Plug & Play Control: Adding Hardware to Online Control Systems

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Outline

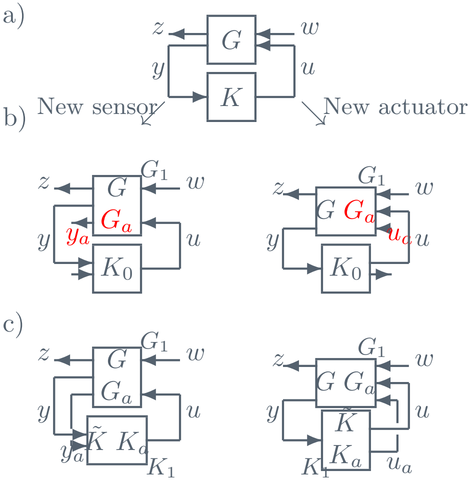
Plug-and-Play Control

Identification Techniques
- Identifying static actuators/sensors
- Identifying new dynamics
- The “Hansen Scheme”

Application Examples
- District heating system model
- Livestock stable

Conclusions

The Plug-and-Play Problem



Outline

Step b) - identifying a new actuator or sensor

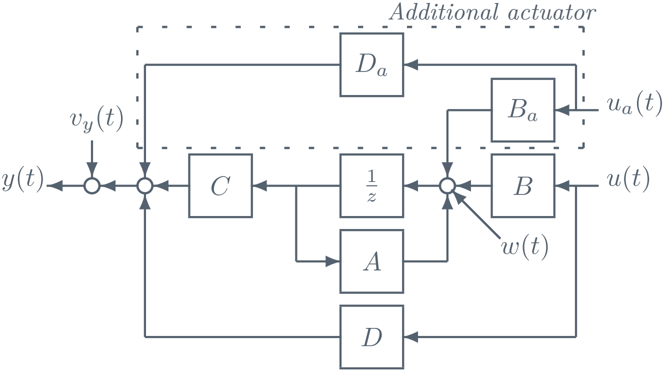
We consider a linear, time invariant system mapping inputs $u(t) \in \mathbb{R}^m$ to outputs $y(t) \in \mathbb{R}^p$ at sample time $t, t = 0, 1, 2, \ldots$ via the state space description

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t) + w(t) \\ y(t) &= Cx(t) + Du(t) + v(t) \end{aligned}$$

where $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, $C \in \mathbb{R}^{p \times n}$ and $D \in \mathbb{R}^{p \times m}$ are constant matrices.

Adding a new actuator or sensor without internal dynamics corresponds to adding extra parameters that need to be identified *without changing the existing parameters*.

Adding a static actuator



Identifying a static actuator

Assume that a (correct) innovation model for the original system is known, and let N prediction and measurement output samples be gathered in vectors as

$$Y^N = \begin{bmatrix} y(1) \\ \vdots \\ y(N) \end{bmatrix} \text{ and } \hat{Y}_i^N = \begin{bmatrix} \hat{y}_i(1) \\ \vdots \\ \hat{y}_i(N) \end{bmatrix}, i = 0, \dots, n+p$$

Define $\Upsilon = \begin{bmatrix} \hat{Y}_1^N & \cdots & \hat{Y}_{p+n}^N \end{bmatrix}$ and $Z = Y^N - \hat{Y}_0^N$.

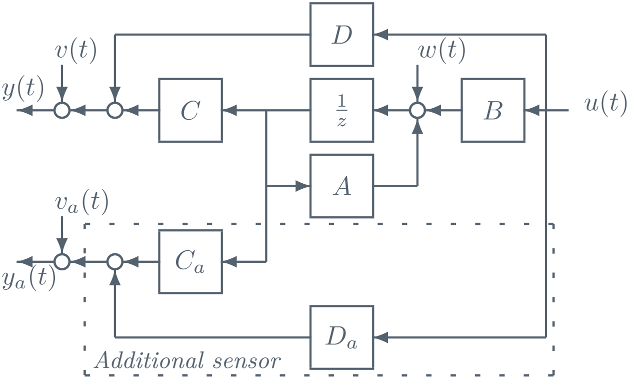
Theorem: Consistent least squares estimator for additional input

Assume the input is persistently exciting; then the LS estimator

$$\hat{\theta} = \begin{bmatrix} \hat{B}_a \\ \hat{D}_a \end{bmatrix} = (\Upsilon^T \Upsilon)^{-1} \Upsilon^T Z$$

is *consistent* in open-loop operation. If there is at least one time delay from output to input, it is consistent in closed-loop operation as well.

