DISCRETE-TIME NONLINEAR OPTIMAL CONTROL OF AN OPEN-CHANNEL HYDRAULIC SYSTEM

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Abstract: A nonlinear optimal control approach based on an implicit finite-dimensional discrete model is presented which determines the optimal opening of a regulator gate at the upstream end in the case of a single reach hydraulic system, in order to minimize the waste water and the variation of the water depth. The nonlinear discrete model is obtained from the well-known Preissmann finite-difference scheme. The effectiveness of the control algorithm is demonstrated with a simulation example. The simulation results show good improvements in reducing the variation of water depth compared to the uncontrolled case.

Keywords: Nonlinear optimal control, distributed parameter systems, open-channel hydraulic systems.

1. INTRODUCTION

Since 1983 optimal control has been introduced for solving the control problem of irrigation canals, which are governed by Saint-Venant equations. The majority of previous works have used LQR approaches based on a linearized finite-dimensional model. That means the Saint-Venant equations are reduced to a set of ordinary differential equations (ODE's) or difference equations (discrete system) by using approximation method (e.g. finite differences, finite elements,...) (Balogun *et al.*, 1988; Hubbard *et al.*, 1987; Reddy, 1996; Malaterre, 1994; Garcia *et al.*, 1992), and then these equations are linearized around an equilibrium. Few previous research works have used nonlinear control for the regulation of canals or rivers (Atanov *et al.*, 1998; Lin and Manz, 1992). Atanov et al. (1998) solved the problem of controlling water level variation in an open channel by using both an infinite-dimensional model and nonlinear optimal control theory. Lin and Manz (1992) have used a linear optimal control in a simulation program based on a nonlinear model.

For many hydraulic engineers, the Preissmann difference scheme (Preissmann, 1963) is an effective numeric method for solving the Saint-Venant equations. There exist many research works which demonstrate the stability and others numerical properties of this scheme (Malaterre, 1994; Cunge, 1966; Cunge *et al.*, 1980). There are also some irrigation simulators (for example SIC developed by the french research center CEMAGREF) which are based on this integration scheme.In this study, we use the Preissmann scheme to obtain a nonlinear finite-dimensional discrete model without using linearization. Then we use nonlinear optimal control theory to deal with this multi-variable discrete system. The main problem here is that the finite-dimensional difference model obtained is implicit.

In this paper, a simple example of a single reach, which is bounded by a underflow gate at the upstream end and a pumping station at the downstream end, is studied (see Fig 1.). The control goal is to determine the optimal opening of the gate in order to minimize the waste water and the variation of the water level distributed along the channel.

2. SYSTEM MODELLING

2.1 *An open-channel hydraulic system*

Throughout this paper, channel flow is assumed to be one-dimensional and unsteady. Flow dynamics is governed by the well-known Saint-Venant equations. These equations describe the open-channel flow dynamics with a set of two coupled first-order nonlinear hyperbolic partial differential equations with two variables: the water level z and the discharge Q at any given canal cross section (see Fig 1.).

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0$$
$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}(\frac{Q^2}{A}) + gA\frac{\partial z}{\partial x} - gA(I - J) = 0 \quad (1)$$

where: z = flow depth,(m); Q=discharge, ($\frac{m^3}{s}$); A = cross sectional flow area, (m^2); I= canal slop, ($\frac{m}{m}$);g=acceleration of gravity, ($\frac{m}{s^2}$); x=distance, (m); t=times, (s); and J=friction slope ($\frac{m}{m}$) which can be evaluated using the Manning equation as follows:

$$J = \frac{\mathcal{M}^2 Q^2}{AR^{\frac{4}{3}}} \quad (2)$$

in which M = Manning friction coefficient,($m^{\frac{1}{6}}$) and R = hydraulic radius, (m). For a canal with a rectangular section, the hydraulic radius will be given by

$$R = \frac{A}{b + 2z} \quad (3)$$

The above equations have been obtained under the following hypotheses: (1) the slope I of the canal bottom is small enough to ensure that the relation $\sin I = I$ holds; (2) the flow of the canal is one-dimensional; (3) the density of the fluid is constant; (4) the pressure in the cross section perpendicular to the flow is hydrostatic; (5) the fluid properties are averaged over the cross section; (6) the internal viscosity is neglected (Chow, 1985).

