# Predicting Medical Insurance Costs in the United States

Phuong Nguyen, Nimra Aamir, Tobi Adelodun, Emma Lait

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## Chapter 1: Introduction

Healthcare is a necessity that many are not able to afford. High medical costs with or without insurance is a problem that is quite apparent in the United States. In the United States there is a connection between healthcare, healthcare insurance costs, and poverty (Hoffman & Paradise, 2008, p. 149). In fact, findings of a survey conducted in 2013 across 11 countries by a team of researchers shows that many people in the United States will not seek medical treatment even if they have insurance due to medical costs (Schoen et al, 2013, p. 2205). Unlike Canada, there is not partially free coverage for healthcare in the US, so private sectors are responsible for covering 100% of medical costs. With medical costs comes the aspect of insurance. According to Riedel, approximately two-thirds of Americans under age 65 have health insurance coverage (2009, p. 439). It is important to note that there is a large number of people who do not have insurance coverage for health care costs because they are not eligible for certain programs or are not able to "afford nongroup coverage" (Riedel, 2009, p. 439), further providing an outlook into issues that individuals face regularly when it comes to medical costs and insurance. Insurance is one of the primary means used to cover medical costs but there are various factors that influence insurance costs which must be analyzed.

To further examine this problem, statistical research on the topic of medical expenses that impact healthcare insurance in the United States will be carried out. For this research, each individual is categorized by seven variables: age, sex, body mass index (BMI), how many kids are covered under the health insurance, region in the United States, and their charges billed by health insurance. We are looking to see how these specific variables impact insurance costs and medical charges as a whole. To complete this analysis, the Medical Cost Personal Dataset: Insurance Forecast by using Linear Regression was taken from Kaggle under an open database license. The dataset itself is taken and cleaned from the book Machine Learning with R by Brett Lantz.

## Chapter 2: Methodology

Our dataset consists of seven variables which are named: age, sex, BMI, children, smoker, region, and charges. Age, BMI, children, and charges are quantitative variable whilst sex, smoker, and region are qualitative variables.

Age = how old the primary beneficiary of the insurance policy is (years)

Sex = the gender of the beneficiary and consists of two factors (male or female)

 $BMI = body mass index of the beneficiary (kg/m^2)$ 

Children = amount of children that is covered by the policy or the amount of dependents that the primary beneficiary has, ranging from 0 to 5 (children)

Smoker = whether the primary beneficiary is a smoker (yes or no)

Region = the area of the United States of America that the beneficiary lives in (southwest, northwest, southeast, northeast)

The response variable is Charges = cost of health insurance that gets billed to the owner of the health insurance policy (dollars)

To begin the process of building our model, we will first build the full model with all the available variables so that we can have a base model to work with. Throughout this whole process of modelling, we will be using a significance level of 0.05. We will look at the summary of the full model and observe the T-statistic and the P-value of each variable to see which ones are considered significant. By having this base model, we are able to remove and add different variables that would improve the accuracy of the model. Once we remove insignificant variables, we will then compare the original model and our first-order model with an ANOVA test to see whether the variables we removed were worth removing or not by observing the F-statistic and P-value of the ANOVA. After, we will then check the interaction terms by building the interaction model from the first order model. The next step is to see if there are any interaction terms that should be added to the first order model. We will also perform a stepwise model selection which will add the variables one by one, and then remove variables if they are not improving the overall model. Another addition to the model is that we are going to check the higher order models, both squared and cubic, to see if there are any terms that we can add to improve the accuracy of our model. After we have confirmed the most accurate model we will then perform all the necessary assumption tests to ensure it meets the needed assumptions. We will use ggplot to graph the residuals against the fitted values of the linear model to check the linearity assumption, the bptest function for the equal variance assumption, shapiro test for the normality assumption, the imcdiag function to test for multicollinearity and also test for outliers using the hatvalues function to ensure that there are no values skewing the dataset. Finally after we perform and confirm all the assumptions, we will perform a Box Cox transformation to see whether we can improve the model further or not.

## Chapter 3: Main results of the analysis

##

Max.

```
## Rows: 1338 Columns: 7
## -- Column specification ------
## Delimiter: ","
## chr (3): sex, smoker, region
## dbl (4): age, bmi, children, charges
##
## i Use `spec()` to retrieve the full column specification for this data.
## i Specify the column types or set `show_col_types = FALSE` to quiet this message.
## # A tibble: 6 x 7
##
                    bmi children smoker region
       age sex
                                                   charges
##
     <dbl> <chr>
                  <dbl>
                           <dbl> <chr>
                                         <chr>
                                                     <dbl>
                   27.9
                                                    16885.
## 1
        19 female
                               0 yes
                                         southwest
##
        18 male
                   33.8
                               1 no
                                         southeast
                                                     1726.
##
  3
        28 male
                   33
                                         southeast
                                                     4449.
                               3 no
## 4
        33 male
                   22.7
                               0 no
                                         northwest
                                                    21984.
## 5
        32 male
                   28.9
                               0 no
                                        northwest
                                                     3867.
## 6
        31 female
                   25.7
                               0 no
                                         southeast
                                                     3757.
                                                           children
##
                                             bmi
         age
                        sex
   Min.
           :18.00
                    Length: 1338
                                       Min.
                                               :15.96
                                                        Min.
                                                               :0.000
##
   1st Qu.:27.00
                    Class : character
                                       1st Qu.:26.30
                                                        1st Qu.:0.000
   Median :39.00
                    Mode
                         :character
                                       Median :30.40
                                                        Median :1.000
##
           :39.21
##
   Mean
                                       Mean
                                               :30.66
                                                        Mean
                                                               :1.095
   3rd Qu.:51.00
                                       3rd Qu.:34.69
                                                        3rd Qu.:2.000
           :64.00
                                       Max.
                                               :53.13
                                                               :5.000
```

Max.

```
##
                          region
       smoker
                                             charges
                       Length: 1338
##
   Length: 1338
                                                 : 1122
                                          \mathtt{Min}.
   Class : character
                                          1st Qu.: 4740
                       Class :character
   Mode :character
                                          Median: 9382
                       Mode :character
##
##
                                          Mean
                                                 :13270
##
                                          3rd Qu.:16640
##
                                          Max.
                                                 :63770
## integer(0)
fullmodel <- lm(charges~age+factor(sex)+</pre>
                  bmi+children+factor(smoker)+
                  factor(region), data=insurance)
summary(fullmodel)
##
## Call:
  lm(formula = charges ~ age + factor(sex) + bmi + children + factor(smoker) +
       factor(region), data = insurance)
##
## Residuals:
##
       Min
                  1Q
                       Median
                                    3Q
                                            Max
## -11304.9 -2848.1
                       -982.1
                                1393.9
                                        29992.8
##
## Coefficients:
##
                           Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                                         987.8 -12.086 < 2e-16 ***
                           -11938.5
## age
                              256.9
                                          11.9
                                                21.587 < 2e-16 ***
## factor(sex)male
                                         332.9 -0.394 0.693348
                             -131.3
## bmi
                              339.2
                                          28.6 11.860 < 2e-16 ***
## children
                              475.5
                                         137.8
                                                3.451 0.000577 ***
## factor(smoker)yes
                            23848.5
                                         413.1 57.723 < 2e-16 ***
## factor(region)northwest
                             -353.0
                                         476.3 -0.741 0.458769
## factor(region)southeast
                           -1035.0
                                         478.7 -2.162 0.030782 *
                                         477.9 -2.009 0.044765 *
## factor(region)southwest
                             -960.0
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 6062 on 1329 degrees of freedom
## Multiple R-squared: 0.7509, Adjusted R-squared: 0.7494
## F-statistic: 500.8 on 8 and 1329 DF, p-value: < 2.2e-16
```

We built an initial first-order model with all of the independent variables and utilized the stepwise regression in order to select the important ones to be included. The maximum model was specified as below:

$$Y_{charges} = \beta_0 + \beta_1 X_{Age} + \beta_2 X_{BMI} + \beta_3 X_{Children} + \beta_4 X_{Smoker} + \beta_5 X_{Region} + \beta_6 X_{Sex}$$

```
library(olsrr)
```

```
##
## Attaching package: 'olsrr'
```

```
## The following object is masked from 'package:datasets':
##
##
     rivers
stepmod=ols_step_both_p(fullmodel, pent=0.1, prem=0.3, details=TRUE)
## Stepwise Selection Method
##
## Candidate Terms:
##
## 1. age
## 2. factor(sex)
## 3. bmi
## 4. children
## 5. factor(smoker)
## 6. factor(region)
##
## We are selecting variables based on p value...
##
##
## Stepwise Selection: Step 1
## - factor(smoker) added
                      Model Summary
                    0.787 RMSE
0.620 Coef. Var
0.619 MSE
## R
                                               7470.216
## R-Squared
                                                 56.292
                                           55804130.200
## Adj. R-Squared
                   0.619
## Pred R-Squared
                   0.618
                            MAE
                                             5662.090
## -----
## RMSE: Root Mean Square Error
## MSE: Mean Square Error
## MAE: Mean Absolute Error
##
                                 ANOVA
##
##
                    Sum of
##
                  Squares
                               DF Mean Square F
                                                               Sig.
## ------
## Regression 121519903621.668
                              1 121519903621.668 2177.615 0.0000
                              1336
             74554317946.700
                                        55804130.200
## Residual
                           1337
## Total
            196074221568.368
##
                                  Parameter Estimates
## -----
          model Beta Std. Error Std. Beta t Sig lower
                                                                        upp
       (Intercept)
                                                 36.829 0.000 7985.002
                  8434.268
                              229.014
                                                                          8883.5
                              506.075
## factor(smoker)yes 23615.964
                                         0.787 46.665 0.000
                                                                22623.175
                                                                          24608.7
##
```

```
##
##
## Stepwise Selection: Step 2
## - age added
##
                       Model Summary
## -----
## R
                     0.849
                               RMSE
                                                 6396.752
## R-Squared
                     0.721
                               Coef. Var
                                                    48.203
## Adj. R-Squared
                     0.721
                               MSE
                                              40918439.071
## Pred R-Squared
                     0.720
                               MAE
                                               4122.078
## RMSE: Root Mean Square Error
## MSE: Mean Square Error
## MAE: Mean Absolute Error
##
##
                                 ANOVA
##
                     Sum of
                                 DF
##
                    Squares
                                          Mean Square F
                               2
## Regression 141448105408.046
                                       70724052704.023
                                                               0.0000
                                                      1728.415
                               1335
## Residual
             54626116160.323
                                          40918439.071
## Total
             196074221568.368
                                1337
##
                                    Parameter Estimates
                 Beta
                                                                             upp
                             Std. Error Std. Beta
                                                 t Sig
       (Intercept)
                   -2391.626
                                528.302
                                                    -4.527
                                                           0.000
                                                                   -3428.019
                                                                              -1355.2
## factor(smoker)yes
                 23855.305
                               433.488
                                           0.795
                                                    55.031 0.000 23004.912
                                                                              24705.6
      age
                   274.871
                                12.455
                                           0.319
                                                    22.069 0.000
                                                                   250.437
                                                                               299.3
##
##
##
##
                       Model Summary
##
  ______
                     0.849
                               RMSE
                                                 6396.752
## R-Squared
                     0.721
                               Coef. Var
                                                   48.203
## Adj. R-Squared
                               MSE
                     0.721
                                              40918439.071
## Pred R-Squared
                   0.720
                              MAE
                                                4122.078
  RMSE: Root Mean Square Error
## MSE: Mean Square Error
##
  MAE: Mean Absolute Error
##
##
                                  ANOVA
##
                     Sum of
##
                     Squares
                             DF Mean Square F Sig.
```

##	Regression	14144	8105408.046		2			28.415	0.0000		
	Residual Total				35	40918439.07	1				
			74221568.368 		37 					_	ļ
##											
##						cameter Estimates	s				
##											
##		model	Beta	Std. E	rror	Std. Beta	t	Sig		lower	uppe
			0201 606				4 FO7				1055 0
##	factor (smoke	cept)	-2391.626	733 520	3.302	0.795					
##	lactor (Smore	age	274.871	12	455	0.795	22.069	0.000	) 250	50.437	299.30
##											
##											
##											
	Stepwise Sel	ection:	Step 3								
##											
## ##	- bmi added										
##			Model	Summary	,						
								-			
##			0.865				092.319				
##	R-Squared		0.747	Coef			45.909				
##	Adj. R-Squar	ed	0.747	MSE		37116	356.457				
##	Pred R-Squar	ed	0.746	MAE			216.776				
								_			
	RMSE: Root	_	_								
##	MSE: Mean A	_									
##		DBOTAGO	LIIOI								
##					NOVA						
##										-	
##			Sum of								
##			Squares		DF	Mean Square	е	F	Sig.		
			1 /65610+11			40052667251 20		246 02		_	
	Regression		1.46561e+11 13219514.179	19	3			316.23	0.0000		
			74221568.368		34 37	3/110330.40	1				
										_	
##											
##					Pa	arameter Estimate					
##		model	Beta	Std.		Std. Beta			-g	lower	1
			11.676, 000				10.4			:0510 100	
##			-11676.830			0.794				13516.100	
## ##						0.794 0.301					
##						7 0.162				268.692	
##											
##											
##											
шш				<b>G</b>	_						

6

Model Summary

R		0.865			92.319		
R-Squared	,	0.747	Coef. Var		15.909		
Adj. R-Squar	red	0.747 0.746	MSE	3711635	6.457 16.776		
RMSE: Root MSE: Mean S	_						
MAE: Mean A	_						
			ANOVA				
		Sum of Squares	DF	Mean Square	F	Sig.	
Regression	10513	.46561e+11	3	48853667351.396 37116356.457	1316.2	3 0.0000	)
		219514.179		3/116356.45/			
			Pa:	rameter Estimates			
	model	Beta	Std. Error	Std. Beta			lower
(Inter	ccept)	 -11676.830	937.569		-12.454	0.000 -	 -13516.100
	er)yes	23823.684	412.867	0.794	57.703	0.000	23013.746
				0.301			
		322.615		0.162	11.737	0.000	268.692 
Stepwise Sel	ection:	Step 4					
buchwipe per		buop i					
- children a	added						
		M - J - J	C				
			Summary 				
R		0.866			57.787		
R-Squared		0.750		4	15.724		
		0.749		3681804	12.098		
=		0.747			78.681		
RMSE: Root		are Error					
MSE: Mean S	Square Er	ror					
MAE: Mean A	Absolute	Error					
			4 NIO17 4				
			ANOVA				_
		Sum of					
			DF	Mean Square			_
				36748942862.937			_
Residual	49078	450116.619	1333	36818042.098			

