# Mathematical modelling for All-solid-state battery

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## Mathematical modelling for the next-generation All-solid-state batteries: Nucleation (SE|SSE)<sup>(\*)</sup>-Interface

Rechargeable Lithium-ion battery (LIB) is at the heart of every electric vehicle (EV), portable electronic device, and energy storage system [5]. Nowadays, LIBs enable human life more efficient and help to solve global environment issues thanks to EVs' zero However, conventional LIB (c-LIB) is emission. sensible to temperature and pressure, hence, flammable and explosive, which is undesirable. This bottleneck is mainly due to liquid-based electrolyte found in c-LIBs.

All-solid-state battery (ASSB) is one of promising candidates to overcome bottlenecks of c-LIBs. Thanks to solid-state electrolyte (SSE), ASSB is highly stable towards temperature and pressure. Nevertheless, Limetal dendrite triggered at (SE|SSE)-Interface is the main drawback of ASSB since these dendritic threads extrapolate into SSE grain boundary network, causing crevice, degradation of ionic conductivity, and the probability of short-circuit, which is unfavorable [10].

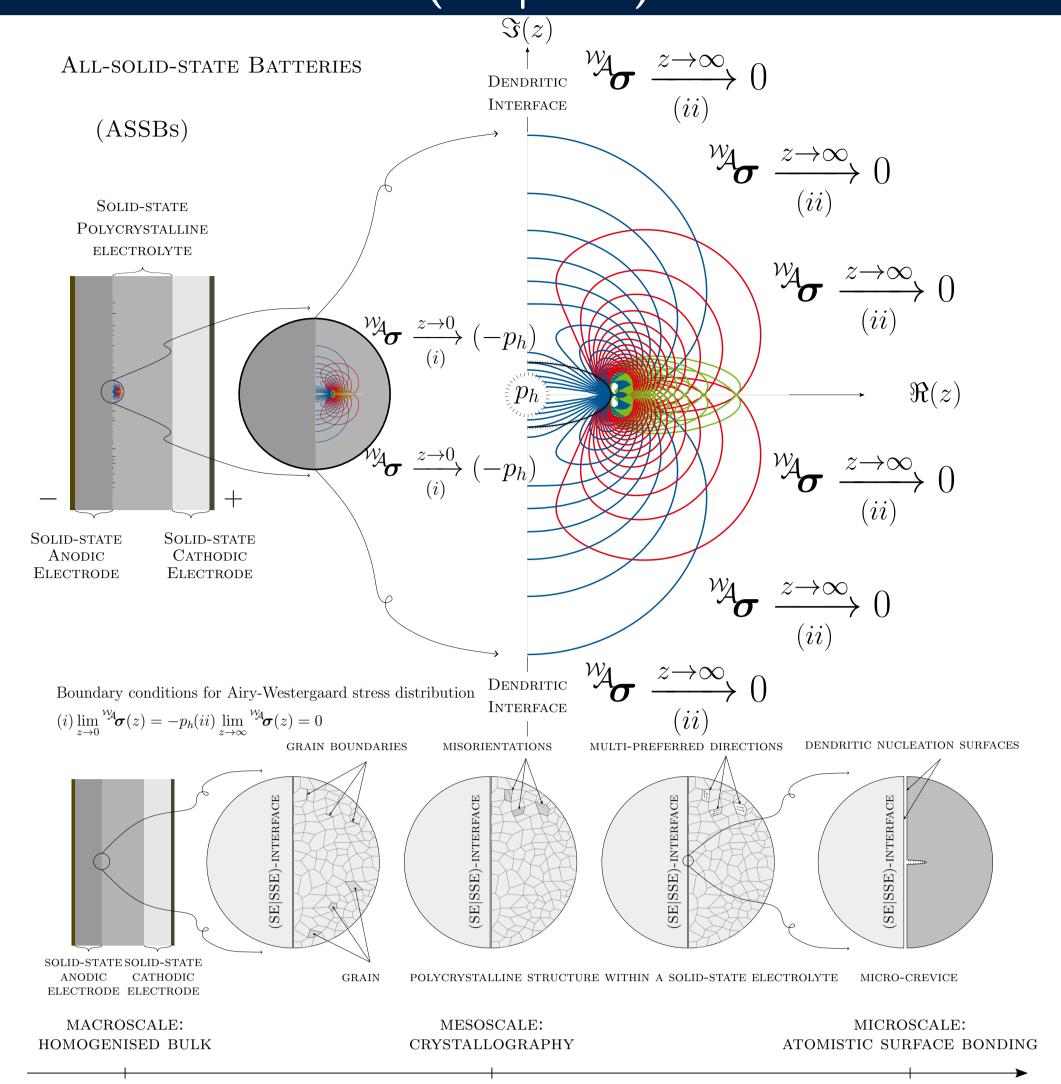
Next-generation All-solid-state battery (ng-ASSB) with a consideration of nucleation criterion defined by

$$\partial_t \boldsymbol{u} + \nabla \cdot \left( \overset{4}{\mathbb{C}} f_{\text{alocation}}(\lambda, \mu, \boldsymbol{d}_{G_i, i=1, \dots, N}^R, \boldsymbol{d}^E; \boldsymbol{x}) : \nabla \boldsymbol{u}^{(s)} \right) + \rho \boldsymbol{b} = -\rho \nabla V_e, \tag{1}$$

s.t. 
$$a_{\text{Griffith}}^{\text{generalised}} := a^* = \arg\min_{a \in \mathbb{R}} \iiint_{\Omega} f(a, \boldsymbol{u}, \theta; \lambda, \mu, \boldsymbol{d}^{(\star)} \otimes \boldsymbol{d}^{(\star)}) d\Omega - \iint_{\Gamma} f(a; \gamma) d\Gamma \Big|_{\bar{\boldsymbol{u}}}$$
 (2)

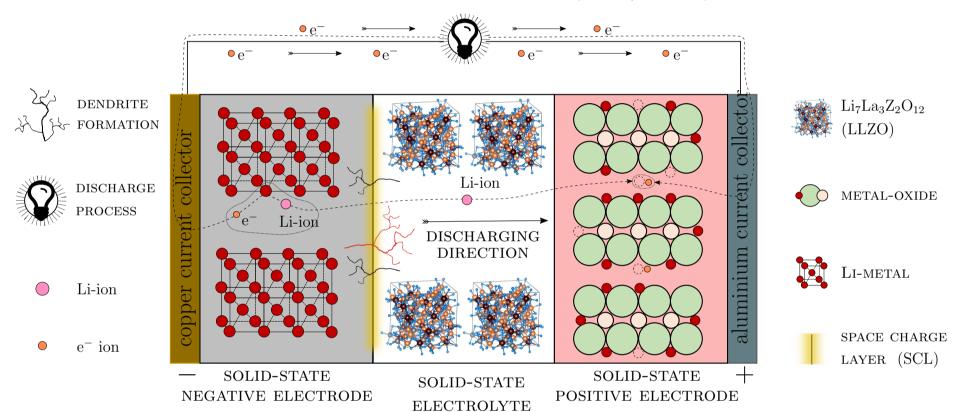
where  $V_e: \mathbb{R}^3 \to \mathbb{R}$  is the electric potential applied globally on ASSB. Due to nature setting of ASSB taking the form (SE|SSE|SE) the electric potential becomes uniform. Additionally,  $\boldsymbol{u}$  is the displacement field,  $\theta$ temperature field, a crevice length,  $\lambda$ ,  $\mu$  Lamé constants,  $\mathbf{d}^{(\star)} \otimes \mathbf{d}^{(\star)}$  embedded misorientation structural tensor, and  $\gamma$  cracking-surface energy density, can help to improve ASSB performance [1][2].

**Aim**: The study is with the purpose of gaining a better insight into dendrite nucleation and formation in ASSB.



### Next-generation All-solid-state battery

**Griffith nucleation criterion** governs (SE|SSE)-Interface.



### Space-charge Layer

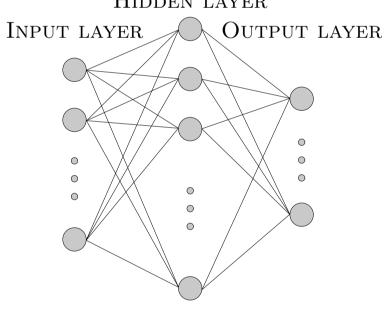
**SCL** presents in ASSB [8], and likewise in Semiconductors. UCLEATION

