kinematic equations

uniform circular motion

kinetic and potential energy

$$\bar{v} = \frac{1}{2}(v_i + v_f)$$
$$\Delta v = a\Delta t + v_i$$
$$x(t) = \frac{1}{2}at^2 + v_i t + x_i$$
$$v_f^2 - v_i^2 = 2a\Delta x$$

$$a = \frac{v^2}{r}$$
$$v = r\omega$$

$$E_{\text{trans}} = \frac{1}{2}mv^2$$
$$E_{\text{rot}} = \frac{1}{2}I\omega^2$$
$$E_{\text{gpe}} = mgh$$
$$E_{\text{sho}} = \frac{1}{2}kx^2$$

potential energy, force, work

gravitational force

angular momentum, torque

$$dU = -\vec{F} \cdot d\vec{l}$$
$$\vec{F} = -\vec{\nabla} U$$
$$dW = \vec{F} \cdot d\vec{l}$$

$$\vec{F} = -G m_1 m_2 r^{-2} \hat{r_{12}}$$

$$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}\vec{\tau} = \vec{r} \times \vec{F} = \frac{d\vec{L}}{dt}$$

constant angular velocity

spherical and cylindrical elements

moment of inertia

$$\vec{F} = -m\Omega^2 r\hat{r}_{\text{centrifuge}} - 2m\vec{\Omega} \times \vec{v}$$

$$\begin{aligned}
d\vec{l} &= dr\hat{r} + rd\theta\hat{\theta} + r\sin\theta d\phi\hat{\phi} \\
d\vec{S} &= r^2 \sin\theta d\theta d\phi\hat{r} + r\sin\theta dr d\phi\hat{\theta} + r dr d\theta\hat{\phi} \\
dV &= r^2 \sin\theta dr d\theta d\phi \\
d\vec{l} &= d\rho\hat{\rho} + \rho d\phi\hat{\phi} + dz\hat{z} \\
d\vec{S} &= \rho d\phi dz\hat{\rho} + d\rho dz\hat{\phi} + d\rho\rho d\phi\hat{z} \\
dV &= \rho d\rho d\phi dz
\end{aligned}$$

$$I = \int s^2 dm$$

$$I = I_{\text{CM}} + Ms^2$$

$I_z = I_x + I_y$ for z perpendicular to the body lying in the $x - y$ plane

Lagrangian

Hamiltonian

effective orbital potential

$$L(\dot{q}_i, q_i, t) = T - U$$
$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} = \frac{\partial L}{\partial q_i}$$
$$p_i = \frac{\partial L}{\partial \dot{q}_i}$$

$$H(p,q) = \sum_i p_i \dot{q}_i - L$$
$$H = T + U \text{ if } U \text{ is independent of } \dot{q} \text{ and } t$$
$$\dot{p} = -\frac{\partial H}{\partial q}$$
$$\dot{q} = \frac{\partial H}{\partial p}$$

$$E = T + V = \frac{1}{2}m\dot{r}^2 + \frac{l^2}{2mr^2} + U(r)$$
$$l = mr^2\dot{\phi}$$
$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$
$$E > 0 \implies \text{hyperbolic}$$
$$E = 0 \implies \text{parabolic}$$
$$E < 0 \implies \text{elliptical}$$
$$V^{'}(r) = 0 \text{ corresponds to circular orbit}$$
$$\text{above equation can be used to solve for radius of any orbit}$$
$$V^{''}(r) \text{ determines stability}$$

Kepler's laws

damped oscillation

driven oscillation

- Planets move in elliptical orbits with one focus at the sun
- Equal orbital area sweeps out equal orbital time
- $T = ka^{\frac{3}{2}}$ for all planets

$$
\begin{aligned}
0 &= m\ddot{x} + b\dot{x} + kx \\
\beta &= \frac{b}{2m} \\
\omega_0 &= (\frac{k}{m})^{\frac{1}{2}}
\end{aligned}
$$

