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Thesis title

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Supervisor of the master thesis: RNDr. Jiří Horák, Ph.D.

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I declare that I carried out this master thesis independently, and only with the cited sources, literature and other professional sources. It has not been used to obtain another or the same degree.

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In date

Author's signature

Dedication. It is nice to say thanks to supervisors, friends, family, book authors and food providers.

Title: Thesis title

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Abstract: This thesis is a continuation of my bachelor thesis. In that work, we examined the boundary layer between a star and the accretion disk. This work led to a single non-linear second-order ODE. In this work, we attempt to examine the equation further. We also include effects of magnetic field into our equations and try to generalise the solution for a general polytropic index.

Keywords: key words

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Introduction

Introduction should answer the following questions, ideally in this order:

1. What is the nature of the problem the thesis is addressing?
2. What is the common approach for solving that problem now?
3. How this thesis approaches the problem?
4. What are the results? Did something improve?
5. What can the reader expect in the individual chapters of the thesis?

Expected length of the introduction is between 1–4 pages. Longer introductions may require sub-sectioning with appropriate headings — use `\section*` to avoid numbering (with section names like ‘Motivation’ and ‘Related work’), but try to avoid lengthy discussion of anything specific. Any “real science” (definitions, theorems, methods, data) should go into other chapters.

It is very advisable to skim through a book about scientific English writing before starting the thesis. I can recommend ‘**glasman2010science**’ by **glasman2010science**.

You may notice that this paragraph briefly shows different “types” of ‘quotes’ in TeX, and the usage difference between a hyphen (‐), en-dash (–) and em-dash (—).

Chapter 1

Important first chapter

First chapter usually builds the theoretical background necessary for readers to understand the rest of the thesis. You should summarize and reference a lot of existing literature and research.

You should use the standard *citations*.

Obtaining bibTeX citation Go to Google Scholar¹, find the relevant literature, click the tiny double-quote button below the link, and copy the bibTeX entry.

Saving the citation Insert the bibTeX entry to the file `refs.bib`. On the first line of the entry you should see the short reference name — from Scholar, it usually looks like `author2015title` — you will use that to refer to the citation.

Using the citation Use the `\cite` command to typeset the citation number correctly in the text; a long citation description will be automatically added to the bibliography at the end of the thesis. Always use a non-breakable space before the citing parenthesis to avoid unacceptable line breaks:

```
Trees utilize gravity to invade ye  
noble sires~\cite{newton1666apple}.
```

Why should I bother with citations at all? For two main reasons:

- You do not have to explain everything in the thesis; instead you send the reader to refer to details in some other literature. Use citations to simplify the detailed explanations.

¹<https://scholar.google.com>

Use `\emph` command like this, to highlight the first occurrence of an important word or term. Reader will notice it, and hopefully remember the importance.

This footnote is an acceptable way to 'cite' webpages or URLs. Documents without proper titles, authors and publishers generally do not form citations. For this reason, avoid citations of wikipedia pages.

- If you describe something that already exists without using a citation, the reviewer may think that you *claim* to have invented it. Expectably, they will demand academic correctness, and, from your perspective, being accused of plagiarism is not a good starting point for a successful defense. Use citations to identify the people who invented the ideas that you build upon.

How many citations should I use? Cite any non-trivial building block or assumption that you use, if it is published in the literature. You do not have to cite trivia, such as the basic definitions taught in the introductory courses.

The rule of thumb is that you should read, understand and briefly review at least around 4 scientific papers. A thesis that contains less than 3 sound citations will spark doubt in reviewers.

There are several main commands for inserting citations, used as follows:

- **knuth1979tex** described a great system for typesetting theses.
- We are typesetting this thesis with \LaTeX , which is based on \TeX and META-FONT [**knuth1979tex**].
- \TeX was expanded to \LaTeX by **lamport1994latex**, hence the name.
- Revered are the authors of these systems! [**knuth1979tex**, **lamport1994latex**]■

1.1 Some extra assorted hints before you start writing English

Word order Strictly adhere to the English word order rules. The sentences follow a fixed structure with a subject followed by a verb and an object (in this order). Exceptions to this rule must be handled specially, and usually separated by commas.

