



# Argonne National Laboratory Seminar: An adjoint of nucleation in All-solid-state batteries

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Applied and  
Computational  
Mathematics

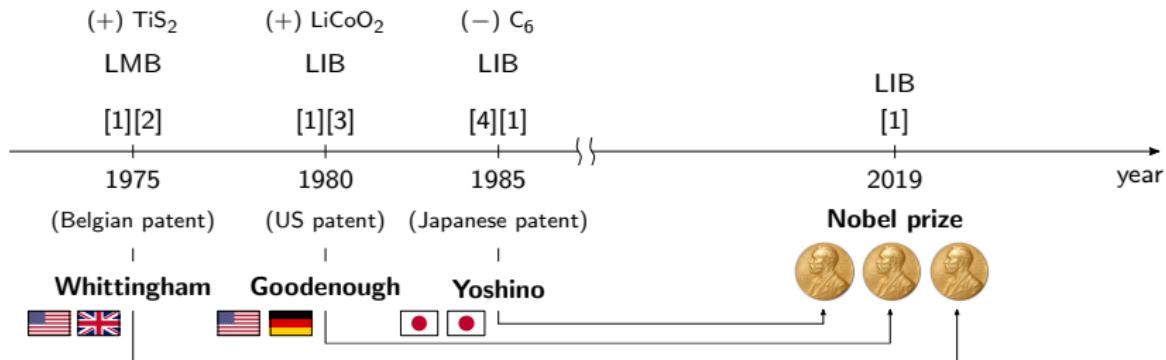
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# Content

- **Part I:** An adjoint of nucleation in All-solid-state batteries
  - ① Motivations
  - ② Mathematical model in summary
  - ③ Adjoint Griffith nucleation criterion
  - ④ Summary: Conclusion, Outlook, and Literature
- **Part II:** Other relevant experience before and during PhD
  - ① Bachelor thesis at BOSCH Germany
  - ② Master thesis at BOSCH Germany

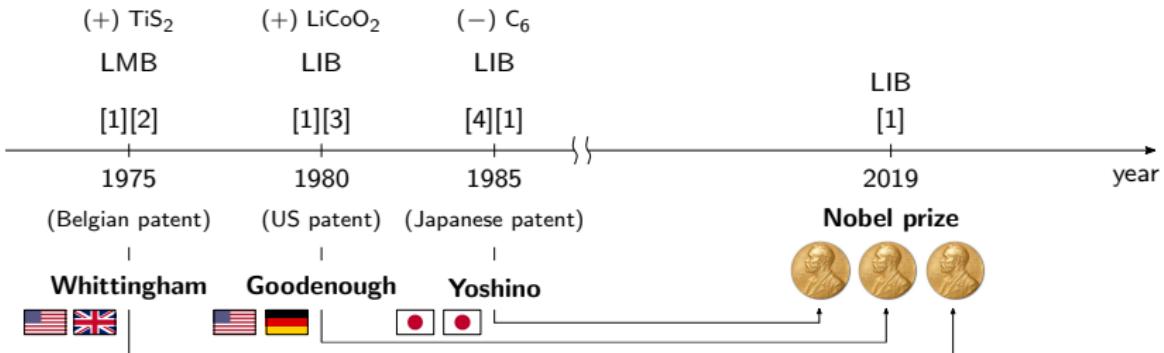
# I-1. Motivation: Lithium-ion battery (LIB) is useful



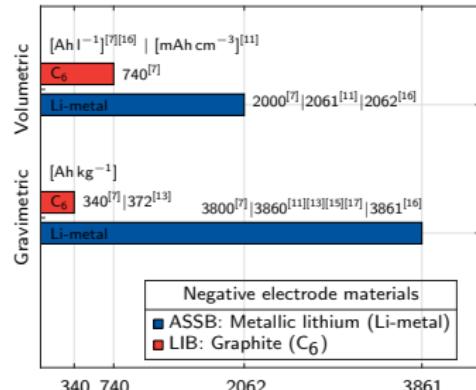
..and LIB has been widely used in various applications, such as

- Electric vehicle (EV)
- Portable electronic device (PED)
- Energy storage system (ESS)