- $\beta^2 > \omega_0^2$: overdamped decays exponentially
- $\beta^2 = \omega_0^2$: critically damed
- $\beta^2 < \omega_0^2$: underdamped decays exponentially, modulated by a sinusoid at $\omega_1^2 = \omega_0^2 - \beta^2$

$$
\begin{aligned}
\omega_{\mathrm{R}} &= (\omega_0^2 - 2\beta^2)^{\frac{1}{2}} \\
D &\propto |\omega_0^2 - \omega^2|^{-1} \text{ for undamped, driven oscillator}
\end{aligned}
$$

oscillator configurations

fluid dynamics

mechanics misc.

- pendulum $\omega = (\frac{g}{l})^{\frac{1}{2}}$
- physical pendulum $\omega = (\frac{mgR}{I})^{\frac{1}{2}}$

$$\frac{v^2}{2} + gz + \frac{p}{\rho} = \text{const}$$

p pressure, ρ fluid density, z height of a point along streamline

$$F = \rho V g$$

F buoyant force, V displaced fluid

- an object falls over when its center of mass is not above its contact with the ground

Maxwell's equations

electrostatics

boundary conditions

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$
$$\vec{\nabla} \times \vec{E} = \frac{-\partial \vec{B}}{\partial t}$$
$$\vec{\nabla} \cdot \vec{B} = 0$$
$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$
$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{J} = 0$$
$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t}$$
$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\vec{E} = -\vec{\nabla}\phi \implies \phi = -\int_a^b \vec{E} \cdot d\vec{l}$$
$$\nabla^2 \phi = -\frac{\rho}{\epsilon_0} \implies \phi = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3\vec{r}'$$
$$\int_S \vec{E}(\vec{r}) \cdot d\vec{S} = \frac{Q_{\text{enc}}}{\epsilon_0}$$
$$\int_C \vec{E}(\vec{r}) \cdot d\vec{l} = 0$$

Electrostatics

$$E^{\parallel}_{\text{out}} - E^{\parallel}_{\text{in}} = 0$$
$$E^{\perp}_{\text{out}} - E^{\perp}_{\text{in}} = \frac{\sigma}{\epsilon_0}$$

ϕ is continuous

$\partial\phi$ continuous when no surface charge is present

Magnetostatics

$$B^{\parallel}_{\text{out}} - B^{\parallel}_{\text{in}} = \mu_0 \vec{k} \times \hat{n}$$
$$B^{\perp}_{\text{out}} - B^{\perp}_{\text{in}} = 0$$

$\vec{k}$ is the surface current density

conductors

work and energy in (electro/magneto)statics

capacitors

$\phi =$ const. throughout conductor

$\implies$ E-field inside is zero

$\implies Q_{\text{inside}} = 0$

$\implies Q_{\text{net}}$ confined to surface

$\implies$ E-field outside is perpendicular to surface

$$W = \frac{1}{2}\int \rho\phi d^3r$$
$$U_E = \frac{\epsilon_0}{2}\int |\vec{E}|^2 d^3r$$
$$U_B = \frac{1}{2\mu_0}\int |\vec{B}|^2 d^3r$$

$$Q = C\phi$$
$$C_{\text{parallel plate}} = \frac{\epsilon_0 A}{d}$$
$$U = \frac{1}{2}Q^2C^{-1} = \frac{1}{2}C\phi^2$$

magnetostatics

magnetostatic configurations

cyclotron

$$\int_S \vec{B} \cdot d\vec{s} = 0$$

$$\int_C \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$$

$$\vec{F_B} = q\vec{v} \times \vec{B}$$

$$\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l} \times \vec{r^{'}}}{|\vec{r^{'}}|^3} d^3 r^{'}$$

$$\vec{B} = \mu_0 \frac{N}{L} I \hat{z} \quad \text{(solenoid)}$$

$$\vec{B} = \frac{\mu_0 N I}{2\pi r} \hat{\phi} \quad \text{(toroid)}$$

There is no field outside a toroid or solenoid.

