Global exercise - GUE11

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12$^{\text{th}}$ January, 2023

Summary of content covered: Analysis

- Review line integral of the 1st kind (scalar field) and 2nd kind (vector field)

- Surface integral of the 1st kind (scalar field) and 2nd kind (vector field)

- Gauss's theorem and its useful applications

1 Course evaluation

The evaluation for the course

22W-11.03530 (L) Mathematische Grundlagen III (CES) (Übung)

has been opened and can be accessed until 20.01.2023 23:59:00 via QR code

Figure 1: Evaluation QR-code.

or via the following link
https://www.campus.rwth-aachen.de/evasys/online.php?pswd=8J5FHC8JH9

2 Analysis: Line integral and Surface integral

2.1 First kind: Scalar field

Recall 1. *Line integral of a **scalar field** $\phi : \Omega \to \mathbb{R}$ is defined as follows*

$$\int_\Gamma \phi \, ds := \int_a^b \phi(\gamma(t)) \big\| \gamma'(t) \big\| \, dt \tag{1}$$

Recall 2. *Surface integral of a **scalar field** $\phi : \Omega \to \mathbb{R}$ is defined as follows*

$$\int_\Gamma \phi \, ds := \int_a^b \phi(\gamma(t)) \big\| \gamma'(t) \big\| \, dt \tag{2}$$

2.2 Second kind: Vector field

Recall 3. *Line integral of a **vector field** $\boldsymbol{f} : \Omega \to \mathbb{R}^n$ is defined as follows*

$$\int_\Gamma \boldsymbol{f} \cdot d\boldsymbol{x} := \int_a^b \big\langle \boldsymbol{f}(\gamma(t)), \gamma'(t) \big\rangle \, dt \tag{3}$$

Recall 4. *Line integral of a **vector field** $\boldsymbol{f} : \Omega \to \mathbb{R}^n$ is defined as follows*

$$\int_\Gamma \boldsymbol{f} \cdot d\boldsymbol{x} := \int_a^b \big\langle \boldsymbol{f}(\gamma(t)), \gamma'(t) \big\rangle \, dt \tag{4}$$

3 Analysis: Surface integral of the first kind

Example 1. *Examine*

Approach:

4   Analysis: Surface integral of the second kind

Example 2. *Examine*

Approach:

5 Analysis: Gauss theorem

Example 3. *Examine*

Approach: