

Global exercise - GUE10

Tuan Vo

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Content covered:

- ✓ Analysis: Surface integral
- ✓ Analysis: Review H9A3, H9A4, and H10A1
- ✓ Numerics: Demo programming exercise 3 (PRU02)

1 Analysis: Surface integral

Example 1. Examine the *Möbius band* given as follows

$$\vec{\gamma} : \begin{cases} (-1, 1) \times (0, 2\pi) \rightarrow \mathbb{R}^3, \\ (t, \phi) \mapsto \vec{\gamma}(t, \phi) := \begin{pmatrix} \cos(\phi) \\ \sin(\phi) \\ 0 \end{pmatrix} + \frac{t}{2} \begin{pmatrix} \cos(\phi) \cos(\phi/2) \\ \sin(\phi) \cos(\phi/2) \\ \sin(\phi/2) \end{pmatrix}. \end{cases} \quad (1)$$

Show that this surface is **not orientable**.

Approach:

Proof. The normal field is computed as follows

$$\begin{aligned} n(t, \phi) &= \pm \frac{\partial_t \vec{\gamma} \times \partial_\phi \vec{\gamma}}{\|\partial_t \vec{\gamma} \times \partial_\phi \vec{\gamma}\|} \\ &= \frac{\mp 1}{a} \begin{pmatrix} \cos(\phi) \sin(\frac{\phi}{2}) - \frac{t}{4} \sin(\phi)(1 - \cos(\phi)) \\ \sin(\phi) \sin(\phi/2) + \frac{t}{4}(\sin^2(\phi) + \cos(\phi)) \\ -\cos(\phi/2) - \frac{t}{4}(1 + \cos(\phi)) \end{pmatrix} \end{aligned}$$

with

$$a = \sqrt{1 + t \cos(\phi/2) + \frac{t^2}{16}(3 + 2 \cos(\phi))}.$$

We choose the positive sign and consider the position $(1, 0, 0)^T = \vec{\gamma}(0, 0) = \vec{\gamma}(0, 2\pi)$:

$$\lim_{\phi \rightarrow 0} n(0, \phi) = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \neq \lim_{\phi \rightarrow 2\pi} n(0, \phi) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

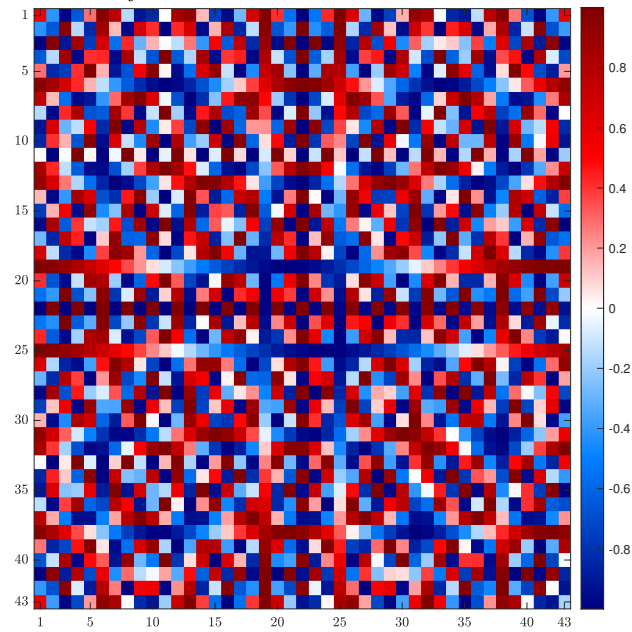
Since the normal field is not continuous on

$$\gamma([-1, 1] \times [0, 2\pi])$$

the surface is **not orientable**. □

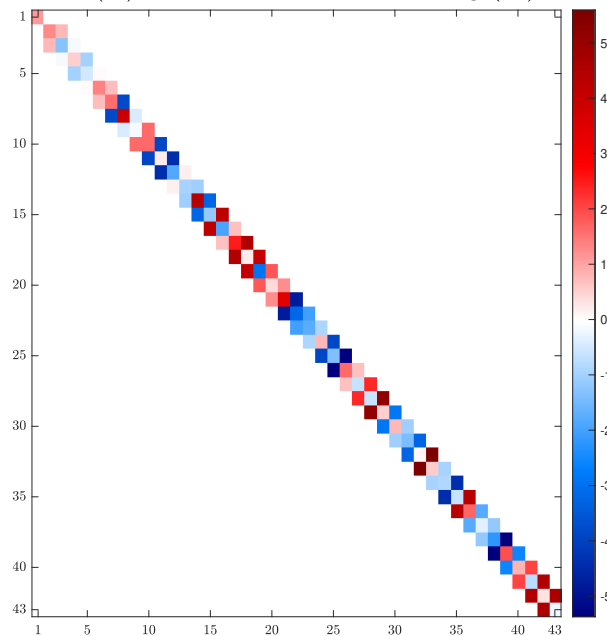
2 Numerics: Demo programming exercise 3 PRU(02)

Matrix $C_{ij} = \cos(ij)$ with $C \in \mathbb{R}^{n \times n}$ and $n = 43$



(a)

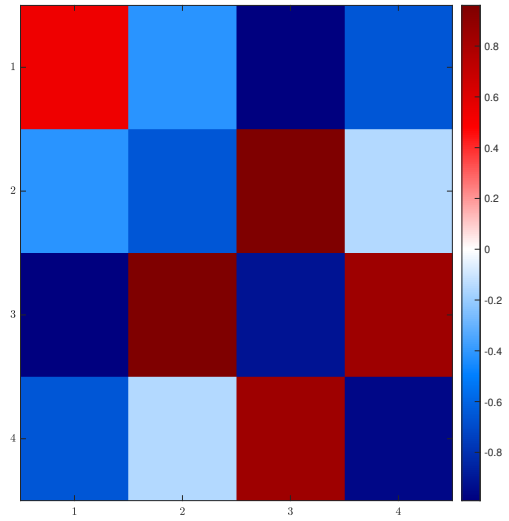
Matrix $(C)_{43 \times 43}$ in forms of Hessenberg $(H)_{43 \times 43}$



(b)

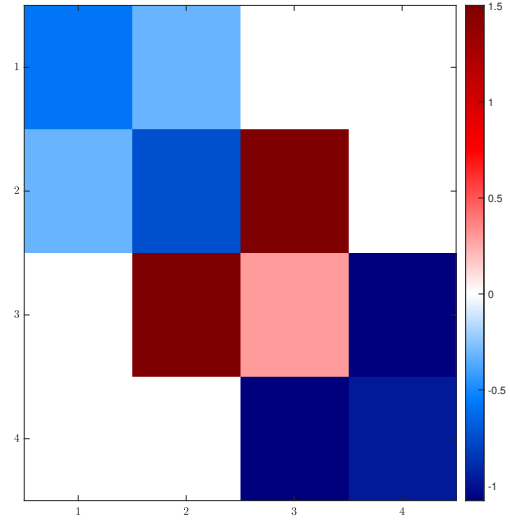
Figure 1: Matrix size 43×43 and its Hessenberg form.

Matrix $C_{ij} = \cos(ij)$ with $C \in R^{n \times n}$ and $n = 4$



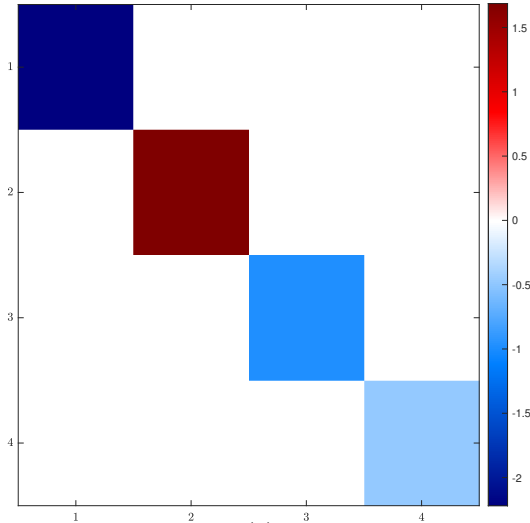
(a)

Matrix $(C)_{4 \times 4}$ in forms of Hessenberg $(H)_{4 \times 4}$



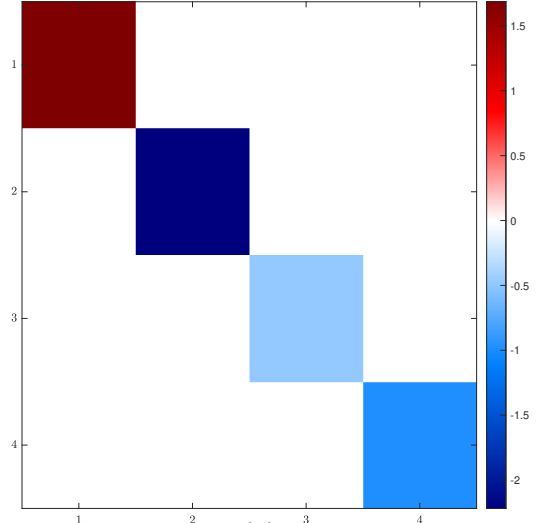
(b)

Resulted matrix $(R)_{4 \times 4}$ w/o Shift
 $\varepsilon = 10^{-8}$; $C_{ij} = \cos(ij)$ with $C \in R^{n \times n}$ and $n = 4$



(c)

Resulted matrix $(R)_{4 \times 4}$ w/i Shift
 $\varepsilon = 10^{-8}$; $C_{ij} = \cos(ij)$ with $C \in R^{n \times n}$ and $n = 4$



(d)

Figure 2: Matrix size 4×4 and its Hessenberg form.