Next-generation all-solid-state battery (#ASSB)

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Mathematical modelling for the next-generation All-solid-state batteries: Nucleation (SE|SSE)^(*)-interface

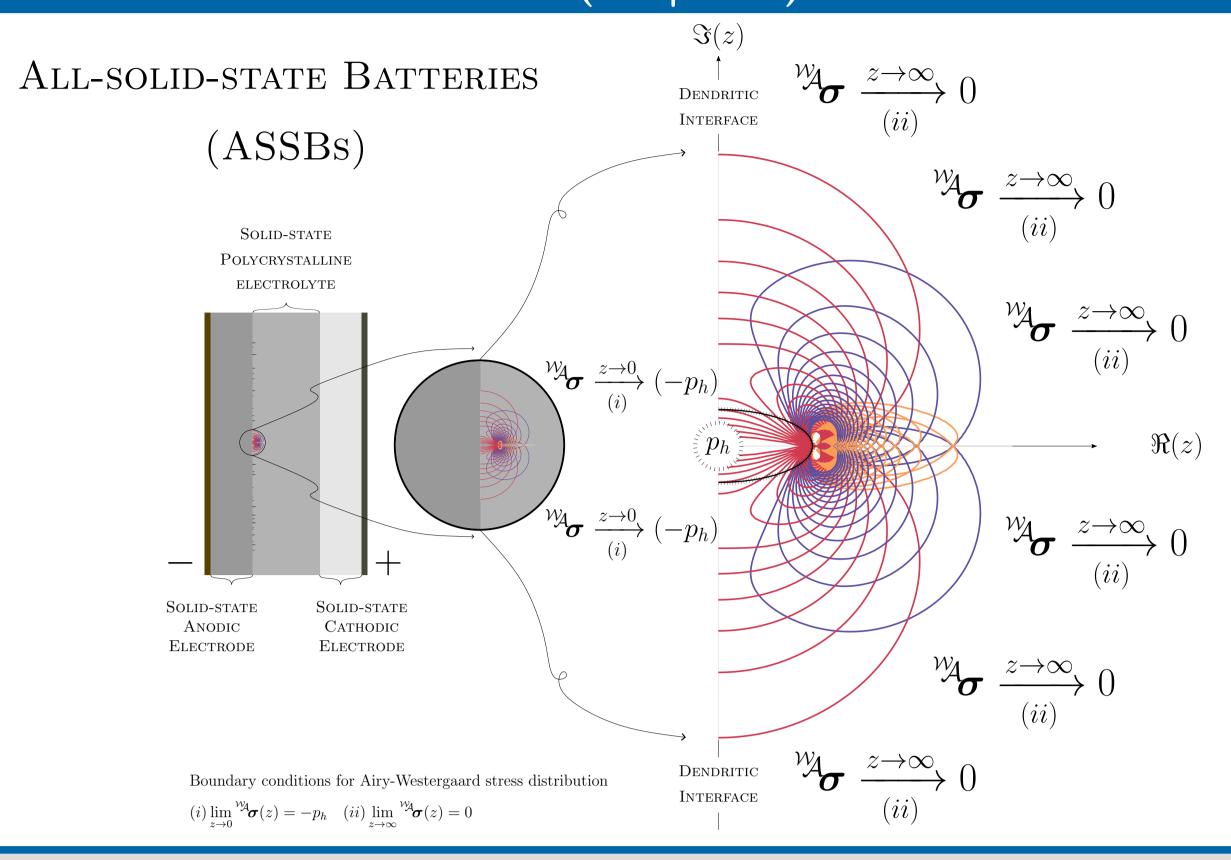
Rechargeable Lithium-ion battery (LIB) is at the heart of every electric vehicle (EV), portable electronic device, and energy storage system [1]. Nowadays, LIBs enable human life more efficient and help to solve global environment issues thanks to EVs' zero emission. However, conventional LIB (c-LIB) is sensible to temperature and pressure, hence, flammable and explosive, which is undesirable. This bottleneck is mainly due to liquid-based electrolyte found in c-LIBs.

