Self-exercise - SRU05 Response to questions

Tuan Vo

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1 Hint for A1-HA05

Example 1. Given the set $M \in \mathbb{R}^2$ bounded by three points A, B, and C, as shown in Figure 1.

- 1. Examine the integral of a function f(x,y) over set \mathcal{M} .
- 2. Compute the area of \mathcal{M} by setting f(x,y) = 1.

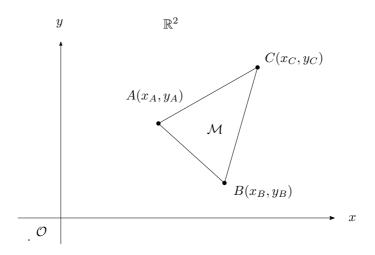


Figure 1: Set $M \in \mathbb{R}^2$.

We will consider two scenarios as depicted in Figure 2 and Figure 3.

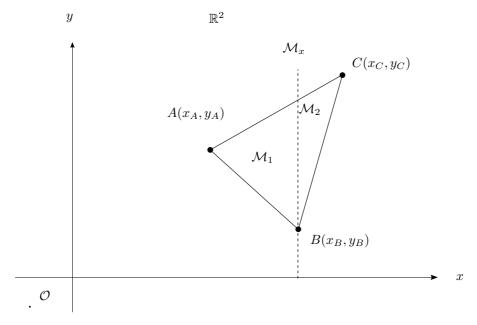


Figure 2: Set $\mathcal{M} \in \mathbb{R}^2$ where $\mathcal{M} = \mathcal{M}_1 \cup \mathcal{M}_2$ and split by \mathcal{M}_x .

Recall 1. Equation of a line (l) in general reads

$$(l): \quad y = a x + b, \tag{1}$$

where a is the slope of the line, and b is the point where the line (l) crosses over the vertical line of the axis. Hence, when this line (l) going through point $P(x_P, y_P)$ and point $Q(x_Q, y_Q)$, it leads to the following two relations

$$y_P = a x_P + b, (2)$$

$$y_Q = a x_Q + b, (3)$$

which leads to the slope a and vertical point b as follows

$$a = \frac{y_P - y_Q}{x_P - x_Q},\tag{4}$$

$$a = \frac{y_P - y_Q}{x_P - x_Q},$$

$$b = \frac{-y_P x_Q + y_Q x_P}{x_P - x_Q}.$$
(5)

Finally, equation of a line (l) passing through point $P(x_P, y_P)$ and point $Q(x_Q, y_Q)$ in general takes the form

$$\therefore \quad y = \frac{y_P - y_Q}{x_P - x_Q} x + \frac{-y_P x_Q + y_Q x_P}{x_P - x_Q}.$$
 (6)

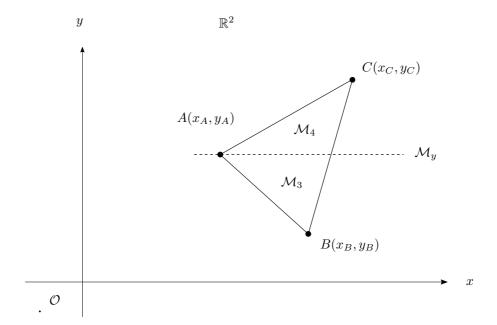


Figure 3: Set $\mathcal{M} \in \mathbb{R}^2$ where $\mathcal{M} = \mathcal{M}_3 \cup \mathcal{M}_4$ and split by \mathcal{M}_y .