$$\vec{F} = q\vec{v} \times \vec{B} = qvB\hat{x}$$

$$R = \frac{mv}{qB}$$

inductance

inductance configurations

multipoles

$$\mathcal{E} = \int_C \vec{E} \cdot d\vec{l} = \frac{-\partial \Phi_B}{\partial t}$$
$$\Phi_{21} = M_{12} I_1$$
$$\Phi_B = LI$$

Induced curents oppose changes in magnetic flux

$$L = \frac{\mu_{0N^2A}}{l} \quad \text{solenoid}$$

$$\vec{p} = \int \vec{r} \rho(\vec{r}) d^3 r$$
$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot \vec{r}}{|\vec{r}|^3}$$
$$\vec{\tau} = \vec{p} \times \vec{E}$$
$$U = -\vec{p} \cdot \vec{E}$$
$$\vec{m} = I\vec{A}$$
$$\vec{\tau} = \vec{m} \times \vec{B}$$
$$U = -\vec{m} \cdot \vec{B}$$

em in media

em waves

radiation

$$\sigma_b = \vec{P} \cdot \hat{n}$$
$$\rho_b = -\vec{\nabla} \cdot \vec{P}$$
$$\epsilon = \kappa \epsilon_0$$

$$\vec{E}(\vec{r}) = E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)} \hat{n}$$
$$\vec{B}(\vec{r}) = B_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)} (\hat{k} \times \hat{n})$$
$$\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B}) = \frac{1}{2\mu_0}\mathrm{Re}[\vec{E} \times \vec{B}^*]$$
$$I = \langle S \rangle = \frac{c\epsilon_0}{2} E_0^2$$

$$P = \frac{\mu_0 q^2 a^2}{6\pi c} \quad \text{accelerating point charge}$$
$$\langle S \rangle = \frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c} \frac{\sin^2(\theta)}{r^2}$$
$$\langle P_E \rangle = \frac{\mu_0 p_0^2 \omega^4}{12\pi c}$$
$$\langle P_B \rangle = \frac{\mu_0 m_0^2 \omega^4}{12\pi c^3}$$

The bottom three equations are for a radiating dipole.

circuit elements

circuit rules

wave equation

$$
\begin{aligned}
V_R &= IR \\
V_C &= \frac{Q}{C} \\
V_L &= L\dot{I} \\
R_{\text{series}} &= R_i \\
\frac{1}{C_{\text{series}}} &= \frac{1}{C_i} \\
L_{\text{series}} &= L_i \\
\frac{1}{R_{\text{parallel}}} &= \frac{1}{R_i} \\
C_{\text{parallel}} &= C_i \\
\frac{1}{L_{\text{parallel}}} &= \frac{1}{L_i}
\end{aligned}
$$

$$
\begin{aligned}
R &= \frac{\rho l}{A} \\
P &= IV \\
U_C &= \frac{1}{2}CV^2 \\
U_L &= \frac{1}{2}LI^2 \\
I &= \frac{V}{R}(1 - e^{-\frac{t}{L/R}}) \quad \text{RL circuit} \\
V &= V_0 e^{-\frac{t}{RC}} \quad \text{RC circuit} \\
\omega_{RLC} &= (LC)^{-\frac{1}{2}}
\end{aligned}
$$

- circuit elements in series see same current
- circuit elements in parallel see same voltage
- current in/out of a node is conserved
- voltage is conserved over elements around a closed loop

$$
\begin{aligned}
\partial_t^2 \psi &= v^2 \partial_x^2 \psi \\
\psi(\vec{r}, t) &= A\cos(\vec{k}\cdot\vec{r} - \omega t + \phi) \\
&= \mathrm{Re}[Ae^{i(\vec{k}\cdot\vec{r}-\omega t)}] \\
v_{\text{phase}} &= \frac{\omega}{k} \\
v_{\text{group}} &= \frac{\partial w}{\partial k} \\
\frac{\omega}{k} &= \frac{c}{n} \implies \lambda \to \frac{\lambda}{n} \quad \text{light} \\
\lambda = \frac{2\pi}{k} \quad T &= \frac{2\pi}{\omega} \quad \omega = 2\pi f
\end{aligned}
$$

