Project 2
Fitting the LMT Pointing Model

Fall 2019

1 Background

The Large Millimeter Telescope (LMT) is a 50m-diameter millimeter wavelength telescope located in the state of Puebla, Mexico. The LMT is the largest telescope of its kind in the world, and the UMass Astronomy Department has access to 30% of the observing time on the instrument. The telescope has been in scientific operation for about four years using the inner 32m-diameter of the antenna. During 2018, we completed the full 50-m diameter surface and started our initial scientific work.

The LMT is an example of an Azimuth-Elevation mounted telescope, and it points to arbitrary locations on the sky by turning around one axis which points up (the Azimuth Axis) and then rotating the dish about the orthogonal axis (the Elevation Axis) to point at different angles upward from the horizon. This kind of mounting system is favored for large structures.

The Azimuth axis is defined as pointing up so that as we rotate the antenna about this axis it can point in any direction, with the North direction as it's zero position. Azimuth increases as the telescope rotates towards the East, and so East is 90 degrees, South is 180 degrees, and West is 270 degrees.

The Elevation axis is orthogonal to the Azimuth axis, and therefore parallel with the ground. Zero elevation corresponds to the antenna pointing at the horizon and elevation increases as the telescope points upward. When pointing straight up, which we call the zenith, the Elevation is 90 degrees.

One of the key technical considerations in observing celestial sources with the LMT is using the Az and El axes to *point* the telescope. We are required to orient the antenna with a precision of about 1 second of arc in order to make best use of the telescope. The mechanical construction of the telescope was not nearly so precise that we could simply enter in the position of a point on the sky and go there to arc second precision. There are many defects that we need to consider:

- Knowledge of the precise location of 0 on the encoder which measures the Azimuth angle.
- Knowledge of the precise location of 0 on the encoder which measures the Elevation angle.
- Tilt of the Azimuth Axis with respect to zenith.
- Alignment of the telescope's optical axis with the mechanical structure.
- Non-orthogonality of the telescope's azimuth and elevation axes.
- Gravitational bending of the mechanical system under changing gravity load as it points to different elevation angles.
- Refraction by the Earth's atmosphere.

Even in a carefully built system (like the LMT) each of these effects is very large compared to our pointing requirement. What is required is a way to measure the effects and determine the values of the parameters that describe these effects.

 

Figure 1: The Large Millimeter Telescope

2 Pointing Model

In order to improve the pointing of the telescope we fit a *Pointing Model* to observations of celestial sources at known positions in the sky. We make careful measurements of position that we read out from the antenna's angle encoders and then compare this to the known position. We call the difference between these angles the *pointing offset*.

We may easily derive formulae for correction of the Azimuth and Elevation angles measured by their respective encoders to the actual Azimuth and Elevation of a point on the sky. Each of the effects listed in the previous section has a different behavior, and by making a lot of observations we can use these formulae to make a least squares fit to find the values of the parameters that describe these imperfections in the structure. Presuming these parameters are stable with time (and for the most part they are) we can then use the model to correct the antenna pointing.

2.1 Azimuth Pointing Model

For the azimuth pointing model, we fit FIVE terms at the LMT.

Function	**Parameter**	**Explanation**
1	A_1	Angle between telescope optical axis and mechanical structure
sin(El)	A_2	term for nonorthogonality of the Az and El axes.
cos(El)	A_3	Zero position of the Azimuth encoder.
sin(El) sin(Az)	A_4	Tilt of Azimuth axis from Zenith
sin(El) cos(Az)	A_5	Tilt of Azimuth axis from Zenith

We can put all these terms together to derive the azimuth pointing offset from our desired position, δAz, in terms of all these properties. (See Equation 1.)

$$\delta Az = A_1 + A_2 \sin El + A_3 \cos El + A_4 \sin El \sin Az + A_5 \sin El \cos Az \quad (1)$$

2.2 Elevation Pointing Model

For the elevation pointing model, we fit SIX terms at the LMT.

Function	Parameter	Explanation
1	E1	Zero position of the Elevation encoder
cot(El)	E2	Refraction error
cos(El)	E3	Gravitational Bending
sin(El)	E4	Gravitational Bending
cos(Az)	E5	Tilt of Azimuth axis from Zenith
sin(Az)	E6	Tilt of Azimuth axis from Zenith

We can put all these terms together to derive the elevation pointing offset from our desired position, δEl, in terms of all these properties. (See Equation 2.)

$$\delta El \;=\; E_1 + E_2 \cot El + E_3 \cos El + E_4 \sin El + E_5 \cos Az + E_6 \sin Az \tag{2}$$

2.3 Fitting the Pointing Model

Note that the model parameters in Equation 1 and Equation 2 are just constants multiplying some function of the Azimuth and/or Elevation. This means that the model we have adopted is *linear* with respect to our model parameters and we can go ahead and make use of our algorithm for doing a least squares fit.

We begin by writing our problem as a linear system (illustrating below with the Azimuth equations although we will have to solve BOTH the Azimuth system and the Elevation system to get our complete pointing model). If I have N observations d_1 through d_N at N Az-El positions Az_1, El_1 through Az_N, El_N, then my set of equations of condition would be:

$$\begin{aligned}
d_1 &= A_1 + A_2 \sin El_1 + A_3 \cos El_1 + A_4 \sin El_1 \sin Az_1 + A_5 \sin El_1 \cos Az_1 \\
d_2 &= A_1 + A_2 \sin El_2 + A_3 \cos El_2 + A_4 \sin El_2 \sin Az_2 + A_5 \sin El_2 \cos Az_2 \\
d_3 &= A_1 + A_2 \sin El_3 + A_3 \cos El_3 + A_4 \sin El_3 \sin Az_3 + A_5 \sin El_3 \cos Az_3 \\
&\dots \\
d_3 &= A_1 + A_2 \sin El_i + A_3 \cos El_i + A_4 \sin El_i \sin Az_i + A_5 \sin El_i \cos Az_i \\
&\dots \\
d_N &= A_1 + A_2 \sin El_N + A_3 \cos El_N + A_4 \sin El_N \sin Az_N + A_5 \sin El_N \cos Az_N
\end{aligned}$$

We can write this system of equations in vector form as follows:

$$\begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ \dots \\ d_i \\ \dots \\ d_N \end{pmatrix} = \begin{bmatrix} 1 & \sin El_1 & \cos El_1 & \sin El_1 \sin Az_1 & \sin El_1 \cos Az_1 \\ 1 & \sin El_2 & \cos El_2 & \sin El_2 \sin Az_2 & \sin El_2 \cos Az_2 \\ 1 & \sin El_3 & \cos El_3 & \sin El_3 \sin Az_3 & \sin El_3 \cos Az_3 \\ \dots & \dots & \dots & \dots & \dots \\ 1 & \sin El_i & \cos El_i & \sin El_i \sin Az_i & \sin El_i \cos Az_i \\ \dots & \dots & \dots & \dots & \dots \\ 1 & \sin El_N & \cos El_N & \sin El_N \sin Az_N & \sin El_N \cos Az_N \end{bmatrix} \begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{pmatrix} \tag{3}$$

or

$$\vec{d} \;=\; H_A \; \vec{A} \tag{4}$$

where $\vec{d}$ contains the observations, $\vec{A}$ contains the values of the parameters A_1 through A_5, and H_A is a matrix with N rows and 5 columns.

Once the equations are written in this form, I hope it is simple to follow Notebook 11 to determine the least squares solution.

3 Project Steps

3.1 Step 1: Assess the Data

3.1.1 Get the Data

First we have to get the data. The data are in the file `Spring2019PointingData.csv`. The file consists of several columns:

- Column 1 - identifies one of 10 groups of pointing data obtained.
- Column 2 - the name of the radio source
- Column 3 - the observation ID number
- Column 4 - the Azimuth of the observation [degrees].
- Column 5 - the Elevation of the observation [degrees].
- Column 6 - the peak flux of the source [K].
- Column 7 - the error in the peak flux measurement [K]
- Column 8 - the signal-to-noise ratio (SNR) of the measurement
- Column 9 - the Azimuth pointing offset measured [arcsec].
- Column 10 - estimate of error in Azimuth pointing offset [arcsec]
- Column 11 - the Elevation pointing offset measured [arcsec]
- Column 12 - estimate of the error in the Elevation pointing offset [arcsec]

To speed things up, I have also provided a short python script, `data_reader.py`, to read these values and create numpy arrays. The script also plots two figures: (1) the Azimuth and Elevation locations of the data points for reference; and (2) the Azimuth and Elevation positions versus ID number. Note that since observations are taken sequentially, this is like a plot of position of the antenna versus time during the experiment. It is easy to see that we followed a pattern of moving the telescope that tried to cover all possible azimuths and elevations.

3.1.2 Graph the Data; Look at Histograms

Make graphs of the Azimuth and Elevation pointing offset measurements versus Azimuth and Elevation. Are there any obvious trends?

Make a histogram of the pointing offset measurements and compute the standard deviation of the data. Note that most of our estimates of the error in the pointing offset measurement were in the vicinity of 1 arcsec. How does the distribution of the measured pointing offsets compare to a normal distribution with a standard deviation of one arcsec? (Note: Make histograms of the measured pointing offsets for azimuth and elevation.) (NOTE: You will see that the observed distribution looks nothing like a nice normal distribution, meaning that there is something going on besides measurement error.)

3.2 Step 2: Fit the Data

Next step is to actually do the least squares fit of the pointing offset data to the pointing model. You have to do two fits: one for the Azimuth model and one for the Elevation model.

