Model Predictive Control of a Sewer System

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Group 1030

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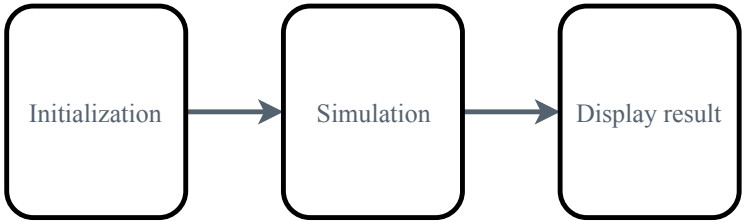
Agenda

- Implementering
- Kontrol
  - Linearisering
  - MPC
- Resultat
- Diskussion/Konklusion

- Implementation
- Kontrol
- Resultater
- Diskussion
- Konklusion

Implementering



1. Pipe
   - længde [m]
   - sektioner
   - S_b (Hældning) [‰]
   - Δx = Længde/Sektioner [m]
   - Diameter [m]
   - Theta
   - $Q_f[m^3/s]$
   - Side inflow
   - Placering i data
2. Tank
   - Størrelse $[m^3]$
   - Højde [m]
   - Areal = Size / Height $[m^2]$
   - Maximum outflow $[m^3/s]$
   - Placering i data

- Rør specifikationer

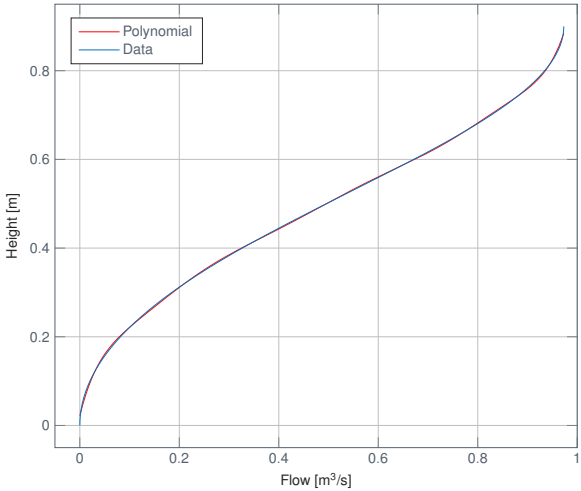
Fields	length	sections	Dx	Sb	d	Theta	Qf	side_inflow	data_location
1	700	35	20	0.0030	0.9000	0.6500	0.9730	0	1
2	303	15	20.2000	0.0030	0.9000	0.6500	0.9730	0	3
3	27	2	13.5000	0.0030	1	0.6500	1.2843	1	4
4	155	8	19.3750	0.0041	1	0.6500	1.5014	0	5
5	295	14	21.0714	0.0122	0.8000	0.6500	1.4386	0	6
6	318	15	21.2000	0.0053	0.9000	0.6500	1.2932	1	8

