Aalborg University

Model Predictive Control of a Sewer System

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STUDENT REPORT

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Model predictive control of flow and concentration of sewage in a sewer system

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Preface

NEW PREFACE !!!!

Units are indicated by sqaure brackets after the parameter has been elaborated e.g. Q is the flow $\left[\frac{m^3}{s}\right]$.

Sources are indicated by [name,year], and can be found in the bibliography list at the given [name,year].

Jacob Naundrup Pedersen

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Nomenclature

Abbreviation

Abbreviation	Definition
GIS	Geographically Information System
WATS	Wastewater of Aerobic/Anaerobic Transformations in Sewers
WWTP	Wastewater Treatment Plant

Symbols

Symbol	Description	Units
A	Area	m^2
Q	Water flow	m^3/s
q	Inflow of water m^2/s	
D	Diameter meter	m
r	Radius	m
F	Force	N
θ	Angle	rad
v	Velocity	m/s
m	Mass	kg
V	Volume	m^3
ρ	Density	kg/m^3
l	Length	m
g	Gravitational acceleration	m/s^2
T	Temperature	$°C$
m_n	Mass flow	kg/s

1 Introduction

Sewers were created to solve the seemingly simple problem of removal of wastewater. The first sewers, registered, dates back to 7000 B.C. in urban settlements and were created to remove wastewater from houses and surface runoff created by precipitation. To avoid clogging and wear of the sewers grit chambers was constructed. They work by slowing the flow of sewage in long narrow channels making the solids, such as sand, end up as sediments in the channels due to gravity. Complexity of sewers increased in ancient Rome where large underground systems were created leading to the main sewer system called "Cloaca Maxima" making it possible to have latrines with running water within households, though mostly made available for the rich.

Waste were still thrown onto streets as the population, without immediate access to a latrine in their household, during night time did not want to put in the effort to properly dispose of the waste. Because of this the ancient Rome suffered from illnesses related to waste lying in the streets. The hygienic aspect of proper disposal of wastewater in relation to drinking water were not considered until the 19th century, where several European cities saw large outbreak of cholera causing the deaths of millions.

The growth in waste furthermore caused the expansion of 26 km. sewer network in Paris to 600 km. during the 19th century. But it is not until the start of the 20th century that the chemical and microbial processes in sewers are considered. The microbial cause of cholera were identified by the German doctor Robert Koch in 1883, a discovery for which he in 1905 received the Nobel Prize in physiology and medicine. The growing industries and technological progress in the 20th century meant that more chemicals were disposed into the sewers having severe consequences for the organic life downstream of the receiving waters. Wastewater treatment plants were introduced to reduce the pollution, but several countries did not have any wastewater treatment plants before after World War II. Today disposal of sewage and setup of wastewater treatment plants is a given part of construction of new settlements, even in poor regions of the world [Hvitved-Jacobsen et al., 2013].

1.1 General sewer construction

This section will elaborate on the general contruction of sewers. Furthermore a brief explanation, of input into the sewer to output from the wastewater treatment plant (WWTP), is given.

Generally sewer construction can be put into two categories which are gravity and pressure sewers. Gravity sewers utilize the topographic advantages of the area in which they are built. But in places where the level of the surface does not accommodate a slope of the sewer pipe such that wastewater flow in the desired direction, wells with pumps are used to transport the wastewater to an elevated level, which can be seen in figure 1.1.

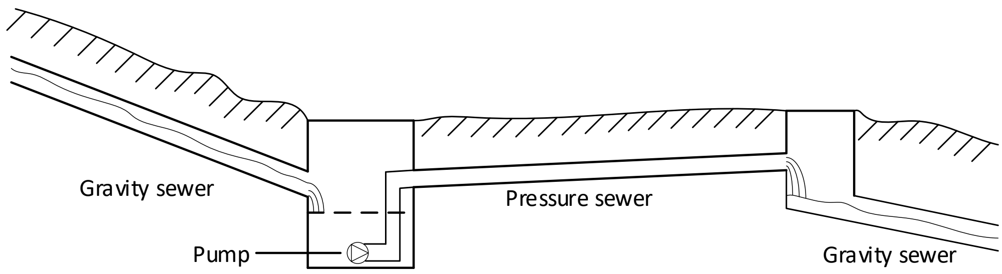

Figure 1.1: Illustrate different methods for transportation of wastewater.

Design of sewer systems involves careful considerations such that as much of the systems utilize the gravity for transport of wastewater to minimize the energy consumption. For this reason the WWTP is typically located in a low topographic area near a river, fjord or the sea. The design process also involves the dimensioning of the pipes to avoid overflow and the depth in which they are placed in the ground such that subzero temperatures does not prevent the flow in the sewers at any time. Furthermore the slope of the pipes must be chosen such that a high enough flow is created to avoid clogging. The material used to create the pipes gives different amount of friction e.g. a concrete surface will be more rough than polyethylene and thereby have a higher friction. This means that a larger slope of a concrete pipe is needed to avoid clogging in the pipes. Typically gravity sewer pipes is made of concrete and pressure sewer pipes of polyethylene.

In figure 1.2 a block diagram of the flow of wastewater is seen.

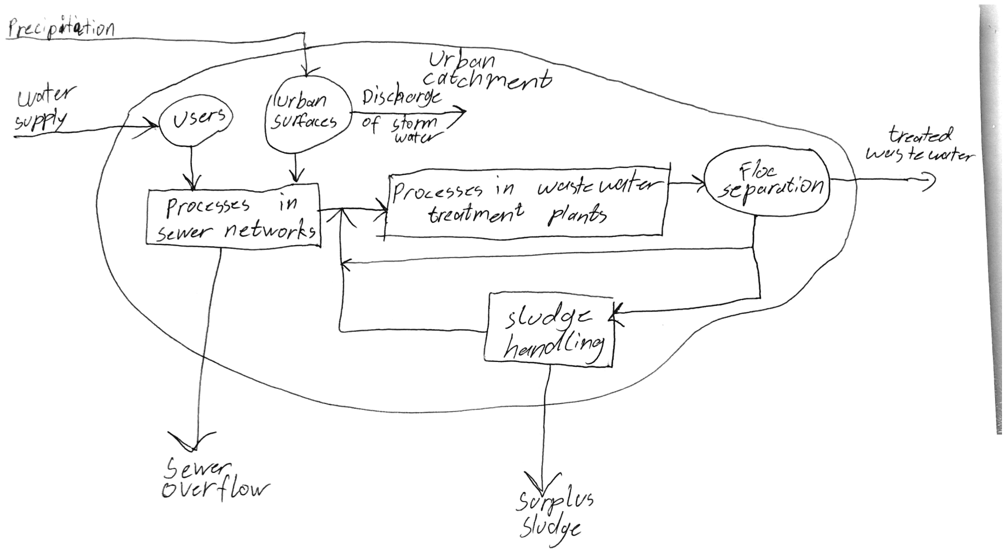

Figure 1.2: General overview of the flow of wastewater from users and surface runoff to the treated water is released into receiving waters. Arbejdsblad billede, inspiration er taget fra hvitved. Fjern sludge handling of floc separation og forklar det nede ved afsnittet.

In the left side of the figure precipitation from urban surfaces and roads are collected in the sewer by inlets placed at the gutter. In recent times separate sewer systems for surface runoff are constructed, which are also called storm water sewers. The water in these sewers are typically led in to storm water basins, rivers or the sea. In areas with older sewer constructions storm water is let into the sewers where it is mixed with wastewater. The wastewater comes from households or industry disposing of substances of varying consistency. Heavy precipitation can cause the sewers to be filled, and to avoid overflow into household or on roads the wastewater is let into rivers or the sea during such events. The reason for designing storm water sewers is partly to avoid letting untreated sewage into the nature, but also to better be able to control the cleaning process in the WWTP. It is desirable to have a steady flow of wastewater with a certain level of chemical concentration flowing into the WWTP. The chemical and microbial reaction happening in the sewer lines is discussed in subsection 1.2.1. When wastewater is received in the treatment plant several processes occurs to separate the unwanted substances from the received wastewater before the clean water is released into nearby rivers or the sea. The processes which the wastewater undergoes at the plant is discussed in subsection 1.2.2.

1.2 Chemical and biological processes

This section will describe the chemical and biological processes that wastewater undergoes from the water is used to it is cleaned at the water treatment plant. The processes in a wastewater treatment plant will be investigated to get an understanding of the different processes.

Within wastewater there is an infinite number of living organisms before entering the WWTP. It contains from around 100.000 to 1.000.000 microorganisms per milliliter. Theses organisms originates from sanitary waste and soil. They are a natural living part of the organic matter and they are an important part of the cleansing, of the wastewater, at the WWTP. To be able to obtain a high water quality at the output of the WWTP it is necessary to have a thorough understanding of these microorganisms [College, 2018].

Nearly all of the microorganisms found in wastewater are not harmful and therefore does not cause a disease for mankind. However a small group of the microorganisms can cause diseases and these are of a great concern in wastewater treatment. The most known diseases are typhoid fever, dysentery, cholera, and hepatitis [College, 2018].

The microorganisms in the WWTP have a specific role in the decomposition of the waste. The three most notable microorganisms in the biological treatment process are bacteria, fungi and protozoa. The bacteria has the primary role of degrading the wastewater compounds, thereby producing sediment solids. Bacteria is a single cell organism and is capable of reproducing rapidly when in contact with water. They consume the waste by taking it through the cell wall [College, 2018]. Fungi like bacteria decomposes the organic waste, however they also pose a significant problem for the wastewater treatment as the fungi can proliferate to an extent where it has detrimental consequences for the effluent quality [AquaEnviro, 2010]. Lastly protozoa acts as predators toward the present bacterial population such that it can be controlled [College, 2018].

In figure 1.3 an illustration is shown of the processes that wastewater undergoes within the sewer.

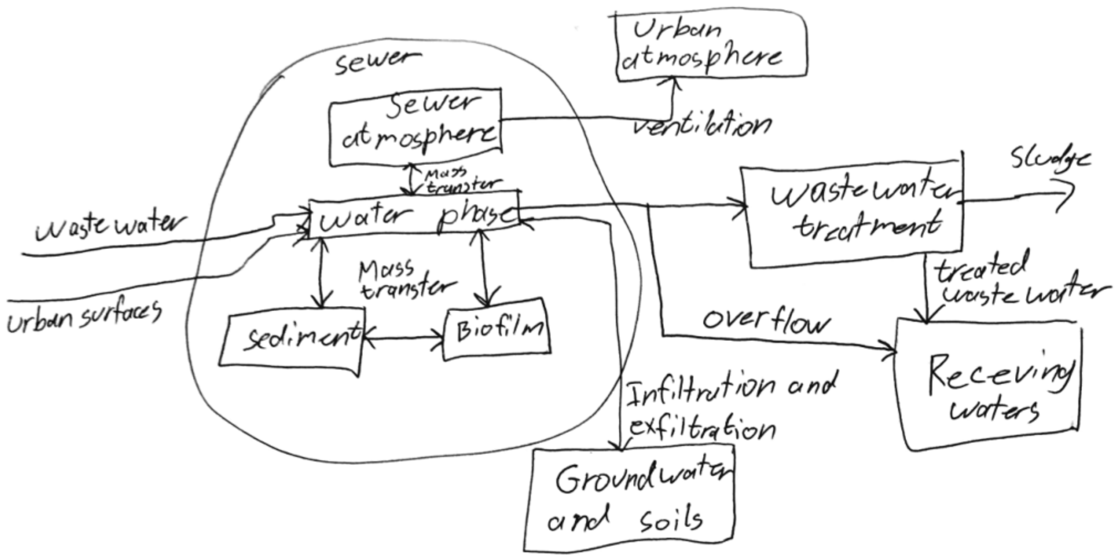

Figure 1.3: General overview of a sewer where the processes are illustrated. arbejdsblad billede. [1]

Wastewater are subject to a variety of mass changes in a sewer. One is due to free electrons in the wastewater, where different kinds of chemical reactions occurs. This results in different compounds being created which will be elaborated in subsection 1.2.1. The concentration level and the chemical compounds, which exists at the inlet of the sewer, undergoes reactions in the sewer before reaching the WWTP. These reactions has the possibility to create gases which are released into the urban atmosphere, which is not ideal as it is typically malodorous. Microorganisms are reproducing on the biofilm that is created on the surface of the pipe. Furthermore the wastewater in the pipes can sink into the groundwater and the soils due to small leaks in the construction of the sewer [2]. The wastewater ends up at the wastewater treatment plant, this process will be explained in subsection 1.2.2. And as previous mentioned in case of heavy precipitation to avoid flooding, the wastewater will be lead into receiving waters.

1.2.1 Chemical reactions in a sewer

A wastewater treatment plant does not only receive what is discharged into the sewer from the industry and households but also the chemical and biological reactions that occurs in a sewer. The chemical reactions occur as redox reactions between the different compounds. Redox reaction is the transfer of electrons between two compounds at an atomic or molecular scale. By transferring electrons from one compound to another new compounds will arise, such as hydrogen sulfide which is know for its malodorous smell of rotten eggs. These reactions are determined by the electron acceptors that are present in the wastewater. The electron acceptor is the compound that receives electrons in a redox reaction. Examples of dissolved acceptors are oxygen (O_2), nitrate (NO_3^-) and sulfate (SO_4^{2-}), which is determined by whether aerobic, anoxic or anaerobic conditions occur in the sewer. The redox reaction reduces these three compounds in the wastewater by changing them to new compounds such as water (H_2O), molecular nitrogen (N_2) and hydrogen sulfide (H_2S) [Hvitved-Jacobsen et al., 2013].

[2] FiXme Note: Måske vi burde have noget med at grundvand faktisk også trænger ind i kloakkerne. Det kan være vigtigt i forbindelse med et statisk offset i vand flow når vi skal til at simulere.

Where redox reactions happens are, to a great extend, determined by the design of the sewer. The aerobic, anaerobic and anoxic conditions does not exist in the same part of the sewer, and the last only occurs if nitrate is artificially added to the wastewater. If the sewer is in an aerobic state then the typical characteristics for the sewer are either partly filled gravity sewer or an aerated pressure sewer. This means that there are free oxygen (O^+) molecules, and these will be connected with hydrogen to create water. If the sewer is in an anoxic state, which occurs in pressure sewers, then the addition of nitrate to the wastewater results in molecular nitrogen. If the sewer is in an anaerobic state the characteristic of the sewer is either a pressure sewer or a full flowing gravity sewer, then the reaction will result in hydrogen sulfide as the sulfate will bind with the hydrogen molecules. With the knowledge of these condition sewers can actively be designed to achieve a specific state [Hvitved-Jacobsen et al., 2013]. The two desired states in the sewers are aerobic in gravity sewers and anoxic in pressure sewers to avoid malodorous dissipation into the urban atmosphere.

[3]

To model these chemical and biological reactions in sewers a model concept, Wastewater Aerobic/Anaerobic Transformation in Sewers (WATS), is used. The WATS model is expressed as differential mass balance equations that is suitable for numerical computation[4] and can therefore be included in simulations for a specific objective e.g. model of water, biofilm and gas phase transformations. The WATS model can be applied to a variety of different sewers as long as a fundamental understanding of the sewer process is available. Whether it is aerobic, anoxic or anaerobic conditions that dominate the sewer, the soil composition and the pH concentration of the wastewater must also be included in the WATS model [Hvitved-Jacobsen et al., 2013].

1.2.2 Wastewater treatment plant

Wasterwater from households and industry contains organic and inorganic matter and if it is release into the water environment it will result in a polluted environment. This can cause oxygen depletion and thereby affect the wildlife in the water environment negatively. Therefore the wastewater is lead to a WWTP to purify it from these substances which are harmful to the environment. In the following section the process that wasterwater undergoes in the WWTP will be elaborated.

In figure 1.4 the process wastewater undergoes is illustrated.

[3] FiXme Note: Lav en table over disse 3 tilstande i kloakken

[4] FiXme Note: Hvad mener du med mere skarp her Thomas? i forhold til numerical computation

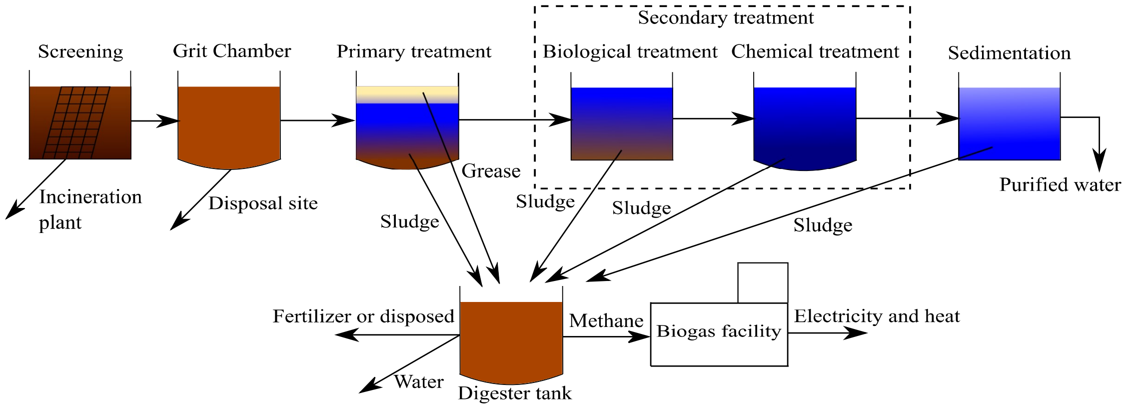

Figure 1.4: General overview of a wastewater treatment plant.

At the WWTP the wastewater undergoes several process before the organic and inorganic matters are removed. The first stage of purifying the wastewater is screening, where the larger objects are removed from the wastewater which would either block the flow or damage the equipment. Examples of objects that are filtered from the wastewater are bottles, plastic bags and diapers [Eschooltoday, 2017].

The wastewater is then lead into the primary treatment where it first will enter a grit chamber where objects as sand and stones will settle to the bottom of the tank. These grit chambers are crucial in WWTP that are connected with combined sewer systems.The storm water may wash sand, stones, and gravel into the sewer and these objects will therefore be sorted out in this process [EPA, 1994]. These objects are collected and disposed at a disposal site.

After the screening and removal of grit the wastewater still contains organic and inorganic matter. By leading the wastewater into the primary treatment tank where the flow of the wastewater is reduced such that the organic matter will sediment in the tank, while the grease will accumulated at the top of the tank. The grease is then scrapped into the digester tank. The matter that have sedimented is now called sludge or raw primary biosolids. At the bottom of the tank large scrappers are moving the sludge to the center of the tank where it is pumped into the digester tank [EPA, 1994]. Water flows or is pumped into the secondary treatment.

In the secondary treatment the water is lead into an aeration tank, this is called the biological treatment. By aeration of the wastewater the bacteria gain optimal conditions for respiration and thereby speed up the process of decomposing the remaining organic matter from the primary treatment process. In the process of decomposing the organic matter the bacterias will produce CO_2 that will dissipate into the atmosphere. Furthermore when the bacteria have consumed the organic matter they will start to produce heavier particles that will sediment in the tank [Rinkesh, 2009]. In figure 1.5 this process is illustrated.

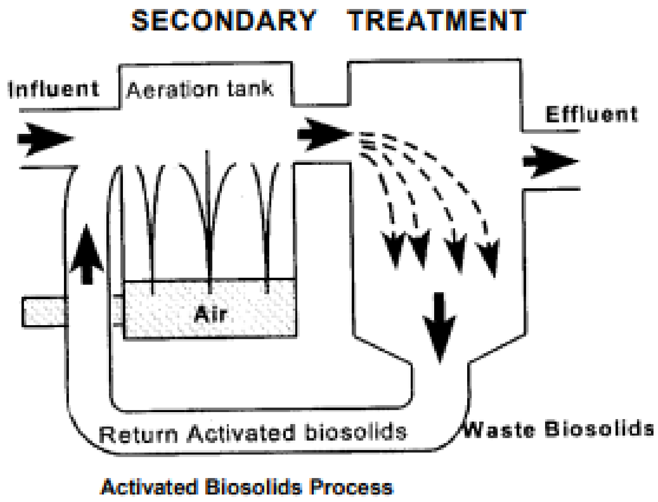

Figure 1.5: Illustration of the secondary treatment process. arbejdsblad billede. [5]

This is called the activated sludge process, because some of the sludge is reused in the aeration phase to keep a high bacteria population. This is done to have ad high s removal rate of pollutants in the wastewater as possible. The reused sludge contains millions of microorganisms that helps purifying the water. The sludge that have sedimented is pumped to the digester tank [EPA, 1994].

Hereafter a chemical treatment is performed to remove the inorganic matter that is left in the wastewater. This will create chemical reactions and thereby create new compounds that will sediment in the tank, which will be pumped into the digester tank. The chemical added to the process does not cause any damage to the environment [Sjøholm, 2016].

After these treatment processes there are still some particle left in the wastewater. It is lead through a sedimentation tank where the remaining bacteria and sludge will settle before released into receiving waters. The sedimented particles will be pumped to the digester tank for further processes.

The sludge that is collected in the digester tank undergoes further treatment, where the remaining water in the sludge is separated from it. The water is lead back to the wastewater treatment process where it will undergo the same process again. The sludge is collected and used as fertilizer or disposed in a landfill and the gas created in the process can be used at a biogas facility to produce electricity and heat [Rinkesh, 2009].

1.3 Problem for the WWTP

In the following section it will be shown how a daily input flow to the WWTP would like and what effects it can have on the purifying process. Furthermore the problems that comes with the industry will be explained.

The WWTP in Fredericia purifies wastewater from the industry, households and urban surfaces. A general daily intake of wastewater is illustrated in figure 1.6

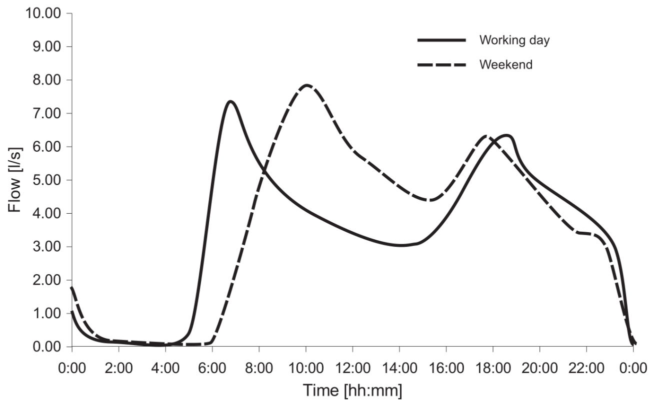

Figure 1.6: Typical daily flow pattern of a household from the city of Frejlev with approximately 2000 habitants [Schlütter, 1999].

From around 06:00 there is a peak, it is due to people preparing for work, where the majority of wastewater comes from toilets visits and baths. Doing the day the flow slows down until people start to return from work where it has a flow increase due to cooking, baths, toilets etc. Doing night the flow is at a minimum as people are sleeping, however there is a constant intake of precipitation, throughout the pipeline, because it will sink through the ground and into sewer through the pipe wall.

These spikes or increased flows at certain times results in a lower quality of the effluent that leaves the WWTP. The aeration phase where the process requires power to drive the pumps that create this oxygenation of the water. The process will have to follow the input flow to accommodate the changes. When the input is high the process requires a higher population of microorganisms to purify the wastewater, and therefore more air is needed in the wastewater.

In Fredericia there are multiple big companies that empty their process out into the sewer. An example is the dairy. They are known to empty their process of ????? tons cream into the sewer at random time intervals. This a huge load for the WWTP to purify. Often the companies does not alert the WWTP about these load changes. This possess a huge problem WWTP ???? as it does not have the capacity to purify the wastewater to achieve a acceptable output quality of the effluent. (Tømmer de bare ud? Eller bliver processen bare speeded up så det ikke bliver renset ligeså godt?). Furthermore the concentration of the matter that comes from the industry is also a problem for the WWTP, as it is in some cases very concentrated.

It is desired to have as constant as possible input into the WWTP to keep the aeration phase as constant as possible. Thereby reducing the energy cost in this phase and gain a more optimal purify process. Thereby obtaining a better water quality at the output of the plant. Furthermore the it is wanted to attenuate the concentration of the matter from the industry to a level where the WWTP purify it to obtain a sufficient water quality.

System description 2

This section will go into details of the structure of the sewer network for which the further work of this project will be based upon.

As mentioned in section 1.2 a steady flow of sewage with a fixed level of contaminants is desired such that an optimal utilization of the wastewater treatment plant can be obtained. An area of interest is Fredericia with a sizable population of approximately 40.000 people and industries where some of the largest consists of a brewery, bottling plant, refinery and a dairy plant [Statistics-Denmark, 2018]. All of these industries is placed in the outskirts of the city, meaning that the wastewater discharged into the sewer goes through populated areas creating an uneven flow of sewage to the wastewater treatment plant. Two main sewer lines separates the northern and southern part of the city. To limit the project only the northern main sewer line is considered, which covers the largest part of the households and the industry located in the city. In figure 2.1 a simplified overview of the northern main sewer line with the various areas of population and industry in Fredericia attached to it. The placement of the sewers shown in 2.1 is obtained from a Geographically Information System (GIS) map publicly available by the municipal of Fredericia [Fredericia-Spildevand, 2018]. The red and the green lines indicate sewers with flows of wastewater and combined wastewater and surface runoff respectively. The populated areas are indicated by two different transparent blue colors, for easier to distinguish between the different parts of the sewer network. The red transparent areas indicate small to medium sized industry. Only the sewer lines out of or between the separate areas are shown. Furthermore the areas connected by a red line has a separate sewer system for surface runoff, which is lead into various ponds or the sea, minimizing the load on the wastewater treatment plant. The bottling plant, refinery and the brewery is marked by the purple, brown and black rings respectively. Several inlets for surface runoff connected directly to the main sewer line exists. The added disturbance from these inlets is neglected on the assumption that the disturbances from the connected areas will be considerable larger.

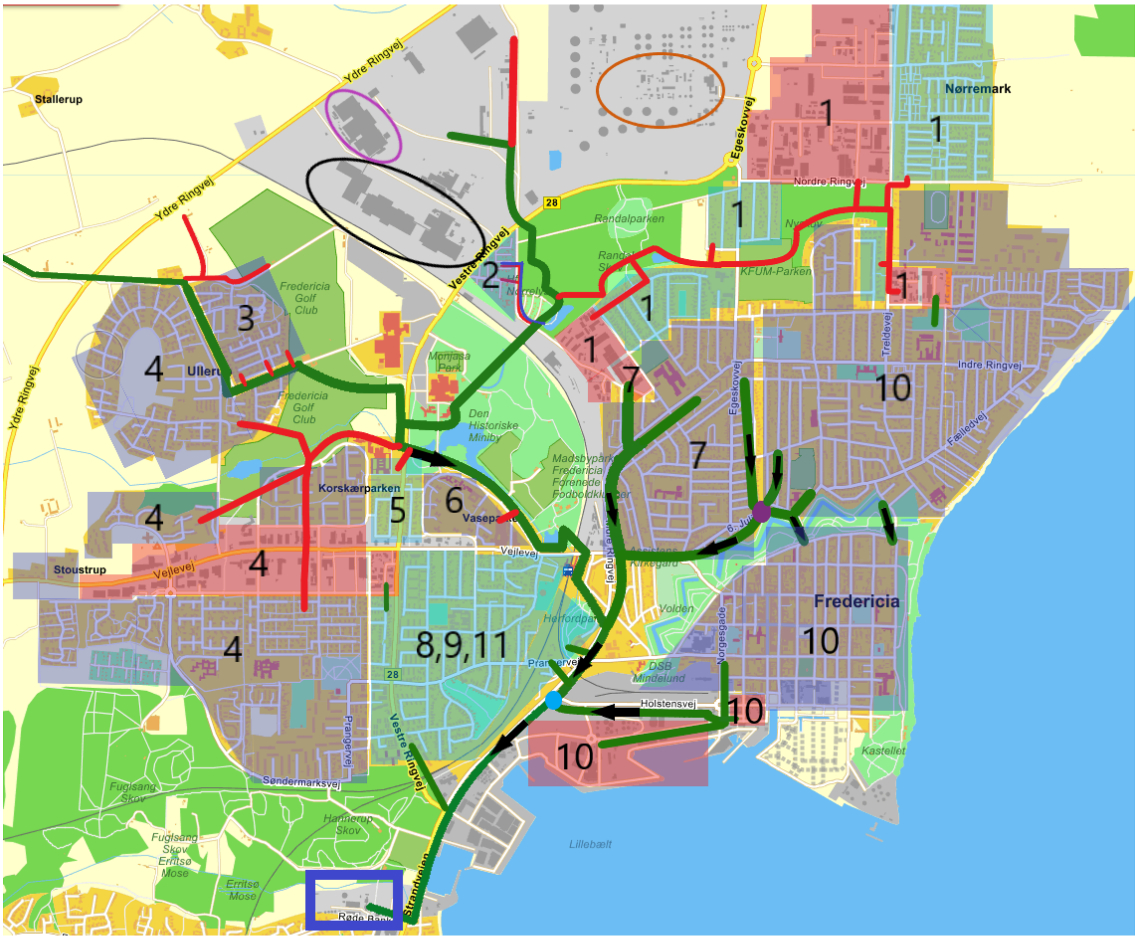

Figure 2.1: Simplified mapping of the northern part of the sewer network in Fredericia. The two blue transparent colors indicate populated areas and the red transparent area indicate industry. Red and green lines is sewers with flows of wastewater and combined wastewater and surface runoff respectively. Bottling plant, refinery and brewery is marked by purple, brown and black circles respectively. The purple dot is a connecting point with two incoming and two outgoing sewer lines. Blue dot is a wastewater pumping station which elevates sewage such that gravity can be utilized for the remaining transport into the treatment plant. Blue rectangle marks the location of the wastewater treatment plant. [Eniro, 2018] [Fredericia-Spildevand, 2018]

The various enumerated parts in figure 2.1 is shown by order of attachment to the main sewer line, together with distance between each attachment, in figure 2.2.

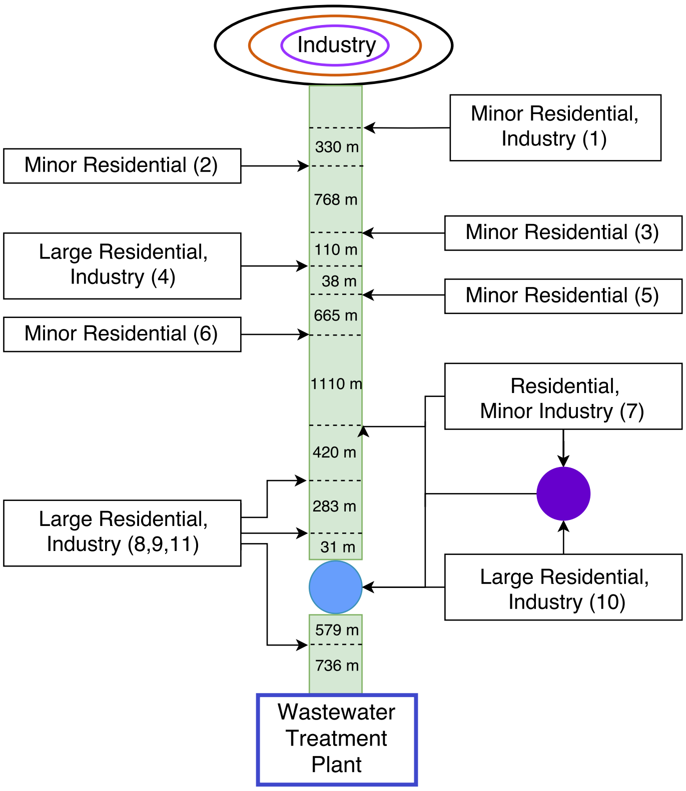

Figure 2.2: Simplification of the attachments to the main sewer line shown in figure 2.1. The numbers correspond to which area is connected to the main sewer line farthest from the wastewater treatment plant, with the distance between them [Fredericia-Spildevand, 2018].

Furthermore the different sections consist of pipe of varying diameters as can be seen in table 2.1.

