ex 2.2

7. (1). $\lim_{x\to 0}\frac{\tan 2x}{\sin 5x} = \lim_{x\to 0}\frac{\sin 2x}{\sin 5x}\cdot\frac{1}{\cos 2x} = \lim_{x\to 0}\frac{2x}{5x} = \frac{2}{5}$

(3). 令 $\arcsin x = t$ $\quad x = \sin t$

$\lim_{x\to 0}\frac{\arcsin x}{x} = \lim_{t\to 0}\frac{t}{\sin t} = 1$

(5). $\lim_{x\to a}\frac{\cos x-\cos a}{x-a}$

8. (1). $\lim_{x\to +\infty}\left(\frac{x+1}{2x-1}\right)^{x} = \lim_{x\to +\infty}\left(\frac{1}{2}\right)^{x}\cdot\left[\left(1+\frac{1}{\frac{1}{3}(2x-1)}\right)^{\frac{1}{3}(2x-1)}\right]^{\frac{3}{2}}\cdot\left(1+\frac{1}{\frac{1}{3}(2x-1)}\right)^{\frac{1}{2}}$

$= e^{\frac{3}{2}}$

(3). $\lim_{x\to \frac{\pi}{2}}(1+\cos x)^{3\sec x} = \lim_{x\to\frac{\pi}{2}}\left(1+\frac{1}{\frac{1}{\cos x}}\right)^{3\cdot\frac{1}{\cos x}} = e^{3}$

(5). $\lim_{x\to 0^+}(\tan x)^{\tan 2x} =$

ex 2.3

1、(1)、 $\lim\limits_{x\to 0} \dfrac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt[8]{x}} = \lim\limits_{x\to 0} \sqrt{\dfrac{x}{\sqrt[4]{x}} + \sqrt{\dfrac{x}{\sqrt{x}} + 1}}$

$= \lim\limits_{x\to 0} \sqrt{x^{\frac{3}{4}} + \sqrt{\sqrt{x}+1}} = 1$

(3)、 $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x} = \lim\limits_{x\to 0} \dfrac{2\sin^2\frac{x}{2}}{x} = \lim\limits_{x\to 0} \dfrac{\sin\frac{x}{2}}{\frac{x}{2}} \cdot \sin\frac{x}{2} = 0$

(5). $\lim\limits_{x\to 0} 4 \dfrac{\sqrt{1+\tan x} - \sqrt{1+\sin x}}{x^3} = \lim\limits_{x\to 0} 4 \cdot \dfrac{\sin x \cdot 2\sin^2\frac{x}{2}}{x^3\cos x(\sqrt{1+\tan x} + \sqrt{1+\sin x})}$

$= \lim\limits_{x\to 0} 4 \cdot \dfrac{1}{4} = 1$

5、(1)、 $\lim\limits_{x\to +\infty} \dfrac{\ln x}{x^k} = \lim\limits_{x\to +\infty} \dfrac{t}{e^{kt}} < \lim\limits_{x\to +\infty} \dfrac{t}{kt+1} = 0$

故 $\lim\limits_{x\to +\infty} \dfrac{\ln x}{x^k} = 0$ ， x^k 更高阶

(3)、 $\dfrac{x^k}{a^x} < \dfrac{([x]+1)^k}{a^{[x]+1}}$ $\quad \lim\limits_{n\to \infty} \dfrac{n^k}{a^n} = 0$

故 $\lim\limits_{x\to +\infty} \dfrac{([x]+1)^k}{a^{[x]+1}} = 0 \Rightarrow \lim\limits_{x\to +\infty} \dfrac{x^k}{a^x} = 0$ $\quad a^x$ 更高阶

8、(1)、 $\lim\limits_{x\to 0} \dfrac{\sin ax}{\sin bx} = \lim\limits_{x\to 0} \dfrac{ax}{bx} = \dfrac{a}{b}$

(3). $\lim\limits_{x\to 0} \dfrac{\tan 2x \cdot \arcsin x}{\sin 3x \cdot \arctan 2x} = \lim\limits_{x\to 0} \dfrac{2x \cdot x}{3x \cdot 2x} = \dfrac{1}{3}$

(5). $\lim\limits_{x\to 0} \dfrac{1-\cos(1-\cos x)}{x^4} = \lim\limits_{x\to 0} \dfrac{(1-\cos x)^2}{2x^4} = \lim\limits_{x\to 0} \dfrac{x^4}{8x^4} = \dfrac{1}{8}$

