

Global Exercise - Gue07

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1 Small remarks about HW08

2 Numerics: FDM · BC · Derivation of A_h & b_h

Example 1. *Discretization of the following convection-diffusion problem:*

$$-u''(x) - 5u'(x) = 3, \quad \text{for } x \in (0, 1) \quad (1)$$

with boundary conditions $u(0) = u(1) = 1$.

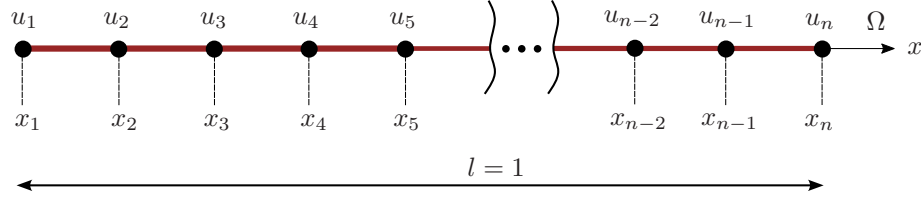


Figure 1: Discretization of domain Ω with n points.

Approach: Central method for $u''(x)$ is obtained as follows

$$u''(x) = \frac{u(x+h) - 2u(x) + u(x-h)}{h^2} + \mathcal{O}(h^2). \quad (2)$$

Forward method for $u'(x)$ is obtained as follows

$$u'(x) = \frac{u(x+h) - u(x)}{h} + \mathcal{O}(h). \quad (3)$$

which leads to the system $A_h u_h = b_h$ where A_h and b_h are recognized as follows

$$A_h = \frac{1}{h^2} \begin{pmatrix} 2+5h & -1-5h & & & & \\ -1 & 2+5h & -1-5h & & & \\ & \ddots & \ddots & \ddots & & \\ & & \ddots & \ddots & \ddots & \\ & & & -1 & 2+5h & -1-5h \\ & & & & -1 & 2+5h \end{pmatrix} \in \mathbb{R}^{(n-2) \times (n-2)}, \quad (4)$$

$$b_h = \begin{pmatrix} 3 + \frac{1}{h^2} \\ 3 \\ \vdots \\ 3 \\ 3 + \frac{1}{h^2} + \frac{5}{h} \end{pmatrix} \in \mathbb{R}^{(n-2)}. \quad (5)$$

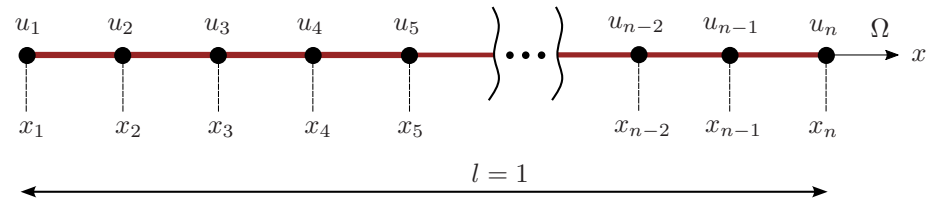


Figure 2: Discretization of domain Ω with n points.

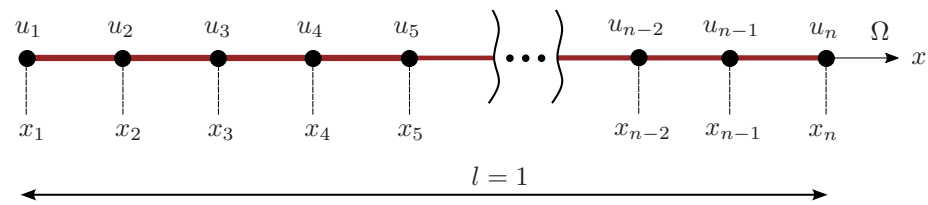


Figure 3: Discretization of domain Ω with n points.

3 Analysis: Spectral theory · Laplace operator application

Example 2. *Examine the following problem*

$$\begin{aligned}\partial_t u(x, t) &= \partial_{xx} u(x, t) + a, & x &\in (0, 1), t > 0, \\ u(0, t) &= 0, & t &> 0, \\ u(1, t) &= 0, & t &> 0, \\ u(x, 0) &= \sin(\pi x), & x &\in (0, 1),\end{aligned}$$

with constant source term $a \in \mathbb{R}$.

1. *Compute the Eigenvalues and the normalized Eigenfunctions from ∂_{xx} .*
2. *Develop the source term $a \in \mathbb{R}$ in these Eigenfunctions.*
3. *Compute the solution of the problem with the Eigenfunction ansatz.*

(Example 2 cont.)

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Example 3. *Examine the following problem*

$$\begin{aligned}\partial_t u(x, t) &= \partial_{xx} u(x, t) + \sin(\pi x), & x &\in (0, 1), t > 0, \\ u(0, t) &= 0, & t &> 0, \\ u(1, t) &= 0, & t &> 0, \\ u(x, 0) &= a, & x &\in (0, 1),\end{aligned}$$

with constant IC $a \in \mathbb{R}$.

1. *Compute the Eigenvalues and the normalized Eigenfunctions from ∂_{xx} .*
2. *Develop the IC and source term in these Eigenfunctions.*
3. *Compute the solution of the problem with the Eigenfunction ansatz.*

Example 4. *Examine the following problem*

$$\begin{aligned}\partial_{tt}u(x,t) &= \partial_{xx}u(x,t) + a, & x &\in (0,1), t > 0, \\ u(0,t) &= 0, & t &> 0, \\ u(1,t) &= 0, & t &> 0, \\ u(x,0) &= x, & x &\in (0,1), \\ u_t(x,0) &= 0, & x &\in (0,1),\end{aligned}$$

with constant source term $a \in \mathbb{R}$.

1. *Compute the Eigenvalues and the normalized Eigenfunctions from ∂_{xx} .*
2. *Develop the IC and source term in these Eigenfunctions.*
3. *Compute the solution of the problem with the Eigenfunction ansatz.*

(Example 4 cont.)