

Global exercise - GUE11

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12th January, 2023

Summary of content covered: Analysis

- Review line integral of the 1st kind (scalar field) and 2nd kind (vector field)
- Surface integral of the 1st kind (scalar field) and 2nd kind (vector field)
- Gauss's theorem and its useful applications

1 Course evaluation

The evaluation for the course

22W-11.03530 (L) Mathematische Grundlagen III (CES) (Übung)

has been opened from now on until **Friday 20.01.2023 23:59:00** via QR code



Figure 1: Evaluation QR-code.

or via the following link

<https://www.campus.rwth-aachen.de/evasys/online.php?pswd=8J5FHC8JH9>

2 Summary: Line integral and Surface integral

2.1 First kind: Scalar field

Recall 1. *Line integral of a **scalar field** $\phi : \Omega \rightarrow \mathbb{R}$ is defined as follows*

$$\int_{\Gamma} \phi \, ds := \int_a^b \phi(\gamma(t)) \|\gamma'(t)\| \, dt \quad (1)$$

→ Mass

Recall 2. *Surface integral of a **scalar field** $\phi : \Omega \rightarrow \mathbb{R}$ is defined as follows*

$$\int_{\Gamma} \phi \, dA := \int_B \phi(\Phi(p, q)) \|\Phi_p \times \Phi_q\| \, d(p, q) \quad (2)$$

→ Mass

2.2 Second kind: Vector field

Recall 3. *Line integral of a **vector field** $\mathbf{f} : \Omega \rightarrow \mathbb{R}^n$ is defined as follows*

$$\int_{\Gamma} \mathbf{f} \cdot d\mathbf{x} := \int_a^b \langle \mathbf{f}(\gamma(t)), \gamma'(t) \rangle \, dt \quad (3)$$

→ Work done

Recall 4. *Surface integral of a **vector field** $\mathbf{f} : \Omega \rightarrow \mathbb{R}^n$ is defined as follows*

$$\int_{\Gamma} \langle \mathbf{f}, \mathbf{v} \rangle \, dA = \int_{\Gamma} \mathbf{f} \cdot d\mathbf{A} := \int_B \langle \mathbf{f}(\Phi(p, q)), \Phi_p \times \Phi_q \rangle \, d(p, q) \quad (4)$$

→

3 Analysis: Surface integral of the first kind

Example 1. *Examine*

Approach:

4 Analysis: Surface integral of the second kind

Example 2. *Examine*

Approach:

5 Analysis: Gauss theorem

Example 3. *Examine*

Approach: