1.

import numpy as np

from scipy import linalg

B = np.random.rand(6, 4)

b = np.random.rand(6, 1)

B\_t = np.transpose(B)

B\_inv = np.linalg.inv(np.matmul(B\_t, B))

x\_ls = np.matmul(np.matmul(B\_inv, B\_t), b)

U, s, Vh = np.linalg.svd(B)

P, L, U = linalg.lu(B)

print("矩阵B:\n", B)

print("向量b:\n", b)

print("线性方程组 Bx=b 的最小二乘解 x:\n", x\_ls)

print("矩阵B的奇异值分解:")

print("U:\n", U)

print("s:\n", s)

print("Vh:\n", Vh)

print("矩阵B的LU分解:")

print("P:\n", P)

print("L:\n", L)

print("U:\n", U)

输出：

矩阵B:

[[0.29158138 0.11126205 0.70744416 0.00940491]

[0.67449821 0.38310905 0.43064873 0.52290008]

[0.83106478 0.2089543 0.43101862 0.900735 ]

[0.335779 0.4423192 0.16958342 0.75612437]

[0.26471285 0.02491465 0.2338597 0.5078168 ]

[0.90687339 0.20854581 0.24212056 0.75094234]]

向量b:

[[0.90416795]

[0.43721004]

[0.96711464]

[0.98653028]

[0.8638027 ]

[0.82719325]]

线性方程组 Bx=b 的最小二乘解 x:

[[-0.85575594]

[-0.48948653]

[ 1.50529381]

[ 1.4655835 ]]

矩阵B的奇异值分解:

U:

[[ 0.90687339 0.20854581 0.24212056 0.75094234]

[ 0. 0.36510301 0.07993584 0.47808041]

[ 0. 0. 0.61991736 -0.28993075]

[ 0. 0. 0. 0.41570866]]

s:

[2.3392961 0.6869068 0.38643333 0.27768122]

Vh:

[[-0.6274105 -0.24863313 -0.34706924 -0.65121469]

[ 0.0923263 -0.03803735 0.84527241 -0.52492244]

[-0.66163552 0.65906153 0.25659826 0.2490656 ]

[ 0.40008794 0.70878249 -0.31498385 -0.48820302]]

矩阵B的LU分解:

P:

[[0. 0. 1. 0. 0. 0.]

[0. 0. 0. 0. 1. 0.]

[0. 0. 0. 0. 0. 1.]

[0. 1. 0. 0. 0. 0.]

[0. 0. 0. 1. 0. 0.]

[1. 0. 0. 0. 0. 0.]]

L:

[[ 1. 0. 0. 0. ]

[ 0.37026007 1. 0. 0. ]

[ 0.3215238 0.12108805 1. 0. ]

[ 0.29189615 -0.0984902 0.27593765 1. ]

[ 0.74376227 0.62448277 0.32367222 -0.57812832]

[ 0.91640662 0.04886712 0.33106267 0.68603166]]

U:

[[ 0.90687339 0.20854581 0.24212056 0.75094234]

[ 0. 0.36510301 0.07993584 0.47808041]

[ 0. 0. 0.61991736 -0.28993075]

[ 0. 0. 0. 0.41570866]]