Adding a static sensor



Identifying a static sensor

Assume that a (correct) innovation model for the original system, providing state estimate $\hat{x}$, is known, and introduce the regression and parameter vectors

$$\phi(t) = \begin{bmatrix} \hat{x}(t) \\ u(t) \end{bmatrix} \text{ and } \theta = \begin{bmatrix} C_a & D_a \end{bmatrix}^T$$

Theorem: Consistent least squares estimator for additional output (deterministic part)
Assume the input is persistently exciting; then the LS estimator

$$\hat{\theta} = \left(\sum_{t=1}^{N} \phi(t)\phi(t)^T \right)^{-1} \sum_{t=1}^{N} \phi(t) y_a(t)$$

is consistent in open-loop operation. It is consistent in closed-loop operation as well, provided there is at least one time delay from output to input.

Example

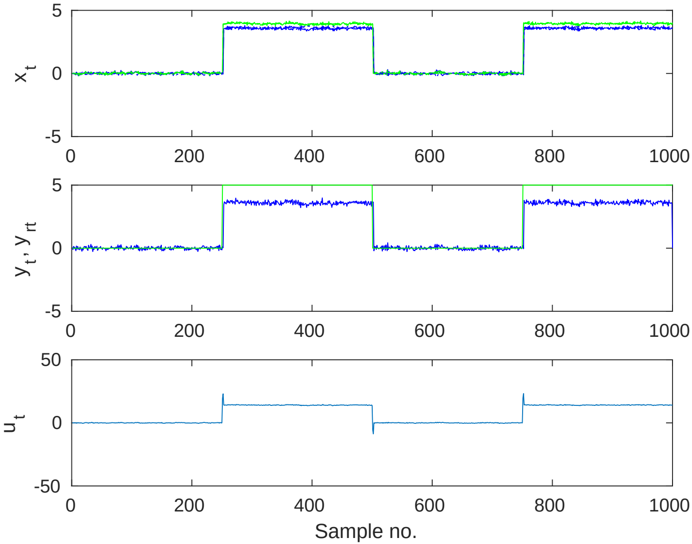
Consider the two-state SISO discrete-time system

$$\begin{aligned} x_{t+1} &= \begin{bmatrix} 0 & 0.91 \\ 0.5 & -0.17 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0.2 \end{bmatrix} u_t + \begin{bmatrix} 0.05 & 0 \\ 0 & 0.05 \end{bmatrix} w_t \\ y_t &= \begin{bmatrix} 1 & 0 \end{bmatrix} x_t + 0.1 v_t \end{aligned}$$

- x, u and y: states, input and output
- w, v: normal distributed white noise sequences

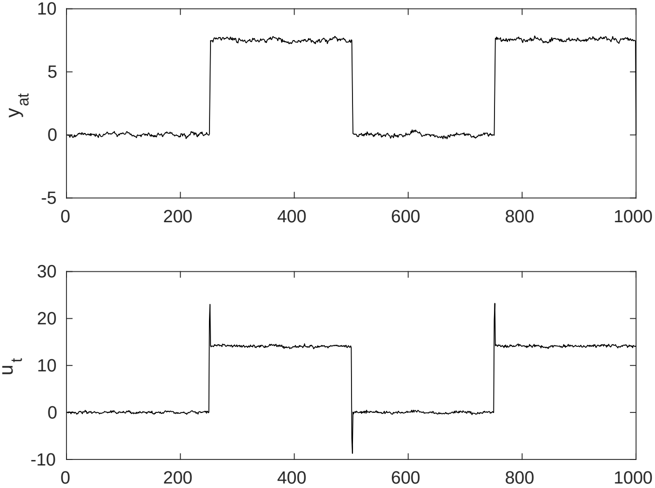
An LQG controller is designed for the system, yielding a feedback gain of $F = \begin{bmatrix} -2.47 & 0.84 \end{bmatrix}$ and an observer gain of $L = \begin{bmatrix} -0.51 & 0.06 \end{bmatrix}^T$.

Example - existing system



Example - additional sensor

An extra sensor, measuring the *sum* of the two states, is added to the system. It has noise spread 0.02.



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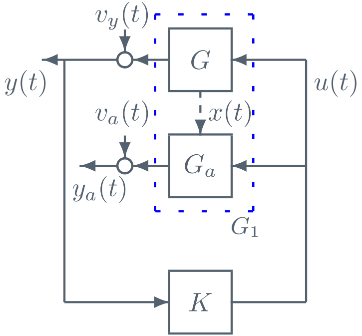
Performing this estimation, we find

$$\hat{\theta} = \begin{bmatrix} \hat{C}_a^T \\ \hat{D}_a^T \end{bmatrix} = \begin{bmatrix} 0.7771 \\ 1.2123 \\ -0.0040 \end{bmatrix}$$

which is actually pretty good.

Adding a new sensor revealing a subsystem



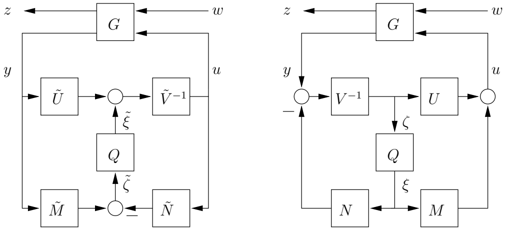
Augmented system model:

$$G_1 = \begin{bmatrix} G_0 \\ G_a \end{bmatrix} = \left[\begin{array}{cc|c} A & 0 & B \\ A_{a1} & A_{a2} & B_a \\ \hline C & 0 & D \\ C_{a1} & C_{a2} & D_a \end{array}\right]$$

Augmented controller model:

$$K_0 = \begin{bmatrix} K & 0 \end{bmatrix} = \left[\begin{array}{c|cc} A + BF + LC + LDF & -L & 0 \\ \hline F & 0 & 0 \end{array}\right]$$

From last time: The Youla-Kucera parameterization of stabilizing controllers

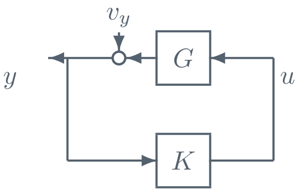


$$K(Q) = U(Q)V(Q)^{-1} = \widetilde{V}(Q)^{-1}\widetilde{U}(Q)$$

where

$$\begin{aligned} U(Q) &= U + MQ\,, \quad V(Q) = V + NQ, \\ \widetilde{U}(Q) &= \widetilde{U} + Q\widetilde{M}\,, \quad \widetilde{V}(Q) = \widetilde{V} + Q\widetilde{N}\,, \quad Q \in \mathcal{RH}_\infty \end{aligned}$$

"Hansen Scheme"



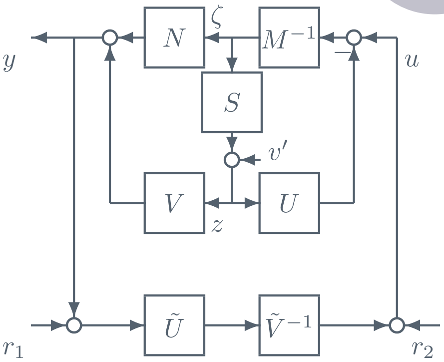
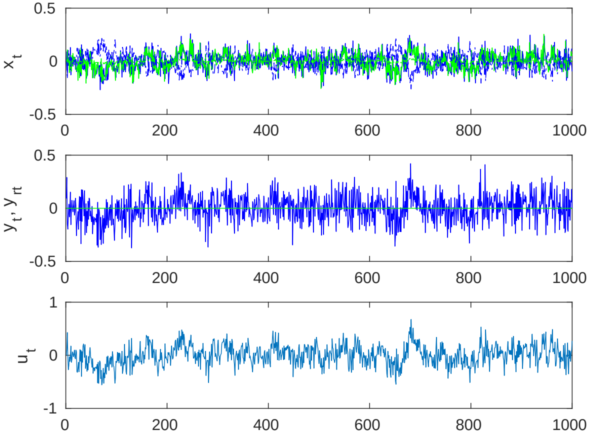