2.2 *The Preissmann scheme*

The Preissmann scheme (Preissmann, 1963) is an implicit conditionally-stable scheme used for solving Saint-Venant equations. If we consider a function $f(x, t)$ and its gradients, the Preissmann scheme is a finite-difference method similar to the Crank-Nicholson method, which is defined as follows:

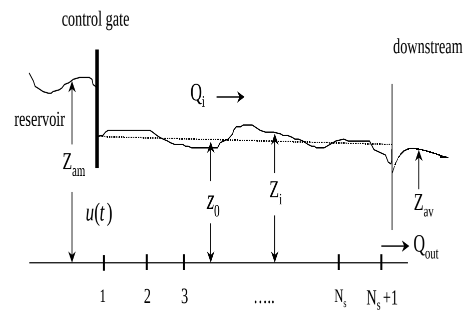

Fig. 1. Geometry of a reach of an open-channel

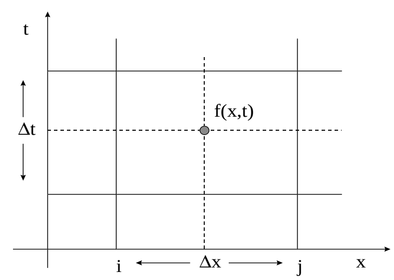

Fig. 2. Notation of the Preissmann scheme

$$\begin{aligned}
f(x,t) &= \frac{1-\theta}{2}[f(x+\Delta x, t) + f(x,t)] \\
&+ \frac{\theta}{2}[f(x+\Delta x, t+\Delta t) + f(x, t+\Delta t)] \\
&= \frac{1-\theta}{2}[f_{i+1}^k + f_i^k] + \frac{\theta}{2}[f_{i+1}^{k+1} + f_i^{k+1}] \\
\frac{\partial f}{\partial x}(x,t) &= \frac{1-\theta}{\Delta x}[f(x+\Delta x, t) - f(x,t)] \\
&+ \frac{\theta}{\Delta x}[f(x+\Delta x, t+\Delta t) - f(x, t+\Delta t)] \\
&= \frac{1-\theta}{\Delta x}[f_{i+1}^k - f_i^k] + \frac{\theta}{\Delta x}[f_{i+1}^{k+1} - f_i^{k+1}] \\
\frac{\partial f}{\partial t}(x,t) &= \frac{1}{2\Delta t}[f(x, t+\Delta t) - f(x,t) \\
&+ f(x+\Delta x, t+\Delta t) - f(x+\Delta x, t)] \\
&= \frac{1}{2\Delta t}[f_i^{k+1} - f_i^k + f_{i+1}^{k+1} - f_{i+1}^k] \qquad (4)
\end{aligned}$$

where i and k are the space and time indices, respectively. θ is a relaxation coefficient defined between 0 and 1.(see Fig 2.) The stability and others properties of this scheme have been discussed in many previous works (Malaterre, 1994; Cunge, 1966; Cunge *et al.*, 1980). In our case, the stability condition is $0.5 \le \theta \le 1$.

Applying this method to equations (1) leads to the following nonlinear implicit expressions:

$$F_i(X^{k+1}, X^k, \Delta t, \Delta x, \theta) = 0$$
$$G_i(X^{k+1}, X^k, \Delta t, \Delta x, \theta) = 0 \quad (5)$$

where $i = 1, \ldots, N_s$, $k = 1, \ldots, N_t, X = [Q_1, z_1, \ldots, Q_{N_s+1}, z_{N_s+1}]^T$, $(N_s+1)\Delta x = L$,and $(N_t+1)\Delta t = T$. When the above equations are applied to the N_s measure points along a reach of channel, a total of $2N_s$ nonlinear equations in $2(N_s+1)$ unknowns are obtained.(see Fig 1.) To complete the formulation, equations describing the upstream and downstream boundary conditions $Q_1 = Q(0,t)$ and $Q_{N_s+1} = Q(L,t)$ must be specified.

