

# Model Predictive Control of a Sewer System

June 14, 2018

Group 1030

Jacob Naundrup Pedersen

Thomas Holm Pilgaard

Department of Electronic Systems

Aalborg University

Denmark



**AALBORG UNIVERSITY**  
DENMARK



# Agenda

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Implementering

Kontrol

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Resultat

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► Implementation

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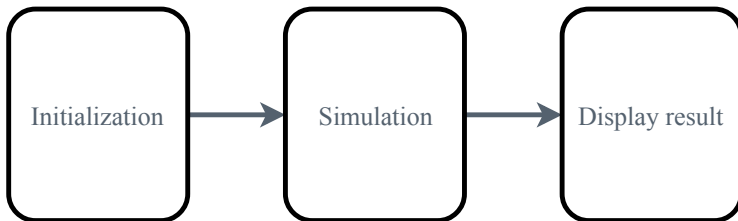
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## 1. Pipe

- ▶ Længde [m]
- ▶ Sektioner
- ▶  $S_b$  (Hældning) [‰]
- ▶  $\Delta x = \text{Længde/Sektioner}$  [m]
- ▶ Diameter [m]
- ▶ Theta
- ▶  $Q_f$  [m<sup>3</sup>/s]
- ▶ Side inflow
- ▶ Placering i data

## 2. Tank

- ▶ Størrelse [m<sup>3</sup>]
- ▶ Højde [m]
- ▶ Areal = Size / Height [m<sup>2</sup>]
- ▶ Maximum outflow [m<sup>3</sup>/s]
- ▶ Placering i data

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### ► Rør specifikationer

Fields	length	sections	Dx	Sb	d	Theta	Qf	side_inflow	data_location
1	700	35	20	0.0030	0.9000	0.6500	0.9730	0	1
2	303	15	20.2000	0.0030	0.9000	0.6500	0.9730	0	3
3	27	2	13.5000	0.0030	1	0.6500	1.2843	1	4
4	155	8	19.3750	0.0041	1	0.6500	1.5014	0	5
5	295	14	21.0714	0.0122	0.8000	0.6500	1.4386	0	6
6	318	15	21.2000	0.0053	0.9000	0.6500	1.2932	1	8

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## ► Tank specifikationer

Fields	 size	 height	 area	 Q_out_max	 data_location
1	90	10	9	0.9730	2
2	90	10	9	1.2932	7

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## ► System specifikationer

Fields	 type	 component	 sections
1	'Pipe'	1	35
2	'Tank'	1	1
3	'Pipe'	4	39
4	'Tank'	1	1
5	'Pipe'	14	206
6	'Total'	21	282



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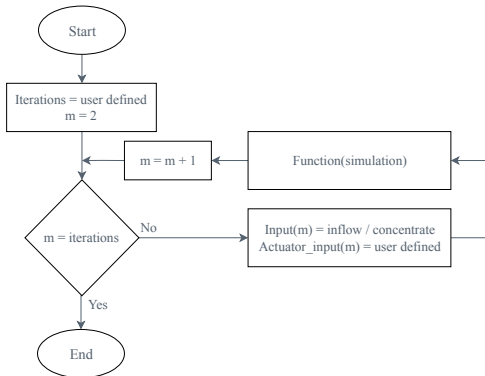
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### ► Itererer igennem rør og tank for hvert tidsskridt



# Implementering Display

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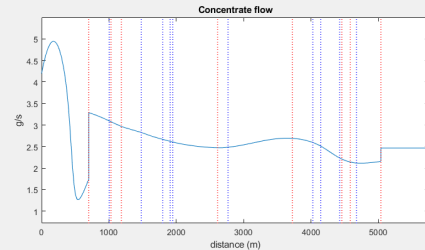
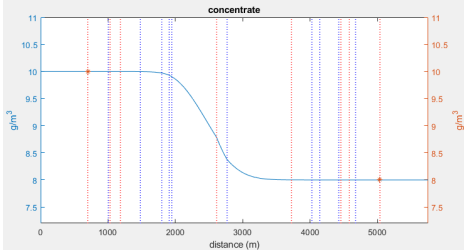
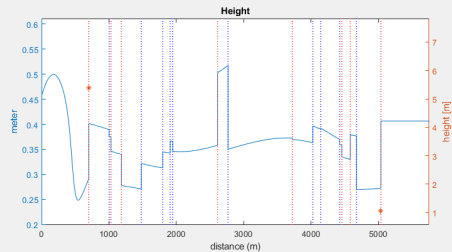
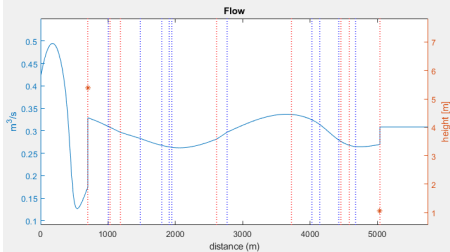
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1 Hours 7 Minutes 40 Seconds - Iteration 200