Figure: *Dual Youla-Kucera parameterization used for closed-loop system identification*

"Hansen Scheme"

Closed-loop identification is harder than open-loop identification because

- There is correlation between y and u via K
- K may limit the frequency content of u



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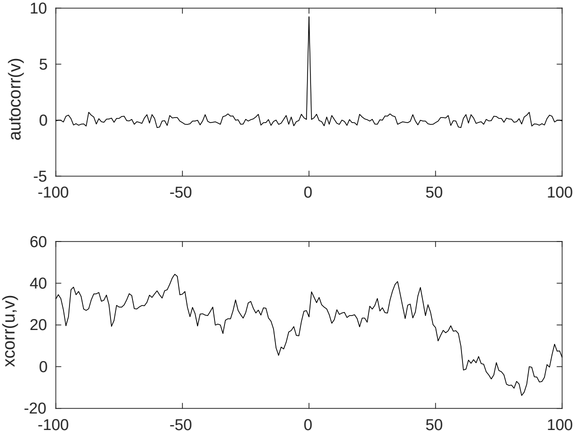
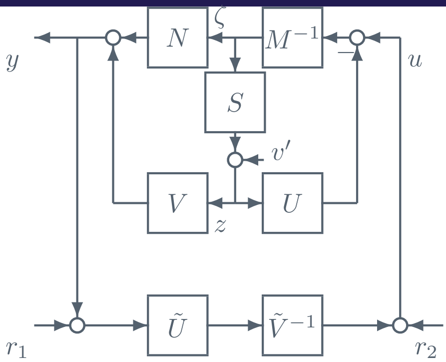


“Hansen Scheme”

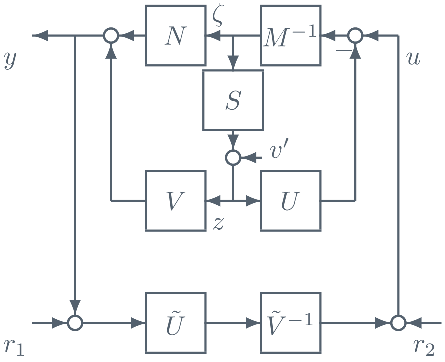


The set of all systems stabilised by K is given by

$$
\begin{aligned}
\{G : G(S) &= (N + VS)(M + US)^{-1} \\
&= (\tilde{M} + S\tilde{U})^{-1}(\tilde{N} + S\tilde{V}), \quad S \in \mathcal{RH}_\infty\}
\end{aligned}
$$

where S is called a *dual Youla parameter.*

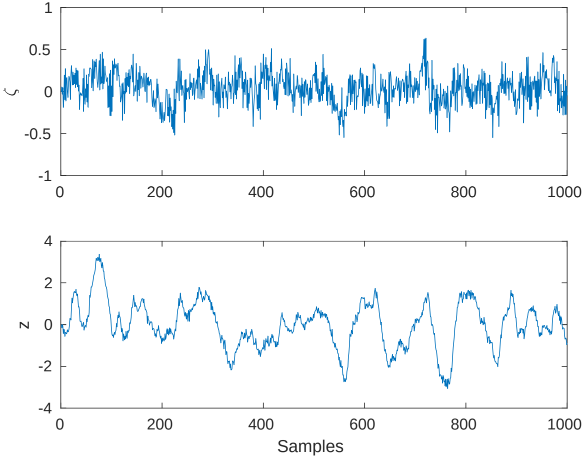
"Hansen Scheme"



- $v' = (\widetilde{M} + S\widetilde{U})v$ - uncorrelated with ζ!
- $\zeta = \widetilde{U}r_1 + \widetilde{V}r_2$ - filtered input references/noise
- $z = \widetilde{M}u - \widetilde{N}y$ - filtered "measurements"

