Seminar 2: Solutions

1. Show in any group of 6 integers there are at least two of them having the same remainder when divided by 5? [3 marks]

Remainders when divided by 5: 0, 1, 2, 3, 4

So there are 5 boxes. If we have 6 integers by the pigeonhole principle, one box must contain more than one item. So two at least two integers share a remainder.

1. How many positive numbers less than 100 are there that are divisible by 5 or 7? [3 marks]

There are ⌊⌋ = 19 divisible by 5

There are ⌊⌋ = 14 divisible by 7

There are ⌊⌋ = 2 divisible by 35

There are 19 + 14 - 2 = 31 divisible by 5 or 7

1. Each student has a password, which is 6-8 characters long and each character is either a digit or a lowercase letter. Each password must contain at least TWO digits. How many possible passwords are there? [5 marks]

6- 8-character long means, 6 or 7 or 8-character long.

Valid passwords: at least 2 digits so invalid ones have less that 2 digits. This means either 1 digit or no digits.

Length 6. All passwords = , No digits: , one digit, (10 x )x6. This is because one place is only digit so we have 10 choices, the remaining 5 characters must be from lowercase letters, so we have 26 choices each. Now the digit can be in the first character or the second, third, fourth, fifth or the last which makes the number of permutations 6.

Length 6: - 60 x =

Length 7: - 70 x =

Length 8: - 80 x =

The number of valid passwords is the sum of the three numbers above.