

## 第五次作业答案

**P183 3(3).**

$$p = 2, \quad q = 3, \quad s = \frac{1}{4}, \quad t = 0.781, \quad Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0.788 & 0.616 \\ 0 & -0.616 & 0.788 \end{pmatrix},$$

$$C^{(1)} = Q^T C Q = \begin{pmatrix} 1 & -0.788 & -0.616 \\ -0.788 & 0.438 & 0 \\ -0.616 & 0 & 4.562 \end{pmatrix},$$

$$p = 1, \quad q = 2, \quad s = 0.357, \quad t = 0.705, \quad Q = \begin{pmatrix} 0.817 & 0.576 & 0 \\ -0.576 & 0.817 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$C^{(2)} = Q^T C^{(1)} Q = \begin{pmatrix} 1.556 & 0 & -0.503 \\ 0 & -0.118 & -0.355 \\ -0.503 & -0.355 & 4.562 \end{pmatrix},$$

$$p = 1, \quad q = 3, \quad s = -2.988, \quad t = -0.163, \quad Q = \begin{pmatrix} 0.987 & 0 & -0.161 \\ 0 & 1 & 0 \\ 0.161 & 0 & 0.987 \end{pmatrix},$$

$$C^{(3)} = Q^T C^{(2)} Q = \begin{pmatrix} 1.474 & -0.057 & 0 \\ -0.057 & -0.118 & -0.350 \\ 0 & -0.350 & 4.644 \end{pmatrix}.$$

$$p = 2, \quad q = 3, \quad s = -6.803, \quad t = -0.073, \quad Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0.997 & -0.073 \\ 0 & 0.073 & 0.997 \end{pmatrix},$$

$$C^{(4)} = Q^T C^{(3)} Q = \begin{pmatrix} 1.474 & -0.057 & 0.004 \\ -0.057 & -0.144 & 0 \\ 0.004 & 0 & 4.667 \end{pmatrix}.$$

特征值为  $1.4795, -0.144, 4.667$ . (更精确的答案是  $1.476, -0.145, 4.669$ , 因保留位数不同, 结果相差不大即可)

**P183 4.** 直接验证即可

补: QR分解

$$H_1 = \begin{pmatrix} 0.1741 & 0.6963 & 0.6963 \\ 0.6963 & 0.4130 & -0.5870 \\ 0.6963 & -0.5870 & 0.4130 \end{pmatrix}, \quad A_1 = H_1 A = \begin{pmatrix} 22.9783 & -3.6556 & -2.6112 \\ 0 & 0.2389 & 0.0445 \\ 0 & -0.7611 & 2.0445 \end{pmatrix}.$$

$$H_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0.2995 & -0.9541 \\ 0 & -0.9541 & -0.2995 \end{pmatrix}, \quad H_2 A_1 = \begin{pmatrix} 22.9783 & -3.6556 & -2.6112 \\ 0 & 0.7977 & -1.9373 \\ 0 & 0 & -0.6547 \end{pmatrix},$$

$$Q = H_1^T H_2 = \begin{pmatrix} 0.1741 & -0.4558 & -0.8729 \\ 0.6963 & 0.6838 & -0.2182 \\ 0.6963 & -0.5698 & 0.4364 \end{pmatrix},$$

$$R = H_2 H_1 A = \begin{pmatrix} 22.9783 & -3.6556 & -2.6112 \\ 0 & 0.7977 & -1.9373 \\ 0 & 0 & -0.6547 \end{pmatrix}.$$

**P45 4.**

$$L_1(x) = \frac{x-b}{a-b}f(a) + \frac{x-a}{b-a}f(b) = 0$$

$$|f(x)| = |f(x) - L_1(x)| = \left| \frac{f''(\xi)}{2}(x-a)(x-b) \right| \leq \frac{M_2}{2} \frac{(b-a)^2}{4} = \frac{(b-a)^2 M_2}{8}$$

**P45 5.**

多项式插值的误差

$$R(x) = f[x_0, x_1, \dots, x_n] \prod_{k=0}^n (x - x_k) = \frac{f^{(n+1)}(\xi)}{(n+1)!} \prod_{k=0}^n (x - x_k).$$

在本题中,

$$\begin{aligned} L_2(x) &= 9 \frac{(x-100)(x-121)}{(81-100)(81-121)} + 10 \frac{(x-81)(x-121)}{(100-81)(100-121)} + 11 \frac{(x-81)(x-100)}{(121-81)(121-100)} \\ &= \frac{9}{760}(x-121)(x-100) + \frac{11}{840}(x-81)(x-100) - \frac{10}{399}(x-121)(x-81) \\ &\approx 10.248120 \end{aligned}$$

$$R_2(x) = \frac{f^{(3)}(\xi)}{3!}(x - x_0)(x - x_1)(x - x_2)$$

$$|R_2(105)| = \left| \frac{f^{(3)}(\xi)}{6}(105 - 81)(105 - 100)(105 - 121) \right| = 120\xi^{-\frac{5}{2}}$$

$$120 \cdot (81)^{-\frac{5}{2}} \approx 0.002032, \quad 120 \cdot (121)^{-\frac{5}{2}} \approx 0.0007451$$

$$L_2(105) - f(105) \approx 0.00117 \in [0.0007451, 0.002032]$$