Model Predictive Control of a Sewer System

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Group 1030

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Group 103

Implement

Linearicer

MPC

Resultat

Diskussion/Konklusion MF

Implementering

Kontrol

Linearisering MPC

Resultat

Diskussion/Konklusion



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Implementering

Kontrol Linearisering

Resultat

Diskussion/Konklusio

- Implementation
- ► Kontrol
- ► Resultater
- Diskussion
- ► Konklusion



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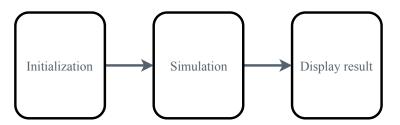
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Lineariserin MPC

Resultat

Diskussion/Konklusion





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Lineariseri

Resultat

Diskussion/Konklusion

1. Pipe

- ► længde [m]
- sektioner
- ► S_b (Hældning) [‰]
- ▶ ∆x = Længde/Sektioner [m]
- ▶ Diameter [m]
- ► Theta
- ▶ Q_f[m³/s]
- ► Side inflow
- ▶ Placering i data

2. Tank

- ► Størrelse [m³]
- ► Højde [m]
- ► Areal = Size / Height [m²]
- ► Maximum outflow [m³/s]
- Placering i data



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► Rør specifikationer

Fields	☐ length	⊞ sections	⊞ Dx	⊞ Sb	⊞ d	H Theta	⊞ Qf	■ side_inflow	data_location
1	700	35	20	0.0030	0.9000	0.6500	0.9730	0	1
2	303	15	20.2000	0.0030	0.9000	0.6500	0.9730	0	3
3	27	2	13.5000	0.0030	1	0.6500	1.2843	1	4
4	155	8	19.3750	0.0041	1	0.6500	1.5014	0	5
5	295	14	21.0714	0.0122	0.8000	0.6500	1.4386	0	6
6	318	15	21.2000	0.0053	0.9000	0.6500	1.2932	1	8



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► Tank specifikationer

Fields	= siz	e	Height	→ area	Q_out_max	data_location
1		90	10	9	0.9730	2
2		90	10	9	1.2932	7



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► System specifikationer

Fields	type type	component	sections
1	'Pipe'	1	35
2	'Tank'	1	1
3	'Pipe'	4	39
4	'Tank'	1	1
5	'Pipe'	14	206
6	'Total'	21	282



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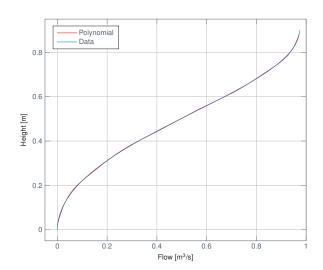
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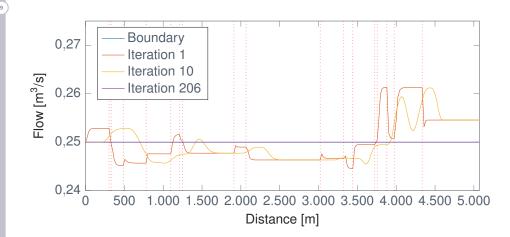
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Lineariseri

Resultat

Dickussion/Konklusia





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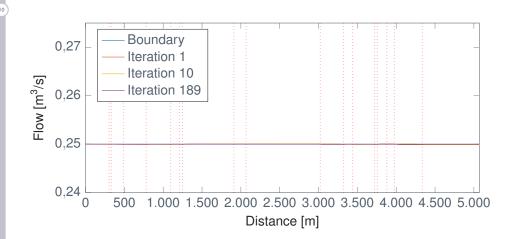
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Implementering

Lineariseri

Resultat

Diskussion/Konklusio





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Implementering

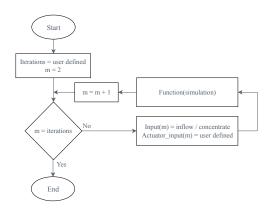
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Resulta

Diskussion/Konklusio

► Itererer igennem rør og tank for hvert tidsskridt





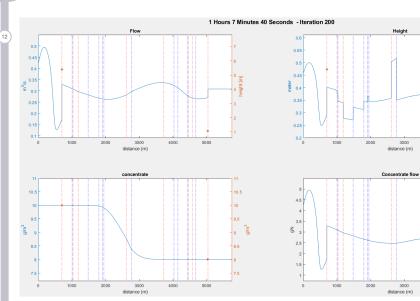
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Kontrol Lineariserin

Resultat

Diskussion/Konklusion



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Kontrol

Linearisering

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Diskussion/Konklusio

► Linearisering af ulineær model

► Opstilles på state space form

$$\frac{\partial A(x,t)}{\partial t} + \frac{\partial Q(x,t)}{\partial x} = 0$$

$$\frac{\partial A(h)}{\partial h} \frac{\partial h(x,t)}{\partial t} + \frac{\partial Q(h)}{\partial h} \frac{\partial h(x,t)}{\partial x} = 0$$



