LAKSHYA

JEE 2025

MATHEMATICS | Lecture - 09

DETERMINANTS

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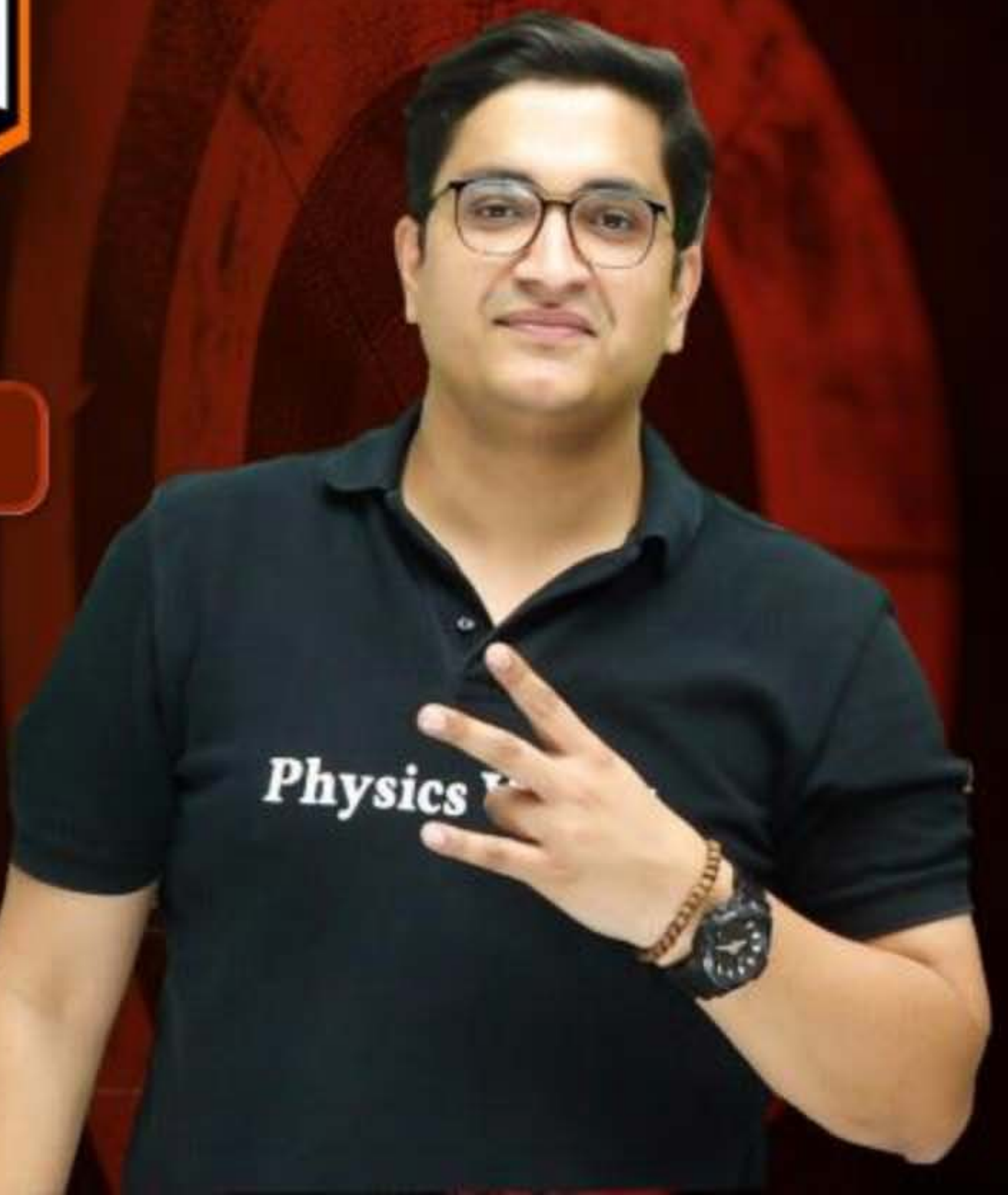


Topics to be covered

1. Determinant Multiplication
2. Sawal hi Sawal
3.
4.

Last Class Recap

Differentiation of Determinant:

$$\boxed{\Delta = \begin{vmatrix} R_1 \\ R_2 \\ R_3 \end{vmatrix} = \begin{vmatrix} R_1' \\ R_2 \\ R_3 \end{vmatrix} + \begin{vmatrix} R_1 \\ R_2' \\ R_3 \end{vmatrix} + \begin{vmatrix} R_1 \\ R_2 \\ R_3' \end{vmatrix}}$$

Determinant Multiplication:

(same order ke det. ke liye hai)

\# 4 ways.