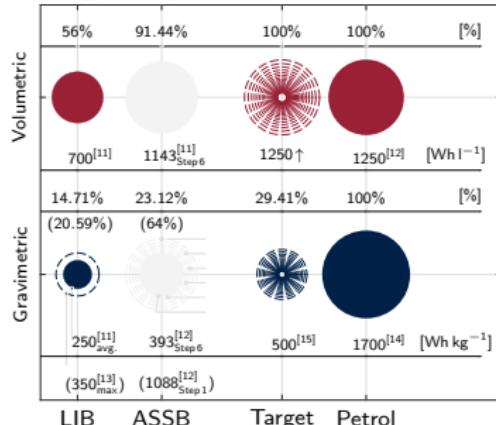
# I-1. Motivation: But LIB exists limited energy density



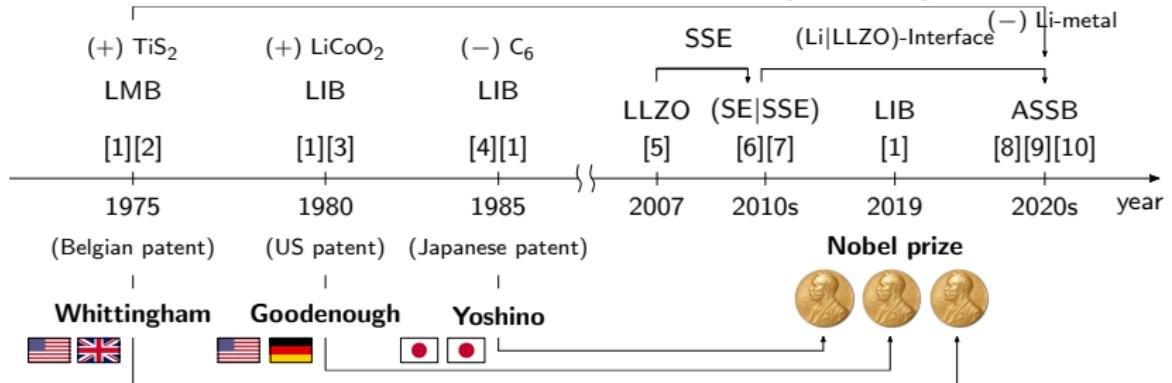
Theoretical capacity of charge:  
pure metallic lithium (Li-metal) versus Graphite (C<sub>6</sub>)



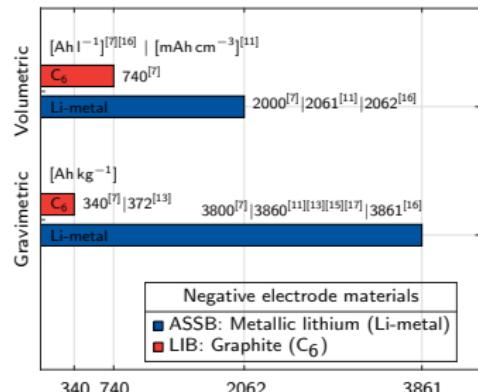
Energy density: ASSB versus LIB versus Petrol



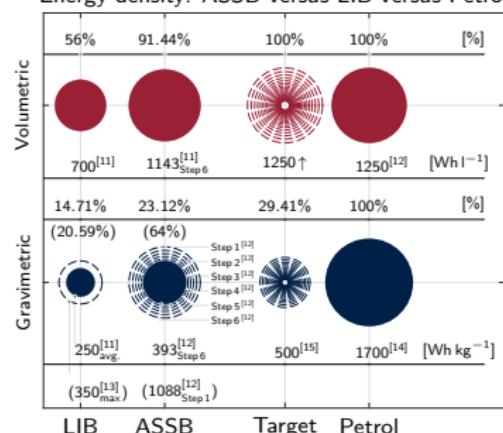
# I-1. Motivation: All-solid-state battery (ASSB) is promising



Theoretical capacity of charge:  
pure metallic lithium (Li-metal) versus Graphite ( $\text{C}_6$ )



Energy density: ASSB versus LIB versus Petrol



# I-1. Motivation: Still, ASSB has drawbacks itself

## Problem:

- ① LIB is useful, but generally limited by its energy density.
- ② ASSB is promising with its energy density, but has drawbacks.
- ③ ASSB drawbacks: **nucleation, dendrite growth, interface**.

