

第八次作业答案

补充材料

P54 3.

(a) 是, (b) 不是, (c) 不是

只需满足距离函数的正定性、对称性与三角不等式 (b,c 不满足三角不等式, 举反例验证即可)。

P54 5.

易验证对称, 线性性, 正定性

P54 7.

取 $[0,1]$ 上三次多项式空间的一组基为

$$\varphi_1 = 1, \varphi_2 = x, \varphi_3 = x^2, \varphi_4 = x^3$$

由 Gram-Schmidt 正交化,

$$e_1 = \varphi_1 = 1,$$

$$e_2 = \varphi_2 - \frac{(\varphi_2, e_1)}{(e_1, e_1)} e_1 = x - \frac{3}{5},$$

$$e_3 = \varphi_3 - \frac{(\varphi_3, e_1)}{(e_1, e_1)} e_1 - \frac{(\varphi_3, e_2)}{(e_2, e_2)} e_2 = x^2 - \frac{10}{9}x + \frac{5}{21},$$

$$e_4 = \varphi_4 - \frac{(\varphi_4, e_1)}{(e_1, e_1)} e_1 - \frac{(\varphi_4, e_2)}{(e_2, e_2)} e_2 - \frac{(\varphi_4, e_3)}{(e_3, e_3)} e_3 = x^3 - \frac{21}{13}x^2 + \frac{105}{143}x - \frac{35}{429}$$

P56 18.

$$f_2(x) = \frac{a_0}{2} + a_1 \cos(x) + a_2 \cos(2x) + b_1 \sin(x) + b_2 \sin(2x)$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} x^2 dx = \frac{2}{3} \pi^2$$

$$a_1 = \frac{1}{\pi} \int_{-\pi}^{\pi} x^2 \cos(x) dx = -4$$

$$a_2 = \frac{1}{\pi} \int_{-\pi}^{\pi} x^2 \cos(2x) dx = 1$$

$$b_1 = \frac{1}{\pi} \int_{-\pi}^{\pi} x^2 \sin(x) dx = 0$$

$$b_2 = \frac{1}{\pi} \int_{-\pi}^{\pi} x^2 \sin(2x) dx = 0$$

$$f_2(x) = \frac{2}{3} \pi^2 - 4 \cos(x) + \cos(2x)$$