# Model Predictive Control of a Sewer System

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Group 1030

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# Agenda



- Implementering
  - Initialisering
  - Simulering
  - Display
- Kontrol
  - Linearisering
  - MPC
- Resultat
- Diskussion/Konklusion

# Overview

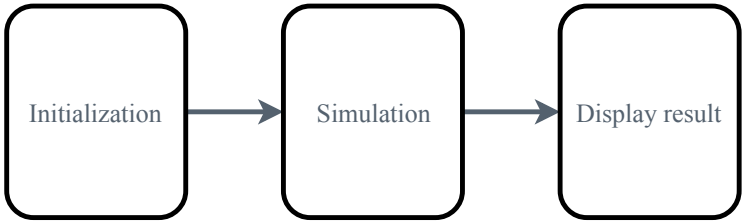
Jacob Naundrup Pedersen



- Implementation
- Kontrol
- Resultater
- Diskussion
- Konklusion

# Implementering





1. Pipe
   - Length [m]
   - Sections (Number of sections the pipe should be split in to)
   - $S_b$ (Slope) [‰]
   - $\Delta x$ = Length/Sections [m]
   - Diameter [meter]
   - Theta (parameter used in Preissmann scheme)
   - $Q_f [m^3/s]$
   - Side/lateral inflow present
   - Section location in data
2. Tank
   - Size $[m^3]$
   - Height [m]
   - Area = Size / Height $[m^2]$
   - Maximum outflow $[m^3/s]$
   - Section location in data

- Rør specifikationer

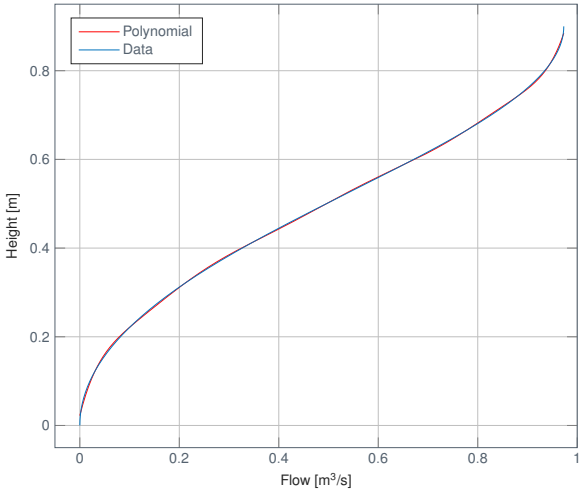
| Fields | length | sections | Dx | Sb | d | Theta | Qf | side_inflow | data_location |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 700 | 35 | 20 | 0.0030 | 0.9000 | 0.6500 | 0.9730 | 0 | 1 |
| 2 | 303 | 15 | 20.2000 | 0.0030 | 0.9000 | 0.6500 | 0.9730 | 0 | 3 |
| 3 | 27 | 2 | 13.5000 | 0.0030 | 1 | 0.6500 | 1.2843 | 1 | 4 |
| 4 | 155 | 8 | 19.3750 | 0.0041 | 1 | 0.6500 | 1.5014 | 0 | 5 |
| 5 | 295 | 14 | 21.0714 | 0.0122 | 0.8000 | 0.6500 | 1.4386 | 0 | 6 |
| 6 | 318 | 15 | 21.2000 | 0.0053 | 0.9000 | 0.6500 | 1.2932 | 1 | 7 |
| 7 | 110 | 5 | 22 | 0.0036 | 0.9000 | 0.6500 | 1.0658 | 1 | 8 |
| 8 | 38 | 2 | 19 | 0.0024 | 1 | 0.6500 | 1.1487 | 1 | 9 |
| 9 | 665 | 30 | 22.1667 | 0.0030 | 1 | 0.6500 | 1.2843 | 1 | 10 |
| 10 | 155 | 7 | 22.1429 | 8.0000e-04 | 1 | 0.6500 | 0.6632 | 0 | 11 |
| 11 | 955 | 47 | 20.3191 | 0.0029 | 1.2000 | 0.6500 | 2.0415 | 1 | 12 |
| 12 | 304 | 15 | 20.2667 | 0.0030 | 1.2000 | 0.6500 | 2.0764 | 0 | 13 |
| 13 | 116 | 5 | 23.2000 | 0.0021 | 1.2000 | 0.6500 | 1.7373 | 1 | 14 |
| 14 | 283 | 12 | 23.5833 | 0.0017 | 1.4000 | 0.6500 | 2.3463 | 1 | 15 |
| 15 | 31 | 2 | 15.5000 | 0.0019 | 1.4000 | 0.6500 | 2.4805 | 1 | 16 |
| 16 | 125 | 6 | 20.8333 | 0.0021 | 1.6000 | 0.6500 | 3.7075 | 0 | 17 |
| 17 | 94 | 4 | 23.5000 | 0.0013 | 1.5000 | 0.6500 | 2.4609 | 0 | 18 |
| 18 | 360 | 18 | 20 | 0.0046 | 1.6000 | 0.6500 | 5.4872 | 1 | 19 |
| 19 | 736 | 38 | 19.3684 | 0.0012 | 1.6000 | 0.6500 | 2.8026 | 0 | 20 |

