Kernel SVM with RBF Kernel: Computing the Kernel Matrix

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1 Introduction

The Support Vector Machine (SVM) with a Radial Basis Function (RBF) kernel is effective for handling non-linearly separable data by transforming input features into a higher-dimensional space. The RBF kernel is defined as:

$$K(x_i, x_j) = \exp\left(-\gamma \|x_i - x_j\|^2\right) \tag{1}$$

where γ is a hyperparameter that controls the influence of each training sample.

2 Kernel Matrix Computation

Given the first three training samples with Age and CreditScore:

$$\begin{aligned} x_1 &= (35, 720) \\ x_2 &= (28, 650) \\ x_3 &= (45, 750) \end{aligned}$$

and setting $\gamma = 0.1$, we compute the pairwise kernel values.

First, compute squared Euclidean distances:

$$\begin{aligned} \|x_1 - x_2\|^2 &= (35 - 28)^2 + (720 - 650)^2 = 7^2 + 70^2 = 49 + 4900 = 4949 \\ \|x_1 - x_3\|^2 &= (35 - 45)^2 + (720 - 750)^2 = 10^2 + 30^2 = 100 + 900 = 1000 \\ \|x_2 - x_3\|^2 &= (28 - 45)^2 + (650 - 750)^2 = 17^2 + 100^2 = 289 + 10000 = 10289 \end{aligned}$$

Now, apply the RBF kernel function:

$$\begin{aligned} K(x_1, x_2) &= \exp(-0.1 \times 4949) \approx \exp(-494.9) \approx 0 \\ K(x_1, x_3) &= \exp(-0.1 \times 1000) \approx \exp(-100) \approx 0 \\ K(x_2, x_3) &= \exp(-0.1 \times 10289) \approx \exp(-1028.9) \approx 0 \end{aligned}$$

Thus, the kernel matrix for the first three samples is approximately:

$$K = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

3 Interpretation and Effect of the RBF Kernel

The RBF kernel effectively maps input features into an infinite-dimensional space, where linearly inseparable data points in the original space become separable. In our case, the high Euclidean distances led to very small kernel values, indicating that these samples are not closely related. This transformation enables SVM to find a more flexible decision boundary that adapts to complex patterns in the dataset.