wave configurations

polarization

interference

$v = (\frac{T}{\mu})^{\frac{1}{2}}$ string w/ tension T and mass density μ

$v = (\frac{\kappa}{\rho})^{\frac{1}{2}}$ sound w/ bulk modulus κ and density ρ

$$I = I_0 \cos^2(\theta)$$

$$\theta_B = \arctan(\frac{n_2}{n_1})$$

$d \sin(\theta) = m\lambda$ max dub-slit

$d \sin(\theta) = (m + \frac{1}{2})\lambda$ min dub-slit

$a \sin(\theta) = m\lambda$ min single-slit

$D \sin(\theta) = 1.22\lambda$ first circular diffraction minimum

$d \sin(\theta) = \frac{n\lambda}{2}$ x-rays on crystal maxima

$n_2 > n_1 \implies \pi$ phase shift

$n_2 < n_1 \implies 0$ phase shift

geometric optics

misc optics

fourier transform and series

$$\theta_i = \theta_r$$
$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$
$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$
$$f = \frac{R}{2} \text{ spherical mirror}$$
$$\frac{1}{f} = (n-1)(\frac{1}{R_1} - \frac{1}{R_2})$$
$$m = \frac{-s'}{s}$$

- s, s' positive distances
  $\Longrightarrow$ same side as light ray
- negative distances
  $\Longrightarrow$ opposite side as light ray
- f positive for converging lense, negative for diverging
- f positive for concave mirror, negative for convex mirror

$$I \propto I_0 \lambda^{-4} a^6 \quad \lambda \gg a$$
$$f = (\frac{v + v_r}{v - v_s}) f_0$$
$$L = \lambda(\frac{1}{4} + \frac{n}{2}) \text{ half-open pipe}$$

$$a_n = \frac{2}{T} \int_0^T f(t) \cos \frac{2\pi n x}{T}$$
$$b_n = \frac{2}{T} \int_0^T f(t) \sin \frac{2\pi n x}{T}$$
$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{2\pi n x}{T} + \sum_{n=1}^{\infty} b_n \sin \frac{2\pi n x}{T}$$
$$c_n = \frac{1}{T} \int_0^T f(t) \exp \frac{-i2\pi n x}{T}$$
$$f(x) = \sum_{n=-\infty}^{\infty} c_n \exp \frac{i2\pi n x}{T}$$

basic stat mech

entropy

equipartition theorem

$$
\begin{aligned}
p_i &= \frac{e^{-\beta E_i}}{Z} \\
Z &= \sum_j e^{-\beta E_j} \\
\beta &= (k_B T)^{-1} \\
\langle O \rangle &= \sum_i p_i O_i \\
\langle E \rangle &= -\partial_\beta \ln Z \\
Z_N &= \frac{1}{N! h^{3N}} \int e^{-\beta H(\vec{p}_1, \ldots \vec{p}_n; \vec{x}_1, \ldots \vec{x}_n)} d^3 p_1 \ldots d^3 p_n d^3 x_1 \ldots d^3 x_n \\
k_B &= 1.3e\text{-}23 \text{ JK}^{-1}
\end{aligned}
$$

$$
\begin{aligned}
S &= k_B \ln \Omega \\
S &= -k_B \sum_i p_i \ln(p_i) = \partial_T (k_B T \ln(Z)) \\
S &= N k_B \ln(\frac{V T^{3/2}}{N}) + \text{const. (monoatomic ideal gas)}
\end{aligned}
$$