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- Linearisering af ulineær model
- Opstilles på state space form

$$\frac{\partial A(x, t)}{\partial t} + \frac{\partial Q(x, t)}{\partial x} = 0$$

$$\frac{\partial A(h)}{\partial h} \frac{\partial h(x, t)}{\partial t} + \frac{\partial Q(h)}{\partial h} \frac{\partial h(x, t)}{\partial x} = 0$$

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- Priessmann scheme
- Opsat på matrix og vektor form

$$\begin{aligned}
 & \left[ \underbrace{\frac{1}{2\Delta t} \frac{\partial A}{\partial h} - \frac{\theta}{\Delta x} \frac{\partial Q}{\partial h}}_a \quad \underbrace{\frac{1}{2\Delta t} \frac{\partial A}{\partial h} + \frac{\theta}{\Delta x} \frac{\partial Q}{\partial h}}_b \right] \begin{bmatrix} h_j^{i+1} \\ h_{j+1}^{i+1} \end{bmatrix} = \\
 & - \left[ \underbrace{\frac{-1}{2\Delta t} \frac{\partial A}{\partial h} - \frac{(1-\theta)}{\Delta x} \frac{\partial Q}{\partial h}}_c \quad \underbrace{\frac{-1}{2\Delta t} \frac{\partial A}{\partial h} + \frac{(1-\theta)}{\Delta x} \frac{\partial Q}{\partial h}}_d \right] \begin{bmatrix} h_j^i \\ h_{j+1}^i \end{bmatrix}
 \end{aligned}$$

# Kontrol

Opsætning på state space

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$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & b_1 & 0 & \cdots & 0 \\ 0 & a_1 & b_2 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & a_{m-1} & b_m \end{bmatrix}}_{\xi} \underbrace{\begin{bmatrix} h_0^{i+1} \\ h_1^{i+1} \\ h_2^{i+1} \\ \vdots \\ h_m^{i+1} \end{bmatrix}}_{x(k+1)} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ c_0 & d_1 & 0 & \cdots & 0 \\ 0 & c_1 & d_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & c_{m-1} & d_m \end{bmatrix}}_A \underbrace{\begin{bmatrix} h_0^i \\ h_1^i \\ h_2^i \\ \vdots \\ h_m^i \end{bmatrix}}_{x(k)} + \\
 \underbrace{\begin{bmatrix} 1 \\ -a_0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}}_B h_0^{i+1} + \underbrace{\begin{bmatrix} \frac{dh}{dQ} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}}_{B_d} d_0^{i+1}$$

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- e - Forøgelse af højde i tank(inflow)
- f - Reducering af højde i tank(Outflow)
- g - Inflow i efterfølgende rør

$$\underbrace{\begin{bmatrix} b_{1,2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & a_{2,1} & b_{2,2} \end{bmatrix}}_{\xi} \underbrace{\begin{bmatrix} h_{1,2}^{i+1} \\ h_{tank}^{i+1} \\ h_{2,0}^{i+1} \\ h_{2,1}^{i+1} \end{bmatrix}}_{x(k+1)} \\
 = \underbrace{\begin{bmatrix} d_{1,2} & 0 & 0 & 0 \\ e & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & c_{2,0} & d_{2,1} \end{bmatrix}}_A \underbrace{\begin{bmatrix} h_{1,2}^i \\ h_{tank}^i \\ h_{2,0}^i \\ h_{2,1}^i \end{bmatrix}}_{x(k)} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & -f \\ 0 & g \\ 0 & 0 \end{bmatrix}}_B \begin{bmatrix} h_0^{i+1} \\ u_{tank} \end{bmatrix}$$