$R \times R$

$C \times C$

$C \times R$

$\boxed{R \times C}$

Factor Theorem:

Δ

$x = a$

$(x-a)$ is a factor of Δ.

$\boxed{\Delta = 0}$

Last Class Recap

Kya hum kisi determinant ko diagonally differentiate kr skte hai?? → No

What is the method to find coeff. of x? → ek baar diff then put $x=0$.

What is the method to find coeff. of x^2? do-baar diff. then put $x=0$.

What is the method to find coeff. of highest power of x ?

$x \to \frac{1}{x}$

⇓

Take LCM

⇓

Remove 'x' from denom.

$$\begin{vmatrix} -1 & 3 & 2 \\ -5 & 1 & 6 \\ 7 & 2 & -1 \end{vmatrix} \times \begin{vmatrix} 0 & 2 & 3 \\ 4 & 0 & 1 \\ -1 & -1 & 5 \end{vmatrix} = \begin{vmatrix} - & - & - \\ - & - & - \\ - & - & - \end{vmatrix}$$

$a_{12} = -4$

$a_{23} = 16$.

$a_{31} = 9$.

R_3 C_1

R × C

Acidic Question

If f, g and h are differentiable functions of x and

$$\Delta = \begin{vmatrix} f & g & h \\ (xf)' & (xg)' & (xh)' \\ (x^2f)'' & (x^2g)'' & (x^2h)'' \end{vmatrix}, \text{ then prove that } \Delta' = \begin{vmatrix} f & g & h \\ f' & g' & h' \\ (x^3f'')' & (x^3g'')' & (x^3h'')' \end{vmatrix}$$

$(xf)' = 1f + xf'$

$(x^2f)'' = (2xf + x^2f')'$

$= 2f + 2xf' + 2xf' + x^2f''$

$= 2f + 4xf' + x^2f''$

$$\Delta = \begin{vmatrix} f & g & h \\ f+xf' & g+xg' & h+xh' \\ 2f+4xf'+x^2f'' & 2g+4xg'+x^2g'' & \ldots \end{vmatrix}$$

$R_2 \to R_2 - R_1$

$$\Delta = \begin{vmatrix} f & g & h \\ xf' & xg' & xh' \\ 2f+4xf'+x^2f'' & \ldots & \ldots \end{vmatrix}$$

$R_3 \to R_3 - 2R_1 - 4R_2$

$$\Delta = \begin{vmatrix} f & g & h \\ xf' & xg' & xh' \\ x^2f'' & x^2g'' & x^2h'' \end{vmatrix}$$

$$\Delta = \begin{vmatrix} f & g & h \\ f' & g' & h' \\ x^3f'' & x^3g'' & x^3h'' \end{vmatrix}$$

Acidic Question

If f, g and h are differentiable functions of x and

$$\Delta = \begin{vmatrix} f & g & h \\ (xf)' & (xg)' & (xh)' \\ (x^2f)'' & (x^2g)'' & (x^2h)'' \end{vmatrix}, \text{ then prove that } \Delta' = \begin{vmatrix} f & g & h \\ f' & g' & h' \\ (x^3f'')' & (x^3g'')' & (x^3h'')' \end{vmatrix}$$

HHPP.

$$\Delta = \begin{vmatrix} f & g & h \\ f' & g' & h' \\ x^3f'' & x^3g'' & x^3h'' \end{vmatrix}$$

$$\Delta' = \overset{R_1' \rightarrow}{\begin{vmatrix} f' & g' & h' \\ f' & g' & h' \\ x^3f'' & x^3g'' & x^3h'' \end{vmatrix}} + \overset{R_2' \rightarrow}{\begin{vmatrix} f & g & h \\ f'' & g'' & h'' \\ x^3f'' & x^3g'' & x^3h'' \end{vmatrix}} + \begin{vmatrix} f & g & h \\ f' & g' & h' \\ (x^3f'')' & (x^3g'')' & (x^3h'')' \end{vmatrix}$$

$0 \qquad x^3 \qquad 0$

Question (ACIDIC)

If $\Delta(x) = \begin{vmatrix} x^2+x & 2x-1 & x+3 \\ 3x+1 & 2+x^2 & x^3-3 \\ x-3 & x^2+4 & 2x \end{vmatrix} = a + bx + cx^2 + dx^3 + ex^4 + fx^5 + gx^6 + hx^7$ then find

(i) a

(ii) $g = -1$ & $h = -1$

(iii) $5f - 6g + 3d - 4e + b - 2c = 53$

(#) Answer.

