P93

19. (1). $y' = f'(x^2)\cdot(x^2)' = 2xf'(x^2)$

$y'' = 2\left(f'(x^2) + xf''(x^2)\cdot 2x\right) = 2\left(f'(x^2) + 2x^2 f''(x^2)\right)$

(2). $y' = f'(e^x+x)\cdot(e^x+x)' = (e^x+1)f'(e^x+x)$

$y'' = e^x f'(e^x+x) + f''(e^x+x)\cdot(e^x+x)'(e^x+1)$

$= e^x f'(e^x+x) + (e^x+1)^2 f''(e^x+x)$

20.

21. (2). $\left((x^2+1)\sin x\right)^{(n)}$ 令 $y = (x^2+1)\sin x$

$= \sin^{(n)}(x)(x^2+1) + \binom{1}{n}\sin^{(n-1)}(x)(x^2+1)' + \binom{2}{n}\sin^{(n-2)}(x)(x^2+1)''$

$n=4k$ $\quad y^{(n)} = (x^2+1)\sin x - 2nx\cos x - n(n-1)\sin x$

$n=4k-1$ $\quad y^{(n)} = -(x^2+1)\cos x - 2nx\sin x + n(n-1)\cos x$

$n=4k-2$ $\quad y^{(n)} = -(x^2+1)\sin x + 2nx\cos x + n(n-1)\sin x$

$n=4k-3$ $\quad y^{(n)} = (x^2+1)\cos x + 2nx\sin x - n(n-1)\cos x$

(3). 令 $y = \frac{1}{x^2-3x+2} = \frac{1}{(x-1)(x-2)}$

$\left(\frac{1}{x-1}\right)^{(n)}\frac{1}{x-2} + n\left(\frac{1}{x-1}\right)^{(n-1)}\left(\frac{1}{x-2}\right)'$

22. $\frac{dy}{dx}\Big|_{x=\frac{\pi}{4}} = -\frac{\sqrt{2}}{2}$ $\quad \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

切线方程为: $y - \frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)$

即 $x + \sqrt{2}y - 1 - \frac{\pi}{4} = 0$

P98

2. (1). $d\left(\ln\left(\frac{\pi}{2}-\frac{x}{4}\right)\right) = \dfrac{d\left(\frac{\pi}{2}-\frac{x}{4}\right)}{\frac{\pi}{2}-\frac{x}{4}}$

$= \dfrac{-\frac{1}{4}dx}{\frac{\pi}{2}-\frac{x}{4}} = \dfrac{dx}{x-2\pi}$

(3). $d\left(\arccos\frac{1}{|x|}\right) = -\dfrac{1}{\sqrt{1-\frac{1}{x^2}}}\,d\frac{1}{|x|}$

$x>0$ 时 $d\frac{1}{|x|} = d\frac{1}{x} = -\frac{dx}{x^2}$

$x<0$ 时 $d\frac{1}{|x|} = d\left(-\frac{1}{x}\right) = \frac{dx}{x^2}$

故 $d\frac{1}{|x|} = -\frac{dx}{x|x|}$

$d\left(\arccos\frac{1}{|x|}\right) = \dfrac{dx}{x|x|\sqrt{1-\frac{1}{x^2}}} = \dfrac{dx}{|x|\sqrt{x^2-1}}$

(5). $d\left(5^{\sqrt{\arctan x^2}}\right) = 5^{\sqrt{\arctan x^2}}\ln 5\, d\left(\sqrt{\arctan x^2}\right)$

$= \dfrac{5^{\sqrt{\arctan x^2}}\ln 5}{2\sqrt{\arctan x^2}}\,d(\arctan x^2)$

$= \dfrac{5^{\sqrt{\arctan x^2}}\ln 5}{2(1+x^4)\sqrt{\arctan x^2}}\,d(x^2)$

$= \dfrac{5^{\sqrt{\arctan x^2}}\,x\ln 5}{(1+x^4)\sqrt{\arctan x^2}}\,dx$

(7). $d\left(e^{-x}\cos(3-x)\right) = e^{-x}d(\cos(3-x)) + \cos(3-x)\,d(e^{-x})$

$= -e^{-x}\sin(3-x)\,d(3-x) + e^{-x}\cos(3-x)\,d(-x)$

$= e^{-x}\sin(3-x)\,dx - e^{-x}\cos(3-x)\,dx$

$= e^{-x}(\sin(3-x)-\cos(3-x))\,dx$

3 (1) $\frac{dx}{dt}=\frac{2t}{1+t^2}$ $\frac{dy}{dt}=1-\frac{1}{1+t^2}=\frac{t^2}{1+t^2}$ $\frac{dy}{dx}=\frac{t}{2}$

$$\frac{d^2y}{dx^2}=\frac{d(\frac{dy}{dx})}{dt}\cdot\frac{dt}{dx}=\frac{d(\frac{t}{2})}{dt}\cdot\frac{2t}{1+t^2}=\frac{t}{1+t^2}$$

(3). $\frac{dx}{d\varphi}=\cos\varphi-\varphi\sin\varphi$ $\frac{dy}{d\varphi}=\sin\varphi+\varphi\cos\varphi$

$$\frac{dy}{dx}=\frac{\sin\varphi+\varphi\cos\varphi}{\cos\varphi-\varphi\sin\varphi}$$

$$\frac{d^2y}{dx^2}=\frac{d\frac{dy}{dx}}{d\varphi}\cdot\frac{d\varphi}{dx}$$

$$=\frac{(2\cos\varphi-\varphi\sin\varphi)(\cos\varphi-\varphi\sin\varphi)+(2\sin\varphi+\varphi\cos\varphi)(\sin\varphi+\varphi\cos\varphi)}{(\cos\varphi-\varphi\sin\varphi)^2(\cos\varphi-\varphi\sin\varphi)}$$

4 (1) $x(t)\big|_{t=\frac{\pi}{4}}=y(t)\big|_{t=\frac{\pi}{4}}=\frac{\sqrt{2}}{2}$

$\frac{dx}{dt}=-\sin t$ $\frac{dy}{dt}=\cos t$ $\frac{dy}{dx}=-\cot t$

$\frac{dy}{dx}\Big|_{x=\frac{\pi}{4}}=-1$ 切线方程: $y-\frac{\sqrt{2}}{2}=-(x-\frac{\sqrt{2}}{2})$ 即 $x+y-\sqrt{2}=0$

P106

4 (4) $0<\alpha<\beta<\frac{\pi}{2}$ $\frac{\beta-\alpha}{\cos^2\alpha}<\tan\beta-\tan\alpha<\frac{\beta-\alpha}{\cos^2\beta}$

考虑函数 $f(x)=\tan x$ $0<x<\frac{\pi}{2}$

$\exists\xi\in(\alpha,\beta)$ 使得 $f'(\xi)=\frac{f(\beta)-f(\alpha)}{\beta-\alpha}$

$f'(\xi)=\frac{1}{\cos^2\xi}$ 即 $\frac{\beta-\alpha}{\cos^2\xi}=\tan\beta-\tan\alpha$

又 $\cos^2\alpha>\cos^2\xi>\cos^2\beta$ 原不等式得证

5 (2) 令 $f(x)=\arctan x+\arctan\frac{1-x}{1+x}$

$$f'(x)=\frac{1}{1+x^2}+\frac{1}{1+\left(\frac{1-x}{1+x}\right)^2}\cdot\frac{(-2)}{(1+x)^2}$$

$$=\frac{1}{1+x^2}+\frac{(1+x)^2}{(1+x)^2+(1-x)^2}\cdot\frac{(-2)}{(1+x)^2}$$

$$=0$$

又 $f(x)$ 分别在 $(-\infty,-1)$, $(-1,+\infty)$ 上连续

故 $x>-1$ 时 $f(x)=f(1)=\frac{\pi}{4}$

$x<-1$ 时 $f(x)=f$

20.(1).

(3). 令 $f(x)=\dfrac{\tan x}{x}$ $\quad f'(x)=\dfrac{\frac{x}{\cos^2 x}-\tan x}{x^2}$

$\exists\,\xi\in(x_1,x_2)$ $\quad \dfrac{\frac{\tan x_2}{x_2}-\frac{\tan x_1}{x_1}}{x_2-x_1}=f'(\xi)$

$x>\sin x$ $\quad$ 即 $2x>\sin 2x=2\sin x\cos x$

$\dfrac{x}{\cos^2 x}>\tan x$ $\quad$ 即 $f'(x)>0$ $\quad$ 故 $f'(\xi)>0$

$\dfrac{\frac{\tan x_2}{x_2}-\frac{\tan x_1}{x_1}}{x_2-x_1}=\dfrac{x_1\tan x_2-x_2\tan x_1}{x_1x_2(x_2-x_1)}>0$

$x_1\tan x_2>x_2\tan x_1$ $\quad$ 原不等式得证

(5).

(7). 即证 $\dfrac{(x-1)^{x-1}(x+1)^{x+1}}{x^{2x}}<4$

令 $f(x)=x\ln x$ $\quad f'(x)=\ln x+1$

$f(x)-f(x-1)=f'(\xi_1)$ $\quad \xi_1\in(x-1,x)$

$f(x+1)-f(x)=f'(\xi_2)$ $\quad \xi_2\in(x,x+1)$

$f'(x)\nearrow$ $\quad$ 故 $f'(\xi_1)<f'(\xi_2)$ $\quad f(x)-f(x-1)<f(x+1)-f(x)$

$f(x+1)+f(x-1)>2f(x)$

$(x+1)\ln(x+1)+(x-1)\ln(x-1)>2x\ln x$