# Motivation: Energy density landscape

ASSB enables energy demand thanks to (i), and followed by (ii). Energy density: ASSB versus LIB versus Petroi THEORETICAL CAPACITY OF CHARGE: PURE METALLIC LITHIUM (Li) VERSUS GRAPHITE  $(C_6)$  $(4h l^{-1})^{[?][?]} | [mAh cm^{-3}]^{[?]}$  $3800^{[?]}|3860^{[?][?][?][?]}|3861^{[?]}$ 

### Artificial Neural Networks

**Application**: Steel's property prediction. HIDDEN LAYER



The ANNs scheme enhances bainitic trafo. temperature prediction, validated by [9].

Semiconductor

**Application**: Start/Stop-System in Starter.

**Use-case**: BMW B47 ( $-25^{\circ}$ C,  $0^{\circ}$ C,  $120^{\circ}$ C).

**Optimisation**: Pareto @BoschForschung.

(Multi-objective optimisation framework)

# Nucleation interface: Taking place at the critically dendritic (SE|SSE)-Interface

Coupled fields are Displacement field u and temperature field  $\theta$ ;

$$m{u}: egin{cases} \Omega imes \mathbb{R}_+ & \to \mathbb{R}^3, \\ (m{x},t) & \mapsto m{u}(m{x},t), \end{cases} \quad heta: egin{cases} \Omega imes \mathbb{R}_+ & \to \mathbb{R}, \\ (m{x},t) & \mapsto m{\theta}(m{x},t), \end{cases}$$

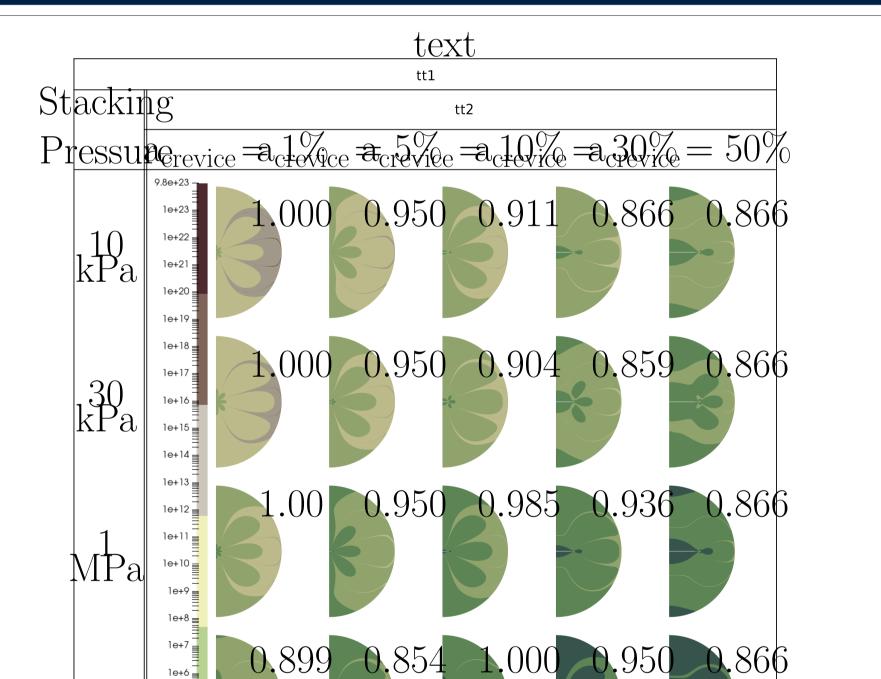
Governing conservation equations used to describe balance of mass, conservation of linear momentum, conservation of angular momentum, and conservation of energy with  $\rho(\boldsymbol{x},t)$  is mass density per unit volume (puv);  $\boldsymbol{b}(\boldsymbol{x},t)$  body force puv;  $\boldsymbol{v}(\boldsymbol{x},t)$  velocity;  $e(\boldsymbol{x},t)$  internal energy puv;  $\boldsymbol{q}(\boldsymbol{x},t)$  heat flux;  $r(\boldsymbol{x},t)$  heat source puv;  $\sigma$  Cauchy stress and  $\varepsilon$  infinitesimal strain.

