第七次作业答案

P45 8. 线性插值误差

$$R(x) = \frac{f''(\xi)}{2}(x - x_0)(x - x_1).$$

$$|R(x)| = \left| \frac{\sin \xi}{2}(x - x_{i-1})(x - x_i) \right| \leq \frac{1}{2}(x - x_{i-1})(x - x_i)$$

$$\leq \frac{1}{8}(x_{i-1} - x_i)^2 = \frac{h^2}{8} < 0.5 \times 10^{-5},$$

$$h < 6.32 \times 10^{-3}.$$

P45 15. Newton 插值公式:

$$N(x) = f[x_0] + \sum_{k=1}^{n} f[x_0, x_1, \dots, x_k](x - x_0)(x - x_1) \dots (x - x_{k-1}).$$

在此题中

$$g(x) = 3.5(x - 2)^2(x - 1)^2 - 1.5(x - 1)^2(x - 2) + 0.5(x - 1) + 0.5$$

$$R(x) = \frac{f^{(5)}(\xi)}{120}(x - 1)^2(x - 2)^3.$$

P45 16. **Step 1:**

$$h_0 = 1.00, \quad h_1 = 2.00, \quad h_2 = 1.00.$$

Step 2:

$$\lambda_1 = \frac{2}{3}, \quad \lambda_2 = \frac{1}{3}, \quad \mu_1 = \frac{1}{3}, \quad \mu_2 = \frac{2}{3}.$$

Step 3:

$$d_1 = -12, \quad d_2 = 12.$$

Step 4:

$$M_0 = M_3 = 0.$$

$$\begin{bmatrix} 2 & \frac{2}{3} \\ \frac{2}{3} & 2 \end{bmatrix} \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} = \begin{bmatrix} -12 \\ 12 \end{bmatrix}$$

$$M_1 = -9, \quad M_2 = 9.$$

Step 5:

$$S(x) = \begin{cases} -1.5x^3 - 9x^2 - 9.5x + 1, & x \in [-2.00, -1.00], \\ 1.5x^3 - 0.5x + 4, & x \in [-1.00, 1.00], \\ -1.5x^3 + 9x^2 - 9.5x + 7, & x \in [1.00, 2.00]. \end{cases}$$

$$S(0) = 4.$$

P59 1.

$$y = a + bx$$

x_i	4800	3700	3400	2800	1900
y_i	3.1	4.0	5.2	6.4	9.6

$$\left(\begin{array}{cc} \sum_i x_i & \sum_i 1 \\ \sum_i x_i^2 & \sum_i x_i \end{array} \right) \left(\begin{array}{c} b \\ a \end{array} \right) = \left(\begin{array}{c} \sum_i y_i \\ \sum_i x_i y_i \end{array} \right)$$

$$b = -0.002255, \quad a = 13.14.$$

P59 6.
预处理

$$\ln(y) = \ln(a) + bx$$

记$\ln(a) = c, \ln(y) = z$, 则$L(a,b) = \sum_{i=1}^{4}(bx_i + c - z_i)^2$, 由最小二乘得$b = 0.037225, c = 0.662143$, 则

$$a = 1.938943, \quad b = -0.037225,$$

$$y = 1.938943e^{-0.037225}.$$

P59 8(2).

$$\left(\begin{array}{cccc} 1 & 1 & 2 & 1 \\ -2 & 5 & 1 & 1 \end{array} \right) \left(\begin{array}{cc} 1 & -2 \\ 5 & 5 \\ 2 & 1 \\ 1 & 1 \end{array} \right) \left(\begin{array}{c} x_1 \\ x_2 \end{array} \right) = \left(\begin{array}{cccc} 1 & 1 & 2 & 1 \\ -2 & 5 & 1 & 1 \end{array} \right) \left(\begin{array}{c} 1 \\ 13.1 \\ 7.9 \\ 5.1 \end{array} \right)$$

$$x_1 = 3.45856, \quad x_2 = 1.79834$$