Sentence structure Do not write long sentences. One sentence should contain exactly one fact. Multiple facts should be grouped in a paragraph to communicate one coherent idea. Both the sentences and paragraphs should include various hints about their relation to the other ideas and paragraphs. These are typically materialized as adverbs or short sentence parts that clarify the cause–outcome and target–method–result relationship of the sentences in a paragraph. Such ‘word glue’ helps the readers to correctly draw the lines that hold their mental

images of your thesis together, and ideally see the big picture of what you were trying to convey right from the first read.

Paragraphs are grouped in labeled sections for a sole purpose of making the navigation in the thesis easier. Do not use the headings as ‘names for paragraphs’ — the text should make perfect sense even if all headings are removed. If a section of your text contains one paragraph per heading, you might have wanted to write an explicit list instead.

Mind the rules for placing commas:

- Do not use the comma before subordinate clauses that begin with ‘that’ (like this one). English does not use subordinate clauses as often as Slavic languages because the lack of a suitable word inflection method makes them hard to understand. In scientific English, try to avoid them as much as possible. Ask doubtfully whether each ‘which’ and ‘when’ is necessary — most of these helper conjunctions can be removed by converting the clause to non-subordinate.

As an usual example, ‘*The sentence, which I wrote, seemed ugly.*’ is perfectly bad; slightly improved by ‘*The sentence that I wrote seemed ugly.*’, which can be easily reduced to ‘*The sentence I wrote seemed ugly.*’. A final version with added storytelling value could say ‘*I wrote a sentence but it seemed ugly.*’

- Use the *Oxford comma* before ‘and’ and ‘or’ at the end of a longer, comma-separated list of items. Certainly use it to disambiguate any possible mixtures of conjunctions: ‘*The car is available in red, red and green, and green versions.*’ Remember that English ‘or’ is typically understood more like ‘either this or that, but not both,’ and the use of ‘and’ is much more appropriate in cases such as possibility overviews and example listings (like in this sentence).
- Consider placing extra commas around any parts of the sentence that break the usual word order, especially if they are longer than a single word.

Nouns Every noun needs a determiner (‘a’, ‘the’, ‘my’, ‘some’, ...); the exceptions to this rule, such as non-adjectivized names and indeterminate plural, are relatively scarce. Without a determiner, a noun can be easily mistaken for something completely different, such as an adjective or a verb.

Name all things with appropriate nouns to help both the reader and yourself, and do not hesitate to invent good names and labels for anything that you will refer to more than once. Proper naming will save you a lot of writing effort because you will not have to repeat descriptions such as ‘*the third output of the second benchmarked method of the improved set,*’ instead you may introduce a

labeling that will allow you to say just something like ‘*output $M2^+ - 3$* ’. At the same time, this will reduce the risk that the reader will confuse the object with another one — for illustration, the long version of the previous example might very easily confuse with the second output of the third method. The same also applies to methods descriptions, algorithms, programs, testing datasets, theorems, use-cases, challenges and other things. As an example, ‘*the algorithm that organizes the potatoes into appropriate buckets*’ shortens nicely as ‘*the potato bucketer*’ and may be labeled as a procedure `BUCKETPOTATOES()`, and ‘*the issue where the robot crashes into a wall and takes significant time to return to the previous task*’ may be called just ‘*the crash-recovery lag*’.

Verbs Although English can express a whopping 65 base verb tenses and their variants, scientific literature often suppresses this complexity and uses only several basic tenses where the meaning is clearly defined. Typically, you state facts in present simple (‘*Theorem 1 proves that Gadget B works as intended.*’), talk about previous work and experiments done in past simple (‘*We constructed Gadget B from Gizmo C, which was previously prepared by Tinkerer et al.*’), and identify achieved results in present perfect (‘*We have constructed Technology T.*’). Avoid using future tense, except for sections that explicitly describe future work — as a typical mistake, if you state that the thesis *will* describe something in later chapters, you imply that the description is not present there yet.

Do not write sentences in passive voice, unless you explicitly need to highlight that something has passively subjected itself to an action. Active voice is more preferable in the theses because it clearly highlights the actors and their contributions — typically, ‘*you did it*’ instead of ‘*it was done*’ by a mysterious entity, which the reviewers rarely envision as yourself. Writing in active voice additionally benefits the explanation of complex processes: There, the word order forces you to identify the acting subject as the first word in the sentence, which further disambiguates how the individual process parts are triggered and ordered.