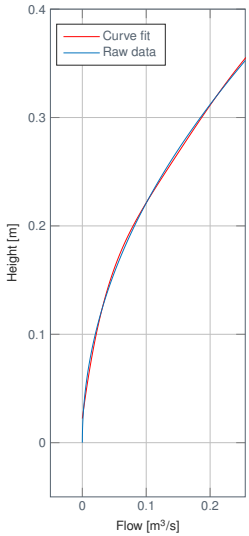
- Tank specifikationer

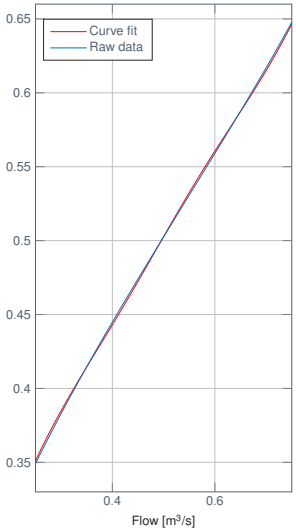
| Field | Value |
|---|---|
| size | 90 |
| height | 10 |
| area | 9 |
| Q_out_max | 0.9730 |
| data_location | 2 |

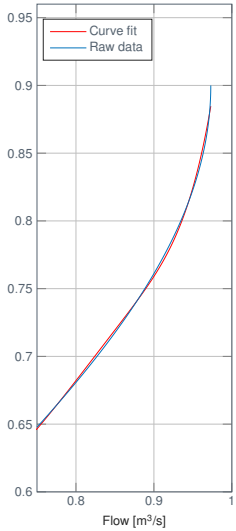
- System specifikationer

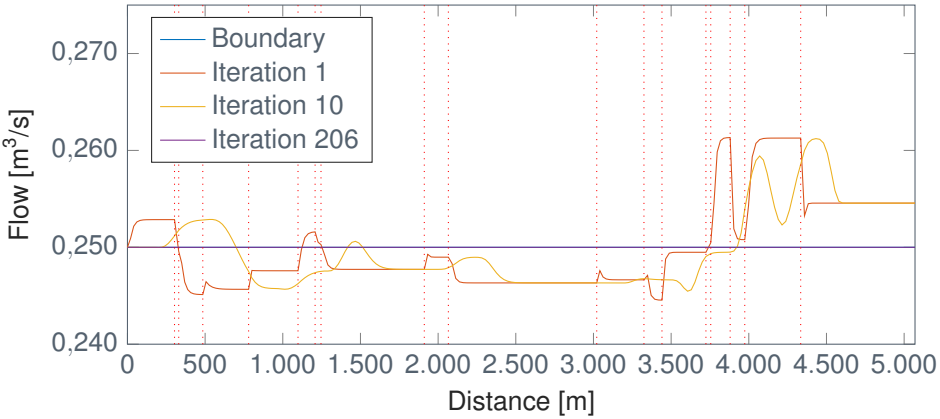
| Fields | type | component | sections |
|---|---|---|---|
| 1 | 'Pipe' | 1 | 35 |
| 2 | 'Tank' | 1 | 1 |
| 3 | 'Pipe' | 18 | 245 |
| 4 | 'Total' | 20 | 281 |

