第九次作业

记 $L(y(t)) = Y(p)$

(1) $\begin{cases} y''(t) + y'(t) = 1 \\ y(0) = y'(0) = 0 \end{cases}$

左右作拉氏变换

$$p^2 Y(p) + pY(p) = \frac{1}{p}$$

$$Y(p) = \frac{1}{p^2(p+1)} \left(= \frac{p+1-p}{p^2(p+1)} = \frac{1}{p^2} - \frac{1}{p(p+1)} = \frac{1}{p^2} - \frac{1}{p} + \frac{1}{p+1} \right)$$

两个奇点 $p=0$，$p=-1$

$$f(t) = \text{Res}(Y(p)e^{pt}, -1) + \text{Res}(Y(p)e^{pt}, 0)$$

$$= e^{-t} + (t-1) = e^{-t} + t - 1$$

(3) $\begin{cases} y'' - (a+b)y' + aby = 0 \\ y(0) = 0 \quad y'(0) = 1 \end{cases}$

左右作拉氏变换

$$(p^2 Y(p) - p \cdot 1) - (a+b)pY(p) + abY(p) = 0$$

$$(p-a)(p-b)Y(p) = p \qquad Y(p) = \frac{p}{(p-a)(p-b)}$$

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$$Y(p) = \frac{1}{a-b}\left(\frac{1}{p-a} - \frac{1}{p-b}\right)$$

记住！ $L(e^{at}) = \frac{1}{p-a}$

故 $y(t) = L^{-1}(Y(p)) = \frac{1}{a-b}(e^{at} - e^{bt})$ （拉氏变换线性性）

(5) $\begin{cases} y'' - y = 4\sin t + 5\cos 2t \\ y(0) = -1 \qquad y'(0) = -2 \end{cases}$

左右做拉氏变换：

$$p^2 Y(p) - p(-1) - (-2) - Y(p) = \frac{4}{p^2+1} + \frac{5p}{p^2+4}$$

（务必记得 $L(\sin \omega t) = \frac{\omega}{p^2+\omega^2}$ $L(\cos \omega t) = \frac{p}{p^2+\omega^2}$ ）

$\omega \in \mathbb{R}$

$$(p^2-1)Y(p) = -p-2 + \frac{4}{p^2+1} + \frac{5p}{p^2+4}$$

$$Y(p) = -\frac{p+2}{p^2-1} + \frac{4}{(p^2+1)(p^2-1)} + \frac{5p}{(p^2+4)(p^2-1)}$$

$$= -\frac{p+2}{p^2-1} + 2\cdot\frac{(p^2+1)-(p^2-1)}{(p^2+1)(p^2-1)} + p\cdot\frac{(p^2+4)-(p^2-1)}{(p^2+4)(p^2-1)}$$

$$= -\frac{p+2}{p^2-1} + \frac{2}{p^2-1} - \frac{2}{p^2+1} + \frac{p}{p^2-1} - \frac{p}{p^2+4}$$

$$= -\frac{2}{p^2+1} - \frac{p}{p^2+4}$$

故 $y(t) = L^{-1}(Y(p)) = -2\cdot L^{-1}\left(\frac{1}{p^2+1}\right) - L^{-1}\left(\frac{p}{p^2+4}\right)$

$$= -2\sin t - \cos 2t$$