EDHEC PhD Finance 2022 - Econometrics Homework

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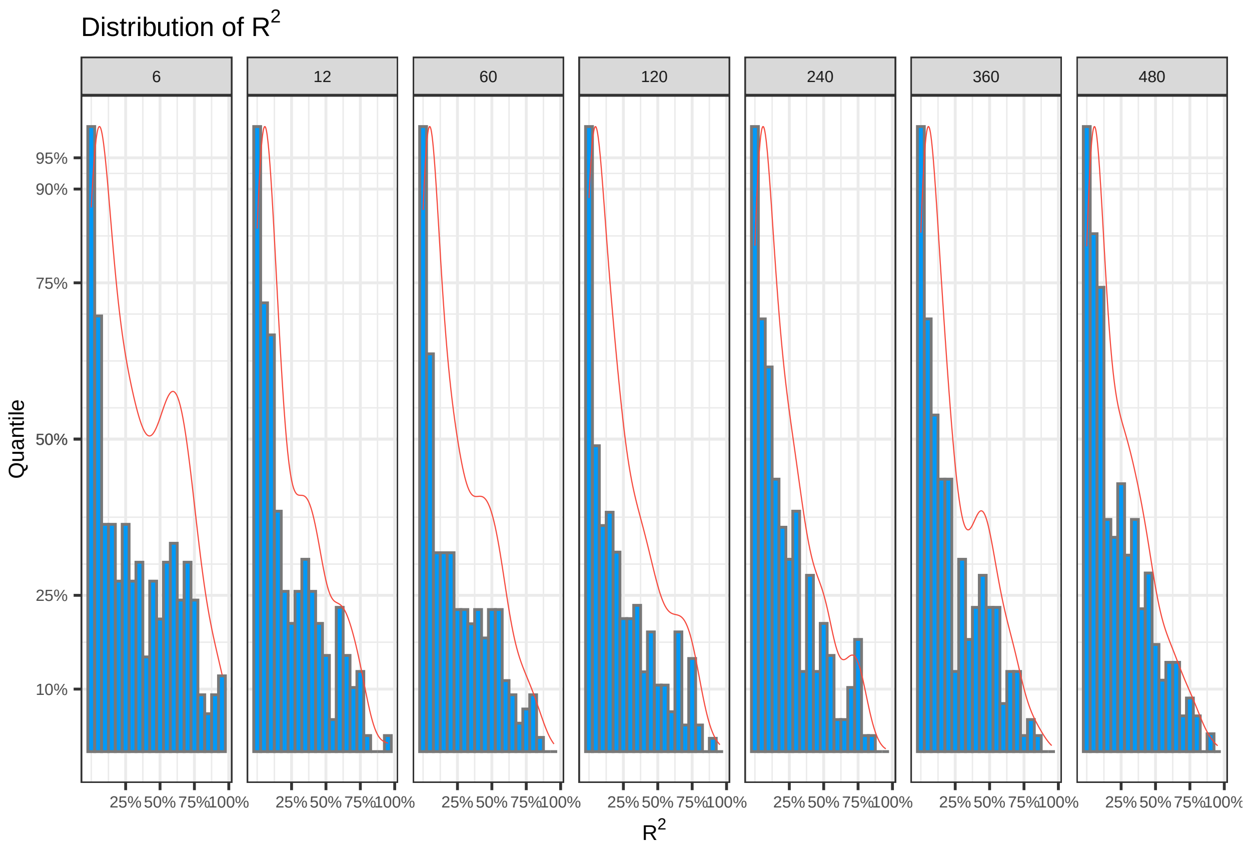
Question 1: Spurious Regressions (2 points)

1.a. [1 Point] Replicate the analysis leading to Figure 14.1 in Davidson MacKinnon (2005, book)