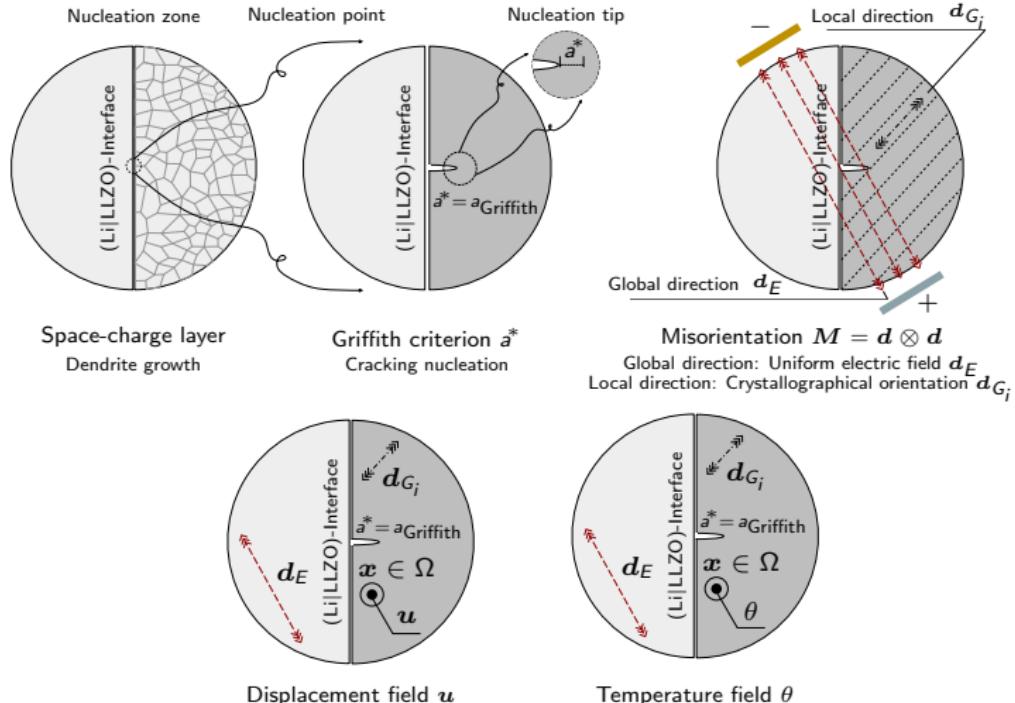
## Aim:

- ① Tackle the **adjoint nucleation** at the ASSB interface stability.

## Study:

- ① Outline the **nucleation criterion** into a mathematical model.
- ② Develop a numerical method to **visualise** the nucleation.
- ③ Examine the results with **relevant material** candidates.

## I-2. Mathematical model in summary (1/3)



- ① Coupled fields include deformation and temperature.
- ② Misorientation tensors present grain disorder structure.
- ③ Thermodynamic consistency is satisfied, closure problem is fulfilled.

## I-2. Mathematical model in summary (2/3)

**Coupled fields** are Deformation field  $\mathbf{u}$  and Temperature field  $\theta$ :

$$\mathbf{u} : \begin{cases} \Omega \times \mathbb{R}_+ \rightarrow \mathbb{R}^3, \\ (\mathbf{x}, t) \mapsto \mathbf{u}(\mathbf{x}, t), \end{cases} \quad \theta : \begin{cases} \Omega \times \mathbb{R}_+ \rightarrow \mathbb{R}, \\ (\mathbf{x}, t) \mapsto \theta(\mathbf{x}, t). \end{cases}$$

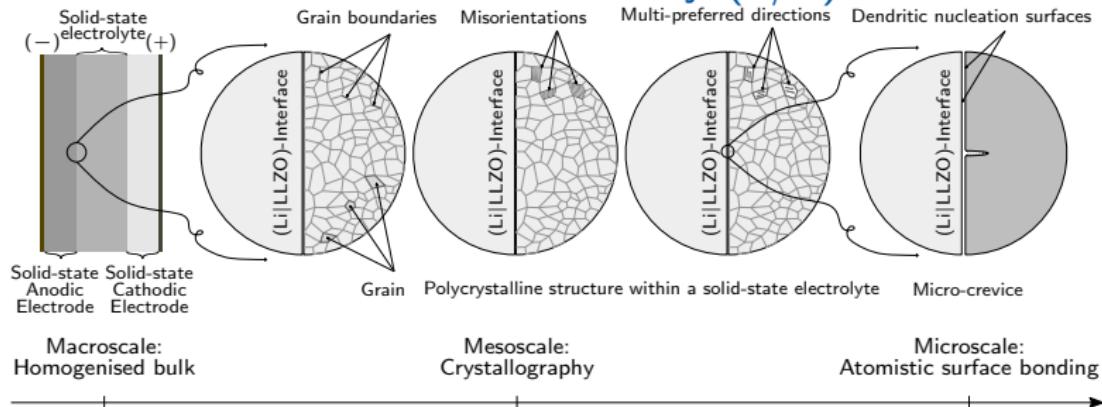
**Governing conservation equation** takes the form

