

# NEXT-GENERATION ALL-SOLID-STATE BATTERY (#ASSB)

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## Mathematical modelling for the next-generation All-solid-state batteries: Nucleation (SE|SSE)<sup>(\*)</sup>-interface

**Rechargeable Lithium-ion battery** (LIB) is at the heart of every electric vehicle (EV), portable electronic device, and energy storage system [1]. Nowadays, LIBs enable human life more efficient and help to solve global environment issues thanks to EVs' zero emission. However, conventional LIB (c-LIB) is sensible to temperature and pressure, hence, flammable and explosive, which is **undesirable**. This bottleneck is mainly due to **liquid-based electrolyte** found in c-LIBs.

**Next-generation All-solid-state battery** (ng-ASSB) with a consideration of **nucleation criterion** defined by

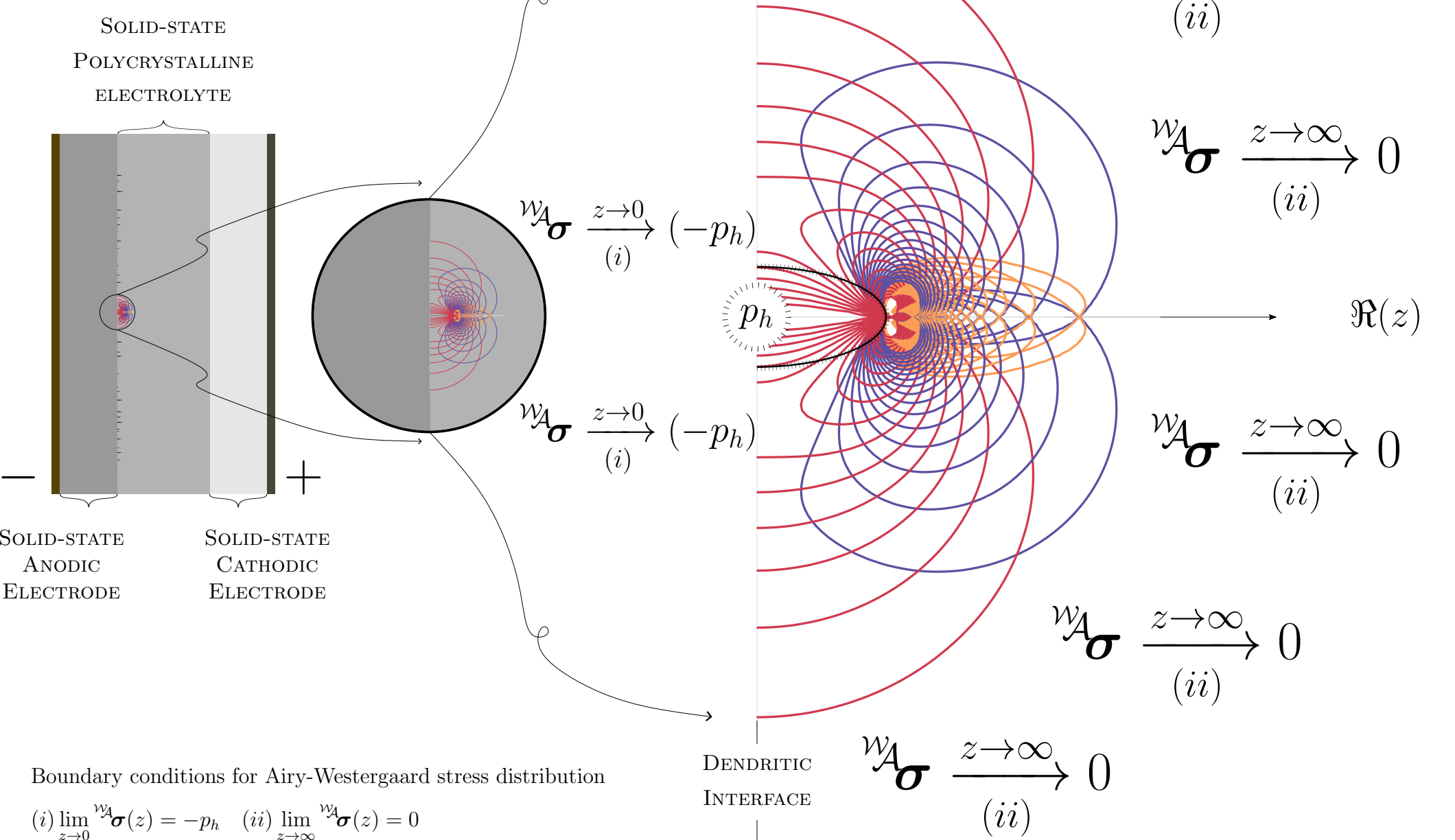
$$a_{\text{Griffith}} := a^* = \arg \min_{a \in \mathbb{R}} \iiint_{\Omega} f(a, \mathbf{u}, \theta; \lambda, \mu, \mathbf{d}^R \otimes \mathbf{d}^R) d\Omega - \iint_{\Gamma} f(a; \gamma) d\Gamma \Big|_{\mathbf{u}^{(s)}}$$

where  $\mathbf{u}$  displacement field,  $\theta$  temperature field,  $a$  crevice length,  $\lambda, \mu$  Lamé constants,  $\mathbf{d}^R \otimes \mathbf{d}^R$  embedded misorientation structural tensor, and  $\gamma$  cracking-surface energy density, **can help** to improve ASSB performance.

**All-solid-state battery** (ASSB) is one of promising candidates to overcome bottlenecks of c-LIBs. Thanks to **solid-state electrolyte** (SSE), ASSB is highly stable towards temperature and pressure. Nevertheless, Li-metal dendrite triggered at (SE|SSE)-interface [5] is the main drawback of ASSB since these dendritic threads extrapolate into SSE grain boundary network, causing crevice, degradation of ionic conductivity, and the probability of short-circuit, which is **unfavorable**.

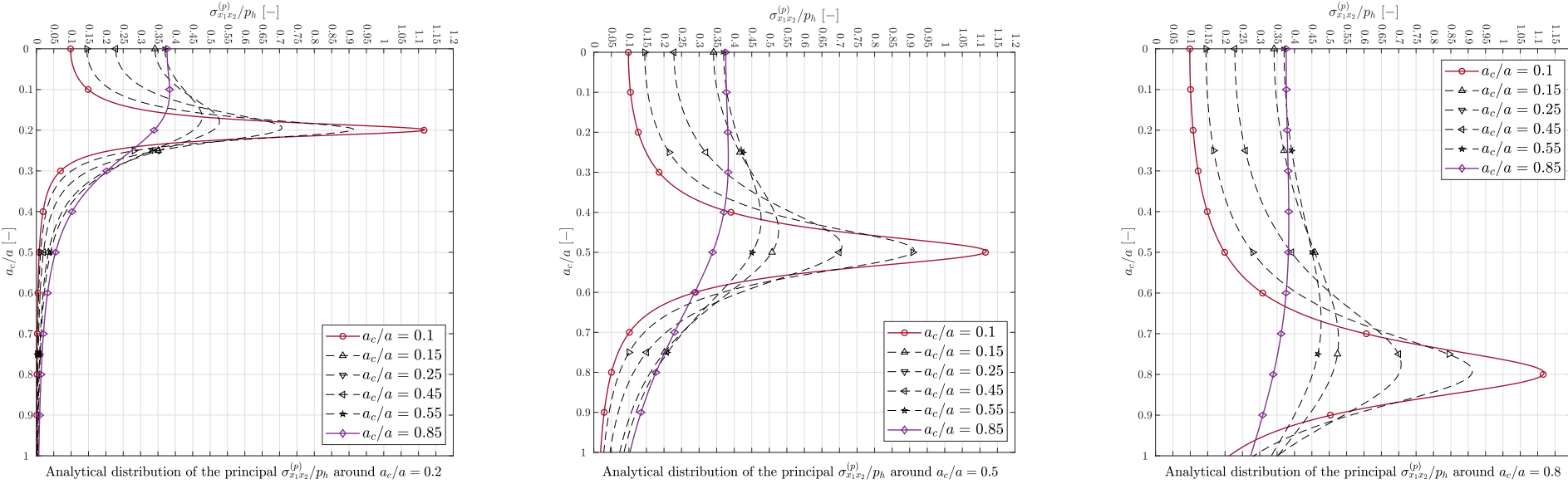
ALL-SOLID-STATE BATTERIES

(ASSBs)



### Interface Analysis

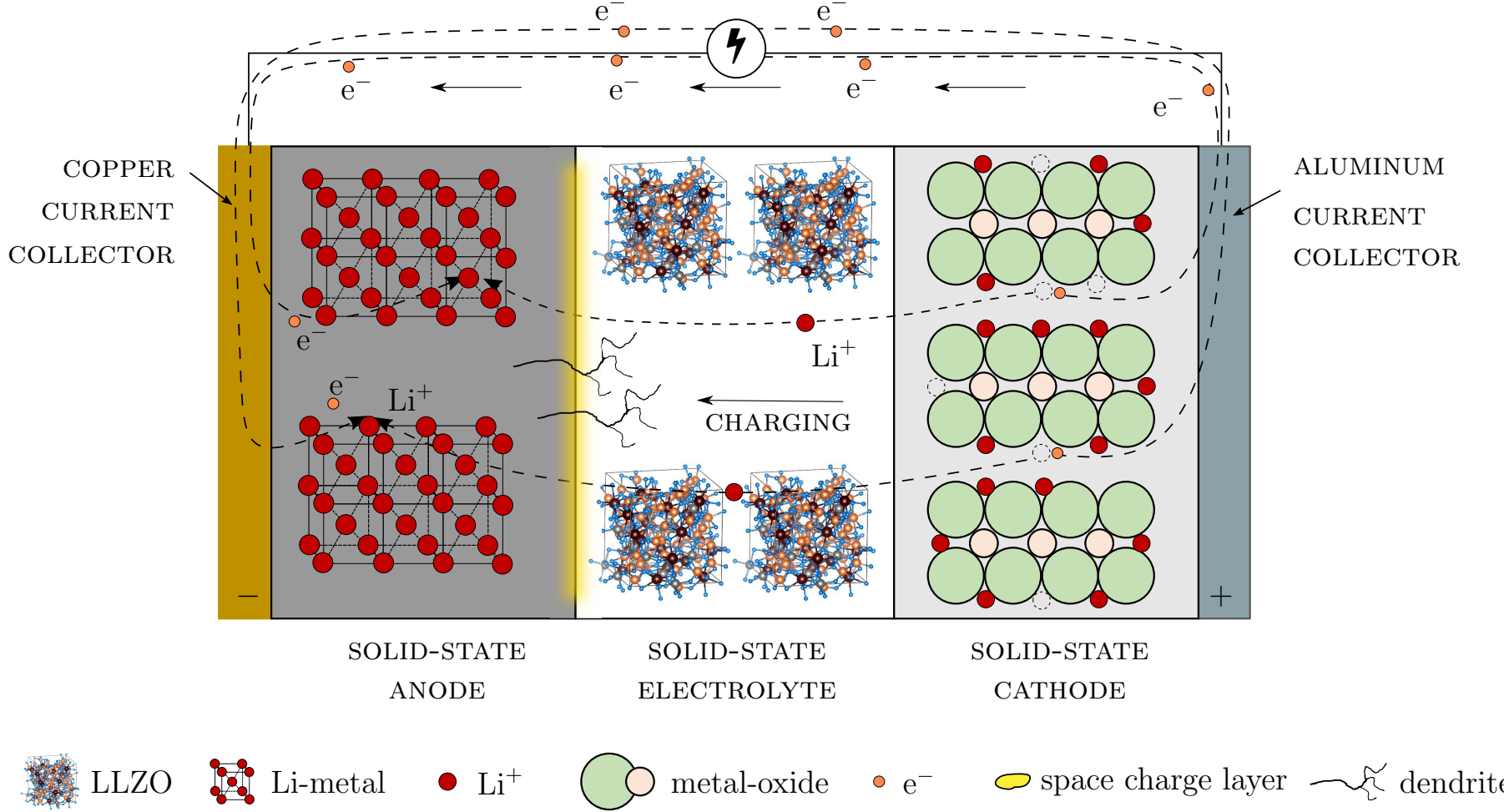
**Interface** between solid electrode and solid-state electrolyte (SE|SSE) taking place at space charge layer (SCL) [2] found in ASSBs critically exhibits mechanical and electrochemical instability [3]. This evidence points directly to the fact that the soft metallic Li anode is erroneously prone to triggering dendrites, under cycles of electric charge & discharge [5].



Distribution: ana. max. shear stress  $w_A \sigma_{11}^{\Pi}$  around crack tip  $a_c$ .

### Next-generation All-solid-state battery

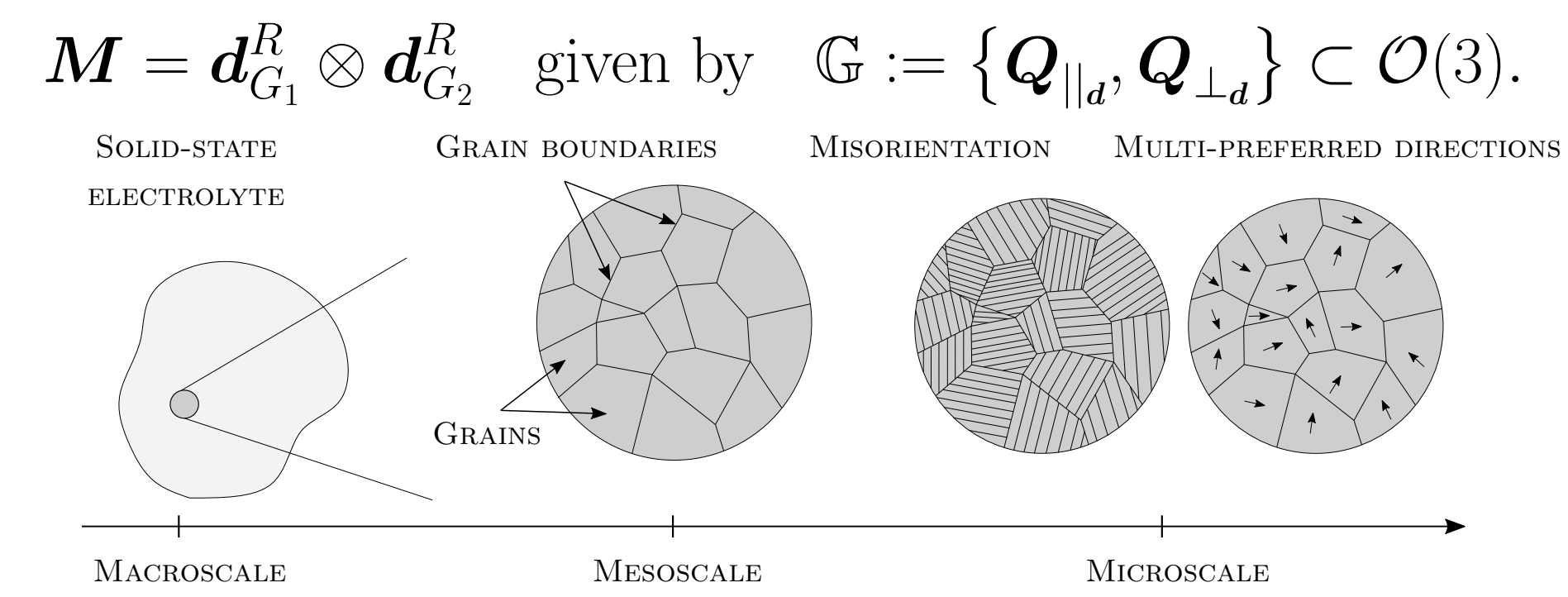
**Nucleation** criterion governs the instable (SE|SSE)-interface [3]



- ✓ **Thermodynamic consistency** is satisfied, followed by [2].
- ✓ **Closure**  $\bar{\Omega}$  is fulfilled by 15 moments, followed by [4].

### Embedded structural-tensor in SSE

**Polycrystalline** garnet-type SSE [5] such as LLZO exhibit grain boundary network, and grains with variation of {size, shape} under microscopic observation. Hence, this microstructure is potentially prone to nuances of destruction.



Consequently, dendrites contribute to degradation of ionic conductivity and tiny-cracks tracing along grain boundaries.

## Nucleation interface: Taking place at the critical dendritic interface

Coupled fields: Displacement field  $\mathbf{u}$  and temperature field  $\theta$ ; structural tensor  $\mathbf{M}$

$$\mathbf{u} : \begin{cases} \Omega \times \mathbb{R}_+ \rightarrow \mathbb{R}^3, \\ (\mathbf{x}, t) \mapsto \mathbf{u}(\mathbf{x}, t), \end{cases} \quad \theta : \begin{cases} \Omega \times \mathbb{R}_+ \rightarrow \mathbb{R}, \\ (\mathbf{x}, t) \mapsto \theta(\mathbf{x}, t), \end{cases} \quad \mathbf{M}^{(R,E)} : \begin{cases} (\mathbf{d}_{\text{Grain } i}^R \otimes \mathbf{d}_{\text{Grain } i}^R) \Big|_{i=1, \dots, N} \\ \mathbf{d}_{\text{Grain } i}^R \otimes \mathbf{d}^E \end{cases}$$

Governing conservation equations

$$\frac{d}{dt} \int_{\Omega} (\cdot) d\Omega = \int_{\Omega} (\cdot)^{\text{action}} d\Omega + \int_{\partial\Omega} (\cdot)^{\text{action}} d\partial\Omega + \int_{\Omega} (\cdot)^{\text{production} + / -} d\Omega$$

$\rho(\mathbf{x}, t)$  is mass density per unit volume (puv);  $\mathbf{b}(\mathbf{x}, t)$  body force puv;  $\mathbf{v}(\mathbf{x}, t)$  velocity;  $e(\mathbf{x}, t)$  internal energy puv;  $\mathbf{q}(\mathbf{x}, t)$  heat flux;  $r(\mathbf{x}, t)$  heat source puv;  $\boldsymbol{\sigma}$  Cauchy stress and  $\boldsymbol{\varepsilon}$  infinitesimal strain. Helmholtz energy functional

$$a_{\text{Griffith}} := a^* = \arg \min_{a \in \mathbb{R}} \iiint_{\Omega} f(a, \mathbf{u}; \lambda, \mu, \mathbf{d} \otimes \mathbf{d}) d\Omega - \iint_{\Gamma} f(a; \gamma) d\Gamma \Big|_{\mathbf{u}^{(s)}}$$

Governing PDE

$$a_{\text{Griffith}} := a^* = \arg \min_{a \in \mathbb{R}} \iiint_{\Omega} f(a, \mathbf{u}; \lambda, \mu, \mathbf{d} \otimes \mathbf{d}) d\Omega - \iint_{\Gamma} f(a; \gamma) d\Gamma \Big|_{\mathbf{u}^{(s)}}$$

abc

**Strain energy:** Interface between solid electrode and solid-state electrolyte (SE|SSE) taking place at space charge

$$\iiint_{\Omega} f(a, \mathbf{u}; \lambda, \mu, \mathbf{d} \otimes \mathbf{d}) d\Omega$$

