kinematic equations

uniform circular motion

kintec and potential energy

$$
\begin{aligned}
\bar{v} &= \frac{1}{2}(v_i + v_f) \\
\Delta v &= a\Delta t + v_i \\
x(t) &= \frac{1}{2}at^2 + v_i t + x_i \\
v_f^2 - v_i^2 &= 2a\Delta x
\end{aligned}
$$

$$
a = \frac{v^2}{r}
$$

$$
\begin{aligned}
E_{\text{trans}} &= \frac{1}{2}mv^2 \\
E_{\text{rot}} &= \frac{1}{2}I\omega^2 \\
E_{\text{gpe}} &= mgh \\
E_{\text{sho}} &= \frac{1}{2}kx^2
\end{aligned}
$$

potential energy, force, work

gravitational force

angular momentum

$$dU = -\vec{F} d\vec{l}$$
$$\vec{F} = -\vec{\nabla} U$$
$$dW = \vec{F} \cdot d\vec{l}$$

$$\vec{F} = -G m_1 m_2 r^{-2} \hat{r_{12}}$$

$$\vec{L} = \vec{r} \times \vec{p} = I \vec{\omega}$$

torque

constant angular velocity

spherical elements

$$\tau = \vec{r} \times \vec{F} = \frac{d\vec{L}}{dt}$$

$$\vec{F} = -m\Omega^2 r\hat{r}_{\text{centrifuge}} - 2m\vec{\Omega} \times \vec{v}$$

$$d\vec{l} = dr\hat{r} + rd\theta\hat{\theta} + r\sin\theta d\phi\hat{\phi}$$
$$d\vec{S} = r^2 \sin\theta d\theta d\phi\hat{r} + r\sin\theta dr d\phi\hat{\theta} + r dr d\theta\hat{\phi}$$
$$dV = r^2 \sin\theta dr d\theta d\phi$$

moment of inertia

Lagrangian

Hamiltonian

$$
\begin{aligned}
I &= \int s^2 dm \\
I &= I_{\text{CM}} + Ms^2 \\
I_z &= I_x + I_y \text{ for } z \text{ perpendicular to the body lying in the } x-y \text{ plane}
\end{aligned}
$$

$$
\begin{aligned}
L(\dot{q}_i, q_i, t) &= T - U \\
\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} &= \frac{\partial L}{\partial q_i} \\
p_i &= \frac{\partial L}{\partial \dot{q}_i}
\end{aligned}
$$

$$
\begin{aligned}
H(p,q) &= \sum_i p_i \dot{q}_i - L \\
H &= T + U \text{ if } U \text{ is independent of } \dot{q} \text{ and } t \\
\dot{p} &= -\frac{\partial H}{\partial q} \\
\dot{q} &= \frac{\partial H}{\partial p}
\end{aligned}
$$

effective orbital potential

Kepler's laws

damped oscillation

$$
\begin{aligned}
E = T + V &= \frac{1}{2}m\dot{r}^2 + \frac{l^2}{2mr^2} + U(r) \\
l &= mr^2\dot{\phi} \\
\mu &= \frac{m_1 m_2}{m_1 + m_2} \\
E > 0 &\implies \text{hyperbolic} \\
E = 0 &\implies \text{parabolic} \\
E < 0 &\implies \text{elliptical}
\end{aligned}
$$

- Planets move in elliptical orbits with one focus at the sun
- Equal orbital area sweeps out equal orbital time
- $T = ka^{\frac{3}{2}}$ for all planets

$$
\begin{aligned}
0 &= m\ddot{x} + b\dot{x} + kx \\
\beta &= \frac{b}{2m} \\
\omega_0 &= (\frac{k}{m})^{\frac{1}{2}}
\end{aligned}
$$

- $\beta^2 > \omega_0^2$: overdamped decays exponentially
- $\beta^2 = \omega_0^2$: critically damed
- $\beta^2 < \omega_0^2$: underdamped decays exponentially, modulated by a sinusoid at $\omega_1^2 = \omega_0^2 - \beta^2$

driven oscillation

oscillators

fluid dynamics

$$\omega_{\mathrm{R}} = (\omega_0^2 - 2\beta^2)^{\frac{1}{2}}$$

$D \propto |\omega_0^2 - \omega^2|^{-1}$ for undamped, driven oscillator

- pendulum $\omega = (\frac{g}{l})^{\frac{1}{2}}$
- physical pendulum $\omega = (\frac{mgR}{I})^{\frac{1}{2}}$

$$\frac{v^2}{2} + gz + \frac{p}{\rho} = \text{const}$$

p pressure, ρ fluid density, z height of a point along streamline

$$F = \rho V g$$

Maxwell's equations

electrostatics

boundary conditions

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$
$$\vec{\nabla} \times \vec{E} = \frac{-\partial \vec{B}}{\partial t}$$
$$\vec{\nabla} \cdot \vec{B} = 0$$
$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$
$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{J} = 0$$
$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t}$$
$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\vec{E} = -\vec{\nabla}\phi \implies \phi = -\int_a^b \vec{E} \cdot d\vec{l}$$
$$\nabla^2 \phi = -\frac{\rho}{\epsilon_0} \implies \phi = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3\vec{r}'$$
$$\int_S \vec{E}(\vec{r}) \cdot d\vec{S} = \frac{Q_{\text{enc}}}{\epsilon_0}$$
$$\int_C \vec{E}(\vec{r}) \cdot d\vec{l} = 0$$

Electrostatics

$$E_{\text{out}}^{\parallel} - E_{\text{in}}^{\parallel} = 0$$
$$E_{\text{out}}^{\perp} - E_{\text{in}}^{\perp} = \frac{\sigma}{\epsilon_0}$$

$\phi$is continuous

$\partial\phi$continuous when no surface charge is present

Magnetostatics

$$B_{\text{out}}^{\parallel} - B_{\text{in}}^{\parallel} = \mu_0 \vec{k} \times \hat{n} B_{\text{out}}^{\perp} - B_{\text{in}}^{\perp} \qquad = 0$$

$\vec{k}$ is the surface current density

conductors

work and energy in (electro/magneto)statics

capacitors

$\phi = \text{const. throughout conductor}$

$\implies$ E-field inside is zero

$\implies$ $Q_{\text{inside}} = 0$

$\implies$ Q_{net}confined to surface

$\implies$ E-field outside is perpendicular to surface

$$W = \frac{1}{2}\int \rho\phi d^3r$$

$$U_E = \frac{\epsilon_0}{2}\int |\vec{E}|^2 d^3r$$

$$U_B = \frac{1}{2\mu_0}\int |\vec{B}|^2 d^3r$$

$$Q = C\phi$$

$$C_{\text{parallel plate}} = \frac{\epsilon_0 A}{d}$$

$$U = \frac{1}{2}Q^2C^{-1} = \frac{1}{2}C\phi^2$$

magnetostatics

magnetostatic fields

cyclotron

$$\int_S \vec{B} \cdot d\vec{s} = 0$$
$$\int_C \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$$
$$\vec{F_B} = q\vec{v} \times \vec{B}$$
$$\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l} \times \vec{r'}}{\left|\vec{r'}\right|^3}$$

$$\vec{B} = \mu_0 \frac{N}{L} I \hat{z} \quad \text{(solenoid)}$$
$$\vec{B} = \frac{\mu_0 N I}{2\pi r} \hat{\phi}$$

There is no field outside a toroid or solenoid.

$$\vec{F} = q\vec{v} \times \vec{B} = qvB\hat{x}$$
$$R = \frac{mv}{qB}$$

inductance

inductance configurations

multipoles

$$\mathcal{E} = \int_C \vec{E} \cdot d\vec{l} = \frac{-\partial \Phi_B}{\partial t}$$
$$\Phi_{21} = M_{12} I_1$$
$$\Phi_B = LI$$

Induced curents oppose changes in magnetic flux

$$L = \frac{\mu_{0N^2A}}{l} \quad \text{solenoid}$$

$$\vec{p} = \int \vec{r} \rho(\vec{r}) d^3 r$$
$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot \vec{r}}{|\vec{r}|^3}$$
$$\tau = \vec{p} \times \vec{E}$$
$$U = -\vec{p} \cdot \vec{E}$$
$$\vec{m} = I\vec{A}$$
$$\tau = \vec{m} \times \vec{B}$$
$$U = -\vec{m} \times \vec{B}$$

em in media

em waves

radiation

$$\sigma_b = \vec{P} \cdot \hat{n}$$
$$\rho_b = -\vec{\nabla} \cdot \vec{P}$$
$$\epsilon = \kappa \epsilon_0$$

$$\vec{E}(\vec{r}) = E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}\hat{n}$$
$$\vec{B}(\vec{r}) = B_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}(\hat{k} \times \hat{n})$$
$$\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B}) = \frac{1}{2\mu_0}\mathrm{Re}[\vec{E} \times \vec{B}^*]$$
$$I = \langle S \rangle = \frac{c\epsilon_0}{2} E_0^2$$

$$P = \frac{\mu_0 q^2 a^2}{6\pi c} \quad \text{accelerating point charge}$$
$$\langle S \rangle = \frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c} \frac{\sin^2(\theta)}{r^2}$$
$$\langle P_E \rangle = \frac{\mu_0 p_0^2 \omega^4}{12\pi c}$$
$$\langle P_B \rangle = \frac{\mu_0 m_0^2 \omega^4}{12\pi c^3}$$

The bottom three equations are for a radiating dipole.

circuits

$$V_R = IR$$
$$V_C = \frac{Q}{C}$$
$$V_L = \frac{L}{\ddot{Q}}$$
$$R_{\text{series}} = R_i$$
$$\frac{1}{C_{\text{series}}} = \frac{1}{C_i}$$
$$L_{\text{series}} = L_i$$
$$\frac{1}{R_{\text{parallel}}} = \frac{1}{R_i}$$
$$C_{\text{parallel}} = C_i$$
$$\frac{1}{L_{\text{parallel}}} = \frac{1}{L_i}$$
$$R = \frac{\rho l}{A}$$
$$P = IV$$
$$U_C = \frac{1}{2}CV^2$$
$$U_L = \frac{1}{2}LI^2$$
$$I = \frac{V}{R}(1 - e^{-\frac{t}{L/R}}) \quad \text{RL circuit}$$
$$V = V_0 e^{-\frac{t}{RC}} \quad \text{RC circuit}$$
$$\omega_{RLC} = (LC)^{-\frac{1}{2}}$$

- circuit elements in series see same current
- circuit elements in parallel see same voltage
- current in/out of a node is conserved
- voltage is conserved over elements around a closed loop