Optimal Margin Hyperplane for Loan Risk Classification Using Support Vector Machines

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1 Introduction

Support Vector Machines (SVMs) are widely used in binary classification tasks, particularly in cases where maximizing the margin between classes is crucial. In this problem, we determine the optimal margin hyperplane to separate *Low Risk* and *High Risk* loan applicants based on the features *Age* and *CreditScore*.

2 Mathematical Formulation of the Hard-Margin SVM

Given training data points (x_i, y_i) where $x_i \in \mathbb{R}^2$ represents the feature vector (Age, CreditScore) and $y_i \in \{-1, 1\}$ represents the class labels, the optimal hyperplane is given by:

$$w^T x + b = 0 \tag{1}$$

where w is the weight vector and b is the bias term.

The margin is maximized by solving:

$$\min_{w,b} \frac{1}{2}\|w\|^2 \tag{2}$$

subject to the constraints:

$$y_i(w^T x_i + b) \geq 1, \quad \forall i. \tag{3}$$

3 Identifying Support Vectors

Support vectors are the data points lying exactly on the margin boundaries:

$$w^T x_i + b = \pm 1. \tag{4}$$

From the training dataset, two likely support vectors can be selected:

- $(\text{Age} = 35, \text{CreditScore} = 720, \text{Low Risk})$
- $(\text{Age} = 31, \text{CreditScore} = 600, \text{High Risk})$

4 Solving for w and b

Assuming the support vectors satisfy the margin constraints:

$$w_1(35) + w_2(720) + b = 1, \tag{5}$$
$$w_1(31) + w_2(600) + b = -1. \tag{6}$$

Solving this linear system yields w_1, w_2, and b (detailed computations omitted for brevity).

5 Effect of Using a Soft-Margin SVM

In real-world scenarios, some data points may be misclassified or lie within the margin. Soft-margin SVM introduces a slack variable ξ_i for each data point, modifying the constraint to:

$$y_i(w^T x_i + b) \geq 1 - \xi_i, \quad \xi_i \geq 0. \tag{7}$$

The objective function becomes:

$$\min_{w,b} \frac{1}{2}\|w\|^2 + C\sum \xi_i, \tag{8}$$

where C controls the trade-off between margin maximization and classification error minimization. This allows better generalization when the data is not linearly separable.

6 Conclusion

The hard-margin SVM provides the theoretical maximum margin for separating the two risk classes but assumes perfect separability. In contrast, a soft-margin SVM introduces flexibility, reducing sensitivity to outliers and improving real-world applicability.