

# MATHEMATICAL MODELLING FOR ALL-SOLID-STATE BATTERY

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## Mathematical modelling for the next-generation All-solid-state batteries: Nucleation (SE|SSE)<sup>(\*)</sup>-Interface

**Rechargeable Lithium-ion battery (LIB)** is at the heart of every electric vehicle (EV), portable electronic device, and energy storage system [5]. Nowadays, LIBs enable human life more efficient and help to solve global environment issues thanks to EVs' zero emission. However, conventional LIB (c-LIB) is sensible to temperature and pressure, hence, flammable and explosive, which is undesirable. This bottleneck is mainly due to **liquid-based electrolyte** found in c-LIBs.

**All-solid-state battery (ASSB)** is one of promising candidates to overcome bottlenecks of c-LIBs. Thanks to **solid-state electrolyte (SSE)**, ASSB is highly stable towards temperature and pressure. Nevertheless, Li-metal dendrite triggered at (SE|SSE)-Interface is the main drawback of ASSB since these dendritic threads extrapolate into SSE grain boundary network, causing crevice, degradation of ionic conductivity, and the probability of short-circuit, which is unfavorable [10].

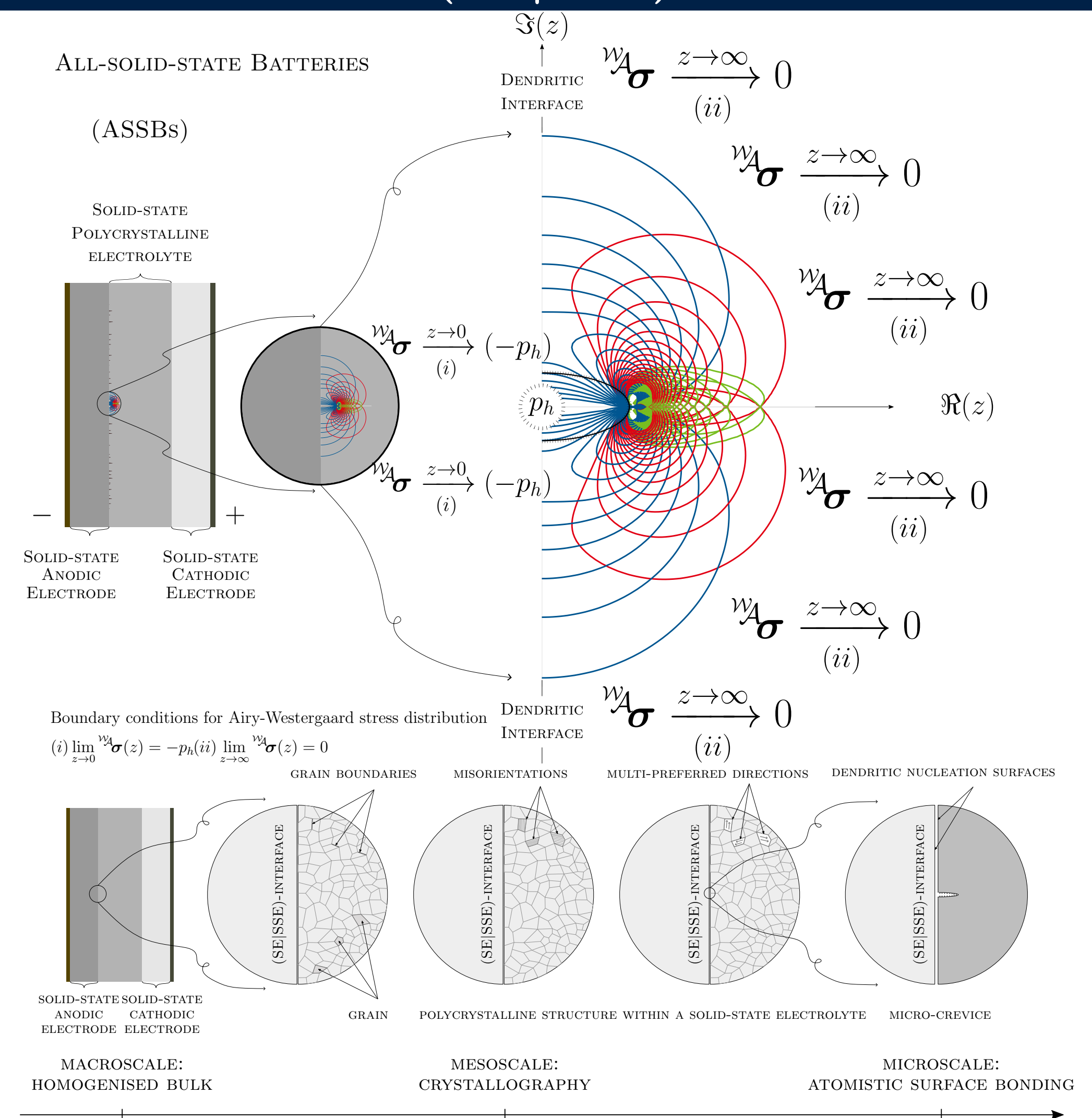
**Next-generation All-solid-state battery (ng-ASSB)** with a consideration of **nucleation criterion** defined by

$$\partial_t \mathbf{u} + \nabla \cdot \left( \mathbf{C}_{\text{allocation}}(\lambda, \mu, \mathbf{d}_{G,i}^R, i=1, \dots, N, \mathbf{d}^E; \mathbf{x}) : \nabla \mathbf{u}^{(s)} \right) + \rho \mathbf{b} = -\rho \nabla V_e, \quad (1)$$

$$\text{s.t. } a_{\text{Griffith}}^{\text{generalised}} := a^* = \arg \min_{a \in \mathbb{R}} \iint_{\Omega} f(a, \mathbf{u}, \theta; \lambda, \mu, \mathbf{d}^{(*)} \otimes \mathbf{d}^{(*)}) d\Omega - \iint_{\Gamma} f(a; \gamma) d\Gamma \Big|_{\mathbf{u}} \quad (2)$$

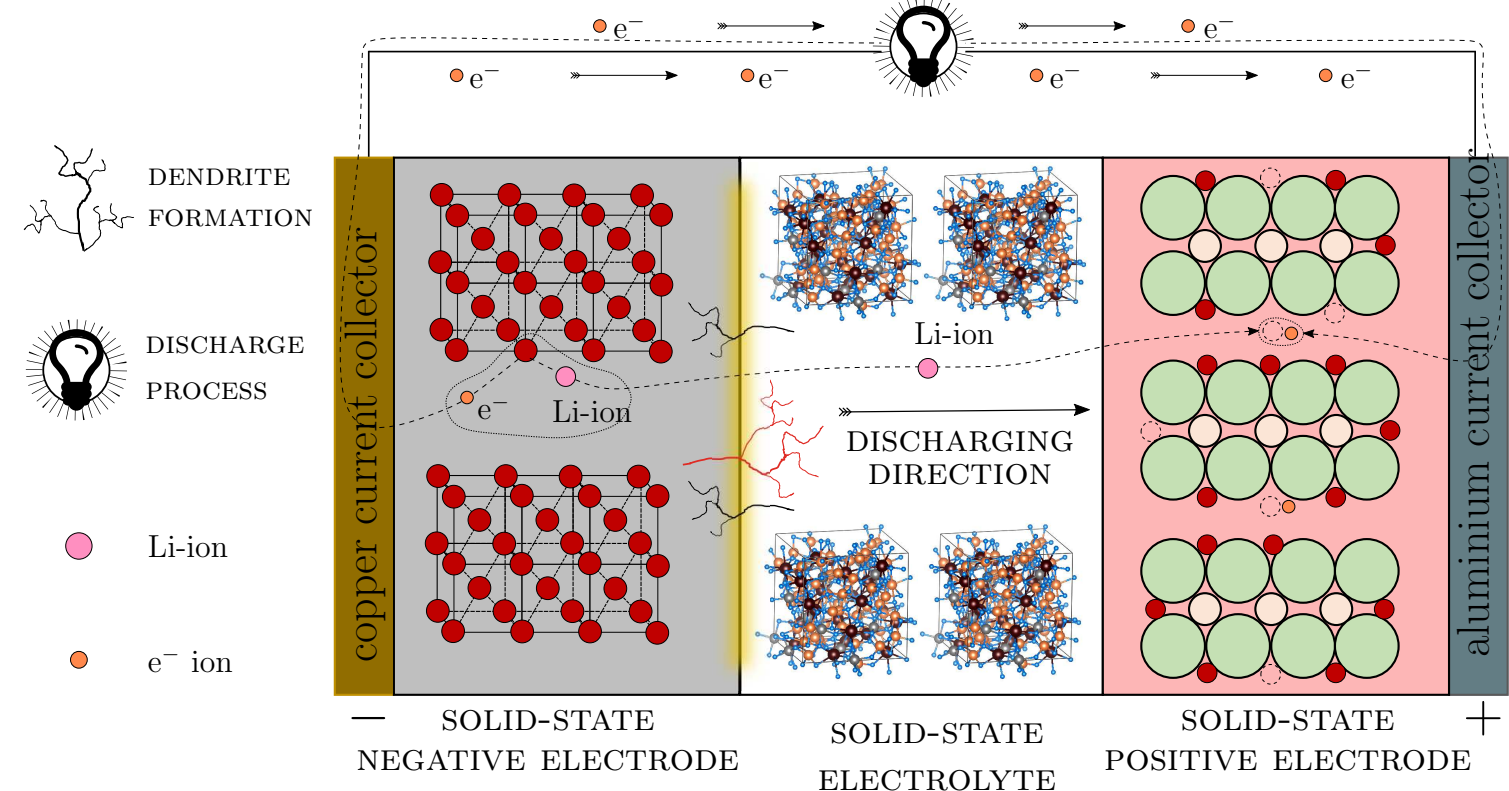
where  $V_e : \mathbb{R}^3 \rightarrow \mathbb{R}$  is the electric potential applied globally on ASSB. Due to nature setting of ASSB taking the form (SE|SSE|SE) the electric potential becomes uniform. Additionally,  $\mathbf{u}$  is the displacement field,  $\theta$  temperature field,  $a$  crevice length,  $\lambda, \mu$  Lamé constants,  $\mathbf{d}^{(*)} \otimes \mathbf{d}^{(*)}$  embedded misorientation structural tensor, and  $\gamma$  cracking-surface energy density, can help to improve ASSB performance [1][2].

