

# Global Exercise - 15

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## 1 Monotone $\rightarrow L_1$ -Contracting $\rightarrow$ TVD $\rightarrow$ Mon.Pre.

**Example 1.** *abc*

Roe's solver:

P.L.Roe [1981], [Approximate Riemann solvers, parameter vectors, and difference schemes.](#)

## 2 Total variation diminishing (TVD)

Example 2. *abc*

### 3 Limiter

**Example 3.** Examine the 1<sup>st</sup>-order-converged LF and the 2<sup>nd</sup>-order-converged LW.

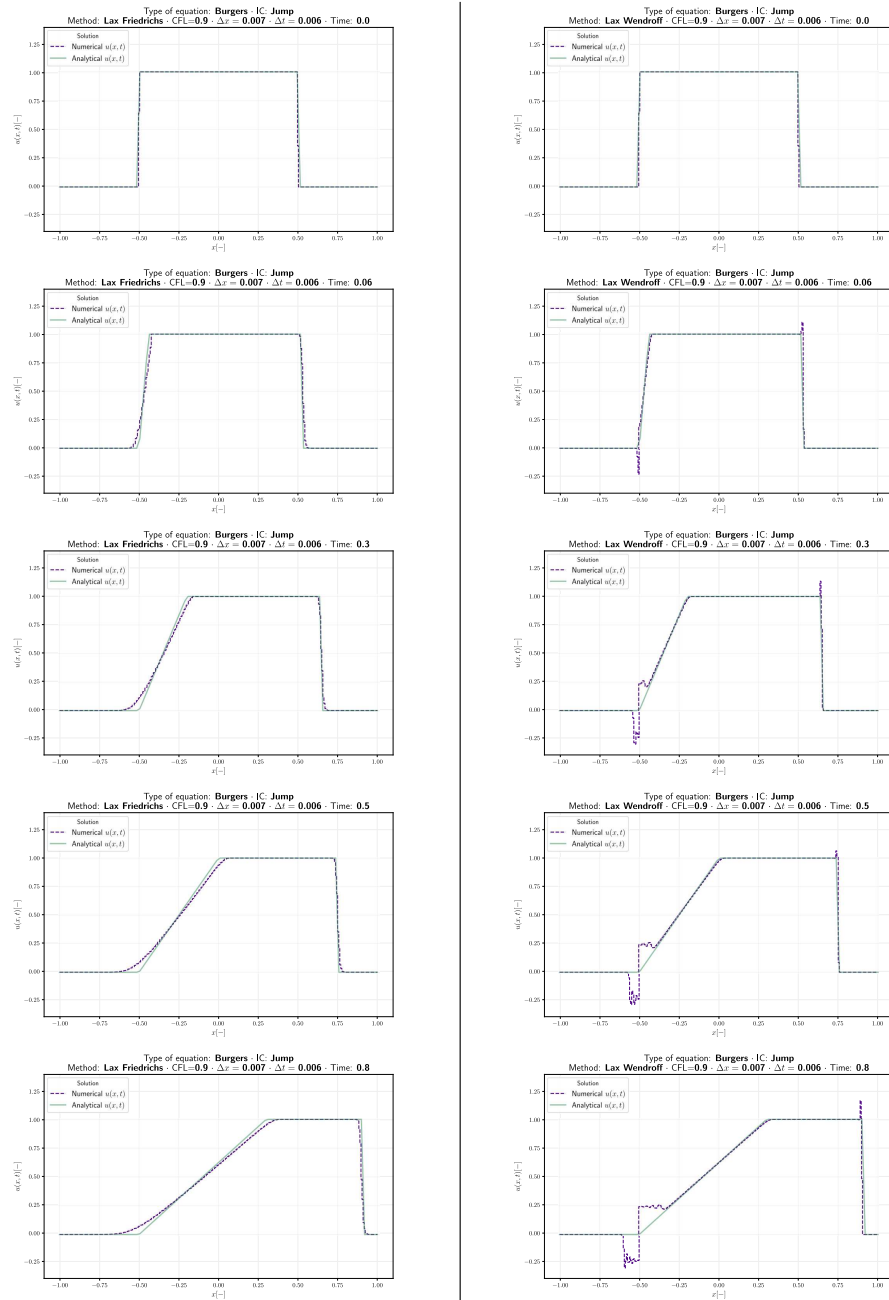
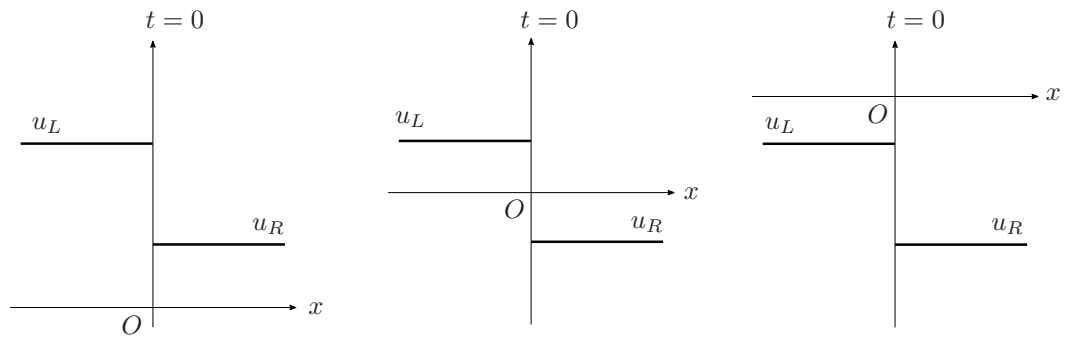


