第二次作业答案

P74 2.

(结果不唯一, 参考例 3.2)

(1)

$$x=\frac{2x^3-5x^2+42}{19}=\varphi(x)$$
$$\left|\varphi'(x)\right|=\left|\frac{6x^2-10x}{19}\right|,\left|\varphi'(3.0)\right|>1,$$

迭代格式不能保证收敛。

(2)

$$x=\sqrt[3]{\frac{5x^2+19x-42}{2}}=\varphi(x)$$
$$\left|\varphi'(3.0)\right|<1$$

迭代格式在 x=3.0 收敛。

(3)

$$x=\sqrt{\frac{2x^3-19x+42}{5}}=\varphi(x)$$
$$\left|\varphi'(3.0)\right|>1$$

迭代格式不能保证收敛。

[注]: 迭代格式在一点 x_0 处收敛性可由 $\left|\varphi'\left(x_0\right)\right|$ 是否小于 1 判定, 因为根据连续性, 若 $\left|\varphi'\left(x_0\right)\right|<1$, 则在 x_0 附近的一个区间内均有 $\left|\varphi'(x)\right|<1$, 大于 1 同理。

P74 7.

$$x_{k+1}=x_k-\tfrac{f(x_k)(x_k-x_{k-1})}{f(x_k)-f(x_{k-1})},$$
$$x_0=1,f\left(x_0\right)=-4$$

$$x_1=2, f\left(x_1\right)=16$$
$$x_2=1.4, f\left(x_2\right)=-3.456$$
$$x_3=1.68421, f\left(x_3\right)=-2.27526$$
$$x_4=2.23188, f\left(x_4\right)=2.42196$$
$$x_5=1.94949, f\left(x_5\right)=-0.439399$$
$$x_6=1.99286, f\left(x_6\right)=-0.0639954$$
$$x_7=2.00025, f\left(x_7\right)=0.00222969$$
$$x_8=2.0, f\left(x_8\right)=-1.06551\times 10^{-5}$$

P74 8.

$$J(x,y)=\begin{pmatrix} 2x & 2y \\ 3x^2 & -1 \end{pmatrix}$$
$$\begin{pmatrix} x_0 \\ y_0 \end{pmatrix}=\begin{pmatrix} 0.8 \\ 0.6 \end{pmatrix}，解得\begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}=\begin{pmatrix} 0.027049 \\ -0.036066 \end{pmatrix}$$
$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}=\begin{pmatrix} 0.827049 \\ 0.563934 \end{pmatrix}，解得\begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}=\begin{pmatrix} -0.001017 \\ -0.000311 \end{pmatrix}$$
$$\begin{pmatrix} x_2 \\ y_2 \end{pmatrix}=\begin{pmatrix} 0.826032 \\ 0.563624 \end{pmatrix}，解得\begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}=\begin{pmatrix} -1.01551\times 10^{-6} \\ 4.85457\times 10^{-7} \end{pmatrix}$$

最终

$$\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 0.826031 \\ 0.563624 \end{pmatrix}$$

P108 3.

(1)

$$\begin{cases} x_1^{(k+1)}=0.1*\left(1+x_2^{(k)}\right) \\ x_2^{(k+1)}=0.1*\left(x_1^{(k)}+x_3^{(k)}\right) \\ x_3^{(k+1)}=0.1*\left(1+x_2^{(k)}+x_4^{(k)}\right) \\ x_4^{(k+1)}=0.1*\left(2+x_3^{(k)}\right) \end{cases}$$
$$X^{(1)}=(0.1,0,0.1,0.2)^T$$
$$X^{(2)}=(0.1,0.02,0.12,0.21)^T$$
$$X^{(3)}=(0.102,0.022,0.123,0.212)^T$$

(2)

$$\begin{cases} x_1^{(k+1)}=0.1*\left(1+x_2^{(k)}\right) \\ x_2^{(k+1)}=0.1*\left(x_1^{(k+1)}+x_3^{(k)}\right) \\ x_3^{(k+1)}=0.1*\left(x_2^{(k+1)}+x_4^{(k)}\right)+0.1 \\ x_4^{(k+1)}=0.1*\left(2+x_3^{(k+1)}\right) \end{cases}$$

$$X^{(1)} = (0.1, 0.01, 0.101, 0.2101)^T$$

$$X^{(2)} = (0.101, 0.0202, 0.123, 0.2123)^T$$

$$X^{(3)} = (0.102, 0.0225, 0.1235, 0.2123)^T$$

(3)

$$R = I - D^{-1}A = \begin{pmatrix} 0 & 0.1 & 0 & 0 \\ 0.1 & 0 & 0.1 & 0 \\ 0 & 0.1 & 0 & 0.1 \\ 0 & 0 & 0.1 & 0 \end{pmatrix}$$

$$S = I - (D+L)^{-1}A = \begin{pmatrix} 0 & 0.1 & 0 & 0 \\ 0 & 0.01 & 0.1 & 0 \\ 0 & 0.001 & 0.01 & 0.1 \\ 0 & 0.0001 & 0.0001 & 0.01 \end{pmatrix}$$

A 严格对角优，故 Jacobi 和 Gauss-Seldel 迭代都收敛。

P108 4(1).

$$\begin{cases} x_1^{(k+1)} = \frac{1}{2} * \left(x_2^{(k)} - x_3^{(k)} - 1 \right) \\ x_2^{(k+1)} = \frac{1}{3} * \left(-3x_1^{(k)} - 9x_3^{(k)} \right) \\ x_3^{(k+1)} = \frac{1}{5} * \left(-3x_1^{(k)} - 3x_2^{(k)} + 4 \right) \end{cases}$$

$$X^{(1)} = (-0.5, 0, 0.8)^T$$

$$X^{(2)} = (-0.9, -1.9, 1.1)^T$$