**Aim:** The study is with the purpose of gaining a better insight into dendrite nucleation and formation in ASSB.



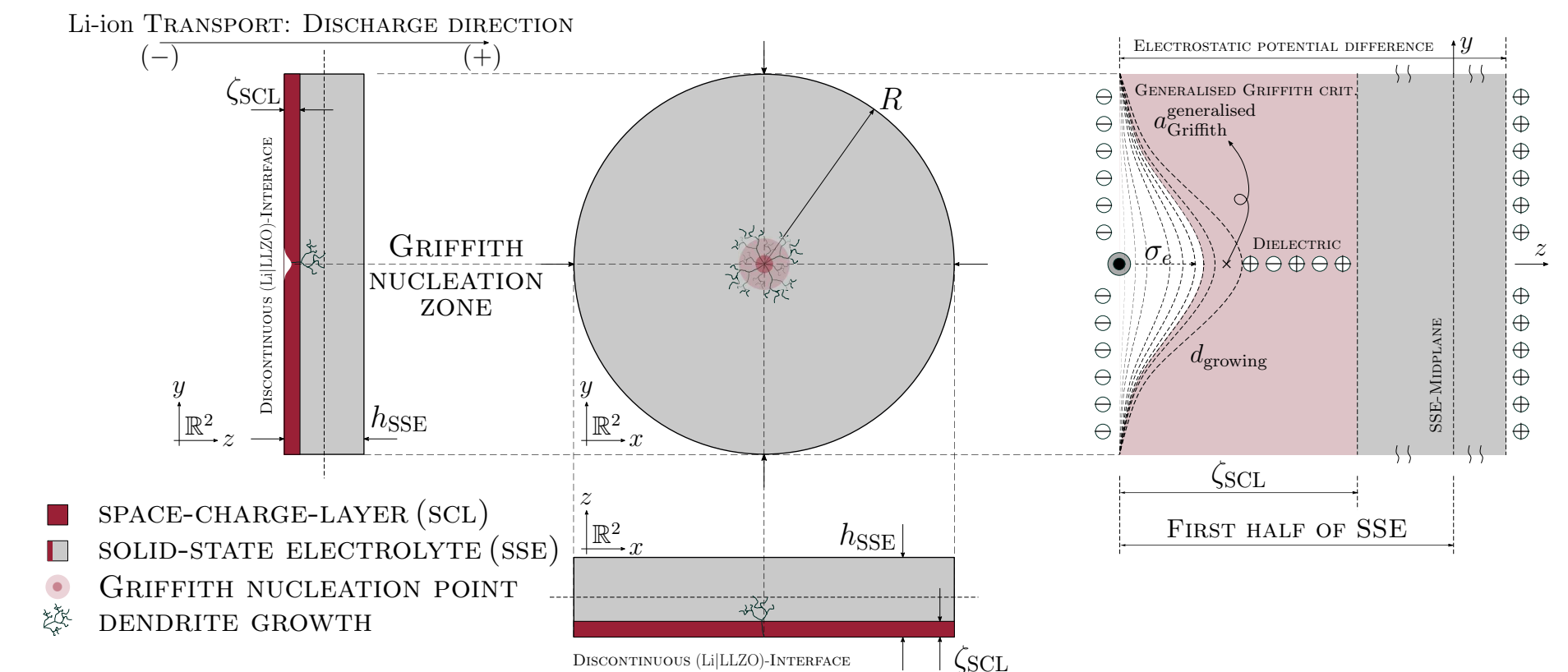
## Next-generation All-solid-state battery

