

Global exercise - GUE11

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Summary of content covered: Analysis

- Review line integral of the 1st kind (scalar field) and 2nd kind (vector field)
- Surface integral of the 1st kind (scalar field) and 2nd kind (vector field)
- Gauss's theorem and its useful applications

1 Course evaluation

2 Analysis: Line integral and Surface integral

2.1 First kind: Scalar field

Recall 1. *Line integral of a **scalar field** $\phi : \Omega \rightarrow \mathbb{R}$ is defined as follows*

$$\int_{\Gamma} \phi \, ds := \int_a^b \phi(\gamma(t)) \|\gamma'(t)\| \, dt \quad (1)$$

Recall 2. *Surface integral of a **scalar field** $\phi : \Omega \rightarrow \mathbb{R}$ is defined as follows*

$$\int_{\Gamma} \phi \, ds := \int_a^b \phi(\gamma(t)) \|\gamma'(t)\| \, dt \quad (2)$$

2.2 Second kind: Vector field

Recall 3. *Line integral of a **vector field** $\mathbf{f} : \Omega \rightarrow \mathbb{R}^n$ is defined as follows*

$$\int_{\Gamma} \mathbf{f} \cdot d\mathbf{x} := \int_a^b \langle \mathbf{f}(\gamma(t)), \gamma'(t) \rangle \, dt \quad (3)$$

Recall 4. *Line integral of a **vector field** $\mathbf{f} : \Omega \rightarrow \mathbb{R}^n$ is defined as follows*

$$\int_{\Gamma} \mathbf{f} \cdot d\mathbf{x} := \int_a^b \langle \mathbf{f}(\gamma(t)), \gamma'(t) \rangle \, dt \quad (4)$$

3 Analysis: Surface integral of the first kind

Example 1. *Examine*

Approach:

4 Analysis: Surface integral of the second kind

Example 2. *Examine*

Approach:

5 Analysis: Gauss theorem

Example 3. *Examine*

Approach: