Adjustable Robust Optimization via Fourier-Motzkin Elimination

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We demonstrate how adjustable robust optimization (ARO) problems with fixed recourse can be cast as static robust optimization problems via Fourier-Motzkin elimination (FME). Through the lens of FME, we characterize the structures of the optimal decision rules for a broad class of ARO problems. A scheme based on a blending of classical FME and a simple Linear Programming technique that can efficiently remove redundant constraints, is developed to reformulate ARO problems. This generic reformulation technique enhances the classical approximation scheme via decision rules, and enables us to solve adjustable optimization problems to optimality. We show via numerical experiments that, for small-size ARO problems our novel approach finds the optimal solution. For moderate or large-size instances, we eliminate a subset of the adjustable variables, which improves the solutions obtained from linear decision rules.

Key words:

Subject classifications: Fourier-Motzkin elimination; adjustable robust optimization; linear decision rules; redundant constraint identification.

Area of review: Optimization.

1. Introduction

In recent years, robust optimization has been experiencing an explosive growth and has now become one of the dominant approaches to address decision making under uncertainty. In robust optimization, uncertainty is described by a distribution free uncertainty set, which is typically a conic representable bounded convex set (see, for instance, El Ghaoui and Lebret (1997), El Ghaoui et al. (1998), Ben-Tal and Nemirovski (1998, 1999, 2000), Bertsimas and Sim (2004), Bertsimas and Brown (2009), Bertsimas et al. (2011)). Among other benefits, robust optimization offers a computationally viable methodology for immunizing mathematical optimization models against parameter uncertainty by replacing probability distributions with uncertainty sets as fundamental primitives. It has been successful in providing computationally scalable methods for a wide variety of optimization problems.

2. Footnotes and Endnotes

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3. Conclusions

We propose a generic FME approach for solving ARO problems with fixed recourse to optimality. Through the lens of FME, we characterize the structures of the ODRs for a broad class of ARO problems. We extend the approach of Bertsimas et al. (2017) for ADRO problems. Via numerical experiments, we show that for small-size ARO problems our approach finds the optimal solution, and for moderate to large-size instances, we successively improve the approximated solutions obtained from LDRs.

On a theoretical level, one immediate future research direction would be to characterize the structures of the ODRs for multistage problems, e.g., see Bertsimas et al. (2010), Iancu et al. (2013). Another potential direction would be to extend our FME approach to ARO problems with integer adjustable variables or non-fixed recourse.

On a numerical level, we would like to investigate the performance of Algorithm with finite adaptability approaches or other decision rules on solving ARO problems. Moreover, many researchers have proposed alternative approaches for computing polytopic projections and identifying redundant constraints in linear programming problems. For instance, Huynh et al. (1992) discusses the efficiency of three alternative procedures for computing polytopic projections, and introduces a new RCI method; Paulraj and Sumathi (2010) compares the efficiency of five RCI methods. Another potential direction would be to adapt and combine the existing alternative procedures to further improve the efficiency of our proposed approach.

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