$$\frac{d}{dt} \int_{\Omega} (\cdot) \, d\Omega = \int_{\Omega} (\cdot)^{\text{action}} \, d\Omega + \int_{\partial\Omega} (\cdot)^{\text{action}} \, d\partial\Omega + \int_{\Omega} (\cdot)^{\text{production}} \, d\Omega$$

which accounts for mass balance, linear and angular momentum, and energy conservation.

**Variables** include mass density  $\rho(\mathbf{x}, t)$ , body force  $\mathbf{b}(\mathbf{x}, t)$ , velocity  $\mathbf{v}(\mathbf{x}, t)$ , internal energy  $e(\mathbf{x}, t)$ , heat flux  $\mathbf{q}(\mathbf{x}, t)$ , heat source  $r(\mathbf{x}, t)$ , Cauchy stress  $\boldsymbol{\sigma}$ , and infinitesimal strain  $\boldsymbol{\varepsilon}$  per unit volume.

## I-2. Mathematical model in summary (3/3)



Find  $a$  such that the following optimisation problem  $\forall a \in \mathcal{V}$  hold:

$$\rho \frac{D^2 \mathbf{U}}{Dt^2} - \nabla \cdot \left( \mathbb{C}_{(\lambda, \mu)}^{T_f \phi} : \nabla \mathbf{U}^{(s)} \right) + \rho V_E = \mathbf{0}, \quad (1)$$

s.t.  $a_{\text{Griffith}}^{\text{generalised}} := a^* = \arg \min_{a \in \mathcal{V}} \left\{ \iiint_{\Omega} f(a, \mathbf{u}, \theta, c^{m|\text{Li}^+|n}; \lambda, \mu, \mathbf{d}_{G_j} \otimes \mathbf{d}_{G_j}) d\Omega - \iint_{\Gamma} f(a; \gamma) d\Gamma \right\} \quad (2)$

# I-3 Adjoint Griffith nucleation criterion (1/4)

Adjoint Griffith nucleation criterion in terms of **critical length** has two forms:

$a_{Griffith}^{generalised}$	$a_{Griffith}^{simplified}$
<ul style="list-style-type: none"><li>• Surface energy</li><li>• Strain energy</li><li>• Grain disorder structure</li></ul>	<ul style="list-style-type: none"><li>• Surface energy</li><li>• Strain energy</li><li>• ×</li></ul>

**Observations:**

- ① **Generalised** Griffith criterion is an optimisation problem, considering **surface energy**, **strain energy**, and **grain disorder structure**.
- ② **Simplified** Griffith criterion requires **surface energy** and **strain energy**.
- ③ **Simplified** Griffith criterion in terms of **critical length** takes the form

$$a_{Griffith}^{simplified} = \frac{2E\gamma_s}{\pi\sigma^2} \Leftrightarrow \sigma_{Griffith} = \sqrt{\frac{2E\gamma_s}{\pi a}}$$

where  $\sigma$  is the Cauchy stress,  $E$  Young's modulus, and  $\gamma_s$  surface energy.

