kinematic equations

uniform circular motion

kintec and potential energy

$$
\begin{aligned}
\bar{v} &= \frac{1}{2}(v_i + v_f) \\
\Delta v &= a\Delta t + v_i \\
x(t) &= \frac{1}{2}at^2 + v_i t + x_i \\
v_f^2 - v_i^2 &= 2a\Delta x
\end{aligned}
$$

$$
a = \frac{v^2}{r}
$$

$$
\begin{aligned}
E_{\text{trans}} &= \frac{1}{2}mv^2 \\
E_{\text{rot}} &= \frac{1}{2}I\omega^2 \\
E_{\text{gpe}} &= mgh \\
E_{\text{sho}} &= \frac{1}{2}kx^2
\end{aligned}
$$

potential energy, force, work

gravitational force

angular momentum

$$dU = -\vec{F}d\vec{l}$$

$$\vec{F} = -\vec{\nabla}U$$

$$dW = \vec{F} \cdot d\vec{l}$$

$$\vec{F} = -Gm_1m_2r^{-2}\hat{r_{12}}$$

$$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$$

torque

constant angular velocity

spherical elements

$$\tau = \vec{r} \times \vec{F} = \frac{d\vec{L}}{dt}$$

$$\vec{F} = -m\Omega^2 r\hat{r}_{\text{centrifuge}} - 2m\vec{\Omega} \times \vec{v}$$

$$\begin{aligned}
d\vec{l} &= dr\hat{r} + rd\theta\hat{\theta} + r\sin\theta d\phi\hat{\phi} \\
d\vec{S} &= r^2 \sin\theta d\theta d\phi\hat{r} + r\sin\theta dr d\phi\hat{\theta} + r dr d\theta\hat{\phi} \\
dV &= r^2 \sin\theta dr d\theta d\phi
\end{aligned}$$

moment of inertia

Lagrangian

Hamiltonian

$$I = \int s^2 dm$$
$$I = I_{\text{CM}} + Ms^2$$
$I_z = I_x + I_y$ for z perpendicular to the body lying in the $x-y$ plane

$$L(\dot{q}_i, q_i, t) = T - U$$
$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} = \frac{\partial L}{\partial q_i}$$
$$p_i = \frac{\partial L}{\partial \dot{q}_i}$$

$$H(p,q) = \sum_i p_i \dot{q}_i - L$$
$H = T + U$ if U is independent of $\dot{q}$ and t
$$\dot{p} = -\frac{\partial H}{\partial q}$$
$$\dot{q} = \frac{\partial H}{\partial p}$$

effective orbital potential

Kepler's laws

damped oscillation

$$E = T + V = \frac{1}{2}m\dot{r}^2 + \frac{l^2}{2mr^2} + U(r)$$
$$l = mr^2\dot{\phi}$$
$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$
$$E > 0 \implies \text{hyperbolic}$$
$$E = 0 \implies \text{parabolic}$$
$$E < 0 \implies \text{elliptical}$$

- Planets move in elliptical orbits with one focus at the sun
- Equal orbital area sweeps out equal orbital time
- $T = ka^{\frac{3}{2}}$ for all planets

$$0 = m\ddot{x} + b\dot{x} + kx$$
$$\beta = \frac{b}{2m}$$
$$\omega_0 = (\frac{k}{m})^{\frac{1}{2}}$$

- $\beta^2 > \omega_0^2$: overdamped decays exponentially
- $\beta^2 = \omega_0^2$: critically damed
- $\beta^2 < \omega_0^2$: underdamped decays exponentially, modulated by a sinusoid at $\omega_1^2 = \omega_0^2 - \beta^2$

driven oscillation

oscillators

fluid dynamics

$\omega_{\mathrm{R}} = (\omega_0^2 - 2\beta^2)^{\frac{1}{2}}$

$D \propto |\omega_0^2 - \omega^2|^{-1}$ for undamped, driven oscillator

- pendulum $\omega = (\frac{g}{l})^{\frac{1}{2}}$
- physical pendulum $\omega = (\frac{mgR}{I})^{\frac{1}{2}}$

$$\frac{v^2}{2} + gz + \frac{p}{\rho} = \mathrm{const}$$

p pressure, ρ fluid density, z height of a point along streamline

$F = \rho V g$

Maxwell's equations

electrostatics

boundary conditions

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$
$$\vec{\nabla} \times \vec{E} = \frac{-\partial \vec{B}}{\partial t}$$
$$\vec{\nabla} \cdot \vec{B} = 0$$
$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$
$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{J} = 0$$
$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t}$$
$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\vec{E} = -\vec{\nabla}\phi \implies \phi = -\int_a^b \vec{E} \cdot d\vec{l}$$
$$\nabla^2 \phi = -\frac{\rho}{\epsilon_0} \implies \phi = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3\vec{r}'$$
$$\int_S \vec{E}(\vec{r}) \cdot d\vec{S} = \frac{Q_{\text{enc}}}{\epsilon_0}$$
$$\int_C \vec{E}(\vec{r}) \cdot d\vec{l} = 0$$

Electrostatics

$$E^{\parallel}_{\text{out}} - E^{\parallel}_{\text{in}} = 0$$
$$E^{\perp}_{\text{out}} - E^{\perp}_{\text{in}} = \frac{\sigma}{\epsilon_0}$$

$\phi$is continuous

$\partial\phi$continuous when no surface charge is present

Magnetostatics

$$B^{\parallel}_{\text{out}} - B^{\parallel}_{\text{in}} = \mu_0 \vec{k} \times \hat{n} B^{\perp}_{\text{out}} - B^{\perp}_{\text{in}} \qquad = 0$$

$\vec{k}$ is the surface current density

conductors

work and energy in (electro/magneto)statics

capacitors

$\phi = $ const. throughout conductor
$\implies$ E-field inside is zero
$\implies$ $Q_{\text{inside}} = 0$
$\implies$ Q_{net}confined to surface
$\implies$ E-field outside is perpendicular to surface

$$W = \frac{1}{2}\int \rho\phi d^3r$$
$$U_E = \frac{\epsilon_0}{2}\int |\vec{E}|^2 d^3r$$
$$U_B = \frac{1}{2\mu_0}\int |\vec{B}|^2 d^3r$$

$$Q = C\phi$$
$$C_{\text{parallel plate}} = \frac{\epsilon_0 A}{d}$$
$$U = \frac{1}{2}Q^2C^{-1} = \frac{1}{2}C\phi^2$$

magnetostatics

magnetostatic fields

cyclotron

$$\int_S \vec{B} \cdot d\vec{s} = 0$$

$$\int_C \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$$

$$\vec{F_B} = q\vec{v} \times \vec{B}$$

$$\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l} \times \vec{r'}}{|\vec{r'}|^3}$$

$$\vec{B} = \mu_0 \frac{N}{L} I \hat{z} \quad \text{(solenoid)}$$

$$\vec{B} = \frac{\mu_0 N I}{2\pi r} \hat{\phi}$$

There is no field outside a toroid or solenoid.

$$\vec{F} = q\vec{v} \times \vec{B} = qvB\hat{x}$$

$$R = \frac{mv}{qB}$$

inductance

inductance configurations

multipoles

$$
\begin{aligned}
\mathcal{E} &= \int_C \vec{E} \cdot d\vec{l} = \frac{-\partial \Phi_B}{\partial t} \\
\Phi_{21} &= M_{12} I_1 \\
\Phi_B &= LI
\end{aligned}
$$

Induced curents oppose changes in magnetic flux

$$
L = \frac{\mu_{0N^2A}}{l} \quad \text{solenoid}
$$

$$
\begin{aligned}
\vec{p} &= \int \vec{r} \rho(\vec{r}) d^3 r \\
V(\vec{r}) &= \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot \vec{r}}{|\vec{r}|^3} \\
\tau &= \vec{p} \times \vec{E} \\
U &= -\vec{p} \cdot \vec{E} \\
\vec{m} &= I\vec{A} \\
\tau &= \vec{m} \times \vec{B} \\
U &= -\vec{m} \times \vec{B}
\end{aligned}
$$

em in media

em waves

radiation

$$\sigma_b = \vec{P} \cdot \hat{n}$$
$$\rho_b = -\vec{\nabla} \cdot \vec{P}$$
$$\epsilon = \kappa \epsilon_0$$

$$\vec{E}(\vec{r}) = E_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)} \hat{n}$$
$$\vec{B}(\vec{r}) = B_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)} (\hat{k} \times \hat{n})$$
$$\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B}) = \frac{1}{2\mu_0}\mathrm{Re}[\vec{E} \times \vec{B}^*]$$
$$I = \langle S \rangle = \frac{c\epsilon_0}{2} E_0^2$$

$$P = \frac{\mu_0 q^2 a^2}{6\pi c} \quad \text{accelerating point charge}$$
$$\langle S \rangle = \frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c} \frac{\sin^2(\theta)}{r^2}$$
$$\langle P_E \rangle = \frac{\mu_0 p_0^2 \omega^4}{12\pi c}$$
$$\langle P_B \rangle = \frac{\mu_0 m_0^2 \omega^4}{12\pi c^3}$$

The bottom three equations are for a radiating dipole.

circuits

$$
\begin{aligned}
V_R &= IR \\
V_C &= \frac{Q}{C} \\
V_L &= \frac{L}{\ddot{Q}} \\
R_{\text{series}} &= R_i \\
\frac{1}{C_{\text{series}}} &= \frac{1}{C_i} \\
L_{\text{series}} &= L_i \\
\frac{1}{R_{\text{parallel}}} &= \frac{1}{R_i} \\
C_{\text{parallel}} &= C_i \\
\frac{1}{L_{\text{parallel}}} &= \frac{1}{L_i} \\
R &= \frac{\rho l}{A} \\
P &= IV \\
U_C &= \frac{1}{2}CV^2 \\
U_L &= \frac{1}{2}LI^2 \\
I &= \frac{V}{R}(1 - e^{-\frac{t}{L/R}}) \quad \text{RL circuit} \\
V &= V_0 e^{-\frac{t}{RC}} \quad \text{RC circuit} \\
\omega_{RLC} &= (LC)^{-\frac{1}{2}}
\end{aligned}
$$

- circuit elements in series see same current
- circuit elements in parallel see same voltage
- current in/out of a node is conserved
- voltage is conserved over elements around a closed loop