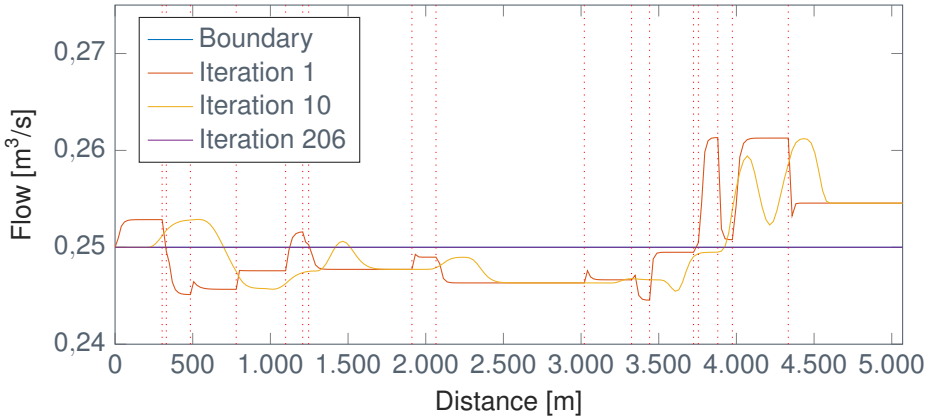
- Tank specifikationer

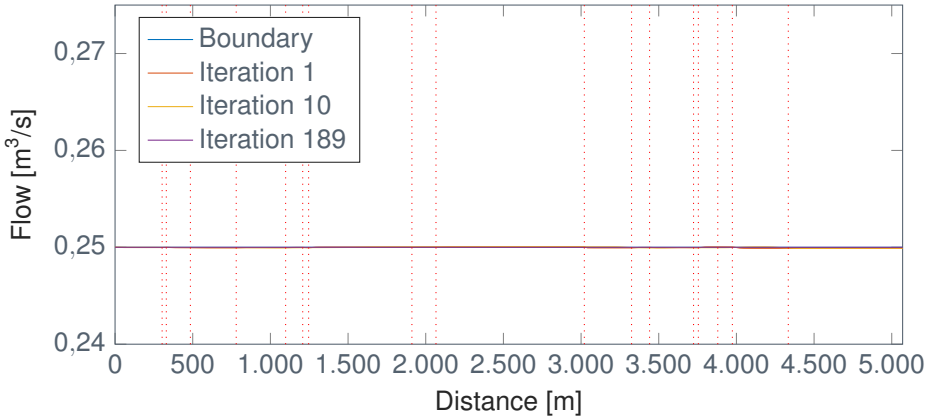
Fields	size	height	area	Q_out_max	data_location
1	90	10	9	0.9730	2
2	90	10	9	1.2932	7

- System specifikationer

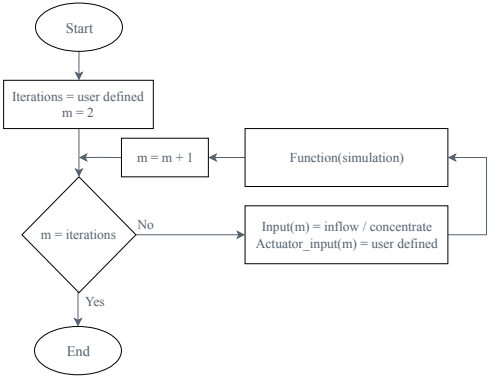
Fields	type	component	sections
1	'Pipe'	1	35
2	'Tank'	1	1
3	'Pipe'	4	39
4	'Tank'	1	1
5	'Pipe'	14	206
6	'Total'	21	282

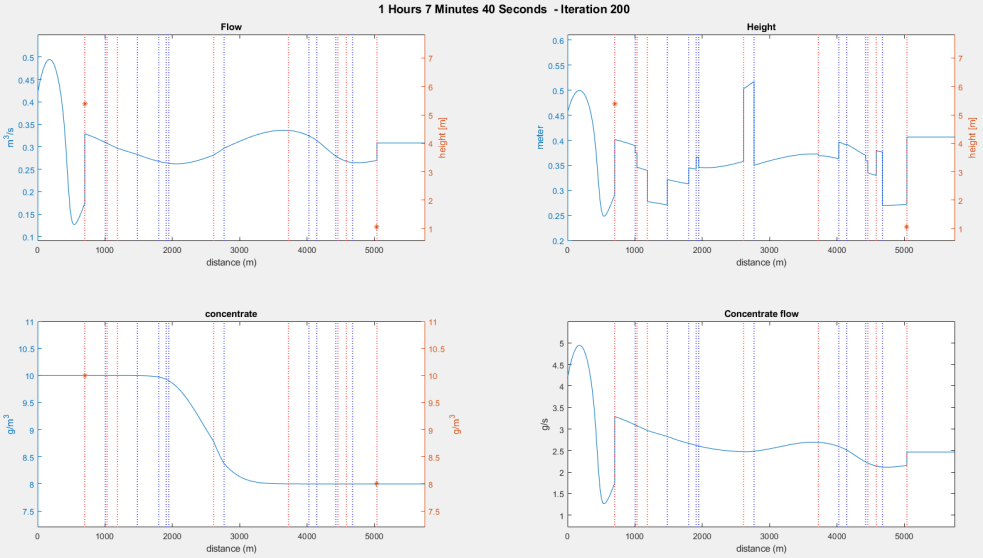






- Itererer igennem rør og tank for hvert tidsskridt





- Linearisering af ulineær model
- Opstilles på state space form

$$\frac{\partial A(x,t)}{\partial t} + \frac{\partial Q(x,t)}{\partial x} = 0$$

$$\frac{\partial A(h)}{\partial h}\frac{\partial h(x,t)}{\partial t} + \frac{\partial Q(h)}{\partial h}\frac{\partial h(x,t)}{\partial x} = 0$$