### Regression 146995771451.750  ### And VA    Regression 1469957714									
Model   Beta   Std. Error   Std. Beta   t   Sig   lower				Par	ameter Estimate	es	<b></b>		<b></b>
Cintercept		model				t	Sig	lower	
### Regression 146995771451.750  ### ABOVA  **Regression 146995771451.75	(Inte	ercept)				-12.848	0.000	-13950.702	-10
Model Summary   Sumary   Sumare   Sum		_			0.794	57.904	0.000		24
Model Summary   Sumary   Sumare   Sum					0.299	21.675	0.000		
Model Summary   Sumary   Sumare   Sum		bmi	321.851	27.378	0.162	11.756	0.000	268.143	
Model Summary	cl	hildren	473.502	137.792	0.047	3.436	0.001	203.190	
Model Summary									
Model Summary									
R 0.866 RMSE 6067.787 R-Squared 0.750 Coef. Var 45.724 Adj. R-Squared 0.749 MSE 36818042.098 Pred R-Squared 0.747 MAE 4178.681  RMSE: Root Mean Square Error MSE: Mean Square Error MAE: Mean Absolute Error  ANOVA  Sum of Squares DF Mean Square F Sig.  Regression 146995771451.750 4 36748942862.937 998.123 0.0000 Residual 49078450116.619 1333 36818042.098 Total 196074221568.368 1337  Parameter Estimates  Parameter Estimates  (Intercept) -12102.769 941.984 -12.848 0.000 -13950.702 -15 factor(smoker)yes 23811.400 411.220 0.794 57.904 0.000 23004.692 25 age 257.850 11.896 0.299 21.675 0.000 244.512 bmi 321.851 27.378 0.162 11.756 0.000 268.143 children 473.502 137.792 0.047 3.436 0.001 203.190									
R Q 0.866 RMSE 6067.787 R-Squared 0.750 Coef. Var 45.724 Adj. R-Squared 0.749 MSE 36818042.098 Pred R-Squared 0.747 MAE 4178.681  RMSE: Root Mean Square Error MAE: Mean Square Error MAE: Mean Absolute Error  ANOVA  Sum of Squares DF Mean Square F Sig.  Regression 146995771451.750 4 36748942862.937 998.123 0.0000 Residual 49078450116.619 1333 36818042.098 Total 196074221568.368 1337  Parameter Estimates  Parameter Estimates  (Intercept) -12102.769 941.984 -12.848 0.000 -13950.702 -15 factor(smoker)yes 23811.400 411.220 0.794 57.904 0.000 23004.692 2 age 257.850 11.896 0.299 21.675 0.000 248.143 children 473.502 137.792 0.047 3.436 0.001 203.190			Model	Summary					
R-Squared 0.750 Coef. Var 45.724 Adj. R-Squared 0.749 MSE 36818042.098 Pred R-Squared 0.747 MAE 4178.681  RMSE: Root Mean Square Error MSE: Mean Square Error MAE: Mean Absolute Error  ANOVA  Sum of Squares DF Mean Square F Sig.  Regression 146995771451.750 4 36748942862.937 998.123 0.0000 Residual 49078450116.619 1333 36818042.098 Total 196074221568.368 1337  Parameter Estimates  Parameter Estimates  (Intercept) -12102.769 941.984 -12.848 0.000 -13950.702 -14 factor(smoker)yes 23811.400 411.220 0.794 57.904 0.000 23004.692 2 age 257.850 11.896 0.299 21.675 0.000 234.512 bmi 321.851 27.378 0.162 11.756 0.000 268.143 children 473.502 137.792 0.047 3.436 0.001 203.190									
# Adj. R-Squared									
Pred R-Squared	_			Coei. Var					
RMSE: Root Mean Square Error MSE: Mean Square Error MAE: Mean Absolute Error  ANOVA  Sum of Squares DF Mean Square F Sig.  Regression 146995771451.750 4 36748942862.937 998.123 0.0000 Residual 49078450116.619 1333 36818042.098 Total 196074221568.368 1337  Parameter Estimates  Parameter Estimates  (Intercept) -12102.769 941.984 -12.848 0.000 -13950.702 -1 factor(smoker)yes 23811.400 411.220 0.794 57.904 0.000 23004.692 2 age 257.850 11.896 0.299 21.675 0.000 234.512 bmi 321.851 27.378 0.162 11.756 0.000 268.143 children 473.502 137.792 0.047 3.436 0.001 203.190	Adj. K-Squa								
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Squares   DF   Mean Square   F   Sig.	Pred R-Squa 	t Mean Sq Square E	uare Error rror						
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Regression 146995771451.750	Pred R-Squa  RMSE: Root MSE: Mean MAE: Mean	t Mean Sq Square E	uare Error rror Error Sum of	ANOVA					
Residual 49078450116.619 1333 36818042.098  Total 196074221568.368 1337  Parameter Estimates  Model Beta Std. Error Std. Beta t Sig lower  (Intercept) -12102.769 941.984 -12.848 0.000 -13950.702 -1  factor(smoker)yes 23811.400 411.220 0.794 57.904 0.000 23004.692 2  age 257.850 11.896 0.299 21.675 0.000 234.512  bmi 321.851 27.378 0.162 11.756 0.000 268.143  children 473.502 137.792 0.047 3.436 0.001 203.190	Pred R-Squa  RMSE: Root MSE: Mean MAE: Mean	t Mean Sq Square E	uare Error rror Error Sum of	ANOVA		F	Sig.		
Parameter Estimates    Model   Beta   Std. Error   Std. Beta   t   Sig   lower	Pred R-Squa 	t Mean Sq Square E Absolute	uare Error rror Error  Sum of Squares	ANOVA DF	Mean Square				
Parameter Estimates  [Intercept] -12102.769	Pred R-Square RMSE: Root MSE: Mean MAE: Mean Regression	t Mean Sq Square E Absolute	uare Error rror Error Sum of Squares	ANOVA  DF  4 3	Mean Square				
Parameter Estimates    model	Pred R-Square	t Mean Sq Square E Absolute	uare Error rror Error Sum of Squares 5771451.750 8450116.619	ANOVA  DF  4 3 1333	Mean Square				
model   Beta   Std. Error   Std. Beta   t   Sig   lower	Pred R-Square	t Mean Sq Square E Absolute	uare Error rror Error Sum of Squares 5771451.750 8450116.619	ANOVA  DF  4 3 1333	Mean Square				
(Intercept) -12102.769 941.984 -12.848 0.000 -13950.702 -12.848 0.000 -13950.702 -13.848 0.000 -13950.702 -13.848 0.000 -13950.702 -13.848 0.000 -13950.702 -13.848 0.000 -13950.702 -13.848 0.0000 0.000 0.	Pred R-Square	t Mean Sq Square E Absolute	uare Error rror Error Sum of Squares 5771451.750 8450116.619	ANOVA  DF  4 3 1333 1337	Mean Square 	998.123			
(Intercept) -12102.769 941.984 -12.848 0.000 -13950.702	Pred R-Square	t Mean Sq Square E Absolute	Sum of Squares 5771451.750 8450116.619 4221568.368	ANOVA  DF  4 3 1333 1337	Mean Square 	998.123			
# factor(smoker)yes 23811.400 411.220 0.794 57.904 0.000 23004.692 2 age 257.850 11.896 0.299 21.675 0.000 234.512 bmi 321.851 27.378 0.162 11.756 0.000 268.143 children 473.502 137.792 0.047 3.436 0.001 203.190	Pred R-Square RMSE: Root MSE: Mean MAE: Mean Regression Residual Total	t Mean Sq Square E Absolute 14699 4907 19607	uare Error rror Error  Sum of Squares 5771451.750 8450116.619 4221568.368 Beta	ANOVA  DF  4 3 1333 1337	Mean Square 6748942862.937 36818042.098	998.123	0.0000	 ) 	
age     257.850     11.896     0.299     21.675     0.000     234.512       bmi     321.851     27.378     0.162     11.756     0.000     268.143       children     473.502     137.792     0.047     3.436     0.001     203.190	Pred R-Square RMSE: Root MSE: Mean MAE: Mean MAE: Mean Regression Residual Total	t Mean Sq Square E Absolute 14699 4907 19607	uare Error rror Error  Sum of Squares  5771451.750 8450116.619 4221568.368  Beta	ANOVA  DF  4 3 1333 1337  Par  Std. Error	Mean Square 6748942862.937 36818042.098	998.123	0.0000 Sig	lower	 
bmi 321.851 27.378 0.162 11.756 0.000 268.143 children 473.502 137.792 0.047 3.436 0.001 203.190	Pred R-Squa	t Mean Sq Square E Absolute 14699 4907 19607 model	uare Error rror Error  Sum of Squares  5771451.750 8450116.619 4221568.368  Beta  -12102.769	ANOVA  DF  4 3 1333 1337  Par  Std. Error  941.984	Mean Square 6748942862.937 36818042.098 cameter Estimate Std. Beta	998.123 es t	0.0000 Sig	lower 	 -10
children 473.502 137.792 0.047 3.436 0.001 203.190	Pred R-Squa	t Mean Sq Square E Absolute 14699 4907 19607 model	uare Error rror Error  Sum of Squares  5771451.750 8450116.619 4221568.368  Beta  -12102.769 23811.400	ANOVA  DF  4 3 1333 1337  Par  Std. Error  941.984 411.220	Mean Square	998.123 es t -12.848 57.904	0.0000 Sig 0.000 0.000	lower -13950.702 23004.692	 -1( 24
	Pred R-Squa	t Mean Sq Square E Absolute 14699 4907 19607 model ercept)	Sum of Squares  5771451.750 8450116.619 4221568.368  Beta  -12102.769 23811.400 257.850	ANOVA  DF  4 3 1333 1337  Par  Std. Error  941.984 411.220 11.896	Mean Square 6748942862.937 36818042.098 cameter Estimate Std. Beta 0.794 0.299	998.123  es  t  -12.848  57.904  21.675	0.0000 Sig 0.000 0.000 0.000	lower -13950.702 23004.692 234.512	
	Pred R-Squa	t Mean Sq Square E Absolute 14699 4907 19607 model ercept) ker) yes age bmi	Sum of Squares 5771451.750 8450116.619 4221568.368  Beta -12102.769 23811.400 257.850 321.851	ANOVA  DF  4 3 1333 1337  Par  Std. Error  941.984 411.220 11.896 27.378	Mean Square 6748942862.937 36818042.098 cameter Estimate Std. Beta 0.794 0.299 0.162	998.123  es  t  -12.848  57.904  21.675  11.756	0.0000 Sig 0.000 0.000 0.000 0.000	lower -13950.702 23004.692 234.512 268.143	
£	Pred R-Squa	t Mean Sq Square E Absolute 14699 4907 19607 model ercept) ker) yes age bmi	Sum of Squares 5771451.750 8450116.619 4221568.368  Beta -12102.769 23811.400 257.850 321.851	ANOVA  DF  4 3 1333 1337  Par  Std. Error  941.984 411.220 11.896 27.378	Mean Square 6748942862.937 36818042.098 cameter Estimate Std. Beta 0.794 0.299 0.162	998.123  es  t  -12.848  57.904  21.675  11.756	0.0000 Sig 0.000 0.000 0.000 0.000	lower -13950.702 23004.692 234.512 268.143	