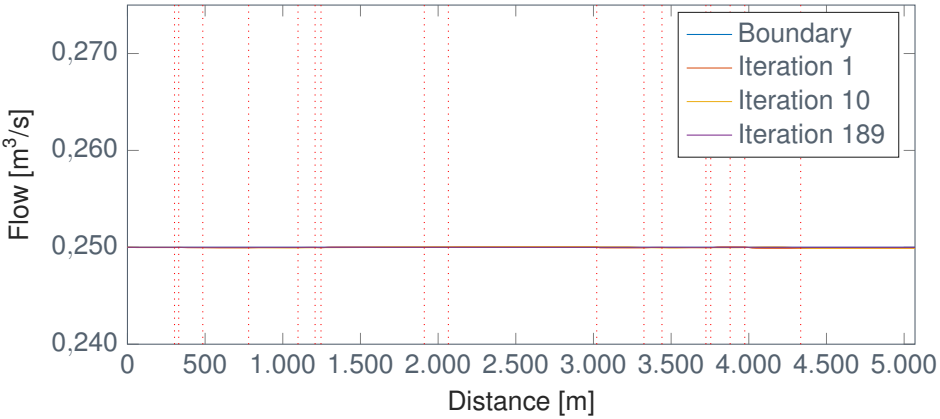




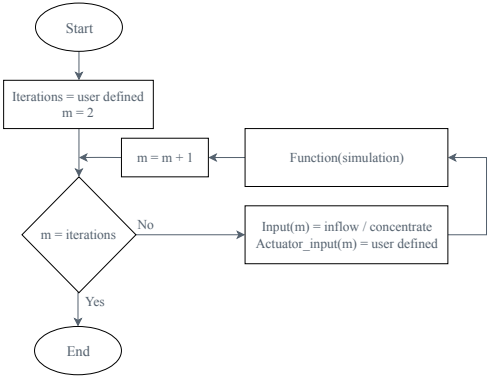



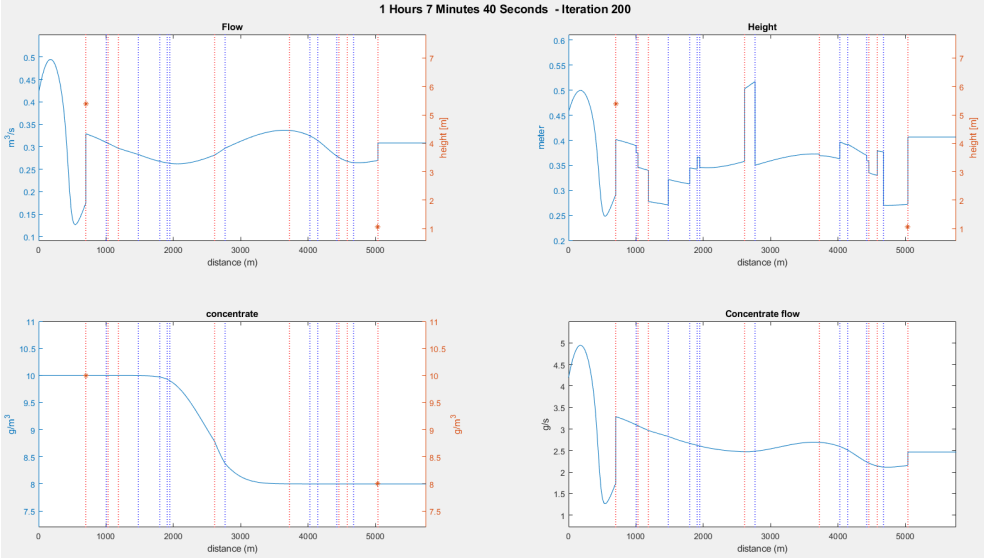







- Itererer igennem rør og tank for hvert tidsskridt





- Linearisering af ulinear model
- Opstilles på state space form

$$\frac{\partial A(x,t)}{\partial t} + \frac{\partial Q(x,t)}{\partial x} = 0 \tag{1}$$

$$\frac{\partial A(h)}{\partial h}\frac{\partial h(x,t)}{\partial t} + \frac{\partial Q(h)}{\partial h}\frac{\partial h(x,t)}{\partial x} = 0 \tag{2}$$

- Priessmann scheme
- Opsat på matrix og vektor form

$$\underbrace{\begin{bmatrix} \frac{1}{2\Delta t}\frac{\partial A}{\partial h} - \frac{\theta}{\Delta x}\frac{\partial Q}{\partial h} & \frac{1}{2\Delta t}\frac{\partial A}{\partial h} + \frac{\theta}{\Delta x}\frac{\partial Q}{\partial h} \end{bmatrix}}_{a\ \ b} \begin{bmatrix} h_j^{i+1} \\ h_{j+1}^{i+1} \end{bmatrix} = - \underbrace{\begin{bmatrix} \frac{-1}{2\Delta t}\frac{\partial A}{\partial h} - \frac{(1-\theta)}{\Delta x}\frac{\partial Q}{\partial h} & \frac{-1}{2\Delta