Ex10.1 1/(1)(3)(5); 2/(1)(2)(5)(6)(8); 4

1.改变下列积分的顺序。

(1)$\int_{-1}^{1} dx \int_{0}^{\sqrt{1-x^2}} f(x,y)dy$

$\int_{0}^{1} dy \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x,y)dx$

(3)$\int_{0}^{a} dy \int_{a-\sqrt{a^2-y^2}}^{a+\sqrt{a^2-y^2}} f(x,y)dx$

$\int_{0}^{2a} dx \int_{0}^{\sqrt{2ax-x^2}} f(x,y)dy$

(5)$\int_{0}^{1} dx \int_{0}^{x} f(x,y)dy + \int_{1}^{2} dx \int_{0}^{2-x} f(x,y)dy$

$\int_{0}^{1} dy \int_{y}^{2-y} f(x,y)dx$

2.计算下列积分。

(1)$\iint_D \frac{y}{(1+x^2+y^2)^{3/2}} dxdy, D=[0,1]\times[0,1]$

$\iint_D \frac{y}{(1+x^2+y^2)^{3/2}} dxdy = \int_0^1 dx \int_0^1 \frac{y}{(1+x^2+y^2)^{3/2}} dy = \int_0^1 dx(-\frac{1}{\sqrt{1+x^2+y^2}})|_0^1$

$= \int_0^1 (\frac{1}{\sqrt{1+x^2}} - \frac{1}{\sqrt{2+x^2}})dx = \ln|x+\sqrt{1+x^2}||_0^1 - \ln|x+\sqrt{2+x^2}||_0^1$

$= \ln(1+\sqrt{2}) - (\ln(1+\sqrt{3}) - \ln(\sqrt{2})) = \ln\frac{2+\sqrt{2}}{1+\sqrt{3}}$

(2)$\iint_D \sin(x+y)dxdy, D=[0,\pi]\times[0,\pi]$

$\iint_D \sin(x+y)dxdy = \int_0^\pi dy \int_0^\pi \sin(x+y)dx = \int_0^\pi dy(-\cos(x+y)|_0^\pi) = \int_0^\pi[\cos(y)-\cos(y+\pi)]dy$

$= \int_0^\pi 2\cos(y)dy = 0$

(5)$\iint_D (x+y-1)dxdy$，D是由$y=x, y=x+a, y=a, y=3a$围成

$a>0$

$\iint_D (x+y-1)dxdy = \int_a^{3a} dy \int_{y-a}^{y} (x+y-1)dx = \int_a^{3a} ay - \frac{1}{2}a^2 + a(y-1)dy$

$= 2a(-\frac{1}{2}a^2 - a) + a\cdot 8a^2 = 7a^3 - 2a^2$

$a<0$

$\iint_D (x+y-1)dxdy = \int_{3a}^{a} dy \int_{y}^{y-a} (x+y-1)dx = \int_{3a}^{a} -ay + \frac{1}{2}a^2 - a(y-1)dy = 7a^3 - 2a^2$

综上结果为$7a^3-2a^2$

(6)$\iint_D \frac{\sin y}{y} dxdy$，$D$是由$y=x$和$x=y^2$围成

$\iint_D \frac{\sin y}{y} dxdy = \int_0^1 dy \int_{y^2}^{y} \frac{\sin y}{y} dx = \int_0^1 (1-y)\sin y dy = -(1-y)\cos y|_0^1 - \int_0^1 \cos y dy = 1-\sin 1$

(8)$\iint_D |\cos(x+y)|dxdy$，D由直线$y=x, y=0, x=\frac{\pi}{2}$围成

$\iint_D |\cos(x+y)|dxdy = \int_0^{\frac{\pi}{4}} dx \int_0^x \cos(x+y)dy + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} dx \int_0^{\frac{\pi}{2}-x} \cos(x+y)dy - \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} dx \int_{\frac{\pi}{2}-x}^{x} \cos(x+y)dy$

$\int_0^{\frac{\pi}{4}} [\sin(2x)-\sin(x)]dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (1-\sin(x))dx - \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\sin(2x)-1)dx = -\int_0^{\frac{\pi}{2}} \sin(x)dx + \frac{\pi}{2} = \frac{\pi}{2} - 1$

4.设函数φ和ψ分别在区间$[a,b]$和$[c,d]$上可积，求证：$f(x,y)=\varphi(x)\psi(y)$在$D=[a,b]\times[c,d]$上可积，且有

$\iint_D f(x,y)dxdy = \int_a^b \varphi(x)dx \int_c^d \psi(x)dx$

由定理10.4, $f(x,y)$在D上可积的充分必要条件是$\inf_T\{\omega(T)\}=0$

然后我们将分割限制为矩形分割T^{rec}，则有$\inf_T\{\omega(T)\} \leq \inf_{T^{rec}}\{\omega(T^{rec})\}$

φ和ψ可积，所以有界，所以设$|\varphi|, |\psi| \leq A$

在一个区域$[x_i, x_{i+1}]\times[y_i, y_{i+1}]$上,$M-m \leq A(M_x-m_x)+A(M_y-m_y)$

而后由定义代入可证。