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Optical absorption, scattering, and multiple scattering: Experimental measurements using food coloring, India ink, and milk

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A series of optical experiments is used to illustrate color effects, absorption, and scattering. In the simple experiments, the intensity of red laser light is measured after the beam passes through samples with increasing concentrations of food dyes, black ink, and milk. The data are fitted with exponential functions, and similar attenuation is obtained with samples which are very different: black ink and white milk. Measured intensities and visual observations serve to illustrate the distinction between absorption, scattering, and attenuation. In more advanced experiments, attenuation coefficients and the mean attenuation length are measured. With milk samples, intensity measurements reveal a non-exponential concentration dependence which is a signature of multiple scattering. Finally, intensity as a function of distance is measured by scanning wedge-shaped samples of ink or milk across a detector. The effects of multiple scattering in milk are very pronounced in this case.
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I. INTRODUCTION

Light beams traveling through milk, ink, or colored dyes can be scattered or absorbed, resulting in a loss of intensity. The intensity loss in an aqueous mixture is proportional to the concentration c, the intensity I, and the path increment dx,

$$dI = -\mu c I \, dx, \tag{1}$$

where μ is the linear attenuation coefficient for an undiluted liquid. The concentration c is the dimensionless volume fraction of the substance added to water and the attenuation coefficient has dimensions of inverse length. Equation (1) is valid in the absence of multiple scattering, corresponding to a sample which is purely absorbing, or for a scatterer in the low-concentration limit. With incident intensity I_0, the intensity is

$$I = I_0 \, e^{-\mu c x}. \tag{2}$$

The form of Eq. (2) indicates that the intensity will decrease exponentially as either the concentration or the path length is increased.

In the simple experiments, students observe the absorption and scattering of a red laser beam traveling through (i) four colored dye solutions, (ii) diluted black ink, and (iii) diluted milk. Drops of these substances are added to water, and the transmitted intensity is measured with a photodiode. The ratio of the transmitted and incident intensity is plotted as a function of the number of added drops, and students analyze the data using exponential functions. This experiment is carried out as a lab exercise in *Physics in Art*, a course for non-science majors.[1]

Absorption and reflection are important in understanding lighting conditions in an art gallery, and in the study of drawings or paint layers which may lie below the visible surface of an art object. An example of absorption in an art context is the use of x radiography to image paintings which have pigments containing lead or cadmium. On the other hand, white paints contain particles such as lead carbonate or titania; these materials are non-absorbing in the visible part of the spectrum and their properties are explained by scattering which occurs at the surfaces of the particles. The absorption lab described here complements other labs, where students measure the emission spectra of different types of light sources, and the infrared reflectance and transmission of dyes and pigments in markers and various types of paints. Prior to beginning the absorption experiments, students in *Physics in Art* make qualitative predictions regarding the strength of absorption by the different food coloring dyes. They are able to complete the absorption data collection, plot the intensities as a function of drop number, and interpret their results during a two-hour lab. Following the lab, the students write a two-page report with a narrative summary for a general audience, similar in style to a science article in a newspaper, but including technical details about the measurement and data analysis. Introducing students to exponential functions and logarithms through the analysis of an absorption experiment is valuable, since these mathematical tools are used in other topics in physics. For example, students in *Physics in Art* learn about radioactive decay and carbon dating of cultural heritage objects, while students in conventional first-year physics courses may learn about Newton's Law of Cooling[2] or the exponential behavior of a discharging *RC* circuit.

In a previous experiment aimed at teaching students about the concentration dependence of optical absorbance, food-dye solutions were analyzed using a spectrometer,[3] and a setup made from discrete components has been tested in absorption and scattering experiments.[4] Results of an experiment on samples with different milk-fat concentrations are presented in Ref. 5, while a theoretical description of multiple scattering suitable for intermediate to advanced undergraduates is given in Ref. 6. The experiments described here complement the ideas and methods presented in Refs. 2–6, as well as other experiments on the optical properties of aqueous solutions and suspensions listed in Ref. 7.

II. INTENSITY AS A FUNCTION OF CONCENTRATION

Our experimental setup is shown in Fig. 1. The experiments start with a 20 ml glass vial containing 10 ml of water which is stirred continuously by a magnetic stirrer. The beam from a 1 mW diode laser[8] with a nominal wavelength of 650 nm passes through the sample, and its intensity is measured with a simple home-made detector. The detector was made by taping

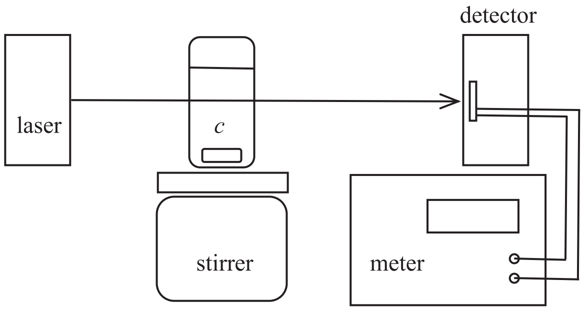

Fig. 1. Experimental setup. A sample with concentration c is held in a 20 ml vial on a magnetic stirrer. The detector is made by taping a 9.7 mm photocell inside the clear cover of a plastic box.

a surplus photodiode inside the clear plastic cover of a project box. The photodiode is relatively large (9.7 mm) and approximately square. The detector size simplifies alignment by allowing students to see when the laser beam is centered on the detector, and is relevant to the multiple-scattering effects discussed below. The detector current (proportional to the intensity) is measured with a benchtop meter[9] which has 0.1 μA resolution. We use a cylindrical glass vial as a sample container because it is inexpensive, easy to handle, and large enough for a small stir bar to be placed inside. The vial acts as a cylindrical lens, causing the beam to spread downstream of the focus. In a typical *Physics in Art* experiment, the sample-to-detector distance is 10 cm or less and all of the transmitted beam falls on the detector. When the detector is 20 cm from the sample, the beam width is approximately twice the width of the detector.

The intensity of the transmitted beam is measured as a function of the number of added drops of dyes, ink, or milk. If one neglects the slight increase in sample volume due to the volume of the added drops, the concentration after n drops are added is given by $c = nc_1$, where c_1 is the concentration increase after one drop is added. Equation (2) becomes

$$I = I_0 e^{-n\mu c_1 L}, \tag{3}$$

where L is the path length through the sample. Given that μ, c_1, and L are fixed as drops are added to the water, Eq. (3) indicates that the normalized intensity I/I_0 decays exponentially as a function of the number of drops n. Undiluted drops of yellow, red, blue, and green food coloring[10] are added to the water, using the built-in droppers which are incorporated in the food-coloring vials. Prior to the lab, the instructor prepares a black ink standard solution of India ink[11] diluted to 1% by volume, while the milk standard solution is prepared with whole milk diluted to 11%. The ink and milk dilutions are chosen so as to produce a similar exponential decrease in the transmitted intensity as 50 μl drops are added with a micropipettor. Using the detector current as a measure of intensity, the background intensity with the laser beam blocked and the room lights dimmed is typically $I_{bkg} = 0.2\,\mu$A. The transmitted intensity through a vial filled with plain water is approximately $I_w = 235\,\mu$A. We define the normalized intensity as

$$\frac{I}{I_0} = \frac{I_n - I_{bkg}}{I_w - I_{bkg}}, \tag{4}$$

where I_n is the detector current after the addition of n drops.

In ambient room light, India ink appears black and milk appears white. With the room lights dimmed and the laser turned on, after several drops of ink are added to water there is some scattering but the laser beam more or less disappears in the dark mixture. On the other hand, when milk is added to water, the mixture appears to glow red. A plot of the normalized intensities vs the added drop number for the various samples is shown in Fig. 2.The attenuation of the laser beam by the ink and milk samples is approximately equal, but the mechanism is different, since the ink is primarily an absorber and the milk scatters the light. In other words, a measurement of the transmitted intensity does not determine whether the sample scatters or absorbs light.[12] The colored dyes show a qualitative but pronounced effect, namely, the red laser light is hardly absorbed by the red and yellow dyes after the addition of ten drops, but is very strongly absorbed after the addition of only one drop of the blue and green dyes.

With students in a more advanced course such as the calculus-based introductory sequence, or in an intermediate-level optics course, one can proceed with a further analysis of the data. Rearranging Eq. (2) and taking the natural logarithm, one obtains

$$\ln\left(\frac{I}{I_0}\right) = -\mu c x. \tag{5}$$

This equation shows that a plot of the logarithm of the normalized intensity can be used to find the attenuation coefficient with two different methods. One can use a container with a fixed path length $x = L$ (such as the vial shown in Fig. 1) and vary the concentration c, or one can use a sample of fixed concentration c in a wedge-shaped container which has variable path length x, as discussed in Sec. III. The concentration c is the volume fraction of ink or milk, with $c = 1$ corresponding to pure ink or pure milk. The concentrations were calculated as accurately as possible by taking into account the increasing volume of the sample as each 50 μl drop is added. Figure 3 shows plots of $\ln(I/I_0)$ vs c, recorded with sample-detector distances d of 6, 10, and 20 cm.

