Analysis of synthetic data using WCLS

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The data here is a synthetic data set mimicking some features of the HeartSteps data set.

1. Preparation

We load the already generated synthetic data set and load some packages and functions required to carry out the WCLS analysis.

```
rm(list = ls())

library(tidyverse)
library(xtable)
library(geepack)
source("xgeepack.R")
source("estimate.R")

synthetic_data <- read.csv("synthetic_data_37subject_210time.csv")

summary(synthetic_data)
```

```
##      userid    decision.index.nogap study.day.nogap jbsteps30.log
##  Min.   : 1   Min.   :  1.0        Min.   : 0.0    Min.   :-0.6931
##  1st Qu.:10   1st Qu.: 53.0        1st Qu.:10.0    1st Qu.: 0.5519
##  Median :19   Median :105.5        Median :20.5    Median : 2.5680
##  Mean   :19   Mean   :105.5        Mean   :20.5    Mean   : 2.7204
##  3rd Qu.:28   3rd Qu.:158.0        3rd Qu.:31.0    3rd Qu.: 4.4883
##  Max.   :37   Max.   :210.0        Max.   :41.0    Max.   : 8.5000
##  jbsteps30.log.lag1 jbsteps30pre.log  location.homework      send
##  Min.   :-0.6931    Min.   :-0.6931   Min.   :0.0000    Min.   :0.000
##  1st Qu.: 0.5156    1st Qu.:-0.6931   1st Qu.:0.0000    1st Qu.:0.000
##  Median : 2.5514    Median : 3.0074   Median :0.0000    Median :0.000
##  Mean   : 2.7068    Mean   : 2.1025   Mean   :0.3683    Mean   :0.497
##  3rd Qu.: 4.4813    3rd Qu.: 4.0007   3rd Qu.:1.0000    3rd Qu.:1.000
##  Max.   : 8.5000    Max.   : 5.3293   Max.   :1.0000    Max.   :1.000
##      avail
##  Min.   :0.0000
##  1st Qu.:1.0000
##  Median :1.0000
##  Mean   :0.8049
##  3rd Qu.:1.0000
##  Max.   :1.0000
```

The variable names are kept consistent with the original HeartSteps data set and the analysis code for that data, the analysis result of which is included in the main manuscript. Below are some explanation for each of the variables:

- userid: id of a user (ranging from 1 to 37)
- decision.index.nogap: decision point index for each user (ranging from 1 to 210)
- study.day.nogap: day in the study for each user (ranging from 0 to 41)
- jbsteps30.log: log-transformed 30-minute step count following each decision point (it is called jbsteps because in HeartSteps the step count was measured by Jawbone tracker)
- jbsteps30.log.lag1: log-transformed 30-minute step count following the previous decision point; i.e., a lagged version of jbsteps30.log
- jbsteps30pre.log: log-transformed step count in the 30-minute window *preceding* each decision point
- location.homework: an indicator of whether the user is currently at home/work (1) or other places (0)
- send: treatment indicator, whether an activity suggestion was sent at the decision point
- avail: availability indicator, whether the person is available at the decision point

We create two additional variables to be used in the WCLS regression below.

```
synthetic_data$"(Intercept)" <- 1
synthetic_data$"I(send - 0.6)" <- synthetic_data$send - 0.6
```

2. Using WCLS to analyze the data

We use the Weighted and Centered Least Squares (WCLS) estimator to analyze the data.