Question

"Module"

If $\Delta(x) = \begin{vmatrix} e^x & \sin x & 1 \\ \cos x & \ln(1+x^2) & 1 \\ x & x^2 & 1 \end{vmatrix}$ expanded in powers of 'x' then find coefficient of x.

$= a + bx + cx^2 + \dots$

diff. then put $x = 0$

$$\begin{vmatrix} e^x & \sin x & 1 \\ -\sin x & \ln(1+x^2) & 1 \\ 1 & x^2 & 1 \end{vmatrix} + \begin{vmatrix} e^x & \cos x & 1 \\ \cos x & \frac{2x}{1+x^2} & 1 \\ x & 2x & 1 \end{vmatrix} + \begin{vmatrix} - & - & 0 \\ - & - & 0 \\ - & - & 0 \end{vmatrix} = b$$

0

$$\begin{vmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{vmatrix} = b.$$

0

$-1 = b.$

QUESTION (JEE Mains-2020)

If $\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix} = Ax^3 + Bx^2 + Cx + D$, then $B + C$ is equal to

(A) 9

(B) −1

(C) 1

(D) −3

Khud Tab tak
try kro jab tak
(D) ना आ जाए !!

DETERMINANT MULTIPLICATION

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \times \begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ b_{21} & b_{22} & b_{23} \\ c_{31} & c_{32} & c_{33} \end{vmatrix}$$

NOTE : ek element then ek row pr focus kre!

EXAMPLE

Prove that $\begin{vmatrix} \boxed{a_1x_1+b_1y_1 & a_1x_2+b_1y_2 & a_1x_3+b_1y_3} \\ a_2x_1+b_2y_1 & a_2x_2+b_2y_2 & a_2x_3+b_2y_3 \\ a_3x_1+b_3y_1 & a_3x_2+b_3y_2 & a_3x_3+b_3y_3 \end{vmatrix} = 0$

$$\begin{vmatrix} a_1 & b_1 & 0 \\ a_2 & b_2 & 0 \\ a_3 & b_3 & 0 \end{vmatrix} \times \begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 0 & & \end{vmatrix}$$

0

zero. HHPP.

QUESTION (IIT-JEE-1994)

For all real values of A, B, C and P, Q, R, show that

$$\begin{vmatrix} \cos(A-P) & \cos(A-Q) & \cos(A-R) \\ \cos(B-P) & \cos(B-Q) & \cos(B-R) \\ \cos(C-P) & \cos(C-Q) & \cos(C-R) \end{vmatrix} = 0$$

$$LHS = \begin{vmatrix} \cos A \cos P + \sin A \sin P & \cos A \cos Q + \sin A \sin Q & \cos A \cos R + \sin A \sin R \\ \cos B \cos P + \sin B \sin P & \cos B \cos Q + \sin B \sin Q & \cos B \cos R + \sin B \sin R \\ \cos C \cos P + \sin C \sin P & \cos C \cos Q + \sin C \sin Q & \cos C \cos R + \sin C \sin R \end{vmatrix}$$

$$\begin{vmatrix} \cos A & \sin A & 0 \\ \cos B & \sin B & 0 \\ \cos C & \sin C & 0 \end{vmatrix} \times \begin{vmatrix} \cos P & \cos Q & \cos R \\ \sin P & \sin Q & \sin R \\ 0 & 0 & 0 \end{vmatrix} = \text{zero}$$

0

QUESTION (JEE Mains-2014)

If $\alpha, \beta \neq 0$ and $f(n) = \alpha^n + \beta^n$ and

$$\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix} = K(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$$, then K is equal to

(A) $\alpha\beta$

(B) $1/\alpha\beta$

(C) 1

(D) -1

$$\begin{vmatrix} 1+1+1 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} \times \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \end{vmatrix}$$

Transp.