8

## - factor(region) added

##		Model Sum						
##	D	0.867	DMCE		6060.1	78		
	R-Squared	0.751 0.750 0.747	Coef.	Var	45.6			
	Adj. R-Squared	0.750	MSE		36725751.3			
##					4171.7			
	RMSE: Root Mean Square MSE: Mean Square Error							
##	-							
##								
## ##				DVA 				
##		Sum of						
##		Squares	Dl	F Mea	an Square	F	Sig.	
##	Regression 1.47 Residual 48845249	229e+11 272 988	1330	7 210327. 0 367	10327.911 25751.333	572.697	0.0000	
	Total 196074221		133		20701.000			
##								
##								
##				Parame	eter Estimate	es		
##		Bet				t	Sig	lower
##								
##		-11990.27				-12.250		-13910.355
##	factor(smoker)yes	23836.30	)1		0.795	57.875 21.610	0.000 0.000	23028.341
## ##	age bmi		'4 SE	11.891	0.298	21.610 11.858	0.000	233.646
##	children	230.00 474 56	56 86	28.559 137.740	0.171	3.445		282.639 204.355
	factor(region)northwest		32	476.120	-0.012	-0.740	0.460	-1286.211
	factor(region) southeast	-1034.36	30	478.537	-0.038	-2.162	0.031	-1973.130
	factor(region)southwest		<b>7</b> 5	477.778	-0.034	-2.008		-1896.656
##								
##								
##								
##		Model Sum	marv					
##			-					
##	R	0.867	RMSE		6060.1	.78		
##	R-Squared		Coef.	Var	45.6			
	Adj. R-Squared	0.750	MSE		36725751.3			
	Pred R-Squared	0.747			4171.7	'10		
## ##	RMSE: Root Mean Square							
##	_	11101						
##	MAE: Mean Absolute Err	or						
##								
##				OVA 				
##		Sum of					<b></b>	
##	1	Squares	DI	F Mea	an Square	F	Sig.	
##	Regression 1.47	229e+11	•	7 210327	10327.911	572.697	0.0000	

	Residual 48845249 Total 196074221		1330 1337	367:	25751.333			
## ##					eter Estimates			
## - ##	model	Beta	Std.	Error	Std. Beta			
"" ## ## ## ## ## f	factor(smoker)yes	-11990.270 23836.301 256.974 338.665 474.566 -352.182 -1034.360 -959.375	9 4 1 1 1 4 4 4	978.762 411.856 11.891 28.559 .37.740 476.120 478.537 477.778	0.795 0.298 0.171 0.047 -0.012 -0.038	21.610 11.858 3.445 -0.740	0.000 0.000 0.000 0.001 0.460 0.031	23028.341 233.646 282.639
## ## ## F ## -	No more variables to be							
## ## -		Model Summ	•					
## A ## F	R-Squared Adj. R-Squared Pred R-Squared	0.747 M	oef. Var ISE IAE		6060.17 45.66 36725751.33 4171.71	7 3		
## ## ## ##	RMSE: Root Mean Square MSE: Mean Square Error MAE: Mean Absolute Err	Error						
‡# ‡# -			ANOVA					
‡# ‡# ‡# -		Sum of Squares	DF	Mea	an Square	F	Sig.	
# F	Regression 1.47 Residual 48845249 Total 196074221		7 1330 1337			72.697	0.0000	
‡# ‡#	Parameter Estimates							
‡# - ‡#	model	. Beta	Std.	Error	Std. Beta	t	Sig	lower
## - ## ## ##	(Intercept) factor(smoker)yes	23836.301	. 4	978.762 111.856 11.891	0.795 0.298	-12.250 57.875 21.610	0.000 0.000 0.000	-13910.355 23028.341 233.646

##	bmi	338.665	28.559	0.171	11.858	0.000	282.639
##	children	474.566	137.740	0.047	3.445	0.001	204.355
##	factor(region)northwest	-352.182	476.120	-0.012	-0.740	0.460	-1286.211
##	factor(region)southeast	-1034.360	478.537	-0.038	-2.162	0.031	-1973.130
##	factor(region)southwest	-959.375	477.778	-0.034	-2.008	0.045	-1896.656

#### summary(stepmod\$model)

```
##
## Call:
  lm(formula = paste(response, "~", paste(preds, collapse = " + ")),
       data = 1)
##
##
## Residuals:
##
        Min
                  1Q
                       Median
                                     3Q
                                             Max
                       -979.7
                                         29935.5
##
  -11367.2
             -2835.4
                                 1361.9
##
## Coefficients:
##
                            Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                            -11990.27
                                          978.76 -12.250
                                                          < 2e-16 ***
## factor(smoker)yes
                             23836.30
                                                  57.875
                                                          < 2e-16 ***
                                          411.86
## age
                              256.97
                                           11.89
                                                  21.610
                                                          < 2e-16 ***
## bmi
                              338.66
                                           28.56
                                                  11.858
                                                          < 2e-16 ***
## children
                              474.57
                                          137.74
                                                   3.445 0.000588 ***
                                                  -0.740 0.459618
## factor(region)northwest
                              -352.18
                                          476.12
## factor(region)southeast
                             -1034.36
                                          478.54
                                                  -2.162 0.030834 *
                              -959.37
                                                  -2.008 0.044846 *
  factor(region)southwest
                                          477.78
##
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 6060 on 1330 degrees of freedom
## Multiple R-squared: 0.7509, Adjusted R-squared:
## F-statistic: 572.7 on 7 and 1330 DF, p-value: < 2.2e-16
```

From the selection, the independent variables that produce the largest absolute t-values were declared. We used the default p-values such as any variables with a p-value lower than 0.1 - the entering threshold will enter the model and higher than p-value = 0.3 will be removed. The output from this procedure suggests the variable Sex to be dropped from the model. Therefore, we decided to include the main effects of both quantitative variables and dummy variables as such: Smoker, Age, BMI, Children, Region in our first-order model.

