

Global Exercise - 15

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1 Monotone $\rightarrow L_1$ -Contracting \rightarrow TVD \rightarrow Mon.Pre.

Example 1. *abc*

Roe's solver:

P.L.Roe [1981], [Approximate Riemann solvers, parameter vectors, and difference schemes.](#)

2 Monotone

Example 2. *abcdef*

3 Total variation diminishing (TVD)

Example 3. *abc*

4 Limiter

Example 4. Examine the 1st-order-converged LF and the 2nd-order-converged LW.

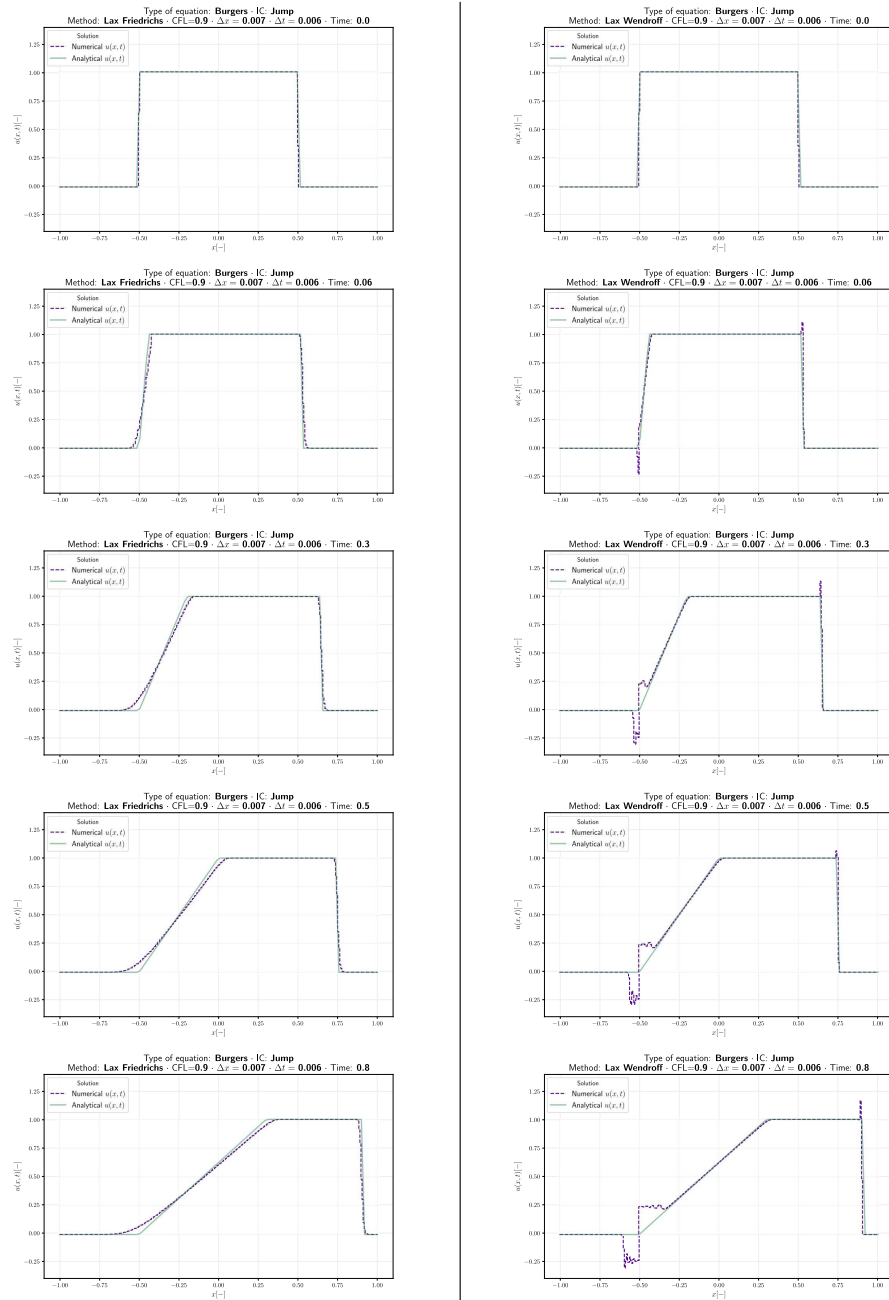
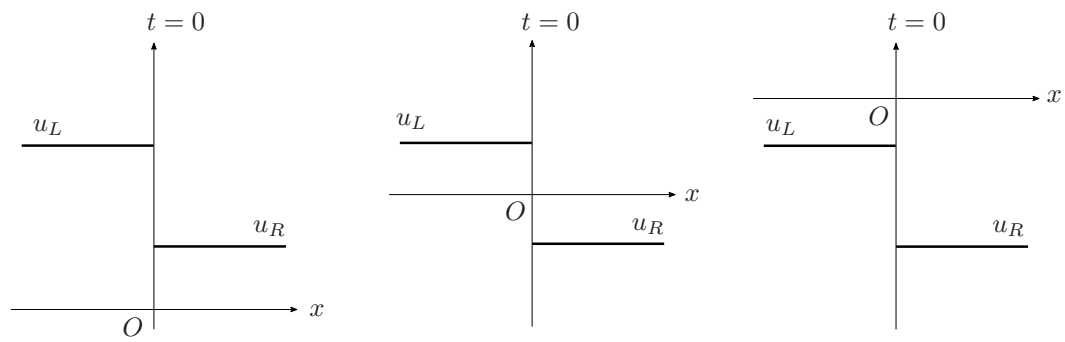


Figure 1: Oscillatory phenomena around discontinuity: (left) none oscillation founded in *Lax-Friedrichs*; (right) oscillation observed in *Lax-Wendroff*.

5 Review Riemann's problem and Godunov's solver

Example 5. *abc*

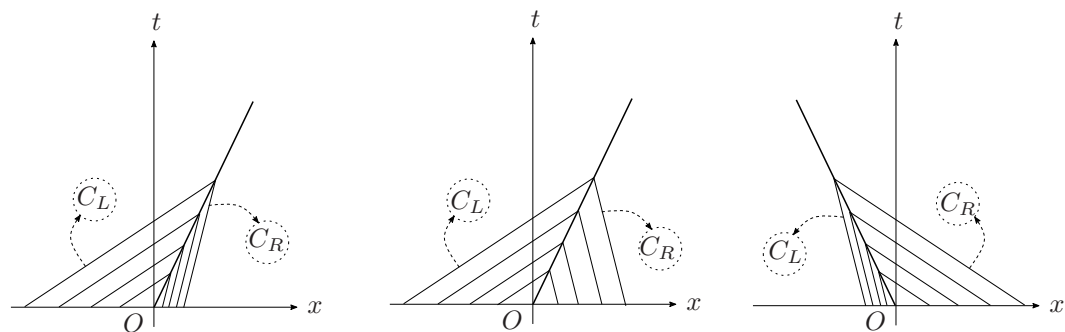
Example 6. Examine $u_L > u_R$.



