

**kinematic equations**

**uniform circular motion**

**kinetic and potential energy**

$$\bar{v} = \frac{1}{2}(v_i + v_f)$$

$$\Delta v = a\Delta t + v_i$$

$$x(t) = \frac{1}{2}at^2 + v_it + x_i$$

$$v_f^2 - v_i^2 = 2a\Delta x$$

$$a = \frac{v^2}{r}$$

$$E_{\text{trans}} = \frac{1}{2}mv^2$$

$$E_{\text{rot}} = \frac{1}{2}I\omega^2$$

$$E_{\text{gpe}} = mgh$$

$$E_{\text{sho}} = \frac{1}{2}kx^2$$

potential energy, force, work

gravitational force

angular momentum

$$dU = -\vec{F}d\vec{l}$$

$$\vec{F} = -\vec{\nabla}U$$

$$dW = \vec{F} \cdot d\vec{l}$$

$$\vec{F} = -Gm_1m_2r^{-2}r\hat{r}_{12}$$

$$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$$

**torque**

**constant angular velocity**

**spherical elements**

$$\tau = \vec{r} \times \vec{F} = \frac{d\vec{L}}{dt}$$

$$\vec{F} = -m\Omega^2 r \hat{r}_{\text{centrifuge}} - 2m\vec{\Omega} \times \vec{v}$$

$$\begin{aligned} d\vec{l} &= dr\hat{r} + r d\theta\hat{\theta} + r \sin\theta d\phi\hat{\phi} \\ d\vec{S} &= r^2 \sin\theta d\theta d\phi\hat{r} + r \sin\theta dr d\phi\hat{\theta} + r dr d\theta\hat{\phi} \\ dV &= r^2 \sin\theta dr d\theta d\phi \end{aligned}$$

**moment of inertia**

**Lagrangian**

**Hamiltonian**

$$I = \int s^2 dm$$

$$I = I_{\text{CM}} + Ms^2$$

$$I_z = I_x + I_y \text{ for } z \text{ perpendicular to the body lying in the } x - y \text{ plane}$$

$$L(\dot{q}_i, q_i, t) = T - U$$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} = \frac{\partial L}{\partial q_i}$$

$$p_i = \frac{\partial L}{\partial \dot{q}_i}$$

$$H(p, q) = \sum_i p_i \dot{q}_i - L$$

$$H = T + U \text{ if } U \text{ is independent of } \dot{q} \text{ and } t$$

$$\dot{p} = -\frac{\partial H}{\partial q}$$

$$\dot{q} = \frac{\partial H}{\partial p}$$



**effective orbital potential**

**Kepler's laws**

**damped oscillation**

$$E = T + V = \frac{1}{2}m\dot{r}^2 + \frac{l^2}{2mr^2} + U(r)$$

$$l = mr^2\dot{\phi}$$

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$E > 0 \implies \text{hyperbolic}$$

$$E = 0 \implies \text{parabolic}$$

$$E < 0 \implies \text{elliptical}$$

- Planets move in elliptical orbits with one focus at the sun
- Equal orbital area sweeps out equal orbital time
- $T = ka^{\frac{3}{2}}$  for all planets

$$0 = m\ddot{x} + b\dot{x} + kx$$

$$\beta = \frac{b}{2m}$$

$$\omega_0 = \left(\frac{k}{m}\right)^{\frac{1}{2}}$$

- $\beta^2 > \omega_0^2$ : overdamped decays exponentially
- $\beta^2 = \omega_0^2$ : critically damped
- $\beta^2 < \omega_0^2$ : underdamped decays exponentially, modulated by a sinusoid  
at  $\omega_1^2 = \omega_0^2 - \beta^2$

**driven oscillation**

**oscillators**

**fluid dynamics**

$$\omega_{\text{R}} = (\omega_0^2 - 2\beta^2)^{\frac{1}{2}}$$

$$D \propto |\omega_0^2 - \omega^2|^{-1} \text{ for undamped, driven oscillator}$$

- pendulum  $\omega = (\frac{g}{l})^{\frac{1}{2}}$
- physical pendulum  $\omega = (\frac{mgR}{I})^{\frac{1}{2}}$

$$\frac{v^2}{2} + gz + \frac{p}{\rho} = \text{const}$$

$p$  pressure,  $\rho$  fluid density,  $z$  height of a point along streamline

$$F = \rho V g$$

**Maxwell's equations**

**electrostatics**

**boundary conditions**

$$\begin{aligned}
\vec{\nabla} \cdot \vec{E} &= \frac{\rho}{\epsilon_0} \\
\vec{\nabla} \times \vec{E} &= \frac{-\partial \vec{B}}{\partial t} \\
\vec{\nabla} \cdot \vec{B} &= 0 \\
\vec{\nabla} \times \vec{B} &= \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \\
\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{J} &= 0 \\
\vec{E} &= -\vec{\nabla} \phi - \frac{\partial \vec{A}}{\partial t} \\
\vec{B} &= \vec{\nabla} \times \vec{A}
\end{aligned}$$

$$\begin{aligned}
\vec{E} &= -\vec{\nabla} \phi \implies \phi = - \int_a^b \vec{E} \cdot d\vec{l} \\
\nabla^2 \phi &= -\frac{\rho}{\epsilon_0} \implies \phi = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3\vec{r}' \\
\int_S \vec{E}(\vec{r}) \cdot d\vec{S} &= \frac{Q_{\text{enc}}}{\epsilon_0} \\
\int_C \vec{E}(\vec{r}) \cdot d\vec{l} &= 0
\end{aligned}$$