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► Priessmann scheme

► Opsat på matrix og vektor form

$$\begin{bmatrix} \frac{1}{2\Delta t} \frac{\partial A}{\partial h} - \frac{\theta}{\Delta x} \frac{\partial Q}{\partial h} & \underbrace{\frac{1}{2\Delta t} \frac{\partial A}{\partial h} + \frac{\theta}{\Delta x} \frac{\partial Q}{\partial h}}_{b} \end{bmatrix} \begin{bmatrix} h_{j+1}^{i+1} \\ h_{j+1}^{i+1} \end{bmatrix} = \\ - \begin{bmatrix} \frac{-1}{2\Delta t} \frac{\partial A}{\partial h} - \frac{(1-\theta)}{\Delta x} \frac{\partial Q}{\partial h} \\ c \end{bmatrix} & \underbrace{\frac{-1}{2\Delta t} \frac{\partial A}{\partial h} + \frac{(1-\theta)}{\Delta x} \frac{\partial Q}{\partial h}}_{d} \end{bmatrix} \begin{bmatrix} h_{j+1}^{i} \\ h_{j+1}^{i} \end{bmatrix}$$



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$$\underbrace{ \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & b_1 & 0 & \cdots & 0 \\ 0 & a_1 & b_2 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & a_{m-1} & b_m \end{bmatrix}}_{\xi} \underbrace{ \begin{bmatrix} h_0^{i+1} \\ h_1^{i+1} \\ h_2^{i+1} \\ \vdots \\ h_m^{i+1} \end{bmatrix}}_{\chi(k+1)} = \underbrace{ \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ c_0 & d_1 & 0 & \cdots & 0 \\ 0 & c_1 & d_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & c_{m-1} & d_m \end{bmatrix}}_{A} \underbrace{ \begin{bmatrix} h_0^i \\ h_1^i \\ h_2^i \\ \vdots \\ h_m^i \end{bmatrix}}_{\chi(k)} + \underbrace{ \begin{bmatrix} h_0^i \\ h_1^i \\ h_2^i \\ \vdots \\ h_m^i \end{bmatrix}}_{\chi(k)} + \underbrace{ \begin{bmatrix} h_0^i \\ h_1^i \\ h_2^i \\ \vdots \\ h_m^i \end{bmatrix}}_{\chi(k)} + \underbrace{ \begin{bmatrix} h_0^i \\ h_1^i \\ h_2^i \\ \vdots \\ h_m^i \end{bmatrix}}_{\chi(k)} + \underbrace{ \begin{bmatrix} h_0^i \\ h_1^i \\ h_2^i \\ \vdots \\ h_m^i \end{bmatrix}}_{\chi(k)} + \underbrace{ \begin{bmatrix} h_0^i \\ h_1^i \\ h_2^i \\ \vdots \\ h_m^i \end{bmatrix}}_{\chi(k)} + \underbrace{ \begin{bmatrix} h_0^i \\ h_1^i \\ h_2^i \\ \vdots \\ h_m^i \end{bmatrix}}_{\chi(k)} + \underbrace{ \begin{bmatrix} h_0^i \\ h_1^i \\ h_2^i \\ \vdots \\ h_m^i \end{bmatrix}}_{\chi(k)} + \underbrace{ \begin{bmatrix} h_0^i \\ h_1^i \\ h_2^i \\ \vdots \\ h_m^i \end{bmatrix}}_{\chi(k)} + \underbrace{ \begin{bmatrix} h_0^i \\ h_1^i \\ h_2^i \\ \vdots \\ h_m^i \end{bmatrix}}_{\chi(k)} + \underbrace{ \begin{bmatrix} h_0^i \\ h_1^i \\ h_2^i \\ \vdots \\ h_m^i \end{bmatrix}}_{\chi(k)} + \underbrace{ \begin{bmatrix} h_0^i \\ h_1^i \\ h_2^i \\ \vdots \\ h_m^i \end{bmatrix}}_{\chi(k)} + \underbrace{ \begin{bmatrix} h_0^i \\ h_1^i \\ h_2^i \\ \vdots \\ h_m^i \end{bmatrix}}_{\chi(k)} + \underbrace{ \begin{bmatrix} h_0^i \\ h_1^i \\ h_2^i \\ \vdots \\ h_m^i \end{bmatrix}}_{\chi(k)} + \underbrace{ \begin{bmatrix} h_0^i \\ h_1^i \\ h_2^i \\ \vdots \\ h_m^i \end{bmatrix}}_{\chi(k)} + \underbrace{ \begin{bmatrix} h_0^i \\ h_1^i \\ h_2^i \\ \vdots \\ h_m^i \end{bmatrix}}_{\chi(k)} + \underbrace{ \begin{bmatrix} h_0^i \\ h_1^i \\ h_2^i \\ \vdots \\ h_m^i \end{bmatrix}}_{\chi(k)} + \underbrace{ \begin{bmatrix} h_0^i \\ h_1^i \\ h_2^i \\ \vdots \\ h_m^i \end{bmatrix}}_{\chi(k)} + \underbrace{ \begin{bmatrix} h_0^i \\ h_1^i \\ h_2^i \\ \vdots \\ h_m^i \end{bmatrix}}_{\chi(k)} + \underbrace{ \begin{bmatrix} h_0^i \\ h_1^i \\ h_2^i \\ \vdots \\ h_m^i \end{bmatrix}}_{\chi(k)} + \underbrace{ \begin{bmatrix} h_0^i \\ h_1^i \\ h_2^i \\ \vdots \\ h_m^i \end{bmatrix}}_{\chi(k)} + \underbrace{ \begin{bmatrix} h_0^i \\ h_1^i \\ h_2^i \\ \vdots \\ h_m^i \end{bmatrix}}_{\chi(k)} + \underbrace{ \begin{bmatrix} h_0^i \\ h_1^i \\ h_2^i \\ \vdots \\ h_m^i \end{bmatrix}}_{\chi(k)} + \underbrace{ \begin{bmatrix} h_0^i \\ h_1^i \\ h_2^i \\ \vdots \\ h_m^i \end{bmatrix}}_{\chi(k)} + \underbrace{ \begin{bmatrix} h_0^i \\ h_1^i \\ h_2^i \\ \vdots \\ h_m^i \end{bmatrix}}_{\chi(k)} + \underbrace{ \begin{bmatrix} h_0^i \\ h_1^i \\ h_2^i \\ \vdots \\ h_m^i \end{bmatrix}}_{\chi(k)} + \underbrace{ \begin{bmatrix} h_0^i \\ h_1^i \\ h_2^i \\ \vdots \\ h_m^i \end{bmatrix}}_{\chi(k)} + \underbrace{ \begin{bmatrix} h_0^i \\ h_1^i \\ h_2^i \\ \vdots \\ h_m^i \end{bmatrix}}_{\chi(k)} + \underbrace{ \begin{bmatrix} h_0^i \\ h_1^i \\ h_2^i \\ \vdots \\ h_m^i \end{bmatrix}}_{\chi(k)} + \underbrace{ \begin{bmatrix} h_0^i \\ h_1^i \\ h_2^i \\ \vdots \\ h_m^i \end{bmatrix}}_{\chi(k)} + \underbrace{ \begin{bmatrix} h_0^i \\ h_1^i \\ h_2^i \\ \vdots \\ h_m^i \end{bmatrix}}_{\chi(k)} + \underbrace{ \begin{bmatrix} h_0^i \\ h_1^i \\ h_2^i \\ \vdots \\ h_m^i \end{bmatrix}}_{\chi(k)} + \underbrace{ \begin{bmatrix} h_0^i \\ h_1^i \\ h_2^i \\ \vdots \\ h_m^i \end{bmatrix}}_{\chi(k)} + \underbrace{ \begin{bmatrix} h_0^i \\ h_1^i \\ h_2^i \\ \vdots \\ h_m^i \end{bmatrix}}$$

$$\underbrace{\begin{bmatrix} 1 \\ -a_0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}}_{B} h_0^{i+1} + \underbrace{\begin{bmatrix} \frac{dh}{dQ} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}}_{B_d} d_0^{i+1}$$



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Kontrol

Linearisering MPC

Resultat

Diskussion/Konklusio

- e Inflow
- ► f Reducering af højden i tank
- ► g Højden i det efterfølge rør

$$\begin{bmatrix} b_{1,2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & a_{2,1} & b_{2,2} \end{bmatrix} \underbrace{\begin{bmatrix} h_{1,2}^{l+1} \\ h_{tank}^{l+1} \\ h_{2,0}^{l+1} \\ h_{2,1}^{l+1} \end{bmatrix}}_{\chi(k+1)}$$

$$= \underbrace{\begin{bmatrix} d_{1,2} & 0 & 0 & 0 \\ e & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & c_{2,0} & d_{2,1} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} h_{1,2}^{l} \\ h_{2,0}^{l} \\ h_{2,0}^{l} \\ h_{2,1}^{l} \end{bmatrix}}_{\chi(k)} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & -f \\ 0 & g \\ 0 & 0 \end{bmatrix}}_{B} \begin{bmatrix} h_{0}^{l+1} \\ u_{tank} \end{bmatrix}$$



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Implemente

Kontrol Linearisering

Lineariserin MPC

Resultat

Diekuseinn/Konklusia

- ► Samligning af ulineær og linear model
- ► System setup
- ► Sinus input

Туре	Components	Sections
Pipe	1	35
Tank	1	1
Pipe	18	227
Total	20	263





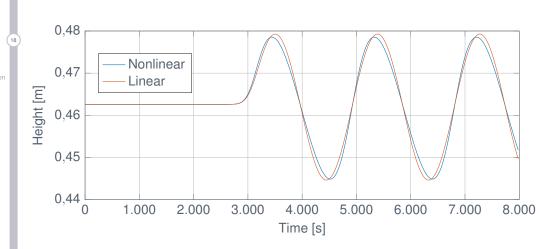
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Kontrol Linearisering

Resultat

Diskussion/Konklusio





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Kontrol

Lineariserin MPC

Resultat

Distance in a West March

- ► Cost function
 - ► Begrænset til minimiere af output
- ▶ Constraints
 - ► Højde
 - ► Kontrol input
- ► Linear model



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Implementer

Kontrol

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Resultat

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▶ Bestemmelse af Prediction horizon

- ▶ Flow profiler
- ► Industri
- ► Begrænsning af Prediction horizon
- ► System setup
- ► Forstyrrelse input

Fields	type type	component	⊞ sections
1	'Pipe'	1	5
2	'Tank'	1	1
3	'Pipe'	1	5
4	'Total'	3	11



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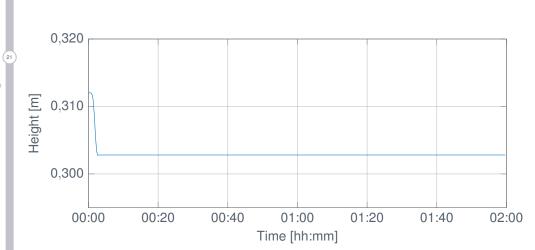
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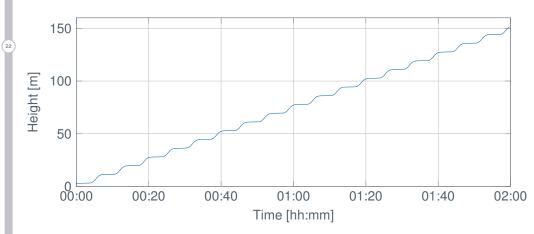
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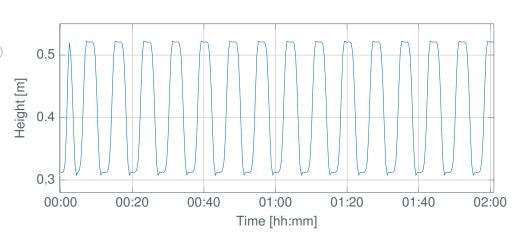
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Resultat

Diskussion/Konklusion

► System setup

► Flow profiler

Туре	Component	Sections
Pipe	1	35
Tank	1	1
Pipe	17	207
Tank	1	1
Pipe	1	38
Total	21	282



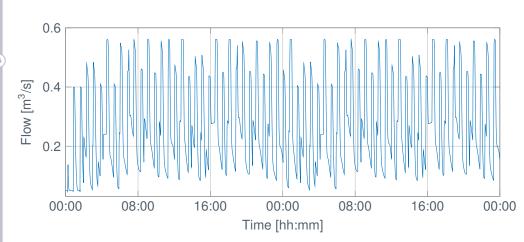
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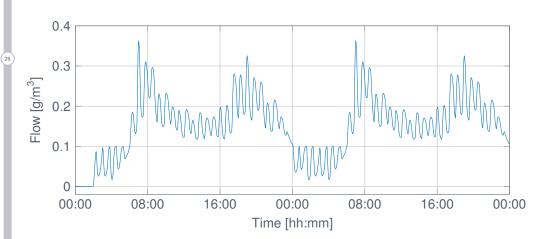
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Diskussion/Konklusion

- ► Over dimensioneret tank
- ► Konstant output af tank



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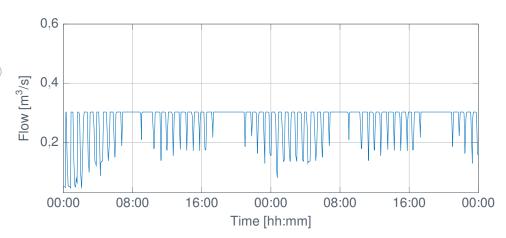
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- ► Courant's number
- ► Model reduction



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Diskussion/Konklusion 30

- ► Simulering
- ► MPC