Figure 1: Oscillatory phenomena around discontinuity: (left) none oscillation founded in *Lax-Friedrichs*; (right) oscillation observed in *Lax-Wendroff*.

## 4 Review Riemann's problem and Godunov's solver

Example 4. *abc*

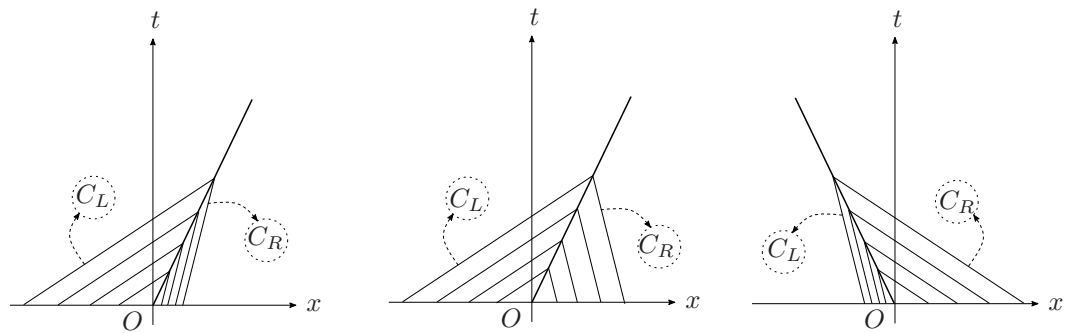
**Example 5.** Examine  $u_L > u_R$ .



Case 1 :  $u_L > u_R > 0$

Case 2 :  $u_L > 0 > u_R$

Case 3 :  $0 > u_L > u_R$



$$(C_L) : t = \frac{1}{u_L}x - \frac{x_0}{u_L}$$

$$(C_R) : t = \frac{1}{u_R}x - \frac{x_0}{u_R}$$

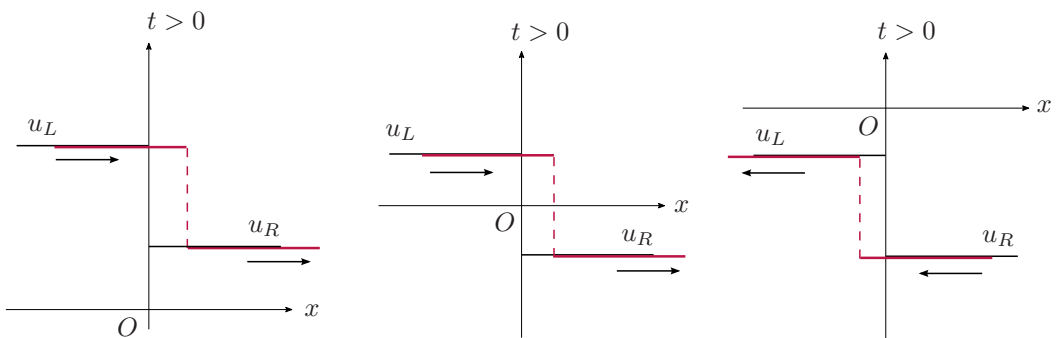
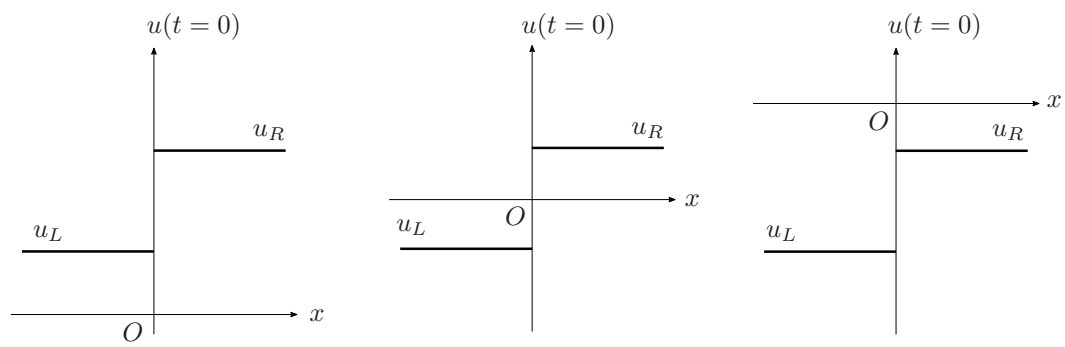


Figure 2: Riemann problem with  $u_L > u_R$ : IC, Characteristics, Solution.

Schock solution

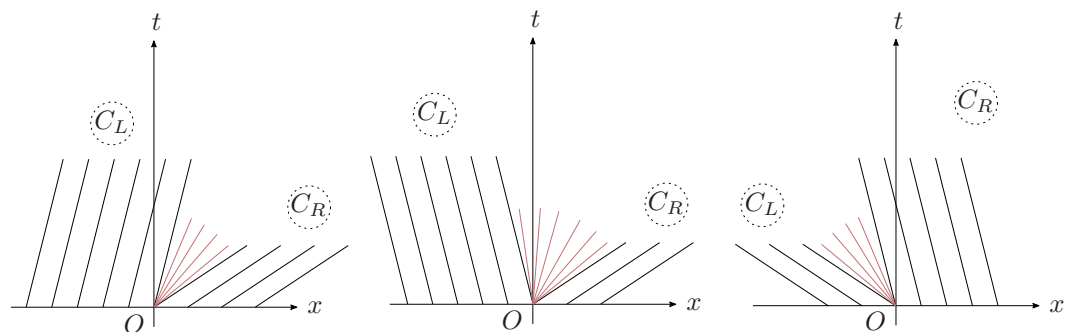
**Example 6.** Examine  $u_L < u_R$ .



Case 1 :  $0 < u_L < u_R$

Case 2 :  $u_L < 0 < u_R$

Case 3 :  $u_L < u_R < 0$



$$(C_L) : t = \frac{1}{u_L}x - \frac{x_0}{u_L}$$

$$(C_R) : t = \frac{1}{u_R}x - \frac{x_0}{u_R}$$

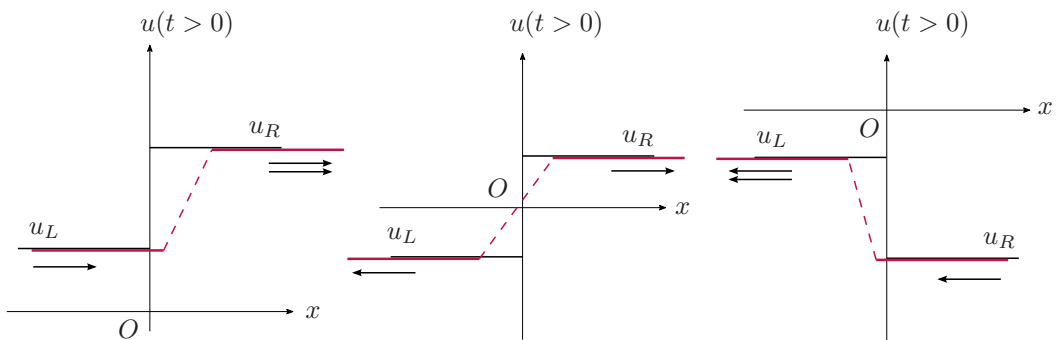


Figure 3: Riemann problem with  $u_L < u_R$ : IC, Characteristics, Solution.

Rarefaction solution