Global Exercise - Gue11

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Content covered:

- ✓ Analysis:
- ✓ [Review HW10] Consistency error (cont.)

1 Analysis

Example 1. *Examine the following problem*

$$abc$$

2 [Review HW10] Consistency error (cont.)

Example 2. *Examine the consistency error of the following problem*

$$u''(x) - u'(x) + u(x) = 2x - 1 - x^2$$

with the exact solution is known, i.e. $u(x) = 1 - x^2$.

Approach: The consistency error reads

$$\left|\left| -\Delta_h u\big|_{\bar{\Omega}_h} - f\big|_{\Omega_h} \right|\right|_\infty = \left|\left| A_h u\big|_{\Omega_h} - f\big|_{\Omega_h} \right|\right|_\infty \tag{1}$$

Observation: The consistency error can be foreseen (and indeed) with the value 0, i.e. the numerical solution resembles the exact/analytical solution, since we have been using the following numerical scheme + the given information about the exact solution:

1. **Second order** discretization scheme is used to approximate $u''(x)$.
2. **Second order** discretization scheme is used to approximate $u'(x)$.
3. The exact solution, which is given, is of **quadratics**.

$$\therefore \quad \boxed{\left|\left| A_h u\big|_{\Omega_h} - f\big|_{\Omega_h} \right|\right|_\infty = 0}$$

Another way is to substitute the given exact solution $u(x) = 1 - x^2$ into (1) and define it on grid point, together with known matrix A and known f, and then compute the consistency error (1) accordingly.