Exercises on Consensus

Exercise 1. *Let the graphs G_1, G_2, and G_3 be given as shown in Figure 1.*

1. *Derive the graphs $G_1 = (V_1, E_1)$, $G_2 = (V_2, E_2)$, and $G_3 = (V_3, E_3)$.*

2. *Derive the Laplacian matrices of the graphs.*

3. *Let G_1, G_2, and G_3 be the topology of a network of agents with integrator dynamics and consensus protocol*

$$u_i = \sum_{j \in N_i} (x_j - x_i). \tag{1}$$

Will the agents reach consensus, and if so, what type of consensus (consensus/average-consensus/min-consensus/max-consensus)?

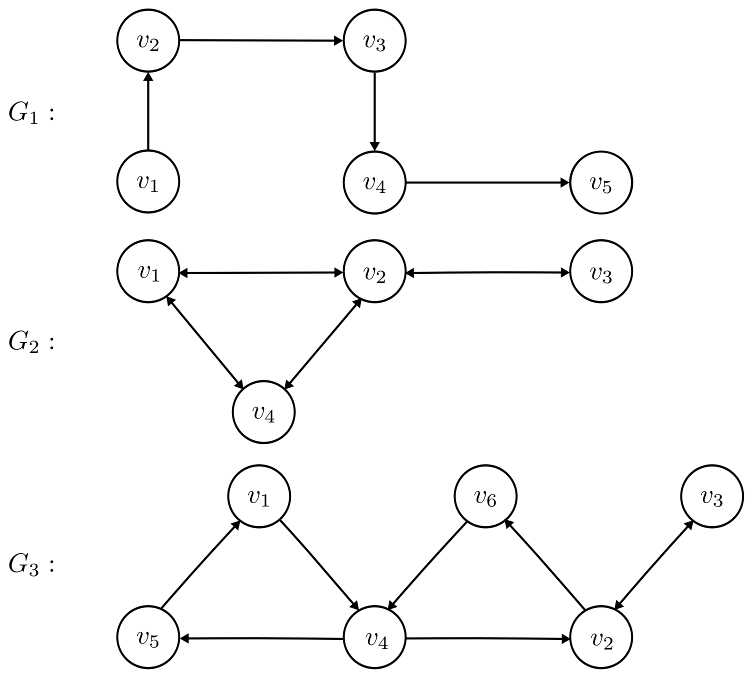

Figure 1: Illustration of a dynamic graph.

Exercise 2. *Consider a simplified version of the clock synchronization algorithm proposed in [1], where we consider offset compensation, and not drift compensation; thus, the dynamics of each clock are given as*

$$\dot{\tau}_i = 1 + u_i, \qquad \tau_i(0) = \beta_i$$

where τ_i *is the value of the* i*th clock,* β_i *is the offset of the clock, and* u_i *is the clock compensation that we must design.*

The offset compensation proposed in equation (15) of [1] can be rephrased as

$$u_i = \alpha \sum_{j \in N_i} (\tau_j - \tau_i), \tag{2}$$

where $\alpha \in \mathbb{R}_{>0}$ *is a gain to be designed.*

1. *Let* G_2 *from Exercise 1 be the topology of a network of agents, and simulate the system.*
2. *Derive the convergence rate of the disagreement of the dynamic network.*
3. *Simulate the dynamic network and compare the simulated results with the convergence bound.*

Exercise 3. *Add a virtual agent to the dynamic network in Figure 2 such that the state of every agent follows the constant reference of the virtual agent.*

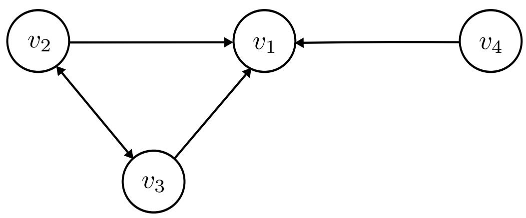

Figure 2: Illustration of a dynamic graph.

Bibliography

[1] L. Schenato and F. Fiorentin. Average timesynch: A consensus-based protocol for clock synchronization in wireless sensor networks. *Automatica*, 47(9):1878 – 1886, 2011.