# Model Predictive Control of a Sewer System

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Group 1030

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Implementer

Simuleri

Kontrol

Lineariserin

Resulta

Diskussion/Konklusion

Implementering
Initialisering
Simulering
Display

Kontrol

Linearisering MPC

Resultat



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#### Implementering

Simuleri

#### Kontro

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Resultat

- Implementation
- ► Kontrol
- ▶ Resultater
- ▶ Diskussion
- ► Konklusion



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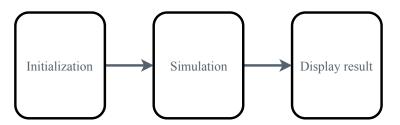
#### Implementering

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#### Initialisering

Diskussion/Konklusion

# Length [m]

- Sections (Number of sections the pipe should be split in to)
- ► S<sub>b</sub> (Slope) [‰]
- $\triangle x = \text{Length/Sections [m]}$
- ▶ Diameter [meter]
- ► Theta (parameter used in Preissmann scheme)
- $ightharpoonup Q_f[m^3/s]$
- Side/lateral inflow present
- Section location in data

# 2. Tank

1. Pipe

- ► Size [m³]
- ► Height [m]
- ► Area = Size / Height [m²]
- ► Maximum outflow [m³/s]
- Section location in data



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# ▶ Rør specifikationer

Fields	length	■ sections	■ Dx	→ Sb	⊞ d	H Theta	■ Qf	$\coprod$ side_inflow	data_location
1	700	35	20	0.0030	0.9000	0.6500	0.9730	0	1
2	303	15	20.2000	0.0030	0.9000	0.6500	0.9730	0	3
3	27	2	13.5000	0.0030	1	0.6500	1.2843	1	4
4	155	8	19.3750	0.0041	1	0.6500	1.5014	0	5
5	295	14	21.0714	0.0122	0.8000	0.6500	1.4386	0	6
6	318	15	21.2000	0.0053	0.9000	0.6500	1.2932	1	7
7	110	5	22	0.0036	0.9000	0.6500	1.0658	1	8
8	38	2	19	0.0024	1	0.6500	1.1487	1	9
9	665	30	22.1667	0.0030	1	0.6500	1.2843	1	10
10	155	7	22.1429	8.0000e-04	1	0.6500	0.6632	0	11
11	955	47	20.3191	0.0029	1.2000	0.6500	2.0415	1	12
12	304	15	20.2667	0.0030	1.2000	0.6500	2.0764	0	13
13	116	5	23.2000	0.0021	1.2000	0.6500	1.7373	1	14
14	283	12	23.5833	0.0017	1.4000	0.6500	2.3463	1	15
15	31	2	15.5000	0.0019	1.4000	0.6500	2.4805	1	16
16	125	6	20.8333	0.0021	1.6000	0.6500	3.7075	0	17
17	94	4	23.5000	0.0013	1.5000	0.6500	2.4609	0	18
18	360	18	20	0.0046	1.6000	0.6500	5.4872	1	19
19	736	38	19.3684	0.0012	1.6000	0.6500	2.8026	0	20



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► Tank specifikationer

Field 📤	Value
ize size	90
Height	10
area area	9
Q_out_max	0.9730
data_location	2



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► System specifikationer

Fields	type type	component	
1	'Pipe'	1	35
2	'Tank'	1	1
3	'Pipe'	18	245
4	'Total'	20	281



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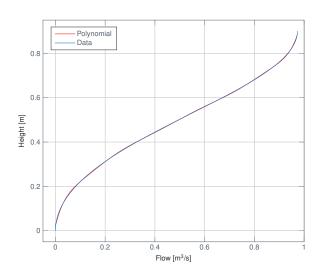
Kontrol

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Initialisering Simulering

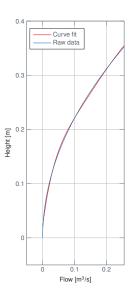
Display

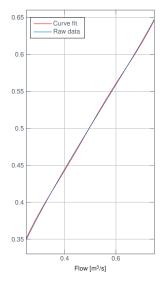
Linearise

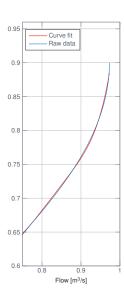
MPC

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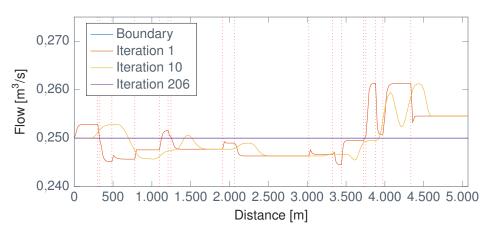
Display