2.1. Marginal Effect

In this analysis, we aim to answer the question: "What is the effect of delivering activity suggestions on individuals' subsequent 30-minute step counts?"

```
xmat <- synthetic_data %>%
    transmute("(Intercept)" = .$"(Intercept)",
              "jbsteps30pre.log" = .$"jbsteps30pre.log",
              "I(send - 0.6)" = .$"I(send - 0.6)")

fit_model1 <- geese.glm(x = as.matrix(xmat),
                        y = synthetic_data$jbsteps30.log,
                        w = synthetic_data$avail,
                        id = as.factor(synthetic_data$user),
                        family = gaussian(), corstr = "independence")
estimate(fit_model1)
```

```
##                  Estimate  95% LCL  95% UCL       SE Hotelling      df1 df2
## (Intercept)      2.01e+00 1.92e+00 2.10e+00 4.57e-02  1.94e+03 1.00e+00  34
## jbsteps30pre.log 3.40e-01 3.00e-01 3.80e-01 1.97e-02  2.98e+02 1.00e+00  34
## I(send - 0.6)    1.57e-01 3.10e-02 2.84e-01 6.22e-02  6.40e+00 1.00e+00  34
##                  p-value
## (Intercept)       <1e-04 ***
## jbsteps30pre.log  <1e-04 ***
## I(send - 0.6)     0.0162 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The estimated marginal effect of is the coefficient for I(send - 0.6).

One can use a different set of control variables (e.g., by additionally including the lag-1 outcome) in the estimation of the same marginal effect:

```r
xmat <- synthetic_data %>%
    transmute("(Intercept)" = .$"(Intercept)",
              "jbsteps30pre.log" = .$"jbsteps30pre.log",
              "jbsteps30.log.lag1" = .$"jbsteps30.log.lag1",
              "I(send - 0.6)" = .$"I(send - 0.6)")

fit_model1.1 <- geese.glm(x = as.matrix(xmat),
                        y = synthetic_data$jbsteps30.log,
                        w = synthetic_data$avail,
                        id = as.factor(synthetic_data$user),
                        family = gaussian(), corstr = "independence")
estimate(fit_model1.1)
```

```
##                      Estimate  95% LCL  95% UCL       SE Hotelling      df1 df2
## (Intercept)          1.90e+00 1.80e+00 2.00e+00 4.91e-02  1.50e+03 1.00e+00  33
## jbsteps30pre.log     3.41e-01 3.01e-01 3.81e-01 1.98e-02  2.98e+02 1.00e+00  33
## jbsteps30.log.lag1 3.97e-02 1.69e-02 6.25e-02 1.12e-02  1.25e+01 1.00e+00  33
## I(send - 0.6)        1.61e-01 3.80e-02 2.85e-01 6.07e-02  7.08e+00 1.00e+00  33
##                      p-value
## (Intercept)          < 1e-04 ***
## jbsteps30pre.log     < 1e-04 ***
## jbsteps30.log.lag1 0.00121 **
## I(send - 0.6)        0.01197 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The estimated marginal effect of is again the coefficient for I(send - 0.6). We see that the estimated marginal effect here is similar to the previous analysis in magnitude, and the standard error is slightly smaller. This illustrates the fact that including control variables that is correlated with the proximal outcome can usually reduce noise and improve estimation precision.

2.2. Effect Change Over Time

In this analysis, we aim to answer the question: "How does the effect of activity suggestions change with each additional day in the study?"

```r
xmat <- synthetic_data %>%
    transmute("(Intercept)" = .$"(Intercept)",
              "jbsteps30pre.log" = .$"jbsteps30pre.log",
              "study.day.nogap" = .$"study.day.nogap",
              "I(send - 0.6)" = .$"I(send - 0.6)",
              "I(send - 0.6):study.day.nogap" =
                  .$"I(send - 0.6)" * .$"study.day.nogap")

fit_model2 <- geese.glm(x = as.matrix(xmat),
                        y = synthetic_data$jbsteps30.log,
                        w = synthetic_data$avail,
                        id = as.factor(synthetic_data$user),
                        family = gaussian(), corstr = "independence")
estimate(fit_model2)
```

```
##                                 Estimate   95% LCL   95% UCL        SE Hotelling
## (Intercept)                      2.18752   2.04533   2.32971   0.06981 982.04017
## jbsteps30pre.log                 0.33967   0.30003   0.37930   0.01946 304.75364
```

```
## study.day.nogap               -0.00852  -0.01398  -0.00305   0.00268  10.08379
## I(send - 0.6)                  0.64860   0.43050   0.86670   0.10707  36.69331
## I(send - 0.6):study.day.nogap  -0.02374  -0.03279  -0.01469   0.00444  28.55599
##                                    df1 df2 p-value
## (Intercept)                    1.00000  32  <1e-04 ***
## jbsteps30pre.log               1.00000  32  <1e-04 ***
## study.day.nogap                1.00000  32  0.0033 **
## I(send - 0.6)                  1.00000  32  <1e-04 ***
## I(send - 0.6):study.day.nogap  1.00000  32  <1e-04 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

We make the plot of the estimated effect over time along with its pointwise 95% confidence interval.

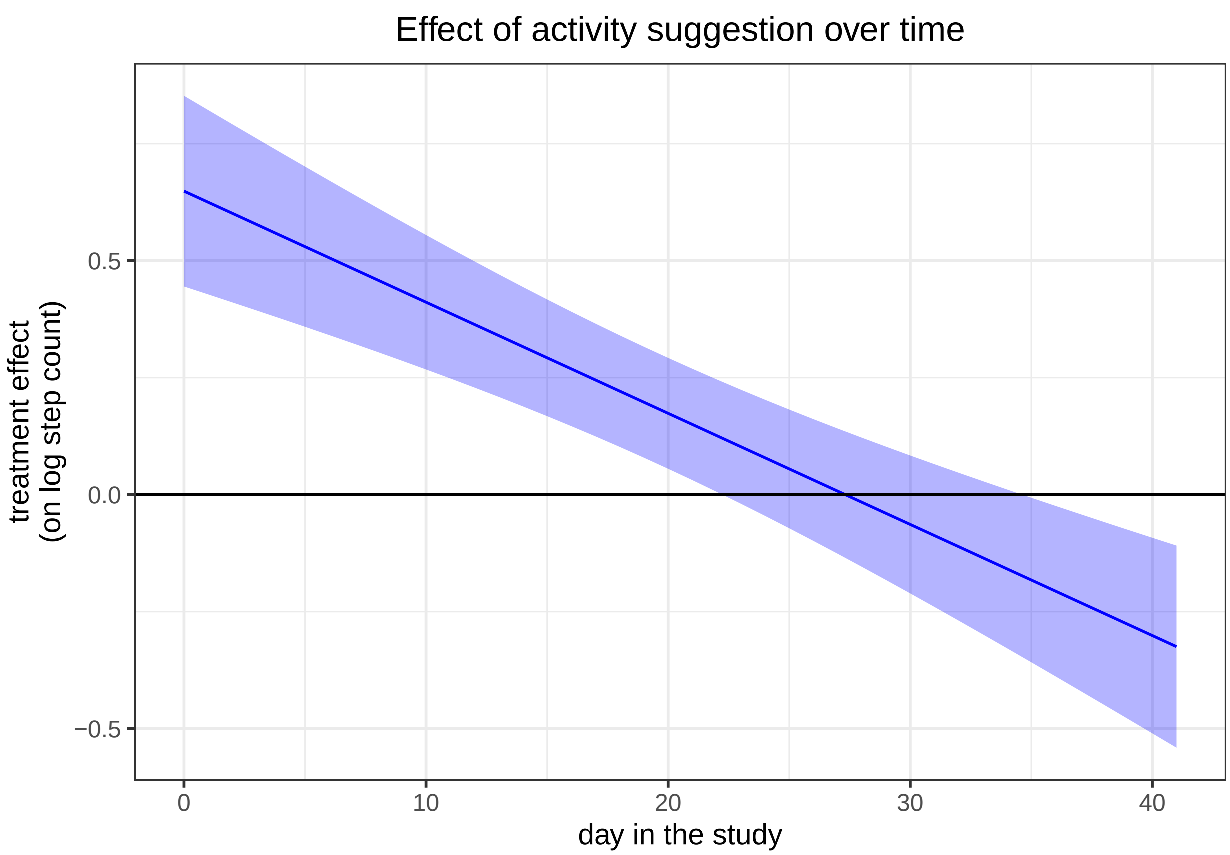
```r
beta_index <- 4:5