# I-3 Adjoint Griffith nucleation criterion (2/4)

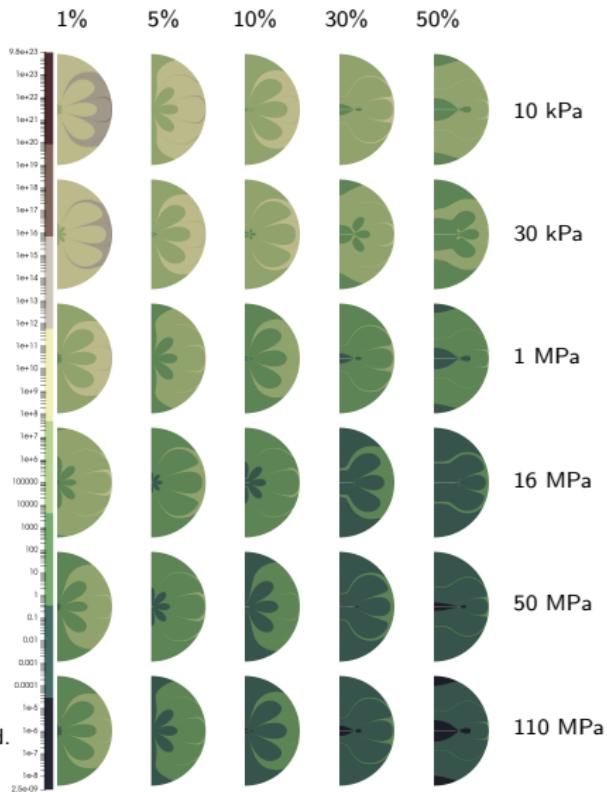
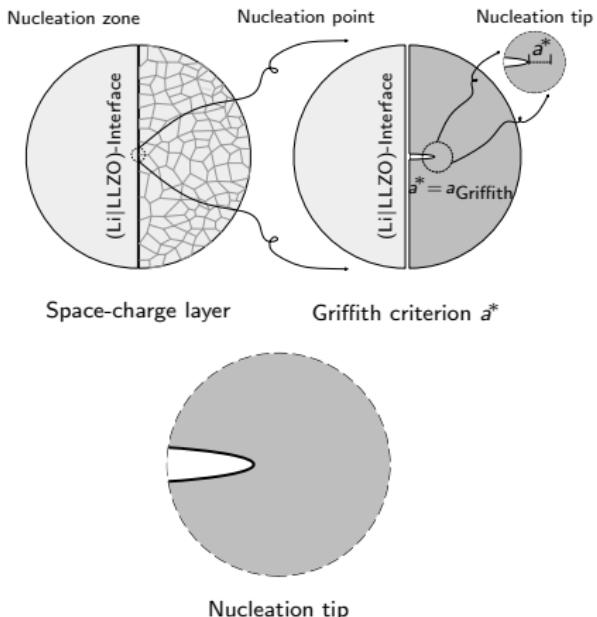
Adjoint Griffith nucleation criterion in terms of critical length has two forms:

$a_{Griffith}^{generalised}$	$a_{Griffith}^{simplified}$
<ul style="list-style-type: none"><li>• Surface energy (DFT)</li><li>• Strain energy (Continuum)</li><li>• Grain disorder structure (FEM)</li></ul>	<ul style="list-style-type: none"><li>• Surface energy (DFT)</li><li>• Strain energy (Continuum)</li><li>• ×</li></ul>

## Approaches:

- ① **Surface energy** is computed using **DFT calculations**.
- ② **Strain energy** is computed using deformation problems in **Continuum**.
- ③ Simplified Griffith nucleation may require **FEM** or **FVM**, along with **DFT**.
- ④ Generalised Griffith nucleation may require **FEM** and **DFT**.
- ⑤ **Grain disorder structure** is a 4th order tensor, and computed using **FEM**.
- ⑥ Study **25Ta<sub>Zr</sub><sup>•</sup>-LLZO**, **50Ta<sub>Zr</sub><sup>•</sup>-LLZO**, **28Al<sub>Li</sub><sup>••</sup>-LLZO**, **36Al<sub>Li</sub><sup>••</sup>-LLZO**.
- ⑦ The results are visualised in the following figures.

### I-3 Adjoint Griffith nucleation criterion (3/4)

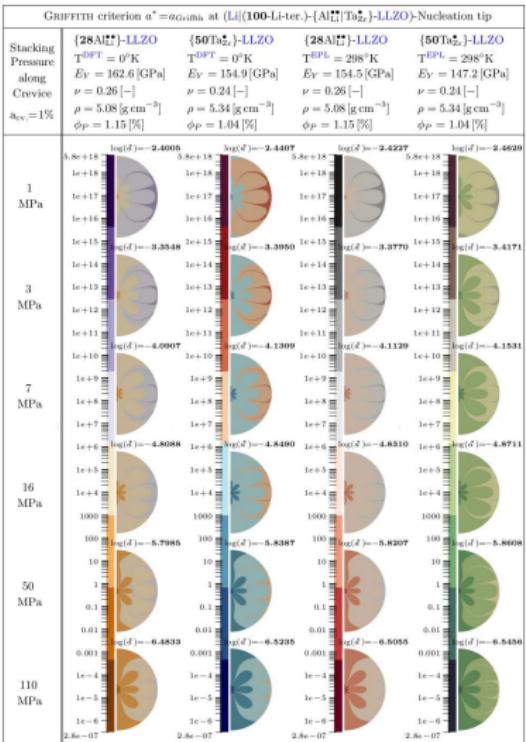
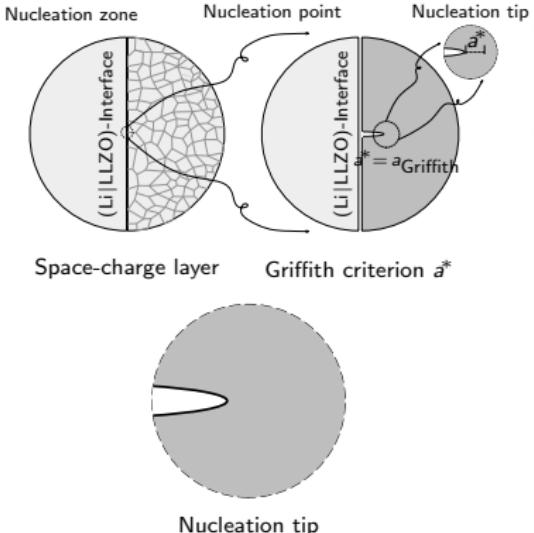