**Griffith nucleation criterion** governs (SE|SSE)-Interface [4].



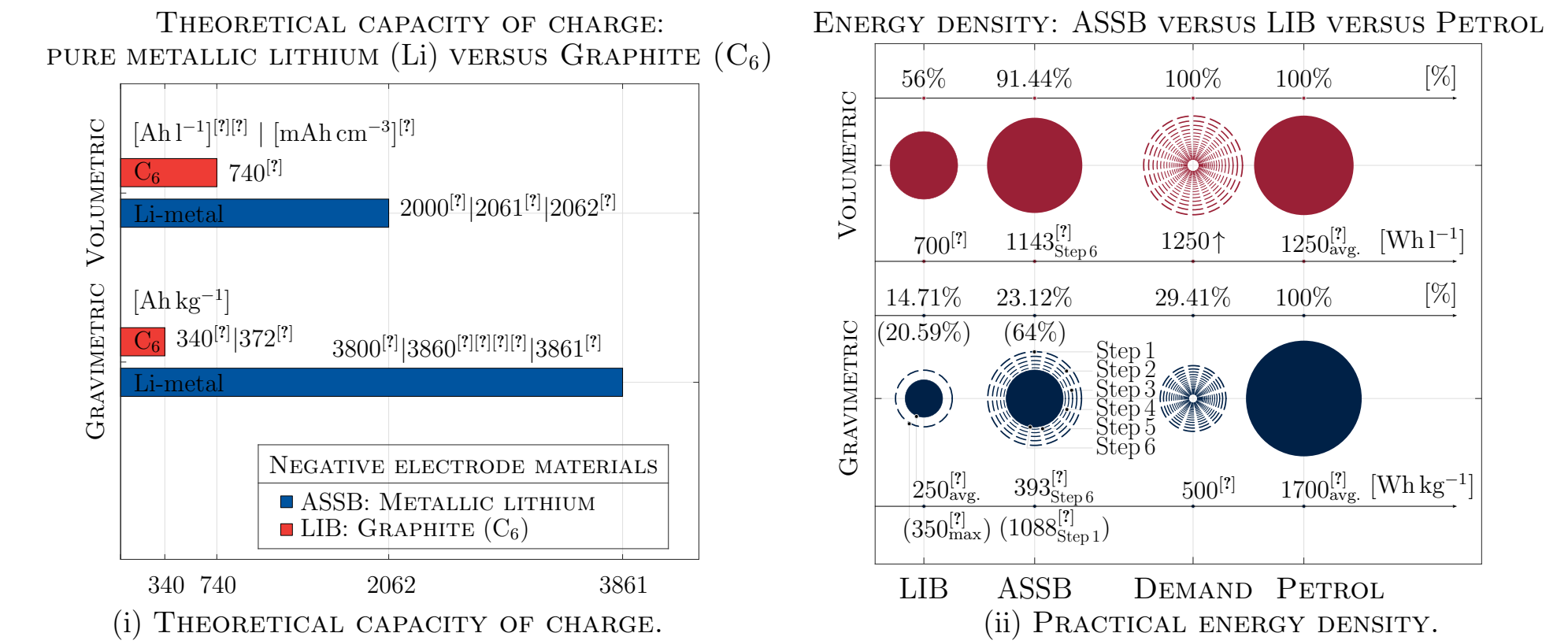
## Observation: Space-charge Layer

**SCL** manifests in ASSB [8], predictably in Semiconductors.



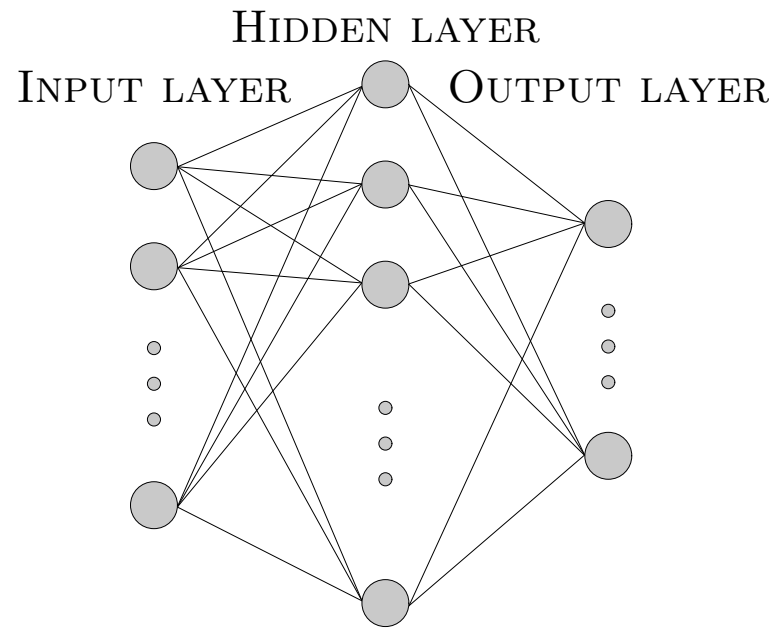