beta_hat <- coef(fit_model2)[beta_index]
vcov <- fit_model2$geese$vbeta[beta_index, beta_index]

newdta <- df_tx <- data.frame(Intercept = 1, study.day.nogap = 0:41)
df_tx$treatment_effect <- as.matrix(newdta) %*% beta_hat
df_tx$tx_se <- NA
for (i in 1:nrow(df_tx)) {
    f_t <- as.numeric(newdta[i, ]) # feature
    df_tx$tx_se[i] <- sqrt(t(f_t) %*% vcov %*% f_t)
}
df_tx$left_ci <- df_tx$treatment_effect - 1.96 * df_tx$tx_se
df_tx$right_ci <- df_tx$treatment_effect + 1.96 * df_tx$tx_se

df_tx_linear <- df_tx


ggplot(df_tx) +
    geom_line(aes(x = study.day.nogap, y = treatment_effect), color = "blue") +
    geom_ribbon(aes(ymin = left_ci, ymax = right_ci, x = study.day.nogap),
                alpha = 0.3, fill = "blue") +
    geom_hline(yintercept = 0, color = "black") +
    xlab(label = "day in the study") +
    ylab(label = "treatment effect\n(on log step count)") +
    ggtitle(paste0("Effect of activity suggestion over time")) +
    theme_bw() +
    theme(plot.title = element_text(hjust = 0.5))
```



plot-1.pdf

2.3. Effect Moderated by Outcome at Previous Time Point

In this analysis, we aim to answer the question: "How does the effect of activity suggestions depend on the logged step count at previous decision point?"

```
xmat <- synthetic_data %>%
    transmute("(Intercept)" = .$"(Intercept)",
              "jbsteps30pre.log" = .$"jbsteps30pre.log",
              "jbsteps30.log.lag1" = .$"jbsteps30.log.lag1",
              "location.homework" = .$"location.homework",
              "I(send - 0.6)" = .$"I(send.active - 0.6)",
              "I(send - 0.6):jbsteps30.log.lag1" =
                  .$"I(send - 0.6)" * .$"jbsteps30.log.lag1")

fit_model3 <- geese.glm(x = as.matrix(xmat),
                        y = synthetic_data$jbsteps30.log,
                        w = synthetic_data$avail,
                        id = as.factor(synthetic_data$user),
                        family = gaussian(), corstr = "independence")
estimate(fit_model3)
```

```
##                                   Estimate   95% LCL   95% UCL        SE
## (Intercept)                       1.85e+00  1.74e+00  1.96e+00  5.34e-02
## jbsteps30pre.log                  3.41e-01  3.01e-01  3.81e-01  1.97e-02
## jbsteps30.log.lag1                3.87e-02  1.55e-02  6.19e-02  1.14e-02
## location.homework                 1.51e-01  3.93e-02  2.63e-01  5.48e-02
## I(send - 0.6):jbsteps30.log.lag1  2.83e-02 -1.66e-03  5.82e-02  1.47e-02
```

```
##                                   Hotelling       df1 df2 p-value
## (Intercept)                        1.20e+03  1.00e+00  32 < 1e-04 ***
## jbsteps30pre.log                   3.01e+02  1.00e+00  32 < 1e-04 ***
## jbsteps30.log.lag1                 1.16e+01  1.00e+00  32 0.00183 **
## location.homework                  7.58e+00  1.00e+00  32 0.00964 **
## I(send - 0.6):jbsteps30.log.lag1  3.70e+00  1.00e+00  32 0.06326 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```