KOHUO

Lineariseri

Regults

Diskussion/Konklusion





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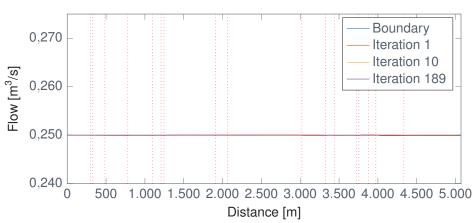
Dioplay

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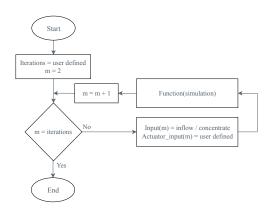


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Display

Diskussion/Konklusion

► Itererer igennem rør og tank for hvert tidsskridt





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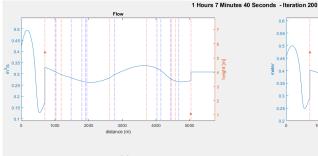
Initialisering

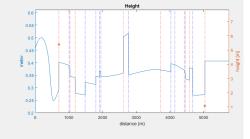
Simulerii Display

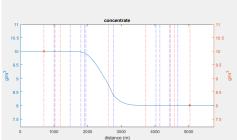
Kontrol Lineariser

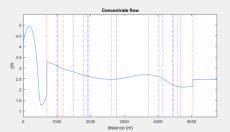
Resultat

Diskussion/Konklusion









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Diskussion/Konklusion

► Linearisering af ulinear model

► Opstilles på state space form

$$\frac{\partial A(x,t)}{\partial t} + \frac{\partial Q(x,t)}{\partial x} = 0$$

$$\frac{\partial A(h)}{\partial h} \frac{\partial h(x,t)}{\partial t} + \frac{\partial Q(h)}{\partial h} \frac{\partial h(x,t)}{\partial x} = 0$$
 (2)

(1)



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Linearisering

Diskussion/Konklusion

Priessmann scheme

Opsat på matrix og vektor form

$$\left[\underbrace{\frac{1}{2\Delta t}\frac{\partial A}{\partial h} - \frac{\theta}{\Delta x}\frac{\partial Q}{\partial h}}_{a} \underbrace{\frac{1}{2\Delta t}\frac{\partial A}{\partial h} + \frac{\theta}{\Delta x}\frac{\partial Q}{\partial h}}_{b}\right] \begin{bmatrix} h_{j+1}^{i+1} \\ h_{j+1}^{i+1} \end{bmatrix} =$$

$$- \left[\underbrace{\frac{-1}{2\Delta t}\frac{\partial A}{\partial h} - \frac{(1-\theta)}{\Delta x}\frac{\partial Q}{\partial h}}_{c} \underbrace{\frac{-1}{2\Delta t}\frac{\partial A}{\partial h} + \frac{\theta}{\Delta x}\frac{\partial Q}{\partial h}}_{d}\right] \begin{bmatrix} h_{j}^{i} \\ h_{j+1}^{i} \end{bmatrix}$$
(3)

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 $\underbrace{\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & b_1 & 0 & \cdots & 0 \\ 0 & a_1 & b_2 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & a_{m-1} & b_m \end{bmatrix}}_{\xi} \underbrace{\begin{bmatrix} h_0^{i+1} \\ h_1^{i+1} \\ h_1^{i+1} \\ \vdots \\ h_m^{i+1} \end{bmatrix}}_{\chi(k+1)} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ c_0 & d_1 & 0 & \cdots & 0 \\ 0 & c_1 & d_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & c_{m-1} & d_m \end{bmatrix}}_{\chi(k)} \underbrace{\begin{bmatrix} h_0^{i} \\ h_1^{i} \\ h_2^{i} \\ \vdots \\ h_m^{i} \end{bmatrix}}_{\chi(k)} + \underbrace{\begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ c_0 & d_1 & 0 & \cdots & 0 \\ 0 & c_1 & d_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & c_{m-1} & d_m \end{bmatrix}}_{\chi(k)} \underbrace{\begin{bmatrix} h_0^{i} \\ h_1^{i} \\ h_2^{i} \\ \vdots \\ h_m^{i} \end{bmatrix}}_{\chi(k)} + \underbrace{\begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & c_1 & d_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & c_{m-1} & d_m \end{bmatrix}}_{\chi(k)} \underbrace{\begin{bmatrix} h_0^{i} \\ h_1^{i} \\ h_2^{i} \\ \vdots \\ h_m^{i} \end{bmatrix}}_{\chi(k)} + \underbrace{\begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & c_1 & d_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & c_{m-1} & d_m \end{bmatrix}}_{\chi(k)} \underbrace{\begin{bmatrix} h_0^{i} \\ h_1^{i} \\ h_2^{i} \\ \vdots \\ h_m^{i} \end{bmatrix}}_{\chi(k)} + \underbrace{\begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & c_1 & d_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & c_{m-1} & d_m \end{bmatrix}}_{\chi(k)} \underbrace{\begin{bmatrix} h_0^{i} \\ h_1^{i} \\ h_2^{i} \\ \vdots \\ h_m^{i} \end{bmatrix}}_{\chi(k)} + \underbrace{\begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & c_1 & d_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & c_{m-1} & d_m \end{bmatrix}}_{\chi(k)} \underbrace{\begin{bmatrix} h_0^{i} \\ h_1^{i} \\ h_2^{i} \\ \vdots \\ h_m^{i} \end{bmatrix}}_{\chi(k)} + \underbrace{\begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & c_1 & d_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & c_{m-1} & d_m \end{bmatrix}}_{\chi(k)} \underbrace{\begin{bmatrix} h_0^{i} \\ h_1^{i} \\ h_2^{i} \\ \vdots \\ h_m^{i} \end{bmatrix}}_{\chi(k)} + \underbrace{\begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & c_1 & d_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & c_{m-1} & d_m \end{bmatrix}}_{\chi(k)} \underbrace{\begin{bmatrix} h_0^{i} \\ h_1^{i} \\ h_2^{i} \\ \vdots \\ h_m^{i} \end{bmatrix}}_{\chi(k)} + \underbrace{\begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & c_1 & d_2 & \cdots & 0 \\ \vdots \\ 0 & 0 & 0 & c_{m-1} & d_m \end{bmatrix}}_{\chi(k)} \underbrace{\begin{bmatrix} h_0^{i} \\ h_1^{i} \\ h_2^{i} \\ \vdots \\ h_m^{i} \end{bmatrix}}_{\chi(k)} + \underbrace{\begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & c_1 & d_2 & \cdots & 0 \\ \vdots \\ 0 & 0 & 0 & c_{m-1} & d_m \end{bmatrix}}_{\chi(k)} + \underbrace{\begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & c_1 & d_2 & \cdots & 0 \\ \vdots \\ 0 & 0 & 0 & c_{m-1} & d_m \end{bmatrix}}_{\chi(k)} + \underbrace{\begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & c_1 & d_2 & \cdots & 0 \\ \vdots \\ 0 & 0 & 0 & c_{m-1} & d_m \end{bmatrix}}_{\chi(k)}$ 

$$\underbrace{\begin{bmatrix}1\\-a_0\\0\\\vdots\\0\end{bmatrix}}_{B}h_0^{j+1}+\underbrace{\begin{bmatrix}\frac{dh}{dQ}\\0\\0\\\vdots\\0\end{bmatrix}}_{B_d}d_0^{j+1}$$

(4)



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► e - Inflow

► f - Reducering af højden i tank

▶ g - Højden i det efterfølge rør

$\begin{bmatrix} b_{1,2} \\ 0 \\ 0 \\ 0 \end{bmatrix}$	0 1 0 0	a <sub>ξ</sub>	0 0 1 <sub>2,1</sub> <i>b</i>	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	$h_{1,2}^{i+1}$ $h_{tank}^{i+1}$ $h_{2,0}^{i+1}$ $h_{2,1}^{i+1}$			
	l <sub>1,2</sub> e 0	0 1 0 0	0 0 0 <i>C</i> <sub>2,0</sub>	0 ]	$\lceil h_{1,2}^i \rceil$	$+\underbrace{\begin{bmatrix}0\\0\\0\\0\end{bmatrix}}$	$\begin{bmatrix} 0 \\ -f \\ g \\ 0 \end{bmatrix}$	$\begin{bmatrix} h_0^{i+1} \\ u_{tank} \end{bmatrix}$

(5)

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Resultat

Diskussion/Konklusion

 Samligning af ulinear og linear model

- ► System setup
- ► Sinus input

Туре	Components	Sections
Pipe	1	35
Tank	1	1
Pipe	18	227
Total	20	263



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Implementeri

Simulering

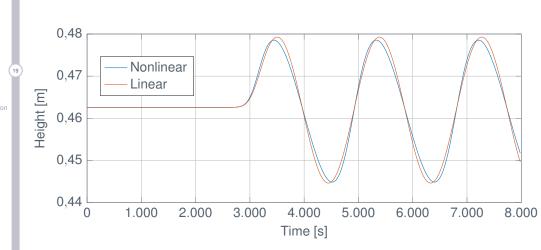
Display

Linearisering

MPC MPC

Resultat

Diskussion/Konklusion





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Simulerin

Kontrol

Lineariserin

MPC

Resultat

Diskussion/Konklusion

► Cost function

- ► Begrænset til minimiere af output
- ▶ Constraints
  - ► Højde
  - ► Kontrol input
- ► Linear model



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Imnlementeri

Simulerin

Display

Kontrol

Lineariserin MPC

Dogultot

Diskussion/Konklusion

- ▶ Bestemmelse af Prediction horizon
  - ► Flow profiler
  - ▶ Industri
- ► Begrænsning af Prediction horizon
- ► System setup
- ► Forstyrrelse input

Fields	type type	component	<b>⊞</b> sections
1	'Pipe'	1	5
2	'Tank'	1	1
3	'Pipe'	1	5
4	'Total'	3	11





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Initialiserin

Display

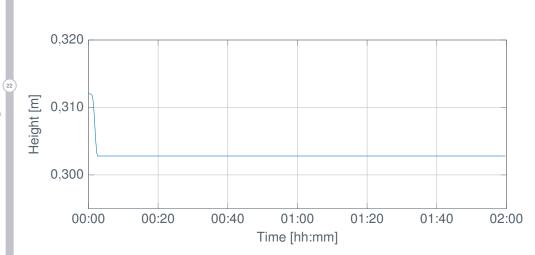
Kontro

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Resultat

Diskussion/Konklusion





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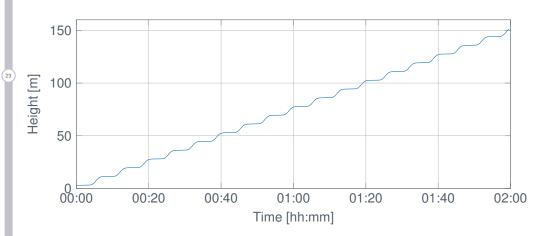
Simulering Display

Kontrol

Lineariserir MPC

Resultat

Diskussion/Konklusion





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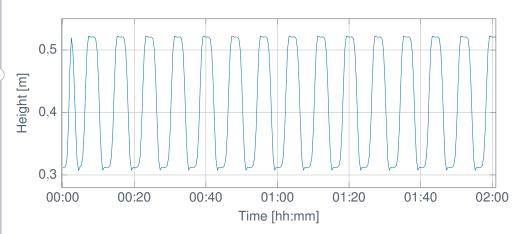
Implementering Initialisering Simulering

Kontrol Lineariserir

Lineariserin MPC

Resultat

Diskussion/Konklusion





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Implementer

Simuleri

Display

Kontrol

Linearisering

Resultat

Diskussion/Konklusion

► System setup

► Flow profiler

Туре	Component	Sections
Pipe	1	35
Tank	1	1
Pipe	17	207
Tank	1	1
Pipe	1	38
Total	21	282



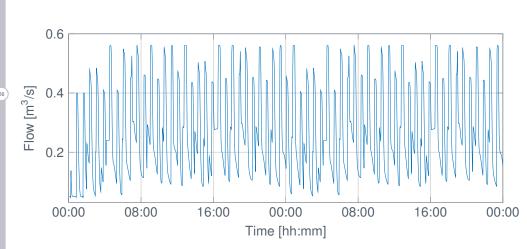
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Implementering Initialisering Simulering

Kontrol Linearisering

## Resultat

Diskussion/Konklusion





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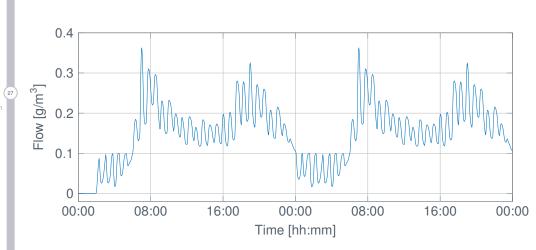
Implementering Initialisering Simulering

Kontrol

Lineariseri

#### Resultat

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Linearisering

#### Resultat

Diskussion/Konklusion

► Over dimensioneret tank

► Konstant output af tank



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Implementerir Initialisering

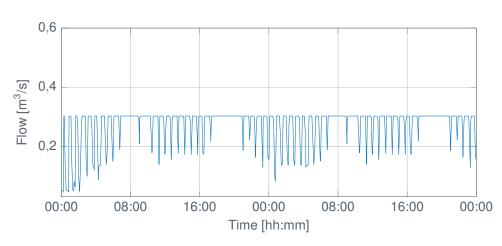
Simulering Display

Kontro

Lineariserir MPC

#### Resultat

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Resultat

- ► Courant's number
- Model reduction



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Kontrol Lineariseri

MPC

Resultat

- Simulering
- ► MPC