2.3 *The boundary conditions*

The problem is subject to the following initial conditions:

$$\begin{cases} Q_i^1 = \phi_i \\ z_i^1 = \varphi_i \quad for \quad i = 1, \ldots, N_s \end{cases} \quad (6)$$

in which ϕ_i and φ_i are initial vectors that be assumed known.

As shown in Fig (1)., the canal is bounded on both ends by an adjustable underflow gate at the upstream end and by a pumping station at the downstream end, respectively. The discharge of the underflow gate can be derived from the energy equation:

$$Q_1^k = Q(0,t) = \mathcal{K}u(t)\sqrt{2g(z_{us} - z_{ds})} \quad (7)$$

where $\mathcal{K}$ represents the product of the discharge coefficient and the length of the gate; the gate opening u is a function of time; z_{us} and z_{ds} are the upstream and downstream water level at the gate, respectively. In our case, z_{us} is the water depth of the reservoir z_{am}, and z_{ds} is specified as another boundary value z_1.

On the other hand we assume that the water demand (discharge) at the downstream end is Q_{out}. So the discharge at the downstream end $Q(L,t)$ will be given by energy equation

$$z_{N_s+1} + \frac{\beta(\frac{Q_{Ns+1}}{bz_{Ns+1}})^2}{2g} = z_{av} + \frac{\beta(\frac{Q_{out}}{bz_{av}})^2}{2g} \quad (8)$$

in which β is discharge coefficient at downstream and z_{av} is water level of next reach. Finally, system (5) is subject to the following boundary conditions:

$$Q_1^k(1-\theta) + Q_1^{k+1}\theta = \mathcal{K}u^k\sqrt{2g[z_{am} - z_1^k(1-\theta) - z_1^{k+1}\theta]}$$
$$Q_{Ns+1}^k(1-\theta) + Q_{Ns+1}^{k+1}\theta = b[z_{N_s+1}^k(1-\theta) + z_{Ns+1}^{k+1}\theta]$$
$$\sqrt{(\frac{Q_{out}}{bz_{av}})^2 + \frac{2g}{\beta}[z_{av} - z_{N_s+1}^k(1-\theta) - z_{Ns+1}^{k+1}\theta]}$$
$$for \quad k = 1, \ldots, N_t \quad (9)$$

in which z_1 and z_{N_s+1} are not specified, but z_{av} should be specified.

2.4 *Statement of the optimal control problem*

Our goal is now to find an optimal gate opening u^* in order to minimize the following cost function:

$$\mathcal{J} = \frac{1}{2}\sum_{k=1}^{N_t}[(u^k)^2 + \sum_{i=1}^{N_s} p(z_i^k - z_0)^2] \quad (10)$$

where z_0 is the desired reference water depth and p is a weighting coefficient. And $(N_t - 1)\Delta t = T$ is the specified final time. Combining water flow dynamics (5) and boundary conditions (9), we have the whole system expressed as a nonlinear implicit finite-dimensional model:

$$\begin{cases} B_0(z_1^k, z_1^{k+1}, Q_1^k, Q_1^{k+1}, u^k) = 0 \\ F_i(X^{k+1}, X^k) = 0 \\ G_i(X^{k+1}, X^k) = 0 \\ B_L(z_{Ns+1}^k, z_{Ns+1}^{k+1}, Q_{Ns+1}^k, Q_{Ns+1}^{k+1}) = 0 \end{cases} \quad (11)$$

in which $i = 1, \ldots, N_s$ and $k = 1, \ldots, N_t$.