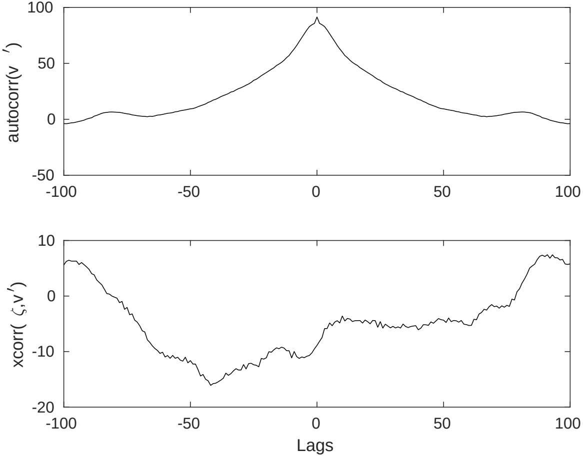
"Hansen Scheme"

Filtered signals



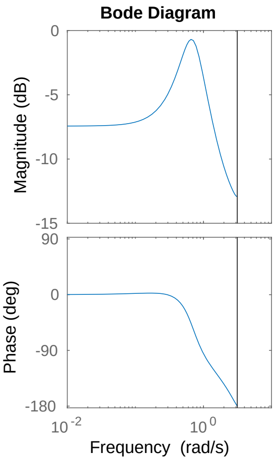
“Hansen Scheme”

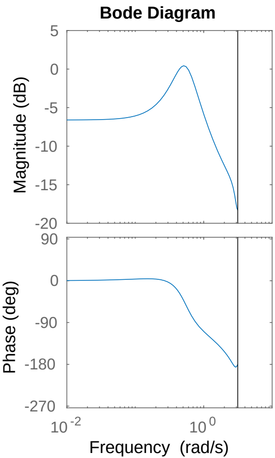
Correlations with filtered noise



“Hansen Scheme”

Identification of S





Left: $G(S) = (\tilde{M} + S\tilde{U})^{-1}(\tilde{N} + S\tilde{V})$ with true S, right: with estimated S

An extended Hansen result

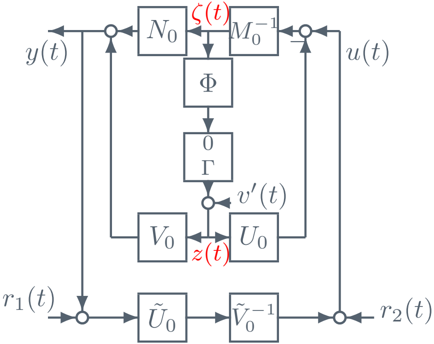
Theorem: Consider the augmented plant in closed loop with the augmented controller. A dual Youla-Kucera parameter representing the new sensor dynamics G_a is given by

$$S_1 = \Gamma\Phi$$

where:

$$\Gamma = \left[\begin{array}{c|cc} A_{a2} & B_a & A_{a1} \\ \hline C_{a2} & D_a & C_{a1} \end{array}\right] \quad and \quad \Phi = \left[\begin{array}{c|c} A+BF & B \\ \hline F & I \\ I & 0 \end{array}\right]$$

An extended Hansen scheme



$$\Gamma = \left[\begin{array}{c|cc} A_{a2} & B_a & A_{a1} \\ \hline C_{a2} & D_a & C_{a1} \end{array}\right]$$

$$\Phi = \left[\begin{array}{c|c} A+BF & B \\ \hline F & I \\ I & 0 \end{array}\right]$$

$\Phi\zeta(t) =$

$\Phi(\tilde{U}_0 r_1(t) + \tilde{V}_0 r_2(t))$

$z(t) =$

$\tilde{M}_0 y(t) - \tilde{N}_0 u(t)$

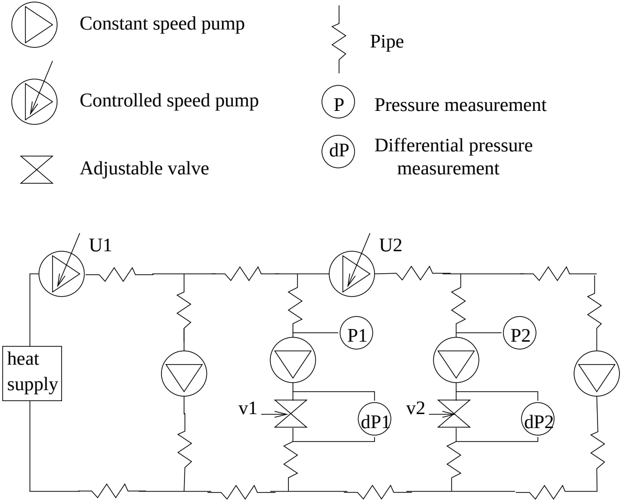
Outline

P & P Techniques

- Identify new sensor gains and dynamics
- Identify new actuator gains
- (LTI control design)
- Gradual transition to new control law
- Terminal addition of new controller blocks

Example: district heating system model

District heating system configuration



State space model

Fifth order model consisting of a third order innovations model from pump speeds and valve settings to pressures and differential pressures. Valves are modeled as first order filtered white noise:

$$
\begin{aligned}
x(t+1) &= Ax(t) + B\begin{bmatrix} U_1^\delta(t) \\ U_2^\delta(t) \end{bmatrix} + \begin{bmatrix} K_P e_P(t) + K_{dP} e_{dP}(t) \\ K_v e_v(t) \end{bmatrix} \\
y_P(t) &= C_P x(t) + e_P(t) \\
y_{dP}(t) &= C_{dP} x(t) + e_{dP}(t)
\end{aligned}
$$

- $x(t)$: state and valve setting estimates (5 in total)
- $e_P(t), e_{dP}(t), e_v(t)$: innovations (one-step prediction errors) for $P(t) = [P_1(t)\ P_2(t)]^T, dP(t) = [dP_1(t)\ dP_2(t)]^T$ and $v(t) = [v_1^\delta(t)\ v_2^\delta(t)]^T$

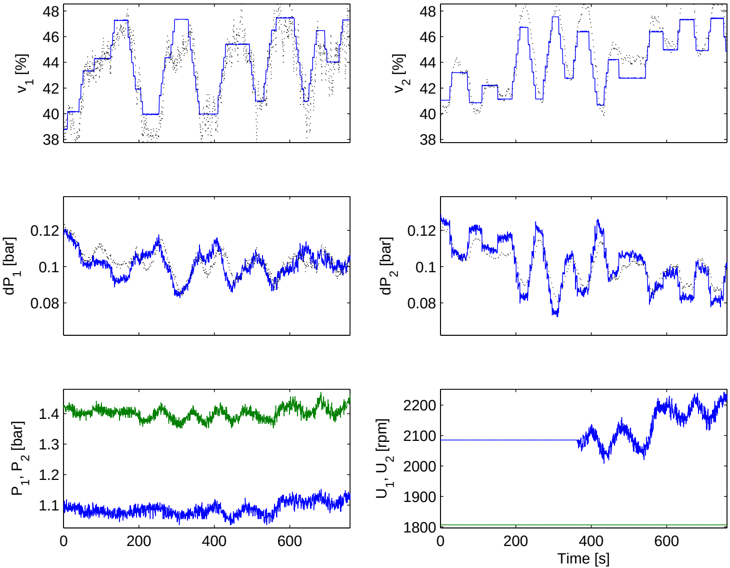
District heating system: initial controller

Initially, the differential pressure measurements are not available to the control system, which relies on $y_P(t) = [P_1(t)\ P_2(t)]^T$ only.

Also, the pump U_2 is a constant-speed pump, so the controlled system has 2 outputs and a single input.

The controller is designed as an LQG controller penalizing the (estimated) differential pressures, i.e. $Q_{SF} = C_{dP}^T C_{dP}$, and the control signal, $R_{SF} = 10^{-4}$.

