Honors Precalculus Summer Packet

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The following problems review material and skills considered to be prerequisite for the PCH course. You should expect to be ready for an assessment over the materials and skills contained herein within the first week of school. Listed below is a breakdown of the topics and skills each set of questions focuses on.

Questions 1-4 deal with basic function structure, factual knowledge of characteristics of basic functions, and simple evaluation and manipulation of the functions.

Question 5 reviews algebraic methods for finding the complex roots of a polynomial, which was a topic of particular focus in A2H.

Questions 6 and 7 review understanding the connections between graphical and algebraic representations of functions.

Questions 8 and 9 focus on more factual knowledge of characteristics of basic functions as well as the function operations of inversion and composition.

Questions 10-13 deal with complex algebraic simplification. You should have the skills necessary to turn the given expression into the desired one through algebraically sound manipulations. These simplifications are representative of the algebra required to perform limits and calculus. Often in a calculus problem, the algebra makes up 90% of the computational work. You should already possess the requisite skillset to succeed with these types of problems based on your A2H class last year.

Questions 14-19 review solving equations involving various different types of functions. Emphasis is placed on solving algebraically and using exact answers (Calculator approximations should not be used until comparing solutions to a graphical check). Checking graphically also reinforces familiarity with technology and the multiple representations of solutions (zeros/roots/intercepts/intersection points).

Question 20 is a great example of the style of question one might find in the PCH course. It combines the concepts of inverse functions and composite functions from an algebraic perspective. The computational skills necessary to complete this problem are limited to addition, subtraction, and multiplication. The challenge lies in understanding the conceptual approach and carefully negotiating the order of operations correctly.

The hope is that you find these questions to be a mixture of factual review that reassures you of your readiness for this course and some interesting questions that challenge you and extend skills you have in reasonable ways.

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Section 1: Functions and their Properties

For the given functions, answer the following questions.

1. $f(x) = \sqrt{x^2 - 4}$

a) Domain $(-\infty, -2] \cup [2, \infty)$

b) Range $[0, \infty)$

c) Zeros (if any) $x = \pm 2$

d) $f(a)$ $\sqrt{a^2 - 4}$

e) $f(x + h)$ $\sqrt{(x+h)^2 - 4}$

2. $f(x) = \left(\frac{1}{2}\right)^x$

a) what type of function? exponential

b) Increasing or Decreasing? decreasing

c) Domain $(-\infty, \infty)$

d) Range $(0, \infty)$

e) Zeros (if any) none

f) y-intercept $(0, 1)$

g) $f(x + h)$ $\frac{1}{2}^{(x+h)}$

3. $f(x) = \log(3x - 2)$

a) what type of function? logarithmic

b) Increasing or Decreasing? increasing

c) Domain $\left(\frac{2}{3}, \infty\right)$

d) Range $(-\infty, \infty)$

e) Zeros (if any) $x = 1$

f) y-intercept none

g) $f(x + h)$ $\log(3(x+h) - 2)$

Honors Precalculus Summer Packet $(x+3)(x+2)$

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4. $f(x) = \frac{x^2+5x+6}{x^2+2x-3}$ $(x+3)(x-1)$

a) what type of function? rational

b) Domain $(-\infty,-3)\cup(-3,1)\cup(1,\infty)$

c) Range $(-\infty,1)\cup(1,\infty)$

d) Zeros (if any) $x=-2$

e) holes (if any) $(-3,\frac{1}{4})$

f) y-intercept $(0,-2)$

g) vertical asymptotes (if any) $x=1$

h) horizontal asymptotes (if any) $y=1$

5. Find all of the complex zeros of $f(x) = x^3 - x^2 - 7x + 15$ algebraically.

$x^3 - x^2 - 7x + 15$

$x^2(x-1) - (7x+15)$

$$\begin{array}{r|rrrr} -1 & 1 & -1 & -7 & 15 \\ & & -1 & 2 & 5 \\ \hline & 1 & -2 & -5 & 20 \end{array}$$

$$\begin{array}{r|rrrr} 1 & 1 & -1 & -7 & 15 \\ & & 1 & 0 & -7 \\ \hline & 1 & 0 & -7 & 8 \end{array}$$

$$\begin{array}{r|rrrr} -3 & 1 & -1 & -7 & 15 \\ & & -3 & 12 & -15 \\ \hline & 1 & -4 & 5 & 0 \end{array}$$

$(x+3)(x^2-4x+5)$

Zeros: $2+i, 2-i$

$\frac{4 \pm \sqrt{-4}}{2}$ $\frac{4 \pm \sqrt{16-20}}{2}$

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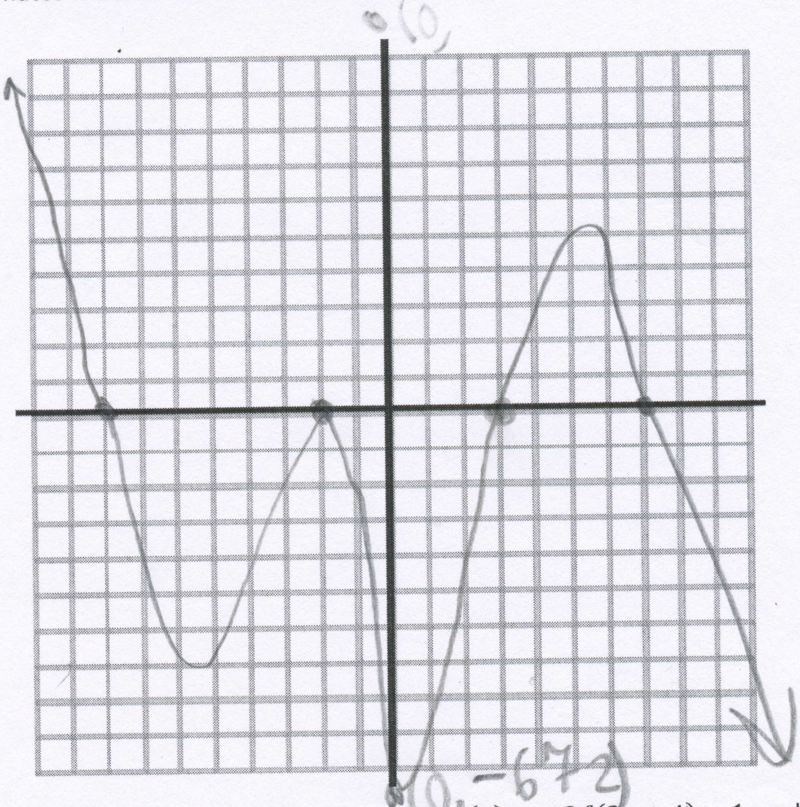
6. Given $f(x) = (x+2)^2(x-3)(7-x)(x+8)$ sketch a graph of the function. You may use your calculator to CHECK but try to sketch the curve from the information you know about the function; zeros, end behavior, y-intercept, etc. X-values and the y-intercept should be accurate, but relative max and min values will be estimates without the use of a calculator.



7. Explain the transformations (shifts, reflections, stretches) that $g(x) = -3f(2x+4) - 1$ applies to the function $f(x)$.

Shrink x by 2 horizontally (divide by 2),
move x to the left by 4units,
then stretch x by 3 vertically
and reflect across X axis (multiply by 3) and move x down 1

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8. Given $f(x) = 3\log_2(x-5)+1$

Find the following:

Domain $(5, \infty)$

Range $(-\infty, \infty)$

$f^{-1}(x)$ $\sqrt[3]{2^{x-1}}+5$

Domain of $f^{-1}(x)$ $(-\infty, \infty)$

Range of $f^{-1}(x)$ $(5, \infty)$

9. Given $f(x) = \sqrt{x+2}$ and $g(x) = x^2 - 6$ Find the following

$f(g(x))$ $\sqrt{(x^2-6)+2}$

Domain of $f(g(x))$ $(-\infty, 2] \cup [2, \infty)$

Range of $f(g(x))$ $[0, \infty)$

$g(f(x))$ $x-4$, $x \geq -2$

Domain of $g(f(x))$ $[-2, \infty)$

Range of $g(f(x))$ $[-6, \infty)$

$f(x^2-6)$

$\sqrt{(x^2-6)+2}$

$(\sqrt{x+2})^2 - 6$

$(x+2) - 6 = x-4$

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Part II: Simplifying expressions and solving equations.