Each quadratic term (d.o.f.) in the Hamiltonian for a particle contributes $k_B T/2$ to the internal energy of a particle

misc combinatorics

thermodynamic laws

ideal gas law

$$(N, M) = \frac{N!}{(N-M)!M!}$$

$$\ln(n!) = n\ln(n) - n \quad n \text{ large}$$

- Energy is neither created nor destoryed
  $\Delta U = Q - W$
- There does not exista process in which the sole effect is to transfer heat from a body at lower temperature to a body at higher temperature
  $\Delta S \geq \int \frac{\delta Q}{T}$
- Entropy is zero at absolute zero temperature. There is only one microstate at absolute zero temperature

$$PV = Nk_BT$$

reversible process

quasistatic

adiabatic

- system is in equilibrium at every instant
- $\delta W = PdV$
- $\delta Q = TdS$

- in thermal equilibrium at every instant, but not necessarily reversible

- $\delta Q = 0$

isentropic / reversible adiabatic

IsoX

free expansion

- reversible and adiabatic
- $\iff \Delta S = 0$
- $PV^\gamma = \text{const.}\,; \gamma = \frac{C_P}{C_V}$

- X held constant

- $\Delta T = 0$
- irreversible
- $PV = P^{'}V^{'}$

thermodynamic identities

heat capacity

model systems

$$dU = TdS - PdV$$
$$T = \frac{\partial U}{\partial S}|_V$$
$$P = -\frac{\partial U}{\partial V}|_S$$
$$\frac{\partial P}{\partial S}|_V = -\frac{\partial T}{\partial V}|_S$$

$$C_V = \frac{\partial Q}{\partial T}|_V = \frac{\partial U}{\partial T}$$
$$C_P = \frac{\partial Q}{\partial T}|_P$$
$$C_P - C_V = Nk_B \text{ (ideal gas)}$$
$$Q = mc\Delta T$$
$$c_{\text{water}} = 4.18 \text{ JK}^{-1}\text{g}^{-1}$$

$$e = 1 - |\frac{Q_C}{Q_H}|$$
$$e = 1 - \frac{T_C}{T_H} \text{ theoretical max, carnot}$$

- use thermodynamic identities to calculate areas
- clockwise paths in P - V and T - S planes do positive work

monoatomic ideal gas

quantum stat mech

schroedinger equation

$$Z_N = \frac{V^N}{N!h^{3N}}(2\pi m k_B T)^{3^N/2}$$

$$U = \frac{3}{2}Nk_BT$$

$$v_{\mathrm{rms}} = (\frac{3k_BT}{m})^{1/2}$$

$$K = \gamma P$$

$$F_{\mathrm{FD}}(E_i) = \frac{1}{e^{(E_i-\mu)/k_BT}+1}$$

$$F_{\mathrm{BE}}(E_i) = \frac{1}{e^{(E_i-\mu)/k_BT}-1}$$

$$\langle N \rangle = \sum_i g(E_i)F(E_i)$$

$$\Psi(x,t) = e^{-iE_nt/\hbar}\psi_n(x) \text{ TDSE}$$

- ψ_n are orthogonal
- ψ continuous, $\psi^{'}$ continuous except when $V(x) = \infty$
- ψ can be taken to be real, momentum expectation value vanishes for singleton state
- ground state has no nodes, n^{th} excited state has n nodes
- for an even potential ψ_n is even for n even, and odd for n odd

uncertainty principle

quantum configurations

QHO

$$\sigma_A^2 \sigma_B^2 \geq (\frac{1}{2i}\langle[A,B]\rangle)^2$$
$$\sigma_A^2 = \langle A^2\rangle - \langle A\rangle^2$$

$E = \dfrac{j(j+1)\hbar^2}{2I}$ rigid rotator

$E_n = \dfrac{\hbar^2\pi^2 n^2}{2ma^2}$ $|\psi_n\rangle = (\dfrac{2}{a})^{1/2}\sin(\dfrac{n\pi x}{a})$ infinite square well