### Individual T-tests:

Hypothesis statement: (the model without Sex variable)  $H_0: \beta_i = 0$   $H_a: \beta_i \neq 0$  (i = Age, BMI, Children, Smoker, Region)

We used the individual T-tests to determine what the best predictors are on the significance level  $\alpha=0.05$ . From the output, the p-values of Age, BMI, Children, Smoker were less than  $\alpha=0.05$  which indicate that we should reject the null hypothesis and these variables are significant in the model. The Region variable had one category above our specified  $\alpha$  among 4 categories. Therefore, we decided to keep Region as one of the predictors from this step for further comparison with interaction and higher order terms. The model from the individual T-tests is:

$$Y_{Charges} = \beta_0 + \beta_1 X_{Age} + \beta_2 X_{BMI} + \beta_3 X_{Children} + \beta_4 X_{Smoker} + \beta_5 X_{Region}$$

```
fullmodel <-lm(charges~age+bmi+children+</pre>
                 factor(smoker)+factor(region),
               data=insurance) #full model without Sex variable
summary(fullmodel)
##
## Call:
## lm(formula = charges ~ age + bmi + children + factor(smoker) +
##
       factor(region), data = insurance)
##
## Residuals:
       Min
                                    30
##
                  1Q Median
                                            Max
## -11367.2 -2835.4 -979.7
                                1361.9 29935.5
##
## Coefficients:
##
                            Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                                        978.76 -12.250 < 2e-16 ***
                           -11990.27
                                          11.89 21.610 < 2e-16 ***
## age
                              256.97
## bmi
                              338.66
                                          28.56 11.858 < 2e-16 ***
## children
                              474.57
                                         137.74
                                                 3.445 0.000588 ***
## factor(smoker)yes
                            23836.30
                                         411.86 57.875 < 2e-16 ***
## factor(region)northwest -352.18
                                         476.12 -0.740 0.459618
## factor(region)southeast -1034.36
                                         478.54 -2.162 0.030834 *
## factor(region)southwest
                             -959.37
                                         477.78 -2.008 0.044846 *
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 6060 on 1330 degrees of freedom
## Multiple R-squared: 0.7509, Adjusted R-squared: 0.7496
## F-statistic: 572.7 on 7 and 1330 DF, p-value: < 2.2e-16
Partial F-test:
Hypothesis statement: (for Region variable) H_0: \beta_{Region} = 0 H_a: \beta_{Region} \neq 0
#F-test for model with or without region
firstordermodel <- lm(charges~age+bmi+children+factor(smoker),</pre>
                      data=insurance) #without Region variable
summary(firstordermodel)
##
## Call:
## lm(formula = charges ~ age + bmi + children + factor(smoker),
##
       data = insurance)
##
## Residuals:
##
       Min
                  1Q Median
                                    3Q
                                            Max
## -11897.9 -2920.8
                       -986.6 1392.2 29509.6
##
## Coefficients:
##
                      Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                     -12102.77
                                 941.98 -12.848 < 2e-16 ***
                                    11.90 21.675 < 2e-16 ***
                        257.85
## age
```

```
## bmi
                        321.85
                                   27.38 11.756 < 2e-16 ***
                        473.50
## children
                                  137.79
                                           3.436 0.000608 ***
## factor(smoker)yes 23811.40
                                  411.22 57.904 < 2e-16 ***
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 6068 on 1333 degrees of freedom
## Multiple R-squared: 0.7497, Adjusted R-squared: 0.7489
## F-statistic: 998.1 on 4 and 1333 DF, p-value: < 2.2e-16
firstordermodel1 <- lm(charges~age+bmi+children+factor(smoker)+factor(region),</pre>
                       data=insurance)
                                        #with Region variable
summary(firstordermodel1)
##
## Call:
## lm(formula = charges ~ age + bmi + children + factor(smoker) +
       factor(region), data = insurance)
##
## Residuals:
##
       Min
                 1Q
                      Median
                                   3Q
                                           Max
## -11367.2 -2835.4
                      -979.7
                               1361.9
                                       29935.5
##
## Coefficients:
                           Estimate Std. Error t value Pr(>|t|)
##
## (Intercept)
                          -11990.27
                                        978.76 -12.250 < 2e-16 ***
## age
                             256.97
                                         11.89 21.610 < 2e-16 ***
                                         28.56 11.858 < 2e-16 ***
## bmi
                             338.66
## children
                             474.57
                                        137.74
                                                 3.445 0.000588 ***
                                        411.86 57.875 < 2e-16 ***
## factor(smoker)yes
                           23836.30
## factor(region)northwest
                            -352.18
                                        476.12
                                                -0.740 0.459618
## factor(region)southeast
                           -1034.36
                                        478.54 -2.162 0.030834 *
                                        477.78 -2.008 0.044846 *
## factor(region)southwest
                            -959.37
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 6060 on 1330 degrees of freedom
## Multiple R-squared: 0.7509, Adjusted R-squared: 0.7496
## F-statistic: 572.7 on 7 and 1330 DF, p-value: < 2.2e-16
anova(firstordermodel, firstordermodel1)
## Analysis of Variance Table
##
## Model 1: charges ~ age + bmi + children + factor(smoker)
## Model 2: charges ~ age + bmi + children + factor(smoker) + factor(region)
    Res.Df
                  RSS Df Sum of Sq
## 1
      1333 4.9078e+10
      1330 4.8845e+10 3 233200844 2.1166 0.09631 .
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

The partial F-test was used in order to check the significance of the Region variable. The goal of this step is to investigate the contribution of this predictor individually. We defined the full model with all the

predictors and the reduced model with the whole set of predictors less the Region one. From the analysis of variance for comparing between these two models, the output shows that F-value=2.1166 with df =  $1330~(p-value=0.09631>\alpha=0.05)$ , indicating that we should clearly not to reject the null hypothesis. We should definitely drop the Region variable off the model. At this point, the model we have is:

```
Y_{Charges} = \beta_0 + \beta_1 X_{Age} + \beta_2 X_{BMI} + \beta_3 X_{Children} + \beta_4 X_{Smoker}
```

Interaction terms individual T-tests:

Hypothesis statement:  $H_0: \beta_i = 0$   $H_a: \beta_i \neq 0$  (i = AgeBMI, AgeChildren, AgeSmoker, BMIChildren, BMISmoker, ChildrenSmoker)

```
itrmodel <- lm(charges~(age+bmi+children+factor(smoker))^2, data=insurance)
summary(itrmodel)</pre>
```

```
##
## Call:
  lm(formula = charges ~ (age + bmi + children + factor(smoker))^2,
##
##
       data = insurance)
##
## Residuals:
##
        Min
                  1Q
                       Median
                                    3Q
                                            Max
## -13996.3 -1947.6 -1331.5
                                -406.4
                                        29570.2
##
## Coefficients:
                                Estimate Std. Error t value Pr(>|t|)
##
                              -6.745e+02 2.106e+03
                                                    -0.320
## (Intercept)
                                                                0.749
## age
                               2.055e+02
                                         4.963e+01
                                                       4.140
                                                              3.7e-05 ***
## bmi
                              -6.339e+01
                                          6.756e+01
                                                     -0.938
                                                                0.348
## children
                               6.957e+02
                                          6.341e+02
                                                       1.097
                                                                0.273
## factor(smoker)yes
                              -1.983e+04
                                          1.861e+03 -10.651
                                                              < 2e-16 ***
## age:bmi
                                          1.561e+00
                               1.912e+00
                                                       1.225
                                                                0.221
## age:children
                               1.201e+00
                                          8.527e+00
                                                       0.141
                                                                0.888
## age:factor(smoker)yes
                                                     -0.038
                                                                0.969
                              -9.141e-01
                                          2.384e+01
## bmi:children
                              -5.334e+00
                                          1.863e+01
                                                     -0.286
                                                                0.775
## bmi:factor(smoker)yes
                               1.437e+03
                                          5.317e+01
                                                      27.029
                                                              < 2e-16 ***
## children:factor(smoker)yes -3.858e+02 2.841e+02
                                                     -1.358
                                                                0.175
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 4874 on 1327 degrees of freedom
## Multiple R-squared: 0.8392, Adjusted R-squared: 0.838
## F-statistic: 692.8 on 10 and 1327 DF, p-value: < 2.2e-16
```

From the output of the T-tests for the interaction terms, there is only one term that is significant for the charge of medical insurance which is between BMI and Smoker (p-value < 0.05). The other interaction terms have small t-value and high p-value compared to our significance level. Therefore, we fail to reject the null hypothesis. The model with the interaction term is shown below:

