

# An Optimization approach for implementing the Sterile Insect Technique in a multi-patch system

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## Abstract

This article explores two complementary approaches to determine an optimal strategy for implementing the Sterile Insect Technique (SIT). SIT involves the release of sterilised male insects that, upon mating with wild females, produce no viable offspring, thereby leading to population suppression or eradication. The first approach is based on a system of coupled ordinary differential equations, enabling a qualitative analysis of release strategies, particularly in the case of constant releases. The second approach incorporates stochasticity at the individual level through multi-agent simulations, providing a more detailed representation of population dynamics. However, such simulations are computationally intensive, especially when exploring a wide range of control strategies. To overcome this limitation, a neural network is trained to emulate the simulation outputs. This surrogate model allows the use of optimisation techniques such as gradient descent to efficiently identify optimal SIT strategies.

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# 1 Introduction

## 1.1 The sterile insect technique

The control of pest and disease vector populations is a pressing issue in both agriculture and public health. Among the various sustainable methods developed to address this challenge, the *Sterile Insect Technique* (SIT) stands out as an environmentally friendly strategy with proven effectiveness in specific contexts. SIT involves releasing sterilized male insects, which, upon mating with wild females, produce no viable offspring. Over time, this leads to the suppression or even eradication of the target population. Although SIT has been successfully applied in several isolated or controlled settings, its implementation in complex, spatially structured environments remains challenging.

In particular, when dealing with heterogeneous landscapes, spatial dynamics can have a profound influence on the outcome of SIT campaigns. In such *multi-patch systems*, a central question arises: *how should sterile insects be released across the different patches to maximise effectiveness while minimising operational costs?* This question becomes especially relevant in scenarios where only partial spatial control is possible due to logistical or financial constraints.

To address this problem, we propose two complementary approaches:

- A **mathematical model** based on coupled ordinary differential equations, which captures the demographic and migratory dynamics of both wild and sterile populations. This approach builds upon the framework introduced in [2] and enables qualitative analysis of control strategies, particularly in the case of constant releases.
- A **multi-agent simulation framework**, inspired by [4], which incorporates stochasticity and heterogeneity at the individual level. To reduce the computational cost associated with exploring a large space of strategies, we train a *deep neural network* to emulate the simulation outputs efficiently. This idea is adapted from [7]. The surrogate model then enables the use of optimisation algorithms, such as gradient descent, to identify near-optimal SIT strategies.

This dual approach allows us to analyse the trade-offs between the number of sterile insects released, the timing and location of the releases, and the intensity of migration between patches.

The paper is organised as follows. Part I focuses on the optimisation problem derived from the system of coupled ordinary differential equations: Section 2 presents analytical results for optimal constant strategies, while Section 3 provides numerical illustrations in different multi-patch scenarios. Part II is devoted to the agent-based simulation framework: Section 4 details the construction of the multi-agent system (MAS), Section 5 explains how the MAS can be approximated by a neural network, and Section 6 shows how this surrogate model can be used to compute optimal release strategies via gradient-based optimisation methods.

## 1.2 Notations

Throughout the article, we will use the following notations:

- $\mathbf{1}_n \in \mathbb{R}^n$  denotes the vector whose each component is equal to 1.
- If  $u = (u_i)_{1 \leq i \leq n}$  and  $v = (v_i)_{1 \leq i \leq n}$  are two vectors in  $\mathbb{R}^n$ , we will write  $u \leq v$  to mean that  $u_i \leq v_i$  for every  $i = 1, \dots, n$ .

# Part I

## optimisation problem

### 2 Towards an optimisation problem

The goal of this section is to formulate the problem as an optimisation problem. To this end, we first model it using a system of coupled differential equations. Analyzing this system will allow us to determine the conditions under which the proposed strategy is effective.

#### 2.1 Modeling issues

Let's consider a system of  $n$  patches. The wild population in patch  $i$  is denoted by  $P_i$ , and the sterile population by  $M_{s,i}$ . We assume that sterile insects are released in each patch at a rate  $u_i$ . Since the time unit is typically a day, we model instantaneous migration between patch  $i$  and patch  $j$ , leading to the following system:

$$\begin{cases} \frac{dP_i}{dt} = P_i f_i(P_i, M_{s,i}) + \sum_{j=1, j \neq i}^n d_{ij} P_j - \sum_{j=1, j \neq i}^n d_{ji} P_i \\ \frac{dM_{s,i}}{dt} = u_i(t) - \mu_S M_{s,i} + \omega \sum_{j=1, j \neq i}^n d_{ij} M_{s,j} - \omega \sum_{j=1, j \neq i}^n d_{ji} M_{s,i} \end{cases} \quad (1)$$

where  $d_{ij}$  is the migration rate from patch  $j$  to patch  $i$ ,  $\mu_S$  represents the mortality rate of sterile insects,  $\omega$  is a parameter representing the impact of radiation on the dispersal ability of the sterile insects and  $f_i(P_i, M_{s,i})$  represents the specific dynamics of the wild population in that patch.

#### Impact of sterile insects on the wild population

Without the sterile population,  $f_i(x, y) = b_i - \mu_{i,1} - \mu_{i,2}x$  where  $b_i > 0$  represents the daily birth-rate per individual,  $\mu_{i,1} > 0$  represents the daily death-rate, and  $\mu_{i,2} > 0$  represents the daily density death-rate in patch  $i$ .

By releasing sterile insects, a mating competition is introduced between wild and sterile individuals, thereby reducing the reproductive potential of the wild population. Therefore, the daily birth-rate becomes  $b_i \frac{P_i}{P_i + \gamma M_{s,i}}$ , where the fraction represents the probability of a fertile mating occurring in the patch  $i$ , and  $\gamma$  is a parameter reflecting the average competitiveness of the sterile males. Thus, we can rewrite the dynamics of the wild population as follows :

$$f_i(x, y) = \frac{b_i x}{\gamma y + x} - \mu_{i,1} - \mu_{i,2}x \quad (2)$$

Releasing sterile insects introduce a strong Allee effect on the population. Indeed, assuming  $b_i - \mu_{i,1} - \mu_{i,2}\gamma y > 2\sqrt{\mu_{i,1}\mu_{i,2}\gamma y}$ ,  $f_i(\cdot, y)$  has two non-negative zeros. The smallest stands for the critical point  $A(y)$  and the biggest for the carrying capacity  $K(y)$ .

#### Study of equilibrium states

Our goal is to reduce the wild population to zero by releasing sterile insects, i.e., we want  $P$  to be as close as possible of  $\mathbf{0}$  and  $\mathbf{0}$  to be an equilibrium state for  $P$  in a way.

Firstly, the differential system (1) can be expressed in the concise form:

$$\begin{cases} \frac{dP}{dt} = P \cdot F(P, M_s) + DP \\ \frac{dM_s}{dt} = U(t) + \mathcal{A}M_s \end{cases} \quad (3)$$

where  $P = (P_i)_i$ ,  $M_s = (M_{s,i})_i$ ,  $U = (u_i)_i$ ,  $F(X, Y) = (f_i(x_i, y_i))_i$ ,  $\mathcal{A} = \omega D - \mu_S I_n$  and :

$$D_{ij} = \begin{cases} d_{ij} & \text{for } i \neq j, \\ -\sum_{k=1, k \neq i}^n d_{ki} & \text{for } i = j. \end{cases}$$

We will focus on the differential equation satisfied by the sterile population by giving some properties of the matrix  $\mathcal{A}$ .

**Proposition 1.** [2, Remark 2.1]

1.  $\mathcal{A}$  is a Metzler matrix, i.e.,  $\mathcal{A}_{ij} \geq 0$  for  $i \neq j$ ,
2. additionally,  $\mathcal{A}_{ii} < 0$ ,
3. since  $\sum_j \mathcal{A}_{ji} = -\mu_S < 0$  for all  $i$ , it follows that  $\sum_{j, j \neq i} \mathcal{A}_{ji} < -\mathcal{A}_{ii}$  for all  $i$ . In other words,  $\mathcal{A}$  is strictly diagonally dominant.

These properties imply that  $\mathcal{A}$  is nonsingular and hence invertible. Moreover,  $\mathcal{A}$  is Hurwitz, which means that every eigenvalue of  $\mathcal{A}$  has a negative real part.

The main result of this part is the following property, devoted to the asymptotic behaviour of  $P(t)$  as  $t \rightarrow +\infty$ .

**Theorem 1.** Let us assume that  $b_i > \mu_{i,1}$  for all  $i = 1, \dots, n$ . Let us introduce  $M_{s,i}^{\text{crit}} = \frac{\mu_{i,1}}{\mu_{i,2}} \left( \sqrt{\frac{b_i}{\mu_{i,1}}} - 1 \right)^2$  works. If  $U \geq 0$  and  $-\mathcal{A}^{-1}U > M_s^{\text{crit}}$ ,  $\mathbf{0}$  is a globally asymptotically stable for  $P$ .

Then, if we want to minimise the amount of sterile insects released, we have to solve the following linear optimisation problem:

$$\inf \left\{ \sum_{1 \leq i \leq n} u_i : -\mathcal{A}^{-1}U \geq M_s^{\text{crit}} \text{ and } U \geq 0 \right\}$$

(S)

## 2.2 Proof of Theorem 1

From now, we assume that the control  $U$  is constant over time. And so the goal is to find conditions on  $U = (u_i)_i$  under which  $\mathbf{0}$  will be an equilibrium state for  $P$ . Yet, as  $\mathcal{A}$  is Hurwitz :

**Proposition 2.**  $M_s^* := -\mathcal{A}^{-1}U \geq 0$  is a globally asymptotically stable (GAS) for  $M_s$ .

Let  $\epsilon > 0$ . Thanks to [Proposition 2](#), there exists  $T > 0$  such as  $M_s(t) > M_s^* - \epsilon$  for all  $t > T$ . We introduce  $P_\epsilon$  such as:

$$\begin{cases} P_\epsilon = P & \text{for } t \leq T \\ \frac{dP_\epsilon}{dt} = P_\epsilon \cdot F(P_\epsilon, M_s^* - \epsilon) + DP_\epsilon & \text{for } t > T \end{cases} \quad (4)$$

By setting,  $r_{i,\epsilon} = \sup_{x \geq 0} f_i(x, M_{s,i}^* - \epsilon)$ , we define the following matrix:

$$A_{r,\epsilon} = D + \text{diag}(r_1, \dots, r_n) \quad (5)$$

**Theorem 2.** [5, Theorem 1] If  $A_{r,\epsilon}$  is Hurwitz,  $P_\epsilon(t) \rightarrow \mathbf{0}$  when  $t \rightarrow +\infty$ .

By calculate the derivative of  $f_i(\cdot, M_{s,i}^* - \epsilon)$ , we find that:

$$r_{i,\epsilon} = \begin{cases} \left( \sqrt{b_i} - \sqrt{\mu_{2,i}\gamma(M_{s,i}^* - \epsilon)} \right)^2 - \mu_{1,i} & \text{when } 0 \leq \gamma(M_{s,i}^* - \epsilon) \leq \frac{b_i}{\mu_{2,i}} \\ -\mu_{1,i} & \text{when } \gamma(M_{s,i}^* - \epsilon) > \frac{b_i}{\mu_{2,i}} \end{cases} \quad (6)$$

Thus, if  $M_{s,i}^{\text{crit}} < \gamma(M_{s,i}^* - \epsilon)$  ensures that:

$$\gamma(M_{s,i}^* - \epsilon) \leq \frac{b_i}{\mu_{2,i}} \implies \left( \sqrt{b_i} - \sqrt{\mu_{2,i}\gamma(M_{s,i}^* - \epsilon)} \right)^2 - \mu_{1,i} < 0$$

We obtain that:

$$\begin{cases} (A_{r,\epsilon})_{ii} = -\mu_{1,i} - \sum_{j,j \neq i} d_{ij} < 0 & \text{when } \gamma(M_{s,i}^* - \epsilon) > \frac{b_i}{\mu_{2,i}} \\ (A_{r,\epsilon})_{ii} = \left( \sqrt{b_i} - \sqrt{\mu_{2,i}\gamma(M_{s,i}^* - \epsilon)} \right)^2 - \mu_{1,i} - \sum_{j,j \neq i} d_{ij} < 0 & \text{when } 0 \leq \gamma(M_{s,i}^* - \epsilon) \leq \frac{b_i}{\mu_{2,i}} \\ 0 \leq \sum_{j,j \neq i} (A_{r,\epsilon})_{ij} = \sum_{j,j \neq i} d_{ij} < |(A_{r,\epsilon})_{ii}| \end{cases}$$

and so, by the Gershgorin circle theorem,  $A_{r,\epsilon}$  is Hurwitz.

