Global exercise - GUE10

Tuan Vo

22nd December, 2022

Content covered:

- ✓ Analysis: Surface integral
- ✓ Analysis: Review H9A3, H9A4, and H10A1
- ✓ Numerics: Demo programming exercise 3 (PRU02)

1 Analysis: Surface integral

Example 1. *Examine the* ***Möbius band*** *given as follows*

$$\vec{\gamma} : \begin{cases} (-1,1) \times (0, 2\pi) \to \mathbb{R}^3, \\ (t,\phi) \mapsto \vec{\gamma}(t,\phi) := \begin{pmatrix} \cos(\phi) \\ \sin(\phi) \\ 0 \end{pmatrix} + \frac{t}{2} \begin{pmatrix} \cos(\phi)\cos(\phi/2) \\ \sin(\phi)\cos(\phi/2) \\ \sin(\phi/2) \end{pmatrix}. \end{cases} \tag{1}$$

Show that this surface is ***not orientable****.*

Approach:

Proof. The normal field is computed as follows

$$\begin{aligned} n(t,\phi) &= \pm \frac{\partial_t \vec{\gamma} \times \partial_\phi \vec{\gamma}}{\|\partial_t \vec{\gamma} \times \partial_\phi \vec{\gamma}\|} \\ &= \frac{\mp 1}{a} \begin{pmatrix} \cos(\phi)\sin(\frac{\phi}{2}) - \frac{t}{4}\sin(\phi)(1-\cos(\phi)) \\ \sin(\phi)\sin(\phi/2) + \frac{t}{4}(\sin^2(\phi) + \cos(\phi)) \\ -\cos(\phi/2) - \frac{t}{4}(1+\cos(\phi)) \end{pmatrix} \end{aligned}$$

with

$$a = \sqrt{1 + t\cos(\phi/2) + \frac{t^2}{16}(3 + 2\cos(\phi))}.$$

We choose the positive sign and consider the position $(1,0,0)^T = \vec{\gamma}(0,0) = \vec{\gamma}(0,2\pi)$:

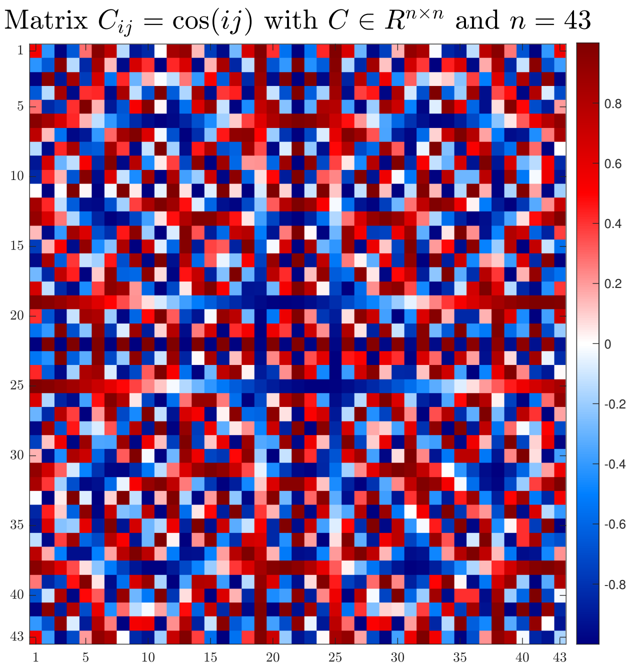
$$\lim_{\phi \to 0} n(0,\phi) = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \neq \lim_{\phi \to 2\pi} n(0,\phi) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

Since the normal field is not continuous on

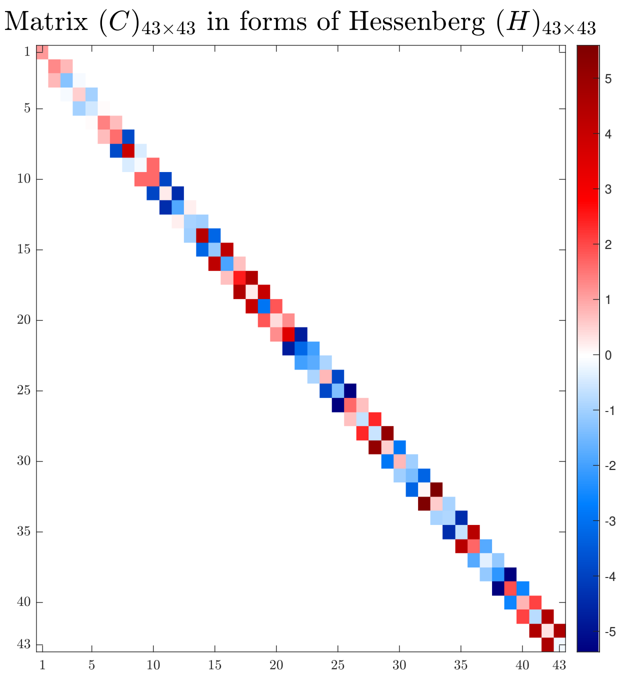
$$\gamma([-1,1] \times [0,2\pi])$$

the surface is **not orientable**. □

2 Numerics: Demo programming exercise 3 PRU(02)



(a)



(b)

Figure 1: Matrix size 43×43 and its Hessenberg form.