Case 1 : $u_L > u_R > 0$

Case 2 : $u_L > 0 > u_R$

Case 3 : $0 > u_L > u_R$



$$(C_L) : t = \frac{1}{u_L}x - \frac{x_0}{u_L}$$

$$(C_R) : t = \frac{1}{u_R}x - \frac{x_0}{u_R}$$

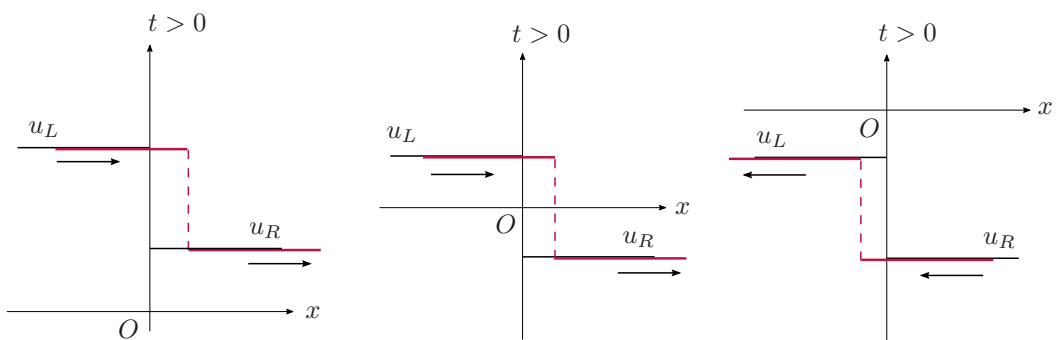
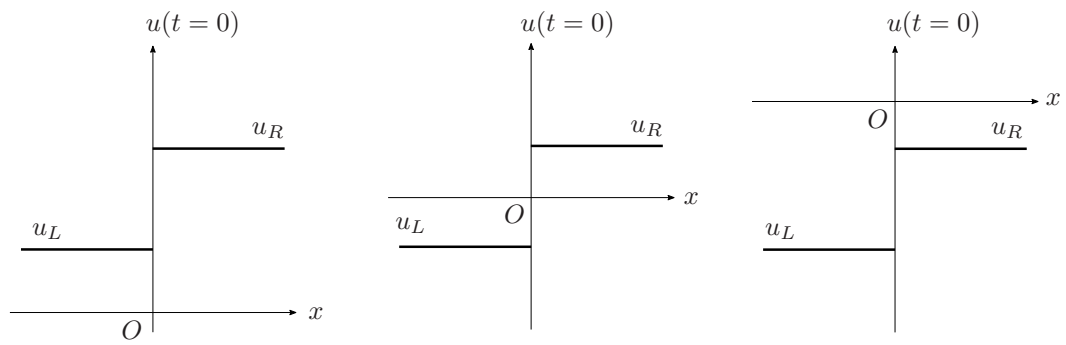


Figure 2: Riemann problem with $u_L > u_R$: IC, Characteristics, Solution.

Schock solution

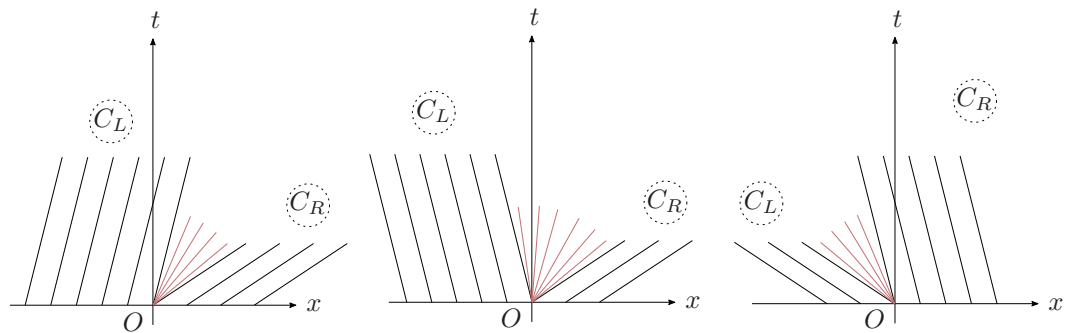
Example 7. Examine $u_L < u_R$.



Case 1 : $0 < u_L < u_R$

Case 2 : $u_L < 0 < u_R$

Case 3 : $u_L < u_R < 0$



$$(C_L) : t = \frac{1}{u_L}x - \frac{x_0}{u_L}$$

$$(C_R) : t = \frac{1}{u_R}x - \frac{x_0}{u_R}$$

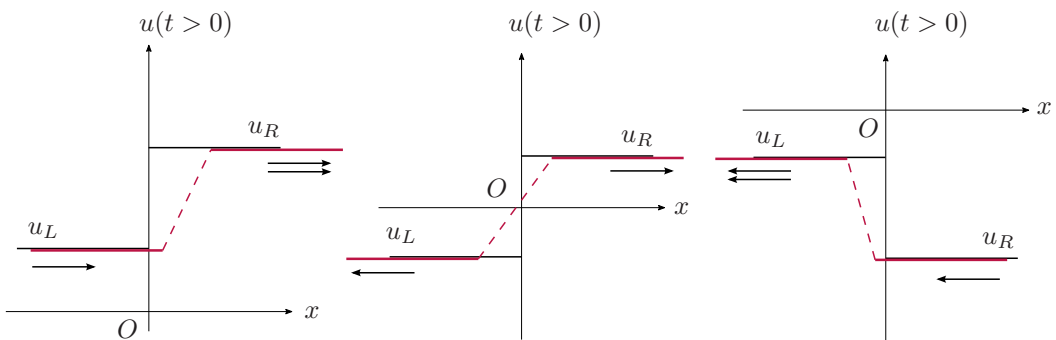


Figure 3: Riemann problem with $u_L < u_R$: IC, Characteristics, Solution.

Rarefaction solution