Universität Stuttgart

Multiphasic modelling of bone-cement injection into vertebral cancellous bone

Authors: Christian Bleiler*, Arndt Wagner, Vincent A. Stadelmann, Markus Windolf, Harald Köstler, Andreas Boger, Boyko Gueorguiev-Rüegg, Wolfgang Ehlers and Oliver Röhrler [2014]

Tuan Vo

February 7, 2017

COMMAS

Table of contents

Motivation and Overview

Motivation

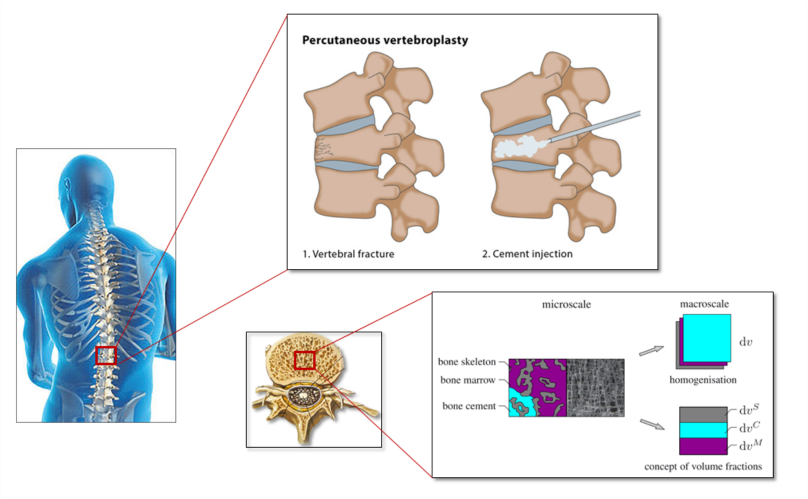

Fig. 1 – Vertebroplasty and Representative elementary volume (REV) [1,2,3,4].

Overview

Main points from the article:

- Biphasic three-constituent model based on Theory of Porous Media:

$$\varphi = \bigcup_{\alpha} \varphi^{\alpha} = \varphi^{S} \cup \varphi^{F} = \varphi^{S} \cup \varphi^{C} \cup \varphi^{M} \text{ where } \begin{cases} \alpha = \{S, C, M\}\,, \\ \varphi^{F} = \bigcup_{\beta} \varphi^{\beta}, \beta = \{C, M\}\,. \end{cases}$$

- Concept: Volumetric homogenization of micro to macro structures.
- Hyperelastic model: Ogden type, i.e. Neo-Hookean.
- Simultaneously modeling fluid flow and solid deformation.
- Thermodynamics consistency satisfied.
- Decomposition of Cauchy stress into isotropic and anisotropic part.
- Permeability tensor extracted from image processing.
- Numerically easy to handle w.r.t computational cost.

Mathematical Model

Mathematical Model (1/6)

Clausius-Planck inequality of the overall aggregate reads:

$$\mathbf{T}^S \cdot \mathbf{D}_S - \rho^S (\Psi^S)'_S + \mathbf{T}^F \cdot \mathbf{D}_F - \rho^F (\Psi^F)'_F - \hat{\mathbf{p}}^F \cdot \mathbf{w}_F \geq 0 \qquad (1)$$

Incompressibility contraint introduced

$$\lambda (n^S + n^F)'_S = -\lambda (n^S \operatorname{div} \mathbf{x}'_S + n^F \operatorname{div} \mathbf{x}'_F + \operatorname{grad} n^F \cdot \mathbf{w}_F) = 0 \qquad (2)$$

Mathematical Model (1/6)

Clausius-Planck inequality of the overall aggregate reads:

$$\mathbf{T}^S \cdot \mathbf{D}_S - \rho^S(\Psi^S)'_S + \mathbf{T}^F \cdot \mathbf{D}_F - \rho^F(\Psi^F)'_F - \hat{\mathbf{p}}^F \cdot \mathbf{w}_F \geq 0 \qquad (1)$$