(7)、 $\lim\limits_{x\to 0} \dfrac{\ln(\sec x + \tan x)}{\sin x} = \lim\limits_{x\to 0} \dfrac{\frac{\sin x+1}{\cos x} - 1}{\sin x} = \lim\limits_{x\to 0} \dfrac{\sin x + (1-\cos x)}{\sin x\cos x}$

$= \lim\limits_{x\to 0} \left(\dfrac{1}{\cos x} + \dfrac{1-\cos x}{\sin x\cos x}\right)$

$$= \lim_{x\to 0} \left(1 + \frac{\frac{1}{2}x^2}{x}\right) = 1$$

(9)、$\lim_{x\to 0} \frac{\sin^2 x(1-\cos x)}{\cos x(e^{\sin x}-1)} = \lim_{x\to 0} \frac{x^2 \cdot \frac{x^2}{2}}{x} = 0$

12. $\lim_{x\to a} \frac{o(f(x)) \cdot o(g(x))}{f(x) \cdot g(x)} = \lim_{x\to 0} \left(\frac{o(f(x))}{f(x)}\right) \cdot \left(\frac{o(g(x))}{g(x)}\right) = 0$

$\Rightarrow o(f(x)) \cdot o(g(x)) = o(f(x) \cdot g(x))$

$\lim_{x\to a} \frac{f(x) \cdot g(x)}{O(f(x)) \cdot O(g(x))} = \lim_{x\to a} \left(\frac{f(x)}{O(f(x))}\right) \cdot \left(\frac{g(x)}{O(g(x))}\right) = 0$

$\Rightarrow O(f(x)) \cdot O(g(x)) = O(f(x)g(x))$

ex 2.4

1. (1)、$y = \frac{(x-1)(x+1)(x+3)}{(x+3)(x-2)}$

$\lim_{x\to -3^-} y = \lim_{x\to -3^+} y = -\frac{8}{5}$ 故为可去间断点，补充定义 $y = -\frac{8}{5}$

$\lim_{x\to 2^-} y = -3$ $\lim_{x\to 2^+} y = 3$ 为跳跃间断点

在 $(-\infty, -\frac{8}{5})$，$(-\frac{8}{5}, 2)$，$(2, +\infty)$ 连续

(3)、在 $[k, k+1)$ 连续 $\lim_{x\to k^-} [x] = k-1$ $\lim_{x\to k^+} [x] = k$

为跳跃间断点

(5). $\lim\limits_{x\to 0^+} y$ 与 $\lim\limits_{x\to 0^-} y$ 均不存在　为第二类间断点

在 $(-\infty,0)$, $(0,+\infty)$ 连续

(7). $\lim\limits_{x\to\frac{\pi}{2}+2k\pi} y=0$, $y\Big|_{x=\frac{\pi}{2}+2k\pi}=1$　故 $\frac{\pi}{2}+2k\pi$ 为可去间断点，补充定义 $y=0$

$\lim\limits_{x\to k\pi^-} y=0$　$\lim\limits_{x\to k\pi^+} y=-1$　$k\pi$ 为跳跃间断点

在 $(2k\pi, \frac{\pi}{2}+2k\pi)$, $(\frac{\pi}{2}+2k\pi, (2k+1)\pi)$, $((2k+1)\pi, 2(k+1)\pi)$ 连续

6. (1). $\lim\limits_{x\to+\infty}(1+\frac{\alpha}{x})^x = \lim\limits_{x\to+\infty}\left((1+\frac{1}{\frac{x}{\alpha}})^{\frac{x}{\alpha}}\right)^{\alpha} = e^{\alpha}$

(3). $\lim\limits_{x\to 0}\frac{\alpha^x-1}{x} =$

7. (1). $\lim\limits_{n\to\infty}(1+\frac{x}{n})^n = \lim\limits_{n\to\infty}(1+\frac{1}{\frac{n}{x}})^{\frac{n}{x}\cdot x} = e^x$

(3). $\lim\limits_{x\to 0}\frac{e^{2x}-1}{\sin x} = \lim\limits_{x\to 0}\frac{2x}{x} = 2$

(5). $\lim\limits_{x\to 0}(2\sin x+\cos x)^{\frac{1}{x}} = \lim\limits_{x\to 0}(2x+1)^{\frac{1}{x}} = \lim\limits_{x\to 0}(1+\frac{1}{\frac{1}{2x}})^{\frac{1}{2x}\cdot 2} = e^2$

(7). $\lim\limits_{x\to 0^+}(\frac{1}{x})^{\tan x} =$