The Lagrange multiplier method is an efficient way to deal with this kind of problem. The Lagrangian functional will incorporate the cost functional and any additional constraints of the problem. Accordingly, we obtain a new cost functional defined as follows:

$$\begin{aligned} &\mathcal{L}(X, \lambda_0, \lambda_L, \lambda_i, \nu_i, u) \\ &= \mathcal{J} + \sum_{k=1}^{N_t}[\lambda_0^{k+1}B_0 + \lambda_L^{k+1}B_L] \\ &+ \sum_{k=1}^{N_t}[\sum_{i=1}^{N_s}\lambda_i^{k+1}F_i + \sum_{i=1}^{N_s}\nu_i^{k+1}G_i] \\ &= \sum_{k=1}^{N_t}\{[\frac{1}{2}(u^k)^2 + \lambda_0^{k+1}B_0 + \lambda_L^{k+1}B_L] \\ &+ \sum_{i=1}^{N_s}[\frac{p}{2}(z_i^k - z_0)^2 + \lambda_i^{k+1}F_i + \nu_i^{k+1}G_i]\} \end{aligned} \quad (12)$$

where λ_i, ν_i ,λ_0 and λ_L are the Lagrangian multipliers vectors. According to Lagrange's theory, the problem now is to minimize this Lagrangian functional without considering the constraints of the system.

2.5 *The adjoint system of the problem*

A sufficient condition for X to be a minimizer of the cost function is that X and λ, ν are a saddle point for $\mathcal{L}$. Then we can deduce the optimality system from the Lagrange conditions. So we have the formulation for the adjoint variable λ_0, λ_L, λ_i and ν_i:

$$\frac{\partial \mathcal{L}}{\partial X^k} = R_j(X^k, \lambda_0^k, \lambda_L^k, \lambda_i^k, \nu_i^k, \lambda_i^{k+1}, \nu_i^{k+1}, u^k) = 0$$
$$for \quad j = 1, \ldots, 2(N_s + 1) \tag{13}$$

with the final time conditions:

$$\begin{cases} \lambda_i^k = 0 \\ \nu_i^k = 0 \\ for \quad k = (N_t + 1), \quad i = 1, \ldots, N_s \end{cases} \tag{14}$$

The gradient of the Lagrangian functional is given by

$$\frac{\partial \mathcal{L}}{\partial u^k} = S(X^k, \lambda_0^k, \lambda_L^k, \lambda_i^k, \nu_i^k, \lambda_i^{k+1}, \nu_i^{k+1}, u^k) = 0$$
$$for \quad i = 1, \ldots, N_s \tag{15}$$

