

Parallel quicksort

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Sorting

- In practice
 - Faster search & look-up routines
 - More efficient data processing
 - Doing stream operations, merging over sequence
- Available algorithms^[1]
 - Slow $O(n^2)$ or worse: Bubble, Insertion, Selection, ...
 - Fast $O(n \log(n))$ Quicksort*, Merge sort, Heapsort, ...
 - Fast*/non-comparison sorts: Counting, Radix, Bucket, ...

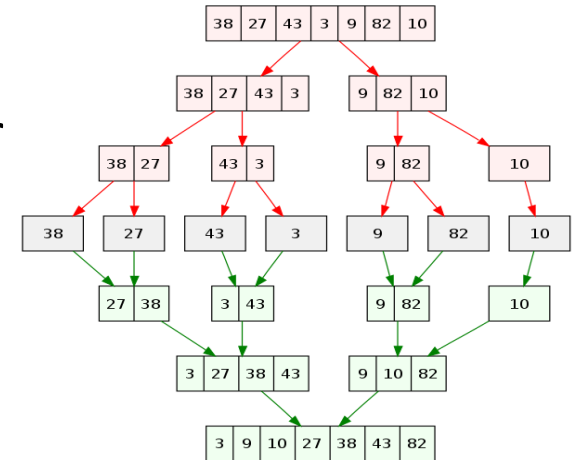


HPC & Sorting

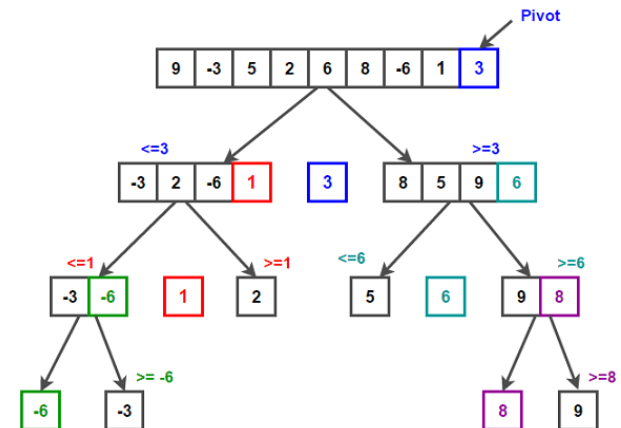
- In practice, Quicksort and Merge sort are most popular sorting algorithms
- Many programming languages and libraries provide [*hybrid*] built-in sorting routines. (C++ STL, Java Collections, etc.)
- Several of the sorting algorithms can be employed in parallel environments and DFS

Merge sort & Quicksort recap

- Merge sort
 - Recursively split the problem into smaller instances and solve them separately
 - Merge sorted lists back



- Quicksort
 - Recursively choose pivots and split data around the pivot
 - Sort split subsequences separately



Parallel quicksort

- There have been several methods proposed to parallelize sorting algorithms.
- Making use of parallelism in the following methods:
 - Sorting local lists recursively and exchanging data according to pivot element → **load balancing issues**
 - Hyperquicksort - similar to the previous idea, approximate median selection done by manager process
 - Regular Sampling method- sample sorted local lists before exchange between different processes

Regular Sampling method

- Initially, each process (machine) has its own initial local list ($N = 27$, $p = 3$, $n = \frac{N}{p} = 9$)

3	14	15	92	65	35	89	79	52
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Process 1

38	46	26	43	42	33	79	50	28
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Process 2

84	19	71	69	40	93	75	10	58
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Process 3

- Each process sorts its own local list in $O(n \log(n))$

3	14	15	35	52	65	79	89	92
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Process 1

26	28	33	38	42	43	46	50	79
----	----	----	----	----	----	----	----	----

Process 2

10	19	40	58	69	71	75	84	93
----	----	----	----	----	----	----	----	----

Process 3

Regular Sampling method

- Each process samples local data at *indices $\frac{n}{p} - 1, 2\frac{n}{p} - 1, 3\frac{n}{p} - 1, 4\frac{n}{p} - 1, \dots$

3	14	15	35	52	65	79	89	92
---	----	----	----	----	----	----	----	----

Process 1

26	28	33	38	42	43	46	50	79
----	----	----	----	----	----	----	----	----

Process 2

10	19	40	58	69	71	75	84	93
----	----	----	----	----	----	----	----	----

Process 3

- Manager process (i.e. process 1) gathers sampled p^2 points from all processes and sorts them in $O(p^2 \log(p))$

15	33	40	43	65	71	79	92	93
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Manager

Idea: If we select $p - 1$ pivots from the above samples, they will be sufficient to route each data element to its correct destination!

$\frac{n}{p}, 2\frac{n}{p}, 3\frac{n}{p}, 4\frac{n}{p}, \dots$ also work in similar fashion.

Regular Sampling method

- Manager selects $p - 1$ pivots and broadcasts them to all the processes

15	33	40	43	65	71	79	92	93
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Manager

40	71
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Pivots

3	14	15	35	52	65	79	89	92
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Process 1

less than 40

less than 71

26	28	33	38	42	43	46	50	79
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Process 2

greater or equal to 71

10	19	40	58	69	71	75	84	93
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Process 3

- Each process sends chunks of its data based on pivots to other processes

3	14	15	35	26	28	33	38	10	19
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Process 1

52	65	42	43	46	50	40	58	69
----	----	----	----	----	----	----	----	----

Process 2

79	89	92	79	71	75	84	93
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Process 3

Regular Sampling method

- At the last step, each process sorts its final data

3	14	15	35	26	28	33	38	10	19
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Process 1

52	65	42	43	46	50	40	58	69
----	----	----	----	----	----	----	----	----

Process 2

79	89	92	79	71	75	84	93
----	----	----	----	----	----	----	----

Process 3

Now, The whole data is sorted and sequentially stored in distributed system.

3	10	14	15	19	26	28	33	35	38
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Process 1

40	42	43	46	50	52	58	65	69
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Process 2

71	75	79	79	84	89	92	93
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Process 3

Notes:

- Except pivots, every data element is sent/received at most once.
- The lengths of final local lists differ at most by $O(p)$.