All-solid-state battery (ASSB) is one of promising candidates to overcome bottlenecks of c-LIBs. Thanks to solid-state electrolyte (SSE), ASSB is highly stable towards temperature and pressure. Nevertheless, Limetal dendrite triggered at (SE|SSE)-interface [5] is the main drawback of ASSB since these dendritic threads extrapolate into SSE grain boundary network, causing crevice, degradation of ionic conductivity, and the probability of short-circuit, which is unfavorable.

Next-generation All-solid-state battery (ng-ASSB) with a consideration of nucleation criterion defined by

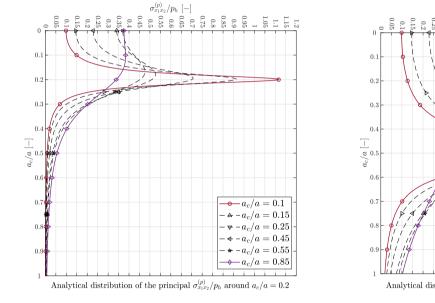
$$a_{\text{Griffith}} := a^* = \arg\min_{a \in \mathbb{R}} \left. \iint_{\Omega} f(a, \boldsymbol{u}, \theta; \lambda, \mu, \boldsymbol{d}^R \otimes \boldsymbol{d}^R) \, d\Omega - \iint_{\Gamma} f(a; \gamma) \, d\Gamma \right|_{\boldsymbol{u}^{(s)}}$$

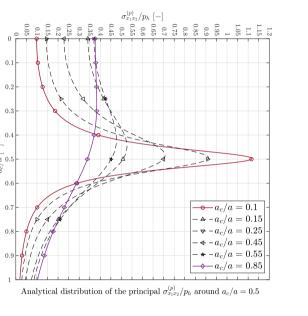
where \boldsymbol{u} displacement field, θ temperature field, a crevice length, λ, μ Lamé constants, $\boldsymbol{d}^R \otimes \boldsymbol{d}^R$ embedded misorientation structural tensor, and γ cracking-surface energy density, can help to improve ASSB performance.

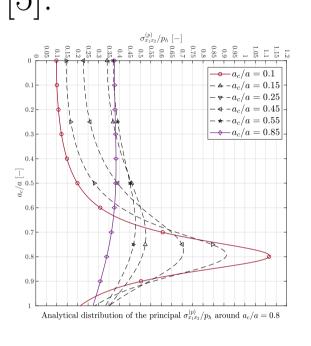


Interface Analysis

Interface between solid electrode and solid-state electrolyte (SE|SSE) taking place at space charge layer (SCL) [2] found in ASSBs critically exhibits mechanical and electrochemical instability [3]. This evidence points directly to the fact that the soft metallic li anode is erroneously prone to triggering dendrites, under cycles of electric charge & discharge [5].



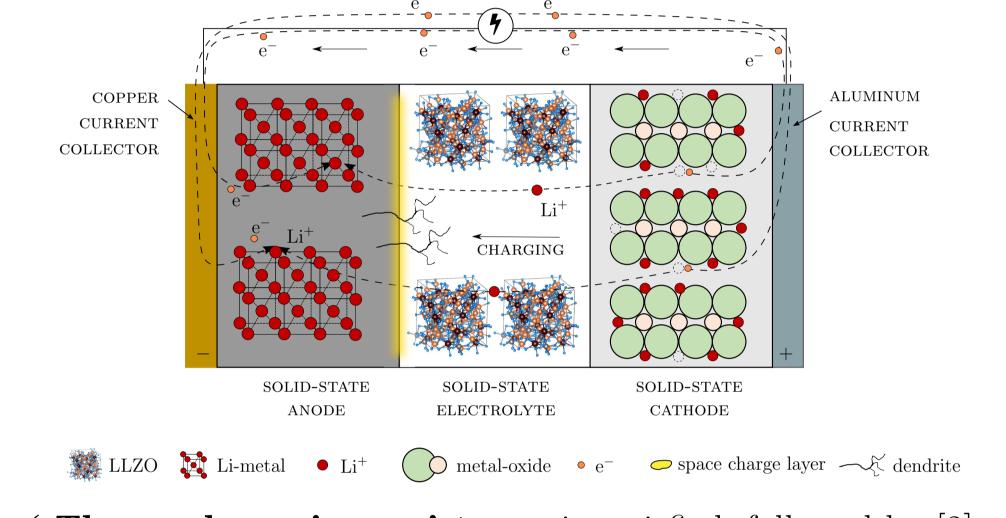




<u>Distribution</u>: ana. max. shear stress ${}^{\mathcal{W}}\!\!\sigma_{x_1x_2}^{\Pi}$ around crack tip a_c .

Next-generation All-solid-state battery

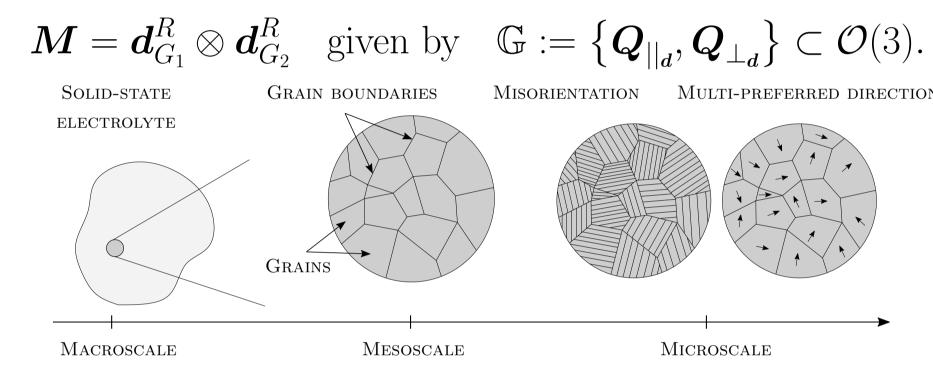
Nucleation criterion governs the instable (SE|SSE)-interface [3]



Thermodynamic consistency is satisfied, followed by [2]. \checkmark Closure Ω is fulfilled by 15 moments, followed by [4].

Embedded structural-tensor in SSE

Polycrystalline garnet-type SSE [5] such as LLZO exhibit grain boundary network, and grains with variation of {size, shape} under microscopic observation. Hence, this microstructure is potentially prone to nuances of destruction.



Consequentially, dendrites contribute to degradation of ionic conductivity and tiny-cracks tracing along grain boundaries.

Nucleation interface: Taking place at the critical dendritic interface

Coupled fields: Displacement field and temperature field; structural tensor

$$\boldsymbol{u}: \begin{cases} \Omega \times \mathbb{R}_{+} \to \mathbb{R}^{3}, \\ (\boldsymbol{x},t) \mapsto \boldsymbol{u}(\boldsymbol{x},t), \end{cases} \quad \theta: \begin{cases} \Omega \times \mathbb{R}_{+} \to \mathbb{R}, \\ (\boldsymbol{x},t) \mapsto \theta(\boldsymbol{x},t), \end{cases} \quad \boldsymbol{M}: \begin{cases} \Omega \times \mathbb{R}_{+} \to \mathbb{R}, \\ (\boldsymbol{x},t) \mapsto \theta(\boldsymbol{x},t), \end{cases}$$

Governing conservation equations

$$\frac{d}{dt} \int_{\Omega}^{\infty} (\cdot) \ d\Omega = \int_{\Omega} (\cdot)^{\text{action}} \ d\Omega + \int_{\partial \Omega} (\cdot)^{\text{action}} \ d\partial\Omega + \int_{\Omega} (\cdot)^{\text{production/source/sink}} \ d\Omega$$

 $\rho(\boldsymbol{x},t)$ is mass density per unit volume (puv); $\boldsymbol{b}(\boldsymbol{x},t)$ body force puv; $\boldsymbol{v}(\boldsymbol{x},t)$ velocity; $e(\boldsymbol{x},t)$ internal energy puv; $\boldsymbol{q}(\boldsymbol{x},t)$ heat flux; $r(\boldsymbol{x},t)$ heat source puv; $\boldsymbol{\sigma}$ Cauchy stress and ε infinitesimal strain. Helmholtz energy functional

$$a_{ ext{Griffith}} := a^* = \arg\min_{a \in \mathbb{R}} \left. \iint_{\Omega} f(a, oldsymbol{u}; \lambda, \mu, oldsymbol{d} \otimes oldsymbol{d}) \, d\Omega - \left. \iint_{\Gamma} f(a; \gamma) \, d\Gamma \right|_{oldsymbol{u}^{(s)}}$$

Governing PDE

$$a_{\text{Griffith}} := a^* = \arg\min_{a \in \mathbb{R}} \left. \iint_{\Omega} f(a, \boldsymbol{u}; \lambda, \mu, \boldsymbol{d} \otimes \boldsymbol{d}) \, d\Omega - \left. \iint_{\Gamma} f(a; \gamma) \, d\Gamma \right|_{\boldsymbol{u}^{(s)}}$$

abc

Strain energy: Interface between solid electrode and solid-state electrolyte (SE|SSE) taking place at space charge

 $\iiint_{\Omega} f(a, \boldsymbol{u}; \lambda, \mu, \boldsymbol{d} \otimes \boldsymbol{d}) d\Omega$

Surface energy: Interface between solid electrode and solid-state electrolyte (SE|SSE) taking place

$$\iint_{\Gamma} f(a;\gamma) \, d\Gamma$$

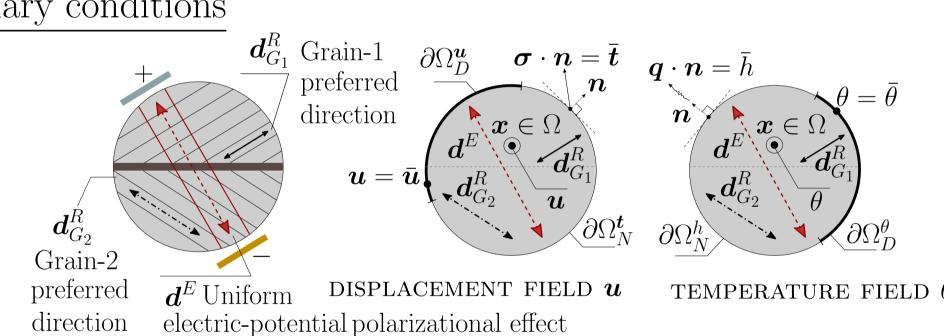
Therefore, the governing problem of dendritic nucleation at (SE|SSE) takes the form

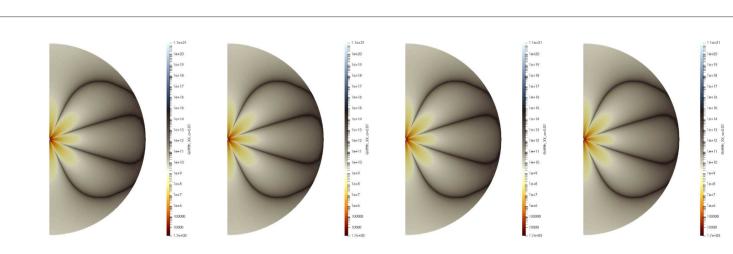
$$\partial_t oldsymbol{u}^{(s)} +
abla \cdot \left(\mathbb{C}^{f_{ ext{alocation}}(\lambda, \mu, oldsymbol{d}_{G_i, i=1,...,N}^R, oldsymbol{d}^E; oldsymbol{x})} :
abla oldsymbol{u}^{(s)}
ight) +
ho oldsymbol{b} = -
ho
abla V_e,$$

s.t.
$$a_{\text{Griffith}} := a^* = \arg\min_{a \in \mathbb{R}} \iiint_{\Omega} f(a, \boldsymbol{u}; \lambda, \mu, \boldsymbol{d} \otimes \boldsymbol{d}) d\Omega - \iint_{\Gamma} f(a; \gamma) d\Gamma \Big|_{\boldsymbol{u}^{(s)}}$$

