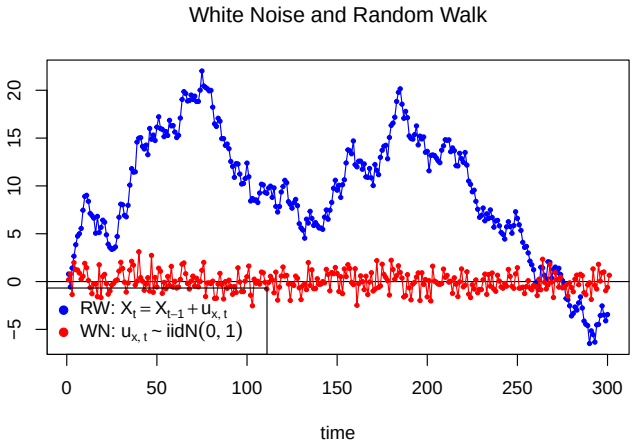
Annex to Econ homework

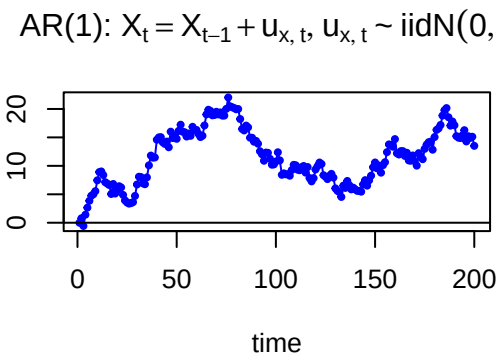
Run required function from main book first.

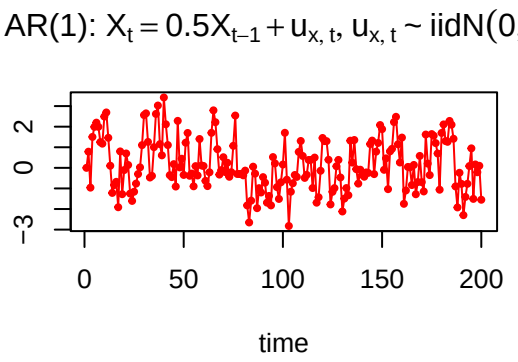
Annex

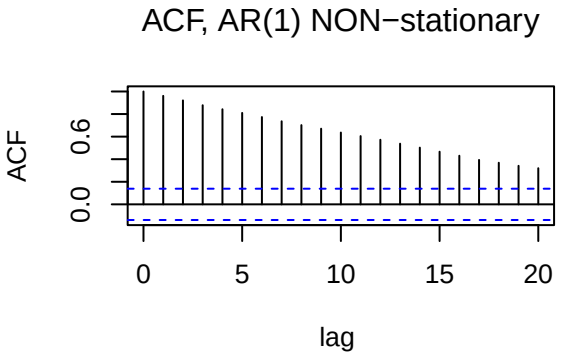
Simulate a random walk processes $y_t = y_{t-1} + e_t, y_0 = 0, e_t \sim IID(0, \sigma^2)$ analog to (14.03)

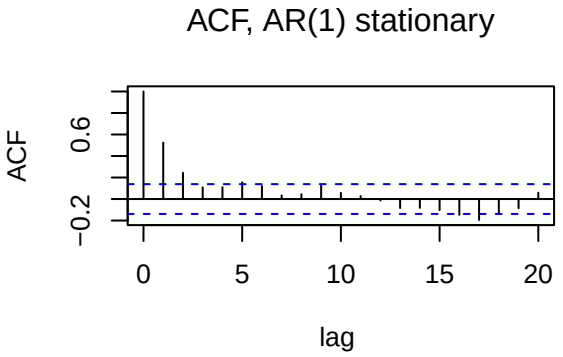


Simulate two AR(1) processes according to $x_t = \phi x_{t-1} + u_t$, with $\phi = 0.5$ and with $\phi = 1$. The ACFs of two processes indicate that in latter significant autocorrelation persists at least up to 20 lags, while the former autocorrelation becomes insignificant after 3 lags. Hence, in the case of AR(1), autocorrelation is a function of the parameter ϕ.



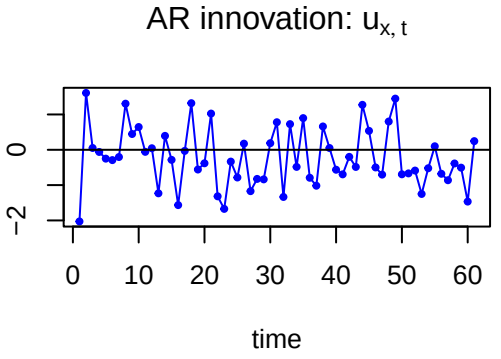


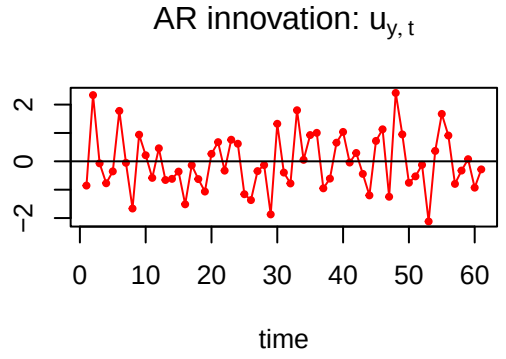


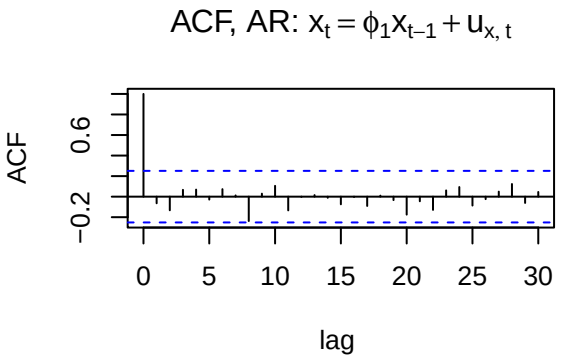


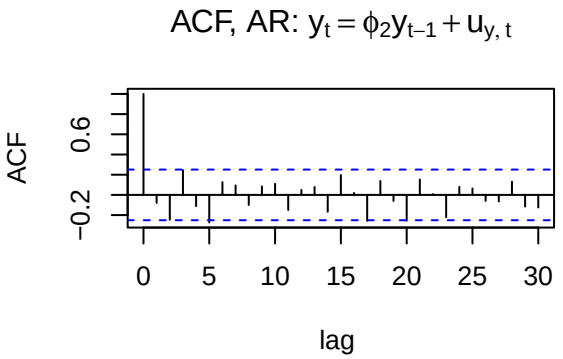
Define a function to produces N AR(1) simulations.

Plot the simulated error terms of the two AR(1)s, that is $u_{x,t}$ and $u_{y,t}$. The respective ACFs indicate that the error terms are not autocorrelated.

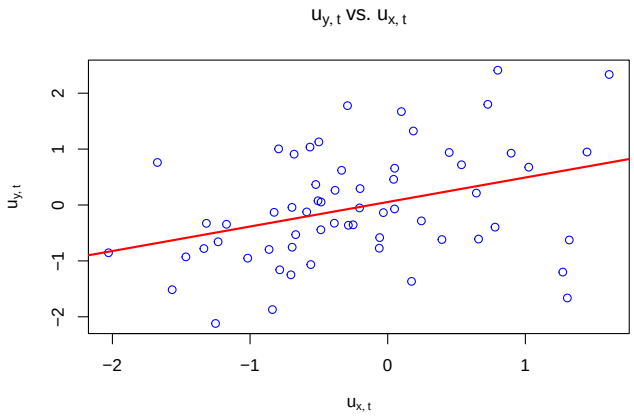








Scatterplot of $u_{x,t}$ and $u_{y,t}$. The error terms, while individually and randomly generated, appear to to correlated, which is confirmed by the regression of $u_{y,t}$ on $u_{x,t}$. The resulting t_β is significant.

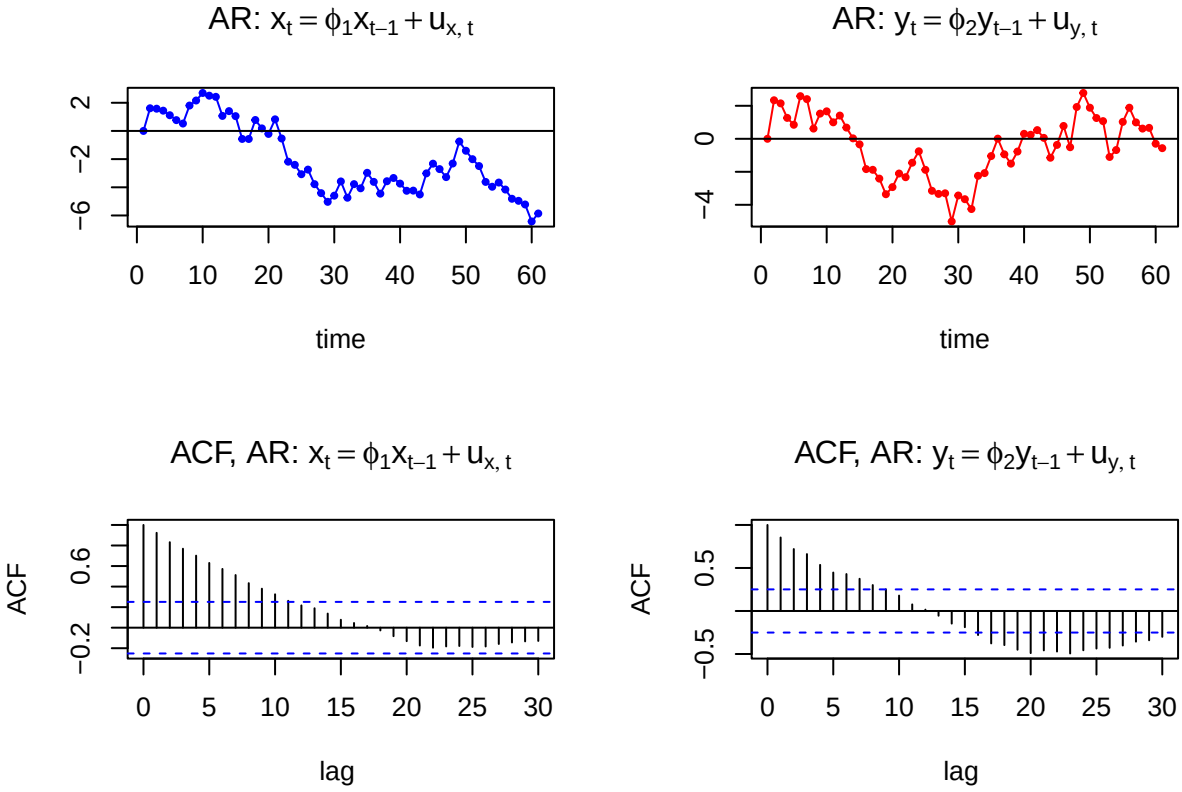



```
##
## ================================================
##                          Dependent variable:
##                      ---------------------------
##                                u_sim[, 2]
## ------------------------------------------------
## u_sim[, 1]                      0.438***
##                                  (0.147)
##
## Constant                         0.053
##                                  (0.125)
##
## ------------------------------------------------
## Observations                       61
## R2                                0.131
## Adjusted R2                       0.116
## Residual Std. Error          0.943 (df = 59)
## F Statistic               8.895*** (df = 1; 59)
## ================================================
## Note:                *p<0.1; **p<0.05; ***p<0.01
```

```
##
## Call:
## lm(formula = u_sim[, 2] ~ u_sim[, 1])
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -2.2890 -0.5590 -0.0190  0.4804  2.0071
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.05336    0.12516   0.426  0.67143
## u_sim[, 1]   0.43838    0.14699   2.982  0.00415 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.9435 on 59 degrees of freedom
## Multiple R-squared:  0.131,  Adjusted R-squared:  0.1163
```

```
## F-statistic: 8.895 on 1 and 59 DF,  p-value: 0.004152
```

