ex 3.1

4. $x\neq 0$ 时 $f'(x) = 2x\sin\frac{1}{x} - \cos\frac{1}{x}$

$$f'\left(\frac{2}{\pi}\right) = \frac{4}{\pi} \qquad f'\left(\frac{1}{\pi}\right) = 1$$

$$f'(0) = \lim_{x\to 0}\frac{f(x)}{x} = \lim_{x\to 0} x\sin\frac{1}{x} = 0$$

9. $\lim\limits_{x\to 1^+} f(x) = f(1) \Rightarrow a+b=1$

$$\lim_{x\to 1^+}\frac{f(x)-f(1)}{x-1} = \lim_{x\to 1^+}\frac{ax+b-1}{x-1} = \lim_{x\to 1^+}\left(a+\frac{a+b-1}{x-1}\right)$$

$a+b=1$ 时该极限存在，值为 1

12. (1). $y' = 3(a+b)x^2 + C$

(3). $y' = \left(x^{\frac{16}{5}}\right)' = \frac{16}{5}x^{\frac{11}{5}}$

(5). $y' = \dfrac{a(cx+d)-c(ax+b)}{(cx+d)^2} = \dfrac{ad-bc}{(cx+d)^2}$

(7). $y' = \cos^3 x + \sin x(-2\cos x\sin x)$

$= \cos^3 x - 2\sin^2 x\cos x$

(9). $(\sin^2 x\cos x)' = 2\sin x\cos^2 x - \sin^3 x$

$$y' = \frac{(x^3+\tan x)(2\sin x\cos^2 x - \sin^3 x) - (3x^2+\frac{1}{\cos^2 x})\sin^2 x\cos x}{(x^3+\tan x)^2}$$

$$= \frac{x^3(2\sin x\cos^2 x - \sin^3 x) + (2-3x^2)\sin^2 x\cos x - \frac{\sin^2 x}{\cos x}(\sin^2 x+1)}{(x^3+\tan x)^2}$$

(11). $y' = \dfrac{x^{a-1} - ax^{a-1}\ln x}{x^{2a}} = \dfrac{1-a\ln x}{x^{a+1}}$

(13). $y' = \dfrac{\left(1+\frac{1}{\sqrt{1-x^2}}\right)\sin x - (x+\arcsin x)\cos x}{\sin^2 x}$

(15). $y' = (a^2+b^2)e^x\left(x^a \arctan x + ax^{a-1}\arctan x + \dfrac{x^a}{1+x^2}\right)$

(17). $y' = 2x\log_3 x + \dfrac{x}{\ln 3}$

(19). $y = x^{\frac{7}{8}}$ $\quad y' = \dfrac{7}{8}x^{-\frac{1}{8}}$

15. (1). $y' = \dfrac{1}{3}\left(\dfrac{1+x}{1-x}\right)^{-\frac{2}{3}} \cdot \dfrac{2}{(1-x)^2} = \dfrac{2}{3}\cdot(1-x)^{-\frac{4}{3}}(1+x)^{-\frac{2}{3}}$

(3). $y' = \dfrac{2}{3}(1+\ln^2 x)^{-\frac{2}{3}} \cdot \dfrac{\ln x}{x}$

(5). $y' = \dfrac{\sqrt{\frac{1+x^2}{1-x^2}} - \frac{x\arcsin x}{\sqrt{1+x^2}}}{1+x^2} - \dfrac{1}{(1+x)^2}$

(7). $y' = \dfrac{1+\frac{1+\frac{1}{2\sqrt{x}}}{2\sqrt{x+\sqrt{x}}}}{2\sqrt{x+\sqrt{x+\sqrt{x}}}} = \dfrac{4\sqrt{x^2+x\sqrt{x}} + 2\sqrt{x}+1}{8\sqrt{(x^2+x\sqrt{x})(x+\sqrt{x+\sqrt{x}})}}$

(9). $y' = \dfrac{1}{3}\left(1+\sqrt[3]{1+\sqrt[3]{x}}\right)^{-\frac{2}{3}} \cdot \dfrac{1}{3}(1+\sqrt[3]{x})^{-\frac{2}{3}} \cdot \dfrac{1}{3}\cdot x^{-\frac{2}{3}}$

$= \dfrac{1}{27}\left[\left(1+\sqrt[3]{1+\sqrt[3]{x}}\right)(1+\sqrt[3]{x})\,x\right]^{-\frac{2}{3}}$

$(\sec x)' = \tan x \sec x$

$(\csc x)' = -\cot x \csc x$

(11). $y' = \cos[\sin(\sin x)] \cdot \cos(\sin x) \cdot \cos x$

(13). $y' = 2\sec\frac{x}{a^2}\cdot\tan\frac{x}{a^2}\cdot\sec\frac{x}{a^2}\cdot\frac{1}{a^2} + 2\tan\frac{x}{b^2}\cdot\sec^2\frac{x}{b^2}\cdot\frac{1}{b^2}$

$= \frac{2}{a^2}\sec^2\frac{x}{a^2}\tan\frac{x}{a^2} + \frac{2}{b^2}\sec^2\frac{x}{a^2}\tan\frac{x}{b^2}$

(15). $y' = a^x\ln a\cdot e^{\sin(\tan x)} + a^x e^{\sin(\tan x)}\cos(\tan x)\cdot\sec^2 x$

(17). $y' = \frac{1}{8}\left(\arcsin^{-\frac{3}{4}}\sqrt{x^2+2x}\right)\sqrt{\frac{x+1}{2x}}$

(19). $y' = \dfrac{2\ln(\ln^3 x)\cdot\frac{1}{\ln^3 x}\cdot 3\ln^2 x\cdot\frac{1}{x}}{\ln^2(\ln^3 x)} = \dfrac{6}{x\ln x\ln(\ln^3 x)}$

(21). $y' = \frac{1+\sin x}{1-\sin x}\cdot\frac{1}{2}\cdot\frac{-2\cos x}{(1+\sin x)^2} = -\frac{1}{\cos x}$

(23). $y' = -\frac{1}{\sqrt{1-x^2}}$

(25). $x^x = e^{x\ln x}$ $\quad (x^x)' = e^{x\ln x}\cdot(1+\ln x) \triangleq A$

$x^{x^x} = e^{x^x\ln x}$ $\quad (x^{x^x})' = e^{x^x\ln x}\left[e^{x\ln x}(1+\ln x)\ln x + x^{x-1}\right] \triangleq B$

$x^{a^x} = e^{a^x\ln x}$ $\quad (x^{a^x})' = e^{a^x\ln x}\cdot\left(a^x\ln a\ln x + \frac{a^x}{x}\right) \triangleq C$

$y' = A+B+C$

(27) $y = e^{\cot\frac{x}{b}\ln(\tan ax)}$

$y' = e^{\cot\frac{x}{b}\ln(\tan ax)}\cdot\left(\frac{a\cot\frac{x}{b}}{\sin ax\cos ax} - \frac{\ln(\tan ax)}{b\sin^2\frac{x}{b}}\right)$

(29). $y = \frac{\cos x^2}{|\cos x^2|}\cdot 2x$

(31). $y' = 2(\arccos x^2)^{-3}\cdot\frac{1}{\sqrt{1-x^4}}$

(33). $\rho' = \frac{2}{\sqrt{a^2-b^2}}\cdot\frac{1}{1+\frac{a-b}{a+b}\cdot\tan^2\frac{\varphi}{2}}\cdot\frac{1}{\cos^2\frac{\varphi}{2}}\cdot\frac{1}{2}$

$$= \sqrt{\frac{a+b}{a-b}} \cdot \frac{a+b}{a+(\cos^2\frac{\varphi}{2}-\sin^2\frac{\varphi}{2})b}$$

(35)、 $y' = \frac{1}{e^x+\sqrt{1+e^{2x}}} \cdot \left(e^x + \frac{e^{2x}}{\sqrt{1+e^{2x}}}\right) = \frac{e^x}{\sqrt{1+e^{2x}}}$

(37)、 $r' = \frac{1}{\arccos\frac{1}{\sqrt{\varphi}}} \cdot \left(-\frac{1}{\sqrt{1-\frac{1}{\varphi}}}\right)$

(39)、 $y' = \frac{1}{\cos(\arctan\frac{e^x-e^{-x}}{2})} \cdot \sin(\arctan\frac{e^x-e^{-x}}{2}) \cdot \frac{1}{1+\frac{(e^x-e^{-x})^2}{4}}$

$$= \frac{e^x-e^{-x}}{2} \cdot \frac{4}{(e^x+e^{-x})^2} = \frac{2(e^x-e^{-x})}{(e^x+e^{-x})^2}$$

16. (1)、 $y' = e^x(x+1)$ $\quad x'(y) = \frac{1}{y'} = \frac{1}{e^x(x+1)} =$

(3)、 $y' = -2e^{-x} + 2e^{-2x}$ $\quad x' = \frac{1}{y'} = \frac{1}{2(e^{-2x}-e^{-x})}$

