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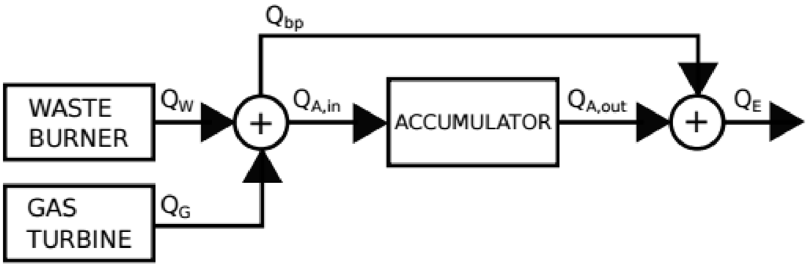
MPC miniproject 17gr833

This miniproject is about optimizing the profit of a power plant burning waste and gas to produce power. MPC is used to optimize the profit. The model of the power plant can be seen on the following figure.



Q_W is the power from burning waste and Q_G is the power from burning gas. Some of the produced energy can be stored in the accumulator, $Q_{A,in}$, or bypass the accumulator, Q_{bp}. The energy leaving the power plant and thereby sold, Q_E, consist of the bypass energy and energy from the accumulator. The system equations are given as the following:

$$Q_W + Q_G = Q_{bp} + Q_{A,in}$$

$$Q_E = Q_{bp} + Q_{A,out}$$

The dynamics for the accumulator is given as a discrete integrator, where the accumulated energy, E_A is limited.

$$E_A[k+1] = E_A[k] + \big(Q_{A,in}[k] - Q_{A,out}[k]\big)T_s$$

$$0 \le E_A \le 200$$

The sampling time, T_s, is in hours. The constraints of the energy in the system is given as the following:

$$0 \le Q_W \le 40$$

$$0 \le Q_G \le 20$$

$$0 \le Q_{A,in} \le 50$$

$$0 \le Q_{A,out} \le 25$$

The prices pr. MWh for burning gas and waste to generate electricity is known and denoted P_G and P_W respectively. The objective is to control the production of electricity to maximize the profit by use of MPC, based on the known selling price of the electricity pr. MWh, P_E. The profit can thereby be expressed over the horizon L as a sum of the income minus the expenses for producing the electricity at a given time.

$$predicted\ profit = \sum_{k=1}^{L}\big(P_E[k]Q_E[k] - (P_G[k]Q_G[k] + P_W[k]Q_W[k])\big)T_s$$

The MPC problem needs to maximize the profit for the interval [k;k+L] hours and use the calculated “control” for time k, and move forward to optimize the problem for [k+1;k+L+1] and so on. Maximizing the profit equals minimizing the negative profit. This sums the following MPC problem implemented by use of CVX in the attached MATLAB script *17gr833.m*

For k=1:M-L+1

Minimize $-(P_E(k{:}L+k-1)' * Q_E - (P_G(k{:}L+k-1)' * Q_G + P_W(k{:}L+k-1)' * Q_W)) * Ts)$

S.T.

$$Q_W + Q_G = Q_{bp} + Q_{A,in}$$

$$Q_E = Q_{bp} + Q_{A,out}$$

$$Q_{W,min} * ones(L,1) \le Q_W \le Q_{W,max} * ones(L,1)$$

$$Q_{G,min} * ones(L,1) \le Q_G \le Q_{G,max} * ones(L,1)$$

$$Q_{A,in,min} * ones(L,1) \le Q_{A,in} \le Q_{A,in,max} * ones(L,1)$$

$$Q_{A,out,min} * ones(L,1) \le Q_{A,out} \le Q_{A,out,max} * ones(L,1)$$

$$E_A(2{:}L+1) = E_A(1{:}L) + (Q_{A,in} - Q_{A,out}) * Ts$$

$$E_{A,min} * ones(L+1,1) \le E_A \le E_{A,max} * ones(L+1,1)$$

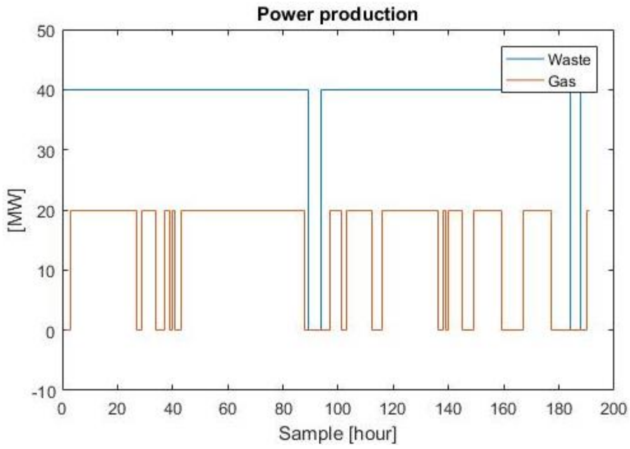
$$E_A(1) = E_{A,sys}(k)$$

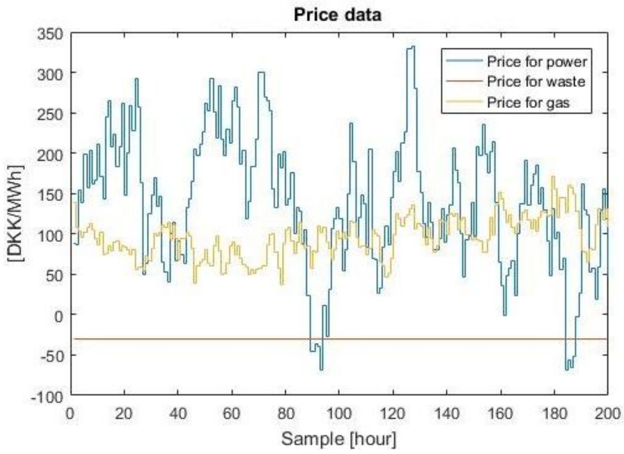
"save all optimized variables for time and calculate the accumulated energy to time k+1"

end

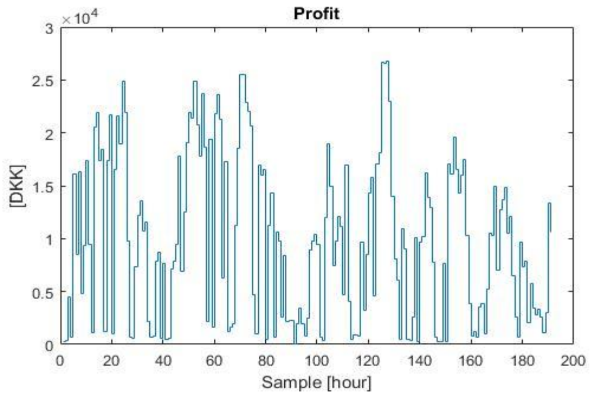
An extra constraint has been added to the problem by the group, in extension to the ones stated on page 1 from the exercise formulation. This constraint is because every optimization in the for loop is done from new empty variables for the optimization algorithm to work on. Because of this, the first entry in the accumulated energy, E_A, is constrained to the known accumulated energy based on previous runs.

The outcome of the MPC algorithm over a prediction horizon of L=10 for both control and prediction horizon, gives the following results. The production of power is determined based on what gives most profit. Since money is earned by burning waste, waste is burned except at the times where the energy price is more negative, than the earned money for burning the waste. The production of power from gas is based on what is most profitable. NB. The following graphs are produced without the rng(1) seed.

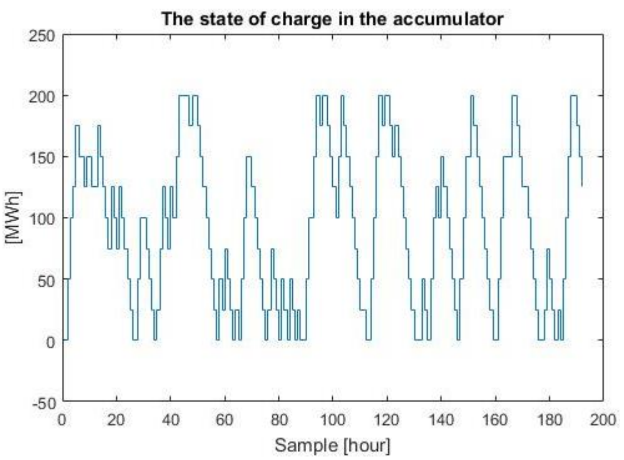


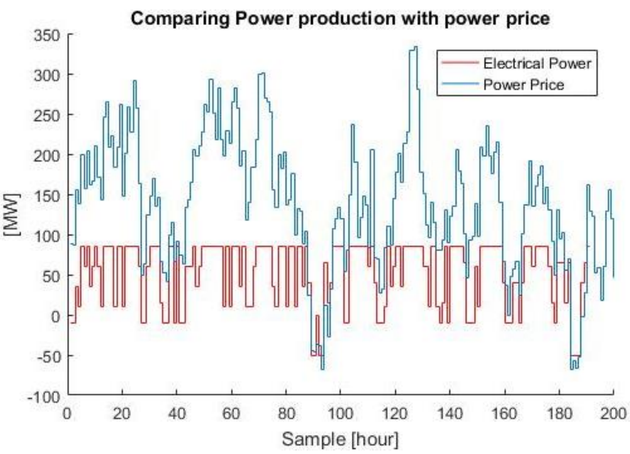


This yields a profit which is positive for all time samples.



It is however very low or even zero at some samples, e.g. because the produced energy is stored in the accumulator instead of sold directly to maximize the profit. It can be seen on the following graphs that the sold power is decreased or even in negative when the power prices are low and instead power is stored in the accumulator and sold when the prices rise again. This behavior can easily be seen based on the price in the prediction/control horizon, as the following graphs illustrate.





In the shown run, the total profit using MPC is $1.86 * 10^6$ DKK. In comparison, if both fuel and gas was burned on full rate to obtain maximum amount of power to sell at all time, the total profit would have been $1.52 * 10^6$ DKK. This is an improvement of 22.4 %.