$$1\left((1-\alpha)(\alpha-\beta)(\beta-1)\right)^2 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \end{vmatrix} \times \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \end{vmatrix}$$

$$1(1-\alpha)^2(\alpha-\beta)^2(1-\beta)^2$$

$(x=1, y=\alpha, z=\beta)$

BUMPER QUESTION

If $\Delta_1 = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$,

$\Delta_2 = \begin{vmatrix} bc-a^2 & ac-b^2 & ba-c^2 \\ ac-b^2 & ab-c^2 & bc-a^2 \\ ab-c^2 & bc-a^2 & ac-b^2 \end{vmatrix}$, → cofactor det. of Δ_1

$\Delta_2 = \Delta_1^2$

$\Delta_3 = \begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ca-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix}$, $= \begin{vmatrix} bc+bc-a^2 & c^2+ab-ab & b^2+ac-ac \\ c^2+ab-ab & ac+ac-b^2 & a^2+bc-bc \\ b^2+ac-ac & a^2+bc-bc & ab+ab-c^2 \end{vmatrix}$

Δ_1^2

$\Delta_4 = \begin{vmatrix} a^2 & c^2 & 2ac-b^2 \\ 2ab-c^2 & b^2 & a^2 \\ b^2 & 2bc-a^2 & c^2 \end{vmatrix} = \Delta_1^2$

DIBY!!

$\begin{vmatrix} b & c & a \\ c & a & b \\ a & b & c \end{vmatrix} \times \begin{vmatrix} c & a & b \\ b & c & a \\ -a & -b & -c \end{vmatrix} = \ominus \begin{vmatrix} c & a & b \\ b & c & a \\ a & b & c \end{vmatrix}$

$R_1 \leftrightarrow R_3$

$\ominus \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix} \rightsquigarrow + \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = \Delta_1$

$\Delta_1 = \ominus\ominus \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ ✓

$\oplus$

then show that $\Delta_2 = \Delta_3 = \Delta_4 = (\Delta_1)^2$

Tumhare mann ke sawal??

Sir iska koi perfect method hai ya sochna hi padega??

No-perfect method.

Kya her sawal mein ye method lga skte hai??

No.

$***$ पता करो check kro!!

To firr kaise pta chalega ki kis question mein lgana hai, hint??

③

② determinant ⇓ Complicated!

$**$ ①

CHALLENGER QUESTION

Without expanding at any stage show that :

$$\begin{vmatrix} 2 & (a+b)+(c+d) & ab+cd \\ (a+b)+(c+d) & 2(a+b)(c+d) & ab(c+d)+cd(a+b) \\ ab+cd & ab(c+d)+cd(a+b) & 2abcd \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & 1 & 0 \\ c+d & a+b & 0 \\ cd & ab & 0 \end{vmatrix} \times \begin{vmatrix} 1 & a+b & ab \\ 1 & c+d & cd \\ 0 & 0 & 0 \end{vmatrix}$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$0 \times 0 = \text{zero}$$

HHPP.

QUESTION (JEE (Adv.)-2015)

Which of the following values of α satisfy the equation

$$\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ (2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2 \\ (3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2 \end{vmatrix} = -648\alpha?$$

H.W.

(A) −4

(B) 9

(C) −9

(D) 4

MODULE QUESTION

$$\Delta = \begin{vmatrix} 1+a^2+a^4 & 1+ab+a^2b^2 & 1+ac+a^2c^2 \\ 1+ab+a^2b^2 & 1+b^2+b^4 & 1+bc+b^2c^2 \\ 1+ac+a^2c^2 & 1+bc+b^2c^2 & 1+c^2+c^4 \end{vmatrix}$$ is equal to

(A) $(a-b)^2(b-c)^2(c-a)^2$

(B) $2(a-b)(b-c)(c-a)$

(C) $4(a-b)(b-c)(c-a)$

(D) $(a+b+c)^3$

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \times \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

$$\left((a-b)(b-c)(c-a)\right)^2$$

SYSTEM OF LINEAR EQUATIONS

100%
ek Question
aayega!!

in both
JEE Mains
&
Adv.

easy

Result
Based.

GENERAL TERMINOLOGY

\# Consistent = when equation has solutions

ex:- $\left.\begin{array}{l} x-y=5 \\ x+y=7 \end{array}\right\} \rightarrow x=6, y=1$ consistent

\# Inconsistent = no-solution

ex:- $\begin{array}{l} x-y=5 \\ 2x-2y=7 \end{array} \rightarrow$ No solⁿ (Inconsis.)