The ink data in Fig. 3 are linear, in agreement with Eq. (5), but the milk samples exhibit positive deviation from

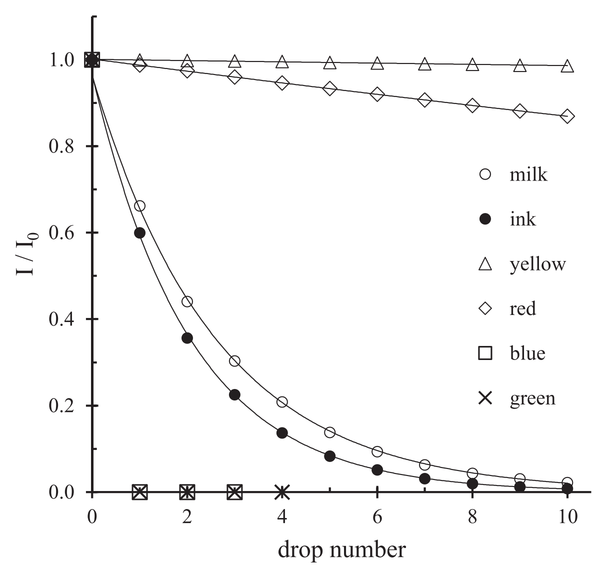

Fig. 2. Normalized intensity vs added drop number for samples of food coloring, milk, and India ink. The curves through the data points are exponential functions. The sample-to-detector distance for these measurements is 10 cm. Students in *Physics in Art* labs obtain similar data.

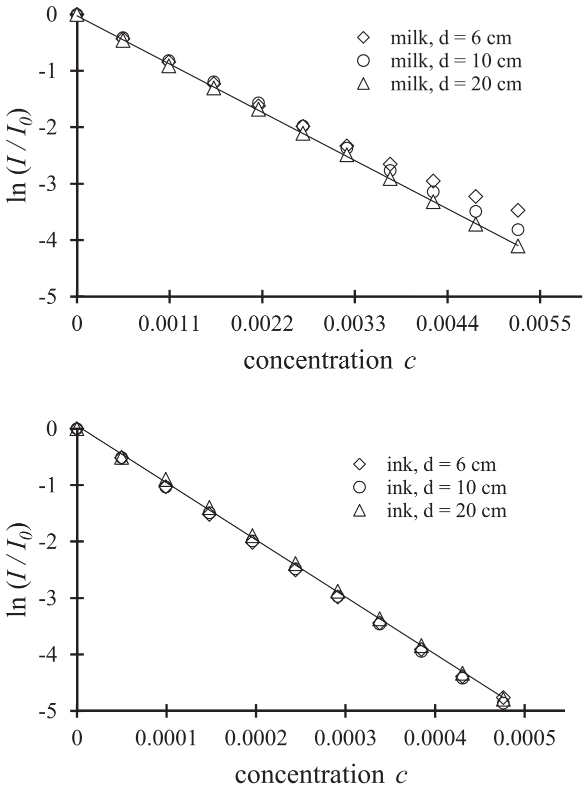

Fig. 3. Log plots of the normalized intensity vs concentration for milk and ink samples, with the detector at varying distance d from the sample. Each succeeding data point corresponds to the addition of a 50 μl drop of 11% milk (above) or 1% ink (below). The horizontal axis ranges in the two plots vary by a factor of 11, which is the ratio of the concentrations.

exponential behavior, with a stronger effect at high concentration and a small detector distance. With a scatterer (such as milk), there is a finite probability that a photon which is initially scattered out of the detector path will be re-scattered into the detector. Multiple scattering tends to increase the number of photons reaching the detector.[6] In the low-concentration limit, multiple scattering is negligible and Eq. (1) is satisfied, with a linear plot of ln (I/I_0) vs c, but the probability of multiple scattering increases with the concentration. The probability of multiple scattering *into the detector path* is larger when the detector subtends a larger solid angle. For a detector of fixed size, the solid angle is proportional to the inverse square of the detector distance. (A close detector subtends a large angle and *vice-versa*.) Figure 3 includes milk data for detector distances $d = 6$ cm and $d = 20$ cm. For a detector which is a 9.7 mm square, the subtended angles in these two cases are 0.026 sr (when $d = 6$ cm) and 0.0024 sr ($d = 20$ cm).

