Global Exercise - 15

Tuan Vo

02nd February 2022

1 Monotone $\rightarrow$ L_1-Contracting $\rightarrow$ TVD $\rightarrow$ Mon.Pre.

Example 1. *abc*

Roe'solver:
P.L.Roe [1981], Approximate Riemann solvers, parameter vectors, and difference schemes.

2 Monotone

Example 2. *abcdefchg*

3 Total variation diminishing (TVD)

Example 3. *abc*

4 Limiter

Example 4. *Examine the 1^{st}-order-converged LF and the 2^{nd}-order-converged LW.*

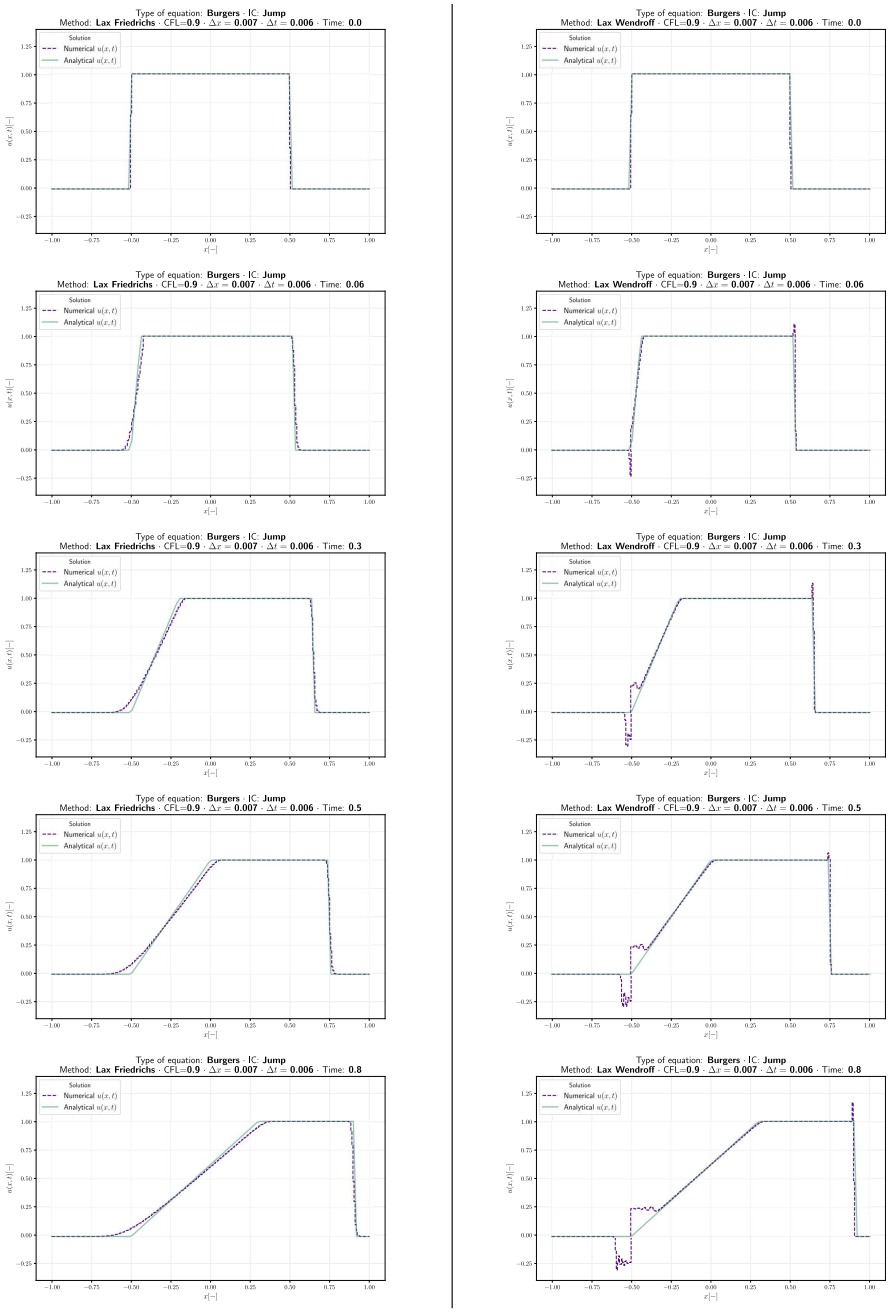


Figure 1: Oscillatory phenomena around discontinuity: (left) none oscillation founded in *Lax-Friedrichs*; (right) oscillation observed in *Lax-Wendroff*.