Pipe section	Pipe length (meter)	Inner pipe diameter (mm)	Bed datum in (m)	Bed datum out (m)	Bed slope (‰)
1 → 2	303	900	11,56	10,65	3,00
	27	1000	10,65	10,57	3,00
2 → 3	155	1000	10,57	9,94	4,10
	295	800	9,94	6,33	12,20
	318	900	6,33	4,71	5,30
3 → 4	110	900	4,71	4,31	3,60
4 → 5	38	1000	4,31	4,40	-2,40
5 → 6	665	1000	4,40	2,43	3,00
6 → 7	155	1000	2,43	2,31	0,80
	955	1200	2,31	-0,48	2,90
7 → 8	293	1200	-0,48	unknown	
	11	1300	unknown	-1,38	
	116	1200	-1,38	-1,62	2,10
8 → 9	283	1400	-1,62	-2,09	1,70
9 → 10	31	1400	-2,09	-2,15	1,90
10 → 11	125	1600	0,31	0,05	2,10
	94	1500	0,05	-0,07	1,30
	360	1600	-0,07	-1,72	4,60
11 → WWTP	736	1600	-1,72	-2,60	1,20
Total length 1 → WWTP	5070				

Table 2.1: Table of the various lengths and the approximate inner diameter of pipe, appearing in order, in the main sewer line. Pipe section indicate the length of pipe between the attachment of the various areas to the main sewer line [Fredericia-Spildevand, 2018].

Some assumptions is made to avoid complications during simulation. The negative slope of the section between connection point four and five is flipped such that no permanent storage of sewage happens. The reason for this assumption is that it will ease the computation, of the free flow in that section, if storage in the pipe sections could be disregarded during simulation. Furthermore the new slope is deemed acceptable based on the obtained slopes for the remaining pipe sections. For the two pipe sections between point seven and eight, where out- and input datum is unknown, are gathered in to a single pipe section. This section will be designated an inner diameter of 1200 mm as the section with the larger diameter is assumed insignificant for the free flow at the end points of the entire section.

Pipe section	Pipe length (meter)	Inner pipe diameter (mm)	Bed slope (‰)
4 → 5	38	1000	2,40
7 → 8	304	1200	3.00
	116	1200	2,10

Table 2.2: New slope values for sections with negative slope and unknown values.

Modeling 3

The approach for this chapter is to obtain a model of the sewer system with its different components comprising of free flow gravity and pressurized sewer lines, interconnections and reservoirs. This is with the goal of obtaining a model of the main sewer line with the various areas as input disturbances, as shown in figure 2.1, such that control of the flow and concentrate into the WWTP can be performed. In the following, methods to model the components such as flow in gravity and pressurized sewer lines (section 3.1), interconnections such that disturbances can be added to the main sewer line (section 3.2), reservoirs in the sewer network (section 3.3) and the concentrate of the wastewater flowing in the main sewer line (section ??), is examined.

3.1 Hydraulics of sewer line

Methods to model hydraulics of gravity and pressurized sewer lines is explained respectively in the following.

3.1.1 Open channel

Modeling fluids is almost always done by considering it as a control volume. The reason is that it is rarely efficient, computational wise, or possible to consider the individual fluid particles. Henceforth the control volume will be denoted by the letter Ω which will correspond to some amount of fluid in a length of sewer line.

The open channel flow in gravity sewer lines can be described by the Saint-Venant equations which gives an expression for conservation of mass and momentum. Some assumptions is made when deriving the Saint-Venant equations:

1. The flow in the channel is one dimensional, and prismatic, and as such any curvature or change in width of the sewer line is considered negligible.
2. Fluid in the sewer line is considered incompressible and the pressure is assumed hydrostatic.
3. The only forces considered is friction, pressure and gravity.
4. The water height and velocity is uniform in the cross-section and only changes horizontally i.e. turbulence in the fluid is not considered.
5. The slope of the channel bed is small

The equation for conservation of mass gives an expression for the amount of fluid flowing in to the control volume and the flow out plus the fluid stored in it. In figure 3.1 a flow in a channel is shown.

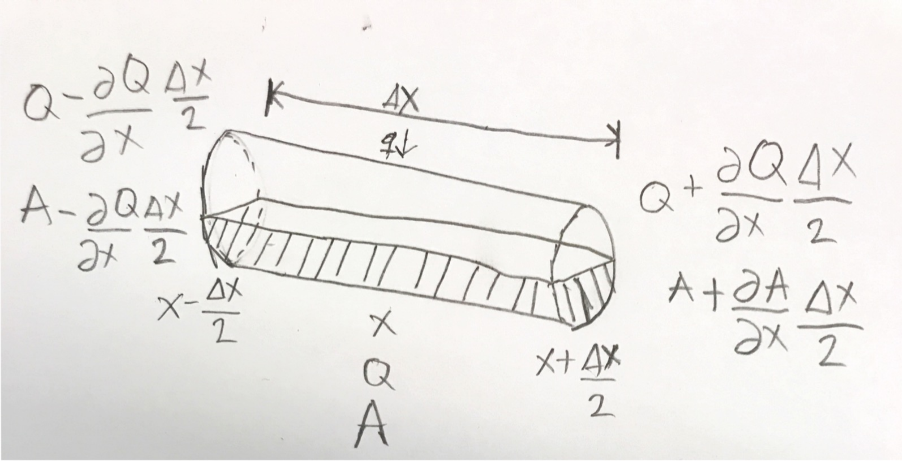

Figure 3.1: Flow in a channel where Q is flow into the channel, q is lateral flow into the channel and the cross section area is given as the area for an circle.

Flows, Q, q and wetted cross section A shown in figure 3.1 is dependent on time and position, but in the following a simpler notation is used for an easier outline.

The flow into the control volume where Q is the flow considered from the middle of the control volume is given as

$$Q_{in} = \left(Q - \frac{\partial Q}{\partial x} \cdot \frac{\Delta x}{2}\right) \cdot \Delta t + q \cdot \Delta x \cdot \Delta t \tag{3.1}$$

Where q is the lateral inflow across the entire channel $[\frac{m^2}{s}]$ and Q is the flow in the channel $[\frac{m^3}{s}]$. Lateral inflow could for example come from adjoint sewer pipes or road drain. The discharge flow of the channel is given as

$$Q_{out} = \left(Q + \frac{\partial Q}{\partial x} \frac{\Delta x}{2}\right) \cdot \Delta t \tag{3.2}$$

Change in stored fluid in the channel is given as:

$$\frac{\partial}{\partial t}\left(\frac{\Delta x}{2} \cdot \left(A - \frac{\partial A}{\partial x}\frac{\Delta x}{2} + A + \frac{\partial A}{\partial x}\frac{\Delta x}{2}\right)\right)\Delta t = \frac{\partial}{\partial t}\left(\frac{\Delta x}{2} \cdot (2A)\right)\Delta t = \frac{\partial A}{\partial t} \cdot \Delta x \Delta t \tag{3.3}$$

This can be simplified to:

As the flow into the channel minus the flow out is equal to the change in stored fluid in the channel, then due to the assumption of incompressible fluid and uniformity the following can be wr.

$$Q_{in} - Q_{out} = \frac{\partial A}{\partial t} \cdot \Delta x \cdot \Delta t \tag{3.4}$$

Thereby inserting equations 3.1 and 3.2 in 3.4 the following is obtained:

$$\begin{aligned}
&\left(Q - \tfrac{\partial Q}{\partial x} \cdot \tfrac{\Delta x}{2}\right) \cdot \Delta t + q \cdot \Delta x \cdot \Delta t - \left(Q + \tfrac{\partial Q}{\partial x} \tfrac{\Delta x}{2}\right) \cdot \Delta t = \tfrac{\partial A}{\partial t} \cdot \Delta t \cdot \Delta x \\
&\Updownarrow \\
&q \cdot \Delta x \cdot \Delta t - \tfrac{\partial Q}{\partial x} \cdot \Delta x \cdot \Delta t = \tfrac{\partial A}{\partial t} \cdot \Delta t \cdot \Delta x
\end{aligned} \tag{3.5}$$

Equation 3.5 can be reduced to the following by isolating and dividing with Δx and Δt, on both sides, yielding the mass conservation part of the Saint-Venant equations.

$$\frac{\partial A(x,t)}{\partial t} + \frac{\partial Q(x,t)}{\partial x} = q(x,t) \tag{3.6}$$

For channel flows without lateral input the mass conservation is given as:

$$\frac{\partial A(x,t)}{\partial t} + \frac{\partial Q(x,t)}{\partial x} = 0 \tag{3.7}$$

Momentum of the control volume Ω shown in figure 3.2 can be found by utilizing Newtons second law which says that force is equal to mass times acceleration. Basically this means that the momentum of the control volume can be found by integrating the sum of forces in the following differential equation.

$$\frac{d\mathcal{M}(t)}{dt} = \sum_i F_i(t) \tag{3.8}$$

Where $\mathcal{M}$(t) is the momentum, given as mass times a velocity vector, of the control volume at time t and F_i(t) is the various external forces affecting the control volume.

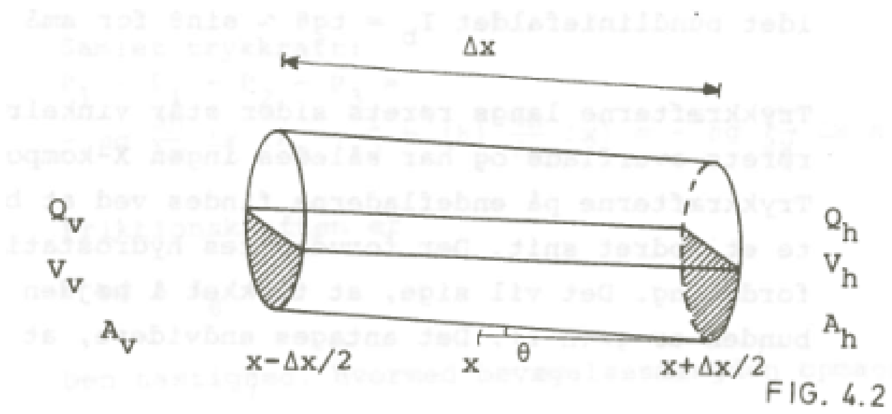

Figure 3.2: Illustration to derive the momentum equation on an open channel.