- Priessmann scheme
- Opsat på matrix og vektor form

$$\underbrace{\begin{bmatrix} \frac{1}{2\Delta t}\frac{\partial A}{\partial h} - \frac{\theta}{\Delta x}\frac{\partial Q}{\partial h} & \frac{1}{2\Delta t}\frac{\partial A}{\partial h} + \frac{\theta}{\Delta x}\frac{\partial Q}{\partial h} \end{bmatrix}}_{a\quad b} \begin{bmatrix} h_j^{i+1} \\ h_{j+1}^{i+1} \end{bmatrix} =$$

$$- \underbrace{\begin{bmatrix} \frac{-1}{2\Delta t}\frac{\partial A}{\partial h} - \frac{(1-\theta)}{\Delta x}\frac{\partial Q}{\partial h} & \frac{-1}{2\Delta t}\frac{\partial A}{\partial h} + \frac{(1-\theta)}{\Delta x}\frac{\partial Q}{\partial h} \end{bmatrix}}_{c\quad d} \begin{bmatrix} h_j^{i} \\ h_{j+1}^{i} \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & b_1 & 0 & \cdots & 0 \\ 0 & a_1 & b_2 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & a_{m-1} & b_m \end{bmatrix}}_{\xi} \underbrace{\begin{bmatrix} h_0^{i+1} \\ h_1^{i+1} \\ h_2^{i+1} \\ \vdots \\ h_m^{i+1} \end{bmatrix}}_{x(k+1)} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ c_0 & d_1 & 0 & \cdots & 0 \\ 0 & c_1 & d_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & c_{m-1} & d_m \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} h_0^{i} \\ h_1^{i} \\ h_2^{i} \\ \vdots \\ h_m^{i} \end{bmatrix}}_{x(k)} + \underbrace{\begin{bmatrix} 1 \\ -a_0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}}_{B} h_0^{i+1} + \underbrace{\begin{bmatrix} \frac{dh}{dQ} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}}_{B_d} d_0^{i+1}$$

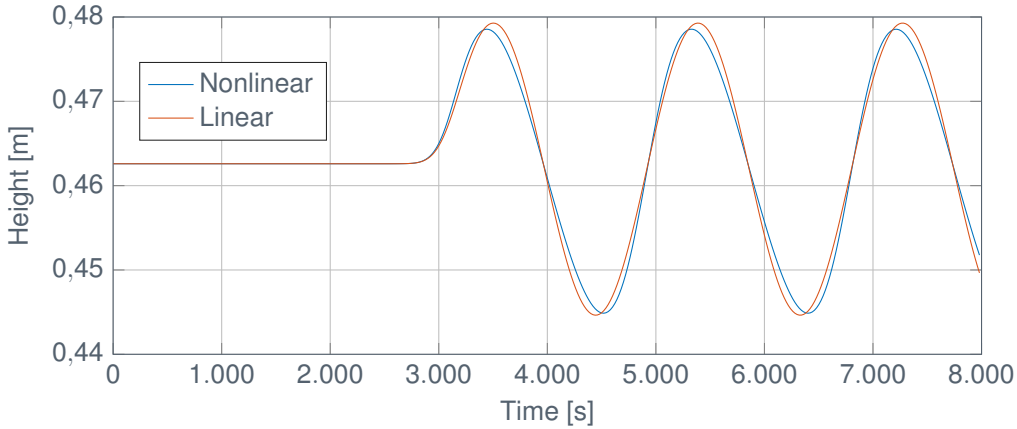
- e - Inflow
- f - Reducering af højden i tank
- g - Højden i det efterfølge rør

$$\underbrace{\begin{bmatrix} b_{1,2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & a_{2,1} & b_{2,2} \end{bmatrix}}_{\xi} \underbrace{\begin{bmatrix} h_{1,2}^{i+1} \\ h_{tank}^{i+1} \\ h_{2,0}^{i+1} \\ h_{2,1}^{i+1} \end{bmatrix}}_{x(k+1)}$$

$$= \underbrace{\begin{bmatrix} d_{1,2} & 0 & 0 & 0 \\ e & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & c_{2,0} & d_{2,1} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} h_{1,2}^{i} \\ h_{tank}^{i} \\ h_{2,0}^{i} \\ h_{2,1}^{i} \end{bmatrix}}_{x(k)} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & -f \\ 0 & g \\ 0 & 0 \end{bmatrix}}_{B} \begin{bmatrix} h_{0}^{i+1} \\ u_{tank} \end{bmatrix}$$

