A (not so) gentle introduction to ecological networks

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Why should I care about networks?

- A good way to harness complexity
- A solid mathematical foundation
- Elegant algorithms
- That guy is going to talk about them for, like, two hours. . .

What is a network?

A mathematical approach

A *graph* is a **representation** of a **set of objects** where some pairs of objects are **connected** by **links**.

Or more formally, $G = (V, E)$, a *graph* G is an ordered pair of *vertices* V linked together by *edges* E.

Each element of E is a two-element subset of V.

The *order* of a graph is $|V|$, and its *size* is $|E|$.

What is a network?

An example

In an omnivory scenario, one top predator P consumes both an intermediate consumer C and a primary producer R. The intermediate consumer also consumes the producer.

This network is specified by

$$G = (\{P, C, R\}, \{\{P, C\}, \{P, R\}, \{C, R\}\})$$

Or for brevity

$$G = (\{P, C, R\}, \{PC, PR, CR\})$$

What is a network?

The adjacency matrix

Networks are often represented by their **adjacency matrix**.

The adjacency matrix $\mathbf{A}$ of a graph $G = (V, E)$ has elements A_{ij} with value 1 if there is an edge between the node V_i and the node V_j, and 0 otherwise.

It follows that $\sum_i \sum_j A_{ij} = |E|$.

What is a network?

Edge direction

Edges can be *directed* (arcs, directed edges) or not. An edge between a vertex and itself (cannibalism) is a *self-loop*.

In an **undirected graph**, there are at most $|V|(|V|-1)/2$ edges if there are no *self-loops*.

In a **directed graph**, there are at most $|V|(|V|-1)$ edges if there are no *self-loops*.

Exercice: What is the maximal size of a graph of order n if there are self-loops?

What is a network?

Edge weight

Edges in a network can have a **weight** (for example, the number of contacts between individuals).

The elements of the adjacency matrix $\mathbf{A}$ can be given *continuous* values.

It's possible to work both on the *weighted* and *unweighted* properties of a graph. However, there are many methods that (as of now) can only be applied to **undirected, unweighted** networks.

Number of partners

The number of vertices *receiving* a link from a focal vertex are called its **successors**

The number of vertices *establishing* a link towards a focal vertex are called its **predecessors**

The *total* number of edges connected to a focal vertex is this vertex **degree**

Where is the ecology in all that?

graph
: The whole community, *i.e.* the populations and their interactions

vertices
: The composition of the community (species present)

edges
: The interactions between the populations

Where is the ecology in all that?

Exemple of “networkable” systems

- Trophic systems
- Plant–pollinators
- Hosts–parasites
- Mutualism
- Social interactions

Any system in which the **same ecological interaction** happens several time in a community can (should) be studied using network theory

Connectance

Connectance is the *proportion of possible interactions realized*

The order of an ecological network is usually called S, and its size L

In a network with directed edges and self-loops, the connectance is $Co = L/S^2$

Exercice: What is the expression of the connectance for directed/undirected networks with/without self-edges?

Number of partners

The number of (*e.g.*) preys of a predator is its **generality** (number of successors)

The number of (*e.g.*) predators of a prey is its **vulnerability** (number of predecessors)

Number of partners

```
web = read.table('web.dat')
generality = rowSums(web)
vulnerability = colSums(web)
degree = generality + vulnerability
```

The niche model of food webs