The force given by the fluid particles at the cross section at each end of the control volume is given as:

$$F = \rho \cdot v \cdot Q \tag{3.9}$$

Where ρ is density $[\frac{kg}{m^3}]$, v is velocity $[\frac{m}{s}]$ and Q is flow $[\frac{m^3}{s}]$. The force given by the in- and output of fluid particles in the control volume, with the assumption that the density of the fluid and the velocity of it in the cross section of the control volume is constant, is given as:

$$F_{in} = \rho \cdot v \cdot Q - \frac{\partial}{\partial x}(\rho \cdot v \cdot Q) \cdot \frac{\Delta x}{2} \tag{3.10}$$

$$F_{out} = \rho \cdot v \cdot Q + \frac{\partial}{\partial x}(\rho \cdot v \cdot Q) \cdot \frac{\Delta x}{2} \tag{3.11}$$

Where subscript *in* denote the force going in through the left side of the channel on figure 3.2 and subscript *out* is the force going out to the right side on figure 3.2. The change of particle momentum in the control volume is given as $F_{in} - F_{out}$ and by replacing velocity with Q/A the following is obtained

$$\begin{aligned}&\rho \cdot \tfrac{Q}{A} \cdot Q - \tfrac{\partial}{\partial x}\left(\rho \cdot \tfrac{Q}{A} \cdot Q\right) \cdot \tfrac{\Delta x}{2} - \\ &\rho \cdot \tfrac{Q}{A} \cdot Q + \tfrac{\partial}{\partial x}\left(\rho \cdot \tfrac{Q}{A} \cdot Q\right) \cdot \tfrac{\Delta x}{2} = -\rho \tfrac{\partial}{\partial x} \tfrac{Q^2}{A} \Delta x\end{aligned} \tag{3.12}$$

The remaining to be found is the forces imposed by gravity, friction and the pressure. The force applied by gravity is given as:

$$F_g = sin(\theta) \cdot g \cdot \rho \cdot \Delta x \cdot A \tag{3.13}$$

where the slope of the pipe bed $\mathrm{S_b} = tan(\theta) \approx sin(\theta)$ for small values of θ resulting in:

$$F_g = S_b \cdot g \cdot \rho \cdot \Delta x \cdot A \tag{3.14}$$

The friction force can be set up similarly as

$$F_f = S_f \cdot g \cdot \rho \cdot \Delta x \cdot A \tag{3.15}$$

where S_f is a friction coefficient. This coefficient can be estimated by different formulas like Manning's or Darcy-Weishbach formula which is seen in equation 3.16 and 3.17 respectively .

$$S_f = \frac{n^2 Q^2}{A^2 R^{4/3}} = \frac{n^2 v^2}{R^{4/3}} \tag{3.16}$$

$$S_f = \frac{f Q^2}{8 g R A^2} = \frac{f v^2}{8 g R} \tag{3.17}$$

Where n is Manning's roughness factor, f is the Weisbach resistance coefficient and R is the hydraulic radius given as wetted area divided by the wetted perimeter [Mays, 2001]. The Weisbach resistance coefficient is found by the Colebrook-White formula seen in equation 3.18.

$$\frac{1}{\sqrt{f}} = -2 \cdot log\left(\frac{k}{14.84 \cdot R} + \frac{2.52}{4Re\sqrt{f}}\right) \tag{3.18}$$

Where f is the Darcy-Weisbach resistance coefficient, k is a pipe roughness coefficient and Re is the Reynolds number.

Last the pressure forces on the x component of the control volume to be found is marked as P_1 to P_3 in figure 3.3.

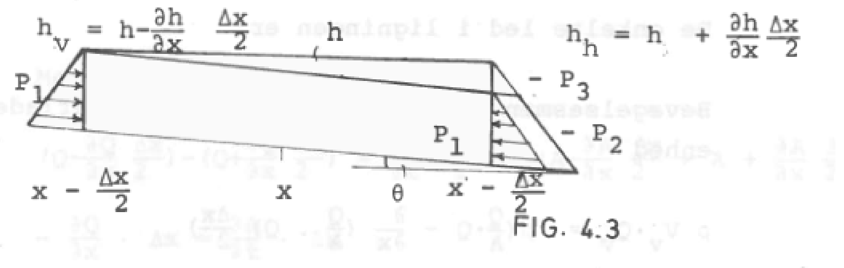

Figure 3.3: Forces acting on the control volume

[1]

The forces acting on the channel are found by taking a sectional view of it. Furthermore it is assumed to be hydrostatic pressure across the channel, thus the pressure in a height z above the bottom of the channel is given as $g\rho(h-z)$. The pressure forces acting on the control volume to the left is given as:

$$P_1 = \int_0^{h_l} \rho \cdot g(h_l - z) \cdot b(z)dz \tag{3.19}$$

The pressure force acting on the control volume to the right is given as:

$$\begin{aligned} &-\textstyle\int_0^{h_r} \rho \cdot g(h_r - z) \cdot b(z)dz = \\ &-\textstyle\int_0^{h_l} \rho \cdot g(h_l - z) \cdot b(z)dz - \int_0^{h_l} \rho \cdot g(h_r - h_l) \cdot b(z)dz - \int_{h_l}^{h_r} \rho \cdot g(h_r - z) \cdot b(z)dz = \\ &-P_1 - P_2 - P_3 \end{aligned} \tag{3.20}$$

The integral of P_2 is:

$$P_2 = -\int_0^{h_l} \rho \cdot g(h_r - h_l) \cdot b(z)dz = -\rho g \frac{\partial h}{\partial x} \Delta x A_v \tag{3.21}$$

By assuming the width, b is constant the following can be approximated:

$$P_3 = -\int_{h_l}^{h_r} \rho \cdot g(h_r - z) \cdot b(z)dz \approx -\rho g b(h) \frac{1}{2} \left(\frac{\partial h}{\partial x} \Delta x \right)^2 \tag{3.22}$$

Taking the sum of forces from equations 3.19 and 3.20:

$$P_1 - P_1 - P_2 - P_3 = -\rho \cdot g \cdot \frac{\partial h}{\partial x} \cdot \Delta x \left(A_v + \frac{1}{2} b(h) \frac{\partial h}{\partial x} \Delta x \right) = -\rho \cdot g \cdot \frac{\partial h}{\partial x} \cdot \Delta x \cdot A \tag{3.23}$$

By summing all the forces from equation 3.12, 3.13, 3.15 and 3.23 they can be inserted into equation 3.8:

$$\frac{d\mathcal{M}(t)}{dt} = -\frac{\partial}{\partial x} \rho \frac{Q^2}{A} \Delta x - S_b \cdot g \cdot \rho \cdot \Delta x \cdot A - S_f \cdot g \cdot \rho \cdot \Delta x \cdot A - \rho \cdot g \cdot \frac{\partial h}{\partial x} \cdot \Delta x \cdot A \tag{3.24}$$

[1]FiXme Note: Ny tegning og sikkert også caption

Lastly the expression for the momentum is:

$$\frac{\partial}{\partial t}\left(\rho\frac{Q}{A}A\Delta x\right) \tag{3.25}$$

Thereby all the sum of forces from equation 3.24 can be set equal to 3.25.

$$\begin{aligned}&\tfrac{\partial}{\partial t}(\rho\tfrac{Q}{A}A\cdot\Delta x) = -\tfrac{\partial}{\partial x}\rho\tfrac{Q^2}{A}\Delta x - S_b\cdot g\cdot\rho\cdot\Delta x\cdot A - S_f\cdot g\cdot\rho\cdot\Delta x\cdot A\\ &-\rho\cdot g\cdot\tfrac{\partial h}{\partial x}\cdot\Delta x\cdot A\end{aligned} \tag{3.26}$$

Dividing with $\Delta x\rho g A$ and isolating then the following definition of the equation is obtained:

$$\frac{1}{gA}\frac{\partial Q}{\partial t} + \frac{1}{gA}\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + \frac{\partial h}{\partial x} + S_f - S_b = 0 \tag{3.27}$$

Some or all of the terms in equation 3.27 can be utilized when simulating free channel flow. An overview of the limitations when excluding parts of the momentum equation is given in table 3.1.

Approximation	Kinematic wave (1)	Noninertia (2)	Quasi-steady dynamic wave (3)	Dynamic wave (4)
momentum equation	$S_b = S_f$	$\frac{\partial h}{\partial x} = S_b - S_f$	$\frac{1}{gA}\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + \frac{\partial h}{\partial x} = S_b - S_f$	equation 3.27
Boundary conditions required	1	2	2	2
Account for downstream backwater effect and flow reversal	No	Yes	Yes	Yes
Damping of flood peak	No	Yes	Yes	Yes
Account for flow acceleration	No	No	only convective acceleration	Yes

Table 3.1: Limitations when excluding, 1.(inertia and pressure terms), 2.(inertia terms), 3.(pressure term relating to local acceleration) and 4.(none), from the momentum equation [Mays, 2001].

The kinematic wave is the simplest approximation and ignores the terms representing changes ~~in~~ inertia and pressure by assuming that the slope of the water surface is identical to that of the channel bed. This means that the flow is uniform across an interval Δx at a given Δt and mechanisms for flood wave peak attenuation is disregarded. Due to the simplicity of the kinematic wave approximation, attenuation which occurs in a real free flowing channel should not be present. Numerical damping can be induced because of the nature of discretization, but mistakenly corrections has been attempted to mitigate the attenuation by choosing smaller step sizes of Δx and Δt, where instead attempts should be made, to fit the sizes, to imitate the natural attenuation of the free flowing channel. Furthermore only one boundary condition is required, which is specified upstream, meaning that any backwater effects such as storing is neglected. This also means that simulation can be done without knowing the lower end boundary of the channel flow. Because of its simplicity the momentum equation in the form of the kinematic wave approximation has been used and researched extensively [Mays, 2001].

3.1.2 Pipe model

In this subsection a model for a pressurized pipe will be elaborated.

In figure 3.4 an illustration of a pressurized pipe is shown.

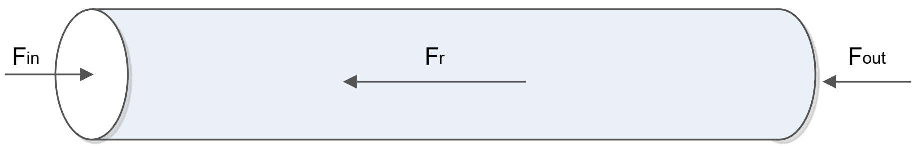

Figure 3.4: Illustrate the forces within a pipe.

A model for a pipe is derivate from Newton second law. Where the three forces shown in figure 3.4, F_{in}, F_r and F_{out}. F_{in} is the force going into the pipe, F_r is the resistance force within the pipe, such as pipe wall and bends. And lastly F_{out} which is the force going out of the pipe.

Newton's second law applied for the pipe,

$$m \cdot \frac{d}{dt} v = F_{in} - F_{out} - F_r \tag{3.28}$$

Where m is the mass of the water $[kg]$, v is the velocity of the water $\left[\frac{m}{s}\right]$ and the resultant forces F_{in}, F_r and F_{out} $[N]$.

The mass of the water can be written as density times the volume of the pipe,

$$m = \rho \cdot V \tag{3.29}$$

Where ρ is the density of liquid $\left[\frac{kg}{m^3}\right]$ and V the volume of the pipe $\left[m^3\right]$.

Assuming the pipe is a cylinder, with a constant cross-sectional area along the pipe, the volume of the pipe can be written as,

$$V = A \cdot l \tag{3.30}$$

Where A is the area of a circle $[m^2]$ and l is the length of the pipe [m].