$|\psi\rangle = e^{\pm ikx}$ $E = \dfrac{\hbar^2 k^2}{2m}$ $p = \hbar k$ free particle

free particle is sum of these wave functions, particle w/ definite momentum is not normalizable, i.e. doesn't

$|\psi\rangle = (\dfrac{m\alpha}{\hbar^2})^{1/2} e^{-m\alpha|x|/\hbar^2}$ $E = \dfrac{-m\alpha^2}{2\hbar^2}$ delta well

The finite square well is symmetric about zero so the states have definite parity.The ground state is even.

$$H = \hbar\omega(a^\dagger a + \frac{1}{2})\ [a, a^\dagger] = 1 \text{ QHO}$$

ground state of QHO is gaussian

$$\langle T\rangle = \langle V\rangle = \frac{E_n}{2} \text{ QHO virial theorem}$$
$$a^\dagger|n\rangle = (n+1)^{1/2}|n+1\rangle$$
$$a|n\rangle = (n)^{1/2}|n-1\rangle$$
$$E_n = \hbar\omega(N + \frac{3}{2})\ N = n_1 + n_2 + n_3 \text{ QHO3D}$$

QM in 3D

angular momentum

spin operators

$$\frac{-\hbar^2}{2m}\partial_r^2 u + (V + \frac{\hbar^2}{2m}\frac{l(l+1)}{r^2})u = Eu$$

$$u(r) = rR(r)$$

$$\int_0^\infty |R(r)|^2 r^2 dr = 1$$

$$[L_i, L_j] = \epsilon^{ijk} i\hbar L_k$$

$$[L^2, L_i] = 0$$

$$L_z Y_l^m = m\hbar Y_l^m$$

$$L^2 Y_l^m = \hbar^2 l(l+1) Y_l^m$$

$$S_\chi = S_\chi^{(1)} + \cdots + S_\chi^{(n)}$$

$$S_\chi^{(m)} = I \odot ... \odot S_\chi \odot ... \odot I$$

$$x_1 = 2^{-1/2}\begin{pmatrix}1\\1\end{pmatrix} \; x_2 = 2^{-1/2}\begin{pmatrix}1 & -1\end{pmatrix}$$

$$y_1 = 2^{-1/2}\begin{pmatrix}1\\i\end{pmatrix} \; y_2 = 2^{-1/2}\begin{pmatrix}1 & -i\end{pmatrix}$$

$$|\Psi\rangle = |\psi\rangle |\chi\rangle$$

$$s_{\text{tot}} = s + s^{'}, s + s^{'} - 1, \ldots, |s - s^{'}|$$

$$m_{\text{tot}} = m_s + m_{s'}$$

- add multiple spins by adding pairs
- boson (fermions) symmetric (anti-symmetric) under exchange of any two particles
- when adding spin-1/2 particles, spin states $s = n/2$ are always symmetric
- 2e helium ground state is spatially symmetric

hydrogen atom

approximation methods

Bohr model

$$
\begin{aligned}
a &= \frac{4\pi\epsilon_0\hbar^2}{\mu e^2} \\
\psi_1 &\propto e^{-r/a} \\
E_n &= \frac{-\hbar^2}{2\mu a^2}\frac{1}{n^2} = \frac{-13.6\text{ev}}{n^2} \\
\alpha &= \frac{e^2}{4\pi\epsilon_0\hbar c} \sim 1/137 \\
\psi|_{r=0} &= 0 \text{ for } l \neq 0
\end{aligned}
$$

$$
\begin{aligned}
E_n &= E_n^0 + \langle\psi_n^0|H_1|\rangle\,\psi_n^0 \\
E_n &= E_n^0 + \sum_{m\neq n}\frac{\langle\psi_m^0|H_1|\psi_n^0\rangle}{E_n^0 - E_m^0}
\end{aligned}
$$

- diagonalize expectation values of degenerate states with perturbation, eigenvalues are first order corrections

