概统第2次作业

8. $P(X\geq 1) = 1 - P(X=0) = 1-(1-0.2)^{20} = 1-0.8^{20}$

≈ 0.99

$$\frac{P(X=k+1)}{P(X=k)} = \frac{\binom{20}{k+1} 0.2^{k+1} 0.8^{20-(k+1)}}{\binom{20}{k} 0.2^{k} 0.8^{20-k}}$$

$$= \frac{20-k}{k+1}\cdot\frac{1}{4} \qquad (*)$$

$k\leq 3$. $(*)>1$. $k\geq 4$ $(*)<1$.

故 $k=4$ 时. $P(X=k)$ 最大.

12. <1> $P(Y=k) = \sum_{m=k}^{\infty} P(Y=k \mid X=m)P(X=m)$

$$= \sum_{m=k}^{\infty} \binom{m}{k} p^k (1-p)^{m-k} \cdot \frac{\lambda^m}{m!} e^{-\lambda}$$

$$= \sum_{m-k=0}^{\infty} \frac{(1-p)^{m-k}\lambda^{m-k}}{(m-k)!}\cdot\frac{1}{k!}\cdot e^{-\lambda}\cdot\lambda^k p^k$$

$k=0,1,2,\ldots$

$$= e^{\lambda(1-p)}\cdot\frac{1}{k!}\cdot e^{-\lambda}\cdot(\lambda p)^k$$

$$= \frac{(\lambda p)^k}{k!} e^{-\lambda p}$$

故 $Y\sim$ Poisson (λp).

同理. $P(Z=k) = \frac{[\lambda(1-p)]^k e^{-\lambda(1-p)}}{k!}$

故 $Z\sim$ Poisson $(\lambda(1-p))$

(2) $P(Y=i,\ Z=j) = P(Y=i,\ Z=j \mid X=i+j)\, P(X=i+j)$

$$= \binom{i+j}{i} p^{i}(1-p)^{j} \cdot \frac{\lambda^{i+j}}{(i+j)!} e^{-\lambda}$$

$$= \frac{(\lambda p)^{i}}{i!} e^{-\lambda p} \cdot \frac{[\lambda(1-p)]^{j}}{j!} e^{-\lambda(1-p)}$$

$$= P(Y=i) \cdot P(Z=j).$$

13. $P\{$系统正常运转$\} = P\{X=0\} = \binom{1000}{0} p^{0}(1-p)^{1000}$

设X为一个月内故障数 $p=0.001$

$$\approx \frac{1^{0}}{0!} e^{-1} = \frac{1}{e}.$$

17. (1) ① $1-(1-\frac{1}{10000})^{100} = 1-0.9999^{100}$

② $np = 0.01$

$$1-\frac{0.01^{0}}{0!} \cdot e^{-0.01} = 1-e^{-0.01}$$

(2) ① $1-0.9999^{x} \geq 0.95$

$$x \geq [\ln 0.05 / \ln 0.9999] + 1 = 29956.$$

② $1-e^{-x \cdot 10^{-4}} \geq 0.95$

$$x \geq [-10^{4} \ln 0.05] + 1 = 29958$$

31. <1> $P(0 \le X \le 4) = P\left(\frac{0-1}{2} \le \frac{X-1}{2} \le \frac{4-1}{2}\right)$

$$= \Phi(1.5) - \Phi(-0.5)$$

$P(X > 2.4) = P(Z > 0.7) = 1 - \Phi(0.7)$

$P(|X| > 2) = 1 - P(-2 < X < 2)$

$$= 1 - P\left(-1.5 < \frac{X-1}{2} < 0.5\right)$$

$$= 1 - \left[\Phi(0.5) - \Phi(-1.5)\right]$$

$$= 1 + \Phi(-1.5) - \Phi(0.5)$$

<2> $P(X > c) = 2P(X \le c) \Leftrightarrow$

$$\Leftrightarrow P\left(\frac{X-1}{2} > \frac{c-1}{2}\right) = 2P\left(\frac{X-1}{2} \le \frac{c-1}{2}\right)$$

$$1 - \Phi\left(\frac{c-1}{2}\right) = 2\Phi\left(\frac{c-1}{2}\right)$$

$$\Phi\left(\frac{c-1}{2}\right) = \frac{1}{3}$$

$$c = 2\Phi^{-1}\left(\frac{1}{3}\right) + 1.$$