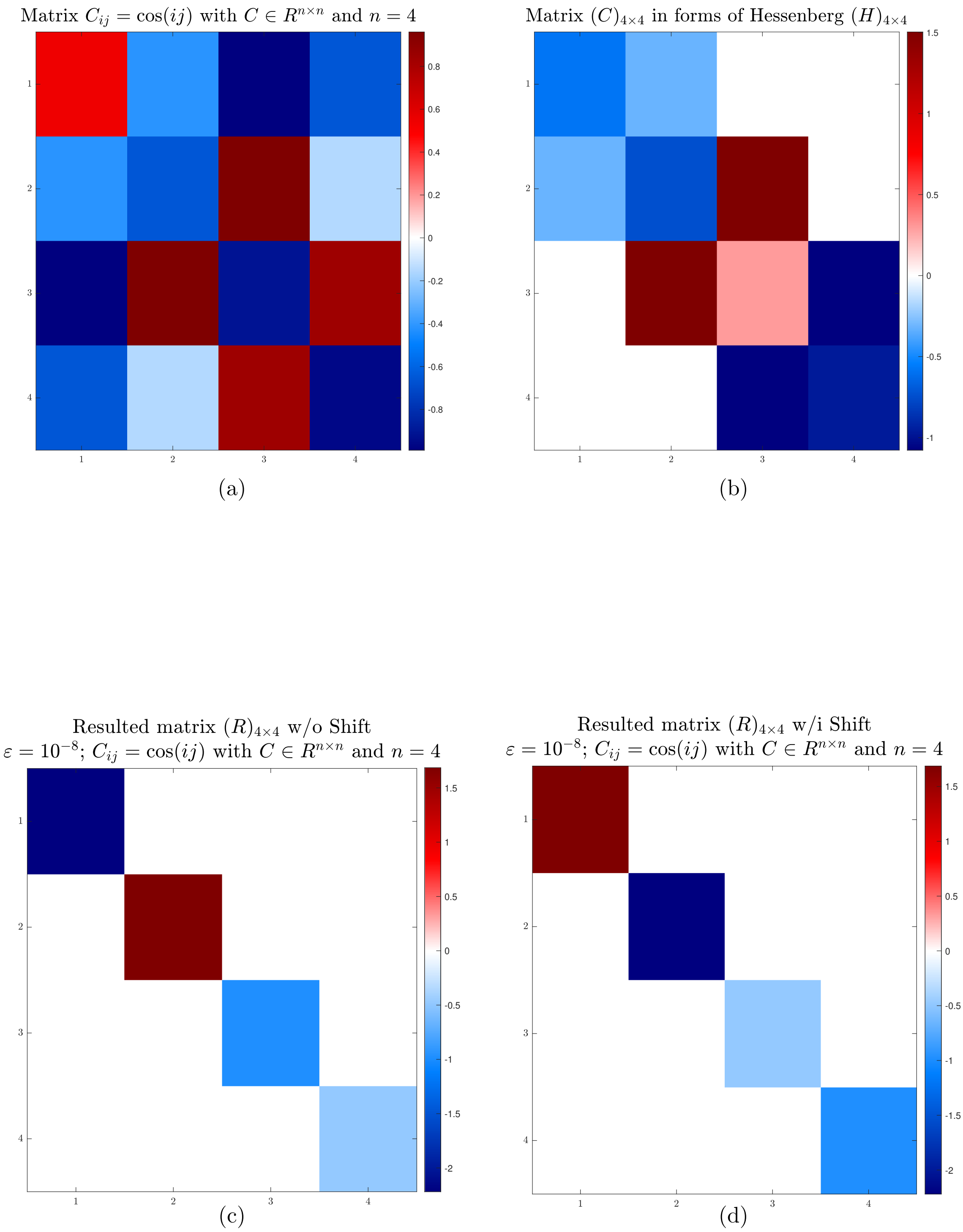


Figure 2: Matrix size 4×4 and its Hessenberg form.