- Electron moves in circular oribt. angular momentum is quantized $L = \hbar n$
- no electron radiation in a given shell (experimentally shown)
- mathces hydrogen transitions
- hydrogen energy $\propto \alpha^2 m_e c^2$

fine structure

Lamb shift

hyperfine structure

- spin-orbit coupling and relativistic momentum
- energy shift $\propto \alpha^4 m_e c^2$
- energy levels get j dependence, m_j conserved, l degeneracy broken
- $J_2 = L^2 + S^2 + 2L\dot{S}$

- QED
- energy shift $\propto \alpha^5 m_e c^2$
- splits s and 2p levels with $j = \frac{1}{2}$ degeneracy

- nucleus-electron spin-spin coupling
- energy shift $\propto \frac{m_e}{m_p} \alpha^4 m_e c^2$
- ground state of hydrogen split depending on singlet or triplet configuration

shell model

stark effect

Zeeman effect

- s, p, d, f $\iff$ $l = 0, 1, 2, 3, \ldots$
- 2^2 orbitals in each shell. $2(2l+1)$ states in each orbital
- 1s, 2s, 2p, 3s, 3p, 3d
- shells fill in order. noble gases have filled shells

- $H_1 = q\vec{E}\vec{x}$
- no change to ground state energy of hydrogenic atom to first order in $|\vec{E}|$
- n = 2, m = 0 states are split
- $\Delta E \propto q|\vec{E}|a_0$

- $H_1 = \frac{q}{2m}(\vec{L} + 2\vec{S}) \cdot \vec{B}$
- $|\vec{B}|$ small, Zeeman perturbs fine. j, l, m_j quantum numbers. j splits based on m_j, spin wants to be anti-aligned with B-field
- $|\vec{B}|$ large, fine perturbs Zeeman. l, m_l, m_s. l splits on m_l, m_s

electric dipole radiation selection rules

blackbody radiation

quantum misc.

- electric dipole approximation $\lambda \gg a \implies$ spatial variation of the field is negligible
- $\Delta m = \pm 1, 0$
- $\Delta l = \pm 1$

$$I(\omega) \propto \frac{h\omega^3}{c^2} \frac{1}{e^{\hbar\omega/k_B T} - 1}$$
$$\frac{dP}{dA} \propto T^4$$
$$\lambda_{\text{peak}} = 3 \cdot 10^{-3} KmT^{-1}$$

$$\lambda_{\text{compton}} = \frac{h}{mc}$$

realtivity basics

relativity misc.

graph reading

$$\beta = \frac{v}{c}$$
$$\gamma = (1-\beta^2)^{-1/2}$$
$$w = \frac{v+u}{1+vu/c^2}$$
$$x^\mu = \begin{pmatrix} ct \\ \vec{x} \end{pmatrix} \; p^\mu = \begin{pmatrix} \frac{E}{c} \\ \vec{p} \end{pmatrix}$$
$$\vec{p} = \gamma m \vec{v}$$
$$\Lambda = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix}$$
$$\lambda^{'} = \lambda\left(\frac{1+\Delta\beta}{1-\Delta\beta}\right)^{1/2}$$

- $p^\mu p_\mu = m^2c^2$
- timelike $\Delta x^\mu \Delta x_\mu > 0$, there exists a rest frame where events occur at the same place
- spacelike $\Delta x^\mu \Delta x_\mu < 0$, there exists a rest frame where events occur at the same time
- lightlike $\Delta x^\mu \Delta x_\mu = 0$ rest frame is spaceship traveling between events