Performance of initial controller, switched on at 360 s



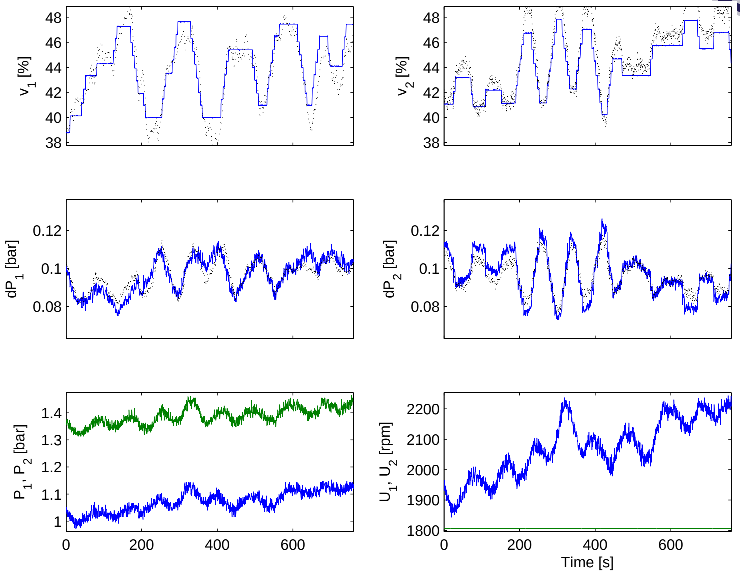
District heating system: adding pressure sensors

Even though the estimates are not very accurate, the controller is able to decrease the variation of the differential pressures somewhat.

However, the consumers still suffer from varying supply rates, so differential pressure sensors are added.

Thus, the measurement vector is now expanded to $y_e(t) = \begin{bmatrix} y_P(t) \\ y_{dP}(t) \end{bmatrix}$ (and the observer gain expanded accordingly).

Performance of controller with additional obs...



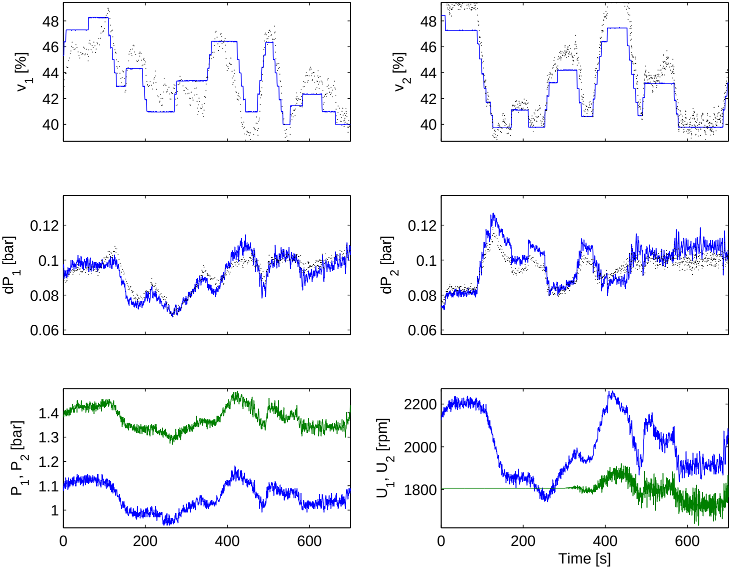
District heating system: adding an actuator

Estimation performance is improved, but not control performance.

Since the additional sensors revealed a problem with the performance, it is decided to replace the U_2 pump with a controllable pump.

An optimal controller K_1 is designed for the system with four measurements and two control inputs. Since the original controller K_0 has an unstable pole, the optimal controller has to be modified accordingly.

Control performance with extra actuator



Example: temperature control in livestock stable

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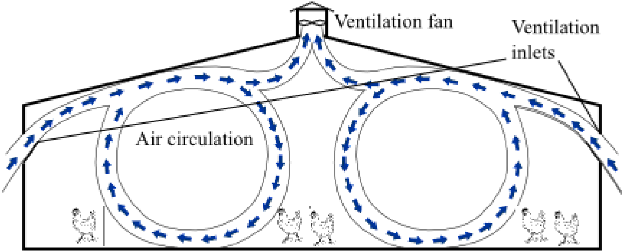

