kinematic equations

uniform circular motion

kintec and potential energy

$$
\begin{aligned}
\bar{v} &= \frac{1}{2}(v_i + v_f) \\
\Delta v &= a\Delta t + v_i \\
x(t) &= \frac{1}{2}at^2 + v_i t + x_i \\
v_f^2 - v_i^2 &= 2a\Delta x
\end{aligned}
$$

$$
a = \frac{v^2}{r}
$$

$$
\begin{aligned}
E_{\text{trans}} &= \frac{1}{2}mv^2 \\
E_{\text{rot}} &= \frac{1}{2}I\omega^2 \\
E_{\text{gpe}} &= mgh \\
E_{\text{sho}} &= \frac{1}{2}kx^2
\end{aligned}
$$

potential energy, force, work

gravitational force

angular momentum

$$dU = -\vec{F} d\vec{l}$$
$$\vec{F} = -\vec{\nabla} U$$
$$dW = \vec{F} \cdot d\vec{l}$$

$$\vec{F} = -G m_1 m_2 r^{-2} \hat{r_{12}}$$

$$\vec{L} = \vec{r} \times \vec{p} = I \vec{\omega}$$

torque

constant angular velocity

spherical elements

$$\tau = \vec{r} \times \vec{F} = \frac{d\vec{L}}{dt}$$

$$\vec{F} = -m\Omega^2 r\hat{r}_{\text{centrifuge}} - 2m\vec{\Omega} \times \vec{v}$$

$$\begin{aligned}
d\vec{l} &= dr\hat{r} + rd\theta\hat{\theta} + r\sin\theta d\phi\hat{\phi} \\
d\vec{S} &= r^2\sin\theta d\theta d\phi\hat{r} + r\sin\theta dr d\phi\hat{\theta} + r dr d\theta\hat{\phi} \\
dV &= r^2\sin\theta dr d\theta d\phi
\end{aligned}$$

moment of inertia

Lagrangian

Hamiltonian

$$I = \int s^2 dm$$
$$I = I_{\mathrm{CM}} + Ms^2$$
$I_z = I_x + I_y$ for z perpendicular to the body lying in the $x - y$ plane

$$L(\dot{q}_i, q_i, t) = T - U$$
$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} = \frac{\partial L}{\partial q_i}$$
$$p_i = \frac{\partial L}{\partial \dot{q}_i}$$

$$H(p, q) = \sum_i p_i \dot{q}_i - L$$
$H = T + U$ if U is independent of $\dot{q}$ and t
$$\dot{p} = -\frac{\partial H}{\partial q}$$
$$\dot{q} = \frac{\partial H}{\partial p}$$

effective orbital potential

Kepler's laws

damped oscillation

$$
\begin{aligned}
E = T + V &= \frac{1}{2}m\dot{r}^2 + \frac{l^2}{2mr^2} + U(r) \\
l &= mr^2\dot{\phi} \\
\mu &= \frac{m_1 m_2}{m_1 + m_2} \\
E > 0 &\implies \text{hyperbolic} \\
E = 0 &\implies \text{parabolic} \\
E < 0 &\implies \text{elliptical}
\end{aligned}
$$

- Planets move in elliptical orbits with one focus at the sun
- Equal orbital area sweeps out equal orbital time
- $T = ka^{\frac{3}{2}}$ for all planets

$$
\begin{aligned}
0 &= m\ddot{x} + b\dot{x} + kx \\
\beta &= \frac{b}{2m} \\
\omega_0 &= (\frac{k}{m})^{\frac{1}{2}}
\end{aligned}
$$

- $\beta^2 > \omega_0^2$: overdamped decays exponentially
- $\beta^2 = \omega_0^2$: critically damed
- $\beta^2 < \omega_0^2$: underdamped decays exponentially, modulated by a sinusoid at $\omega_1^2 = \omega_0^2 - \beta^2$

driven oscillation

oscillators

fluid dynamics

$\omega_{\mathrm{R}} = (\omega_0^2 - 2\beta^2)^{\frac{1}{2}}$

$D \propto |\omega_0^2 - \omega^2|^{-1}$ for undamped, driven oscillator

- pendulum $\omega = (\frac{g}{l})^{\frac{1}{2}}$
- physical pendulum $\omega = (\frac{mgR}{I})^{\frac{1}{2}}$

