Ex 1-1

2、假设$\sqrt{6}$是有理数，$\sqrt{6}=\frac{p}{q}$，$p,q\in N^*$，$(p,q)=1$

5. (1)、无上界及最大值　$\inf N=0$　$\min=0$

(3)、$A=\{\frac{n}{m+n+1}\mid m,n\in N\}$　$\sup A=1$　$\inf A=0$

无最大、最小值

(5) $B=\{(1-\frac{1}{n+1})\mid n\in N\}$　$\inf B=\min B=\frac{1}{2}$

$\sup B=1$　无最大值

7. $\forall\varepsilon>0$，$\exists x_0\in A\cup B$，s.t. $x_0<\min\{\inf A,\inf B\}+\varepsilon$

故 $\inf(A\cup B)=\min\{\inf A,\inf B\}$

同理可知 $\sup(A\cup B)=\max\{\sup A,\sup B\}$

假设 $\inf(A\cap B)<\max\{\inf A,\inf B\}$

不妨设 $\inf A<\inf B$　则 $\inf(A\cap B)<\inf B$

取 $\varepsilon=\frac{\inf B-\inf(A\cap B)}{2}$，$\exists x_0\in A\cap B$，即 $x_0\in B$，

s.t. $x_0<\inf(A\cap B)+\varepsilon=\frac{\inf B+\inf(A\cap B)}{2}<\inf B$　矛盾

故 $\inf(A\cap B)\geq\max\{\inf A,\inf B\}$

同理可知 $\sup(A\cap B)\leq\min\{\sup A,\sup B\}$

12. $\forall\, 0<\varepsilon<\inf A+\inf B$, $\exists x_1\in A, y_1\in B$, s.t. $x_1<\inf A+\varepsilon$ $y_1<\inf B+\varepsilon$

$x_1y_1<(\inf A+\varepsilon)(\inf B+\varepsilon)=\inf A\cdot\inf B+\varepsilon(\inf A+\inf B)+\varepsilon^2$

$<\inf A\cdot\inf B+2\varepsilon(\inf A+\inf B)$

令 $\varepsilon'=2\varepsilon(\inf A+\inf B)$

则对 $\forall\varepsilon'>0$, $\exists x_1y_1\in AB$, s.t. $x_1y_1<\inf A\cdot\inf B+\varepsilon'$

即 $\inf(AB)=\inf A\cdot\inf B$

同理可得 $\sup(AB)=\sup A\cdot\sup B$

14. 即证 $\left(\frac{n}{n-1}\right)^n>\left(\frac{n+1}{n}\right)^{n+1}$ 令 $f(n)=\left(\frac{n}{n-1}\right)^n$

即证 $f(n)>f(n+1)$ 令 $f(x)=x(\ln x-\ln(x-1))$

$f'(x)=\ln x-\ln(x-1)-\frac{1}{x-1}$ $f''(x)=\frac{1}{x}-\frac{1}{x-1}+\frac{1}{(x-1)^2}=\frac{1}{x(x-1)^2}>0$

$\lim\limits_{x\to+\infty}f'(x)=0$ $f'(2)<0$ $f'(x)<0$ $f(x)\downarrow$

ex. 1-2

2. $f(x) = 2(x-1)^2 - (x-1)+1 = 2x^2 - 5x + 4$

$g(x+\frac{1}{x}) = (x+\frac{1}{x})^2 - 2 \qquad g(x) = x^2 - 2$

$f\circ g = f(g(x)) = 2g^2(x) - 5g(x) + 4 = 2x^4 - 13x^2 + 22$

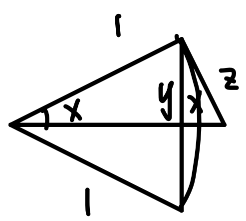
$g\circ f = g(f(x)) = f^2(x) - 2 = 4x^4 - 20x^3 + 26x^2 - 40x + 14$

6. $f(x) = \ln(\sqrt{x^2+1} + x)$

$f(-x) = \ln(\sqrt{x^2+1} - x) = \ln\left(\frac{1}{\sqrt{x^2+1}+x}\right) = -\ln(\sqrt{x^2+1}+x) = -f(x)$

$$\begin{cases} e^y = \sqrt{x^2+1} + x \\ e^{-y} = \sqrt{x^2+1} - x \end{cases} \Rightarrow x = \frac{e^y - e^{-y}}{2}$$

15.



$y = \sin x \qquad z = \tan x$

$\sin x < x < \tan x$