According to the preceding equations, the adjoint system of the system (11) can be written as follows:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial Q_1^k} &= \lambda_0^{k+1}(1-\theta) + \lambda_0^k \theta + \frac{\partial}{\partial Q_1^k} \\
&\quad (\lambda_1^{k+1} F_1^k + \lambda_1^k F_1^{k-1} + \nu_1^{k+1} G_1^k + \nu_1^k G_1^{k-1}) \\
\frac{\partial \mathcal{L}}{\partial z_1^k} &= p(z_1^k - z_0) + \lambda_0^{k+1} \frac{\partial B_0^k}{\partial z_1^k} + \lambda_0^k \frac{\partial B_0^{k-1}}{\partial z_1^k} \\
&\quad + \frac{\partial}{\partial z_1^k}(\lambda_1^{k+1} F_1^k + \lambda_1^k F_1^{k-1} + \nu_1^{k+1} G_1^k + \nu_1^k G_1^{k-1}) \\
\frac{\partial \mathcal{L}}{\partial Q_i^k} &= \frac{\partial}{\partial Q_i^k}(\lambda_{i-1}^{k+1} F_{i-1}^k + \lambda_{i-1}^k F_{i-1}^{k-1} \\
&\quad + \nu_{i-1}^{k+1} G_{i-1}^k + \nu_{i-1}^k G_{i-1}^{k-1}) \\
&\quad + \frac{\partial}{\partial Q_i^k}(\lambda_i^{k+1} F_i^k + \lambda_i^k F_i^{k-1} + \nu_i^{k+1} G_i^k + \nu_i^k G_i^{k-1}) \\
\frac{\partial \mathcal{L}}{\partial z_i^k} &= p(z_i^k - z_0) + \frac{\partial}{\partial z_i^k}(\lambda_{i-1}^{k+1} F_{i-1}^k \\
&\quad + \lambda_{i-1}^k F_{i-1}^{k-1} + \nu_{i-1}^{k+1} G_{i-1}^k + \nu_{i-1}^k G_{i-1}^{k-1}) \\
&\quad + \frac{\partial}{\partial z_i^k}(\lambda_i^{k+1} F_i^k + \lambda_i^k F_i^{k-1} + \nu_i^{k+1} G_i^k + \nu_i^k G_i^{k-1}) \\
\frac{\partial \mathcal{L}}{\partial Q_{N_s+1}^k} &= \lambda_L^{k+1}(1-\theta) + \lambda_L^k \theta + \frac{\partial}{\partial Q_{N_s+1}^k} \\
&\quad (\lambda_{N_s}^{k+1} F_{N_s}^k + \lambda_{N_s}^k F_{N_s}^{k-1} + \nu_{N_s}^{k+1} G_{N_s}^k + \nu_{N_s}^k G_{N_s}^{k-1}) \\
\frac{\partial \mathcal{L}}{\partial z_{N_s+1}^k} &= \lambda_L^{k+1} \frac{\partial B_L^k}{\partial z_{N_s+1}^k} + \lambda_L^k \frac{\partial B_L^{k+1}}{\partial z_{N_s+1}^k} + \frac{\partial}{\partial z_{N_s+1}^k} \\
&\quad (\lambda_{N_s}^{k+1} F_{N_s}^k + \lambda_{N_s}^k F_{N_s}^{k-1} + \nu_{N_s}^{k+1} G_{N_s}^k + \nu_{N_s}^k G_{N_s}^{k-1})
\end{aligned} \tag{16}$$

where $i = 2, \ldots, N_s$. And the nominal control should satisfy the stationary condition.

$$\frac{\partial \mathcal{L}}{\partial u^k} = u^k + \lambda_0 \frac{\partial B_0}{\partial u^k} = 0 \tag{17}$$

3. NUMERICAL SOLUTION

The optimal control input u depends on λ_0 (the state of the adjoint system) and boundary condition at upstream end. For that reason, the system (11) and the preceding adjoint system have to be computed for $1 \leq k \leq N_t + 1$ and $1 \leq i \leq N_s + 1$. But the above two systems cannot be computed together, because these equations subject to the initial condition (6) and final condition (14), respectively. The following iterative procedure can be used to solve this two-point boundary value problem.

S(1): Guess an initial control sequence $u^0(k), k = 1, \ldots, N_t + 1$.

S(2): Solve the system equations (11) forward from $k = 1$ to $k = N_t + 1$ with the specified initial conditions (6). Record the solution Q_i and z_i.

S(3): Determine the adjoint state λ_0, λ_L, λ_i and $\nu_i$from a backward integration of the adjoint system (16) from $k = N_t + 1$ to $k = 1$, using Q_i and z_i obtained from S(2).

S(4): Compute the gradient of the cost functional (17) using the solutions of S(2) and S(3).