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- Samling af ulineær og linear model
- System setup
- Sinus input

Type	Components	Sections
Pipe	1	35
Tank	1	1
Pipe	18	227
Total	20	263

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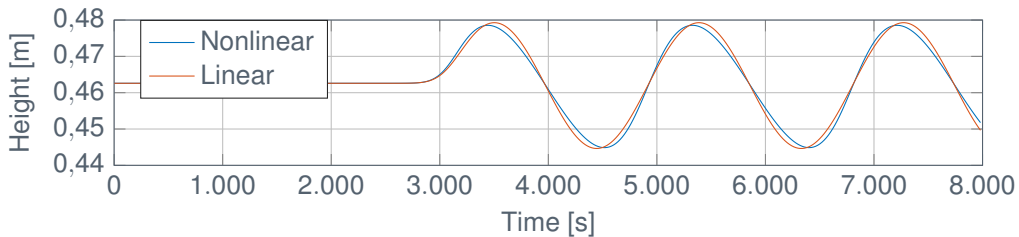
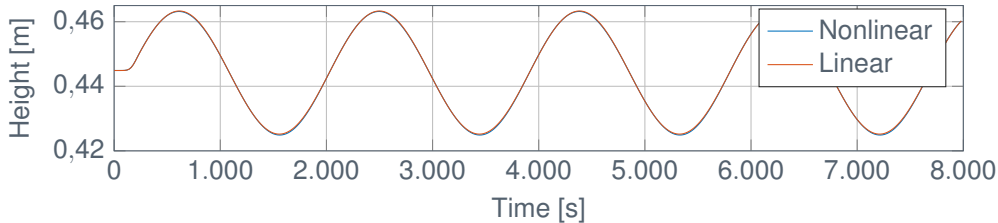
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- ▶ Cost function
  - ▶ Afgrænset til at minimiere output variationer
- ▶ Constraints
  - ▶ Højde
  - ▶ Kontrol input
- ▶ Prediction model

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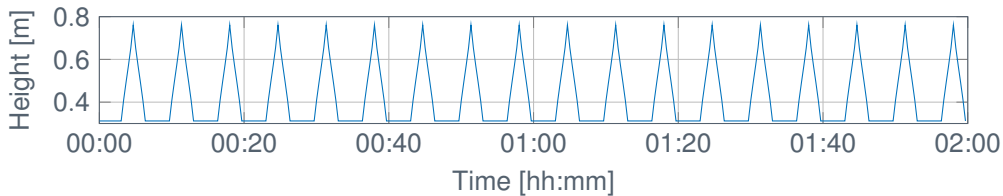
Resultat

Diskussion/Konklusion

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- ▶ Bestemmelse af Prediction horizon
  - ▶ Flow profiler
  - ▶ Industri
- ▶ Begrænsning af Prediction horizon
- ▶ System setup
- ▶ Forstyrrelses input

Fields	ch type	component	sections
1	'Pipe'	1	5
2	'Tank'	1	1
3	'Pipe'	1	5
4	'Total'	3	11



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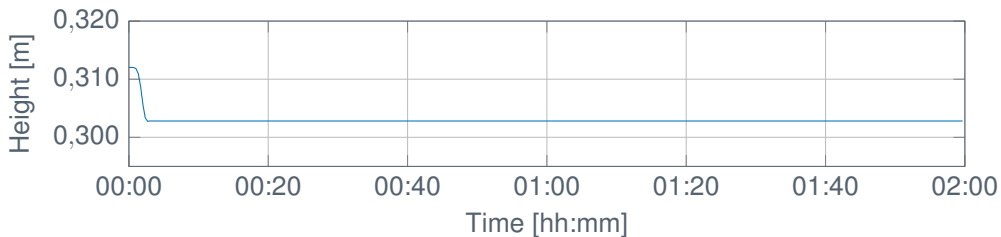
MPC

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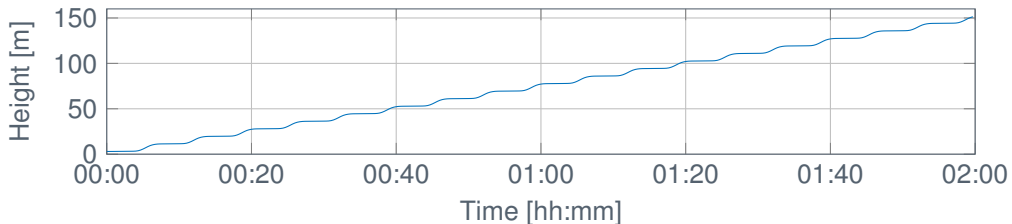
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### Output height



### Tank height



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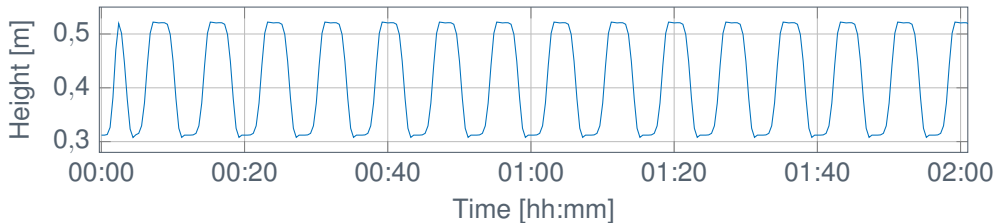
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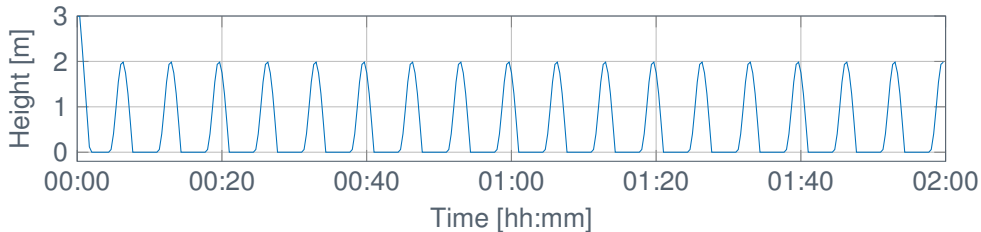
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### Output height



### Tank height



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- System setup, efterligning af Fredericia
- Flow profiler

Type	Component	Sections
Pipe	1	35
Tank	1	1
Pipe	17	207
Tank	1	1
Pipe	1	38
Total	21	282

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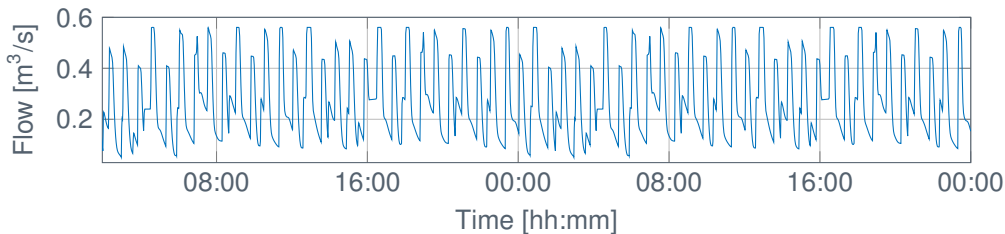
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**Resultat**

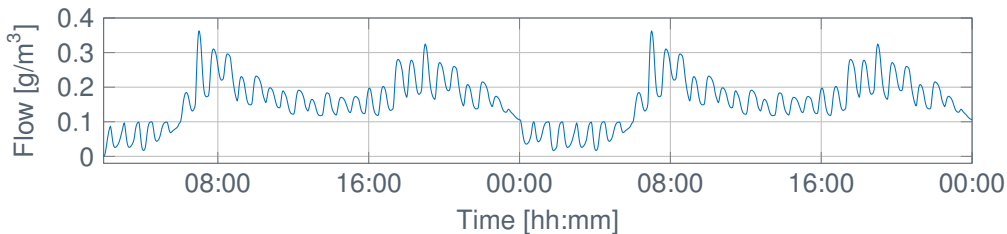
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### Output flow



### Output concentration



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# Diskussion

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- ▶ Courant's tal
- ▶ Model reduction

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- ▶ Simulering
- ▶ MPC