- Horizontal axis: Crevice length.
  - Vertical axis: Stacking pressure.
  - OpenFOAM **solidDisplacementFoam.C** solver is developed.
  - Nucleation criterion is shown using **logarithmic scale**.

## Nucleation at the adjoint interfaces in (100Li)-50Ta<sub>7<sub>r</sub></sub>-LLZO

# I-3 Adjoint Griffith nucleation criterion (4/4)

## Further studies



### Observations:

- Stacking pressure influences nucleation.
- Pre-existing crevice presents grain boundary.
- Griffith criterion is observed on log-scale.

# I-4 Summary

## Conclusion:

- ① A generalised Griffith criterion is developed for nucleation adjoint in ASSB.
- ② Both **DFT** and **Continuum** methods are used for nucleation criterion.
- ③ DFT calculation results for **LLZO-surfaces** are obtained using literature [6].
- ④ Continuum calculations have been studied and presented in [1][2][3][4].
- ⑤ **Griffith** criterion has been studied and presented in [5].
- ⑥ **Simplified** Griffith criterion is used to study LLZO adjoint nucleation.

## Outlook:

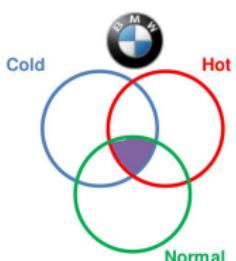
- ① Would metallic Li **anode-less** ASSB be a solution eventually?

## Literature:

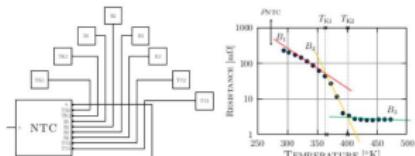
- 1 T.Vo, C.Hüter, S.Braun, Poster, *BOSCH Spring School on AI in Industry*, Robert Bosch GmbH, 2024.
- 2 T.Vo, C.Hüter, S.Braun, M.Torrilhon, Poster, *Oxford Battery Modelling Symp.*, University of Oxford, 2023.
- 3 T.Vo, C.Hüter, S.Braun, M.Torrilhon, Next-gen ASSB Poster, *SIAM Conference on Comp. Sci. Eng.*, 2023.
- 4 T.Vo, NUMAP-FOAM Summer School, OpenFOAM Pres., Maxwell Centre, University of Cambridge, 2022.
- 5 T.Vo, C.Hüter, S.Braun, R.Spatschek, ASSB Griffith, IEK Doc. Pres., Forschungszentrum Jülich, 2020.
- 6 P.Barai, A.Ngo, B.Narayanan, K.Higa, L.Curtiss, V.Srinivasan, LLZO, *J. of the electrochemical soc.*, 2020.
- 7 C.Hüter, S.Fu, M.Finsterbusch, E.Figgemeier, L.Wells, R.Spatschek, E-E interface, *AIMS Mat. Sci.*, 2017.
- 8 S.Braun, C.Yada, A.Latz, Space-charge layer ASSB, *J. Physical Chem. C*, 119, 22281-22288, 2015.

# II-1. Other relevant experience: BOSCH Bachelor thesis

## Semiconductor<sup>[I]</sup>



**Application:** Start/Stop-System in Starter.  
**Use-case:** BMW B47 (-25°C, 0°C, 120°C).  
**Optimisation:** Pareto @BoschForschung.  
(Multi-objective optimisation framework)



Nd/Gd Negative-Temperature Coefficient (NTC) semiconductor model validated [7].