Present your results for the parameters of the Azimuth model and the Elevation model, including an estimate of the errors in these quantities.

Present the values of the RMS of the fit to the Azimuth and Elevation models.

3.3 Step 3: Assess the Fit

Once the fit is complete, it is important to look at how well we did in fitting the measured pointing offsets. Here are some things to do in order to assess the fit:

- Compute the residuals to the model fit
- Make graphs of the residuals versus Azimuth and Elevation.
- Make graphs of the residuals versus ID number (which corresponds roughly to time)
- Create histograms of the residuals for Azimuth pointing and Elevation pointing.

With these figures, look carefully at residuals and address questions:

- Have you improved the pointing errors by fitting the model? (Compare the residuals to the model with the standard deviation of the measured pointing offsets before the fit.)
- Do the residuals look like they follow a normal distribution?
- Are the residuals in Azimuth the same as the residuals in Elevation?
- Do points with larger signal-to-noise (SNR) ratio have smaller residuals than ones with large values of SNR?
- In a graph of the residuals versus Azimuth and Elevation, do you see any systematic trends that might indicate unmodeled effects? (Note: Here we create four plots: (1) azimuth pointing offset residuals versus azimuth; (2) azimuth pointing offset residuals versus Elevation; (3) elevation pointing offset residuals versus Azimith; and (4) elevation pointing offset residuals versus Elevation.)
- Are the residuals correlated with ID, meaning that something about the model is changing with time?

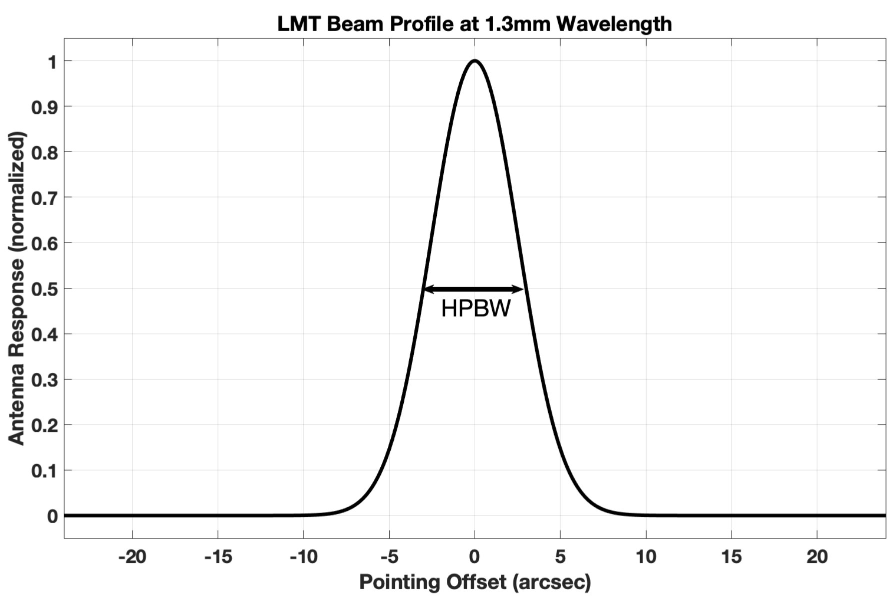

Figure 2: Antenna Beam Profile for the LMT at a wavelength of 1.3 millimeters. The HPBW is 6 seconds of arc.

3.4 Step 4: Your Conclusion

Our main goal, of course, is to predict how well we will be able to point the antenna. The antenna response (B) as a function of the angular offset of the source from the beam peak (θ) is well approximated by a gaussian function, where:

$$B(\theta) \; = \; e^{-4\ln 2 \frac{\theta^2}{H^2}}$$

and H is the half power beam width (commonly abbreviated as HPBW), which is the distance across the beam peak where the response is greater than half the peak value.

At 1.3 millimeters wavelength, the HPBW of the LMT is 6 arcsec (see 3.3). Compare the residuals to the pointing model to the antenna beam. What kinds of errors might we make in the measurement of radio sources given the residuals that remain after our fit? Do you think the antenna can point well enough to support observations with a 6 arcsecond beam?

4 Extra Credit

When I reduced this data, it looked to me like the pointing of the telescope was actually drifting during the observations. I modeled this by dividing the data into 10 groups (column 1 in the data file) and allowing the model to have a different constant term (A_1 and E_1) for each group. All other model terms: $A_2 through A_5$ and E_2 through E_6 were assumed to be the same for all groups. Modify your program to allow the different groups to have different values constants A_1 and E_1. This means you will now have 14 terms for the azimuth model and 15 terms for the elevation model.

Show the residuals plotted versus ID number and make a graph of the azimuth and elevation residuals to this model. How much do we improve the pointing?