Example 13.22 Reconsider the pendulum equation of the previous example with the nominal parameters $a = c = 10$ and $b = 1$. Suppose the pendulum is resting at the open-loop equilibrium point $x = 0$ and we want to move it to a new equilibrium point at $x_1 = \pi/2$ and $x_2 = 0$. Taking the reference signal r as the output of the second-order transfer function $1/(\tau s + 1)^2$ driven by a step input w will provide the desired motion if the jump in w is taken as $\pi/2$. The tracking control is taken as

$$u = 0.1(10 \sin x_1 + x_2 + \ddot{r} - k_1 e_1 - k_2 e_2)$$

where $k_1 = 400$ and $k_2 = 20$. Taking the initial conditions of the reference model to be zero, we find that the tracking error $e(t) = x(t) - \mathcal{R}(t)$ will be identically zero and the motion of the pendulum will track the desired reference signal for all t. The choice of the time constant τ determines the speed of motion from the initial to the final position. If there were no constraint on the magnitude of the control u, we could have chosen τ arbitrarily small and achieved arbitrarily fast transition from $x_1 = 0$ to $x_1 = \pi/2$. However, the control input u is the torque of a motor and there is a maximum torque that the motor can supply. This constraint puts a limit on how quick we can move the pendulum. By choosing τ to be compatible with the torque constraint, we can achieve better performance. Figure 13.3 shows two different choices of τ when the control is constrained to $|u| \leq 2$. For $\tau = 0.05$ sec, the output $y(t)$ deviates from the reference $r(t)$, reflecting the fact that the reference signal demands a control effort that cannot be delivered by the motor. On the other hand, with $\tau = 0.25$ sec, the output signal achieves a good tracking of the reference signal. In both cases, we could not achieve a settling time better than about 1.2 seconds, but by choosing $\tau = 0.25$, we were able to avoid the overshoot that took place when $\tau = 0.05$. △

13.5 Exercises

13.1 Consider the third-order model of a synchronous generator connected to an infinite bus from Exercise 1.8. Consider two possible choices of the output:

$$\textbf{(1)}\ \ y = \delta; \qquad \textbf{(2)}\ \ y = \delta + \gamma\dot{\delta},\ \ \gamma \neq 0$$

In each case, study the relative degree of the system and transform it into the normal form. Specify the region over which the transformation is valid. If there are nontrivial zero dynamics, find whether or not the system is minimum phase.

13.2 Consider the system

$$\dot{x}_1 = -x_1 + x_2 - x_3, \qquad \dot{x}_2 = -x_1 x_3 - x_2 + u, \qquad \dot{x}_3 = -x_1 + u, \qquad y = x_3$$

(a) Is the system input–output linearizable?

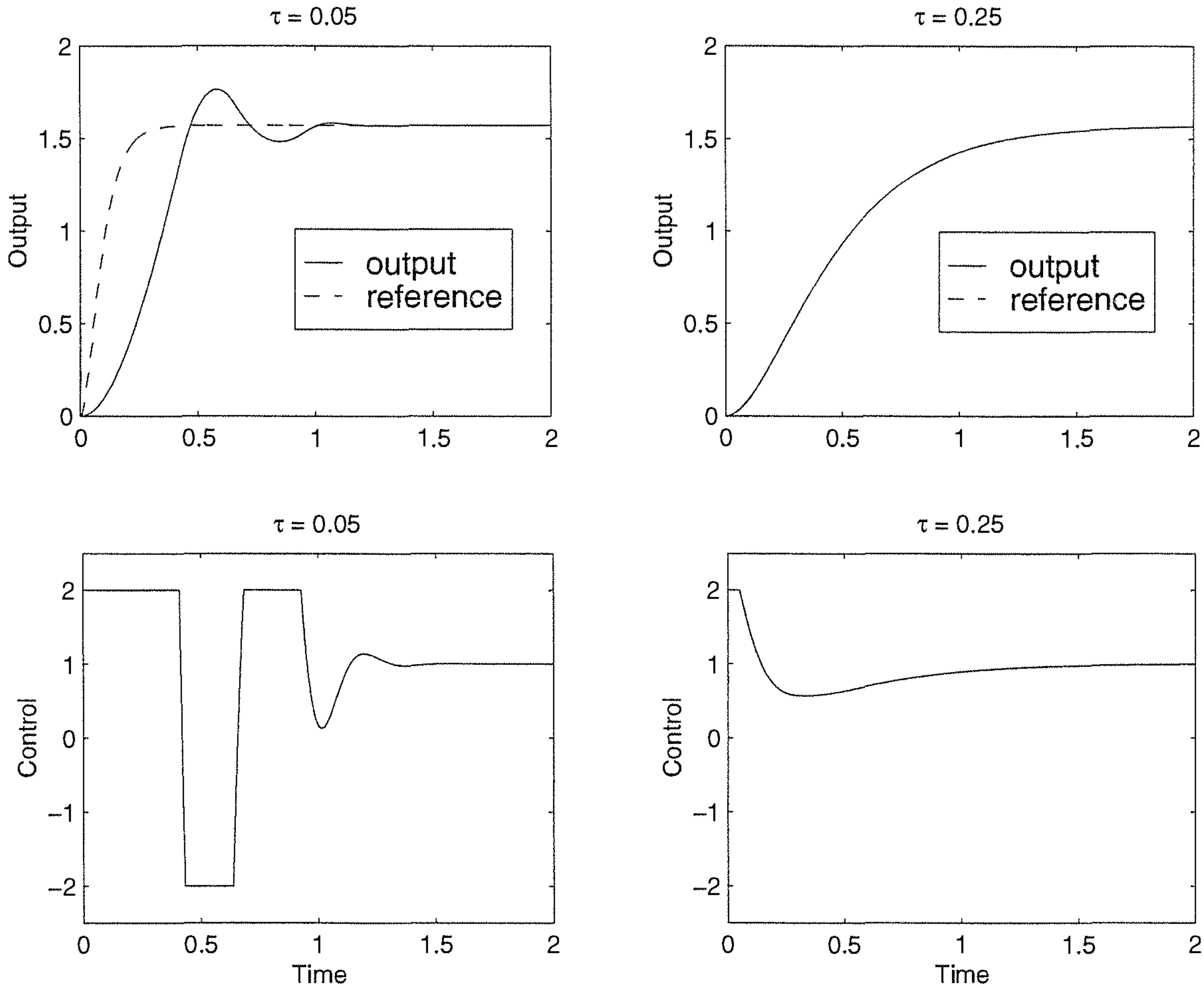

Figure 13.3: Simulation of the tracking control of Example 13.22.

(b) If yes, transform it into the normal form and specify the region over which the transformation is valid.

(c) Is the system minimum phase?

13.3 Consider the inverted pendulum of Exercise 1.15 and let θ be the output. Is the system input–output linearizable? Is it minimum phase?

13.4 Consider the system of Example 12.6. Is the system input–output linearizable? Is it minimum phase?

13.5 With reference to Example 13.8, consider the partial differential equations (13.26). Suppose $q(x)$ is independent of ζ_m and ξ_n. Show that $\phi_i = \zeta_i$ for $1 \leq i \leq m-1$ and $\phi_m = \zeta_m - \xi_n/q(x)$ satisfy the partial differential equations.

13.6 Show that the state equation of Exercise 6.11 is feedback linearizable.