5 Review Riemann's problem and Godunov's solver

Example 5. *abc*

Example 6. *Examine* $u_L > u_R$.

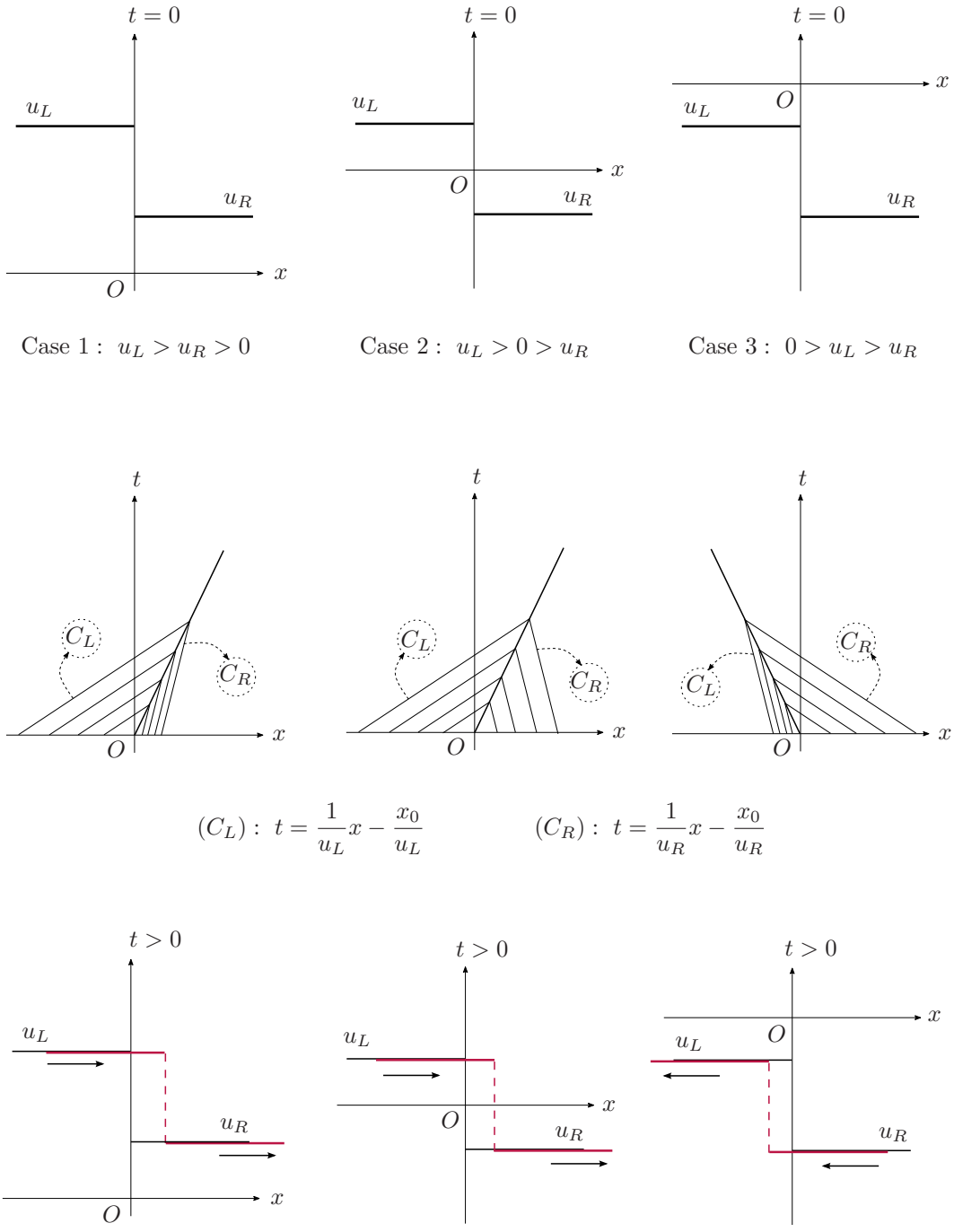
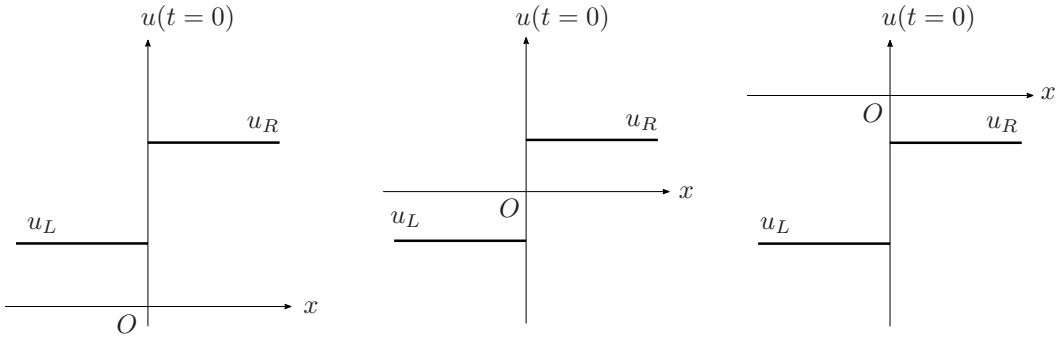


