Multiple Linear Regression for Predicting CreditScore

Tran Quoc Thai
Student ID: 2370759

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1 Introduction

In this report, we apply multiple linear regression to predict `CreditScore` using `Age` and `Education` as input features. The model assumes the following linear relationship:

$$y = \theta_0 + \theta_1 x_1 + \theta_2 x_2, \tag{1}$$

where:

- y represents `CreditScore`.
- x_1 represents `Age`.
- x_2 represents `Education`.
- $\theta_0, \theta_1, \theta_2$ are the regression coefficients to be estimated.

To compute the optimal values of θ, we use the **Normal Equation**:

$$\theta = (X^T X)^{-1} X^T y. \tag{2}$$

2 Constructing the Design Matrix X and Target Vector y

The training dataset consists of the following records:

To handle the missing value in Education for x_2, we apply mean imputation:

$$x_2^{\text{missing}} = \frac{16 + 14 + 12 + 18 + 14 + 16 + 12}{7} = 14.57. \tag{3}$$

The design matrix X (including a bias column of ones) is:

Age (x_1)	**Education** (x_2)	**CreditScore** (y)
35	16	720
28	14	650
45	missing	750
31	12	600
52	18	780
29	14	630
42	16	710
33	12	640

Table 1: Training Dataset

$$X = \begin{bmatrix} 1 & 35 & 16 \\ 1 & 28 & 14 \\ 1 & 45 & 14.57 \\ 1 & 31 & 12 \\ 1 & 52 & 18 \\ 1 & 29 & 14 \\ 1 & 42 & 16 \\ 1 & 33 & 12 \end{bmatrix}. \tag{4}$$

The target vector y is:

$$y = \begin{bmatrix} 720 \\ 650 \\ 750 \\ 600 \\ 780 \\ 630 \\ 710 \\ 640 \end{bmatrix}. \tag{5}$$

3 Computing the Normal Equation Solution

3.1 Compute $X^T X$

$$X^T X = \begin{bmatrix} \sum x_0 & \sum x_1 & \sum x_2 \\ \sum x_1 & \sum x_1^2 & \sum x_1 x_2 \\ \sum x_2 & \sum x_1 x_2 & \sum x_2^2 \end{bmatrix}. \tag{6}$$

$$
\begin{aligned}
\sum x_0 &= 1+1+1+1+1+1+1+1 = 8,\\
\sum x_1 &= 35+28+45+31+52+29+42+33 = 295,\\
\sum x_2 &= 16+14+14.57+12+18+14+16+12 = 116.57,\\
\sum x_1^2 &= 35^2+28^2+45^2+31^2+52^2+29^2+42^2+33^2 = 11267,\\
\sum x_2^2 &= 16^2+14^2+14.57^2+12^2+18^2+14^2+16^2+12^2 = 2031.66,\\
\sum x_1x_2 &= (35)(16)+(28)(14)+(45)(14.57)+(31)(12)+(52)(18)+(29)(14)+(42)(16)+(33)(12)\\
&= 4760+392+656+372+936+406+672+396 = 8952.
\end{aligned}
$$

Thus,

$$
X^TX = \begin{bmatrix} 8 & 295 & 116.57 \\ 295 & 11267 & 8952 \\ 116.57 & 8952 & 2031.66 \end{bmatrix}. \tag{7}
$$

3.2 Compute the Determinant $\det(X^TX)$

The determinant of a 3×3 matrix:

$$
\det(A) = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}
$$

is computed as:

$$
\det(A) = a(ei - fh) - b(di - fg) + c(dh - eg).
$$

Applying this to our matrix:

$$
\det(X^TX) = 8\begin{vmatrix} 11267 & 8952 \\ 8952 & 2031.66 \end{vmatrix} - 295\begin{vmatrix} 295 & 116.57 \\ 8952 & 2031.66 \end{vmatrix} + 116.57\begin{vmatrix} 295 & 11267 \\ 8952 & 8952 \end{vmatrix}.
$$

Computing the 2×2 determinants:

$$
\begin{aligned}
\begin{vmatrix} 11267 & 8952 \\ 8952 & 2031.66 \end{vmatrix} &= (11267 \times 2031.66) - (8952 \times 8952)\\
&= 22885643.22 - 80150814\\
&= -57265170.78
\end{aligned}
$$

$$\begin{vmatrix} 295 & 116.57 \\ 8952 & 2031.66 \end{vmatrix} = (295 \times 2031.66) - (116.57 \times 8952)$$
$$= 599370.7 - 1043947.04$$
$$= -444576.34$$

$$\begin{vmatrix} 295 & 11267 \\ 8952 & 8952 \end{vmatrix} = (295 \times 8952) - (11267 \times 8952)$$
$$= 2646240 - 1314660.32$$
$$= 1331580$$

Thus,

$$\begin{aligned} \det(X^T X) &= (8 \times -57265170.78) - (295 \times -444576.34) + (116.57 \times 1331580) \\ &= -458121366.24 + 131149015.3 + 155073267.66 \\ &= -171899083.28 \end{aligned}$$

3.3 Compute the Adjugate Matrix Adj$(X^T X)$

The cofactor matrix is computed by taking the determinant of each minor matrix and adjusting the signs:

$$\text{Adj}(X^T X) = \begin{bmatrix} C_{11} & -C_{12} & C_{13} \\ -C_{21} & C_{22} & -C_{23} \\ C_{31} & -C_{32} & C_{33} \end{bmatrix}.$$

Using the previously computed determinants:

$$\begin{aligned} C_{11} &= \begin{vmatrix} 11267 & 8952 \\ 8952 & 2031.66 \end{vmatrix} \\ &= (11267 \times 2031.66) - (8952 \times 8952) \\ &= 22885643.22 - 80150814 \\ &= -57265170.78. \end{aligned}$$

$$\begin{aligned} C_{12} &= \begin{vmatrix} 295 & 116.57 \\ 8952 & 2031.66 \end{vmatrix} \\ &= (295 \times 2031.66) - (116.57 \times 8952) \\ &= 599370.7 - 1043947.04 \\ &= -444576.34. \end{aligned}$$

$$C_{13} = \begin{vmatrix} 295 & 11267 \\ 8952 & 8952 \end{vmatrix}$$
$$= (295 \times 8952) - (11267 \times 8952)$$
$$= 2646240 - 1314660.32$$
$$= 1331580.$$

$$C_{21} = \begin{vmatrix} 8 & 116.57 \\ 8952 & 2031.66 \end{vmatrix}$$
$$= (8 \times 2031.66) - (116.57 \times 8952)$$
$$= 16253.28 - 1043947.04$$
$$= -253206.56.$$

$$C_{22} = \begin{vmatrix} 8 & 295 \\ 116.57 & 2031.66 \end{vmatrix}$$
$$= (8 \times 2031.66) - (295 \times 116.57)$$
$$= 16253.28 - 34220.15$$
$$= 2032.6.$$

$$C_{23} = \begin{vmatrix} 8 & 295 \\ 8952 & 8952 \end{vmatrix}$$
$$= (8 \times 8952) - (295 \times 8952)$$
$$= 71616 - 2681715.9$$
$$= -253207.9.$$

$$C_{31} = \begin{vmatrix} 8 & 295 \\ 11267 & 8952 \end{vmatrix}$$
$$= (8 \times 8952) - (295 \times 11267)$$
$$= 71616 - 3314738.1$$
$$= -291312.1.$$

$$C_{32} = \begin{vmatrix} 8 & 116.57 \\ 11267 & 8952 \end{vmatrix}$$
$$= (8 \times 8952) - (116.57 \times 11267)$$
$$= 71616 - 130451.1$$
$$= 1164.9.$$

$$
\begin{aligned}
C_{33} &= \begin{vmatrix} 8 & 116.57 \\ 295 & 11267 \end{vmatrix} \\
&= (8 \times 11267) - (116.57 \times 295) \\
&= 90136 - 34422.15 \\
&= 5581.9.
\end{aligned}
$$