- straight line on log-log is $y = ax^b$ where b is the slope
- straight line on log-linear is $y = Ca^{bx}$ where $b\log_{10}(a)$ is the slope
- straight line on linear-log is $y = C\log_a(bx)$ where $C/\log_{10}(a)$ is the slope

statistics

electronics

radiation detection

$$
\begin{aligned}
\sigma_s^2 &= \frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2 \text{ sample variance, use } 1/n \text{ for pop}\\
\delta(aA) &= a\delta A\\
\delta(A+B) &= ((\delta A)^2+(\delta B)^2)^{1/2}\\
\delta(AB^{-1}) &= \delta(AB) = AB((\frac{\delta A}{A})^2+(\frac{\delta B}{B})^2)^(1/2)\\
X &= \frac{\sum_i x_i\sigma_{x_i}^{-2}}{\sum_i \sigma_{x_i}^{-2}}\\
\sigma^2 &= \frac{1}{\sum_i \sigma_{x_i}^{-2}}\\
P(n) &= \frac{\lambda^n e^{-\lambda}}{n!}\\
\sigma &= N^{1/2} \text{ Poisson distribution with } N \text{ large}\\
P(t) &= \lambda e^{-\lambda t} \text{ time between events}
\end{aligned}
$$

$$
\begin{aligned}
V &= V_0 e^{i\omega t}\\
V &= IZ\\
Z_{\text{cap}} &= \frac{1}{i\omega C}\\
Z_{\text{ind}} &= i\omega L\\
Z_{\text{res}} &= R\\
Z_{\text{series}} &= \sum_i Z_i\\
Z_{\text{parallel}}^{-1} &= \sum_i Z_i^{-1}
\end{aligned}
$$

- resonant frequency is where $Im(Z)=0$
- diode - current only flows in one direction once bias voltage is met -—¿—-
- op-amp - output voltage is proportional to difference between inputs =—¿-
- nuclei are stopped faster than electrons alpha (He^4) $\propto$ 10^{-5}m, $e \propto$ 10^{-3}m
- nuceli interact w/ electrons. electrons interact w/ electrons or nuclei
- nuclei tend to have straight paths, electrons bounce around
- nuclei lose energy due to collisions, electrons lose energy by collision or bremsstrahlung radiation (photon emission due to deceleration)

photon interactions

lasers

lab misc.

- photon absorbption

  dominant at a few keV

  photon absorbed by atom and electron released

  $E_{max} = E\gamma - \phi$ work function used for photon on material

- compton scattering

  photon scatters elastically off atomic electron, scattered electron is ejected from atom

  dominant for 10s of keV to a few Mev

  $\Delta\lambda = \frac{h}{mc}(1 - \cos(\theta))$

- pair production

  If $E_\gamma > 2m_ec^2$, dominant process for tens of MeV

  photon produces electron-positron pair

- decay channels compound like $\tau^{-1} = \tau_1^{-1} + \tau_2^{-1}$
- 3-level ΔE_{01} small, 4-level ΔE_{01} large
- solid-state laser

  crystal, glass, transitions between atomic energy levels Nd:YAG, YAG E-field splits Nd

- collisional gas laser

  gas excited by KE from collisions, light filtered with conducting cavity

- molecular gas laser

  vibrational energy levels

- dye laser

  organic dye dissolved in water, electron transport chain

- semiconductor (dipole) laser

  excites semiconductor conduction band energy due to e-hole annihilation

- free electron laser

  accelerate with E-field

- michelson-morley interferometer $d\sin\theta = m\lambda$

- a confidence interval is a range of values for a parameter (typically the mean), associated to a confidence level. The confidence level gives the reliability of the estimation procedure, not to be confused with the long run probability of measuring the parameter within the interval. The upper limit on the parameter is the value at which the statement doesn't hold that the long run probability of measuring the parameter within the interval.

fermi gas

misc misc.

$$E = \frac{\hbar^2}{2m}(3\frac{Nd}{V}\pi^2)^{2/3}$$
$$P = \frac{(3\pi^2)^{2/3}}{5m}(\frac{Nd}{V})^{5/3}$$
$$T = \frac{E}{k_B}$$
$$p = (2mE)^{1/2}$$

$$z_{\text{redshift}} = \frac{\lambda_{\text{observed}-\lambda_{\text{emitted}}}}{\lambda\text{emitted}}$$

- p-type junction: electron holes, positive net charge
- n-type junction: no electron holes, negative net charge