$$\frac{v^2}{2} + gz + \frac{p}{\rho} = \mathrm{const}$$

p pressure, ρ fluid density, z height of a point along streamline

$F = \rho V g$

Maxwell's equations

electrostatics

boundary conditions

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$
$$\vec{\nabla} \times \vec{E} = \frac{-\partial \vec{B}}{\partial t}$$
$$\vec{\nabla} \cdot \vec{B} = 0$$
$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$
$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{J} = 0$$
$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t}$$
$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\vec{E} = -\vec{\nabla}\phi \implies \phi = -\int_a^b \vec{E} \cdot d\vec{l}$$
$$\nabla^2 \phi = -\frac{\rho}{\epsilon_0} \implies \phi = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}^{\,\prime})}{|\vec{r} - \vec{r}^{\,\prime}|} d^3\vec{r}^{\,\prime}$$
$$\int_S \vec{E}(\vec{r}) \cdot d\vec{S} = \frac{Q_{\text{enc}}}{\epsilon_0}$$
$$\int_C \vec{E}(\vec{r}) \cdot d\vec{l} = 0$$

Electrostatics

$$E^{\parallel}_{\text{out}} - E^{\parallel}_{\text{in}} = 0$$
$$E^{\perp}_{\text{out}} - E^{\perp}_{\text{in}} = \frac{\sigma}{\epsilon_0}$$

$\phi$is continuous

$\partial\phi$continuous when no surface charge is present

Magnetostatics

$$B^{\parallel}_{\text{out}} - B^{\parallel}_{\text{in}} = \mu_0 \vec{k} \times \hat{n} B^{\perp}_{\text{out}} - B^{\perp}_{\text{in}} \qquad = 0$$

$\vec{k}$ is the surface current density

conductors

work and energy in (electro/magneto)statics

capacitors

$\phi =$ const. throughout conductor

$\implies$ E-field inside is zero

$\implies$ $Q_{\text{inside}} = 0$

$\implies$ Q_{net}confined to surface

$\implies$ E-field outside is perpendicular to surface

$$W = \frac{1}{2}\int \rho\phi d^3r$$
$$U_E = \frac{\epsilon_0}{2}\int |\vec{E}|^2 d^3r$$
$$U_B = \frac{1}{2\mu_0}\int |\vec{B}|^2 d^3r$$

$$Q = C\phi$$
$$C_{\text{parallel plate}} = \frac{\epsilon_0 A}{d}$$
$$U = \frac{1}{2}Q^2C^{-1} = \frac{1}{2}C\phi^2$$

magnetostatics

magnetostatic fields

cyclotron

$$
\begin{aligned}
\int_S \vec{B} \cdot d\vec{s} &= 0 \\
\int_C \vec{B} \cdot d\vec{l} &= \mu_0 I_{\text{enc}} \\
\vec{F_B} &= q\vec{v} \times \vec{B} \\
\vec{B}(\vec{r}) &= \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l} \times \vec{r'}}{\left|\vec{r'}\right|^3}
\end{aligned}
$$

$$
\begin{aligned}
\vec{B} &= \mu_0 \frac{N}{L} I \hat{z} \quad \text{(solenoid)} \\
\vec{B} &= \frac{\mu_0 N I}{2\pi r} \hat{\phi}
\end{aligned}
$$

There is no field outside a toroid or solenoid.

$$
\begin{aligned}
\vec{F} &= q\vec{v} \times \vec{B} = qvB\hat{x} \\
R &= \frac{mv}{qB}
\end{aligned}
$$

inductance

inductance configurations

multipoles

$$\mathcal{E} = \int_C \vec{E} \cdot d\vec{l} = \frac{-\partial \Phi_B}{\partial t}$$
$$\Phi_{21} = M_{12} I_1$$
$$\Phi_B = LI$$

