第一次作业答案

P107 1.

(1).

$$A = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$$

$$||A||_1 = \max_{1 \leq j \leq 2} \sum_{i=1}^{2} |a_{ij}| = \max\{2, 1\} = 2$$

$$||A||_\infty = \max_{1 \leq i \leq 2} \sum_{j=1}^{2} |a_{ij}| = \max\{1, 2\} = 2$$

$$A^T A = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$$

$$det(\lambda I - A^T A) = \lambda^2 - 3\lambda + 1$$

$A^T A$ 特征值

$$\lambda_1 = \frac{3 + \sqrt{5}}{2}, \lambda_2 = \frac{3 - \sqrt{5}}{2}.$$

$$\rho(A^T A) = \max_{i=1,2} \lambda_i = \frac{3 + \sqrt{5}}{2},$$

$$||A||_2 = \sqrt{\rho(A^T A)} = \frac{1 + \sqrt{5}}{2}.$$

(2).

$$||A||_1 = 7, \ ||A||_2 \approx 6.22614, \ ||A||_\infty = 8.$$

(3).

$$||A||_1 = 2, \ ||A||_2 = \frac{1 + \sqrt{5}}{2}, \ ||A||_\infty = 2.$$

P107 2.

(1).

B 特征值

$$\lambda_1 = 2, \lambda_2 = 4.$$

$$\rho(B) = \max\{\lambda_1, \lambda_2\} = 4.$$

(2).

$$\rho(B) = 8.$$