t}\frac{\partial A}{\partial h} + \frac{(1-\theta)}{\Delta x}\frac{\partial Q}{\partial h} \end{bmatrix}}_{c\ \ d} \begin{bmatrix} h_j^{i} \\ h_{j+1}^{i} \end{bmatrix} \tag{3}$$

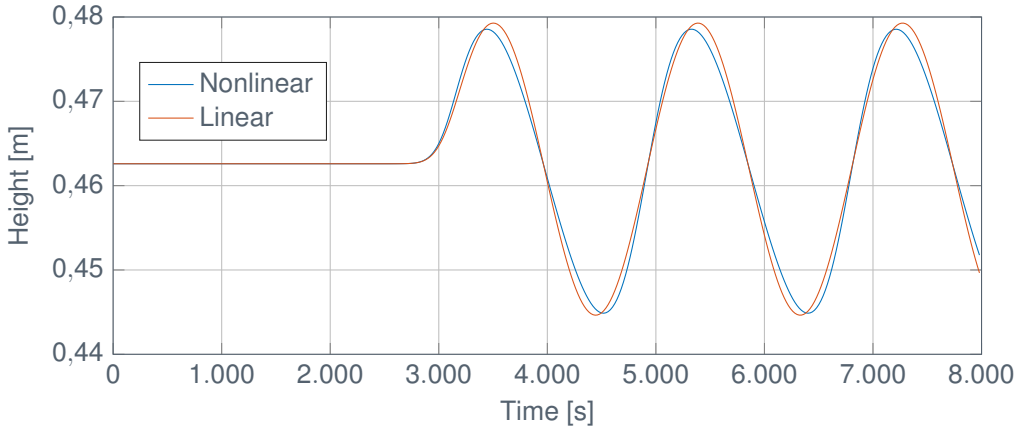
$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & b_1 & 0 & \cdots & 0 \\ 0 & a_1 & b_2 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & a_{m-1} & b_m \end{bmatrix}}_{\xi} \underbrace{\begin{bmatrix} h_0^{i+1} \\ h_1^{i+1} \\ h_2^{i+1} \\ \vdots \\ h_m^{i+1} \end{bmatrix}}_{x(k+1)} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ c_0 & d_1 & 0 & \cdots & 0 \\ 0 & c_1 & d_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & c_{m-1} & d_m \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} h_0^{i} \\ h_1^{i} \\ h_2^{i} \\ \vdots \\ h_m^{i} \end{bmatrix}}_{x(k)} + \underbrace{\begin{bmatrix} 1 \\ -a_0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}}_{B} h_0^{i+1} + \underbrace{\begin{bmatrix} \frac{dh}{dQ} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}}_{B_d} d_0^{i+1} \tag{4}$$