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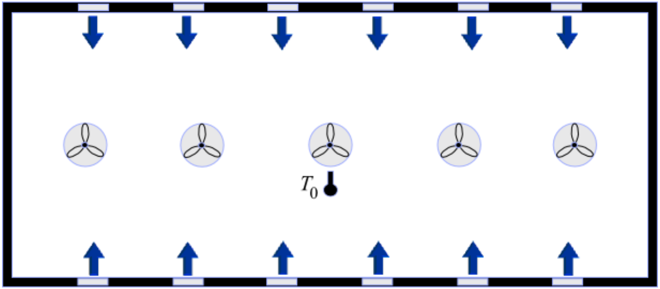

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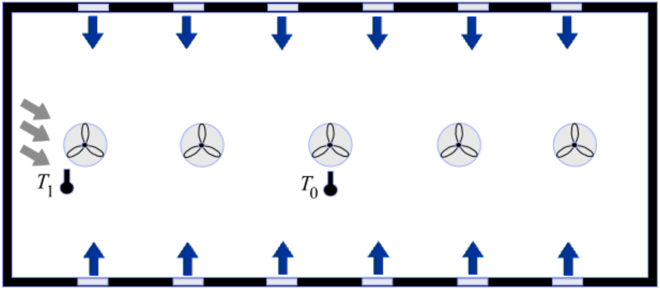

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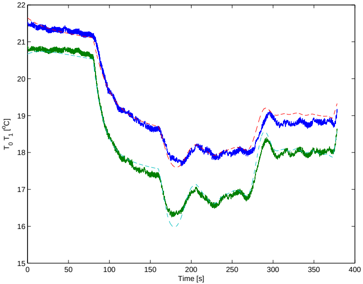

Identified TF model from ventilation rate to temperatures T_1, T_2:

$$\begin{bmatrix} G_0 \\ G_1 \end{bmatrix} = \begin{bmatrix} \frac{-0.72}{s/1200+1} \\ \frac{-0.88}{s/800+1} \end{bmatrix}.$$

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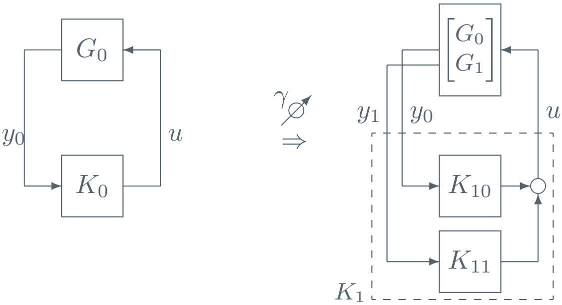
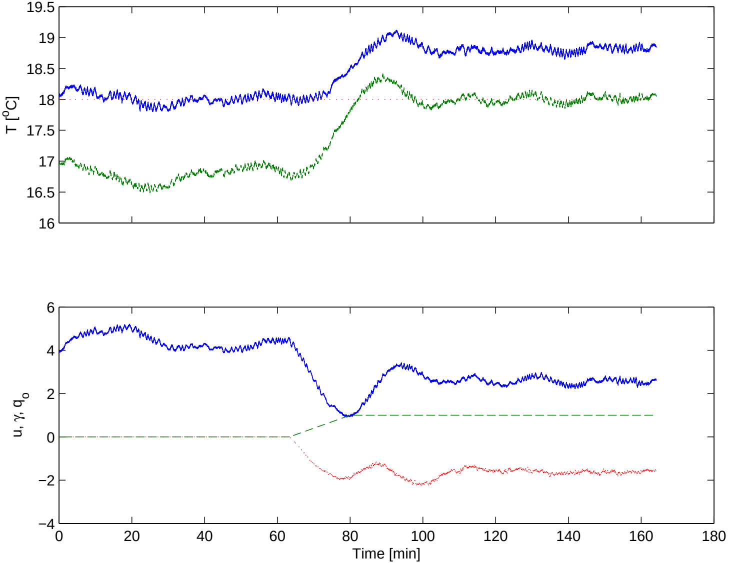


Livestock stable: using the new sensor



Outline

Conclusions

- Plug-and-Play control was discussed for LTI plants
- New sensors, actuators and subsystems are identified by appropriate system identification algorithms
- Once a new component has been identified, it may be included in the existing control system, preferably in an *automatic* and *reversible* manner
- We proposed some techniques and showed that they could be used in practice