

Self-exercise - SRU05

Response to questions

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1 Hint for A1-HA05

Example 1. Given the set $\mathcal{M} \in \mathbb{R}^2$ bounded by three points A , B , and C , as shown in Figure 1.

1. Examine the integral of a function $f(x, y)$ over set \mathcal{M} .
2. Compute the area of \mathcal{M} by setting $f(x, y) = 1$.

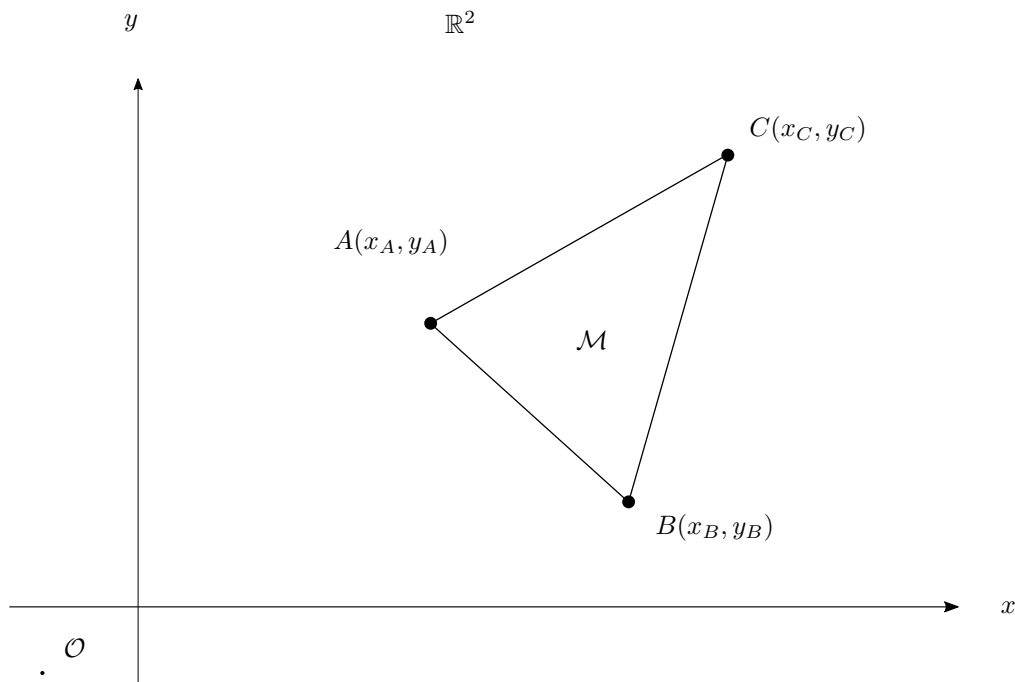


Figure 1: Set $\mathcal{M} \in \mathbb{R}^2$.

Approach:

We will look at the Figure 1 and consider it in two scenarios as follows

1. Scenario 1: Figure 2,
2. Scenario 2: Figure 3,

which we will see later on that the two approaches will actually lead to the same solution, thanks to Fubini's theorem [1].

Recall 1. Equation of a line (l) in general reads

$$(l) : \quad y = a x + b, \quad (1)$$

where a is the slope of the line, and b is the point where the line (l) crosses over the vertical line of the axis. Hence, when this line (l) going through point $P(x_P, y_P)$ and point $Q(x_Q, y_Q)$, it leads to the following two relations

$$y_P = a x_P + b, \quad (2)$$

$$y_Q = a x_Q + b, \quad (3)$$

which leads to the slope a and vertical point b as follows

$$a = \frac{y_P - y_Q}{x_P - x_Q}, \quad (4)$$

$$b = \frac{-y_P x_Q + y_Q x_P}{x_P - x_Q}. \quad (5)$$

Finally, equation of a line (l) passing through point $P(x_P, y_P)$ and point $Q(x_Q, y_Q)$ in general takes the form

$$\therefore \quad \boxed{y = \frac{y_P - y_Q}{x_P - x_Q} x + \frac{-y_P x_Q + y_Q x_P}{x_P - x_Q}}. \quad (6)$$

By taking advantage of the formula (6) we are ready to obtain the following three line functions for (AB) , (AC) , and (BC) as follows

$$(AB) : \quad y = -x + 3, \quad (7)$$

$$(AC) : \quad y = \frac{1}{2}x + \frac{3}{2}, \quad (8)$$

$$(BC) : \quad y = 2x - 3, \quad (9)$$

which we, alternatively, we can also write those expressions in terms of x , as follows

$$(AB) : \quad x = -y + 3, \quad (10)$$

$$(AC) : \quad x = 2y - 3, \quad (11)$$

$$(BC) : \quad x = \frac{1}{2}y + \frac{3}{2}, \quad (12)$$

1. Scenario 1: Figure 2

Followed by the \mathcal{M}_x split we obtain the set \mathcal{M}_1

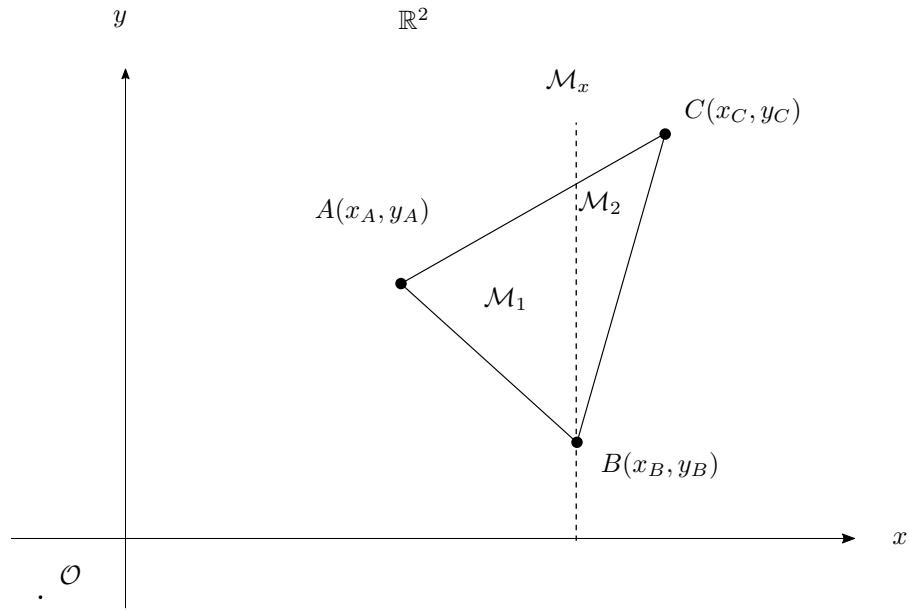


Figure 2: Set $\mathcal{M} \in \mathbb{R}^2$ where $\mathcal{M} = \mathcal{M}_1 \cup \mathcal{M}_2$ and split by \mathcal{M}_x .

$$\mathcal{M}_1 = \left\{ y \in \mathbb{R} \mid -x + 3 \leq y \leq \frac{1}{2}x + \frac{3}{2} \right\}, \quad (13)$$

which is due to the fact that while following the y -axis we first come across the line (AB) and then the line (AC) . Similarly, the set \mathcal{M}_2 is obtained as follows

$$\mathcal{M}_2 = \left\{ y \in \mathbb{R} \mid 2x - 3 \leq y \leq \frac{1}{2}x + \frac{3}{2} \right\}, \quad (14)$$

2. Scenario 2: Figure 3

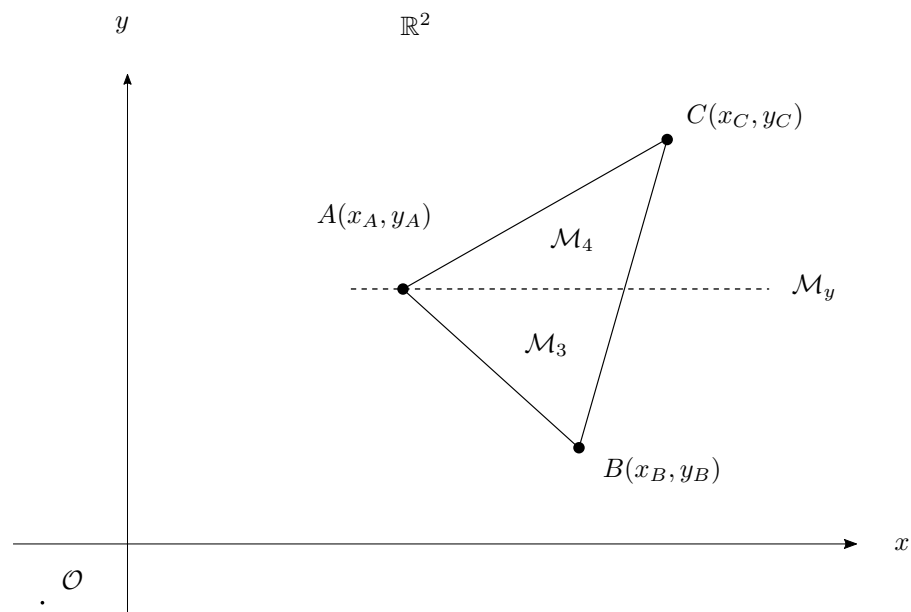


Figure 3: Set $\mathcal{M} \in \mathbb{R}^2$ where $\mathcal{M} = \mathcal{M}_3 \cup \mathcal{M}_4$ and split by \mathcal{M}_y .

References

- [1] Fubini's theorem. https://en.wikipedia.org/wiki/Fubini%27s_theorems. Accessed: 2022-11-19.