Game theory – part 2

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Outlook

Pure strategies - Mixed strategies

Static games – Dynamic games

Perfect information – Imperfect information

Method to solve simultaneous move games:

Best Response approach → Nash equilibrium

Method to solve sequential-move games:

Backward Induction → Subgame Perfect equilibrium

Mixed strategies

- **Def** A mixed strategy *p* is a randomization over *i*th player pure strategies.
- Example: Paper-Scissors-Rock game

	$s_2 = \text{Paper}$	$s_2 = \text{Rock}$	$s_2 = \text{Scissors}$
$s_1 = \text{Paper}$	(0,0)	(1,−1)	(−1,1)
$s_1 = \text{Rock}$	(−1,1)	(0,0)	(1,−1)
$s_1 = \text{Scissors}$	(1,−1)	(−1,1)	(0,0)

- The expected pay-off of a mixed strategy is a weighted average of the expected payoffs of each of the pure strategy in the mix.

Existence of NE

- *NE is a outcome of the game such that no player has an incentive to choose a different strategy, if all other players do not deviate.*

- **Theorem.** Every finite game in strategic form has a Nash equilibrium in either mixed or pure strategies.

Dynamic games: sequential and repeated games

- Some of single-stage games result in NE that are suboptimal from the point of view of other players.
- The same games, when played repeatedly or sequentially, may yield different equilibrium.

- A game in extensive form is represented as a tree, where each node of the tree represents a decision point of one of the players, and the edges represent possible actions available to that player. Payoffs are specified at the end nodes.

Backward induction

- BI is an iterative technique for finding equilibrium:
  - Determine the optimal choice of the player who makes the last move of the game
  - Determine the optimal action of the player moving next-to-last, taking the last player's action as given
  - …
  - Determine the optimal choice for the first player

- **Theorem.** Every finite extensive-form game with perfect information has a pure-strategy NE.

Imperfect information

- **Definition**. An information set of player *i* is a collection of player *i*'s nodes among which *i* cannot distinguish.

- **Definition**. Perfect information: all information sets in the tree contain just one node. Imperfect information: not perfect information.

- **Statement**. Any game in a form of a tree can be re-written as a matrix

Subgame

- **Definition**. A subgame is a part of a game that looks like a game within the tree. It satisfies 3 properties:

  (1) it starts from a single node

  (2) it comprises all successors of that node

  (3) it does not break up any information sets

Subgame perfect equilibrium

- **Definition**. A NE is a subgame perfect equilibrium SPE if it induces a NE in any subgame of the game.