Since  $b_i > \mu_{i,1}$ , a straightforward computation shows that  $M_{s,i}^{\text{crit}} = \frac{\mu_{i,1}}{\mu_{i,2}} \left( \sqrt{\frac{b_i}{\mu_{i,1}}} - 1 \right)^2$  works.

**Corollary 1.** Assuming  $b_i > \mu_{i,1}$  for all  $i$ . If  $M_{s,i}^{\text{crit}} < \gamma M_{s,i}^*$ ,  $\mathbf{0}$  is a globally asymptotically stable for  $P$ .

*Proof.* If  $M_{s,i}^{\text{crit}} < \gamma M_{s,i}^*$ , there is  $\epsilon > 0$  such as  $M_{s,i}^{\text{crit}} < \gamma(M_{s,i}^* - \epsilon)$ . Then  $A_{r,\epsilon}$  is Hurwitz and by [Theorem 2](#),  $P_\epsilon(t) \rightarrow \mathbf{0}$ . However  $0 \leq P \leq P_\epsilon$ , so  $P(t) \rightarrow \mathbf{0}$ .  $\square$

### 2.3 Analysis of Problem (S)

**Proposition 3.** *Problem (S) has a solution.*

*Proof.* It is straightforward that the existence issue comes to the existence of a vector  $U$  satisfying both

$$-\mathcal{A}^{-1}U \geq M_s^{\text{crit}} \quad \text{and} \quad U \geq 0.$$

To this aim, let us introduce

$$\bar{U} = \frac{1}{m_{\mathcal{A}}} \max_{1 \leq i \leq n} M_{s,i}^{\text{crit}} \mathbf{1}_n, \quad \text{where} \quad m_{\mathcal{A}} = \min_{1 \leq i,j \leq n} \{(-\mathcal{A}^{-1})_{i,j} \mid (-\mathcal{A}^{-1})_{i,j} > 0\}$$

Notice that  $-\mathcal{A}$  enjoys the following property.

**Lemma 1.** *Each coefficient of  $\mathcal{A}^{-1}$  is non-positive.*

For the sake of clarity, we postponed the proof of Lemma 1 to the end of this proof.

According to Lemma 1, for  $i_0 \in \{1, \dots, n\}$ , one has

$$\begin{aligned} (-\mathcal{A}^{-1}\bar{U})_{i_0} &= \sum_{j=1}^n (-\mathcal{A}^{-1})_{i_0,j} \bar{U}_j \\ &\geq \frac{\sum_{j=1}^n (-\mathcal{A}^{-1})_{i_0,j}}{m_{\mathcal{A}}} \max_{1 \leq i \leq n} M_{s,i}^{\text{crit}} \geq \max_{1 \leq i \leq n} M_{s,i}^{\text{crit}} \end{aligned}$$

so that  $-\mathcal{A}^{-1}\bar{U} \geq M_{s,i}^{\text{crit}}$ . The conclusion follows.  $\square$

*Proof of Lemma 1.* We will prove this result by contraction. Assume  $(\mathcal{A}^{-1})_{i_0j_0} = \max_{i,j} (\mathcal{A}^{-1})_{ij} > 0$ . By considering that  $\mathcal{A}^{-1}\mathcal{A} = I_n$ , we obtain that:

$$(\mathcal{A}^{-1}\mathcal{A})_{i_0j_0} = \sum_k (\mathcal{A}^{-1})_{ik} \mathcal{A}_{kj} = \omega \sum_{k \neq j_0} (\mathcal{A}^{-1})_{i_0k} d_{kj_0} - (\mathcal{A}^{-1})_{i_0j_0} (\omega \sum_{k \neq j_0} d_{kj_0} + \mu_s) = 0 \text{ or } 1 \geq 0.$$

Therefore, as  $\mu_s > 0$ ,  $\sum_{k \neq j_0} (\mathcal{A}^{-1})_{i_0k} d_{kj_0} > (\mathcal{A}^{-1})_{i_0j_0} \sum_{k \neq j_0} d_{kj_0}$  i.e.  $\sum_{k \neq j_0} ((\mathcal{A}^{-1})_{i_0k} - (\mathcal{A}^{-1})_{i_0j_0}) d_{kj_0} > 0$ .

Moreover,  $d_{ij} \geq 0$ , so there is  $j$  such as  $(\mathcal{A}^{-1})_{i_0j} > (\mathcal{A}^{-1})_{i_0j_0}$  and we are done.  $\square$

For many reasons, one may not be able to release sterile insects in all patches. In case of constant control, we can find necessary and sufficient conditions under which there is an optimal solution of Problem (S) strictly positive.

**Theorem 3.** *Problem (S) has a positive optimal solution  $U$  if and only if  $-\mathcal{A}M_s^{\text{crit}} > 0$ . In this case  $U = -\mathcal{A}M_s^{\text{crit}}$ .*

*Proof.* First, let us write Problem (S) in a standard form. Let us introduce the function  $g_i$  for  $i = 1 \dots n$  as

$$g_i(x_1, \dots, x_n) = (\mathcal{A}^{-1}X)_i + M_{s,i}^{\text{crit}}$$

Therefore, Problem (S) rewrites

$$\begin{cases} \min u_1 + \dots + u_n \\ g_i(u_1, \dots, u_n) \leq 0 \text{ for } i = 1 \dots n \\ -u_i \leq 0 \text{ for } i = 1 \dots n \end{cases}$$

As all constraints are differentiable convex functions, necessary Karush-Kuhn-Tucker conditions are also sufficient. Thus,  $U$  is optimal if, and only if there are  $\lambda_1, \dots, \lambda_n, \tilde{\lambda}_1, \dots, \tilde{\lambda}_n \geq 0$  such as

$$\begin{cases} \sum_{i=1}^n \lambda_i \nabla g_i(u_1, \dots, u_n) + (1 - \tilde{\lambda}_i) e_i = \mathbf{0} \\ \lambda_i g_i(u_1, \dots, u_n) = 0 & \text{for } i = 1 \dots n \\ \tilde{\lambda}_i u_i = 0 & \text{for } i = 1 \dots n \end{cases}$$

with  $(e_i)_i$  the canonical basis of  $\mathbb{R}^n$ , which rewrites

$$\begin{cases} -{}^t \mathcal{A}^{-1} \Lambda = \mathbf{1}_n - \tilde{\Lambda} \\ \lambda_i g_i(u_1, \dots, u_n) = 0 & \text{for } i = 1 \dots n \\ \tilde{\lambda}_i u_i = 0 & \text{for } i = 1 \dots n \end{cases}$$

with  $\Lambda = (\lambda_i)_i$ ,  $\tilde{\Lambda} = (\tilde{\lambda}_i)_i$ .

Thus,  $U$  is optimal if, and only if there is  $\Lambda = (\lambda_i)_i \geq 0$  such that

$$\begin{cases} -{}^t \mathcal{A}^{-1} \Lambda = \mathbf{1}_n \\ \lambda_i g_i(u_1, \dots, u_n) = 0 & i = 1 \dots n \\ U > 0 \end{cases} \Leftrightarrow \begin{cases} \Lambda = -{}^t \mathcal{A} \mathbf{1}_n = \mu_S \mathbf{1}_n > 0 \\ g_i(u_1, \dots, u_n) = 0 & i = 1 \dots n \\ U > 0 \end{cases}$$

which means that  $U = -\mathcal{A}M_s^{\text{crit}}$  and  $-\mathcal{A}M_s^{\text{crit}} > 0$ .  $\square$

**Remark 4.** If  $U = -\mathcal{A}M_s^{\text{crit}}$ ,  $\sum_{1 \leq i \leq n} u_i = {}^t U \mathbf{1}_n = {}^t M_s^{\text{crit}} (-{}^t \mathcal{A} \mathbf{1}_n) = {}^t M_s^{\text{crit}} (\mu_S \mathbf{1}_n) = \mu_S \sum_{1 \leq i \leq n} M_{s,i}^{\text{crit}}$ . And so, when the optimal strategy is to release sterile males in all patches, the global amount of insects released depends only of  $\mu_S$  and  $(b_i, \mu_{1,i}, \mu_{2,i})$  for  $i = 1 \dots n$ , i.e, depends only of birth and mortality rates and not of migration rates.

### 3 Numerical results and discussion

As established in Section 2.3, Problem (S) is a linear optimisation problem. Consequently, classical methods such as the simplex algorithm can be employed to solve it numerically. This allows us to assess whether the resulting control strategy effectively reduces the wild population, and to analyze its qualitative properties. In this section, we focus on case studies involving 2, 3, and 10 patches.

#### 3.1 Case of 2 patches.

We focus on the case with 2 patches, patch 1 and 2. A schematic representation of this configuration is provided in Figure 1.

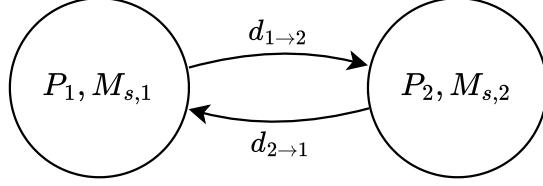


Figure 1: A two patches system

We aim to study the impact of migration rates on the optimal constant control. The corresponding migration matrix is given by:

$$D = \begin{pmatrix} -d_{1 \rightarrow 2} & d_{2 \rightarrow 1} \\ d_{1 \rightarrow 2} & -d_{2 \rightarrow 1} \end{pmatrix}$$

So the condition from [Theorem 3](#) can be rewritten as

$$-\mathcal{A}M_s^{\text{crit}} > 0 \iff d_{2 \rightarrow 1} \in I_{2,d} = (d_{2,\text{inf}}^{\text{crit}}, d_{2,\text{sup}}^{\text{crit}}),$$

where

$$\begin{aligned} d_{2,\text{inf}}^{\text{crit}} &= d_{1 \rightarrow 2} \cdot \frac{M_{s,1}^{\text{crit}}}{M_{s,2}^{\text{crit}}} - \frac{\mu_s}{\omega}, \\ d_{2,\text{sup}}^{\text{crit}} &= d_{1 \rightarrow 2} \cdot \frac{M_{s,1}^{\text{crit}}}{M_{s,2}^{\text{crit}}} + \frac{\mu_s}{\omega} \cdot \frac{M_{s,1}^{\text{crit}}}{M_{s,2}^{\text{crit}}}. \end{aligned}$$

**Remark 5.** Due to the symmetric roles of patch 1 and patch 2, the condition can equivalently be written with respect to  $d_{1 \rightarrow 2}$  as

$$d_{1 \rightarrow 2} \in I_{1,d} = (d_{1,\text{inf}}^{\text{crit}}, d_{1,\text{sup}}^{\text{crit}}),$$

where

$$\begin{aligned} d_{1,\text{inf}}^{\text{crit}} &= d_{2 \rightarrow 1} \cdot \frac{M_{s,2}^{\text{crit}}}{M_{s,1}^{\text{crit}}} - \frac{\mu_s}{\omega}, \\ d_{1,\text{sup}}^{\text{crit}} &= d_{2 \rightarrow 1} \cdot \frac{M_{s,2}^{\text{crit}}}{M_{s,1}^{\text{crit}}} + \frac{\mu_s}{\omega} \cdot \frac{M_{s,2}^{\text{crit}}}{M_{s,1}^{\text{crit}}}. \end{aligned}$$

**Remark 6.** The length of the interval  $I_{2,d}$  is given by

$$d_{2,\text{sup}}^{\text{crit}} - d_{2,\text{inf}}^{\text{crit}} = \frac{\mu_S}{\omega} \left( \frac{M_{s,1}^{\text{crit}}}{M_{s,2}^{\text{crit}}} + 1 \right) > \frac{\mu_S}{\omega},$$

and notably, it does not depend on  $d_{1 \rightarrow 2}$ .