Incompressibility contraint introduced

$$\lambda(n^S + n^F)'_S = -\lambda(n^S \mathrm{div}\mathbf{x}'_S + n^F \mathrm{div}\mathbf{x}'_F + \mathrm{grad}\, n^F \cdot \mathbf{w}_F) = 0 \qquad (2)$$

via Lagrangean multiplier λ method, i.e. by (2) +(1), reads:

$$\begin{aligned}(\overbrace{\mathbf{T}^S + n^S\lambda\mathbf{I}}^{\mathbf{T}_E^S}) \cdot \mathbf{D}_S - \rho^S(\Psi^S)'_S + (\overbrace{\mathbf{T}^F + n^F\lambda\mathbf{I}}^{\mathbf{T}_E^F}) \cdot \mathbf{D}_F - \rho^F(\Psi^F)'_F& \\ - (\underbrace{\hat{\mathbf{p}}^F - \lambda \mathrm{grad}\, n^F}_{\hat{\mathbf{p}}_E^F}) \cdot \mathbf{w}_F \geq 0& \qquad (3)\end{aligned}$$

The overall Cauchy stress then yields:

$$\mathbf{T} = \mathbf{T}_E - \lambda\mathbf{I} \qquad \text{with} \qquad \mathbf{T}_E = \mathbf{T}_E^S + \mathbf{T}_E^F \qquad (4)$$

Mathematical Model (1/6)

Clausius-Planck inequality of the overall aggregate reads:

$$\mathbf{T}^S \cdot \mathbf{D}_S - \rho^S(\Psi^S)'_S + \mathbf{T}^F \cdot \mathbf{D}_F - \rho^F(\Psi^F)'_F - \hat{\mathbf{p}}^F \cdot \mathbf{w}_F \geq 0 \qquad (1)$$

Incompressibility contraint introduced

$$\lambda(n^S + n^F)'_S = -\lambda(n^S \mathrm{div}\mathbf{x}'_S + n^F \mathrm{div}\mathbf{x}'_F + \mathrm{grad}\, n^F \cdot \mathbf{w}_F) = 0 \qquad (2)$$

via Lagrangean multiplier λ method, i.e. by (2) +(1), reads:

$$\overbrace{(\mathbf{T}^S + n^S\lambda\mathbf{I})}^{\mathbf{T}^S_E} \cdot \mathbf{D}_S - \rho^S(\Psi^S)'_S + \overbrace{(\mathbf{T}^F + n^F\lambda\mathbf{I})}^{\mathbf{T}^F_E} \cdot \mathbf{D}_F - \rho^F(\Psi^F)'_F - \underbrace{(\hat{\mathbf{p}}^F - \lambda\,\mathrm{grad}\, n^F)}_{\hat{\mathbf{p}}^F_E} \cdot \mathbf{w}_F \geq 0 \qquad (3)$$

The overall Cauchy stress then yields:

$$\mathbf{T} = \mathbf{T}_E - \lambda\mathbf{I} \qquad \text{with} \qquad \mathbf{T}_E = \mathbf{T}^S_E + \mathbf{T}^F_E \qquad (4)$$

With neglected fluid extra stress, i.e. $\mathbf{T}^F_E \approx 0$, it is concluded:

$$\therefore \mathbf{T} = \mathbf{T}^S_E - \lambda\mathbf{I} \qquad (5)$$

Mathematical Model (2/6)

Solid extra stress $\mathbf{T}_E^S$ by an isotropic finite Neo-Hookean elasticity law $\mathbf{T}_{E,\mathrm{iso}}^S$ and an anisotropic contribution $\mathbf{T}_{E,\mathrm{aniso}}^S$:

$$\mathbf{T}_E^S = \mathbf{T}_{E,\mathrm{iso}}^S + \mathbf{T}_{E,\mathrm{aniso}}^S \tag{6}$$