## Motivation: Energy density landscape

ASSB enables **energy demand** due to (i), and followed by (ii).



## Artificial Neural Networks

**Application:** Steel's property prediction.



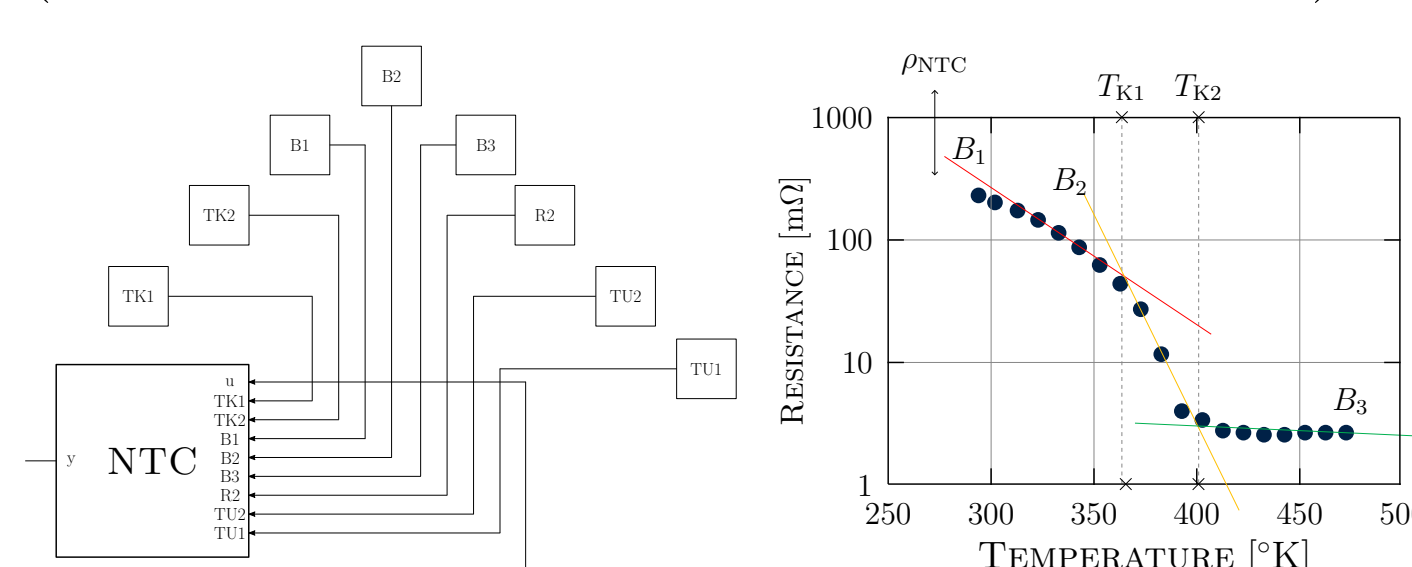
The ANNs scheme enhances bainitic trafo. temperature prediction, validated by [9].

## Semiconductor

**Application:** Start/Stop-System in Starter.

**Use-case:** BMW B47 (-25°C, 0°C, 120°C).

**Optimisation:** Pareto @BoschForschung. (Multi-objective optimisation framework)



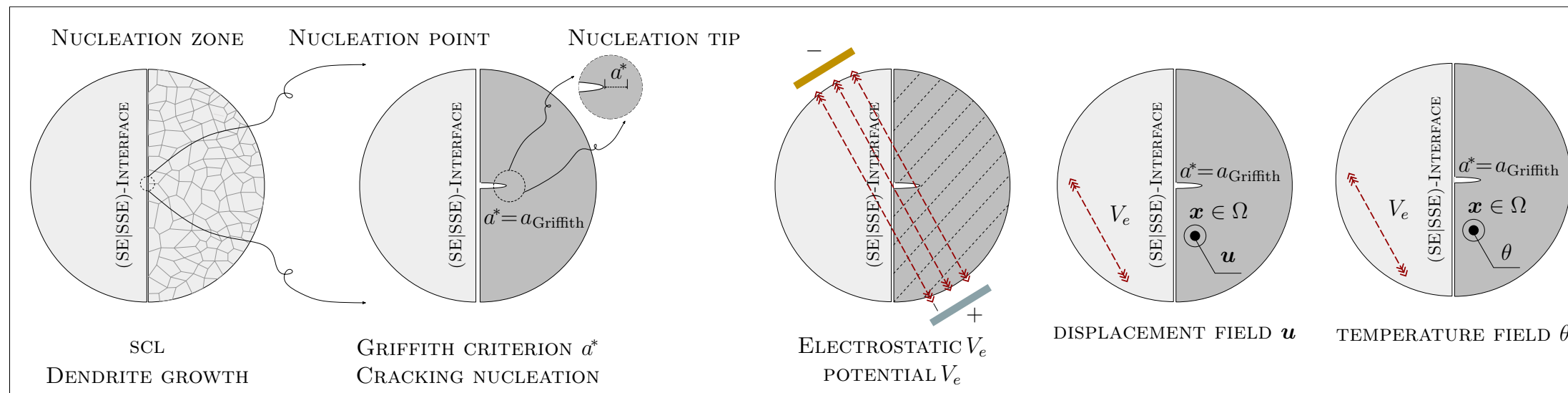
Nd/Gd Negative-Temperature Coefficient (NTC) semiconductor model validated [7].

## Lithium-ion battery

**Modelling:** Swelling phenomena @FEM [5].

**Use-case:** Bosch-48-V-Battery.

## Nucleation interface: Taking place at the critically dendritic (SE|SSE)-Interface



**Coupled fields** are Displacement field  $\mathbf{u}$  and temperature field  $\theta$ :

$$\mathbf{u} : \begin{cases} \Omega \times \mathbb{R}_+ \rightarrow \mathbb{R}^3, \\ (\mathbf{x}, t) \mapsto \mathbf{u}(\mathbf{x}, t), \end{cases} \quad \theta : \begin{cases} \Omega \times \mathbb{R}_+ \rightarrow \mathbb{R}, \\ (\mathbf{x}, t) \mapsto \theta(\mathbf{x}, t). \end{cases}$$

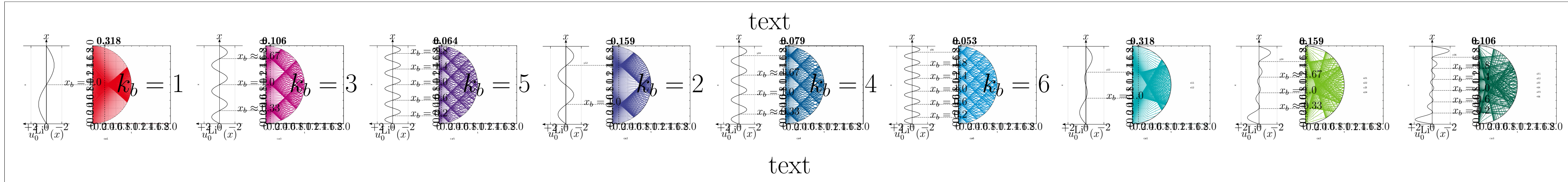
Governing conservation equations account for mass balance, linear and angular momentum, and energy conservation. These equations include variables such as mass density  $\rho(\mathbf{x}, t)$ , body force  $\mathbf{b}(\mathbf{x}, t)$ , velocity  $\mathbf{v}(\mathbf{x}, t)$ , internal energy  $e(\mathbf{x}, t)$ , heat flux  $\mathbf{q}(\mathbf{x}, t)$ , heat source  $r(\mathbf{x}, t)$ , Cauchy stress  $\boldsymbol{\sigma}$ , and infinitesimal strain  $\boldsymbol{\varepsilon}$  per unit volume.

**Strain energy** ( $E_{\text{st}}$ ) is derived from the SSE deformation due to dendrite formation at the unstable (SE|SSE)-Interface:

$$E_{\text{st}} := \iiint_{\Omega} f(a, \mathbf{u}; \lambda, \mu, \mathbf{d} \otimes \mathbf{d}) d\Omega$$

**Surface energy** ( $E_{\text{sf}}$ ) is assessed through the analysis of crevices at the (SE|SSE)-Interface under specific pressure conditions:

$$E_{\text{sf}} := \iint_{\Gamma} f(a; \gamma) d\Gamma$$



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Scan me

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## References

- [1] T. Vo, C. Hüter, S. Braun, M. Torrilhon, *Math. modelling for ASSB: (SE|SSE)-Interface*. Poster, Oxford Batt. Modelling Symp., University of Oxford, OBMS23, P. 66, **2023**.
- [2] T. Vo, C. Hüter, S. Braun, M. Torrilhon, *Next-gen. All-solid-state Battery (#ASSB)*. Poster, SIAM Computational Science and Eng. Conference, CSE23, Amsterdam, **2023**.
- [3] T. Vo, *Math. modelling for ASSB*. Presentation, Numerical Modelling in Applied Physics with OpenFOAM Summer School, NUMAP-FOAM, University of Cambridge, **2022**.
- [4] T. Vo, C. Hüter, S. Braun, R. Spatschek, *Mathematical modelling for All-solid-state battery: Griffith criterion*. Doctoral presentation, IEK-2, Forschungszentrum Jülich, **2020**.
- [5] T. Vo, *Modeling the swelling phenomena of li-ion batt. cells based on a numerical chemo-mech. coupled approach*. Master thesis, Robert Bosch Battery Systems GmbH, **2018**.
- [6] T. Vo, A. Gallet-Segarra, C. Bertsch, *Integration of Modelica powertrain models into Hardware in the loop env.*. PreMaster, Robert Bosch GmbH, FEBER, EAM-P2695, **2015**.
- [7] T. Vo, *Simulation environment for NTC-based voltage drop reduction in Start/Stop appl. and its optimization*. Semiconductor. Bachelor thesis, Robert Bosch GmbH, **2014**.
- [8] S. Braun, C. Yada, A. Latz, *Thermodynamically consistent model for Space-Charge-Layer formation in a solid electrolyte*. J. Physical Chemistry C, 119, 22281-22288, **2015**.
- [9] C. Hüter, X. Yin, T. Vo, S. Braun, *A pragmatic dataset augmentation approach for transformation temp. prediction in steels*. Comp. Mat. Science, Vol. 176, 109488, **2020**.
- [10] C. Hüter, S. Fu, M. Finsterbusch, E. Figgemeier, L. Wells, and R. Spatschek, *Electrode-electrolyte interface stability in SSE system*. AIMS Materials Science, 4(4):867-877, **2017**.

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