As  $\omega$  increases, the size of  $I_{2,d}$  decreases. Since  $\omega$  reflects the ability of sterile insects to migrate between patches, a high value of  $\omega$  implies easier dispersal. In such cases, it becomes less critical to release sterile insects in every patch.

Conversely,  $\mu_S$  denotes the mortality rate of sterile insects. When  $\mu_S$  is high, sterile insects tend to die quickly and are therefore less likely to migrate between patches, making local releases in all patches more necessary.

The ratio  $\frac{M_{s,1}^{\text{crit}}}{M_{s,2}^{\text{crit}}}$  serves as a measure of asymmetry between the two patches. Indeed, the size of  $I_{2,d}$  is directly proportional to this ratio, while the size of the corresponding interval  $I_{1,d}$  is inversely proportional.

## Numerical results

The parameter values associated with each patch are summarized in [Table 1](#), based on data from [2, Table 1].

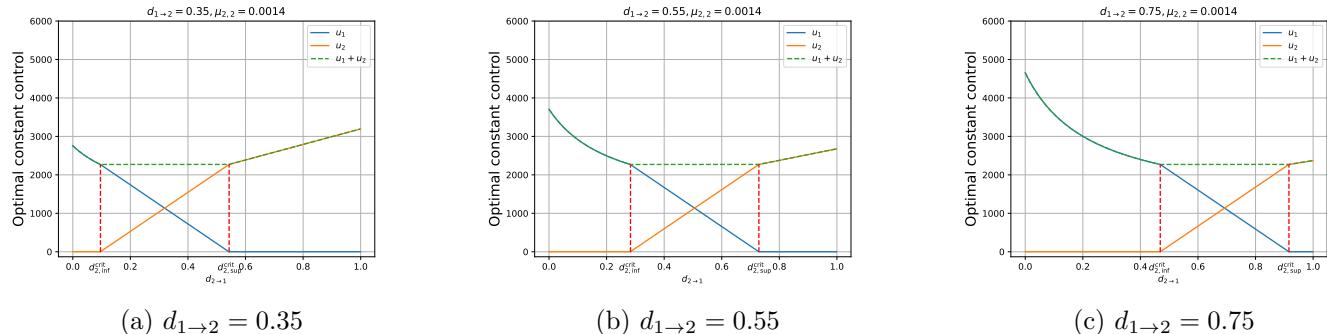
	$b_i$	$\mu_{1,i}$	$\mu_{2,i}$	$\omega$	$\mu_s$	$\gamma$
Patch 1	8.15	0.035	0.0015	1.0	0.231	1.0
Patch 2			0.0014			

Table 1: Parameter values for Patch 1 and Patch 2

As indicated in [Table 1](#), the parameters  $b_i$  and  $\mu_{1,i}$  are invariant across patches. Consequently, the ratio of the critical sterile insect populations is given by  $\frac{M_{s,1}^{\text{crit}}}{M_{s,2}^{\text{crit}}} = \frac{\mu_{2,2}}{\mu_{2,1}}$ .

In the first row of [Figure 2](#), the value of  $\mu_{2,2}$  is fixed at 0.0014 to investigate the effects of varying  $d_{1 \rightarrow 2}$ . Conversely, in the second row of [Figure 2](#), the value of  $d_{1 \rightarrow 2}$  is set to 0.5 to assess the impact of changes in  $\mu_{2,2}$ .

(i) Variation of  $d_{1 \rightarrow 2}$  with fixed  $\mu_{2,2} = 0.0014$



(ii) Variation of  $\mu_{2,2}$  with fixed  $d_{1 \rightarrow 2} = 0.5$

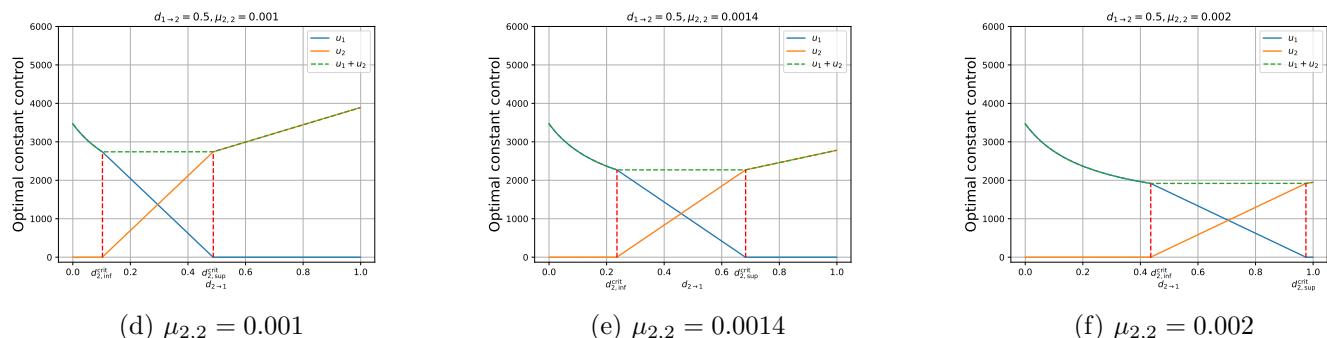


Figure 2: Combined figure showing the variation of parameters in a two-patch system.

We find that the length of the interval  $I_{2,d}$  is independent of migration rates and depends solely on  $\mu_s$ ,  $\omega$ , and  $M_s^{\text{crit}}$ . Specifically, as noted in [Remark 6](#), for  $\mu_{2,2} = 0.0014$ , we have

$$d_{2,\text{sup}}^{\text{crit}} - d_{2,\text{inf}}^{\text{crit}} = 0.4466,$$

which is consistent with the value observed in [Figure 2](#).

[Figure 2](#) also demonstrates that both  $d_{2,\text{sup}}^{\text{crit}}$  and  $d_{2,\text{inf}}^{\text{crit}}$  increase with  $d_{1 \rightarrow 2}$ . A linear regression analysis reveals that both  $d_{2,\text{sup}}^{\text{crit}}$  and  $d_{2,\text{inf}}^{\text{crit}}$  exhibit a linear relationship with  $d_{1 \rightarrow 2}$ , sharing the same slope. This aligns with the expressions derived in [Section 3.1](#); refer to [Table 2](#) for the linear regression details.

Across [Figure 2a](#), [Figure 2b](#), and [Figure 2c](#), it is evident that the total amount of sterile insects released, denoted by  $\tilde{u}$ , remains constant over  $d_{2 \rightarrow 1} \in I_{2,d}$  and  $d_{1 \rightarrow 2}$ . Furthermore, [Figure 2](#) indicates that  $\tilde{u}$  decreases with  $\mu_{2,2}$ . A linear regression analysis shows that  $\tilde{u}$  is, in fact, linearly related to  $\frac{1}{\mu_{2,2}}$ ; see [Table 2](#) for details.

interest	slope	intercept	R-value	p-value
$d_{2,\text{sup}}^{\text{crit}}(d_{1 \rightarrow 2})$	0.93	0.22	1.00	0.00
$d_{2,\text{inf}}^{\text{crit}}(d_{1 \rightarrow 2})$	0.93	-0.23	1.00	0.00
$\tilde{u}(\frac{1}{\mu_{2,2}})$	1.65	1096	1.00	0.00

Table 2: Three linear regressions

### 3.2 An interesting case with 3 patches

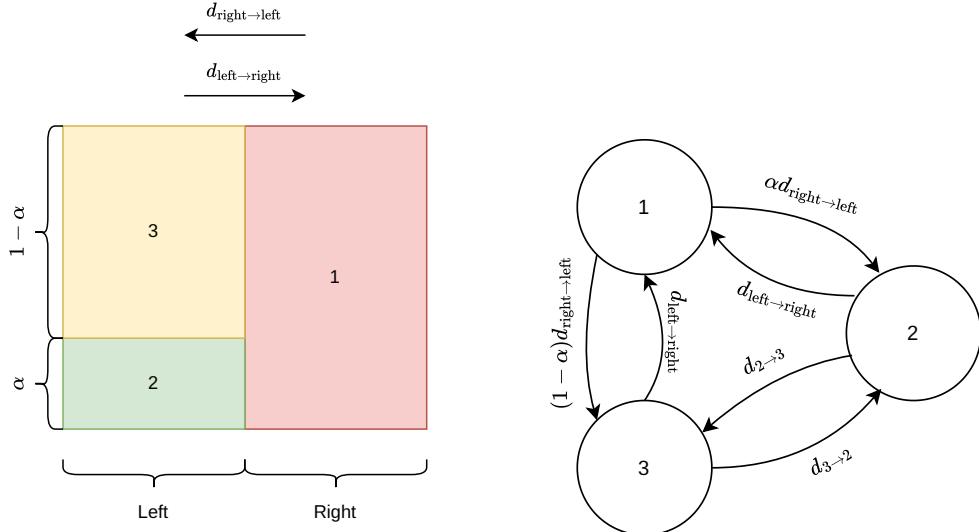


Figure 3: Combined figure

We consider a spatial configuration comprising two primary regions: the *Left* and the *Right*. The *Right* region is composed of a single patch, denoted as patch 1, whereas the *Left* region is divided into two distinct sub-patches: patch 3 (upper section) and patch 2 (lower section), as depicted in [Figure 4](#).

Mosquito migration between the *Left* and *Right* regions occurs at distinct directional rates:  $d_{\text{right} \rightarrow \text{left}}$  and  $d_{\text{left} \rightarrow \text{right}}$ .

It is assumed that migration from the *Right* region to the *Left* region is proportionally distributed between patches 2 and 3, based on the relative size of their shared border. Specifically, the migration rate

from patch 1 to patch 2 is  $\alpha d_{\text{right} \rightarrow \text{left}}$ , while the migration rate from patch 1 to patch 3 is  $(1 - \alpha)d_{\text{right} \rightarrow \text{left}}$ , where  $\alpha \in (0, 1)$ .

Conversely, the migration rates from patch 2 to patch 1 and from patch 3 to patch 1 are both equal to  $d_{\text{left} \rightarrow \text{right}}$ .

Additionally, it is assumed that patches 2 and 3 share identical environmental conditions, thus  $M_{s,2}^{\text{crit}} = M_{s,3}^{\text{crit}} = M_{s,\text{Left}}^{\text{crit}}$ .

The matrix  $\mathcal{A}$  can be explicitly expressed as follows:

$$\mathcal{A} = \begin{pmatrix} -\mu_S - \omega d_{\text{right} \rightarrow \text{left}} & \omega d_{\text{left} \rightarrow \text{right}} & \omega d_{\text{left} \rightarrow \text{right}} \\ \omega \alpha d_{\text{right} \rightarrow \text{left}} & -\mu_S - \omega(d_{\text{left} \rightarrow \text{right}} + d_{2 \rightarrow 3}) & \omega d_{3 \rightarrow 2} \\ \omega(1 - \alpha)d_{\text{right} \rightarrow \text{left}} & \omega d_{2 \rightarrow 3} & -\mu_S - \omega(d_{\text{left} \rightarrow \text{right}} + d_{3 \rightarrow 2}) \end{pmatrix}.$$

According to [Theorem 3](#), if  $U > 0$ :

$$U = -\mathcal{A}M_s^{\text{crit}} = \begin{pmatrix} \mu_S + \omega d_{\text{right} \rightarrow \text{left}} & -\omega d_{\text{left} \rightarrow \text{right}} & -\omega d_{\text{left} \rightarrow \text{right}} \\ -\omega \alpha d_{\text{right} \rightarrow \text{left}} & \mu_S + \omega(d_{\text{left} \rightarrow \text{right}} + d_{2 \rightarrow 3}) & -\omega d_{3 \rightarrow 2} \\ -\omega(1 - \alpha)d_{\text{right} \rightarrow \text{left}} & -\omega d_{2 \rightarrow 3} & \mu_S + \omega(d_{\text{left} \rightarrow \text{right}} + d_{3 \rightarrow 2}) \end{pmatrix} \begin{pmatrix} M_{s,\text{Right}}^{\text{crit}} \\ M_{s,\text{Left}}^{\text{crit}} \\ M_{s,\text{Left}}^{\text{crit}} \end{pmatrix}.$$

The first row yields:

$$u_1 = M_{s,\text{Right}}^{\text{crit}} (\mu_S + \omega d_{\text{right} \rightarrow \text{left}}) - 2M_{s,\text{Left}}^{\text{crit}} \omega d_{\text{left} \rightarrow \text{right}}.$$

The last two rows provide:

$$\begin{aligned} u_2 &= \frac{u_{\text{left}}}{2} + M_{s,\text{Right}}^{\text{crit}} \omega \left( \frac{1}{2} - \alpha \right) d_{\text{right} \rightarrow \text{left}} - M_{s,\text{Left}}^{\text{crit}} \omega \Delta d, \\ u_3 &= \frac{u_{\text{left}}}{2} + M_{s,\text{Right}}^{\text{crit}} \omega \left( \alpha - \frac{1}{2} \right) d_{\text{right} \rightarrow \text{left}} + M_{s,\text{Left}}^{\text{crit}} \omega \Delta d, \end{aligned}$$

where  $u_{\text{left}} = u_2 + u_3 = 2M_{s,\text{Left}}^{\text{crit}} (\mu_S + \omega d_{\text{left} \rightarrow \text{right}}) - M_{s,\text{Right}}^{\text{crit}} \omega d_{\text{right} \rightarrow \text{left}}$  and  $\Delta d = d_{3 \rightarrow 2} - d_{2 \rightarrow 3}$ .

**Remark 7.** Consider a system comprising two patches, L and R, with migration rates and environmental conditions defined as follows:

$$(d_{L \rightarrow R}, d_{R \rightarrow L}) = (d_{\text{left} \rightarrow \text{right}}, d_{\text{right} \rightarrow \text{left}})$$

and

$$M_{s,L}^{\text{crit}} = 2M_{s,\text{Left}}^{\text{crit}}, \quad M_{s,R}^{\text{crit}} = M_{s,\text{Right}}^{\text{crit}}.$$

According to [Theorem 3](#), if  $U > 0$ , then  $U$  will be equal to  ${}^t(u_1 \ u_{\text{left}})$ .

**Remark 8** (Impact of  $\Delta d$  and  $\alpha$ ). These parameters both represent migration dynamics between patches. Specifically,  $\alpha$  and  $\Delta d$  increase with migration from patch 1 to patch 2 and from patch 3 to patch 2, respectively. Conversely, they decrease with migration from patch 1 to patch 3 and from patch 2 to patch 3.

- When  $\Delta d > 0$  and  $\alpha > \frac{1}{2}$ , insects are more likely to migrate to patch 2 rather than to patch 3. In this scenario, similar to [Section 3.1](#), following the optimal constant strategy results in a greater release of sterile insects in patch 3 than in patch 2. In extreme cases of asymmetry, sterile insects may need to be released exclusively in patch 3.

- Conversely, if the situation is perfectly symmetric, with  $\Delta d = 0$  and  $\alpha = \frac{1}{2}$ , we find that:

$$u_2 = u_3 = \frac{u_{left}}{2}.$$

## Numerical Results

To numerically validate these findings, we utilize the parameters for the three patches as detailed in [Table 3](#). In the first row of [Figure 4](#), we set the value of  $\Delta d$  and plot the optimal constant control, determined by solving the linear program, over values of  $\alpha \in (0, 1)$ . In the second row of [Figure 4](#), the value of  $\alpha$  is fixed, and the optimal constant control is plotted over values of  $\Delta d$ .

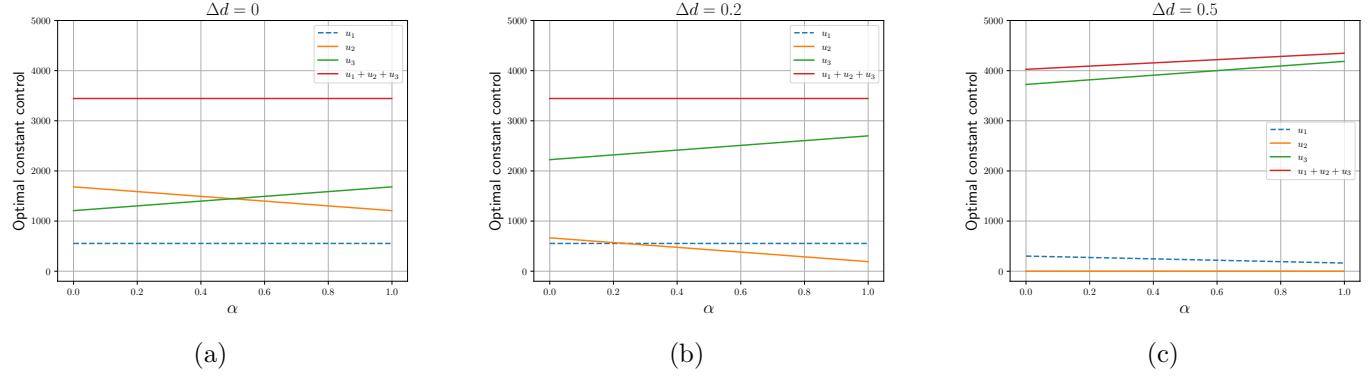
	$b_i$	$\mu_{1,i}$	$\mu_{2,i}$	$\omega$	$\mu_s$	$\gamma$
Patch 1			0.0015			
Patch 2	8.15	0.035	0.0014	1.0	0.231	1.0
Patch 3			0.0014			

Table 3: Parameter values for Patch 1, Patch 2, and Patch 3

These numerical results confirm [Theorem 8](#). Indeed, when  $\Delta d = 0$ , as shown in [Figure 4a](#),  $u_2$  and  $u_3$  exhibit symmetric roles with respect to  $\alpha$ . Consequently, when  $\alpha = \frac{1}{2}$ ,  $u_2 = u_3$ . As mentioned in [Theorem 8](#), the larger  $\Delta d$  and  $\alpha$  are, as depicted in [Figure 4b](#) and [Figure 4c](#), the more sterile insects are released in patch 3 and the fewer in patch 2. Moreover, when  $\Delta d = 0.5$ , sterile insects are not released at all in patch 2.

As in [Section 3.1](#), there appears to exist a  $\Delta d_{sup}^{crit}$  such that, when  $\Delta d > \Delta d_{sup}^{crit}$ , sterile insects are not released in all patches according to the optimal constant control. According to [Figure 4](#),  $\Delta d_{sup}^{crit}$  seems to decrease as  $\alpha$  increases.

**Optimal constant control over  $\alpha \in (0, 1)$  for different values of  $\Delta d$**



**Optimal constant control over  $\Delta d \in (0, \frac{1}{2})$  for different values of  $\alpha$**

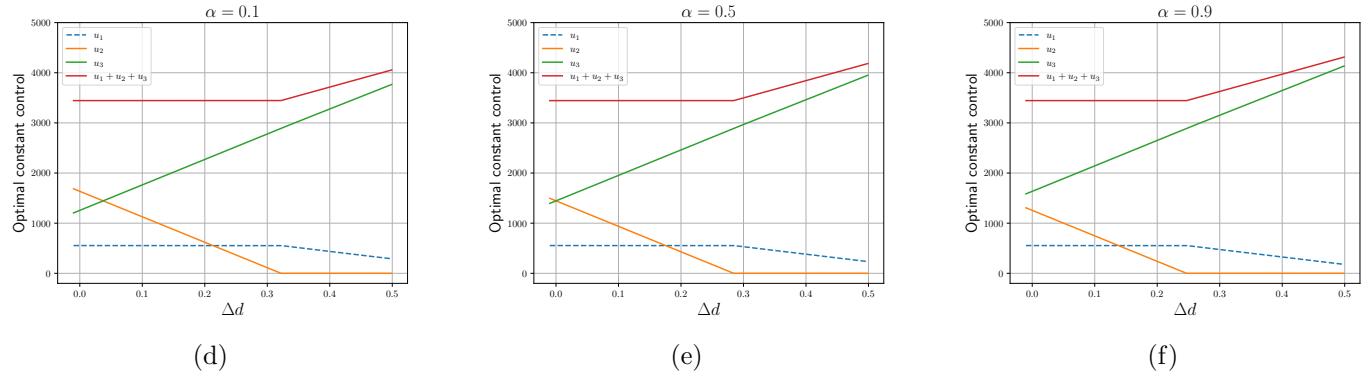


Figure 4: Optimal constant control for different parameters.

### 3.3 A case with 9 patches

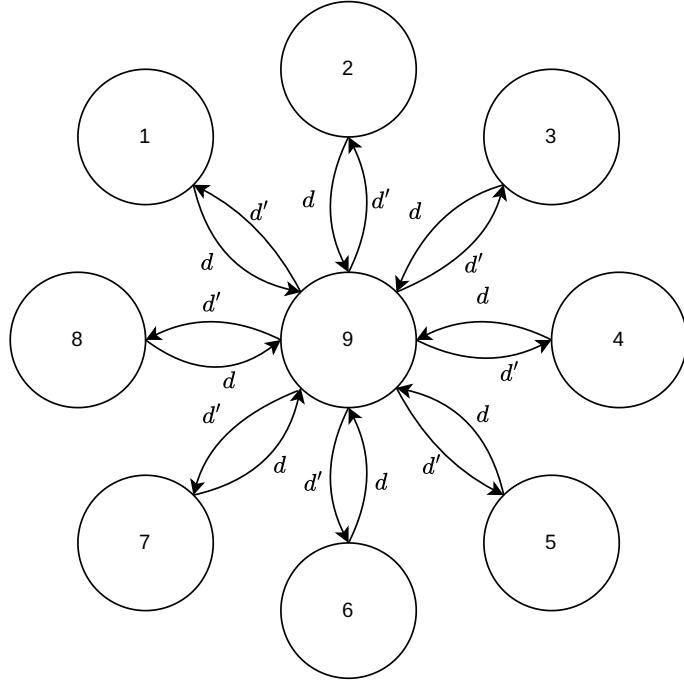


Figure 5: Configuration with 9 patches and a central patch.

We now consider a scenario involving 9 patches with a central patch. As illustrated in Figure 5, there is one central patch, patch 9, connected to the other patches, while patches 1 through 8 are each linked solely to patch 9. The parameters  $d$  and  $d'$  denote the migration rates from patch  $i$  to patch 9 and from patch 9 to patch  $i$ , respectively, for  $i \in \{1, 2, \dots, 8\}$ .

We assume that all patches share identical environmental parameters, which are detailed in Table 4.

	$b_i$	$\mu_{1,i}$	$\mu_{2,i}$	$\omega$	$\mu_s$	$\gamma$
Patch $i \in \{1, 2, \dots, 9\}$	8.15	0.035	0.0015	1.0	0.231	1.0

Table 4: Parameter values for the patches.

To investigate the impact of  $d$  and  $d'$  on the optimal constant strategy, we fix  $d'$  and plot the optimal constant strategy over varying values of  $d$ . The results are presented in Figure 6.

According to the numerical results, three distinct scenarios emerge:

- $u_9 > 0$  and  $u_i = 0$  for  $i \in \{1, 2, \dots, 8\}$ : This scenario occurs when  $d'$  is significantly larger than  $d$ . It is notable that  $u_9$  increases substantially as  $d$  decreases.
- $u_9 > 0$  and  $u_i > 0$  for  $i \in \{1, 2, \dots, 8\}$ : This case arises when  $d$  and  $d'$  are relatively close to each other. All  $u_i$  appear to vary linearly with  $d$ , with  $u_9$  decreasing as  $d$  increases, while the other  $u_i$  increase.
- $u_9 = 0$  and  $u_i > 0$  for  $i \in \{1, 2, \dots, 8\}$ : This situation occurs when  $d$  is significantly larger than  $d'$ .

**Remark 9.** The values of  $d$  that delineate these three domains can be computed using the same methodology as described in Section 3.1.

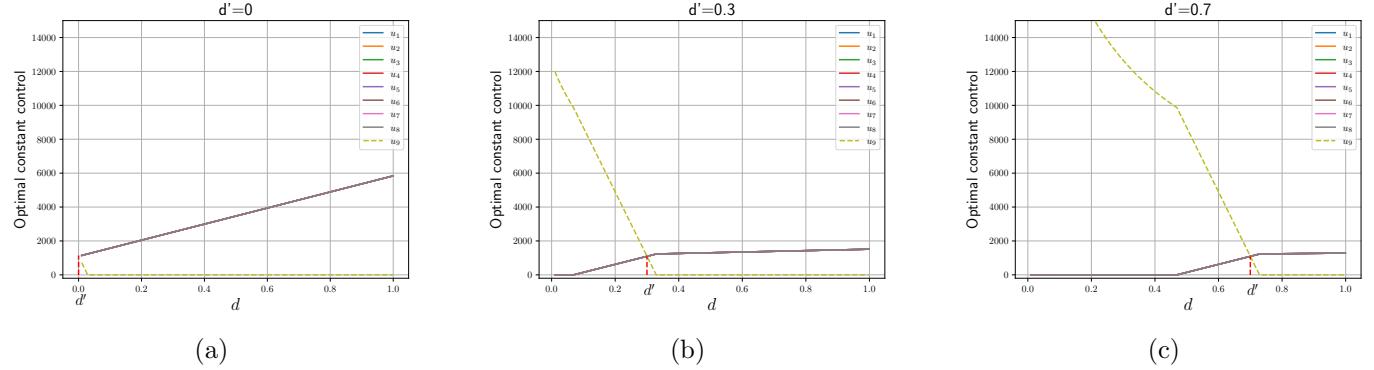


Figure 6: Optimal constant control over  $d \in (0, 1)$ , according to different values of  $d'$ .

## Part II

# Multi-agent simulation

An alternative method for developing an effective strategy is to simulate the life of each insect using a multi-agent simulation. Previously, in equation (1), we modeled insect evolution with an emphasis on the behavior of patches. In this section, we shift our focus to individual insect behavior to gain a more detailed understanding of the overall population dynamics. This approach allows us to identify strategies that incorporate additional constraints.

## 4 Multi-Agent Simulation

To implement a multi-agent simulation (MAS), it is essential to understand the life cycle of *Aedes albopictus*, the mosquito species considered in this study. We use biological parameters and developmental data from Delatte et al. [1], and our agent-based modeling framework is inspired by Isidoro et al. [4, Section 4.1].

The mosquito life cycle is divided into four distinct stages: egg, larva, pupa, and adult. Among adult males, we distinguish between wild males and sterile males, the latter being those released as part of the control strategy. For adult females, we account for their reproductive status as determined by mating events.

In this simulation, we focus solely on the mosquitoes and their ecological environment. Interactions with humans are not modeled; we assume that adult female mosquitoes can acquire sufficient blood meals to complete their gonotrophic cycles and lay eggs without limitation.

### 4.1 Agents

We can enumerate the types of agents included in our simulation as follows. Figure 7 provides an overview of the life cycle of a wild female mosquito.

**Female Mosquito (WF).** This is by far the most complex agent. We distinguish three types of female mosquitoes: young females (YF), fertile females (FF), and sterile females (SF). Young females are individuals that have never mated with a male. Fertile and sterile females are those that have mated with, respectively, a wild male or a sterile male. Fertile females are able to lay eggs, while sterile females not. The life cycle of a wild female mosquito is illustrated in Figure 7. To account for the competitiveness between wild and sterile males, we introduce the parameter  $\gamma$ , as defined in Section 2.1. This parameter will allow us to determine the probability that a given WF engages in a fertile mating, as discussed in Section ??.

**Wild Male Mosquito (WM).** This agent is significantly simpler than the female. A WM undergoes the same developmental stages—from egg to adult—and then moves randomly among patches. At each stage, the mosquito faces a probability of death.

**Sterile Male Mosquito (SM).** An SM is simply a WM that has been released as part of the control strategy and marked as sterile. Otherwise, it behaves identically to a WM.

**Egg, Larva, and Pupa.** These are nonmobile stages of the mosquito life cycle. Individuals in these stages progress to the next stage according to stage-specific survival rates and transition times.

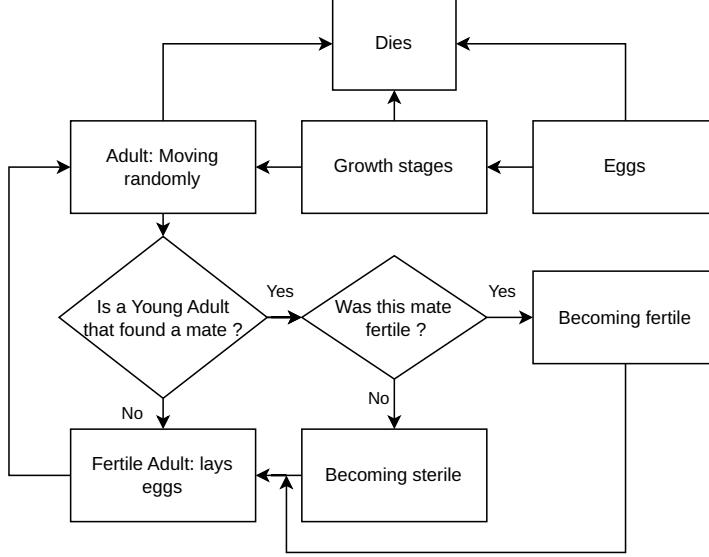


Figure 7: WF lifecycle

## 4.2 Stochastic model

We can model agents and their interactions using random variables by representing each mosquito as a tuple of random variables. Each agent (mosquito) is reduced to its characteristics modeled by random variables.

We denote the set of adult mosquitoes on date  $t$  by  $\mathcal{M}_{\text{adult}}^t = (\mathcal{M}_{\text{adult},k})_{k \in \mathcal{I}^t}$  with  $\mathcal{M}_{\text{adult},k} = (t_{0,k}, p_{0,k}, g_k, \mathcal{L}_k, \mathcal{P}_k, \mathcal{R}_k, \mathcal{T}_k)$  that represents an adult mosquito, where :

- $t_{0,k}$  is the date on which the mosquito became adult.
- $p_{0,k}$  is the patch where the mosquito is born.
- $g_k$  is the gender of the mosquito.
- $\mathcal{I}^t = \{k : 0 \leq t - t_{0,k} \leq \mathcal{L}_k\}_{k \in \mathbb{N}}$  is the set of alive mosquito indices
- $\mathcal{P}_k = (\mathcal{P}_k^t)_{t \in \mathbb{N}}$  is a discrete-time Markov chain, with,  $\mathcal{P}_k^{t_{0,k}} = p_{0,k}$  and  ${}^t D$  its transition matrix.  $\mathcal{P}_k^t$ , with  $t \geq t_{0,k}$ , corresponds to the patch where the mosquito  $k$  is on date  $t$ .
- $\mathcal{L}_k \sim \text{Weibull}(w_1, w_2)$  is the lifespan of the mosquito  $k$ , where  $(w_1, w_2)$  are parameters that depend on whether the mosquito  $k$  is a WM, an SM or a female.
- $\mathcal{N}_{\text{WM},i}^t = \text{card}(\{k : \mathcal{P}_k^t = i \text{ and } \mathcal{R}_k^t = 1\}_{k \in \mathcal{I}^t})$  is the number of WM in the patch  $i$  at the date  $t$ .
- $\mathcal{N}_{\text{SM},i}^t = \text{card}(\{k : \mathcal{P}_k^t = i \text{ and } \mathcal{R}_k^t = -1\}_{k \in \mathcal{I}^t})$  is the number of SM in the patch  $i$  at the date  $t$ .
- $\mathcal{R}_k = (\mathcal{R}_k^t)_{t \in \mathbb{N}}$  is a stochastic process that represents the reproductive ability of the mosquito  $k$  over time. For  $t \in \mathbb{N}$ ,

if  $t \leq t_{0,k}$ ,

$$\mathcal{R}_k^t = \begin{cases} 1 & \text{if } \mathcal{M}_{\text{adult},k} \text{ is a WM,} \\ 0 & \text{if } \mathcal{M}_{\text{adult},k} \text{ is a YF,} \\ -1 & \text{if } \mathcal{M}_{\text{adult},k} \text{ is a SM,} \end{cases}$$

if  $t \geq t_{0,k}$ ,

$$\mathcal{R}_k^{t+1} = \begin{cases} \mathcal{R}_k^t & \text{if } \mathcal{R}_k^t \neq 0 \text{ or } X_{\text{mating}} = 0, \\ 2 & \text{if } X_{\text{mating}} = X_{\text{fertile mating}} = 1, \\ -2 & \text{otherwise.} \end{cases}$$

with  $X_{\text{mating}} \sim \text{Bernoulli}(p_{\text{mating}})$  and  $X_{\text{fertile mating}} \sim \text{Bernoulli}\left(\frac{\mathcal{N}_{\text{WM}, \mathcal{P}_k^t}^t}{\mathcal{N}_{\text{WM}, \mathcal{P}_k^t}^t + \gamma \mathcal{N}_{\text{SM}, \mathcal{P}_k^t}^t}\right)$

- We denote by  $\tau_k \in \mathbb{N} \cup \{+\infty\}$  the date a YF become fertile, thus  $\tau_k = \inf\{t \in \mathbb{N} : \mathcal{R}_k^t = 2\}$
- $\mathcal{T}_k$  represents how gonotrophic cycles of the mosquito  $k$  are spaced. We denote by  $n_{\text{cycle}}$  the number of gonotrophic cycles. Therefore,

$$\mathcal{T}_k = \{0, Y_{1,k}, Y_{1,k} + Y_{2,k}, \dots, Y_{1,k} + Y_{2,k} + \dots + Y_{n_{\text{cycles}}-1,k}\}$$

with  $Y_{1,k}, Y_{2,k}, \dots, Y_{n_{\text{cycles}}-1,k}$  independent and identically distributed random variables.