Mathematical Model (2/6)

Solid extra stress $\mathbf{T}_E^S$ by an isotropic finite Neo-Hookean elasticity law $\mathbf{T}_{E,\text{iso}}^S$ and an anisotropic contribution $\mathbf{T}_{E,\text{aniso}}^S$:

$$\mathbf{T}_E^S = \mathbf{T}_{E,\text{iso}}^S + \mathbf{T}_{E,\text{aniso}}^S \tag{6}$$

$$\mathbf{T}_{E,\text{iso}}^S = \frac{\mu^S}{J_S}(\mathbf{F}_S\mathbf{F}_S^T - \mathbf{I}) + \lambda^S(1-n_{0S}^S)^2\left(\frac{1}{1-n_{0S}^S} - \frac{1}{J_S - n_{0S}^S}\right)\mathbf{I} \tag{7}$$

$$\mathbf{T}_{E,\text{aniso}}^S = \frac{\tilde{\mu}^S}{J_S} I_4^{-1}\left(I_4^{\frac{\tilde{\gamma}^S}{2}} - 1\right)(\mathbf{a}^S \otimes \mathbf{a}^S) \tag{8}$$

- $\mathbf{T}_{E,\text{iso}}^S$ $\rightarrow$ Non-linear elastic behavior of biological tissues.
- $\mathbf{T}_{E,\text{aniso}}^S$ $\rightarrow$ Vertebral structure considered as transversely isotropic.

Mathematical Model (3/6)

1^{st} Consideration: Overall momentum balance:

$$\boxed{0 = \mathrm{div}\mathbf{T} + \rho\mathbf{b}} \tag{9}$$

1^{st} Consideration: Overall momentum balance:

$$\boxed{0 = \mathrm{div}\mathbf{T} + \rho \mathbf{b}} \tag{9}$$

Overall Cauchy stress:

$$\mathbf{T} = \mathbf{T}_E^S - p\mathbf{I} \tag{10}$$

where

$$\mathbf{T}_E^S = \mathbf{T}_{E,\mathrm{iso}}^S + \mathbf{T}_{E,\mathrm{aniso}}^S \tag{11}$$

$$\mathbf{T}_{E,\mathrm{iso}}^S = \frac{\mu^S}{J_S}(\mathbf{F}_S\mathbf{F}_S^T - \mathbf{I}) + \lambda^S(1 - n_{0S}^S)^2 \left(\frac{1}{1 - n_{0S}^S} - \frac{1}{J_S - n_{0S}^S} \right) \mathbf{I} \tag{12}$$

$$\mathbf{T}_{E,\mathrm{aniso}}^S = \frac{\tilde{\mu}^S}{J_S} I_4^{-1} \left(I_4^{\frac{\tilde{\gamma}^S}{2}} - 1 \right) (\mathbf{a}^S \otimes \mathbf{a}^S) \tag{13}$$

Hydrostatic overall pore pressure:

$$p = s^C p^{CR} + s^M p^{MR} \tag{14}$$

Mathematical Model (4/6)

2^{nd} Consideration: Mass balance of a constituent

$$(\rho^\alpha)'_\alpha + \rho^\alpha \mathrm{div}\mathbf{x}'_\alpha = 0 \tag{15}$$

Mathematical Model (4/6)

2^{nd} Consideration: Mass balance of a constituent

$$(\rho^\alpha)'_\alpha + \rho^\alpha \mathrm{div}\mathbf{x}'_\alpha = 0 \tag{15}$$

with materially incompressible constraint yields volume balance of liquid constituents:

$$\boxed{0 = (n^\beta)'_S + \mathrm{div}(n^\beta \mathbf{w}_\beta) + n^\beta \mathrm{div}(\mathbf{u}_S)'_S} \tag{16}$$

