Global exercise - GUE10

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Content covered:

- ✓ Analysis: Surface integral
  1. Area
  2. Orientable surface
- ✓ Numerics: Optimization

1 Analysis: Surface integral

Example 1. *Examine the* ***Möbius band*** *given as follows*

$$\vec{\gamma} : (-1, 1) \times (0, 2\pi) \to \mathbb{R}^3$$

with

$$\vec{\gamma}(t, \phi) = \begin{pmatrix} \cos(\phi) \\ \sin(\phi) \\ 0 \end{pmatrix} + \frac{t}{2} \begin{pmatrix} \cos(\phi)\cos(\phi/2) \\ \sin(\phi)\cos(\phi/2) \\ \sin(\phi/2) \end{pmatrix}.$$

Show that this surface is ***not orientable****.*

Approach:

Proof. The normal field is computed as follows

$$\begin{aligned} n(t, \phi) &= \pm \frac{\partial_t \vec{\gamma} \times \partial_\phi \vec{\gamma}}{\|\partial_t \vec{\gamma} \times \partial_\phi \vec{\gamma}\|} \\ &= \frac{\mp 1}{a} \begin{pmatrix} \cos(\phi)\sin(\frac{\phi}{2}) - \frac{t}{4}\sin(\phi)(1 - \cos(\phi)) \\ \sin(\phi)\sin(\phi/2) + \frac{t}{4}(\sin^2(\phi) + \cos(\phi)) \\ -\cos(\phi/2) - \frac{t}{4}(1 + \cos(\phi)) \end{pmatrix} \end{aligned}$$

with

$$a = \sqrt{1 + t\cos(\phi/2) + \frac{t^2}{16}(3 + 2\cos(\phi))}.$$

We choose the positive sign and consider the position $(1, 0, 0)^T = \vec{\gamma}(0, 0) = \vec{\gamma}(0, 2\pi)$:

$$\lim_{\phi \to 0} n(0, \phi) = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \neq \lim_{\phi \to 2\pi} n(0, \phi) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

Since the normal field is not continuous on

$$\gamma([-1, 1] \times [0, 2\pi])$$

the surface is **not orientable**. □