Explanation of $a \leq \frac{s-3}{3}$ and $b < \frac{s-a}{2}$

Problem 9. Find the only Pythagorean triplet (a, b, c), for which a + b + c = 1000.

We are given:

- a, b, and c are positive integers.
- $a < b < c$.

We want to find the range of possible values for a and b.

Deriving $a \leq \frac{s-3}{3}$

Since $a < b < c$, the smallest possible value for a is 1. To find the maximum possible value for a, we consider the smallest possible values for b and c:

- The smallest b can be is $a + 1$ (since $a < b$).
- The smallest c can be is $b + 1 = a + 2$ (since $b < c$).

Substitute these into the equation $a + b + c = s$:

$$a + (a + 1) + (a + 2) = s$$

$$3a + 3 = s$$

$$3a = s - 3$$

$$a = \frac{s-3}{3}$$

This gives the maximum possible value for a. Therefore:

$$a \leq \frac{s-3}{3}$$

Deriving $b < \frac{s-a}{2}$

Next, we find the range for b. Given $a < b < c$ and $a + b + c = s$, we can express c as:

$$c = s - a - b$$

Since $b < c$, we have:

$$b < s - a - b$$

$$2b < s - a$$

$$b < \frac{s-a}{2}$$

This inequality gives the upper bound for b in terms of a and s.