Darcy-like filter velocities:

$$n^C \mathbf{w}_C = -\frac{\kappa_r^C \mathbf{K}^S}{\mu^{CR}} \left(\mathrm{grad} p^{CR} - \rho^{CR}\mathbf{b}\right) \tag{17}$$

$$n^M \mathbf{w}_M = -\frac{\kappa_r^M \mathbf{K}^S}{\mu^{MR}} \left(\mathrm{grad} p^{MR} - \rho^{MR}\mathbf{b} - \frac{p_{\mathrm{dif.}}}{s^M}\mathrm{grad} s^M\right) \tag{18}$$

Mathematical Model (5/6): Summarized equations

Volume balance of liquid constituents and overall momentum balance:

$$\boxed{\begin{aligned} 0 &= (n^\beta)'_S + \operatorname{div}(n^\beta \mathbf{w}_\beta) + n^\beta \operatorname{div}(\mathbf{u}_S)'_S \\ 0 &= \operatorname{div}\mathbf{T} + \rho\mathbf{b} \end{aligned}} \qquad \begin{aligned} (19) \\ (20) \end{aligned}$$

Compute: $\mathbf{u}_S, p^{CR}$ and p^{MR}.

Mathematical Model (5/6): Summarized equations

Volume balance of liquid constituents and overall momentum balance:

$$\boxed{\begin{aligned} 0 &= (n^\beta)'_S + \operatorname{div}(n^\beta \mathbf{w}_\beta) + n^\beta \operatorname{div}(\mathbf{u}_S)'_S \\ 0 &= \operatorname{div}\mathbf{T} + \rho\mathbf{b} \end{aligned}} \qquad \begin{aligned} (19) \\ (20) \end{aligned}$$

Compute: $\mathbf{u}_S, p^{CR}$ and p^{MR}.

$$\boxed{\begin{aligned} n^C \mathbf{w}_C &= -\frac{\kappa_r^C \mathbf{K}^S}{\mu^{CR}} \left(\operatorname{grad} p^{CR} - \rho^{CR}\mathbf{b}\right) \\ n^M \mathbf{w}_M &= -\frac{\kappa_r^M \mathbf{K}^S}{\mu^{MR}} \left(\operatorname{grad} p^{MR} - \rho^{MR}\mathbf{b} - \frac{p_{\text{dif.}}}{s^M} \operatorname{grad} s^M\right) \\ \mathbf{T} &= \mathbf{T}^S_{E,\text{iso}} + \mathbf{T}^S_{E,\text{aniso}} - p\mathbf{I} \quad \text{where} \quad p = s^C p^{CR} + s^M p^{MR} \\ \mathbf{T}^S_{E,\text{iso}} &= \frac{\mu^S}{J_S}(\mathbf{F}_S \mathbf{F}_S^T - \mathbf{I}) + \lambda^S (1 - n^S_{0S})^2 \left(\frac{1}{1 - n^S_{0S}} - \frac{1}{J_S - n^S_{0S}}\right) \mathbf{I} \\ \mathbf{T}^S_{E,\text{aniso}} &= \frac{\tilde{\mu}^S}{J_S} I_4^{-1} \left(I_4^{\frac{\tilde{\gamma}^S}{2}} - 1\right)(\mathbf{a}^S \otimes \mathbf{a}^S) \qquad (21) \end{aligned}}$$

Mathematical Model (6/6)

Volume balance of liquid constituents and overall momentum balance:

$$\boxed{\begin{aligned} 0 &= (n^\beta)'_S + \mathrm{div}(n^\beta \mathbf{w}_\beta) + n^\beta \mathrm{div}(\mathbf{u}_S)'_S \\ 0 &= \mathrm{div}\mathbf{T} + \rho \mathbf{b} \end{aligned}} \quad \begin{aligned} (22) \\ (23) \end{aligned}$$