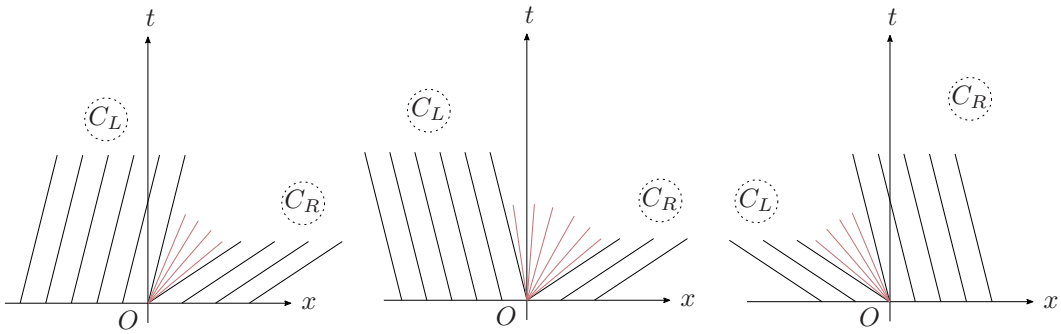
Figure 2: Riemann problem with $u_L > u_R$: IC, Characteristics, Solution.

Schock solution

Example 7. *Examine* $u_L < u_R$.



Case 1 : $0 < u_L < u_R$ Case 2 : $u_L < 0 < u_R$ Case 3 : $u_L < u_R < 0$



$$(C_L): \; t = \frac{1}{u_L}x - \frac{x_0}{u_L} \qquad (C_R): \; t = \frac{1}{u_R}x - \frac{x_0}{u_R}$$

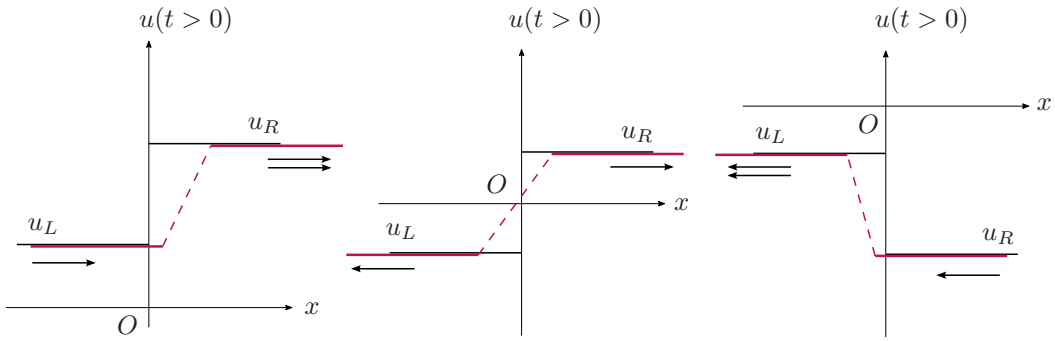


Figure 3: Riemann problem with $u_L < u_R$: IC, Characteristics, Solution.

Rarefaction solution