**Surface energy:** Interface between solid electrode and solid-state electrolyte (SE|SSE) taking place

$$\iint_{\Gamma} f(a; \gamma) d\Gamma$$

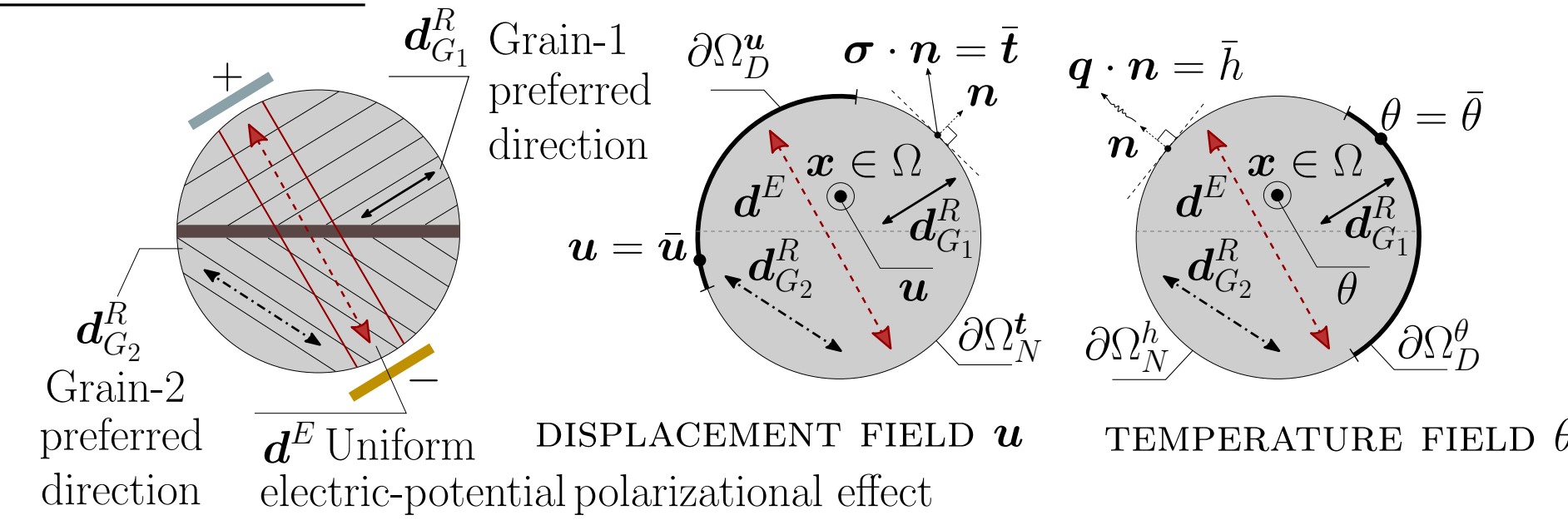
Therefore, the governing problem of dendritic nucleation at (SE|SSE) takes the form

$$\partial_t \mathbf{u}^{(s)} + \nabla \cdot \left( \mathbb{C}_{\text{allocation}}^{(4)}(\lambda, \mu, \mathbf{d}_{G_i}^R, i=1, \dots, N, \mathbf{d}^E; \mathbf{x}) : \nabla \mathbf{u}^{(s)} \right) + \rho \mathbf{b} = -\rho \nabla V_c,$$