\# Trivial Solution = jab sabhi variables ki zero है

ex:- $\begin{array}{l} x+2y=0 \\ x-y=0 \end{array}$ $\left\{\begin{array}{l} x=0 \\ y=0 \end{array}\right.$ Trivial solⁿ

$(x,y,z) \equiv (0,0,0)$

\# Non-Trivial/Non-Zero Solution = Kum se kum ek variable ki value non-zero है

$(x,y,z) \equiv (-1,2,3)$ ✓

$\equiv (-1,0,0)$ ✓

$\equiv (2,4,0)$ ✓

Non-Triv. solⁿ

\# Homogeneous System of equation = when constant part of each eqⁿ is zero.

\# Non-Homogeneous System = when const. part in atleast 1 eqⁿ is non-zero

LINEAR EQUATION IN TWO VARIABLES

Class – $\underset{\sim}{X}^{th}$ & $\underset{\sim}{XI}^{th}$

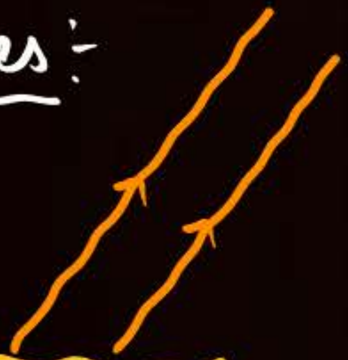
$$\boxed{\begin{aligned} a_1x + b_1y &= c_1 \\ a_2x + b_2y &= c_2 \end{aligned}}$$

$a_1x + b_1y = c_1 \longrightarrow$ line$_1$

$a_2x + b_2y = c_2 \longrightarrow$ line$_2$

Solution ≡ No. of point of intersecⁿ

Possibilities :-



Parallel

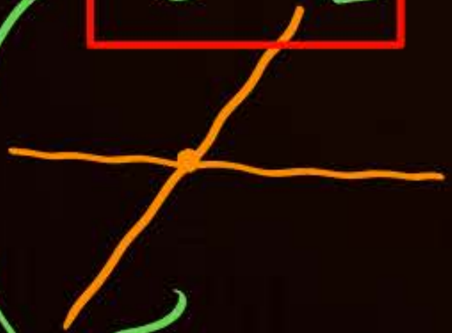
⇓

no-solⁿ

condⁿ $\boxed{\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} \neq \dfrac{c_1}{c_2}}$

ex:- $2x + y = 7$

$2x + y = 3$



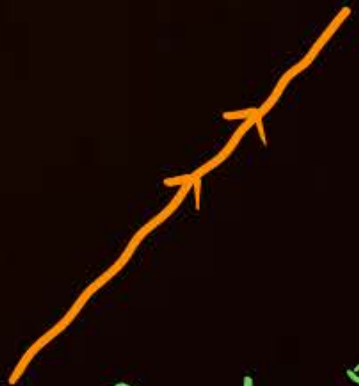
Intersecting

$\boxed{\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}}$

ex:- $2x + y = 7$

$x + y = 3.$

1 solⁿ ⇒ unique solⁿ



Overlapping / same

∞-solⁿ

ex:- $2x + y = 7$

$4 \times \quad 8x + 4y = 28$

coincident

condⁿ $\boxed{\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}}$

QUESTION

Find value of 'a' if following system of equations has NO SOLUTION

$$\left.\begin{aligned} a^2x + (2-a)y &= 4 \\ ax + (2a-1)y &= a^5 - 2 \end{aligned}\right\}$$

Parallel.

$\Downarrow$

$$\boxed{\frac{a^2}{a} = \frac{2-a}{2a-1}} \neq \frac{4}{a^5-2}$$

$$\cancel{2}a^3 - \cancel{a^2} = \cancel{2}a - \cancel{a^2}$$

$$a^3 = a$$

$$a^3 - a = 0$$

$$a(a^2-1) = 0$$

$$\boxed{a = 0, 1, -1}$$

check??

$\times$ $\underbrace{a=0}_{\infty\text{-sol}^n}$ $\Rightarrow$ $\left.\begin{aligned} 2y &= 4 \\ -y &= -2 \end{aligned}\right\} \to \left.\begin{aligned} y &= 2 \\ y &= 2 \end{aligned}\right\} \to$ coincident.