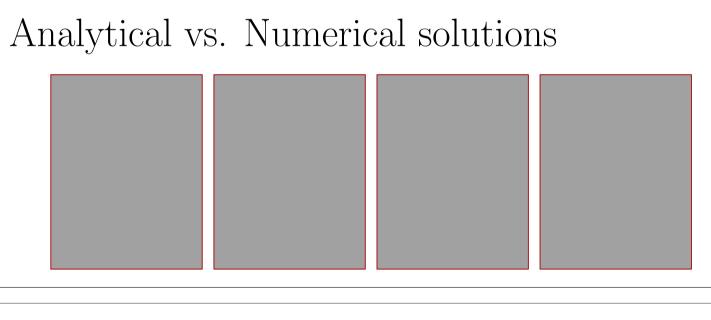
where

Boundary conditions





Comparison: Analytical vs. Numerical solutions



Partial differential equation (PDE) $abla \cdot \left(\overset{4}{\mathbb{C}} f^{\mathbb{D}(\Omega)}_{(\lambda,\mu)} \,
abla^{(s)} oldsymbol{u} \,
ight) +
ho \, oldsymbol{b} = oldsymbol{0}$ Displacement vector field solution $oldsymbol{u_i} \leftarrow oldsymbol{u} = oldsymbol{K}^{-1} oldsymbol{f}$ Strain tensor $arepsilon_{ij} = rac{1}{2} \left(\partial_{x_j} u_i + \partial_{x_i} u_j
ight)$ Stress tensor $\sigma_{ij} = \sum_{k,l} \overset{4}{\mathbb{C}}_{ijkl}^{f_{(\lambda,\mu)}^{(a,l)}} \, arepsilon_{kl}$ Strain energy density $\mathcal{E}_{ ext{strain}} := rac{1}{2} \sum_{i,j} \sigma_{ij} \, arepsilon_{ij}$

FEM: Strain energy density

Analysis: Airy-Westergaard function used for stress analysis: (i) max. shear stress and (ii) principal stresses

$$\left\| \mathcal{Y}_{\mathcal{A}} : \begin{cases} \mathbb{C} \to \mathbb{C}, \\ z \mapsto \mathcal{Y}_{\mathcal{A}}(z) := \Re(\iint_{\Gamma} \mathcal{K}^{(\star)} dz) + x_2 \Im(\oint_{\Gamma} \mathcal{K}^{(\star)} dz), \end{cases} \right\| \mathcal{K}^{(\star)} : \left\{ \begin{aligned} \mathbb{C} \to \mathbb{C}, \\ z \mapsto \mathcal{K}^{(\star)} := -p_h + p_h / \sqrt{1 - a^2 / z^2}, \end{aligned} \right. \right\}$$

where a the crevice length, p_h pressure at the opening crevice on dendritic interface, and $\forall \{p_h, a\} \in \mathbb{R}_+$.

Numerics \rightarrow FEM: element matrix \mathbf{K}^e approx. by Gauss quadrature; indices imply 4+2=6 for-loop: $K_{ik}^{e^{\alpha\beta}} = \int_{\Omega^{\mathcal{E}}} \left(\mathcal{L}_{1}^{\alpha} \, \mathbb{C}_{i1k1}^{fGL}(\boldsymbol{x}) \, \mathcal{R}_{1}^{\beta} + \mathcal{L}_{1}^{\alpha} \, \mathbb{C}_{i1k2}^{fGL}(\boldsymbol{x}) \, \mathcal{R}_{2}^{\beta} + \mathcal{L}_{2}^{\alpha} \, \mathbb{C}_{i2k1}^{fGL}(\boldsymbol{x}) \, \mathcal{R}_{1}^{\beta} + \mathcal{L}_{2}^{\alpha} \, \mathbb{C}_{i2k2}^{fGL}(\boldsymbol{x}) \, \mathcal{R}_{2}^{\beta} \right) \det(\boldsymbol{J}) \, d\Omega^{\xi}$

where \mathcal{L}_i^{α} and \mathcal{R}_l^{β} are gradients of basis functions at node α^{th} and β^{th} , respectively.

Contact

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