the deformation of SSE due to dendrite formation at (SE|SSE)interface

 $\iiint_{\Omega} f(a, \boldsymbol{u}; \lambda, \mu, \boldsymbol{d} \otimes \boldsymbol{d}) d\Omega$ 

Strain energy is based on Surface energy is analysised based on the open crevice cracking at (SE|SSE)-interface affected by prescribed pressure

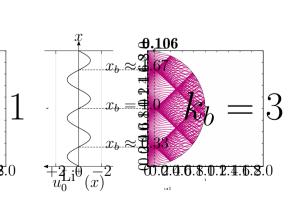
$$\iint f(a;\gamma) \, d\Gamma$$

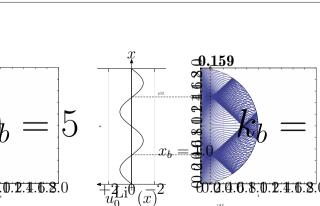


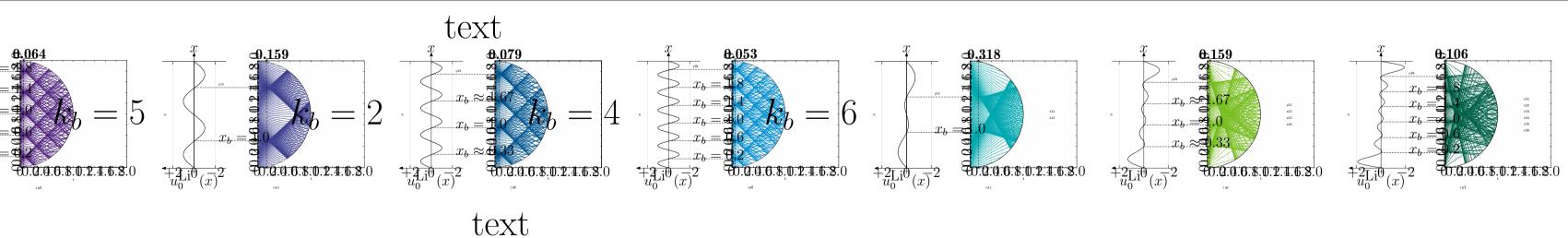
(i) Theoretical capacity of charge

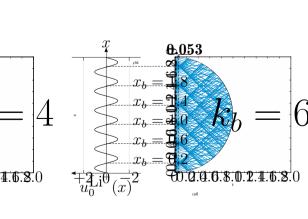
0.948 0.900 1.000 0.950 0.866

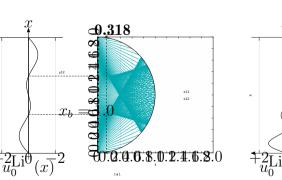
text

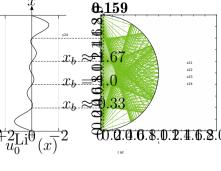


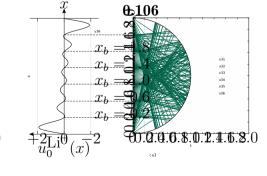












### Lithium-ion battery

Negative-temperature Coefficient (NTC)

Nd/Gd semiconductor is modeled and

**Application**: Swelling phenomena @FEM. **Use-case**: Bosch-48-V-Battery [5].

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### Acknowledgments

T. Vo expresses sincere gratitude to the JARA-CSD research project, led by Dr. S. Braun and Dr. C. Hüter, for their crucial support. Special thanks to Prof. Dr. R. Spatschek for hosting at IEK-2, FZ Jülich. Moreover, T. Vo's role as a teaching assistant in the courses of Mathematische Grundlagen I, II, III, IV, and NumPDE at Computational Engineering Science (CES) RWTH Aachen University (2019-2023), was accomplished and enriched thanks to the guidance and mentorship of Prof. Dr. M. Torrilhon, Prof. Dr. M. Schlottke-Lakemper, Prof. Dr. B. Berkels, Dr. R. Speck, Dr. S. Braun. Prof. Dr. J. Kowalski, Dr. A. Jha, Prof. Dr. B. Stamm, Dr. A. Litvinenko, and Dr. M. Kirchhart.

# Contact

 $P NTC \stackrel{13}{\cancel{1}}$ 

validated [7].

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Temperature [°K]