```
##
## Call:
        lm(formula = charges ~ age + bmi + factor(smoker) + bmi * factor(smoker) +
                     children, data = insurance)
##
##
## Residuals:
##
                        Min
                                                       10
                                                                       Median
                                                                                                                30
                                                                                                                                         Max
        -14598.6 -1924.4 -1321.4
##
                                                                                                    -465.6
                                                                                                                            29892.4
##
## Coefficients:
##
                                                                                    Estimate Std. Error t value Pr(>|t|)
                                                                                                                                                    -3.283 0.00105 **
## (Intercept)
                                                                                 -2729.002
                                                                                                                         831.270
                                                                                       264.948
                                                                                                                               9.553
                                                                                                                                                    27.735
                                                                                                                                                                             < 2e-16 ***
## age
                                                                                                                            24.873
## bmi
                                                                                             5.656
                                                                                                                                                        0.227
                                                                                                                                                                            0.82014
## factor(smoker)yes
                                                                              -20194.709
                                                                                                                      1654.505 -12.206 < 2e-16 ***
## children
                                                                                       508.924
                                                                                                                         110.615
                                                                                                                                                        4.601 4.61e-06 ***
## bmi:factor(smoker)yes
                                                                                    1433.788
                                                                                                                            52.823
                                                                                                                                                 27.143 < 2e-16 ***
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 4871 on 1332 degrees of freedom
## Multiple R-squared: 0.8388, Adjusted R-squared: 0.8382
## F-statistic: 1387 on 5 and 1332 DF, p-value: < 2.2e-16
Y_{Charges} = \beta_0 + \beta_1 X_{Age} + \beta_2 X_{BMI} + \beta_3 X_{Children} + \beta_4 X_{Smoker} + \beta_5 X_{BMI} * X_
Higher order model:
Age^3, BMI^3) ( i = Age^2, BMI^2, BMI^3)
hm <- lm(charges~age+bmi+factor(smoker)+children +
```

We want to check for the significance of the higher order terms. Firstly, we included the quadratic terms of three quantitative variables which are Age, BMI and Children. All of the values are significant except the higher order of the Children variable (p-value = 0.0661). Therefore, we removed the higher order of Children and increased the order of Age and BMI to cubic terms. The cubic term of Age was insignificant at p-value = 0.7437. Therefore, we finalized our higher order terms as such: Age^2 (t = 3.838, p-value = 0.00013) BMI^2 (t = 2.020, p-value = 0.04361) BMI^3 (t = -2.253, p-value = 0.0241) The model with higher order terms:

$$Y_{Charges} = \beta_0 + \beta_1 X_{Age} + \beta_2 X_{BMI} + \beta_3 X_{Children} + \beta_4 X_{Smoker} + \beta_5 X_{Age^2} + \beta_6 X_{BMI^2} + \beta_7 X_{BMI^3}$$

Interaction terms and higher order model:

Hypothesis statement:  $H_0: \beta_i = 0$   $H_a: \beta_i \neq 0$  (i = BMI\*factor(Smoker))

```
highermodel <- lm(charges~age+bmi+factor(smoker)+
                    children + I(age^2) + I(bmi^2) +
                    I(bmi^3) + bmi*factor(smoker),
                  data=insurance)
summary(highermodel)
##
## Call:
##
  lm(formula = charges ~ age + bmi + factor(smoker) + children +
       I(age^2) + I(bmi^2) + I(bmi^3) + bmi * factor(smoker), data = insurance)
##
## Residuals:
##
      Min
                1Q Median
                                30
                                       Max
## -8511.4 -1849.2 -1318.2 -635.3 30538.3
##
## Coefficients:
                           Estimate Std. Error t value Pr(>|t|)
##
## (Intercept)
                          2.752e+04
                                    8.667e+03
                                                 3.175 0.001531 **
                                     6.435e+01
                                                -0.407 0.683833
## age
                         -2.621e+01
                         -2.774e+03
                                    8.335e+02
                                               -3.328 0.000898 ***
## bmi
                                     1.628e+03 -12.577 < 2e-16 ***
## factor(smoker)yes
                         -2.048e+04
## children
                                                 5.867 5.60e-09 ***
                          6.683e+02
                                     1.139e+02
## I(age^2)
                          3.645e+00
                                     8.027e-01
                                                 4.541 6.11e-06 ***
## I(bmi^2)
                          9.714e+01
                                     2.614e+01
                                                 3.716 0.000211 ***
## I(bmi^3)
                                                -4.068 5.01e-05 ***
                         -1.085e+00
                                     2.666e-01
## bmi:factor(smoker)yes 1.444e+03
                                    5.198e+01 27.781 < 2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4788 on 1329 degrees of freedom
## Multiple R-squared: 0.8446, Adjusted R-squared: 0.8437
## F-statistic: 903.2 on 8 and 1329 DF, p-value: < 2.2e-16
anova(hm2, highermodel)
## Analysis of Variance Table
##
## Model 1: charges ~ age + bmi + factor(smoker) + children + I(age^2) +
##
       I(bmi^2) + I(bmi^3)
## Model 2: charges ~ age + bmi + factor(smoker) + children + I(age^2) +
##
       I(bmi^2) + I(bmi^3) + bmi * factor(smoker)
##
     Res.Df
                   RSS Df Sum of Sq
                                              Pr(>F)
## 1
       1330 4.8151e+10
```

We conducted the ANOVA test to confirm the contribution of the interaction term with the higher order model. The reduced model is the one with the main effects and without the interaction term. Meanwhile, the full model includes the main effects, the interaction term and higher order terms. The result of ANOVA with F = 771.78 and p-value < 0.05 indicates that we should reject the hypothesis. We finalized our model with higher order and interaction terms as shown below:

1329 3.0462e+10 1 1.769e+10 771.78 < 2.2e-16 \*\*\*

## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.05 '.' 0.1 ' ' 1

$$Y_{Charges} = \beta_0 + \beta_1 X_{Age} + \beta_2 X_{BMI} + \beta_3 X_{Children} + \beta_4 X_{Smoker} + \beta_5 X_{Age^2} + \beta_6 X_{BMI^2} + \beta_7 X_{BMI^3} + \beta_8 X_{BMI} * X_{Smoker}$$

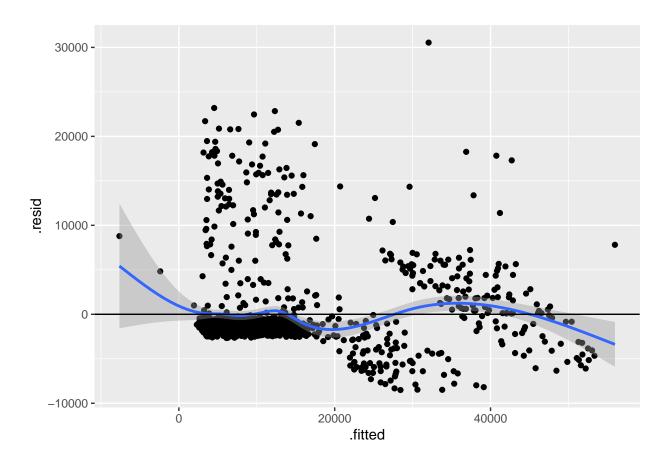
Multiple regression assumptions

#### 1. Linearity assumption:

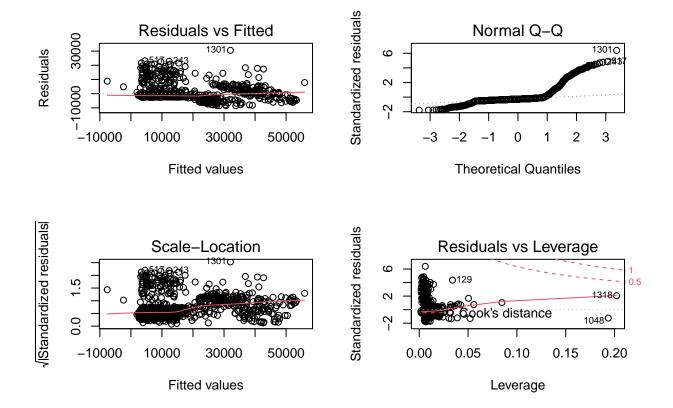
The best predicted model assumes that there is a straight-line relationship between all the predictors and the response. We expect to see the linear pattern when plotting the residuals and fitted values from the model. Using the residual plot as shown below, there are no discernible patterns detected. Therefore, this model passes the linearity assumption.