Try to avoid overusing gerunds (verbs that end with ‘-ing’). It is convenient to write shorter sentences by using gerunds as adjectives, but these are typically quite hard to understand because the readers may easily confuse the intended adjectives with verbs. If your sentence contains two gerunds close to each other, it may need a rewrite.

Scientific writing resources Consult the book by [glasman2010science](#) for more useful details and recommended terminology for writing about the scientific research. Very pragmatically, the book by [sparling1989english](#) describes many common mistakes that Czech and Slovak (and generally Slavic) writers make when writing English.

Chapter 2

More complicated chapter

After the reader gained sufficient knowledge to understand your problem in chapter 1, you can jump to your own advanced material and conclusions.

You will need definitions (see definition 1 below in section 2.1), theorems (theorem 1), general mathematics, algorithms (algorithm 1), and tables (table 2.1). Figures 2.1 and 3.1 show how to make a nice figure. See figure 2.2 for an example of TikZ-based diagram. Cross-referencing helps to keep the necessary parts of the narrative close — use references to the previous chapter with theory wherever it seems that the reader could have forgotten the required context. Conversely, it is useful to add a few references to theoretical chapters that point to the sections which use the developed theory, giving the reader easy access to motivating application examples.

See documentation of package `booktabs` for hints on typesetting tables. As a main rule, *never* draw a vertical line.

2.1 Example with some mathematics

Definition 1 (Triplet). *Given stuff X , Y and Z , we will write a triplet of the stuff as (X, Y, Z) .*

Theorem 1 (Car coloring). *All cars have the same color. More specifically, for any set of cars C , we have*

$$(\forall c_1, c_2 \in C) \text{ COLOUR}(c_1) = \text{COLOUR}(c_2).$$

Proof. Use induction on sets of cars C . The statement holds trivially for $|C| \leq 1$. For larger C , select 2 overlapping subsets of C smaller than $|C|$ (thus same-colored). Overlapping cars need to have the same color as the cars outside the overlap, thus also the whole C is same-colored. □

This is plain wrong though.

Column A	Column 2	Numbers	More
Asd	QWERTY	123123	–
Asd qsd 1sd	BAD	234234234	This line should be helpful.
Asd	INTERESTING	123123123	
Asd qsd 1sd	PLAIN WEIRD	234234234	–
Asd	QWERTY	123123	–
Asd qsd 1sd	GOOD	234234299	–
Asd	NUMBER	123123	–
Asd qsd 1sd	DANGEROUS	234234234	(no data)

Table 2.1 An example table. Table caption should clearly explain how to interpret the data in the table. Use some visual guide, such as boldface or color coding, to highlight the most important results (e.g., comparison winners).

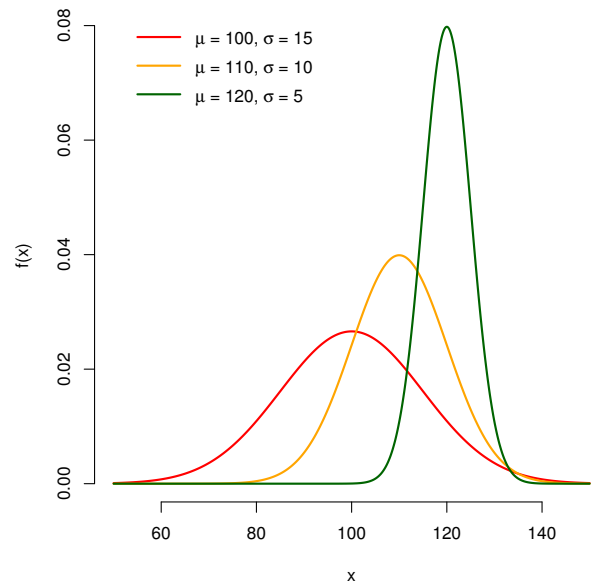
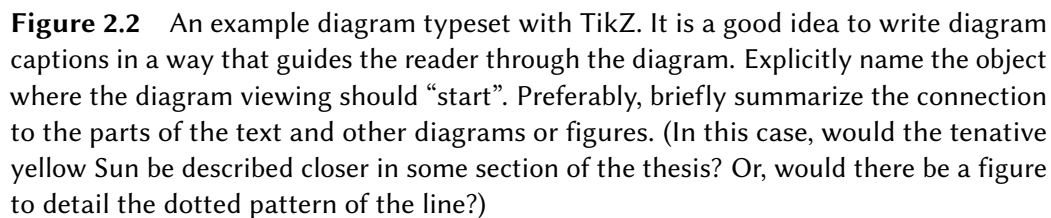


