ex 2.2

3. (1). $\forall \varepsilon>0$, $\exists \delta = \sqrt[n]{\varepsilon}$, s.t. $|x-0| = |x| < \delta$ 时,

$$\left|x^n \sin\frac{1}{x} - 0\right| \le |x^n - 0| < \delta^n = \varepsilon$$

(3). $\forall \varepsilon>0$, $\exists \delta = \varepsilon$ $|x-a|<\delta$ 时

$$|\cos x - \cos a| < |x-a| < \delta = \varepsilon$$

5. (1). $\lim\limits_{x\to 1} \dfrac{x-\sqrt{x}}{\sqrt{x}-1} = \lim\limits_{x\to 1} \dfrac{\sqrt{x}(\sqrt{x}-1)}{\sqrt{x}-1} = \lim\limits_{x\to 1} \sqrt{x} = 1$

(3). $\sqrt{x} - \sqrt{x+1} < \sin\sqrt{x+1} - \sin\sqrt{x} < \sqrt{x+1} - \sqrt{x}$

$$\lim_{x\to+\infty}\left(\sqrt{x+1} - \sqrt{x}\right) = \lim_{x\to+\infty} \frac{1}{\sqrt{x+1}+\sqrt{x}} = 0 = \lim_{x\to+\infty}\left(\sqrt{x} - \sqrt{x+1}\right)$$

$$\lim_{x\to+\infty}\left(\sin\sqrt{x+1} - \sin\sqrt{x}\right) = 0$$

(5). $-\dfrac{1}{x} \le \dfrac{\sin x}{x} \le \dfrac{1}{x}$

$$\lim_{x\to+\infty}\frac{1}{x} = \lim_{x\to+\infty}\left(-\frac{1}{x}\right) = 0 \qquad \lim_{x\to+\infty}\frac{\sin x}{x} = 0$$

4. (1). $x = k\pi \to +\infty$ $\sin x = 0$ $\qquad y = (2k+\frac{1}{2})\pi \to +\infty$ $\sin y = 1$

(3). $\lim\limits_{x\to 0^+}\dfrac{|x|}{x} = 1$ $\qquad \lim\limits_{x\to 0^-}\dfrac{|x|}{x} = -1 \neq \lim\limits_{x\to 0^+}\dfrac{|x|}{x}$

(5) $\cos x = 1 - 2\sin^2\dfrac{x}{2}$ $\qquad \dfrac{x}{\sqrt{1-\cos x}} = \begin{cases} \sqrt{2}\,\dfrac{\frac{x}{2}}{\sin\frac{x}{2}}, & x>0 \\ -\sqrt{2}\cdot\dfrac{\frac{x}{2}}{\sin\frac{x}{2}}, & x<0 \end{cases}$

$\lim\limits_{x\to 0^+} f(x) \neq \lim\limits_{x\to 0^-} f(x)$ 极不存在

6. (1). $\lim_{x\to+\infty} \frac{\ln x}{x} = \lim_{t\to+\infty} \frac{t}{e^t} < \lim_{t\to+\infty} \frac{t}{t^2} = 0$

故 $\lim_{x\to+\infty} \frac{\ln x}{x} = 0$

(3). $\frac{x^k}{a^x} < \frac{([x]+1)^k}{a^{[x]+1}}$ $\lim_{n\to\infty} \frac{n^k}{a^n} = 0$

故 $\lim_{x\to+\infty} \frac{([x]+1)^k}{a^{[x]+1}} = 0 \Rightarrow \lim_{x\to+\infty} \frac{x^k}{a^x} = 0$