- Samligning af ulineær og linear model
- System setup
- Sinus input

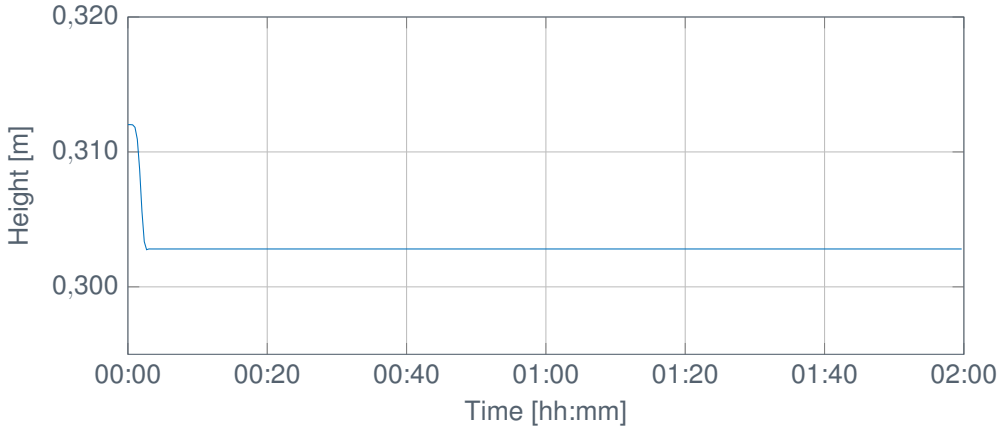
Type	Components	Sections
Pipe	1	35
Tank	1	1
Pipe	18	227
Total	20	263

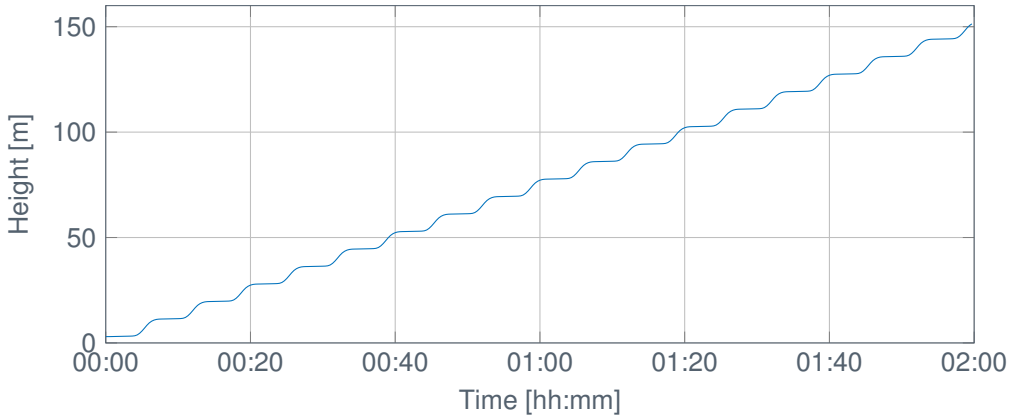


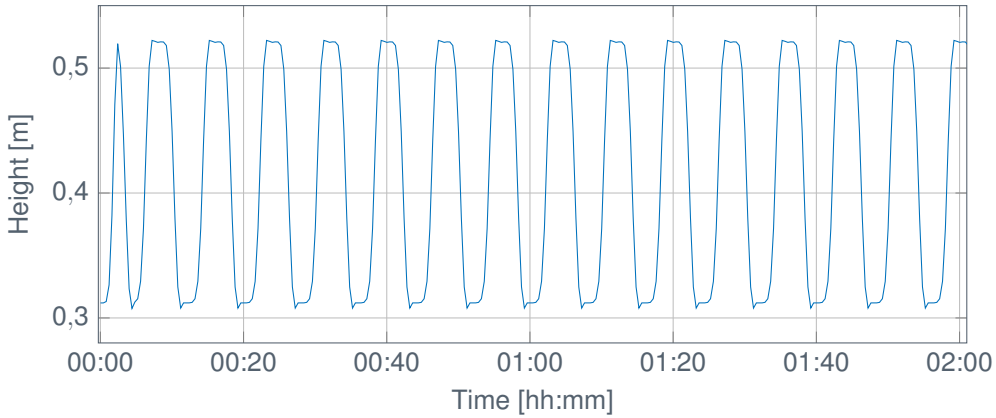
- Cost function
  - Begrænset til minimiere af output
- Constraints
  - Højde
  - Kontrol input
- Linear model

- Bestemmelse af Prediction horizon
  - Flow profiler
  - Industri
- Begrænsning af Prediction horizon
- System setup
- Forstyrrelse input

Fields	type	component	sections
1	'Pipe'	1	5
2	'Tank'	1	1
3	'Pipe'	1	5
4	'Total'	3	11