Figure 2.1 A figure with a plot, not entirely related to anything. If you copy the figures from anywhere, always refer to the original author, ideally by citation (if possible). In particular, this picture — and many others, also a lot of surrounding code — was taken from the example bachelor thesis of MFF, originally created by Martin Mareš and others.



```
function EXECUTEWITHHIGHPROBABILITY( $A$ )  
     $r \leftarrow$  a random number between 0 and 1  
     $\varepsilon \leftarrow 0.0000000000000000000000000000000000042$   
    if  $r \geq \varepsilon$  then  
        execute  $A$  ..... We discard the return value  
    else  
        print: Not today, sorry.  
    end if  
end function
```

2.2 Extra typesetting hints

Do not overuse text formatting for highlighting various important parts of your sentences. If an idea cannot be communicated without formatting, the sentence probably needs rewriting anyway. Imagine the thesis being read aloud as a podcast — the storytellers are generally unable to speak in boldface font.

Most importantly, do not overuse bold text, which is designed to literally **shine from the page** to be the first thing that catches the eye of the reader. More precisely, use bold text only for ‘navigation’ elements that need to be seen and located first, such as headings, list item leads, and figure numbers.

Use underline only in dire necessity, such as in the previous paragraph where it was inevitable to ensure that the reader remembers to never typeset boldface text manually again.

Use *emphasis* to highlight the first occurrences of important terms that the reader should notice. The feeling the emphasis produces is, roughly, “Oh my — what a nicely slanted word! Surely I expect it be important for the rest of the thesis!”

Finally, never draw a vertical line, not even in a table or around figures, ever. Vertical lines outside of the figures are ugly.

Chapter 3

Results and discussion

You should have a separate chapter for presenting your results (generated by the stuff described previously, in our case in chapter 2). Remember that your work needs to be validated rigorously, and no one will believe you if you just say that ‘it worked well for you’.

Instead, try some of the following:

- State a hypothesis and prove it statistically
- Show plots with measurements that you did to prove your results (e.g. speedup). Use either R and `ggplot`, or Python with `matplotlib` to generate the plots.¹ Save them as PDF to avoid printing pixels (as in figure 3.1).
- Compare with other similar software/theses/authors/results, if possible
- Show example source code (e.g. for demonstrating how easily your results can be used)
- Include a ‘toy problem’ for demonstrating the basic functionality of your approach and detail all important properties and results on that
- Include clear pictures of ‘inputs’ and ‘outputs’ of all your algorithms, if applicable

It is sometimes convenient (even recommended by some journals, including Cell) to name the results sub-sections so that they state what exactly has been achieved. Examples follow.

¹Honestly, the plots from `ggplot` look much better.

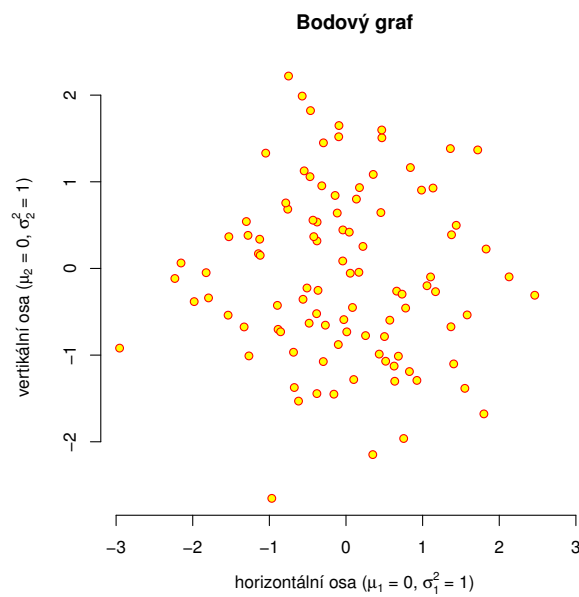


Figure 3.1 This caption is a friendly reminder to never insert figures “in text,” without a floating environment, unless explicitly needed for maintaining the text flow (e.g., the figure is small and developing with the text, like some of the centered equations, as in theorem 1). All figures *must* be referenced by number from the text (so that the readers can find them when they read the text) and properly captioned (so that the readers can interpret the figure even if they look at it before reading the text — reviewers love to do that).