S(5): Repeat S(2) through S(4) using an updated estimate of u which can be determined by

$$u^{(i+1)}(k) = u^{(i)}(k) - \alpha(k) S(u^{(i)})$$
$$k = 1, \ldots, Nt \tag{18}$$

where i is the iteration index, the direction $S(u^{(i)})$ is chosen in order to reduce the cost functional value. this algorithm updates control vector with an optimal step length $\alpha(i) > 0$ (Reklaitis and Ragsdell, 1983). There are many ways to choose $S(u^{(i)})$ and $\alpha(i)$. In this study, we choose α is variable and depend on the gradient of cost function and

$$S(u^{(i)}) = \frac{\partial \mathcal{L}^{(i)}}{\partial u}(k) \quad k = 1, \ldots, Nt \tag{19}$$

S(6): This iteration process terminates when $S(u^{(i)})$ less than some specified tolerance for any $1 \leq k \leq N_t + 1$.

4. SIMULATION RESULTS

We consider a single reach open-channel system as show in Fig.1. The input flow is given by an underflow gate at the upstream end. At the downstream end, a pumping station is introduced to present the water demand. The control problem is to adjust the gate opening in order to minimize the water level deviation from the desired value and also to satisfy the discharge demand at the downstream end. The geometric parameters of the canal are: length $L = 30km$, width $b = 5m$, bed slope $I = 0.001$, manning friction coefficient $M = 0.4$, width of the gate $b_g = b - 0.5m$, min-max gate opening 0.01 $7m$, water level of reservoir $z_{am} = 7m$, water level of downstream $z_{av} = 3m$. The simulation parameters are: space

Table 1 Summary of the iteration procedure

Initial guess $u^0(t)$	Initial step length α	Number of iterations	Minimum cost value $\mathcal{J}^*$
5	0.2	40	0.9423
5	0.5	13	0.9273

step $dx = 5km$, time step $dt = 20min$, coefficient of Preissmann $\theta = 0.75$, pumping station demand $Q_{out} = 10\frac{m^3}{s}$, initial discharge $\phi_i = 18\frac{m^3}{s}$, initial water level $\varphi_i = 3.4m$ the desired water level $z_0 = 3.47m$ and the weighting coefficient $p = 10$. In Fig. 3 and Fig. 4, we show the simulation results with constant gate opening. As shown in Fig. 5 and Fig. 6, the water variation is much small than the preceding case compared with the desired water level $z_0 = 3.47m$, and the discharges at downstream are also enough to provide for Q_{out}. The Fig. 7 illustrate the convergence of the cost values. The cost values are much reduced from 230 for the case with constant gate opening to 0.9423 with control. The iteration procedure terminates when $\mid J^{k+1} - J^k \mid \leq 0.001$ which is decided by perform several trial computations of the same problem.

To illustrate the effects of various iteration step sizes and different initial guesses for this problem, we summarized some results of computation runs in Table[1]. The simulation program is written in MATLAB. In order to solve the implicit equations 5, we use a nonlinear programming algorithm based on the Gauss-Newton method with a mixed quadratic and cubic line search procedure.

5. CONCLUSIONS

In this paper we propose a nonlinear optimal control approach for the regulation of an open channel based on an implicit finite-dimensional discrete model. This model is obtained from the well-known implicit Preissmann finite-difference scheme. The control goal is to determine the optimal opening of a regulator gate at the upstream end in order to minimize the waste water and the variation of the water depth. The problem solution is based on the computation of the associated adjoint model which is used to compute the gradient of cost function.

The simulation results demonstrates some good improvements in reducing the variation of water depth compared to the uncontrolled case.

We should mention here that the here-proposed optimal control is an open-loop control law. That means that the optimal control law depends on initial conditions. However, this approach can be the main part of predictive control scheme based on a receding horizon, which is actually a closed-loop control law.