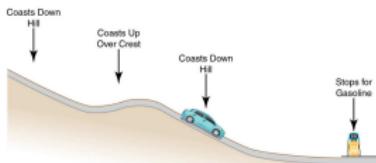


Figure 1.2.: Coasting function of St/St Starter [14].

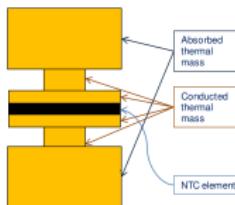


Figure 5.1.: Symmetry model of a NTC component.

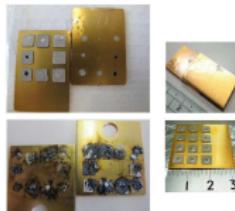
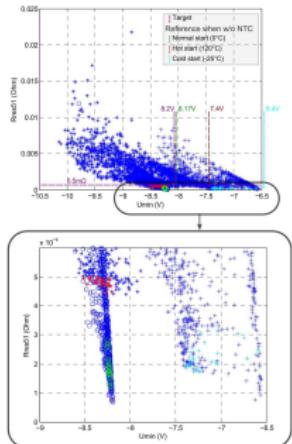


Table 6.4.: Testing conditions and Requirements of BMW B47 4 Cylinders Engine.

Conditions	-25°C	0°C	120°C
$U_{\text{Starter}}$	11V	12.05	12
$M_{\text{Starter}}$	3.348N	4.3	6.3
$R_{\text{Starter}}$	6.664Ω	5.7	5.4
$I_{\text{Starter}}$	1.63A	1.05A	0.95A
$T_{\text{Starter}}$	33.3	75	75
$M_{\text{Generator}}$	0.037Nm	0.76	1.04
$\Delta M_{\text{Generator}}$	0.0004Nm <sup>2</sup>	0.0004	0.0004
$\Delta \dot{Q}_{\text{Generator}}$	0.0005W/m <sup>2</sup> /A	0.0002	0.0005
$I_{\text{Generator}}$	5.3A	5.8	6.0
Requirements	$\omega = 120\text{min}^{-1}$	$U_{\text{St}} = 8.2V$	$U_{\text{St}} = 8.2V$