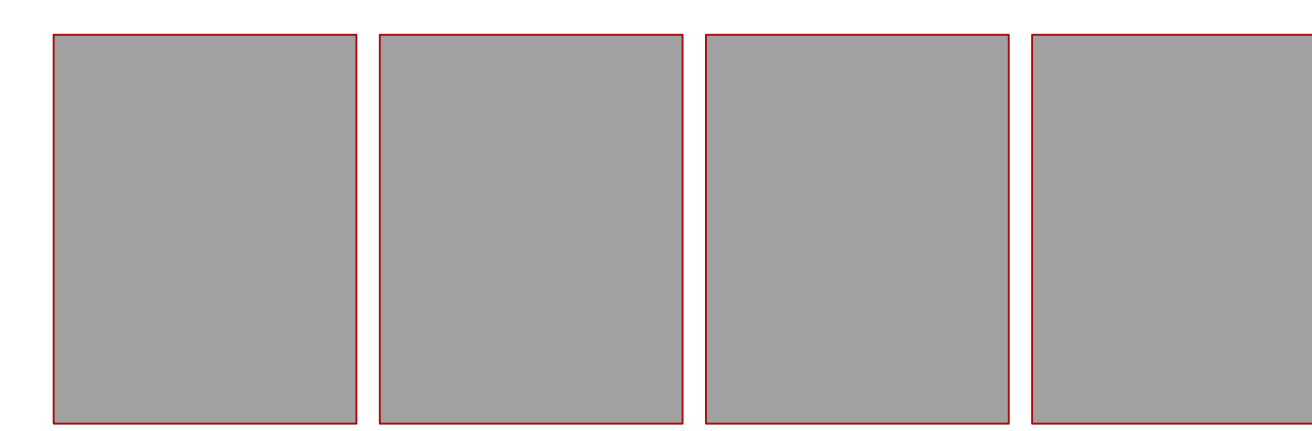
$$\text{s.t. } a_{\text{Griffith}} := a^* = \arg \min_{a \in \mathbb{R}} \iiint_{\Omega} f(a, \mathbf{u}; \lambda, \mu, \mathbf{d} \otimes \mathbf{d}) d\Omega - \iint_{\Gamma} f(a; \gamma) d\Gamma \Big|_{\mathbf{u}^{(s)}}$$

where

Boundary conditions



Comparison: Analytical vs. Numerical solutions



Analysis: Airy-Westergaard function used for stress analysis: (i) max. shear stress and (ii) principal stresses

$$w_A : \begin{cases} \mathbb{C} \rightarrow \mathbb{C}, \\ z \mapsto w_A(z) := \Re \left( \oint_{\Gamma} \mathcal{K}^{(*)} dz \right) + x_2 \Im \left( \oint_{\Gamma} \mathcal{K}^{(*)} dz \right), \end{cases} \quad \mathcal{K}^{(*)} : \begin{cases} \mathbb{C} \rightarrow \mathbb{C}, \\ z \mapsto \mathcal{K}^{(*)} := -p_h + p_h / \sqrt{1 - a^2/z^2}, \end{cases}$$

where  $a$  the crevice length,  $p_h$  pressure at the opening crevice on dendritic interface, and  $\forall \{p_h, a\} \in \mathbb{R}_+$ .

Numerics  $\rightarrow$  FEM: element matrix  $\mathbf{K}^e$  approx. by *Gauss quadrature*; indices imply  $4 + 2 = 6$  for-loop:

$$K_{ik}^{e\alpha\beta} = \int_{\Omega^e} \left( \mathcal{L}_1^\alpha \mathbb{C}_{i1k1}^{fGL}(\mathbf{x}) \mathcal{R}_1^\beta + \mathcal{L}_1^\alpha \mathbb{C}_{i1k2}^{fGL}(\mathbf{x}) \mathcal{R}_2^\beta + \mathcal{L}_2^\alpha \mathbb{C}_{i2k1}^{fGL}(\mathbf{x}) \mathcal{R}_1^\beta + \mathcal{L}_2^\alpha \mathbb{C}_{i2k2}^{fGL}(\mathbf{x}) \mathcal{R}_2^\beta \right) \det(\mathbf{J}) d\Omega^\xi$$

where  $\mathcal{L}_j^\alpha$  and  $\mathcal{R}_i^\beta$  are gradients of basis functions at node  $\alpha^{th}$  and  $\beta^{th}$ , respectively.

### Contact

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### References

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- [2] **S.Braun**, C.Yada and A.Latz, *Thermodynamically consistent model for Space-Charge-Layer formation in a solid electrolyte*. Jr. Phys. Chem., 119, 22281-22288, **2015**.
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