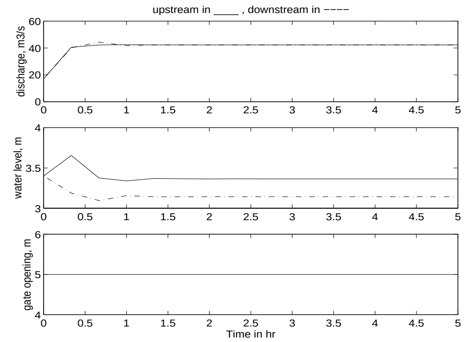

Fig. 3. Discharge, water level and control gate opening without control ($p = 10$)

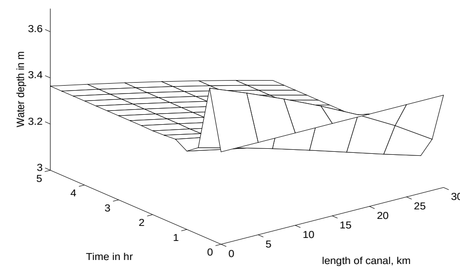

Fig. 4. System behavior without control ($z_0 = 3.47m$)

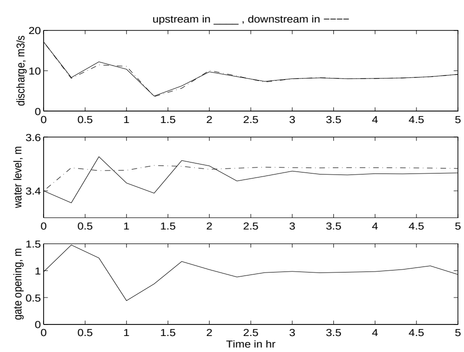

Fig. 5. Discharge, water level and control gate opening with control

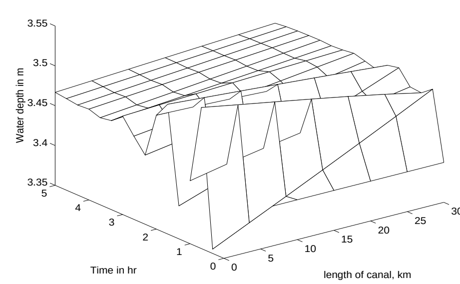

Fig. 6. System behavior with control ($z_0 = 3.47m$ and $p = 10$)

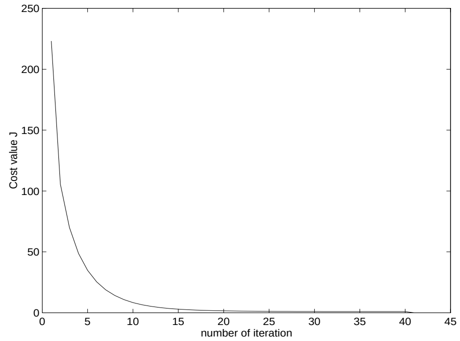

Fig. 7. Convergence results ($\alpha(k) = 0.2$)

6. APPENDIX

Here we derive the Preissmann scheme expressions:

$$\begin{aligned}
F_i^k &= \frac{b}{2\Delta t}[z_i^{k+1} - z_i^k + z_{i+1}^{k+1} - z_{i+1}^k] \\
&+ \frac{\theta}{\Delta x}[Q_{i+1}^{k+1} - Q_i^{k+1}] + \frac{1-\theta}{\Delta x}[Q_{i+1}^k - Q_i^k] \\
G_i^k &= \frac{1}{2\Delta t}[Q_i^{k+1} - Q_i^k + Q_{i+1}^{k+1} - Q_{i+1}^k] \\
&+ \frac{\theta}{\Delta x}[\frac{(Q_{i+1}^{k+1})^2}{bz_{i+1}^{k+1}} - \frac{(Q_i^{k+1})^2}{bz_i^{k+1}}] \\
&+ \frac{1-\theta}{\Delta x}[\frac{(Q_{i+1}^k)^2}{bz_{i+1}^k} - \frac{(Q_i^k)^2}{bz_i^k}] \\
&+ \frac{gb}{2}\{\frac{1-\theta}{\Delta x}[(z_{i+1}^k)^2 - (z_i^k)^2] \\
&+ \frac{\theta}{\Delta x}[(z_{i+1}^{k+1})^2 - (z_i^{k+1})^2]\} \\
&- gbI\{\frac{1-\theta}{2}[z_{i+1}^k + z_i^k] + \frac{\theta}{2}[z_{i+1}^{k+1} + z_i^{k+1}]\} \\
&+ \frac{gM^2}{R^{\frac{4}{3}}}\{\frac{1-\theta}{2}[(Q_{i+1}^k)^2 + (Q_i^k)^2] \\
&+ \frac{\theta}{2}[(Q_{i+1}^{k+1})^2 + (Q_i^{k+1})^2]\} \qquad (20)
\end{aligned}$$

