Trey Bryant
11/25/20

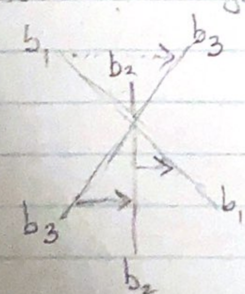
HW #5
CSC 433

1. From the slides we can see that if we want to rotate around a direction (which in this case is $\vec{v}$) we have to compute the orthogonal directions $\vec{u}$ & $\vec{w}$ which we've already done in past projects using the camera parameters, and then we must apply a rotate-z() matrix. So, our rotation matrix will be the product of

$$R = \begin{bmatrix} v \\ u \\ w \end{bmatrix}^T \begin{bmatrix} \cos 30 & -\sin 30 & 0 \\ \sin 30 & \cos 30 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2. To create an image equivalent to the mirror's reflection, we would need to start by modifying line 13 to, instead of performing orthographic projection of point Q, we must calculate a normal vector from point Q and project it out to find the first surface it intersects and then make that the color of the pixel that is added to the buffer.

3. This is indeed possible if they intersect (which I believe is legal) as so:



As can be seen from this top view, b_3 occludes b_1 near the top of the diagram, & the first two conditions are met at the bottom

4. For matrix T we want to follow a similar process as in #1 except our direction vector is different this time, it's $\vec{w}$ instead of $\vec{v}$ so T would

$$T = \begin{bmatrix} w \\ v \\ v \end{bmatrix} \begin{bmatrix} \cos 45 & -\sin 45 & 0 \\ \sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

5. Let $P_{us} - P_{ta} = \vec{u}$, $P_{us} - P_{ta} = \vec{v}$ and $\vec{w} = \vec{u} \times \vec{v}$. From the slides we will have transformation matrix M equal to

$$M = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ w_x & w_y & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

6a. $$g^2 = \begin{bmatrix} \cos(5) & \sin(5) \\ -\sin(5) & \cos(5) \end{bmatrix} \begin{bmatrix} \cos(5) & \sin(5) \\ -\sin(5) & \cos(5) \end{bmatrix} = \begin{bmatrix} \cos^2(5) - \sin^2(5) & 2\cos(5)\sin(5) \\ -2\cos(5)\sin(5) & \cos^2(5) - \sin^2(5) \end{bmatrix}$$

$$= \begin{bmatrix} \cos(2 \cdot 5) & \sin(2 \cdot 5) \\ -\sin(2 \cdot 5) & \cos(2 \cdot 5) \end{bmatrix}$$ Extrapolating: $$\begin{bmatrix} \cos(2^n \cdot 5) & \sin(2^n \cdot 5) \\ -\sin(2^n \cdot 5) & \cos(2^n \cdot 5) \end{bmatrix}$$

b. $$b^2 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Extrapolating: $$b^{2020} = \begin{pmatrix} 1 & 0 & 2020 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$