# Global exercise - GUE10

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Content covered:

- ✓ Analysis: Surface integral
    1. Area
    2. Orientable surface
- ✓ Numerics: Optimization

## 1 Analysis: Surface integral

**Example 1.** *Examine the **Möbius band** given as follows*

$$\vec{\gamma} : (-1,1) \times (0,2\pi) \to \mathbb{R}^3$$

*with*

$$\vec{\gamma}(t,\phi) = \begin{pmatrix} \cos(\phi) \\ \sin(\phi) \\ 0 \end{pmatrix} + \frac{t}{2} \begin{pmatrix} \cos(\phi)\cos(\phi/2) \\ \sin(\phi)\cos(\phi/2) \\ \sin(\phi/2) \end{pmatrix}.$$

*Show that this surface is **not orientable**.*

Approach:

*Proof.* The normal field is computed as follows

$$\begin{aligned} n(t,\phi) &= \pm \frac{\partial_t \vec{\gamma} \times \partial_\phi \vec{\gamma}}{||\partial_t \vec{\gamma} \times \partial_\phi \vec{\gamma}||} \\ &= \frac{\mp 1}{a} \begin{pmatrix} \cos(\phi)\sin(\frac{\phi}{2}) - \frac{t}{4}\sin(\phi)(1-\cos(\phi)) \\ \sin(\phi)\sin(\phi/2) + \frac{t}{4}(\sin^2(\phi)+\cos(\phi)) \\ -\cos(\phi/2) - \frac{t}{4}(1+\cos(\phi)) \end{pmatrix} \end{aligned}$$

with

$$a = \sqrt{1 + t\cos(\phi/2) + \frac{t^2}{16}(3 + 2\cos(\phi))}.$$

We choose the positive sign and consider the position $(1,0,0)^T = \vec{\gamma}(0,0) = \vec{\gamma}(0,2\pi)$:

$$\lim_{\phi\to 0} n(0,\phi) = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \neq \lim_{\phi \to 2\pi} n(0,\phi) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

Since the normal field is not continuous on

$$\gamma([-1,1]\times[0,2\pi])$$

the surface is **not orientable**. □