Induced curents oppose changes in magnetic flux

$$L = \frac{\mu_{0N^2A}}{l} \quad \text{solenoid}$$

$$\vec{p} = \int \vec{r} \rho(\vec{r}) d^3 r$$
$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot \vec{r}}{|\vec{r}|^3}$$
$$\tau = \vec{p} \times \vec{E}$$
$$U = -\vec{p} \cdot \vec{E}$$
$$\vec{m} = I\vec{A}$$
$$\tau = \vec{m} \times \vec{B}$$
$$U = -\vec{m} \times \vec{B}$$

em in media

em waves

radiation

$$\sigma_b = \vec{P} \cdot \hat{n}$$
$$\rho_b = -\vec{\nabla} \cdot \vec{P}$$
$$\epsilon = \kappa \epsilon_0$$

$$\vec{E}(\vec{r}) = E_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} \hat{n}$$
$$\vec{B}(\vec{r}) = B_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)} (\hat{k} \times \hat{n})$$
$$\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B}) = \frac{1}{2\mu_0} \mathrm{Re}[\vec{E} \times \vec{B}^*]$$
$$I = \langle S \rangle = \frac{c\epsilon_0}{2} E_0^2$$

$$P = \frac{\mu_0 q^2 a^2}{6\pi c} \quad \text{accelerating point charge}$$
$$\langle S \rangle = \frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c} \frac{\sin^2(\theta)}{r^2}$$
$$\langle P_E \rangle = \frac{\mu_0 p_0^2 \omega^4}{12\pi c}$$
$$\langle P_B \rangle = \frac{\mu_0 m_0^2 \omega^4}{12\pi c^3}$$

The bottom three equations are for a radiating dipole.

circuits

wave equation

wave configurations

$$V_R = IR$$
$$V_C = \frac{Q}{C}$$
$$V_L = \frac{L}{\ddot{Q}}$$
$$R_{\text{series}} = R_i$$
$$\frac{1}{C_{\text{series}}} = \frac{1}{C_i}$$
$$L_{\text{series}} = L_i$$
$$\frac{1}{R_{\text{parallel}}} = \frac{1}{R_i}$$
$$C_{\text{parallel}} = C_i$$
$$\frac{1}{L_{\text{parallel}}} = \frac{1}{L_i}$$
$$R = \frac{\rho l}{A}$$
$$P = IV$$
$$U_C = \frac{1}{2} C V^2$$
$$U_L = \frac{1}{2} L I^2$$
$$I = \frac{V}{R}(1 - e^{-\frac{t}{L/R}}) \quad \text{RL circuit}$$
$$V = V_0 e^{-\frac{t}{RC}} \quad \text{RC circuit}$$
$$\omega_{RLC} = (LC)^{-\frac{1}{2}}$$

- circuit elements in series see same current
- circuit elements in parallel see same voltage
- current in/out of a node is conserved
- voltage is conserved over elements around a closed loop

$$\partial_t^2 \psi = v^2 \partial_x^2 \psi$$
$$\psi(\vec{r}, t) = A \cos(\vec{k} \cdot \vec{r} - \omega t + \phi)$$
$$= \text{Re}[A e^{i(\vec{k} \cdot \vec{r} - \omega t)}]$$
$$v_{\text{phase}} = \frac{\omega}{k}$$
$$v_{\text{group}} = \frac{\partial w}{\partial k}$$
$$\frac{\omega}{k} = \frac{c}{n} \quad \lambda \to \text{light}$$
$$\lambda = \frac{2\pi}{k}$$
$$T = \frac{2\pi}{\omega}$$
$$\omega = 2\pi f$$

$$v = (\frac{T}{\mu})^{\frac{1}{2}} \text{ string w/ tension } T \text{ and mass density } \mu$$
$$v = (\frac{\kappa}{\rho})^{\frac{1}{2}} \text{ sound w/ bulk modulus } \kappa \text{ and density } \rho$$

polarization

interference

geometric optics

$$I = I_0 \cos^2(\theta)$$

$$\theta_B = \arctan(\frac{n_2}{n_1})$$

$d\sin(\theta) = m\lambda$ max dub-slit

$d\sin(\theta) = (m + \frac{1}{2})\lambda$ min dub-slit

$a\sin(\theta) = m\lambda$ min single-slit

$D\sin(\theta) = 1.22\lambda$ first circular diffraction minimum$d\sin(\theta) = \frac{n\lambda}{2}$ x-rays on crystal maxima

$n_2 > n_1 \implies \pi$ phase shift

$n_2 < n_1 \implies 0$ phase shift

$$\theta_i = \theta_r n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$f = \frac{R}{2} \text{ spherical mirror}$$

$$\frac{1}{f} = (n-1)(\frac{1}{R_1} - \frac{1}{R_2})$$

$$m = \frac{-s'}{s}$$

misc optics

basic stat mech

entropy

$$I \propto I_0 \lambda^{-4} a^6$$
$$f = (\frac{v + v_r}{v - v_s}) f_0$$

$$p_i = \frac{e^{-\beta E_i}}{Z}$$
$$Z = \sum_j e^{-\beta E_j}$$
$$\beta = (k_B T)^{-1}$$
$$\langle O \rangle = \sum_i p_i O_i$$
$$\langle E \rangle = -\partial_\beta \mathrm{ln} Z$$
$$Z_N = \frac{1}{N! h^{3N}} \int e^{-\beta H(\vec{p}_1, \ldots \vec{p}_n; \vec{x}_1, \ldots \vec{x}_n)} d^3 p_1 \ldots d^3 p_n d^3 x_1 \ldots d^3 x_n$$
$$k_B = 1.380649e - 23 \mathrm{JK}^{-1}$$

$$S = k_B \mathrm{ln} \Omega$$
$$S = -k_B \sum_i p_i \mathrm{ln} p_i = \partial_T (k_B T \mathrm{ln} Z)$$
$$S = N k_B \mathrm{ln} \frac{V T^{3/2}}{N} + \text{const. (monoatomic ideal gas)}$$

equipartition theorem

misc combinatorics

thermodynamic laws

Each quadratic term (d.o.f.) in the Hamiltonian for a particle contributes $k_B T/2$ to the internal energy of a particle

$$(N, M) = \frac{N!}{(N-M)!M!}$$

$$\ln n! = n \ln n - n \ n \text{ large}$$

- Energy is neither created nor destoryed
  $\Delta U = Q - W$
- There does not exista process in which the sole effect is to transfer heat from a body at lower temperature to a body at higher temperature
  $\Delta S \geq \int \frac{\delta Q}{T}$
- Entropy is zero at absolute zero temperature. There is only one microstate at absolute zero temperature

ideal gas law

reversible process

quasistatic

$$PV = Nk_BT$$

- system is in equilibrium at every instant
- $\delta W = PdV$
- $\delta Q = TdS$

- in thermal equilibrium at every instant, but not necessarily reversible

adiabatic

isentropic / reversible adiabatic

IsoX

- $\delta Q = 0$

- reversible and adiabatic
- $\iff \Delta S = 0$
- $PV^{\gamma} = \text{const.}\, ; \gamma = \frac{C_P}{C_V}$

- X held constant

free expansion

thermodynamic identities

heat capacity

- $\Delta T = 0$
- irreversible
- $PV = P^{'}V^{'}$

$$dU = TdS - PdV$$
$$T = \frac{\partial U}{\partial S}|_V$$
$$P = -\frac{\partial U}{\partial V}|_S$$
$$\frac{\partial P}{\partial S}|_V = -\frac{\partial T}{\partial V}|_S$$

$$C_V = \frac{\partial Q}{\partial T}|_V = \frac{\partial U}{\partial T}$$
$$C_P = \frac{\partial Q}{\partial T}|_P$$
$$C_P - C_V = Nk_B \text{ (ideal gas)}$$
$$Q = mc\Delta T$$
$$c_{\text{water}} = 4.18 \text{ JK}^{-1}\text{g}^{-1}$$

model systems

monoatomic ideal gas

quantum stat mech

$$e = 1 - |\frac{Q_C}{Q_H}|$$

$$e = 1 - \frac{T_C}{T_H} \text{ theoretical max, carnot}$$

- use thermodynamic identities to calculate areas
- clockwise paths in P - V and T - S planes do positive work

$$Z_N = \frac{V^N}{N!h^{3N}}(2\pi m k_B T)^{3^N/2}$$

$$U = \frac{3}{2}N k_B T$$

$$v_{\text{rms}} = (\frac{3k_B T}{m})^{1/2}$$

$$K = \gamma P$$

$$F_{\text{FD}}(E_i) = \frac{1}{e^{(E_i-\mu)/k_B T} + 1}$$

$$F_{\text{BE}}(E_i) = \frac{1}{e^{(E_i-\mu)/k_B T} - 1}$$

$$\langle N \rangle = \sum_i g(E_i) F(E_i)$$