10. Simplify $(2x^2-3x+1)(4)(3x+2)^3(3)+(3x+2)^4(4x-3)$ into $(3x+2)^3(36x^2-37x+6)$

$$(3x+2)^3\left[(2x^2-3x+1)(12)+(3x+2)(4x-3)\right]$$

$$(3x+2)^3\left[(24x^2-36x+12)+(12x^2+8x-9x-6)\right]$$

$$(3x+2)^3(36x^2-37x+6)$$

11. Simplify $\frac{(6x+1)^3(27x^2+2)-(9x^3+2x)(3)(6x+1)^2(6)}{(6x+1)^6}$ into $\frac{27x^2-24x+2}{(6x+1)^4}$

$$\frac{\cancel{(6x+1)^2}\left[(6x+1)(27x^2+2)-(9x^3+2x)(18)\right]}{\cancel{(6x+1)^6}}$$

$$\frac{(6x+1)(27x^2+2)-(162x^3+36x)}{(6x+1)^4}$$

$$\frac{27x^2-24x+2}{(6x+1)^4}$$

Back →

$$\frac{(6x+1)(27x^2+2)-(162x^3+36x)}{(6x+1)^4}$$

$$\frac{(\cancel{162x^3}+12x+27x^2+2)-(\cancel{162x^3}+36x)}{(6x+1)^4}$$

$$\frac{27x^2-24x+2}{(6x+1)^4}$$

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$(x^2+4) = u$

12. Simplify $\frac{(x^2+4)^{1/3}(3)-(3x)\left(\frac{1}{3}\right)(x^2+4)^{-2/3}(2x)}{[(x^2+4)^{1/3}]^2}$ into $\frac{x^2+12}{(x^2+4)^{4/3}}$

$$\frac{(u)^{\frac{1}{3}}(3)-(3x)\left(\frac{1}{3}\right)(u)^{-\frac{2}{3}}(2x)}{\left((u)^{\frac{4}{3}}\right)}$$

$$\frac{3x^2+12-2x^2}{(x^2+4)^{\frac{4}{3}}}$$

$$\frac{3u-2x^2}{u^{\frac{4}{3}}} \qquad \frac{3(x^2+4)-2x^2}{(x^2+4)^{\frac{4}{3}}} \qquad \frac{x^2+12}{(x^2+4)^{\frac{4}{3}}}$$

13. Solve for $\frac{dy}{dx}$: $9x^2y^2\frac{dy}{dx} + 6xy^3 + 4(x-y)\left(1+\frac{dy}{dx}\right) = 5x\frac{dy}{dx} + 5y$ Hint: treat $\frac{dy}{dx}$ as a unique variable that you are solving for.

$\frac{dy}{dx} = u$

$$9x^2y^2u + 6xy^3 + 4(x-y)(1+u) = 5xu + 5y$$

$$9x^2y^2u + 6xy^3 + (4x-4y)(1+u) = 5xu + 5y$$

$$9x^2y^2u + 6xy^3 + (4x + 4xu - 4y - 4yu) = 5xu + 5y$$

$$-5y + 9x^2y^2u + 6xy^3 + 4x + 4xu - 4y - 4yu = 5xu$$

$$9x^2y^2u - 9y - xu + 4x + 6xy^3 - 4yu = 0$$

$$\frac{dy}{dx} = \frac{9y-4x-6xy^3}{9x^2y^2-x-4y}$$

$\frac{dy}{dx}$:

$$u(9x^2y^2 - x - 4y) = 9y - 4x - 6xy^3$$

$$u = \frac{9y-4x-6xy^3}{9x^2y^2-x-4y}$$

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14. Solve algebraically (Exact answers). Check your answers graphically.

$$\sqrt{2x-3}-\sqrt{x+7}=2$$

$2x-3+(x+7)-2\sqrt{2x^2-21+14x-3x}$

$3x+4-2\sqrt{2x^2+11x-21}=4$

$3x-2\sqrt{2x^2+11x-21}=0$

$-2\sqrt{2x^2+11x-21}=-3x$

$\sqrt{2x^2+11x-21}=-\frac{3x}{2}$

$x^2-44x+84=0$

$(x-42)(x-2)=0$

$2x^2+11x-21=\frac{9x^2}{4}$

$8x^2+44x-84=9x^2$

Solutions $x=42$, 2 is extraneous

15 Find the solutions algebraically (Exact answers). Check graphically.

$$3\log_4(x-1)-7=5$$

$3\log_4(x-1)=12$

$\log_4(x-1)=4$

$4^4=x-1$

$256=x-1$

$x=257$

Solutions: $x=257$

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16. Solve for x algebraically (Exact answers). Check your answers graphically.

$$5^{2x-1} = 2^{x+3}$$

$\log 5^{2x-1} = \log 2^{x+3}$

$(2x-1)\log 5 = (x+3)\log 2$

$2x\log 5 - \log 5 = x\log 2 + 3\log 2$

$2x\log 5 - x\log 2 = 3\log 2 + \log 5$

$x(2\log 5 - \log 2) = 3\log 2 + \log 5$

$x = \dfrac{3\log 2 + \log 5}{2\log 5 - \log 2}$

Solutions

$x = 1.46052$

17. Solve for x algebraically (Exact answers). Check your answer graphically.

$$-3 \cdot 10^{2+3x} + 9 = -6$$

$10^{2+3x} - 3 = -2$

$10^{2+3x} = 5$

$2+3x = \log 5$

$x = -.43367$

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Solutions:

18. Solve for x algebraically. Write your answers in **interval notation**. Check your answers graphically.

$$x^3 + 2x^2 \geq 4x + 8$$

$x^2(x+2) \geq 4(x+2)$

$x^2(x+2) - 4(x+2) \geq 0$

$(x+2)(x^2-4)$

$(x+2)(x-2)(x+2) \geq 0$

Solutions: $[2,\infty) \cup [-2,-2]$

$(x+2)^2(x-2) \geq 0$

19. Solve for x algebraically. Check your answers graphically.

$$\ln(2x+3) - \ln(x-5) = 2$$

$\ln \frac{2x+3}{x-5} = 2$

$e^2 = \frac{2x+3}{x-5}$

$(e^2)(x-5) = (2x+3)$

Solutions: $e^2x - 5e^2 = 2x+3$

$e^2x - 2x = 5e^2+3$

$x(e^2-2) = 5e^2+3$

$x = \frac{5e^2+3}{e^2-2}$

$x = 7.41229$

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20. You are told that $\boldsymbol{H}(x)$ and $\boldsymbol{K}(x)$ are inverse functions of one another.

Prove that $g(x) = -2\boldsymbol{H}\left(\frac{1}{3}x+1\right) - 4$ and $f(x) = 3\boldsymbol{K}\left(-2-\frac{1}{2}x\right) - 3$ are also inverse functions using composition. Hint: Realize that $\boldsymbol{H}\left(\frac{1}{3}x+1\right)$ is read as H **of** $\left(\frac{1}{3}x+1\right)$.

$f(g(x)) = x$

$g(f(x)) = x$

$f(g(x)) = 3K\left(-2 - \frac{1}{2}\left(-2H\left(\frac{1}{3}x+1\right)-4\right)\right)-3$

$= 3K\left(-2 + H\left(\frac{1}{3}x+1\right)+2\right)-3$

$= 3K\left(H\left(\frac{1}{3}x+1\right)\right)-3$

$= 3\left(\frac{1}{3}x+1\right)-3$

$= x+3-3$

$= x$

$g(f(x)) = -2H\left(\frac{1}{3}\left(3K\left(-2-\frac{1}{2}x\right)-3\right)+1\right)-4$

$= -2H\left(K\left(-2-\frac{1}{2}x\right)-1+1\right)-4$

$= -2\left(-2-\frac{1}{2}x\right)-4$

$= 4+x-4$

$= x$