$\boxed{a=1} \Rightarrow \left.\begin{aligned} x + y &= 4 \\ x + y &= -1 \end{aligned}\right\}$ ✓ $\parallel^{\text{el}}$.

$\boxed{a=-1} \to \left.\begin{aligned} x + 3y &= 4 \\ -x - 3y &= -3. \end{aligned}\right\}$ $\parallel^{\text{el}}$ ✓

QUESTION (JEE (Adv.)-2016)

Let $a, \lambda, \mu \in R$. Consider the system of linear equations

$$\boxed{\begin{aligned} ax + 2y &= \lambda \\ 3x - 2y &= \mu \end{aligned}} \qquad \boxed{\frac{a}{3} = \frac{2}{-2} = \frac{\lambda}{\mu}}$$

Which of the following statement(s) is(are) correct?

(A) If $a = -3$, then the system has infinitely many solutions for all values of λ and μ $\rightarrow a=-3 \Rightarrow \left.\begin{aligned} -3x+2y&=\lambda \\ 3x-2y&=\mu \end{aligned}\right\} \rightarrow \begin{aligned} 3x-2y&=-\lambda \checkmark \\ 3x-2y&=\mu \checkmark \end{aligned}$

(B) If $a \neq -3$, then the system has a unique solution for all values of λ and μ $\hookrightarrow \frac{a}{3} \neq \frac{2}{-2} \Rightarrow a \neq -3$

(C) If $\lambda + \mu = 0$, then the system has infinitely many solution for $a = -3$ ✓

$\hookrightarrow \lambda=0, \mu=0 \Rightarrow \Big| \ \lambda/\mu = -1 \Rightarrow \frac{a}{3} = -1 = -1 \rightsquigarrow \infty$-sol^n ✓

(D) If $\lambda + \mu \neq 0$, then the system has no solution for $a = -3$

$\hookrightarrow \lambda/\mu \neq -1$

QUESTION (JEE Mains-2013)

The number of values of k, for which the system of equations

$(k+1)x+8y=4k$

$kx+(k+3)y=3k-1$ has no solution, is

H.W.

(A) Infinite

(B) 1

(C) 2

(D) 3

DIBY (DO IT BY YOURSELF)

Mathematics is all about Questions

Sawal ka mja tab tak hai jab tak Jawab pta na chale!!

"Determinant Multiplication"

DIBY-21

Prove that in a $\triangle ABC$, $\begin{vmatrix} \sin 2A & \sin C & \sin B \\ \sin C & \sin 2B & \sin A \\ \sin B & \sin A & \sin 2C \end{vmatrix} = 0$.

DIBY-22

after makeup before makeup

Show: $\begin{vmatrix} (a_1-b_1)^2 & (a_1-b_2)^2 & (a_1-b_3)^2 \\ (a_2-b_1)^2 & (a_2-b_2)^2 & (a_2-b_3)^2 \\ (a_3-b_1)^2 & (a_3-b_2)^2 & (a_3-b_3)^2 \end{vmatrix} = \begin{vmatrix} (1+a_1b_1)^2 & (1+a_1b_2)^2 & (1+a_1b_3)^2 \\ (1+a_2b_1)^2 & (1+a_2b_2)^2 & (1+a_2b_3)^2 \\ (1+a_3b_1)^2 & (1+a_3b_2)^2 & (1+a_3b_3)^2 \end{vmatrix}$

uske-saath uske-bina

DIBY-23

Show $\Delta = \begin{vmatrix} \sin(x+A) & \cos(x+A) & a+x\sin A \\ \sin(x+B) & \cos(x+B) & b+x\sin B \\ \sin(x+C) & \cos(x+C) & c+x\sin C \end{vmatrix}$ is independent of 'x'.

Homework

Re-attempt all the Questions - jo apke hisab se ache hai of Lecture.

DPP

Module:

Exercise (Prarambh) : Ques: 1 to 34

Exercise (Prabal) : Ques: 18,19,27,28,29,30,31,32,34,35

Exercise (Parikshit) : Ques: 11,12,19,20,21,22,23

Note: Jaha Matrix likha hua dikhe unn Questions ko leave kr de!

NCERT:

Exercise 4.1 : COMPLETE

Exercise 4.2 : COMPLETE

Exercise 4.3 : COMPLETE

It's not about End Result,
It is all about JOURNEY

THANK YOU

#futureIITians