

Global Exercise - Gue09

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Content covered:

- ✓ Programming exercise 02: Explanation + Hints.
- ✓ Analysis:
 - * Expansion of $f(x, y)$ in eigenfunctions of Laplace operator (2D).
 - * Distribution & its derivative.
- ✓ Numerics: Ghost points in FDM + Neumann BCs.

1 Programming exercise 02: Explanation + Hints

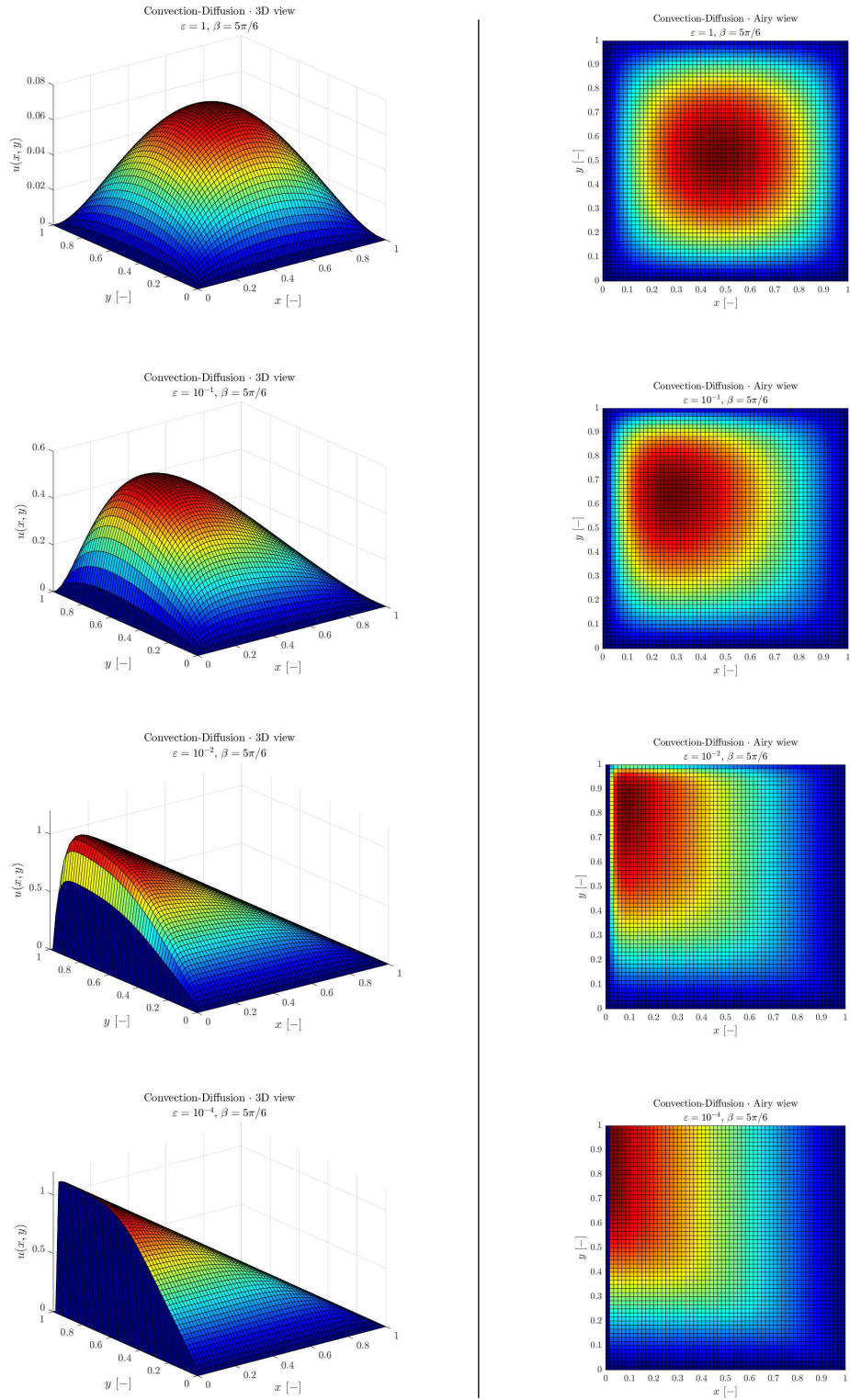


Figure 1: Comparison among different values of $\varepsilon = [1, 10^{-1}, 10^{-2}, 10^{-4}]$: Upwind differences used for discretizing convection terms $\partial_x u$ and $\partial_y u$; fixed $\beta = 5\pi/6$.

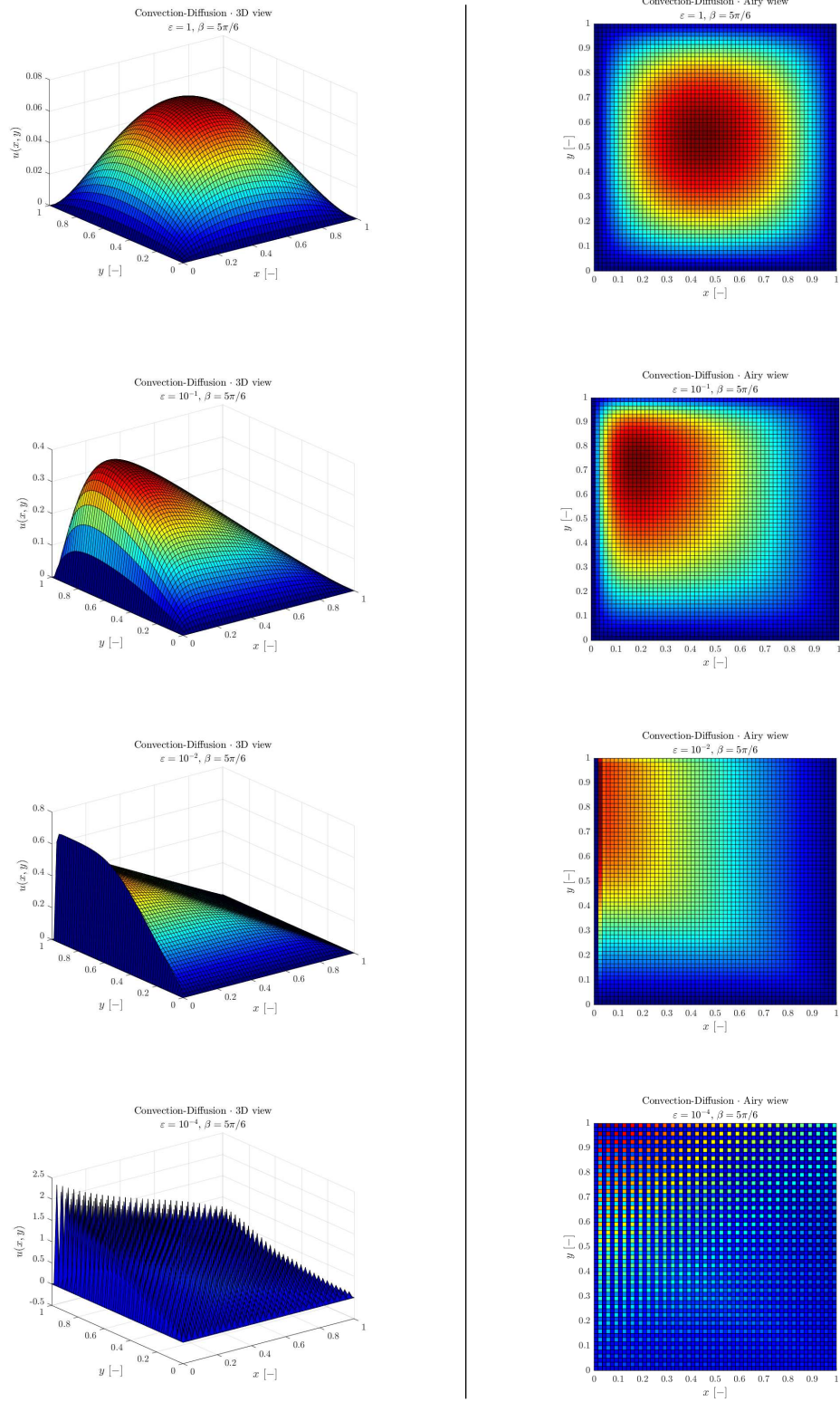
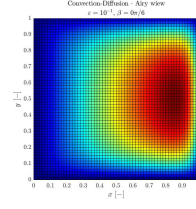
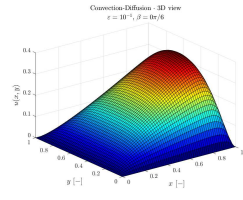
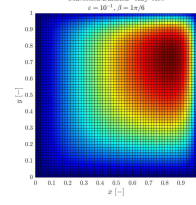
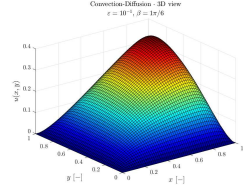


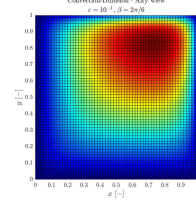
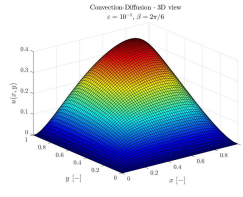
Figure 2: Comparison among different values of $\varepsilon = [1, 10^{-1}, 10^{-2}, 10^{-4}]$: Central difference used for discretizing convection terms $\partial_x u$ and $\partial_y u$; fixed $\beta = 5\pi/6$.



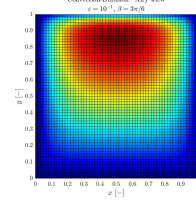
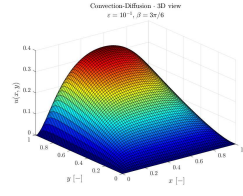
$$\beta = \frac{0\pi}{6} = 0$$



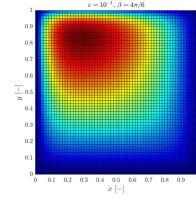
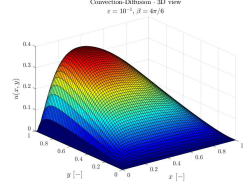
$$\beta = \frac{1\pi}{6}$$



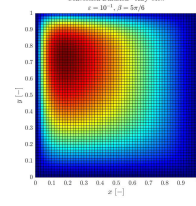
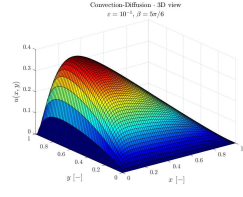
$$\beta = \frac{2\pi}{6} = \frac{\pi}{3}$$



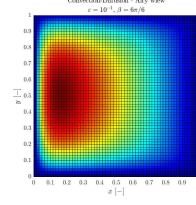
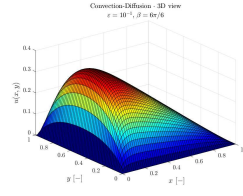
$$\beta = \frac{3\pi}{6} = \frac{\pi}{2}$$



$$\beta = \frac{4\pi}{6} = \frac{2\pi}{3}$$



$$\beta = \frac{5\pi}{6}$$



$$\beta = \frac{6\pi}{6} = \pi$$

Figure 3: Comparison among different values of $\beta = \left[0, \frac{\pi}{6}, \frac{2\pi}{6}, \frac{3\pi}{6}, \frac{4\pi}{6}, \frac{5\pi}{6}, \pi\right]$: Upwind differences used for discretizing convection terms $\partial_x u$ and $\partial_y u$; fixed $\varepsilon = 10^{-1}$.

2 Analysis: 2D Eigenfuctions of Laplace operator

Example 1. *Examine the following*

3 Analysis: Distribution and its derivative

Example 2. *Examine the following*

4 Numerics: Ghost points in FDM

Example 3. *Examine the following*