kinematic equations

uniform circular motion

kintec and potential energy

$$\begin{aligned}
\bar{v} &= \frac{1}{2}(v_i + v_f) \\
\Delta v &= a\Delta t + v_i \\
x(t) &= \frac{1}{2}at^2 + v_i t + x_i \\
v_f^2 - v_i^2 &= 2a\Delta x
\end{aligned}$$

$$a = \frac{v^2}{r}$$

$$\begin{aligned}
E_{\text{trans}} &= \frac{1}{2}mv^2 \\
E_{\text{rot}} &= \frac{1}{2}I\omega^2 \\
E_{\text{gpe}} &= mgh \\
E_{\text{sho}} &= \frac{1}{2}kx^2
\end{aligned}$$

potential energy, force, work

gravitational force

angular momentum

$$dU = -\vec{F} d\vec{l}$$

$$\vec{F} = -\vec{\nabla} U$$

$$dW = \vec{F} \cdot d\vec{l}$$

$$\vec{F} = -G m_1 m_2 r^{-2} \hat{r_{12}}$$

$$\vec{L} = \vec{r} \times \vec{p} = I \vec{\omega}$$

torque

constant angular velocity

spherical elements

$$\tau = \vec{r} \times \vec{F} = \frac{d\vec{L}}{dt}$$

$$\vec{F} = -m\Omega^2 r\hat{r}_{\text{centrifuge}} - 2m\vec{\Omega} \times \vec{v}$$

$$\begin{aligned}
d\vec{l} &= dr\hat{r} + rd\theta\hat{\theta} + r\sin\theta d\phi\hat{\phi} \\
d\vec{S} &= r^2\sin\theta d\theta d\phi\hat{r} + r\sin\theta drd\phi\hat{\theta} + rdrd\theta\hat{\phi} \\
dV &= r^2\sin\theta drd\theta d\phi
\end{aligned}$$

moment of inertia

Lagrangian

Hamiltonian

$$I = \int s^2 dm$$

$$I = I_{\mathrm{CM}} + M s^2$$

$I_z = I_x + I_y$ for z perpendicular to the body lying in the $x - y$ plane

$$L(\dot{q}_i, q_i, t) = T - U$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} = \frac{\partial L}{\partial q_i}$$

$$p_i = \frac{\partial L}{\partial \dot{q}_i}$$

$$H(p, q) = \sum_i p_i \dot{q}_i - L$$

$H = T + U$ if U is independent of $\dot{q}$ and t

$$\dot{p} = -\frac{\partial H}{\partial q}$$

$$\dot{q} = \frac{\partial H}{\partial p}$$

effective orbital potential

Kepler's laws

damped oscillation

$$E = T + V = \frac{1}{2}m\dot{r}^2 + \frac{l^2}{2mr^2} + U(r)$$
$$l = mr^2\dot{\phi}$$
$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$
$$E > 0 \implies \text{hyperbolic}$$
$$E = 0 \implies \text{parabolic}$$
$$E < 0 \implies \text{elliptical}$$

- Planets move in elliptical orbits with one focus at the sun
- Equal orbital area sweeps out equal orbital time
- $T = ka^{\frac{3}{2}}$ for all planets

$$0 = m\ddot{x} + b\dot{x} + kx$$
$$\beta = \frac{b}{2m}$$
$$\omega_0 = (\frac{k}{m})^{\frac{1}{2}}$$

- $\beta^2 > \omega_0^2$: overdamped decays exponentially
- $\beta^2 = \omega_0^2$: critically damed
- $\beta^2 < \omega_0^2$: underdamped decays exponentially, modulated by a sinusoid at $\omega_1^2 = \omega_0^2 - \beta^2$

driven oscillation

oscillators

fluid dynamics

$\omega_{\mathrm{R}} = (\omega_0^2 - 2\beta^2)^{\frac{1}{2}}$

$D \propto |\omega_0^2 - \omega^2|^{-1}$ for undamped, driven oscillator

- pendulum $\omega = (\frac{g}{l})^{\frac{1}{2}}$
- physical pendulum $\omega = (\frac{mgR}{I})^{\frac{1}{2}}$

$$\frac{v^2}{2} + gz + \frac{p}{\rho} = \text{const}$$

p pressure, ρ fluid density, z height of a point along streamline

$F = \rho V g$