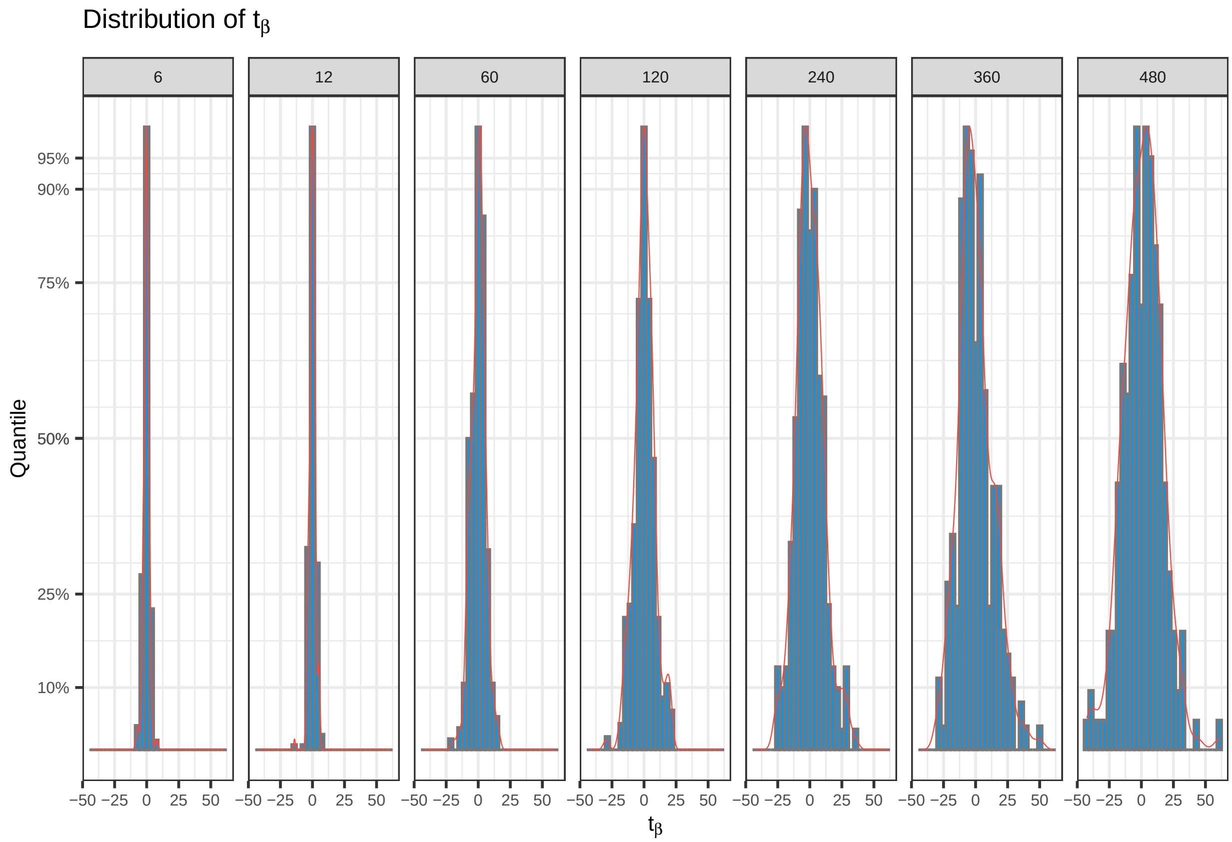
1.a.i

Compute also for each sample size T the distribution of the R^2 of the MC simulations with either 7 separate histograms, or one unique figure where you report on the y-axis the 5%, 10% 25%, 50%, 75%, 90% and 95% quantiles of the distributions of the simulated R^2, and on the x-axis you have T = 6, 12, 60, 120, 240, 360, 480.



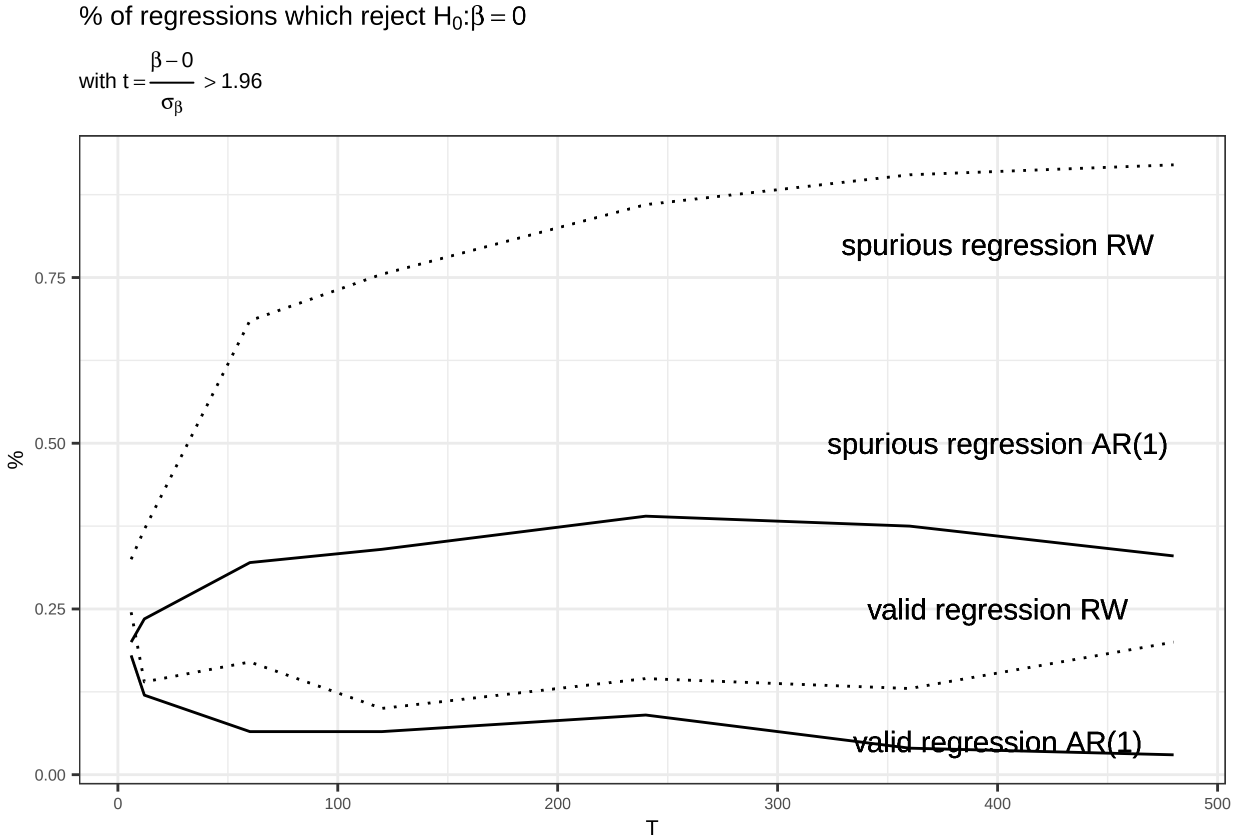
1.a.ii

Similarly (either with histograms, or with one plot of the quantiles) report the distributions of the estimates t-statistics for the test of the null H0 : $\beta_2 = 0$



1.a.iii

Compute the empirical rejection frequencies (that is the empirical size of the tests), which is exactly the figure 14.1 in Davidson MacKinnon (2005, book).



1.b [1 Point] Based on the results obtained by answering to point (a) summarize the problems of spurious regressions in econometrics.

Spurious regression as outlined in Davidson MacKinnon occurs for two reasons:

1. incorrectly specified H_0 and
2. standard asymptotic results do not hold whenever at least one of the regressors is I(1), even when a model is correctly specified

The $H_0 : \beta_2 = 0$ tested with the model (14.12)

$$y_t = \beta_1 + \beta_2 y_{t-1} + v_t$$

implies a DGP': $y_t = \beta_1 + v_t$, when y_t is actually generated using DGP (14.01) $y_t = y_{t-1} + v_t, y_0 = 0, v_t \sim iidN(0,1)$.

The wrongly specified H_0 is rejected with increasing frequency in n. This merely confirms that y_t is not generated by the implied DGP'. Correctly specifying the model as (14.13)

$$y_t = \beta_1 + \beta_2 x_t + \beta_3 y_{t-1} + v_t$$

and testing $H_0 : \beta_2 = 0$, implying $\beta_3 = 1$ reduces the model to the actual DGP (14.01). This treatment, however, does not completely eliminate the problem i.e. leaves the rejection rate still significantly above 0.

For $\hat{\beta}$ to converge to β_0 asymtotically, the bias $(\hat{\beta} - \beta_0)$ must be $O_p(1)$:

$$
\begin{aligned}
(\hat{\beta} - \beta_0) &= (X'X)^{-1}X'u, with \\
(X'X)^{-1} &\in O_p(n^{-1}) and \\
X'u &\in O_p(n^{.5}). consequently: \\
n^{.5}(\hat{\beta} - \beta_0) &= n^{.5}(X'X)^{-1}X'u = n^{.5}O_p(n^{-1})O_p(n^{.5}) = O_p(1)
\end{aligned}
$$

The relevant assumption to be tested is therefore is $(X'X) \in n^{.5}O_p(n^{-1})$.

The random walk (14.01) is I(1), due to:

$$
\begin{aligned}
w_t &= w_{t-1} + \epsilon_t \\
w_t - w_{t-1} &= \epsilon_t \\
(1 - L)w_t &= \epsilon_t \\
(1 - \phi(z))w_t &= \epsilon_t \\
\phi(z) &= 1
\end{aligned}
$$

Consequently, both x_t and y_t are I(1).

Further, (14.01) reduces to $w_t = \sum_{s=1}^{t} \epsilon_s$, which enters as X'X or:

$$
\begin{aligned}
\sum_{t=1}^{n} \left(\sum_{r=1}^{t} \sum_{s=1}^{t} \right) \epsilon_r \epsilon_s &= \sum_{t=1}^{n} \sum_{r=1}^{t} E(\epsilon_r^2), \epsilon_r \epsilon_s = 0 \; \forall r \neq s \\
\sum_{t=1}^{n} \sum_{r=1}^{t} \sigma^2 &= \sum_{t=1}^{n} \sum_{r=1}^{t} 1 \\
\sum_{t=1}^{n} t &= \frac{1}{2} n(n+1)
\end{aligned}
$$

Consequently, given (14.01) being I(1), $X'X \in O_p(n^2)$ and therefore cannot possibly converge to a finite probability limit. The bias $(\hat{\beta} - \beta_0)$ therefore does not converge asymptotically in probability.