Thus, the cofactor matrix is:

$$
\mathrm{Cof}(X^T X) = \begin{bmatrix} -57265170.78 & 444576.34 & 1331580 \\ 253206.56 & 2032.6 & 253207.9 \\ 291312.1 & 1164.9 & 5581.9 \end{bmatrix}.
$$

Taking the transpose:

$$
\mathrm{Adj}(X^T X) = \begin{bmatrix} -57265170.78 & 253206.56 & 291312.1 \\ 444576.34 & 2032.6 & 1164.9 \\ 1331580 & 253207.9 & 5581.9 \end{bmatrix}.
$$

3.4 Compute $(X^T X)^{-1}$

$$
(X^T X)^{-1} = \frac{1}{\det(X^T X)} \cdot \mathrm{Adj}(X^T X).
$$

Dividing each element by -171899083.28:

$$
(X^T X)^{-1} = \begin{bmatrix} 0.572 & -0.015 & -0.017 \\ -0.015 & 0.0012 & 0.0007 \\ -0.017 & 0.0007 & 0.0032 \end{bmatrix}.
$$

Computing $X^T y$:

$$
X^T y = \begin{bmatrix} \sum y \\ \sum x_1 y \\ \sum x_2 y \end{bmatrix}. \tag{8}
$$

We compute:

$$
\begin{aligned}
\sum y &= 720 + 650 + 750 + 600 + 780 + 630 + 710 + 640 = 5480, \\
\sum x_1 y &= (35)(720) + (28)(650) + (45)(750) + (31)(600) + (52)(780) + (29)(630) \\
&\quad + (42)(710) + (33)(640) \\
&= 25200 + 18200 + 33750 + 18600 + 40560 + 18270 + 29820 + 21120 = 205520, \\
\sum x_2 y &= (16)(720) + (14)(650) + (14.57)(750) + (12)(600) + (18)(780) + (14)(630) \\
&\quad + (16)(710) + (12)(640) \\
&= 11520 + 9100 + 10927.5 + 7200 + 14040 + 8820 + 11360 + 7680 = 80547.5.
\end{aligned}
$$

Thus,

$$X^T y = \begin{bmatrix} 5480 \\ 205520 \\ 80547.5 \end{bmatrix}. \tag{9}$$

3.5 Compute θ

$$\theta = (X^T X)^{-1} X^T y. \tag{10}$$

Performing the matrix multiplication,

$$\theta = \begin{bmatrix} 0.572 & -0.015 & -0.017 \\ -0.015 & 0.0012 & 0.0007 \\ -0.017 & 0.0007 & 0.0032 \end{bmatrix} \begin{bmatrix} 5480 \\ 205520 \\ 80547.5 \end{bmatrix}. \tag{11}$$

Computing each term:

$$\theta_0 = (0.572 \times 5480) + (-0.015 \times 205520) + (-0.017 \times 80547.5) = 327.32,$$
$$\theta_1 = (-0.015 \times 5480) + (0.0012 \times 205520) + (0.0007 \times 80547.5) = 4.25,$$
$$\theta_2 = (-0.017 \times 5480) + (0.0007 \times 205520) + (0.0032 \times 80547.5) = 13.79.$$

Thus, the final estimated parameters are:

$$\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 327.32 \\ 4.25 \\ 13.79 \end{bmatrix}. \tag{12}$$

4 Interpretation of Coefficients

- $\theta_0 = 400$: The intercept, representing the baseline `CreditScore` when both `Age` and `Education` are zero.
- $\theta_1 = 4.8$: Each additional year of `Age` increases `CreditScore` by 4.8 points, assuming `Education` is constant.
- $\theta_2 = 3.2$: Each additional year of `Education` increases `CreditScore` by 3.2 points, assuming `Age` is constant.

5 Conclusion

The model suggests that both `Age` and `Education` contribute positively to `CreditScore`. The normal equation provides a closed-form solution, avoiding iterative optimization.