and the partial derivatives of the adjoint system (16)

$$\begin{aligned}
\frac{\partial F_i^k}{\partial Q_i^k} &= -\frac{1-\theta}{\Delta x} \qquad \frac{\partial F_i^{k-1}}{\partial Q_i^k} = -\frac{\theta}{\Delta x} \\
\frac{\partial F_{i-1}^k}{\partial Q_i^k} &= \frac{1-\theta}{\Delta x} \qquad \frac{\partial F_{i-1}^{k-1}}{\partial Q_i^k} = \frac{\theta}{\Delta x} \\
\frac{\partial F_i^k}{\partial z_i^k} &= -\frac{b}{2\Delta t} \qquad \frac{\partial F_i^{k-1}}{\partial z_i^k} = \frac{b}{2\Delta t} \\
\frac{\partial F_{i-1}^k}{\partial z_i^k} &= -\frac{b}{2\Delta t} \qquad \frac{\partial F_{i-1}^{k-1}}{\partial z_i^k} = \frac{b}{2\Delta t} \\
\frac{\partial G_i^k}{\partial Q_i^k} &= -\frac{1}{2\Delta t} - \frac{2(1-\theta)}{\Delta x}\frac{Q_i^k}{bz_i^k} + \frac{gM^2}{R^{\frac{4}{3}}}(1-\theta)Q_i^k \\
\frac{\partial G_i^{k-1}}{\partial Q_i^k} &= \frac{1}{2\Delta t} - \frac{2\theta}{\Delta x}\frac{Q_i^k}{bz_i^k} + \frac{gM^2}{R^{\frac{4}{3}}}\theta Q_i^k \\
\frac{\partial G_{i-1}^k}{\partial Q_i^k} &= -\frac{1}{2\Delta t} + \frac{2(1-\theta)}{\Delta x}\frac{Q_i^k}{bz_i^k} + \frac{gM^2}{R^{\frac{4}{3}}}(1-\theta)Q_i^k \\
\frac{\partial G_{i-1}^{k-1}}{\partial Q_i^k} &= \frac{1}{2\Delta t} - \frac{2\theta}{\Delta x}\frac{Q_i^k}{bz_i^k} + \frac{gM^2}{R^{\frac{4}{3}}}\theta Q_i^k \\
\frac{\partial G_i^k}{\partial z_i^k} &= \frac{1-\theta}{b\Delta x}(\frac{Q_i^k}{z_i^k})^2 - gb\frac{1-\theta}{\Delta x}z_i^k - gbI\frac{1-\theta}{2} \\
\frac{\partial G_i^{k-1}}{\partial z_i^k} &= \frac{\theta}{b\Delta x}(\frac{Q_i^k}{z_i^k})^2 - gb\frac{\theta}{\Delta x}z_i^k - gbI\frac{\theta}{2} \\
\frac{\partial G_{i-1}^k}{\partial z_i^k} &= -\frac{1-\theta}{b\Delta x}(\frac{Q_i^k}{z_i^k})^2 + gb\frac{1-\theta}{\Delta x}z_i^k - gbI\frac{1-\theta}{2} \\
\frac{\partial G_{i-1}^{k-1}}{\partial z_i^k} &= -\frac{\theta}{b\Delta x}(\frac{Q_i^k}{z_i^k})^2 + gb\frac{\theta}{\Delta x}z_i^k - gbI\frac{\theta}{2} \qquad (21)
\end{aligned}$$

7. REFERENCES

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