17. (1)、 $\frac{dy}{dx} = f'(x^3) \cdot 3x^2$

(3)、 $\frac{dy}{dx} = f'(e^x+x^e) \cdot (e^x + ex^{e-1})$

(5)、 $\frac{dy}{dx} = f'(f(f(\sin x+\cos x))) \cdot f'(f(\sin x+\cos x)) \cdot f'(\sin x+\cos x) \cdot (\cos x-\sin x)$

18. (1)、 $x'(t) = a(1-\cos t)$ $\quad y'(t) = a\sin t$

$k_t = \frac{y'(t)}{x'(t)} = \frac{\sin t}{1-\cos t} = 0$

$t=\pi \Rightarrow x = a\pi \quad y = 2a$

切线： $y=2a$ $\quad$ 法线： $x=a\pi$

(3)、 $x'(t)=\frac{3a(1+t^2)-6at^2}{(1+t^2)^2}=\frac{3a(1-t^2)}{(1+t^2)^2}$ $y'(t)=\frac{6at}{(1+t^2)^2}$

$x'(2)=-\frac{9a}{25}$ $y'(2)=\frac{12a}{25}$ $k_t=\frac{y'(2)}{x'(2)}=\frac{4}{3}$ $k_n=-\frac{3}{4}$

$x(2)=\frac{6a}{5}$ $y(2)=\frac{12a}{5}$

切线： $y-\frac{12a}{5}=\frac{4}{3}(x-\frac{6a}{5})$

法线： $y-\frac{12a}{5}=-\frac{3}{4}(x-\frac{6a}{5})$

28. (1)、 $y'=-2xe^{-x^2}$ $y''=2e^{-x^2}(2x^2-1)$

(3)、 $y'=xa^x(2+x\ln a)$ $y''=(a^x+x\cdot a^x\cdot\ln a)(2+x\ln a)+xa^x\ln a$

$=a^x[(x\ln a)^2+4x\ln a+2]$

(5)、 $y'=2x\arctan x+1$ $y''=2(\arctan x+\frac{x}{1+x^2})$

30. (1)、 $\frac{dy}{dt}=\frac{t^2}{1+t^2}$ $\frac{dx}{dt}=\frac{2t}{1+t^2}$

$\frac{dy}{dx}=\frac{t}{2}$ $\frac{d}{dx}\cdot\frac{dy}{dx}=\frac{\frac{d}{dt}\cdot\frac{dy}{dx}}{\frac{dx}{dt}}=\frac{1+t^2}{4t}$

(3)、 $\frac{dx}{d\varphi}=a(\cos\varphi-\varphi\sin\varphi)$ $\frac{dy}{d\varphi}=a(\sin\varphi+\varphi\cos\varphi)$

$\frac{dy}{dx}=\frac{\sin\varphi+\varphi\cos\varphi}{\cos\varphi-\varphi\sin\varphi}$ $\frac{d}{dx}\cdot\frac{dy}{dx}=\frac{\frac{d}{d\varphi}\cdot\frac{dy}{dx}}{\frac{dx}{d\varphi}}=\frac{2+\varphi^2}{a(\cos\varphi-\varphi\sin\varphi)^3}$

32. (1)、 $y'=f'(x^2)\cdot 2x$ $y''=4x^2f''(x^2)+2f'(x^2)$

$y'''=2x^3f'''(x^2)+6xf''(x^2)$

(3)、 $y'=\frac{f'(x)}{f(x)}$ $y''=\frac{f''(x)f(x)-(f'(x))^2}{f^2(x)}$

$$y''' = \frac{[f'''(x)f(x) - f''(x)f'(x)]f^2(x) - 2f(x)f'(x)[f''(x)f(x) - (f'(x))^2]}{f^2(x)}$$

$$= \frac{f'''(x)}{f(x)} - \frac{3f'(x)f''(x)}{f^2(x)} + \frac{2(f'(x))^3}{f^3(x)}$$

35. (1). $(e^x x^2)^{(50)} = e^x(x^2+100x+2450)$

(3). $f'(x) = 2x\sin x + (x^2+1)\cos x$

$f''(x) = 2(\sin x + x\cos x) + 2x\cos x - (x^2+1)\sin x$

$= 2(\sin x + 2x\cos x) - (x^2+1)\sin x$

(5). $f(x) = \frac{1}{(x+1)(x+2)} = \frac{1}{x+1} - \frac{1}{x+2}$

$f^{(n)}(x) = (-1)^n n! \, [(x+1)^{-n-1} - (x+2)^{-n-1}]$

(7). $f(x) = \frac{1-x}{1+x} = \frac{2}{x+1} - 1$

$f^{(n)}(x) = 2(-1)^n n! \, (x+1)^{-n-1}$