```
library(ggplot2)
ggplot(highermodel, aes(x=.fitted, y=.resid))+geom_point()+geom_smooth()+geom_hline(yintercept = 0)
```

## `geom\_smooth()` using method = 'gam' and formula 'y ~ s(x, bs = "cs")'



```
par(mfrow=c(2,2))
plot(highermodel)
```



#### 2. Equal variance assumption:

### $H_0$ : Heteroscedasticity is not present $H_a$ : Heteroscedasticity is present

The model is also assumed to have the error terms that have a constant variance. In order to verify whether our model is homoscedastic, a scale-location between fitted value and standardized residuals as well as the studentized Breusch-Pagan test were utilized. From the plot, we can see the horizontal line with equally spread points. The output of the Breusch-Pagan (BP = 6.8338 and p-value = 0.5547) indicates that we should not reject the null hypothesis. Therefore, it suggests that the predicted model passes the assumption and is homoscedastic.

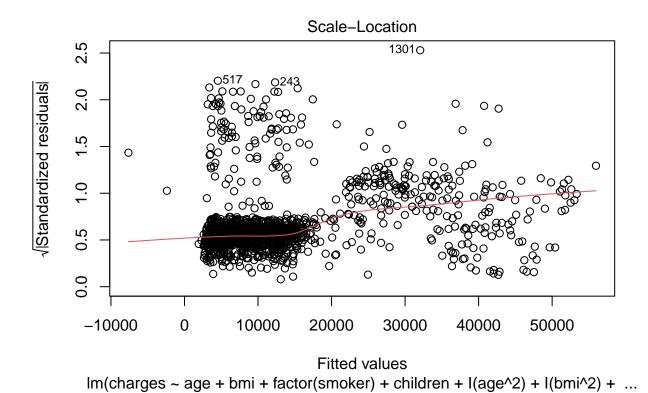
### library(lmtest)

```
## Warning: package 'lmtest' was built under R version 4.1.2
## Loading required package: zoo
##
## Attaching package: 'zoo'
## The following objects are masked from 'package:base':
##
## as.Date, as.Date.numeric
```

### bptest(highermodel)

```
##
## studentized Breusch-Pagan test
##
## data: highermodel
## BP = 6.8338, df = 8, p-value = 0.5547

plot(highermodel, which=3)
```



### 3. Multicollinearity test:

There is a chance that independent variables are correlated with each other. To test for multicollinearity, we performed the test with the variance inflation factor (VIF). Since our model has 3 higher-order terms and 1 interaction term, the high values of VIF are more likely to happen. The diagnostic shows that the multicollinearity may be due to the variables with higher order, the categorical variable Smoker and the interaction term between BMI and Smoker. Therefore, we are safe to ignore the high VIF and conclude that there are no extreme multicollinearities detected.

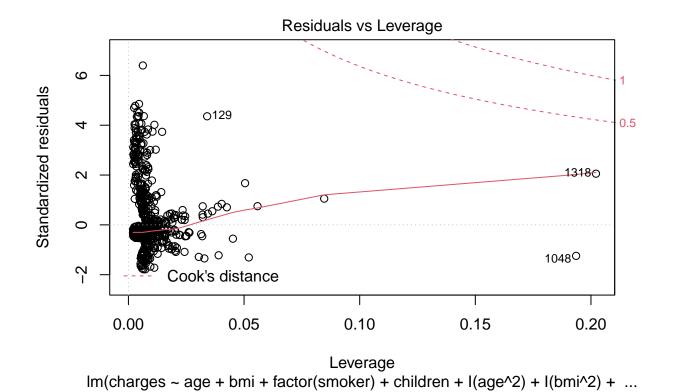
```
library(mctest)
imcdiag(highermodel, method="VIF")
```

```
## Call:
## imcdiag(mod = highermodel, method = "VIF")
##
##
##
   VIF Multicollinearity Diagnostics
##
                              VIF detection
##
## age
                          47.6852
                                          1
## bmi
                        1506.8357
                                          1
## factor(smoker)yes
                          25.1962
                                          1
## children
                           1.0999
                                          0
## I(age^2)
                                          1
                          47.6361
## I(bmi^2)
                        5996.5695
                                          1
## I(bmi^3)
                        1574.5771
                                          1
## bmi:factor(smoker)yes
                          25.5082
                                          1
## Multicollinearity may be due to age bmi factor(smoker)yes I(age^2) I(bmi^2) I(bmi^3) bmi:factor(smok
##
## 1 --> COLLINEARITY is detected by the test
  0 --> COLLINEARITY is not detected by the test
##
##
  _____
```

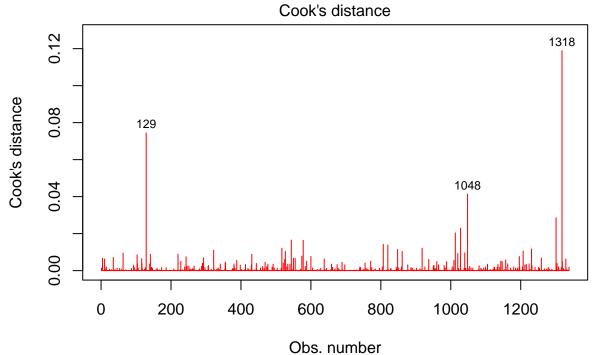
#### 4. Influential points and outliers:

We used residuals and leverage plot to detect outliers and influential points. We observed no points beyond Cook's distance, which means all of the points do not have high Cook's distance scores. In the plot showing the Cook's distance  $D_i$  of each observation, there are no points that have a distance greater than 0.5. Therefore, they are not influential. The leverage plot shows multiple influential points that are beyond the 2p/n threshold. However, when we removed these points from the data, the adjusted R square decreased and there are no reasons to delete these points. Therefore, we kept the original data and the predicted model.

```
plot(highermodel, which=5)
```

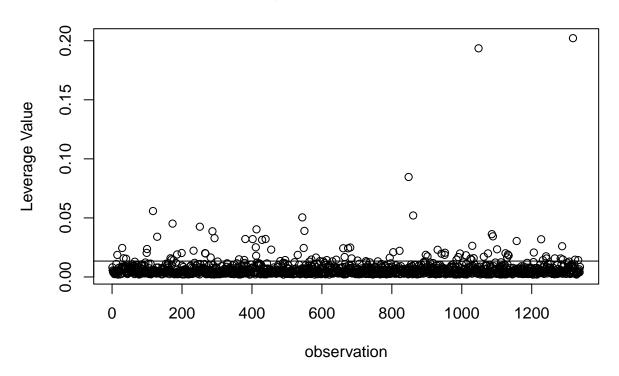


plot(highermodel, pch=18, col="red", which=c(4))



Im(charges ~ age + bmi + factor(smoker) + children + I(age^2) + I(bmi^2) + ...

## **Leverage in Insurance Dataset**



```
influential <- as.numeric(names(outlier))
data <- insurance[-influential, ]
testmodel <- lm(charges~age+bmi+I(age^2)+I(bmi^2)+I(bmi^3)+children+factor(smoker)+bmi*factor(smoker),
summary(testmodel)</pre>
```

```
##
## Call:
## lm(formula = charges ~ age + bmi + I(age^2) + I(bmi^2) + I(bmi^3) +
       children + factor(smoker) + bmi * factor(smoker), data = data)
##
##
##
  Residuals:
##
     Min
              1Q Median
                            3Q
                                  Max
   -8540 -1741 -1332
                          -795
                                30391
##
##
## Coefficients:
##
                           Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                          1.592e+04 1.872e+04
                                                 0.850
                                                          0.395
## age
                         -3.878e+01 6.675e+01
                                                -0.581
                                                           0.561
## bmi
                         -1.560e+03
                                    1.866e+03
                                                -0.836
                                                           0.403
## I(age^2)
                          3.831e+00 8.311e-01
                                                 4.609 4.46e-06 ***
## I(bmi^2)
                          5.727e+01
                                    6.070e+01
                                                 0.944
                                                           0.346
## I(bmi^3)
                         -6.680e-01 6.453e-01
                                                -1.035
                                                           0.301
## children
                          7.022e+02 1.197e+02
                                                 5.866 5.73e-09 ***
## factor(smoker)yes
                         -3.314e+04 2.383e+03 -13.907 < 2e-16 ***
## bmi:factor(smoker)yes 1.867e+03 7.758e+01 24.060 < 2e-16 ***
```

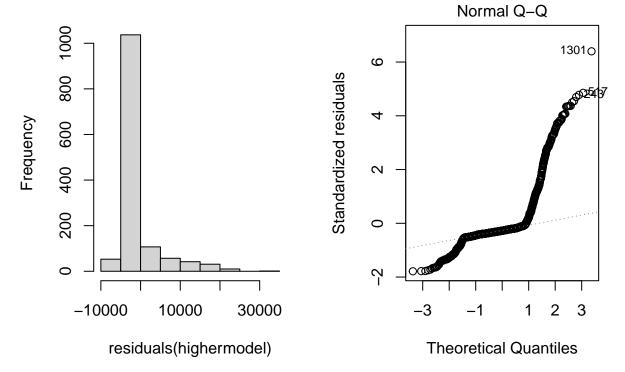
```
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4761 on 1240 degrees of freedom
## Multiple R-squared: 0.8299, Adjusted R-squared: 0.8288
## F-statistic: 756.2 on 8 and 1240 DF, p-value: < 2.2e-16</pre>
```

#### 5. Normality assumption:

 $H_0$ : The sample data are significantly normally distributed  $H_a$ : The sample data are not significantly normally distributed Another assumption of multiple linear regression is that the errors between observed and predicted values should be normally distributed. Looking at the histogram of residuals and the Q-Q plot, we fail to observe the normal trend of the distribution of the residuals and the points falling close to the diagonal reference line respectively. We used the Shapiro-Wilk test to confirm the normality assumption. On the significance level  $\alpha=0.05$ , the result from the test (W = 0.6393, p-value < 2.2e-16), we reject the null hypothesis. Overall, our data does not meet the normality assumption.

```
par(mfrow=c(1,2))
hist(residuals(highermodel))
plot(highermodel, which=2)
```

## Histogram of residuals(highermoc



```
shapiro.test(residuals(highermodel))
```

```
## Shapiro-Wilk normality test
##
## data: residuals(highermodel)
## W = 0.63931, p-value < 2.2e-16</pre>
```

6. Independence assumption: Our data for both dependent and independent variables are not observed sequentially over a period of time (time-series data). The response Y - charges is not related to time. Therefore, we do not check the model with independence assumption for this dataset.

### Transformation for normality

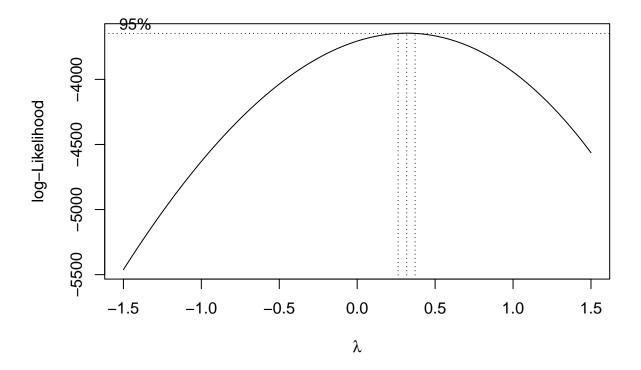
We made a transformation on Y - the response variable for nonnormality of the error terms by using a Box Cox transformation. The method determines the  $\lambda$  in order to transform Y to a replacement response variable Y^(lambda) with the expectation that the regression residuals become normally distributed. The optimal transformation is  $\lambda=0.31818$ . However, after the transformation, we then run the Shapiro-Wilk normality test again where the result (W = 0.6393 and p-value < 2.2e-16) indicates that the null hypothesis is rejected. The transformed data still does not distribute normally and the model fails the normality assumption.

## library(MASS)

```
##
## Attaching package: 'MASS'

## The following object is masked from 'package:olsrr':
##
## cement

bc=boxcox(highermodel,lambda=seq(-1.5,1.5))
```



```
bestlambda=bc$x[which(bc$y==max(bc$y))]
bestlambda
```

## [1] 0.3181818

### Conclusion

The best fit model for our dataset is:

$$Y_{Charges} = \beta_0 + \beta_1 X_{Age} + \beta_2 X_{BMI} + \beta_3 X_{Children} + \beta_4 X_{Smoker} + \beta_5 X_{Age^2} + \beta_6 X_{BMI^2} + \beta_7 X_{BMI^3} + \beta_8 X_{BMI} * X_{Smoker}$$

This model includes the main effects that were shown to have a significant impact on insurance charges and the interaction & higher order terms that also significantly affect insurance charges. The main effects included in this model were all determined to be significant through individual t-tests, stepwise selection, and a partial F-test. Both the interaction term and the higher order terms were kept in the model based on the results of their individual t-tests, and insignificant interaction terms and higher order terms were left out of the final model.

Independent Variable effects:

Intercept = 27520.39: When all other variables equal zero, the predicted insurance cost would be \$27,520.39.

Age = -26.21: When all other predictor variables are held constant, the insurance cost decreases by \$26.21 when the age of the insurance holder increases by one year.

BMI = -2773.974: When all other predictor variables are held constant, insurance cost decreases by \$2773.97 when BMI of the insurance holder increases by one 1 kg/m $^2$ .

Smoker (Yes) = -20476.48: When all other predictor variables are held constant, insurance cost decreases by \$20,476.48 when the insurance holder is a smoker.

Children = 668.2903: When all other predictor variables are held constant, insurance cost increases by \$668.29 with every additional child covered by the health insurance.

BMI & Smoker interaction = 1444.106: When all the other predictors are held constant and the insurance is a smoker, then the insurance cost will increase by \$1444.11 when the BMI of the insurance holder increases by 1 kg/m<sup>2</sup>.

The R^2 adjusted value for the best fit model obtained is 0.8437, which indicates that 84.37% of the variation in insurance cost is explained by this model. The RMSE is 4788 on 1329 degrees of freedom, which is the lowest RMSE value obtained in all the models that were tested. The minimized RMSE value indicates that this model is the best fit to the data.

#### Discussion

Our model had both expected and unexpected results. Region did not affect medical insurance costs, which isn't surprising considering that the regional variable is very broad and only considers four regions across the United States. For the region variable to have a noticeable impact on insurance costs, it would be effective to look at costs on a state or county level. Sex was also not a significant predictor, and the presence of influential points in the data made it difficult to draw conclusions with this variable. The results of the children variable were straightforward, the more children covered by the insurance policy, the higher the insurance costs.

The coefficient obtained in the model for the smoker variable was unusual, having a \$20,476 decrease in insurance costs when the beneficiary is a smoker. This result was strange because when medical costs are considered with only smoker status as a variable, medical costs were higher when the beneficiary was a smoker. BMI in the model also returned a strange result, decreasing insurance cost by \$2774 for every 1 kg/m^2 increase in BMI. Like smoker status, when medical costs are only analyzed with the BMI variable, medical costs increase as BMI increases. Age had the same effect as BMI and smoker status, having a negative effect in the model but a positive correlation when modeled on its own.

An aspect of this model that could be changed is changing the children variable from a quantitative variable to a qualitative variable where children are present or absent in the beneficiary's health insurance. This would likely yield a different result and would likely make predicting insurance costs with children more accurate. Another aspect that could be improved is working with the violation of the normality assumption. Despite transformations, the assumption was not met, and another type of modelling may be more appropriate for this dataset.

#### References

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