### Electrostatics

$$\begin{aligned}
E_{\text{out}}^{\parallel} - E_{\text{in}}^{\parallel} &= 0 \\
E_{\text{out}}^{\perp} - E_{\text{in}}^{\perp} &= \frac{\sigma}{\epsilon_0} \\
\phi &\text{is continuous} \\
\partial\phi &\text{continuous when no surface charge is present}
\end{aligned}$$

### Magnetostatics

$$\begin{aligned}
B_{\text{out}}^{\parallel} - B_{\text{in}}^{\parallel} &= \mu_0 \vec{k} \times \hat{n} B_{\text{out}}^{\perp} - B_{\text{in}}^{\perp} = 0 \\
\vec{k} &\text{ is the surface current density}
\end{aligned}$$

**conductors**

**work and energy in (electro/magneto)statics**

**capacitors**

$\phi = \text{const. throughout conductor}$

$\implies$  E-field inside is zero

$\implies Q_{\text{inside}} = 0$

$\implies Q_{\text{net}}$  confined to surface

$\implies$  E-field outside is perpendicular to surface

$$W = \frac{1}{2} \int \rho \phi d^3r$$

$$U_E = \frac{\epsilon_0}{2} \int |\vec{E}|^2 d^3r$$

$$U_B = \frac{1}{2\mu_0} \int |\vec{B}|^2 d^3r$$

$$Q = C\phi$$

$$C_{\text{parallel plate}} = \frac{\epsilon_0 A}{d}$$

$$U = \frac{1}{2} Q^2 C^{-1} = \frac{1}{2} C \phi^2$$



**magnetostatics**

**magnetostatic fields**

**cyclotron**

$$\int_S \vec{B} \cdot d\vec{s} = 0$$

$$\int_C \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$$

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

$$\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l} \times \vec{r}'}{|\vec{r}'|^3}$$

$$\vec{B} = \mu_0 \frac{N}{L} I \hat{z} \quad (\text{solenoid})$$

$$\vec{B} = \frac{\mu_0 N I}{2\pi r} \hat{\phi}$$

There is no field outside a toroid or solenoid.

$$\vec{F} = q\vec{v} \times \vec{B} = qvB\hat{x}$$

$$R = \frac{mv}{qB}$$

**inductance**

**inductance configurations**

**multipoles**

$$\mathcal{E} = \int_C \vec{E} \cdot d\vec{l} = \frac{-\partial\Phi_B}{\partial t}$$

$$\Phi_{21} = M_{12}I_1$$

$$\Phi_B = LI$$

Induced currents oppose changes in magnetic flux

$$L = \frac{\mu_0 N^2 A}{l} \quad \text{solenoid}$$

$$\vec{p} = \int \vec{r} \rho(\vec{r}) d^3r$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot \vec{r}}{|\vec{r}|^3}$$

$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$U = -\vec{p} \cdot \vec{E}$$

$$\vec{m} = I\vec{A}$$

$$\vec{\tau} = \vec{m} \times \vec{B}$$

$$U = -\vec{m} \cdot \vec{B}$$

**em in media**

**em waves**

**radiation**

$$\begin{aligned}\sigma_b &= \vec{P} \cdot \hat{n} \\ \rho_b &= -\vec{\nabla} \cdot \vec{P} \\ \epsilon &= \kappa \epsilon_0\end{aligned}$$

$$\begin{aligned}\vec{E}(\vec{r}) &= E_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} \hat{n} \\ \vec{B}(\vec{r}) &= B_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} (\hat{k} \times \hat{n}) \\ \vec{S} &= \frac{1}{\mu_0} (\vec{E} \times \vec{B}) = \frac{1}{2\mu_0} \text{Re}[\vec{E} \times \vec{B}^*] \\ I = \langle S \rangle &= \frac{c\epsilon_0}{2} E_0^2\end{aligned}$$

$$\begin{aligned}P &= \frac{\mu_0 q^2 a^2}{6\pi c} \quad \text{accelerating point charge} \\ \langle S \rangle &= \frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c} \frac{\sin^2(\theta)}{r^2} \\ \langle P_E \rangle &= \frac{\mu_0 p_0^2 \omega^4}{12\pi c} \\ \langle P_B \rangle &= \frac{\mu_0 m_0^2 \omega^4}{12\pi c^3}\end{aligned}$$

The bottom three equations are for a radiating dipole.

**circuits**

$$V_R = IR$$

$$V_C = \frac{Q}{C}$$

$$V_L = \frac{L}{\ddot{Q}}$$

$$R_{\text{series}} = R_i$$

$$\frac{1}{C_{\text{series}}} = \frac{1}{C_i}$$

$$L_{\text{series}} = L_i$$

$$\frac{1}{R_{\text{parallel}}} = \frac{1}{R_i}$$

$$C_{\text{parallel}} = C_i$$

$$\frac{1}{L_{\text{parallel}}} = \frac{1}{L_i}$$

$$R = \frac{\rho l}{A}$$

$$P = IV$$

$$U_C = \frac{1}{2}CV^2$$

$$U_L = \frac{1}{2}LI^2$$

$$I = \frac{V}{R}(1 - e^{-\frac{t}{L/R}}) \quad \text{RL circuit}$$

$$V = V_0 e^{-\frac{t}{RC}} \quad \text{RC circuit}$$

$$\omega_{RLC} = (LC)^{-\frac{1}{2}}$$

- circuit elements in series see same current
- circuit elements in parallel see same voltage
- current in/out of a node is conserved
- voltage is conserved over elements around a closed loop