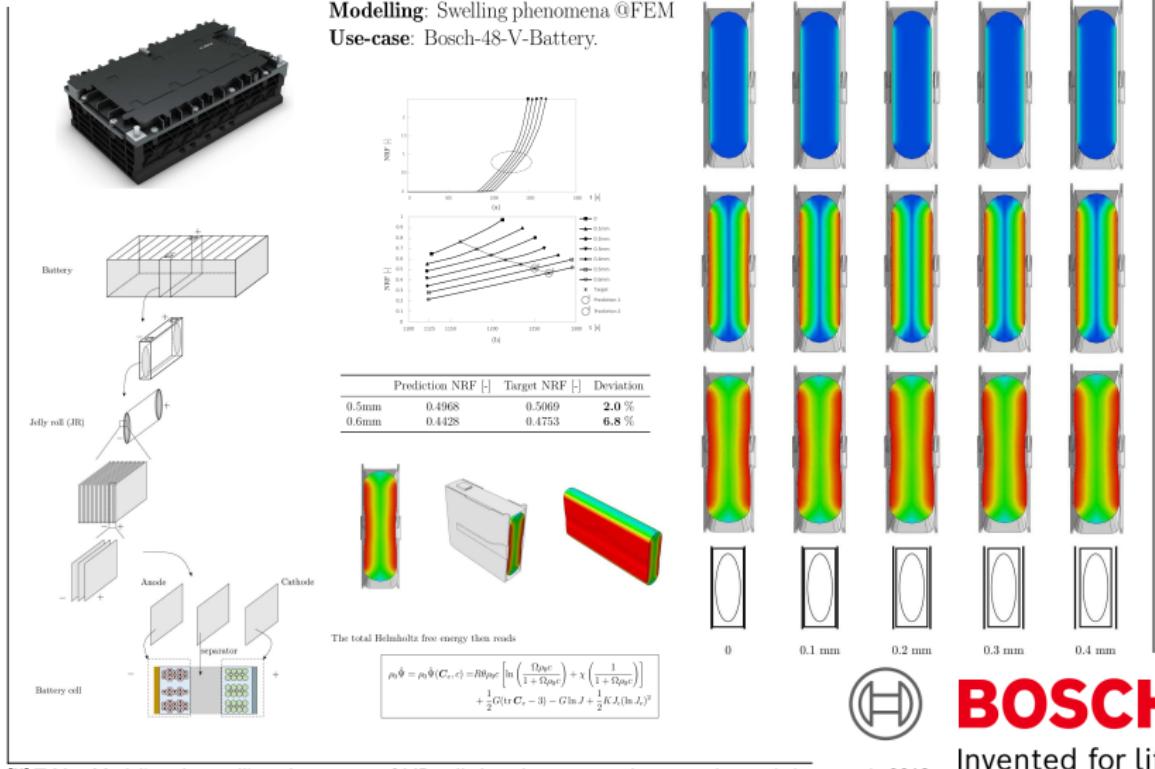
# BOSCH

Invented for life

[I] T.Vo, Sim. environment for NTC-based vol. drop reduction in Start/Stop appl. and its optimisation, 2014.

## II-2. Other relevant experience: BOSCH Master thesis

### Battery<sup>[II]</sup>



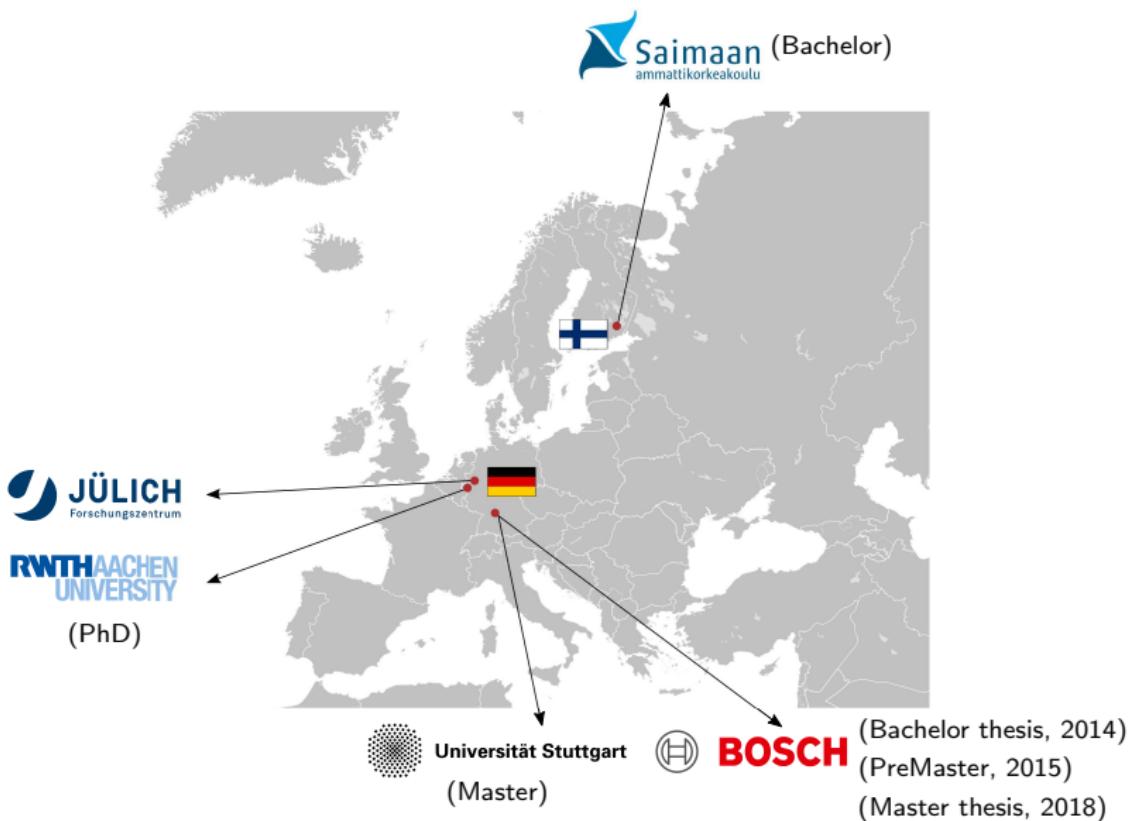
[II] T.Vo, Modeling the swelling phenomena of LIB cells based on a num. chemo-mech. coupled approach, 2018.



**BOSCH**

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## II-3. Other relevant experience: Summary



Thank you for your listening.