- $\mathcal{E}_i^t$  represents the number of FF that will lay eggs at date  $t$  in the patch  $i$ . So,

$$\mathcal{E}_i^t = \text{card}(\{k : \mathcal{P}_k^t = i \text{ and } t - \tau_k \in \mathcal{T}_k\}_{k \in \mathcal{I}^t})$$

For others stages than adult, the representation is much simpler. It is just a tuple  $(t_{\text{stage},k}, p_{\text{stage},k}, g_{\text{stage},k}, \mathcal{S}_{\text{stage},k})$  where:

- $t_{\text{stage},k}$  is the date the mosquito entered in this stage,
- $p_{\text{stage},k}$  the patch where it is,
- $g_{\text{stage},k}$  its gender,
- $\mathcal{S}_{\text{stage},k}$  represents the duration of this stage.

So we have three sets  $\mathcal{M}_{\text{egg}}^t$ ,  $\mathcal{M}_{\text{larva}}^t$  and  $\mathcal{M}_{\text{pupa}}^t$  for respectively egg, larva and pupa. And we have that :

$$\begin{aligned} \mathcal{M}_{\text{adult}}^{t+1} &= \mathcal{M}_{\text{adult}}^t \cup \{\mathcal{M}_{\text{pupa},k} : t+1 > \mathcal{S}_{\text{pupa},k} \geq t \text{ and } X_{\text{pupa},k} = 1\}_{k \in \mathbb{N}} \\ \mathcal{M}_{\text{pupa}}^{t+1} &= \mathcal{M}_{\text{pupa}}^t \cup \{\mathcal{M}_{\text{larva},k} : t+1 > \mathcal{S}_{\text{larva},k} \geq t \text{ and } X_{\text{larva},k} = 1\}_{k \in \mathbb{N}} \\ \mathcal{M}_{\text{larva}}^{t+1} &= \mathcal{M}_{\text{larva}}^t \cup \{\mathcal{M}_{\text{egg},k} : t+1 > \mathcal{S}_{\text{egg},k} \geq t \text{ and } X_{\text{egg},k} = 1\}_{k \in \mathbb{N}} \\ \mathcal{M}_{\text{egg}}^{t+1} &= \mathcal{M}_{\text{egg}}^t \cup \bigcup_{i=1}^n \{(t+1, i, 0, \mathcal{S}_{\text{egg},j}) : j = 1 \dots \mathcal{N}_{\text{male egg}}^t \mathcal{E}_i^t\} \cup \{(t+1, i, 1, \mathcal{S}_{\text{egg},j}) : j = 1 \dots \mathcal{N}_{\text{female egg}}^t \mathcal{E}_i^t\} \end{aligned}$$

with  $X_{\text{stage},k} \sim \text{Bernoulli}(p_{\text{stage} \rightarrow \text{next stage}})$  represents if the mosquito  $k$  survive to its next stage and  $\mathcal{N}_{\text{male egg}}^t$  and  $\mathcal{N}_{\text{female egg}}^t$  the average number of respectively male and female eggs layed at date  $t$  by a FF.

## Implementation

We decide for a first time to implement this simulation in Python for simplicity's sake. We define a Mosquito class whose different mosquitoes stages, YF, FF, SF, WM and SM are inherited. Mosquito instances are stored in two queue, one for mosquitoes that will be dealt with at time  $t$  and other at  $t + 1$ .

---

### Algorithm 1 Multi-agents main loop

---

```

 $t \leftarrow 1$ 
Queues  $\leftarrow [\emptyset, \emptyset]$ 
while  $t \leq T$  do
    if Queues[ $t \bmod 2$ ] is empty then
        Add sterile males according to the release strategy
         $t \leftarrow t + 1$ 
    end if
     $M \leftarrow \text{Dequeue}(\text{Queues}[t \bmod 2])$ 
     $M$  grows old and potentially lays eggs
    if  $M$  is died then
        Next
    end if
    if  $M$  is a YF then
         $M$  tries to mate
    end if
    if  $M$  is an adult then
         $M$  migrates randomly
    end if
    Enqueue(Queues[ $t + 1 \bmod 2$ ],  $M$ )
end while

```

---

## Numerical Results

We employ biological parameters from [1] at a temperature of 25°C. Migration rates are consistent with those described in Section 3.3, with  $d = 0.01$  and  $d' = 0$ . Initially, each patch contains 1000 male eggs and 1000 female eggs. The strategy for releasing sterile insects involves a weekly release starting from day 20, except in the central patch where no sterile insects are released. The results of the multi-agent simulation (MAS) are presented in Figure 8.

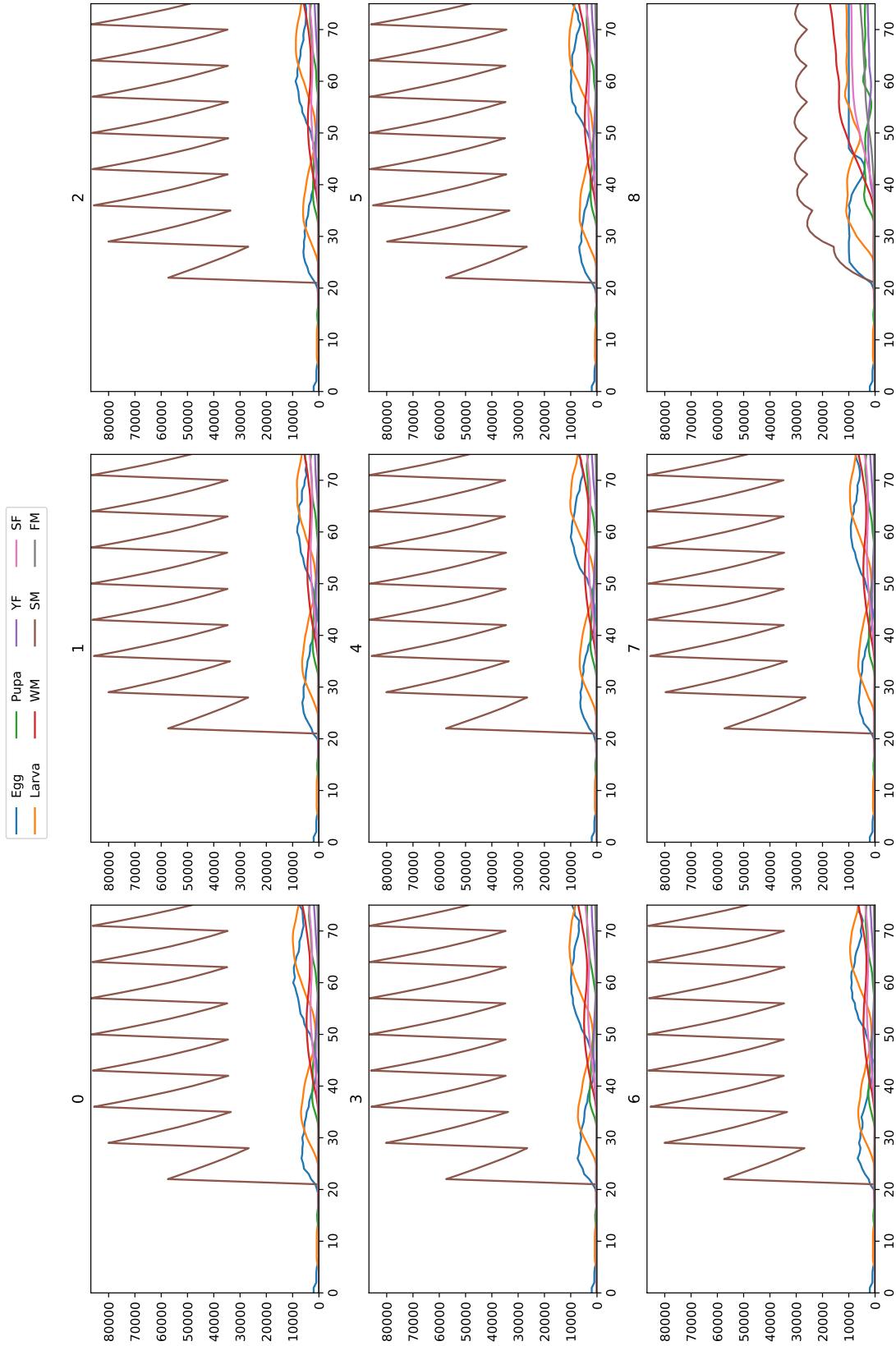


Figure 8: Multi-agent simulation results.

Parameter	Value
$S_{\text{egg}}$	Normal(4.5, 0.7)
$p_{\text{egg} \rightarrow \text{larva}}$	0.492
$S_{\text{larva}}$	Normal(7.3, 0.6)
$p_{\text{larva} \rightarrow \text{pupa}}$	0.812
$S_{\text{pupa}}$	Normal(2.7, 0.1)
$p_{\text{pupa} \rightarrow \text{adult}}$	0.938
$\mathcal{L}$ for WM	Weibull(2.19, 22.8)

Parameter	Value
$\gamma$	0.5
$\mathcal{L}$ for SM	Weibull(2.19, 16)
$\mathcal{L}$ for female	Weibull(2.21, 25.48)
$n_{\text{cycle}}$	5
$Y_{i,k}$	Normal(5, 1)
$\mathcal{N}_{\text{male egg}}^t$	Normal(25, 5)
$\mathcal{N}_{\text{female egg}}^t$	Normal(23, 5)

Table 5: Model parameters and their corresponding values.

## 5 Replacing MAS by a Neural Network

Multi-Agent Simulation (MAS) enables us to explore release strategies aimed at reducing mosquito populations below a given threshold. However, the primary challenge with MAS is the computational time required. Although frameworks such as the JADE framework can reduce computation time, they are not sufficient for efficiently exploring the space of release strategies. A significant reduction in computational time can be achieved by replacing the MAS with a neural network. The core idea is to train a neural network for a given environment to predict the evolution of the wild mosquito population, drawing substantial inspiration from the work presented in [7].

### 5.1 Multi-Agent Simulation

We consider the MAS as described by the stochastic model in Section 4.2. We assume that all parameters related to the environment and mosquito behavior are predefined. Thus, the only variable parameter of the MAS is the release strategy. We denote by  $\mathfrak{U}$  the set of release strategies and by  $\mathfrak{M}$  the set of mosquito configurations. Let  $T$  be a positive integer representing the duration of the simulation. A release strategy

$$\mathcal{U} = (u_i^t)_{\substack{0 \leq t \leq T \\ 1 \leq i \leq n}} \in \mathfrak{U}$$

is a  $(T + 1) \times n$  matrix in  $\mathbb{R}_+$  where  $u_i^t$  represents the quantity of sterile mosquitoes released in patch  $i \in \{1, 2, \dots, n\}$  at time  $t \in \{0, 1, \dots, T\}$ .

Therefore, the MAS simulation can be represented by a mapping  $\mathbf{Sim} : \mathfrak{U} \times \mathfrak{M} \rightarrow \mathfrak{M}$ . For each release strategy  $\mathcal{U} \in \mathfrak{U}$ , the execution of the simulation up to the time-step  $1 \leq \tilde{T} \leq T$  is represented by the sequence  $(\mathcal{M}^0, \hat{\mathcal{M}}_{\mathcal{U}}^1, \dots, \hat{\mathcal{M}}_{\mathcal{U}}^{\tilde{T}})$  such that  $\hat{\mathcal{M}}_{\mathcal{U}}^t = \mathbf{Sim}(\mathcal{U}, \hat{\mathcal{M}}_{\mathcal{U}}^{t-1})$  for  $t = 1, \dots, \tilde{T}$ , with  $\mathcal{M}^t = (\mathcal{M}_{\text{egg}}^t, \mathcal{M}_{\text{larva}}^t, \mathcal{M}_{\text{pupa}}^t, \mathcal{M}_{\text{adult}}^t)$ . For each strategy  $\mathcal{U} \in \mathfrak{U}$ , the state sequence yielded by  $\mathbf{Sim}$  is  $\hat{\mathcal{M}}_{\mathcal{U}}^{t_0:T} = (\mathcal{M}^0, \hat{\mathcal{M}}_{\mathcal{U}}^1, \dots, \hat{\mathcal{M}}_{\mathcal{U}}^T)$ .

### 5.2 Aggregated information

The dimension of each state of the simulation,  $\mathcal{M} \in \mathfrak{M}$ , is substantial. Therefore, when predicting the simulation, we focus on aggregated information derived from each state. We denote this aggregated information by  $\mathbf{agr} : \mathfrak{M} \rightarrow \mathfrak{N}$ , where  $\mathfrak{N}$  represents the set of aggregated information.

For a state  $\mathcal{M} \in \mathfrak{M}$ , the aggregated information is given by:

$$\text{agr}(\mathcal{M}) = \begin{pmatrix} \mathcal{N}_{\text{egg},0} & \mathcal{N}_{\text{larva},0} & \mathcal{N}_{\text{pupa},0} & \mathcal{N}_{\text{WM},0} & \mathcal{N}_{\text{SM},0} & \mathcal{N}_{\text{YF},0} & \mathcal{N}_{\text{FF},0} & \mathcal{N}_{\text{SF},0} \\ \mathcal{N}_{\text{egg},1} & \mathcal{N}_{\text{larva},1} & \mathcal{N}_{\text{pupa},1} & \mathcal{N}_{\text{WM},1} & \mathcal{N}_{\text{SM},1} & \mathcal{N}_{\text{YF},1} & \mathcal{N}_{\text{FF},1} & \mathcal{N}_{\text{SF},1} \\ \vdots & \vdots \\ \mathcal{N}_{\text{egg},n} & \mathcal{N}_{\text{larva},n} & \mathcal{N}_{\text{pupa},n} & \mathcal{N}_{\text{WM},n} & \mathcal{N}_{\text{SM},n} & \mathcal{N}_{\text{YF},n} & \mathcal{N}_{\text{FF},n} & \mathcal{N}_{\text{SF},n} \end{pmatrix} \in \mathbb{R}_+^{n \times 8},$$

where  $\mathcal{N}_{\text{STAGE},i}$  represents the number of mosquitoes in patch  $i$  at stage STAGE.

For each release strategy  $\mathcal{U} \in \mathfrak{U}$  and time step  $1 \leq \tilde{T} \leq T$ , the aggregated information of  $\hat{\mathcal{M}}_{\mathcal{U}}^{t_0:\tilde{T}}$  is  $\tilde{\mathcal{N}}_{\mathcal{U}}^{t_0:\tilde{T}} = (\text{agr}(\mathcal{M}^0), \text{agr}(\hat{\mathcal{M}}_{\mathcal{U}}^1), \dots, \text{agr}(\hat{\mathcal{M}}_{\mathcal{U}}^{\tilde{T}}))$ .

### 5.3 MAS network

We denote by  $f_{\theta}$  the model with parameters  $\theta$ . For each releasing strategy  $\mathcal{U} \in \mathfrak{U}$ , the prediction of the model until the time-step  $1 \leq \tilde{T} \leq T$  is represented by  $(\mathcal{N}^0, \tilde{\mathcal{N}}_{\mathcal{U}}^1, \dots, \tilde{\mathcal{N}}_{\mathcal{U}}^{\tilde{T}}) \in \mathfrak{N}^{\tilde{T}+1}$  such that

$$\tilde{\mathcal{N}}_{\mathcal{U}}^t = f_{\theta}(\mathcal{U}, (\mathcal{N}^0, \tilde{\mathcal{N}}_{\mathcal{U}}^1, \dots, \tilde{\mathcal{N}}_{\mathcal{U}}^{t-1}))$$

for  $t = 1 \dots \tilde{T}$ . For each releasing strategy, the state sequence yielded by  $f_{\theta}$  is  $\tilde{\mathcal{N}}_{\mathcal{U}}^{t_0:T}(f_{\theta}) = (\mathcal{N}^0, \tilde{\mathcal{N}}_{\mathcal{U}}^1, \dots, \tilde{\mathcal{N}}_{\mathcal{U}}^T)$ .

Our goal is to find a model that can approximate the state sequence yielded by the simulation and the aggregated function,  $\tilde{\mathcal{N}}_{\mathcal{U}}^{t_0:T}$ . We obtain the model by solving,

$$\min_{\theta} L(\tilde{\mathcal{N}}_{\mathcal{U}}^{t_0+1:T_1}, \tilde{\mathcal{N}}_{\mathcal{U}}^{t_0+1:T_1}(f_{\theta})) \quad (7)$$

where  $T_0$  and  $T_1$  are two timesteps such as  $1 \leq T_0 < T_1 \leq T$  and  $L$  is a loss function that calculates a difference between state sequence yielded by **Sim** and **agr**,  $\tilde{\mathcal{N}}_{\mathcal{U}}^{t_0+1:T_1}$ , and state sequence predicted by  $f_{\theta}$ ,  $\tilde{\mathcal{N}}_{\mathcal{U}}^{t_0+1:T_1}(f_{\theta})$ .

### 5.4 Dataset Creation

To determine the optimal model  $f_{\theta}$  according to [Equation \(7\)](#), we need to create a dataset. For this purpose, we establish the following:

- An initial state of mosquitoes  $\mathcal{M}^0 \in \mathfrak{M}$ ,
- A timestep  $1 \leq T_0 < T$  such that we do not require the model to predict states before  $T_0$ ,
- A timestep  $T_0 < T_1 \leq T$  such that we aim to predict states between timesteps  $T_0 + 1$  and  $T_1$ ,
- A subset of release strategies  $\mathfrak{U}_{\text{sub}} \subseteq \mathfrak{U}$ .

Therefore, a dataset of size  $N$  will be a set  $D_N = \{(X_i, Y_i)\}_{1 \leq i \leq N}$  with:

$$X_i = (\tilde{\mathcal{N}}_{\mathcal{U}_i}^{t_0:T_0}, \mathcal{U}_i) \text{ and } Y_i = \tilde{\mathcal{N}}_{\mathcal{U}_i}^{t_0+1:T_1}$$

for  $\mathcal{U}_i \in \mathfrak{U}_{\text{sub}}$  and  $i \in \{1, 2, \dots, N\}$ .

To ensure the model is sensitive to the release strategy,  $N$  must be sufficiently large so that  $\{\mathcal{U}_i\}_{1 \leq i \leq N}$  is representative of  $\mathfrak{U}_{\text{sub}}$ .

### Example of subset of releasing strategies

For  $i \in \{1, 2, \dots, n\}$ , we define the  $(T + 1) \times n$  matrix  $M_i$  as follows:

$$M_i = \begin{pmatrix} 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & & \vdots & \vdots & \vdots & & \vdots \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & \mathbf{1} & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & & \vdots & \vdots & \vdots & & \vdots \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & \mathbf{1} & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & & \vdots & \vdots & \vdots & & \vdots \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & \mathbf{1} & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & & \vdots & \vdots & \vdots & & \vdots \end{pmatrix} \quad \begin{array}{l} \leftarrow \text{row } T_0 + 1 \\ \leftarrow \text{row } T_0 + 8 \\ \leftarrow \text{row } T_0 + 15 \end{array}$$

↑  
column  $i$

Therefore, the subset of releasing strategies

$$\mathfrak{U}_{\text{weekly}} = \text{span}\{M_1, M_2, \dots, M_n\} \cap \mathbb{R}_+^{(T+1) \times n} \subseteq \mathfrak{U}$$

represents the releasing strategies such that a constant amount of sterile insects is released in each patch once a week starting from timestep  $T_0 + 1$ .

Notice that  $\mathfrak{U}_{\text{weekly}}$  is a subset of a vector space of dimension  $n$ , thus we can characterize a strategy  $\mathcal{U} \in \mathfrak{U}_{\text{weekly}}$  by an  $n$ -tuple,  $u = (u_1, u_2, \dots, u_n)$ , that represents the amount of sterile insects released weekly in each patch.

## 5.5 Neural Network Architectures

To identify the optimal model  $f_\theta$  for forecasting the wild population, we employ a recurrent neural network (RNN) architecture known as sequence-to-sequence [6]. A sequence-to-sequence (seq2seq) model comprises two RNNs: an encoder network and a decoder network. In our implementation, we utilize Long Short-Term Memory (LSTM) networks [3] for both components. The encoder is tasked with encoding the past, that is, states from date 0 to  $T_0$ , while the initial states of the decoder are derived from the release strategy.

## 5.6 Results

We consider a system of two patches, as described in Section 3.1. We utilized biological data from Table 5 for multiagent simulation. The migration rates between these two patches are  $d_{1 \rightarrow 2} = 0.7$  and  $d_{2 \rightarrow 1} = 0.35$ .

We focus on release strategies within the set  $\mathfrak{U}_{\text{weekly}}$ , as defined in 5.4. We generated a data set comprising 900 simulations, with the amount of sterile insects released randomly varying from 0 to 125,000.

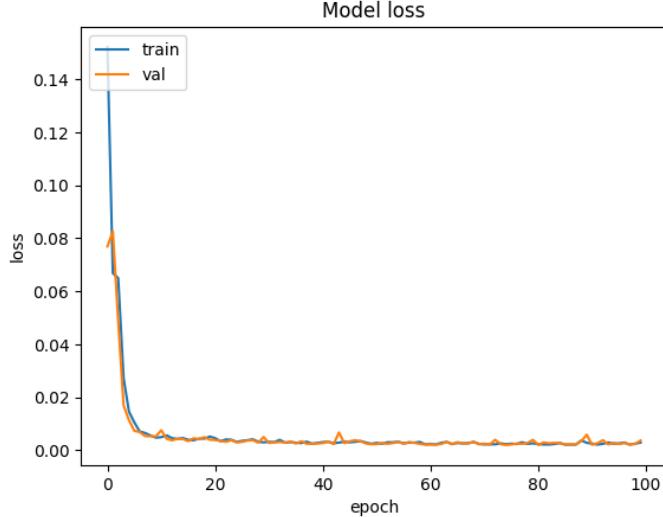


Figure 9: Loss (MSE) of the Seq2Seq model over epoch

Data have been scaled between 0 and 1 in order to train the neural networks.

We plot the loss (MSE) over the epoch in Figure 9. We can view that loss converge relatively rapidly to a small value, the few fluctuations can be fixed by made more simulations.

## 6 Optimization on the Neural Network

Once the model  $f_\theta$  has been trained to approximate the Multi-Agent Simulation (MAS), we can leverage the regularity of  $f_\theta$  to identify an optimal releasing strategy.

### 6.1 Cost Function

To determine an optimal releasing strategy, we introduce a cost function  $J$  to evaluate the effectiveness of a given strategy. Thus,  $J$  is a non-negative function of  $\mathcal{U} \in \mathfrak{U}_{\text{sub}}$ . For optimization purposes, we desire  $J$  to be as smooth as possible, enabling the use of algorithms such as gradient descent.

Here are some examples of interesting cost functions:

- $J_1(\mathcal{U}) = S_{\mathcal{U}}^{T_1}$ : The total number of mosquitoes at time  $T_1$  across all patches.
- $J_2(\mathcal{U}) = S_{\mathcal{U}}^{T_1} + \frac{\lambda}{2} \|\mathcal{U}\|^2$ : Similar to  $J_1$ , but with an added regularization factor concerning the total number of sterile insects released.
- $J_3(\mathcal{U}) = \sum_{t=T_0+1}^{T_1} S_{\mathcal{U}}^t + \frac{\lambda}{2} \|\mathcal{U}\|^2$ : The cumulative number of mosquitoes from  $T_0 + 1$  to  $T_1$ , including a regularization factor.

where  $S_{\mathcal{U}}^t = \sum_{i=1}^8 \sum_{j=1}^n (\tilde{\mathcal{N}}_{\mathcal{U}}^t)_{i,j}$  represents the total number of mosquitoes at time  $t \in \{1, 2, \dots, n\}$  across all patches, and  $\lambda \in \mathbb{R}_+$ .

## 6.2 Optimal Strategy Algorithm

The cost function depends on predictions made by the model. Since these predictions are differentiable with respect to  $\mathcal{U}$  (as they are generated by a neural network), we can compute numerical values of  $\nabla_{\mathcal{U}} J(\mathcal{U})$  using automatic differentiation. This capability allows us to perform gradient descent over the values of  $\mathcal{U}$ , thereby identifying an optimal strategy.

## 6.3 Results

We focus on the subset  $\mathfrak{U}_{\text{weekly}}$ , as mentioned in [Section 5.4](#), with only 2 patches and use TensorFlow for automatic differentiation. The dataset used is the same as in [Section 5.6](#).

A strategy  $\mathcal{U} \in \mathfrak{U}_{\text{weekly}}$  can be characterised by a pair  $(u_1, u_2) \in \mathbb{R}_+^2$ , which represents the amount of sterile insects released weekly. Thus,  $J$  can be viewed as a function of  $u = (u_1, u_2)$ , we denote by  $\phi(u_1, u_2) \in \mathfrak{U}_{\text{weekly}}$  the releasing strategy associate to the couple  $(u_1, u_2)$ , and  $\nabla_u J(u)$  is a pair, rather than a  $(T + 1) \times n$  matrix like  $\mathcal{U}$ . As said in [Section 5.6](#), all data have been scaled from 0 to 1, and so rather to optimise  $J(u_1, u_2)$  with  $0 \leq u_1, u_2 \leq 1$  with optimise  $J(\sigma(z_1), \sigma(z_2))$  with  $z_1, z_2 \in \mathbb{R}$  and  $\sigma$  the sigmoid function.

Here's, we optimise with four different cost function  $J$ :

- $J_0(u_1, u_2) = S_{\phi(u_1, u_2)}^T$
- $J_1(u_1, u_2) = S_{\phi(u_1, u_2)}^T + u_1$
- $J_2(u_1, u_2) = S_{\phi(u_1, u_2)}^T + u_2$
- $J_{12}(u_1, u_2) = S_{\phi(u_1, u_2)}^T + u_1 + u_2$

Results of gradient descent algorithm are plotted in [Figure 10](#), learning rate is set to 0.1 for  $J_1$  and  $J_2$  and to 0.3 for  $J_0$  and  $J_{12}$ . MAS results using these four optimal strategies are plotted in [Figure 11](#).

## 7 Conclusion and open issues

In this study, we have explored two complementary approaches to optimize the implementation of the Sterile Insect Technique (SIT) in multi-patch systems. By employing a system of coupled ordinary differential equations and multi-agent simulations, we have provided a comprehensive analysis of the dynamics involved in SIT strategies. The use of a neural network to emulate simulation outputs has proven to be an effective method for reducing computational intensity, thereby facilitating the exploration of a wide range of control strategies.

Our findings demonstrate the potential of using deep neural networks to emulate multi-agent simulations and, moreover, to utilize them in identifying optimal release strategies. Although this method has only been applied to a system of two patches, the primary advantage of this approach is its adaptability to more complex systems. These could include systems with numerous patches, more intricate strategy structures, and even more detailed multi-agent simulations. This adaptability underscores the method's potential for broader applications in pest management and ecological modelling.

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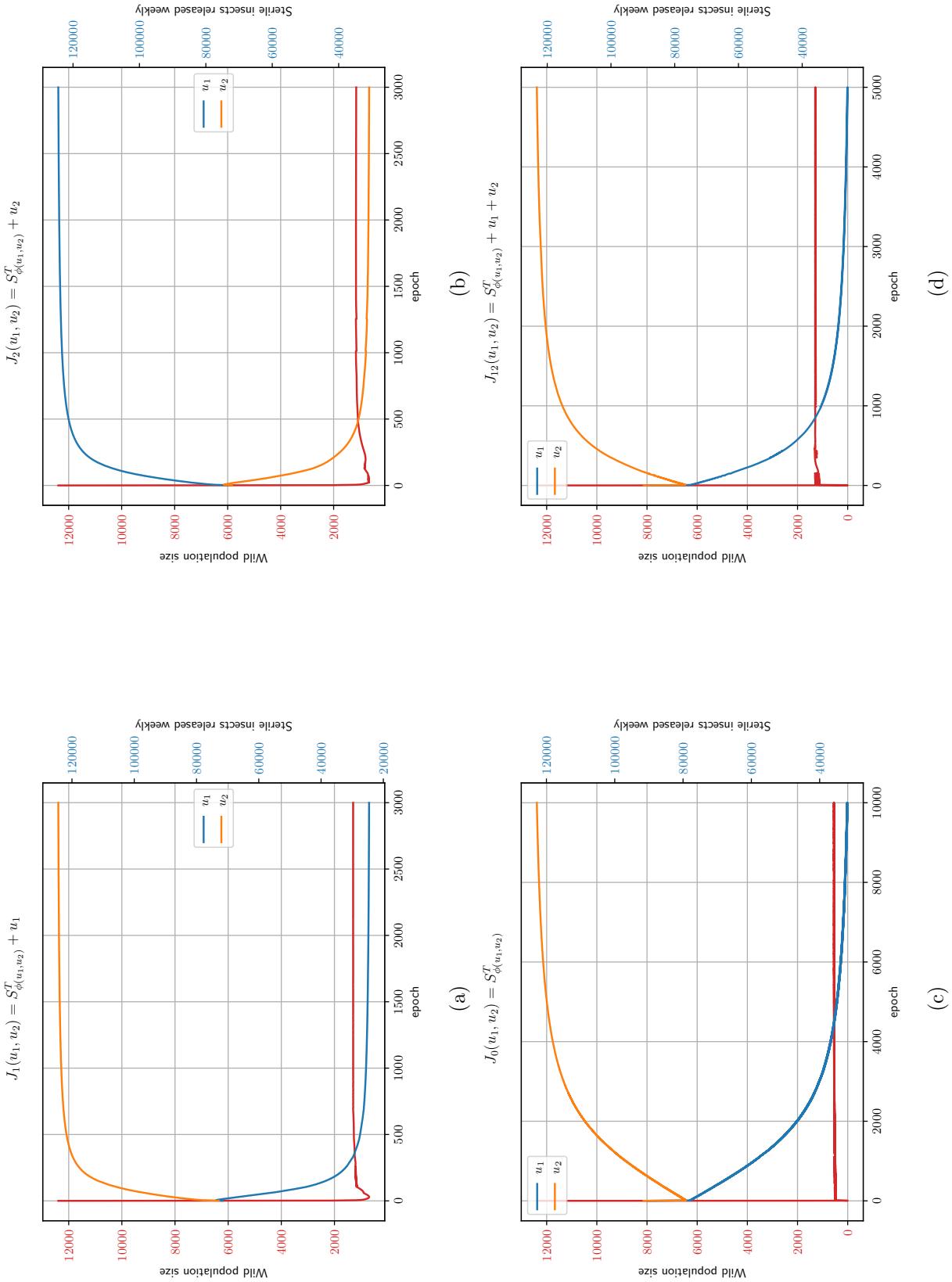


Figure 10: Evolution of wild population predicted over epochs for different cost functions.

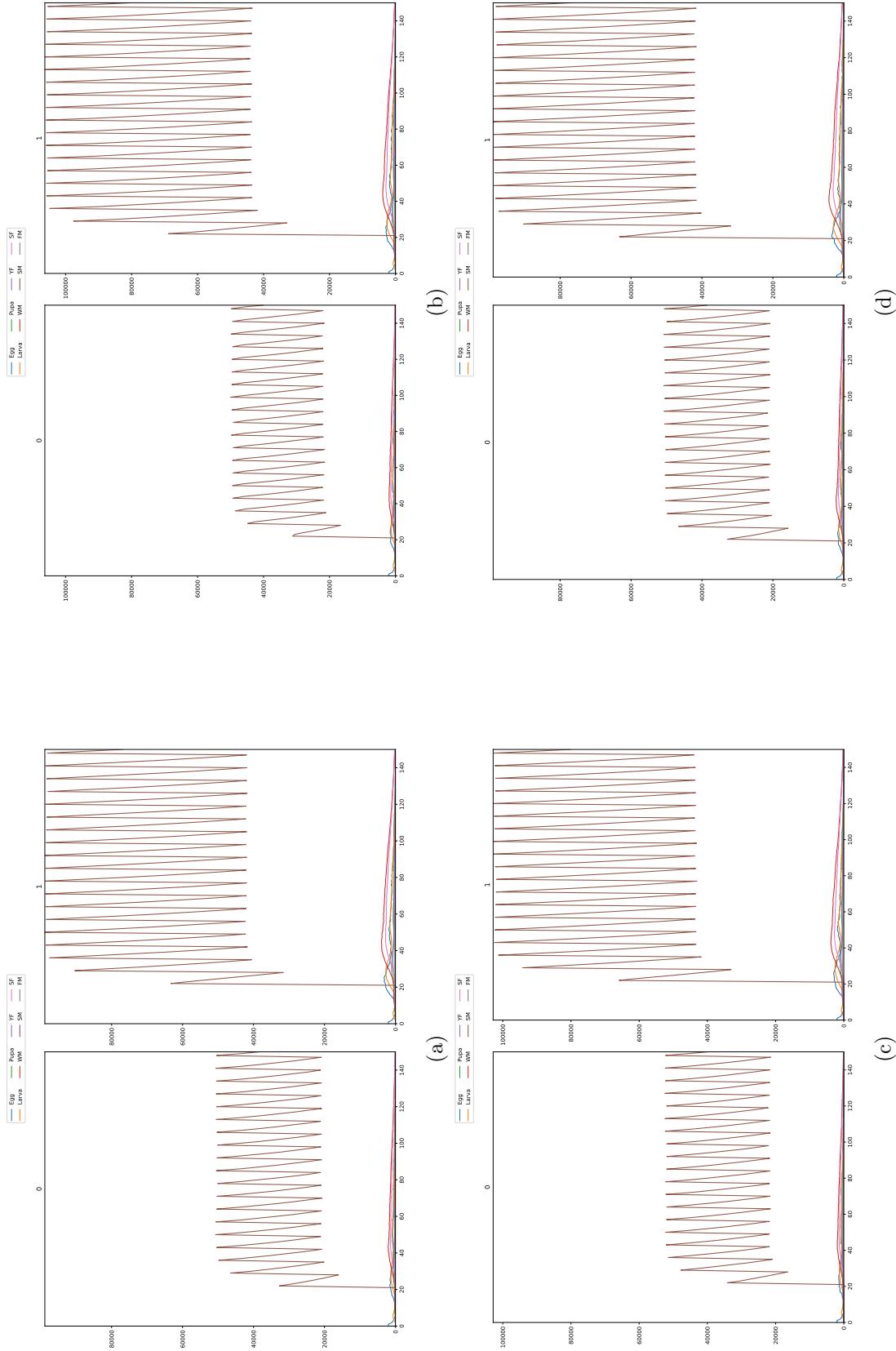


Figure 11: MaaS results with optimal strategies found in Figure 10