Inserting the previous expressions in equation 3.28 the following is obtained,

$$\rho \cdot A \cdot l \cdot \frac{d}{dt} v = F_{in} - F_{out} - F_r \tag{3.31}$$

The velocity of the water is written as,

$$v = \frac{Q}{A} \tag{3.32}$$

Where Q is the flow through the pipeline $\left[\frac{m^3}{s}\right]$.

The input, output and resistance force are written as $F = p \cdot A$, where p is the pressure [Pa]. Inserting the expression in 3.31 the following is obtained,

$$\rho \cdot A \cdot l \cdot \frac{d}{dt}\left(\frac{Q}{A}\right) = p_{in} \cdot A - p_{out} \cdot A - p_r \cdot A \tag{3.33}$$

Simplified to,

$$\frac{\rho \cdot l}{A} \cdot \frac{d}{dt}Q = \Delta p - p_r \tag{3.34}$$

Where the pressure difference is expressed as Δp and the area is divided on both sides.

The resistance pressure can now be divided into two resistance, namely form resistance and surface resistance [Prabhata K. Swamee, 2008].

Surface resistance:
The surface resistance describes the pressure loss across a straight pipe.

$$h_f = \frac{8 \cdot f \cdot l}{\pi^2 \cdot g \cdot d^5} \cdot |Q| \cdot Q \tag{3.35}$$

Where h_f is the head loss for the surface resistance of the pipe $[m]$, f is a coefficient for surface resistance, also knows as the friction factor, g is the gravitational acceleration $\left[\frac{m}{s^2}\right]$ and d is the diameter of a circular pipe $[m]$.

Form resistance:
The form resistance describes the pressure losses from fittings in a hydraulic network,

$$h_l = k_L \frac{|v| \cdot v}{2 \cdot g} \tag{3.36}$$

Where h_l is the head loss for the form resistance of the pipe [m] and k_L is the form-loss coefficient.

The form-loss coefficient varies for different components e.g. threaded elbows and tees. The coefficient for the different components can be found in [Munson et al., 1998].

The pressure resistance in the pipe is expressed as:

$$p_r = \rho \cdot g \cdot h \tag{3.37}$$

By substituting p_r in equation 3.34 with equation 3.37 the following is obtained,

$$\frac{\rho \cdot l}{A} \cdot \frac{d}{dt}Q = \Delta p - \rho \cdot g \cdot h \tag{3.38}$$

The surface and form, equations 3.35 and 3.36, can be inserted into equation 3.38,

$$\frac{\rho \cdot l}{A} \cdot \frac{d}{dt}Q = \Delta p - \rho \cdot g \left(k_L \frac{|v| \cdot v}{2 \cdot g} + \frac{8 \cdot f \cdot l}{\pi^2 \cdot g \cdot d^5} \cdot |Q| \cdot Q\right) \tag{3.39}$$

Substitute the head loss factors with k_v, a coefficient describing the pressure losses in the pipes, and the pipe inertance J instead of $\frac{\rho \cdot l}{A}$, the final pipe model is expressed as,

$$J \cdot \frac{d}{dt} Q = \Delta p - \rho \cdot g \cdot k_v \cdot |Q| \cdot Q \tag{3.40}$$

3.1.3 Pump model

In this section a model of a pump will be elaborated. The model for the pump will be used in the simulation to rise the wastewater from a lower position to a higher one.

The model for a pump is expressed as [Kallesøe, 2005].

$$h = -a_{n2} Q^2 + a_{n1} Q \omega + a_{n0} \omega^2 \tag{3.41}$$

Where a_{n2}, a_{n1} and a_{n0} are the constant parameters of the pump, $[Q]$ is the flow of the water, $[\omega]$ is the angular velocity of the pump $[\frac{rad}{s}]$ and h is the pressure delivered by the pump [m]. In table 3.2 the parameters for the pump can be found.

Parameters:	a_{n2}	a_{n1}	a_{n0}
Values:	-0,1830	-1,8004	7,0579

Table 3.2: Parameters for the pump model [Kallesøe, 2005].

3.2 Sewer interconnection

This section will explain how interconnection between channels are modeled and how the mixing between wastewater and wastewater with chemicals.

In figure 3.5 an illustration of a interconnection between two flows are shown.

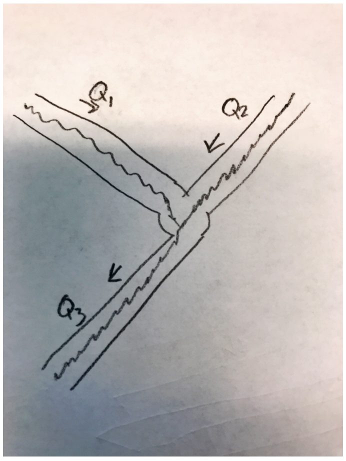

Figure 3.5: Illustration of an interconnection between two flow inputs and one output.

[2]

The two channels Q_1 and Q_2 are connected at the end of each channel here the wastewater from the two channels will be added up. The flow Q_3 is calculated as:

$$Q_3 = Q_1 + Q_2 \tag{3.42}$$

For the mixing between two flows where one of them is transporting chemicals will be done in similar way as for flows.

$$C_3 = \frac{C_1 Q_1 + C_2 Q_2}{Q_3} \tag{3.43}$$

This is done to keep the same amount of chemicals within the wastewater, as the algorithm for transporting chemicals is depended on flow, as seen in section 3.4. If it is not calculated like this, thus when the flow is increasing the same will the chemicals within the wastewater, hence it is needed to divided by Q_3 to get the right amount of chemicals in the wastewater.

3.3 Sewer reservoir

In this section the model for a reservior will be derived.

In figure 3.6 an illustration of a reservoir is shown.

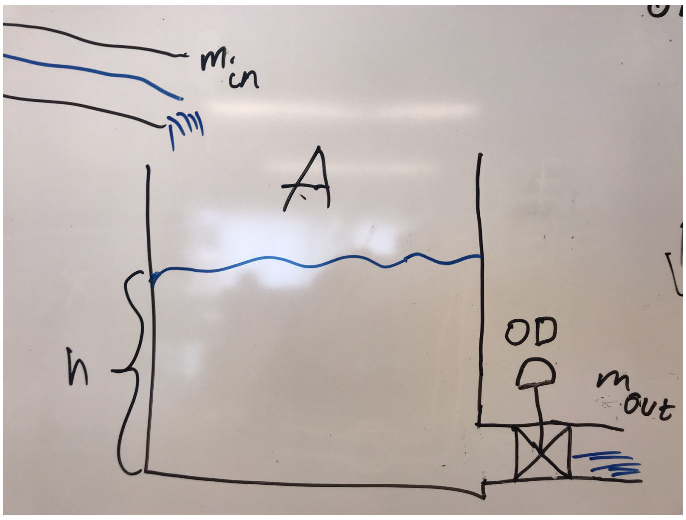

Figure 3.6: An illustration of a reservoir.

This illustration will be used to derive the model for the reservior. From the left an open channel that discharges fluid into the reservoir is shown. The fluid going into the reservoir has a mass flow rate, m_{in} $[\frac{kg}{\cancel{sec}}]$. Within the reservoir the stored fluid has a height, h $[m]$, and the reservoir has a surface area, A. To the bottom right the fluid is discharged, m_{out}.

[2] FiXme Note: Ny tegning

The output mass flow is depended on the openings degree (OD) of the valve. The mass balance equation is derived in [Vojtesek et al.,] and is given as:

$$\frac{dM_{cv}(t)}{dt} = \dot{m}_{in}(t) - \dot{m}_{out}(t) \tag{3.44}$$

Where M_{cv} is the total mass within the control volume $[kg]$, and m_{in} and m_{out} is the mass in and outflow rate of the tank $\left[\frac{kg}{s}\right]$. The mass balance can be written as $M_{cv} = \rho Ah$ where ρ is the densisty $\left[\frac{kg}{m^3}\right]$, A is the area $\left[m^2\right]$ and h is the height [m]. The mass flow rate can be written as $m = \rho Q$, where Q is the flow $\left[\frac{m^3}{s}\right]$. Inserting this into equation 3.44 the following is obtained:

$$\frac{d(\rho Ah(t))}{dt} = \rho Q_{in}(t) - \rho Q_{out}(t) \tag{3.45}$$

By assuming incompressible fluid such that density is constant then:

$$\rho A\frac{dh(t)}{dt} = \rho\left(Q_{in}(t) - Q_{out}(t)\right) \tag{3.46}$$

Simplifying equation 3.46 by dividing with ρA:

$$\frac{dh(t)}{dt} = \frac{1}{A}\left(Q_{in}(t) - Q_{out}(t)\right) \tag{3.47}$$

Because the output flow is depending on the OD of the valve, a model is needed to described the output flow of it. The model for the valve is derived by [Boysen, 2011] and is given as:

$$Q_{out} = kv(OD)\sqrt{\Delta p} \tag{3.48}$$

Where kv(OD) is a function that describes the flow for different ODs. Δp is the pressure drop across the valve [Pa]. The pressure at the bottom of the tank can be found with:

$$P_{bot} = P_{top} + \rho gh \tag{3.49}$$

Where P_{top} is the atmospheric pressure $[Pa]$ and g is the gravitational constant $\left[\frac{m}{s^2}\right]$. The pressure on the output side of the valve is assumed to be approximately 101,375 kPA as it is an open channel. Inserting equation 3.49 into equation 3.48 the following is obtained:

$$Q_{out} = kv(OD)\sqrt{(P_0 + \rho gh) - P_0} = kv(OD)\sqrt{\rho gh} \tag{3.50}$$

Equation 3.50 can be inserted into equation 3.47 and thereby the following model for the reservoir is obtained.

$$\frac{dh(t)}{dt} = \frac{1}{A}\left(Q_{in}(t) - \left(kv(OD)\sqrt{\rho gh(t)}\right)\right) \tag{3.51}$$

Equation 3.51 then gives an expresion where change in height is given as a function of inflow, pressure and opening degree of the valve.

3.4 Transport of concentrate

A model for transport of concentrate in sewer pipes is derrived in the following.

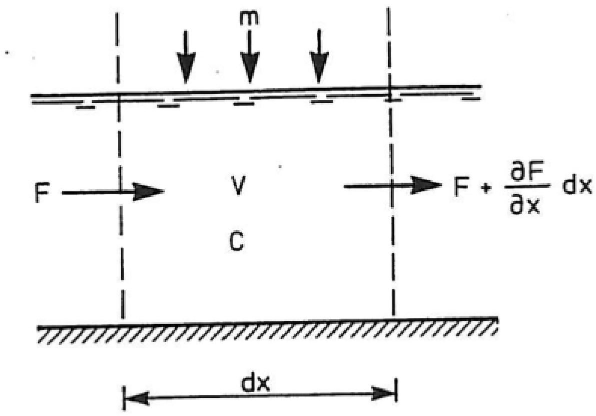

Figure 3.7: Illustration of a control volume containing concentrate.

A derivation of a one dimensional model for transport of concentration, in channel flow, is given. As in section 3.1 certain assumptions is made when deriving the model.

