T.

$$x(k+i+1) = Ax(k+i) + Bu(k+i) + B_d u_d(k+i)$$

$$y(k+i) = Cx(k+i)$$

$$J = \| \gamma(k) - \gamma(k-1) \|_{Q(k)}^2 = (\gamma(k)^T - \gamma(k-1)^T) Q (\gamma(k) - \gamma(k-1)) \quad (2)$$

$$\gamma(k)^T Q \gamma(k) + \gamma(k-1)^T Q \gamma(k-1) - \gamma(k)^T Q \gamma(k-1) - \gamma(k-1)^T Q \gamma(k)$$

$$\begin{aligned} \gamma(k)^T Q \gamma(k) &= (x(k|k)^T \psi(k)^T + u(k)^T \gamma(k)^T + u_d(k)^T \theta^T) Q (\psi(k)x(k|k) + \gamma u(k) + \theta u_d(k)) \\ &= x(k|k)^T \psi(k)^T Q \psi(k)x(k|k) + x(k|k)^T \psi(k)^T Q \gamma u(k) + x(k|k)^T \psi(k)^T Q \theta u_d(k) + \\ &u(k)^T \gamma(k)^T Q \psi(k)x(k|k) + u(k)^T \gamma(k)^T Q \gamma u(k) + u(k)^T \gamma(k)^T Q \theta u_d(k) + \\ &u_d(k)^T \theta^T Q \psi(k)x(k|k) + u_d(k)^T \theta^T Q \gamma u(k) + u_d(k)^T \theta^T Q \theta u_d(k) \end{aligned}$$

$$\gamma(k-1)^T Q \gamma(k-1) =$$

$$\begin{aligned} &= x(k-1|k)^T \psi(k-1)^T Q \psi(k-1)x(k-1|k) + x(k-1|k)^T \psi(k-1)^T Q \gamma u(k-1) + x(k-1|k)^T \psi(k-1)^T Q \theta u_d(k-1) + \\ &u(k-1)^T \gamma(k-1)^T Q \psi(k-1)x(k-1|k) + u(k-1)^T \gamma(k-1)^T Q \gamma u(k-1) + u(k-1)^T \gamma(k-1)^T Q \theta u_d(k-1) + \\ &u_d(k-1)^T \theta^T Q \psi(k-1)x(k-1|k) + u_d(k-1)^T \theta^T Q \gamma u(k-1) + u_d(k-1)^T \theta^T Q \theta u_d(k-1) \end{aligned}$$

$$- \gamma(k)^T Q \gamma(k-1) =$$

$$\begin{aligned} &= -(x(k|k)^T \psi(k)^T Q \psi(k-1)x(k-1|k) + x(k|k)^T \psi(k)^T Q \gamma u(k-1) + x(k|k)^T \psi(k)^T Q \theta u_d(k-1) + \\ &u(k)^T \gamma(k)^T Q \psi(k-1)x(k-1|k) + u(k)^T \gamma(k)^T Q \gamma u(k-1) + u(k)^T \gamma(k)^T Q \theta u_d(k-1) + \\ &u_d(k)^T \theta^T Q \psi(k-1)x(k-1|k) + u_d(k)^T \theta^T Q \gamma u(k-1) + u_d(k)^T \theta^T Q \theta u_d(k-1)) \end{aligned}$$

$$- \gamma(k-1)^T Q \gamma(k) =$$

$$\begin{aligned} &= -(x(k-1|k)^T \psi(k-1)^T Q \psi(k)x(k|k) + x(k-1|k)^T \psi(k-1)^T Q \gamma u(k) + x(k-1|k)^T \psi(k-1)^T Q \theta u_d(k) + \\ &u(k-1)^T \gamma(k-1)^T Q \psi(k)x(k|k) + u(k-1)^T \gamma(k-1)^T Q \gamma u(k) + u(k-1)^T \gamma(k-1)^T Q \theta u_d(k) + \\ &u_d(k-1)^T \theta^T Q \psi(k)x(k|k) + u_d(k-1)^T \theta^T Q \gamma u(k) + u_d(k-1)^T \theta^T Q \theta u_d(k)) \end{aligned}$$

Constant

Linear

Quadratic

(3)

$$H = \gamma^T Q \gamma$$

$$f = 2(x(k|k)^T \Psi^T Q \gamma u(k) + 2(u_d^T \Theta^T Q \gamma u(k) -$$

$$(2(x(k-1|k)^T \Psi^T Q \gamma u(k)) - (2(u(k-1)^T \gamma^T Q \gamma u(k)))$$

$$- 2(u_d(k-1)^T \Theta^T Q \gamma u(k)))$$

$$\min_u J = \min_u u^T H u + f u + c$$

$c = \text{constant}$

$$x(k|k) \text{ } 263 \times 1$$

$$\Psi \text{ } 10 \times 263$$

$$\gamma \text{ } 10 \times 20$$

$$\Theta \text{ } 10 \times 120$$

$$Q \text{ } 10 \times 10$$

$$u(k-1) \text{ } 12 \times 12$$