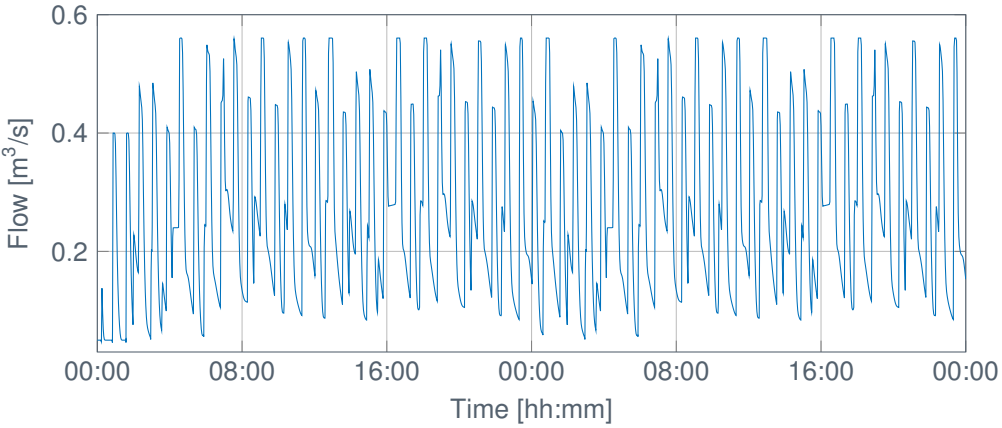


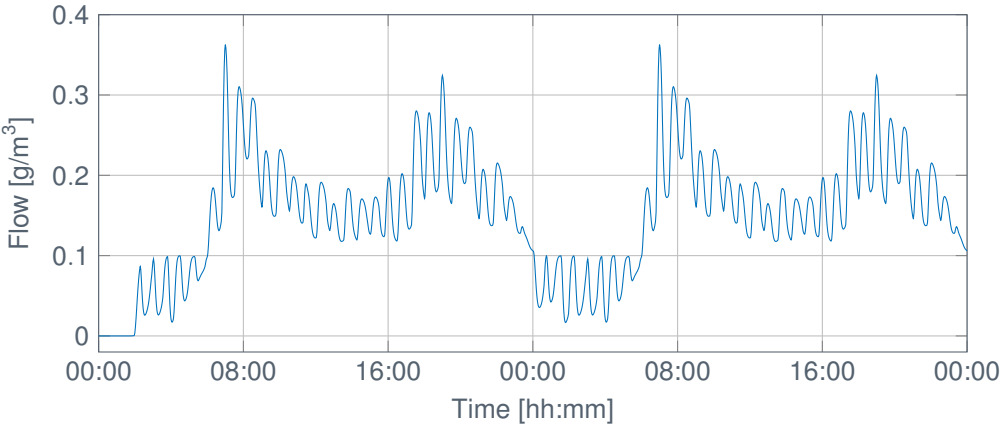




- System setup
- Flow profiler

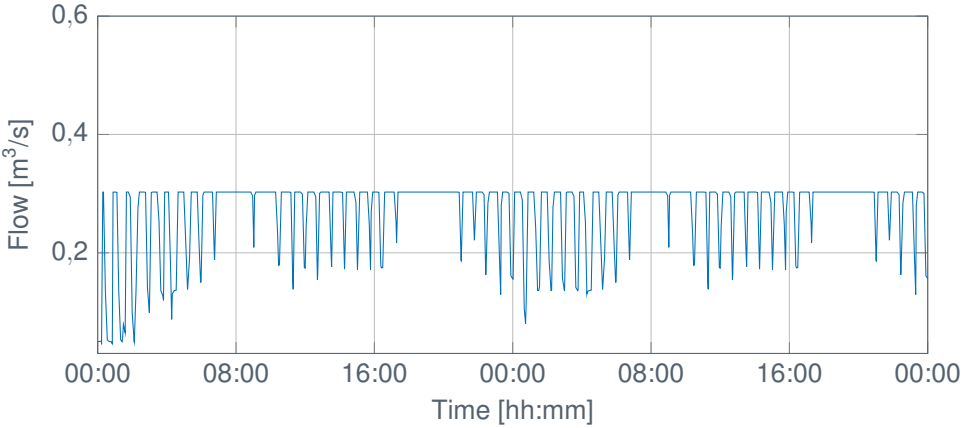
Type	Component	Sections
Pipe	1	35
Tank	1	1
Pipe	17	207
Tank	1	1
Pipe	1	38
Total	21	282





- Over dimensioneret tank
- Konstant output af tank

Resultat
Anden test



Diskussion

- Courant's number
- Model reduction

Konklusion

- Simulering
- MPC