1. The flow of concentrate is assumed to be steady and uniform in the cross section.
2. The anoxic, anaerobic or aerobic processes occurring in the sewer line is neglected

In figure 3.7 the control volume is seen.

From the control volume an equation for conservation of continuity can be derived as in section 3.1. The change of stored concentrate in the control volume is given as:

$$\left(V \cdot C + \frac{\partial(V \cdot C)}{\partial t}\Delta t\right) - V \cdot C = \frac{\partial(V \cdot C)}{\partial t}\Delta t \tag{3.52}$$

Where V is the volume $[m^3]$ and C is the concentrate $[\frac{g}{m^3}]$.

The flux of concentrate in and out of the control volume at a given time Δt is given as:

$$(\phi_{in} - \phi_{out} + \phi_{lateral})\,\Delta t = \phi\cdot\Delta t - \left(\phi + \frac{\partial \phi}{\partial x}\Delta x\right)\Delta t + m\cdot\Delta x\cdot\Delta t = -\frac{\partial \phi}{\partial x}\cdot\Delta x\cdot\Delta t + m\cdot\Delta x\cdot\Delta t \tag{3.53}$$

Where ϕ is the flux $[\frac{g}{sec}]$ and m is lateral flux input to the control volume $[\frac{g}{sec\cdot m}]$.

The continuity equation can now be stated as the change in stored concentration equal to the sum of flux in- and outflow from the control volume as given by equation 3.52 and 3.53:

$$\frac{\partial(V \cdot C)}{\partial t}\Delta t = -\frac{\partial \phi}{\partial x} \cdot \Delta x \cdot \Delta t + m \cdot \Delta x \cdot \Delta t \tag{3.54}$$

When replacing V with A $\cdot\Delta$x and dividing with $\Delta x \cdot \Delta t$ then the basic continuity equation of conservation is obtained:

$$\frac{\partial(A \cdot C)}{\partial t} = -\frac{\partial \phi}{\partial x} + m \tag{3.55}$$

Depending on the desired approximation the flux and lateral inflow terms can be expanded. The expanded lateral term describes a dead zone at the bottom the channel, which can be useful to model if dealing with rugged channel bed. Due to the prismatic assumption in 3.1 of the sewer channel the dead zone in the channel is not investigated further. The flux terms describing convective flow and dispersion can be seen in table 3.3.

Approximation	Convective flow	Convective + (dispersion)
Flux term	$\phi = Q \cdot C$	$\phi = Q \cdot C + \left(-\epsilon \cdot A\frac{\partial C}{\partial x}\right)$

Table 3.3: Table of convective flux term without and with dispersion where Q if flow, C is concentrate, A is area and ϵ is a dispersion coefficient $[\frac{m^2}{sec}]$.

The dispersion term shown in the above table, also known as Fickian diffusion, gives an expression for how the molecules of the concentrate spreads. On a molecular level the the concentrate will to some degree disperse upstream and downstream as shown in figure 3.8.

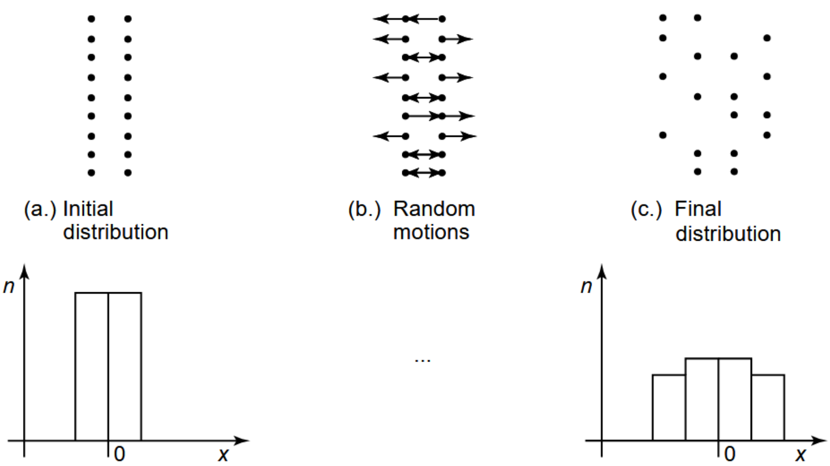

Figure 3.8: Illustration of distribution of convective flow without dispersion (a) and with (c), where dots illustrate molecules of the concentrate within a control volume [Institue of hydromechanics,].

For various concentrates the dispersion coefficient ϵ which varies with temperature can be found in lookup tables [Institue of hydromechanics,].

Inserting the terms in table 3.3 into equation 3.55 then the following expressions of the continuity equation is obtained:

$$\frac{\partial(A \cdot C)}{\partial t} + \frac{\partial(Q \cdot C)}{\partial x} = m \tag{3.56}$$

$$\frac{\partial(A \cdot C)}{\partial t} + \frac{\partial(Q \cdot C)}{\partial x} - \epsilon \cdot \frac{\partial^2(A \cdot C)}{\partial x^2} = m \tag{3.57}$$

4 Simulation

In this chapter the schemes utillized to be able to simulate the non linear parts of the sewage flow with its various concentrations is explained.

4.1 Preissmann scheme

In this section the Saint Venant equations are solved using a numerical method.

The numerical method used for solving the Saint Venant equations, described in section 3.1, is the Preissmann scheme which is based on the box scheme. Other method exist such as, Lax scheme, Abbot-Ionescu scheme, leap-frog scheme, Vasiliev scheme, however the Preissmann scheme is commonly known as the most robust. Basically by using the Preissmann scheme the Saint Venant equations can be discretized, and thereby utilized to simulate the flow and height throughout an open channel.

From section 3.1 the Saint Venant equations for conservation of mass and momentum are derived, they are also shown below.

$$\frac{\partial A(x,t)}{\partial t} + \frac{\partial Q(x,t)}{\partial x} = q(x,t) \tag{4.1}$$

$$\frac{1}{gA}\frac{\partial Q}{\partial t} + \frac{1}{gA}\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + \frac{\partial h}{\partial x} + S_f - S_b = 0 \tag{4.2}$$

In figure 4.1 a single mesh for the Preissmann scheme is illustrated.

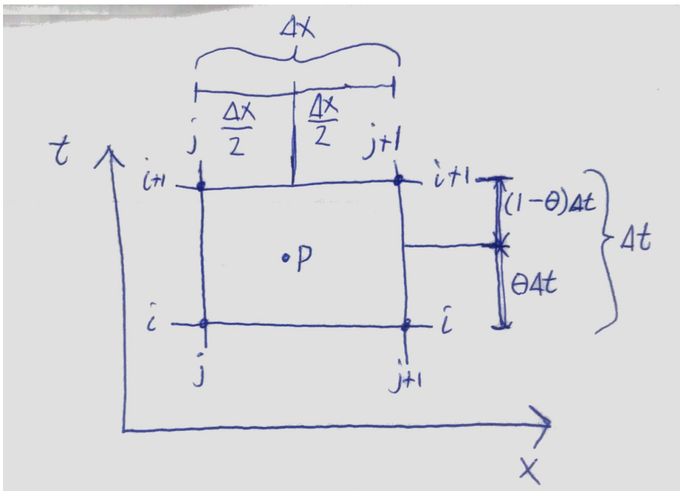

Figure 4.1: Preissmann non-staggered grid scheme.

Where θ is a weighting parameter ranging between zero and one, j is index of cross section and i is index of time. The mesh contains four nodes, (j,i), (j+1,i), (j,i+1) and (j+1,i+1), however in the implementation the dimension of the grid is $\Delta t \times \Delta x$ for $0 \leq x \leq L$ and $0 \leq t$. Where L defines the length of the open channel section. The derivatives in equations 4.1 and 4.2 are calculated as approximation at the point P, which is in the middle of the interval of Δx.The difference between the box scheme and the Preissmann scheme is that the point P is always found at the middle of Δx and the point can only move along the time axis within this mesh by adjusting the weighting parameter θ. This weighting parameter will be elaborated on later. An arbitrary function $f_p(x,t)$ calculated at point P is approximated by [Szymkiewiczi, 1998].

$$f_P \approx \frac{1}{2}(\theta \cdot f_j^{i+1} + (1-\theta) f_j^i) + \frac{1}{2}(\theta \cdot f_{j+1}^{i+1} + (1-\theta) f_{j+1}^i) \tag{4.3}$$

The numerical approximation for the derivatives in equations 4.1 and 4.2 for time and length are shown below [Szymkiewiczi, 1998].

$$\left.\frac{\partial f}{\partial t}\right|_P \approx \frac{1}{2}\left(\frac{f_j^{i+1} - f_j^i}{\Delta t} + \frac{f_{j+1}^{i+1} - f_{j+1}^i}{\Delta t}\right) \tag{4.4}$$

$$\left.\frac{\partial f}{\partial x}\right|_P \approx (1-\theta)\frac{f_{j+1}^i - f_j^i}{\Delta x} + \theta \frac{f_{j+1}^{i+1} - f_j^{i+1}}{\Delta x} \tag{4.5}$$

These approximations from equations 4.4 and 4.5 can therefore be inserted for the derivatives in the Saint Venant equations 4.1 and 4.2 and thereby achieve the following,

$$\theta \frac{Q_{j+1}^{i+1} - Q_j^{i+1}}{\Delta x} + (1-\theta)\frac{Q_{j+1}^i - Q_j^i}{\Delta x} + \frac{1}{2}\frac{A_{j+1}^{i+1} - A_{j+1}^i}{\Delta t} + \frac{1}{2}\frac{A_j^{i+1} - A_j^i}{\Delta t} = q_p \tag{4.6}$$

$$\frac{1}{gA_p}\left(\frac{1}{2}\left(\frac{Q_{j+1}^{i+1}-Q_{j+1}^{i}}{\Delta t}+\frac{Q_{j}^{i+1}-Q_{j}^{i}}{\Delta t}\right)\right)+\frac{1}{gA_p}\left(\frac{\theta}{\Delta x}\left(\left(\frac{Q^2}{A}\right)_{j+1}^{i+1}-\left(\frac{Q^2}{A}\right)_{j}^{i+1}\right)\right)+$$
$$\frac{1-\theta}{\Delta x}\left(\left(\frac{Q^2}{A}\right)_{j+1}^{i}-\left(\frac{Q^2}{A}\right)_{j}^{i}\right)+\theta\left(\frac{h_{j+1}^{i+1}-h_{j}^{i+1}}{\Delta x}\right)+$$
$$(1-\theta)\left(\frac{h_{j+1}^{i}-h_{j}^{i}}{\Delta x}\right)+S_f-S_b=0 \quad (4.7)$$

By discretized the Saint Venant equations they can be used in a simulation to calculate parameters for the open channel model. The mesh shown in figure 4.1 is used to calculate the node (j+1,i+1) by knowing the previous values in time and length (j,i), (j+1,i) and (j,i+1). Therefore some initial condition must be known to calculate the parameters for the open channel in the first iteration. The flow, at t=0, must be known throughout the pipe, furthermore the flow that will enter the channel for $t \leq 0$ must be known, as shown in figure 4.2.