Compare ACF of the two AR processes. Autocorrelation is present and takes a fading wave pattern.

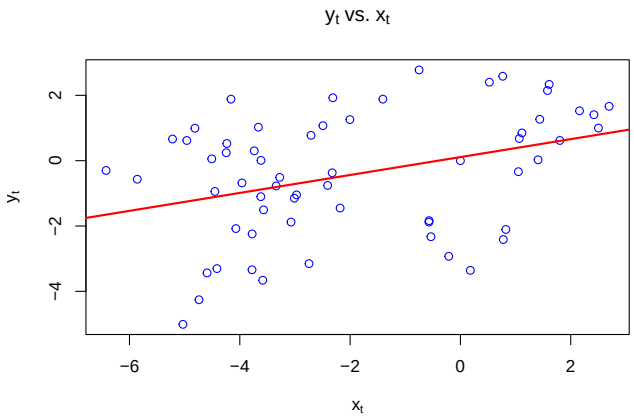


Run a regression of two AR(1) processes.

```
##
## ================================================
##                          Dependent variable:
##                      ---------------------------
##                                AR_sim[, 2]
## ------------------------------------------------
## AR_sim[, 1]                    0.274***
##                                 (0.091)
##
## Constant                        0.110
##                                 (0.290)
##
## ------------------------------------------------
## Observations                      61
## R2                               0.132
## Adjusted R2                      0.118
## Residual Std. Error         1.777 (df = 59)
## F Statistic              8.997*** (df = 1; 59)
## ================================================
## Note:                *p<0.1; **p<0.05; ***p<0.01

##
## Call:
## lm(formula = AR_sim[, 2] ~ AR_sim[, 1])
##
```

```
## Residuals:
##     Min      1Q  Median      3Q     Max
## -3.7395 -1.1474  0.2007  1.4091  2.9137
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.11036    0.29011    0.38  0.70500
## AR_sim[, 1]  0.27438    0.09148    3.00  0.00396 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.777 on 59 degrees of freedom
## Multiple R-squared:  0.1323, Adjusted R-squared:  0.1176
## F-statistic: 8.997 on 1 and 59 DF,  p-value: 0.003956
```



Compare the regression results. The errors as well as the AR(1) are correlated given significant βs

```
##
## ============================================================
##                                   Dependent variable:
##                               ---------------------------
##                                  u_sim[, 2]    AR_sim[, 2]
##                                     (1)            (2)
## ------------------------------------------------------------
## u_sim[, 1]                        0.438***
##                                   (0.147)
##
## AR_sim[, 1]                                      0.274***
##                                                   (0.091)
##
## Constant                           0.053          0.110
##                                   (0.125)        (0.290)
##
## ------------------------------------------------------------
## Observations                         61             61
## R2                                  0.131          0.132
## Adjusted R2                         0.116          0.118
## Residual Std. Error (df = 59)       0.943          1.777
## F Statistic (df = 1; 59)          8.895***       8.997***
## ============================================================
## Note:                          *p<0.1; **p<0.05; ***p<0.01
```