Improbable Happenings in the *Among Us* Cinematic Universe

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Introduction & Assumptions

Among Us is a super popular computer game nowadays in which a pre-selected number of impostors (usually two) attempt to kill all of the remaining players before either (a) the other players discover their identities and banish them, or (b) the other players complete a given set of tasks.

We played *Among Us* for a hot minute with a group of 8-10 players this evening, and Danny was really mad that he was *never* selected as impostor. (Alex was also never selected as an impostor, but he didn't like constantly complain about it). On the other hand, Will and I (tpmon) were paired as impostors on three separate occasions. We conjecture that each of the above events is highly improbable, and will calcuate the probabilities using basic principles of probability theory.

To simplify these calculations & make them uniform, we will assume that there were 15 games with exactly 9 players in each game. We will also assume that the selection of the two impostors each round by the game's random number generator was **independent and identically distributed**.

Lastly, and most importantly, we will assume that all players are identical, and we will consider the probability of these events happening to any of the 9 players. This is theoretically relevant because it will decrease the total population of potential scenarios ninefold, and therefore increase probabilities accordingly; it is empirically reasonable because we assume that these events occur randomly, not due to any grudge or affinity which the random number generator may bear toward Danny and Alex or Tpmon and Will, respectively.

Case 1: Danny is not selected

Because we have assumed that impostor selection is independent across rounds, we begin by considering the probability that Player D is not selected in a single round. In a given round R_i, we have 9 players; 2 of them are selected as impostors and 7 are not. The probability $P(\bar{D})$ of outcome $\bar{D}$, that Player D is not selected, is $7/9 \approx 0.7778$.

We have assumed that the distribution of the impostor selector is identical across rounds. The probability of the intersection of n independent outcomes is the product of their probabilities; therefore, we have

$$P(\bar{D}_1 \cap \bar{D}_2 \cap \cdots \cap \bar{D}_{15}) = \prod_{i=1}^{15} P_i = P(\bar{D})^{15} \approx 0.0231$$

We will not consider the trivial case in which Alex is not selected, since we have assumed that Alex and Danny are identical.

Case 2: Neither Danny nor Alex is selected

We will approach this case as we did the first, beginning by finding the probability that neither Player D nor Player A is selected in a single round: $P(\bar{D} \cap \bar{A})$. I don't really remember how to do this, so I'll find it using its complement:

$$P(\bar{D} \cap \bar{A}) = 1 - P(\overline{\bar{D} \cap \bar{A}}) = 1 - P(D \cup A)$$

Recall that the probability of the union of two non-independent events is equal to the sum of the individual probabilities minus the probability of their intersection: $P(D \cup A) = P(D) + P(A) - P(D \cap A)$. Naturally, $P(D) = P(A) = 2/9$, and by the definition of a combination, $P(D \cup A) = 1/\binom{10}{2} = 1/45$. Then $P(D \cup A) = 2/9 + 2/9 - 1/45 \approx 0.422$ and $P(\bar{D} \cap \bar{A}) \approx 1 - 0.422 = 0.578$.

The rest follows exactly from (1):

$$P((\bar{D}_1 \cap \bar{A}_1) \cap (\bar{D}_2 \cap \bar{A}_2) \cap \cdots \cap (\bar{D}_{15} \cap \bar{A}_{15})) = \prod_{i=1}^{15} P_i = P(\bar{D} \cap \bar{A})^{15} \approx 0.00027$$

Case 3: Thomas and Will are simultaneously selected on three different occasions

The probability that both Player T and Player W are selected to be the impostors in the same round is $P(T \cap W)$. This is equivalent to $P(D \cap A)$, a probability which we determined in (2) to have a value of 1/45.

We can form a game in which there are two possible outcomes: outcome P, in which Player T and Player W are selected, and outcome Q, in which one or both of them is not. Observe that these events are complements; that is, $P + Q = 1$. We have created a binomial distribution in which $P = 1/45$ and $Q = 44/45$.

Then, by the definition of the binomial distribution, the probability that outcome P occurs exactly 3 times in 15 trials is

$$P(3, 15, 1/45) = \binom{15}{3}(1/45)^3(44/45)^{12} \approx 0.0038$$

Empirically, however, we really aren't looking for the probability that this outcome occurs *exactly three* times, but that it occurs more than twice:

$$P(3) + P(4) + \cdots + P(15) = 1 - (P(0) + P(1) + P(2))$$

$$= 1 - \left(\binom{15}{0}(1/45)^0(44/45)^{15} + \binom{15}{1}(1/45)^1(44/45)^{14} + \binom{15}{2}(1/45)^2(44/45)^{13}\right)$$

$$\approx 0.0041$$

Conclusion

As much as *some of us* (cough cough Danny) would love to refute the randomness of the random number generator on these grounds, these limited observations almost certainly do not give us enough statistical evidence to do so. An authoritative conclusion would require us to analyze the full set of impostors selected during the entire evening, and no one bothered to write all of that down. Our observation of these data suffers from the **improbability principle**, whereby, due largely to the law of large numbers, improbable events are perceived to be much more common than they are in reality, simply because they are more interesting or memorable. I hope you had as much fun reading this as I did writing it. Thanks for reading; please like, comment, and subscribe.