For a system with exponential intensity as in Eq. (2), the mean attenuation length is $1/\mu$, the inverse of the attenuation coefficient. Equation (5) implies that the slopes of the plots in Fig. 3 are equal to $-\mu L$ for a container with path length L. The sample vials used in our experiments have inner diameter $L = 25$ mm. The India ink data in Fig. 3 have a slope of -10130, yielding attenuation coefficient $\mu_{ink} = 405\,\mathrm{mm}^{-1}$ and mean attenuation length $1/\mu_{ink} = 2.47\,\mu$m. For milk, we use the data with a sample-detector distance of 20 cm, where multiple scattering is relatively insignificant. In this case, the slope is -777, from which we obtain $\mu_{milk} = 31.1\,\mathrm{mm}^{-1}$ and $1/\mu_{milk} = 32.2\,\mu$m. The attenuation coefficient for milk can be compared to Ref. 13, where milk samples with varying fat content were measured at several wavelengths. In our case, we assume that whole milk[14] has a fat content of 3.25%. Using the data shown in Fig. 5(a) of Ref. 13, the attenuation coefficient for 600 nm photons is approximately 38 mm^{-1}, a value 22% higher than that which we obtained. We also compared our data to the results of Ref. 5. The attenuation coefficient for whole milk was not reported explicitly, but we can estimate it using the data shown in Fig. 3 of that paper. With $\ln(I/I_0) \approx -6$ and a reported path length of 75 μm, we obtain an attenuation coefficient of 80 mm^{-1}, significantly larger than our measured value and that of Ref. 13.

III. INTENSITY AS A FUNCTION OF DISTANCE

Using Eq. (5) with fixed concentration c, the slope of a plot of $\ln(I/I_0)$ vs path length x is equal to $-\mu c$. One could perform variable-path-length measurements by using a set of containers with varying thickness, or by using a wedge-shaped sample holder. In our setup (Fig. 4), a vertical laser beam passes through a wedge made from a pair of glass microscope slides with a spacer at one end; the spacer is made from a piece of a slide. It was not necessary to seal the edges of the wedge since the ink and milk samples were held in place by capillary forces. A plastic mm scale was taped to the upper slide, with the scale zero aligned with the right-hand end where the slides were in contact. The same detector as in Fig. 1 was covered with a pinhole, made by passing a 1.1 mm-diameter sewing needle through a piece of aluminum foil. A fiducial line was drawn to mark the position of the pinhole. Intensity readings were taken as the wedge was pushed across the pinhole. At a given wedge position, the scale reading at the pinhole fiducial line is x_{sc}, corresponding to the distance from the pinhole to the right-hand end of the wedge. The path length through the sample is given by $x = x_{sc} \sin\theta$, where $\theta \approx 0.89°$ is the wedge angle. The intensity was measured at 2 mm increments in x_{sc}, corresponding to 31 μm increases in the path length x.

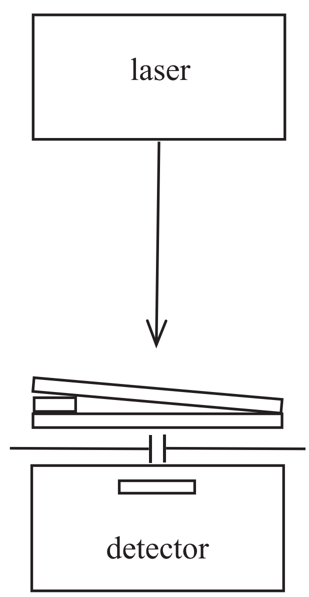

Fig. 4. Setup for measurements of intensity as a function of sample thickness. A liquid sample is held in a wedge made of two glass microscope slides. The detector is covered by foil with a 1.1 mm pinhole. A small plastic scale is taped to the upper slide, and the wedge is scanned across the pinhole. Intensities are recorded at 2 mm intervals in the horizontal position of the wedge, corresponding to sample thickness steps of approximately 31 μm.