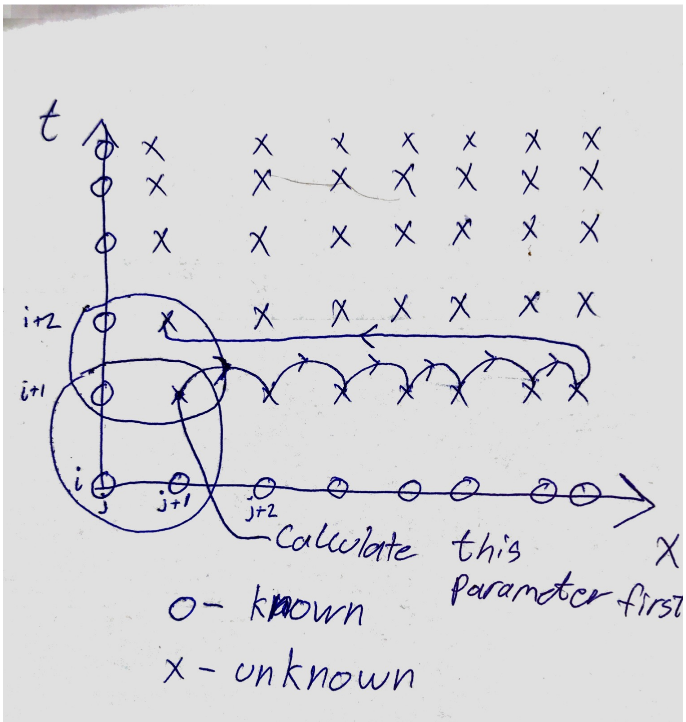

Figure 4.2: Preissmann non-staggered grid scheme example of calculation pattern.

By knowing the flow and parameters for the pipe, the height can be calculated in the initialization nodes, which will be elaborated on later. With equation 4.6, the flow (j+1,i+1) can be calculated by knowing the flow and height in the previous nodes (j,i), (j+1,i) and (j,i+1) as illustrated with the box in the left bottom corner in figure 4.2.

For the Preissmann scheme to be numerical stable the θ parameter is $\theta \leq 0,5$. Does closer θ is to 0,5 the more accurate it is, however it is also more likely to be unstable, therefore from [Krvavica and Travaš, 2014] it is suggested to place θ in the range of $0,55 \leq \theta \leq 0.65$ for practical analysis.

In the following section the calculating of the flow and height in each node will be explained.

Iteration scheme

In this section the scheme for calculating the flow and height in each iteration will be explained [Michelsen, 1976].

The discretized continuity equation 4.6 from section 4.1 is solved for the desired flow,

$$Q_{j+1}^{i+1} = -\frac{1}{2\theta}\left(A_{j+1}^{i+1} - H\right)\frac{\Delta x}{\Delta t} \tag{4.8}$$

Where H is a parameter where all the previous flows and areas are known,

$$H = \left(2(1-\theta)Q_j^i - 2(1-\theta)Q_{j+1}^i + 2\theta Q_j^{i+1} + 2q(x,t)\Delta x\right)\frac{\Delta t}{\Delta x} - A_j^{i+1} + A_j^i + A_{j+1}^i \tag{4.9}$$

In equation 4.8 the flow Q_{j+1}^{i+1} is a function of the unknown area A_{j+1}^{i+1}, and by subtracting the flow on each side the following is achieved,

$$0 = -Q_{j+1}^{i+1} - \frac{1}{2\theta}\left(A_{j+1}^{i+1} - H\right)\frac{\Delta x}{\Delta t} \tag{4.10}$$

By calling the right hand side of equation 4.10 for V the following is obtained,

$$V = -Q_{j+1}^{i+1} - \frac{1}{2\theta}\left(A_{j+1}^{i+1} - H\right)\frac{\Delta x}{\Delta t} \tag{4.11}$$

Equation 4.11 can now be solved by finding the zero for the function V using newton method. However there are still two unknowns in equation 4.11, Q_{j+1}^{i+1} and A_{j+1}^{i+1}. Q_{j+1}^{i+1} can be replaced with the following,

$$Q = \left(0.46 - 0.5 \cdot cos\left(\pi\frac{h}{d}\right) + 0.04 \cdot cos\left(2\pi\frac{h}{d}\right)\right)Q_f \tag{4.12}$$

Which describes the flow in an open channel by knowing the height, diameter of the pipe and the flow for a fully filled pipe, Q_f. Q_f can be found with the following equation,

$$Q_f = -3.02 \cdot ln\left(\frac{0.74 \cdot 10^{-6}}{d\sqrt{d \cdot I_e}} + \frac{k}{3.71 \cdot d}\right)d^2\sqrt{d \cdot I_e} \tag{4.13}$$

Where k is a roughness factor found in?? [1] and I_e is a friction term. Equation 4.12 is inserted into equation 4.11 and thereby the following is obtained,

$$V = -Q_f\left(0,46 - 0,5 \cdot cos\left(\pi\frac{h_{j+1}^{i+1}}{d}\right) + 0,04 \cdot cos\left(2\pi\frac{h_{j+1}^{i+1}}{d}\right)\right)\frac{\Delta t}{\Delta x} - \frac{1}{2\theta}\left(A_{j+1}^{i+1} - H\right)$$

[1] FiXme Note: kilde til hvor vi finder den

$$(4.14)$$

Furthermore Q_f from equation 4.13 is inserted into equation 4.14,

$$V = -72\left(\frac{d}{4}\right)^{0.635}\pi\left(\frac{d}{2}\right)^{2} I_e^{0,5}\left(0,46 - 0,5\cdot cos\left(\pi\frac{h_{j+1}^{i+1}}{d}\right) + 0,04\cdot cos\left(2\pi\frac{h_{j+1}^{i+1}}{d}\right)\right)\frac{\Delta t}{\Delta x} - \frac{1}{2\theta}\left(A_{j+1}^{i+1} - H\right) \quad (4.15)$$

V is a function of the height h_{j+1}^{i+1} as the height is the only unknown in finding the area A_{j+1}^{i+1} for the channel. The area for a circular channel is calculated with the following,

$$A = \frac{d^2}{4}\cdot acos\left(\frac{\frac{d}{2} - h}{\frac{d}{2}}\right) - \sqrt{h(d-h)}\left(\frac{d}{2} - h\right) \quad (4.16)$$

As mention the continuity equation 4.15 can be solved by finding the zero for the function V. Newtons method is used to find better approximations to the roots/zeres of a real valued fuction. By using the newton method the roots of equation 4.15 can be found, which will correspond to the height in the channel. The approximation is,

$$(h_{j+1}^{i+1})_{k+1} = (h_{j+1}^{i+1})_k - \frac{V_k}{V_k'} \quad (4.17)$$

Where k is the number of iterations, V′ is the differentiated of V with respect to height, $(h_{j+1}^{i+1})_k$ is an initial guess of the root and $(h_{j+1}^{i+1})_{k+1}$ is a better approximate of the height. This calculation is iterated until a satisfied approximation is achieved which fulfills the requirement,

$$\left(h_{j+1}^{i+1}\right)_k - (h_{j+1}^{i+1})_{k-1} < (\epsilon\cdot h_{j+1}^{i+1})_k \quad (4.18)$$

Where ϵ is a small tolerance number, e.g. 5 centimeter variation in water height. Thereby the water height can be found and the area of the water can be calculated with equation 4.16 [2] and thereafter equation 4.8 can be used to calculate the flow of the node.

This calculation is performed for each node in the Preissmann scheme, therefore it is an iterative method of solving the flow for an open channel.

In the following figures the flow 4.3a and the water height 4.3b for the output of the channel is plotted

[2] FiXme Note: afsnit om de forskellige ligninger til beregning af Areal, Ie,Ib,Qf,Q

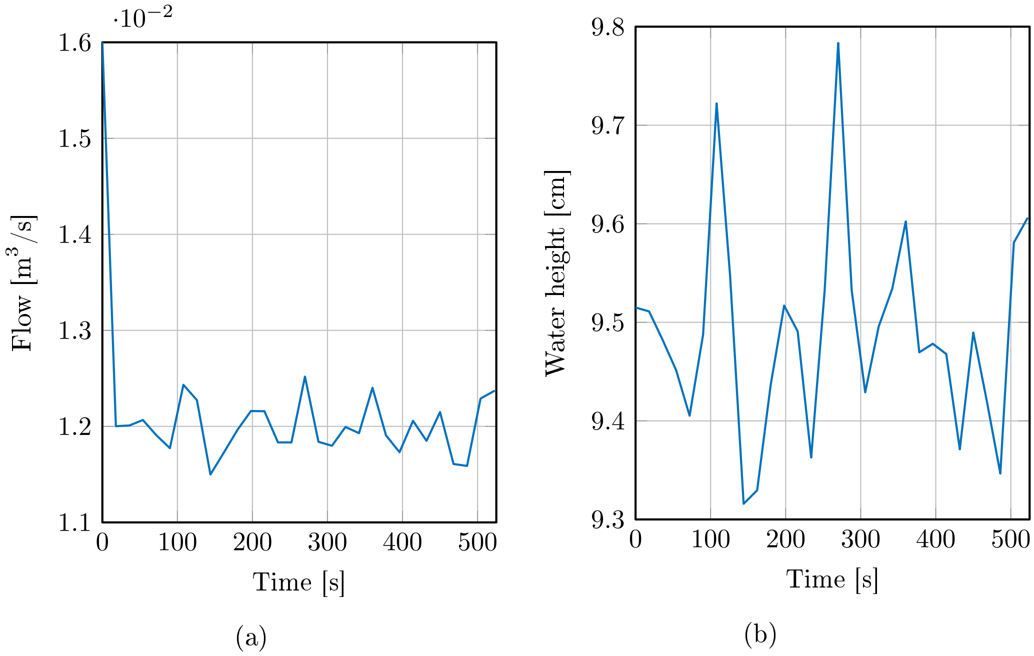

Figure 4.3: (a) Output flow, (b) the water height at the output of the channel.

These figures shows that whether the height is increasing or decreasing the flow is following, which is what is expected, because a higher water level will result in a higher flow rate and vice versa.

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Appendix A

Rettelser

Note: Ny tegning 4

Note: Måske vi burde have noget med at grundvand faktisk også trænger ind i kloakkerne. Det kan være vigtigt i forbindelse med et statisk offset i vand flow når vi skal til at simulere. 4

Note: Lav en table over disse 3 tilstande i kloakken 5

Note: Hvad mener du med mere skarp her Thomas? i forhold til numerical computation 5

Note: Ny tegning 7

Note: Ny tegning og sikkert også caption 17

Note: Ny tegning 22

Note: kilde til hvor vi finder den 30

Note: afsnit om de forskellige ligninger til beregning af Areal, Ie,Ib,Qf,Q 31