1 A small note about monotonicity: *Upwind* scheme

$$(a > 0) \text{ CFL: } 0 \le f'(u) \frac{\Delta t}{\Delta x} \le 1$$

$$\mathcal{H}_{\Delta t} = u_j^n + \frac{\Delta t}{\Delta x} \left(f(u_{j-1}^n) - f(u_j^n) \right)$$

$$A_1 = \partial_{u_{j-1}^n} \mathcal{H}_{\Delta t} = \frac{\Delta t}{\Delta x} f'(u_{j-1}^n)$$

$$B_1 = \partial_{u_j^n} \mathcal{H}_{\Delta t} = 1 - \frac{\Delta t}{\Delta x} f'(u_j^n)$$

$$C_1 = \partial_{u_{j+1}^n} \mathcal{H}_{\Delta t} = 0$$

$$f'(u)$$

$$a$$

$$\mathcal{H}_{\Delta t} \text{ Monotonically increasing?}$$

$$A_1 \ge 0 \Leftrightarrow f'(u_{j-1}^n) \ge 0 \checkmark$$

$$B_1 \ge 0 \Leftrightarrow f'(u_{j-1}^n) \le 0 \checkmark$$

$$B_1 \ge 0 \Leftrightarrow f'(u_{j-1}^n) \ge 0 \checkmark$$

$$A_1 \ge 0 \Leftrightarrow f'(u_{j-1}^n) \ge 0 \checkmark$$

$$A_2 \ge 0 \Leftrightarrow f'(u_{j-1}^n) \ge 0 \checkmark$$

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$$A_2 \ge 0 \Leftrightarrow f'(u_{j-1}^n) \ge 0 \Leftrightarrow f'(u_{j-1}^n) \ge 0 \Rightarrow f'(u_$$

Figure 1: Monotonicity of *Upwind* scheme: 8 different cases.

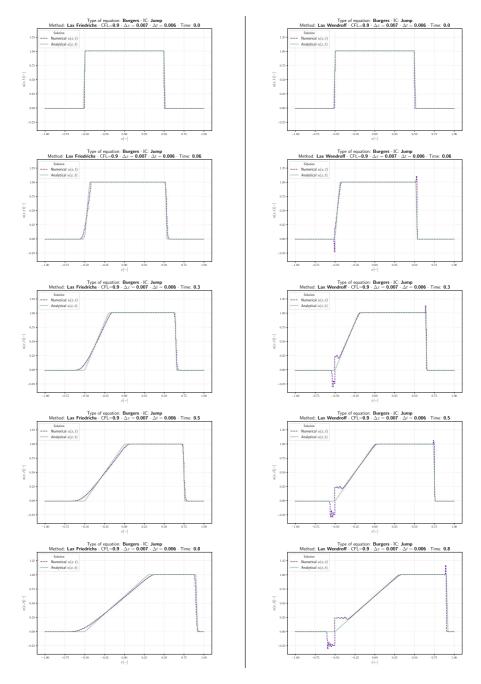


Figure 2: Oscillatory phenomena around discontinuity: (left) none oscillation founded in *Lax-Friedrichs*; (right) oscillation observed in *Lax-Wendroff*.