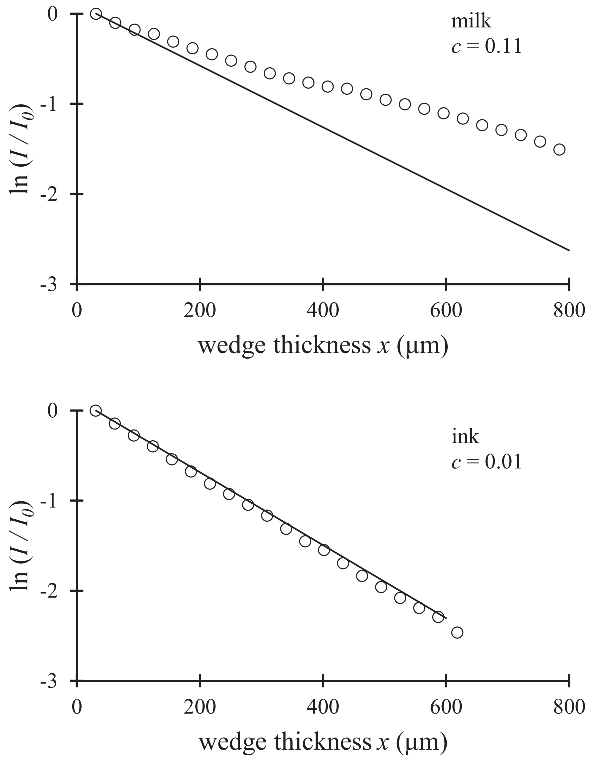

Fig. 5. Measurement of normalized intensity vs wedge thickness for milk and ink standard solutions, using the setup shown in Fig. 4. The straight lines are drawn with slopes corresponding to attenuation coefficients $\mu_{ink} = 405\,\text{mm}^{-1}$ and $\mu_{milk} = 31.1\,\text{mm}^{-1}$. The large deviation of the milk data is consistent with strong multiple scattering. The ink data show that the same value of the attenuation coefficient is obtained by measuring intensity as a function of concentration or intensity as a function of sample thickness.

Plots of ln (I/I_0) vs x are shown in Fig. 5, for 1% ink ($c = 0.01$) and 11% milk ($c = 0.11$). The straight lines in the Fig. 5 plots are made using Eq. (5) with the attenuation coefficients μ_{ink} and μ_{milk} obtained from Fig. 3. The wedge data for ink are linear and appear to correspond to the same attenuation coefficient ($\mu_{ink} = 405\,\text{mm}^{-1}$) as in the concentration experiments. In the case of milk, the wedge data show a large divergence from the line corresponding to the Fig. 3 attenuation coefficient ($\mu_{milk} = 31.1\,\text{mm}^{-1}$). The large discrepancy between the milk intensities and the straight line is a multiple-scattering effect, presumably due to the large angular acceptance of the detector in the wedge experiments. The diameter of the laser beam in these experiments is approximately 2.5 mm, and we estimate that the distance from the illuminated portion of the fluid wedge to the pinhole is about 2 mm. These values yield a detector acceptance of roughly 1.2 sr, much larger than the values corresponding to the Fig. 3 concentration data. The large detector acceptance angle in the wedge experiments increases the probability that a multiply-scattered photon will find its way into the detector, accounting for the discrepancy between the milk data of Fig. 5 and Fig. 3.

IV. SUMMARY

Absorption and scattering experiments can be used to compare colored dyes, absorbing black ink, and milk, which is a scatterer. At the introductory level, one can measure the transmitted intensity as drops are added to stirred solutions and plot the normalized intensity as a function of drop number. Students are able to connect the color of a dye solution with the strength of its absorption of a red laser beam. With appropriate concentrations of the added drops, one can investigate an absorber (ink) and a scatterer (milk) with similar attenuation as a function of drop number. Students learn to distinguish between absorption, scattering, and attenuation. In more advanced labs, one can determine the attenuation coefficient by plotting the logarithm of the normalized intensity as a function of concentration. If one uses a detector with an area of approximately $1\,\text{cm}^2$ or larger, one can observe multiple-scattering effects in milk by making measurements at various sample-to-detector distances, corresponding to different detector acceptance angles. The wedge experiments allow one to measure intensity as a function of distance within a fluid, confirming that the attenuation coefficient for ink can be determined from either a plot of intensity vs concentration or a plot of intensity vs position.

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