Weak forms:

$$\begin{aligned} 0 = &\int_\Omega \left[(n^\beta)'_S + n^\beta \mathrm{div}(\mathbf{u}_S)'_S\right] \delta p^{\beta R} \mathrm{d}v \\ &- \int_\Omega n^\beta \mathbf{w}_\beta \cdot \mathrm{grad}\delta p^{\beta R} \mathrm{d}v + \int_{\Gamma_{v^\beta}} n^\beta \mathbf{w}_\beta \cdot \mathbf{n} \delta p^{\beta R} \mathrm{d}a \end{aligned} \tag{24}$$

$$0 = \int_\Omega \mathbf{T} \cdot \mathrm{grad}\delta \mathbf{u}_S \, \mathrm{d}v - \int_\Omega \rho \mathbf{b} \cdot \delta \mathbf{u}_S \, \mathrm{d}v - \int_{\Gamma_t} \mathbf{T}\mathbf{n} \cdot \delta \mathbf{u}_S \, \mathrm{d}a \tag{25}$$

Take variational $\rightarrow$ Weak forms $\rightarrow$ Discretization.

Result

Result: In short

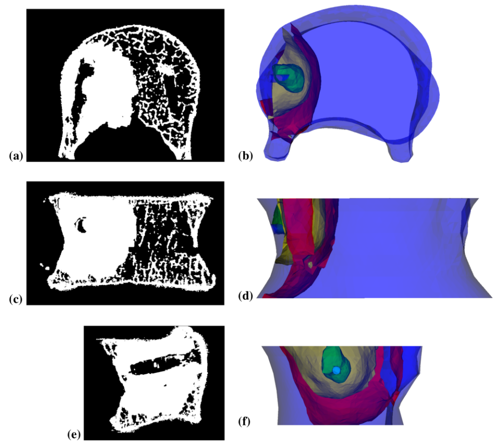

Fig. 2 – Comparison between Experiment (left) and Modeling (right) result;
View: Transversal (a,b), coronal (c,d) and sagittal (e,f)[1].

→ Qualitative comparable to experimental data.

Conclusion

Conclusion

Conclusion:

- Qualitative biphasic three-constituent model.
- Elegant approach of image-processing permeability tensor.
- Potential for clinical applications and investigations/predictions.

Future work:

- User-interface and database needed before ready use at clinics.
- Further interesting extensions, i.e. fracture modeling.

Questions

References I

1. C. Bleiler et al. Multiphasic modelling of bone-cement injection into vertebral cancellous bone; 2014.
2. Cardiovascular and Interventional Radiological Society of Europe (CIRSE). Vertebral Augmentation.
3. Neurospinal Associates. What to expect when you have a Vertebroplasty Procedure.
4. Dr. Bruce McFarlane. Notes on Anatomy and Physiology: Spinal Stenosis; June 2010.

Backup slide 1: Why $p = s^{\beta} p^{\beta R}$ or $p = s^{C} p^{CR} + s^{M} p^{MR}$

Given saturation function:

$$s^{\beta} = \frac{n^{\beta}}{n^{F}} \quad \text{with} \quad \sum_{\beta} s^{\beta} = 1 \tag{26}$$

In general we have:

$$p = \frac{F}{A} = \frac{Fd}{Ad} = \frac{\text{Energy}}{\text{Volume}} \tag{27}$$

Here

$$\left.\begin{aligned} p &= \frac{\text{Energy}}{dv^{F}} \\ p^{\beta R} &= \frac{\text{Energy}}{dv^{\beta}} \end{aligned}\right\} \Rightarrow p dv^{F} = p^{\beta R} dv^{\beta} \tag{28}$$

Volume fractions give:

$$n^{F} = \frac{dv^{F}}{dv} \Rightarrow dv^{F} = n^{F} dv \tag{29}$$

$$n^{\beta} = \frac{dv^{\beta}}{dv} \Rightarrow dv^{\beta} = n^{\beta} dv \tag{30}$$

Insertion of (29) and (30) into (28) yields: $\boxed{p = s^{\beta} p^{\beta R}}$