Global exercise - GUE08

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Content covered:

- ✓ Numerics:
  1. Review: QR-decomposition by using
     (a) Givens-Rotation
     (b) Householder-Reflection
  2. QR algorithm to find all eigenvalues of a matrix A.
- ✓ Analysis: Application of Hölder's inequality to approximate integral

1 Numerics: Review of QR-decomposition

There are two main methods used to decompose a matrix into an orthogonal matrix Q and a right upper triangular matrix R

1. Givens-Rotation: ideally for **sparse** matrices
   (a) Detect those non-zero entries standing below the diagonal
   (b) Apply an orthogonal matrix so-called Givens-Rotation to **clean up** those non-zero entries.
   (c) This method works ideally for those sparse matrices.
2. Householder-Reflection: ideally for **dense** matrices
   (a) This method works ideally for dense matrices.
   (b) This method can work well with parallel implementation.

2 Numerics: QR-algorithm

Observation 1 (**QR-algorithm** vs. **QR-decomposition**)**.** *QR-algorithm (as seen in Maths III) is not QR-decomposition (as seen in Maths II). QR-algorithm (Maths III) is an algorithm used to find* ***all*** *eigenvalues of a matrix A* ***numerically****. Meanwhile, QR-decomposition is a technique in linear algebra used to decompose a matrix A into an orthogonal matrix Q and a right upper triangular matrix R. Nevertheless, we will still need QR-decomposition for QR-algorithm.*

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3 Analysis: Application of Hölder's inequality

Observation 2. *Sometimes we would like to estimate whether an integral, from complicated to very complicated, is bounded or not, without actually compute it. The Hölder's inequality is an ideally mathematical tool for such situation.*

Example 1. *Examine the following integral*

$$\int_{\Omega} (x+2)^{-3/5} \exp\left(-2x/3\right) dx$$

Approach: