(iii) *total-variation-diminishing (TVD)*, if we have

$$TV\left(u_{\Delta x}^{n+1}\right) \leq TV\left(u_{\Delta x}^{n}\right)$$

(iv) *monotonicity-preserving*, if we have

$$u_j^n \leq u_{j+1}^n \quad \Rightarrow \quad H_{\Delta t}\left(u_{\Delta x}^n, j\right) \leq H_{\Delta t}\left(u_{\Delta x}^n, j+1\right) \quad \forall j$$

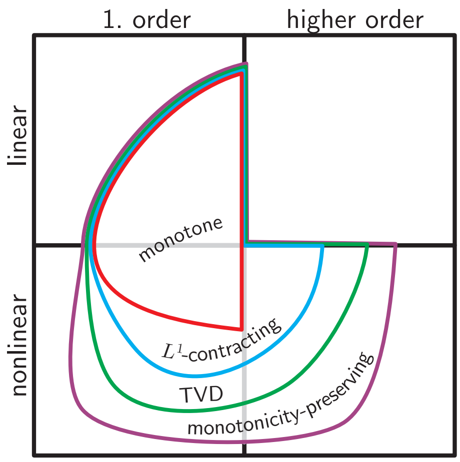
Remarks II.28:

(a) These kind of definitions we already had in the context of Krushkov's theorem. We would like to copy this to the numerical level.

(b) For a piece-wise constant grid function $u_{\Delta x}$ we have

$$TV(u_{\Delta x}) = \sum_{j \in \text{grid}} \left|u_{j+1}^n - u_j^n\right|$$

(b) In the next section we will prove the following picture, where the entire box represents all FV methods.



II.5.2 From Monotone to Monotonicity-Preserving

Theorem II.17: Structure of FV Methods

For a 3-point-stencil method $u_j^{n+1} = H_{\Delta t}\left(u_{j-1}^n, u_j^n, u_{j+1}^n, j\right)$ in conservative form we have

$$\text{monotone} \overset{\text{(i)}}{\Rightarrow} L^1\text{-contracting} \overset{\text{(ii)}}{\Rightarrow} \text{TVD} \overset{\text{(iii)}}{\Rightarrow} \text{monotonicity-preserving}$$