- e - Inflow
- f - Reducering af højden i tank
- g - Højden i det efterfølge rør

$$\underbrace{\begin{bmatrix} b_{1,2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & a_{2,1} & b_{2,2} \end{bmatrix}}_{\xi} \underbrace{\begin{bmatrix} h_{1,2}^{i+1} \\ h_{tank}^{i+1} \\ h_{2,0}^{i+1} \\ h_{2,1}^{i+1} \end{bmatrix}}_{x(k+1)} = \underbrace{\begin{bmatrix} d_{1,2} & 0 & 0 & 0 \\ e & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & c_{2,0} & d_{2,1} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} h_{1,2}^{i} \\ h_{tank}^{i} \\ h_{2,0}^{i} \\ h_{2,1}^{i} \end{bmatrix}}_{x(k)} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & -f \\ 0 & g \\ 0 & 0 \end{bmatrix}}_{B} \begin{bmatrix} h_{0}^{i+1} \\ u_{tank} \end{bmatrix} \tag{5}$$

- Samligning af ulinear og linear model
- System setup
- Sinus input

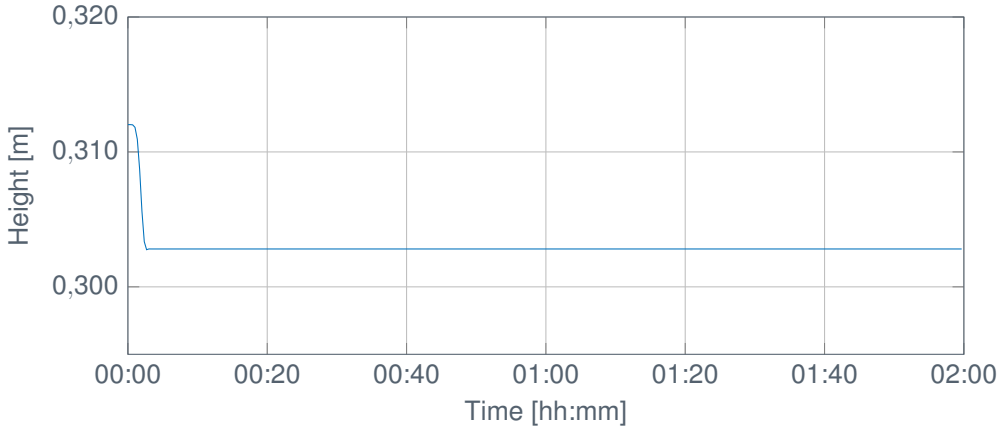
| Type | Components | Sections |
|---|---|---|
| Pipe | 1 | 35 |
| Tank | 1 | 1 |
| Pipe | 18 | 227 |
| Total | 20 | 263 |

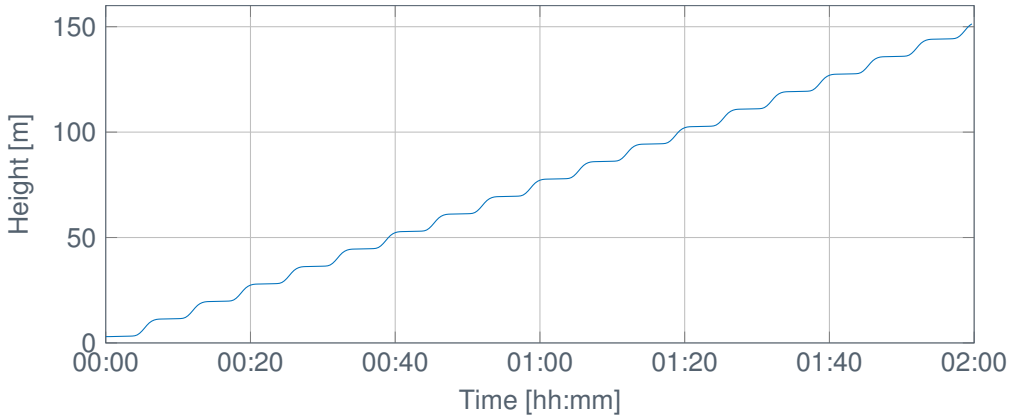


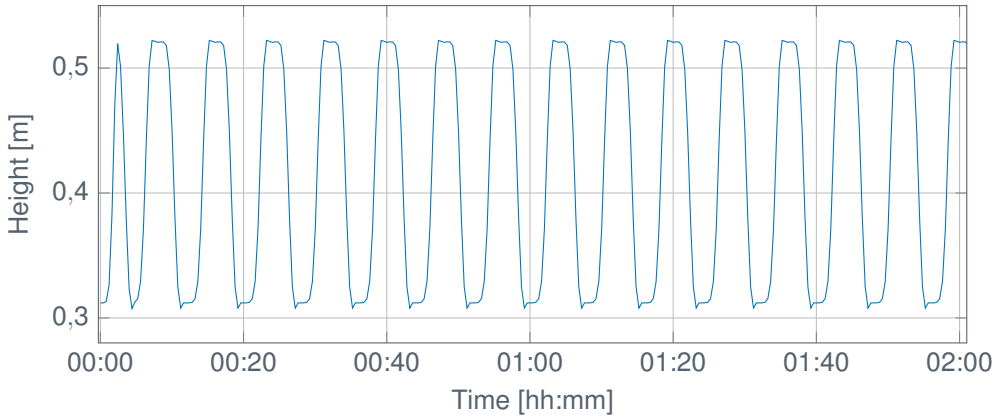
- Cost function
  - Begrænset til minimiere af output
- Constraints
  - Højde
  - Kontrol input
- Linear model

- Bestemmelse af Prediction horizon
  - Flow profiler
  - Industri
- Begrænsning af Prediction horizon
- System setup
- Forstyrrelse input

| Fields | type | component | sections |
|---|---|---|---|
| 1 | 'Pipe' | 1 | 5 |
| 2 | 'Tank' | 1 | 1 |
| 3 | 'Pipe' | 1 | 5 |
| 4 | 'Total' | 3 | 11 |





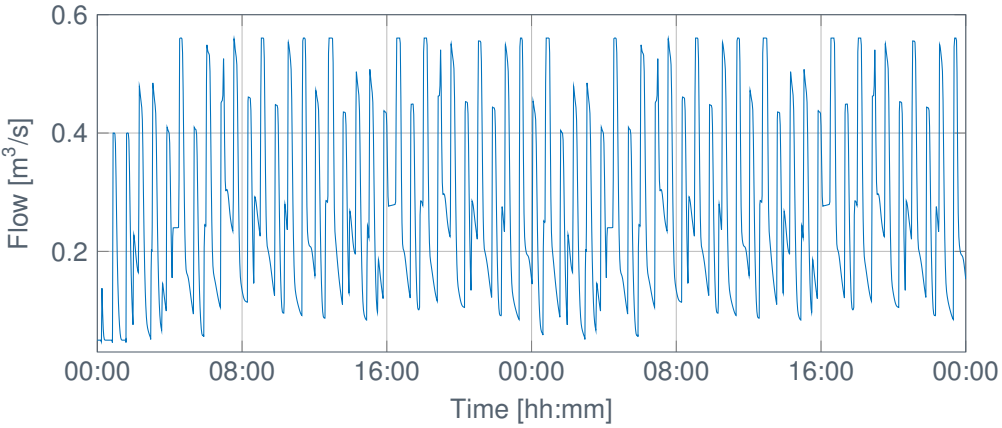


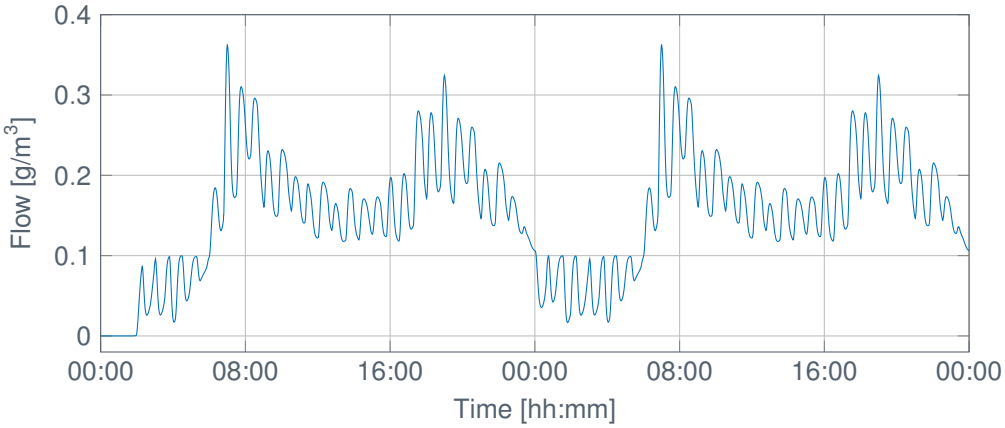
- System setup
- Flow profiler

| Type | Component | Sections |
|---|---|---|
| Pipe | 1 | 35 |
| Tank | 1 | 1 |
| Pipe | 17 | 207 |
| Tank | 1 | 1 |
| Pipe | 1 | 38 |
| Total | 21 | 282 |

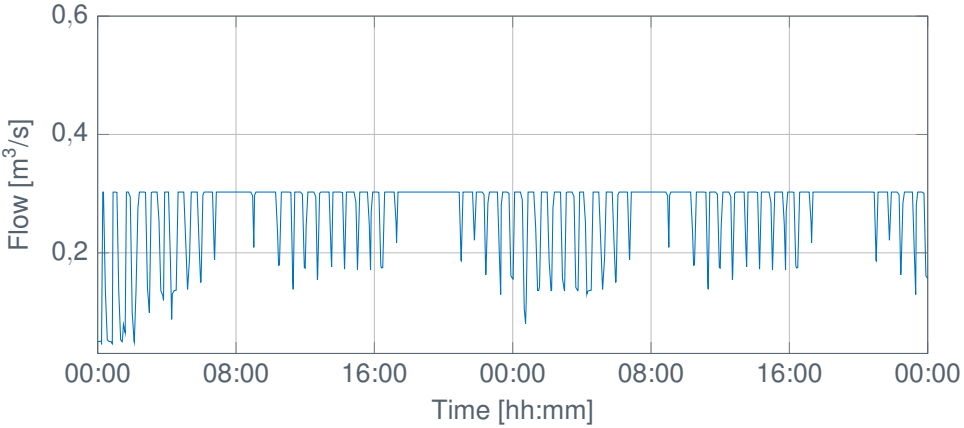
# Resultat
## Første test







- Over dimensioneret tank
- Konstant output af tank



# Diskussion



- Courant's number
- Model reduction

# Konklusion



- Simulering
- MPC