3.1 SuperProgram is faster than OldAlgorithm

3.1.1 Scalability estimation

3.1.2 Precision of the results

3.2 Weird theorem is proven by induction

3.3 Amount of code reduced by CodeRedTool

3.3.1 Example

3.3.2 Performance on real codebases

3.4 NeuroticHelper improves neural network learning

3.5 Graphics and figure quality

No matter how great the text content of your thesis is, the pictures will always catch the attention first. This creates the very important first impression of the thesis contents and general quality. Crucially, that also decides whether the thesis is later read with joy, or carefully examined with suspicion.

Preparing your thesis in a way such that this first impression gets communicated smoothly and precisely helps both the reviewer and you: the reviewer will not have a hard time understanding what exactly you wanted to convey, and you will get a better grade.

Making the graphics ‘work for you’ involves doing some extra work that is often unexpected. At the same time, you will need to fit into graphics quality constraints and guidelines that are rarely understood before you actually see a bad example. As a rule of thumb, you should allocate at least the same amount of time and effort for making the figures look good as you would for writing, editing and correcting the same page area of paragraph text.

3.5.1 Visualize all important ideas

The set of figures in your thesis should be comprehensive and complete. For all important ideas, constructions, complicated setups and results there should be a visualization that the reader can refer to in case the text does not paint the ‘mental image’ sufficiently well. At the bare minimum, you should have at least 3

figures (roughly corresponding to the 3 chapters) that clearly and unambiguously show:

1. the context of the problem you are solving, optionally with e.g. question marks and exclamation marks placed to highlight the problems and research questions
2. the overall architecture of your solution (usually as a diagram with arrows, such as in figure 2.2, ideally with tiny toy examples of the inputs and outputs of each box),
3. the advancement or the distinctive property of your solution, usually in a benchmark plot, or as a clear demonstration and comparison of your results.

3.5.2 Make the figures comprehensible

The figures should be easily comprehensible. Surprisingly, that requires you to follow some common “standards” in figure design and processing. People are often used to a certain form of the visualizations, and (unless you have a very good reason) deviating from the standard is going to make the comprehension much more complicated. The common standards include the following:

- caption everything correctly, place the caption at an expectable position
- systematically label the plots with ‘main’ titles (usually in boldface, above the plot), plot axes, axis units and ticks, and legends
- lay out the diagrams systematically, ideally follow a structure of a bottom-up tree, a left-to-right pipeline, a top-down layered architecture, or a center-to-borders mindmap
- use colors that convey the required information correctly


Although many people carry some intuition for color use, achieving a really correct utilization of colors is often very hard without previous experience in color science and typesetting. Always remember that everyone perceives color hues differently, therefore the best distinction between the colors is done by varying lightness of the graphics elements (i.e., separating the data by dark vs. light) rather than by using hues (i.e., forcing people to guess which one of salmon and olive colors means “better”). Almost 10% of the population have their vision impaired by some form of color vision deficiency, most frequently by deuteranomaly that prevents interpretation of even the most ‘obvious’ hue differences, such as green vs. red. Finally, printed colors look surprisingly different from the on-screen

colors. You can prevent much of these problems by using standardized palettes and well-tested color gradients, such as the ones from ColorBrewer² and ViridisLite³. Check if your pictures still look good if converted to greyscale, and use a color deficiency simulator to check how the colors are perceived with deuteranomaly.

Avoid large areas of over-saturated and dark colors:

- under no circumstances use dark backgrounds for any graphical elements, such as diagram boxes and tables — use very light, slightly desaturated colors instead
- avoid using figures that contain lots of dark color (as a common example, heatmaps rendered with the ‘magma’ color palette often look like huge black slabs that are visible even through the paper sheet, thus making a dark smudge on the neighboring page)
- increase the brightness of any photos to match the average brightness of the text around the figure

Remember to test your figures on other people — usually, just asking ‘What do you think the figure should show?’ can help you debug many mistakes in your graphics. If they think that the figure says something different than what you planned, then most likely it is your figure what is wrong, not the understanding of others.

Finally, there are many magnificent resources that help you arrange your graphics correctly. The two books by Tufte [**tufte1990envisioning**, **tufte1983visual**] are arguably classics in the area. Additionally, you may find many interesting resources to help you with technical aspects of plotting, such as the ggplot-style ‘Fundamentals’ book by **wilke2019fundamentals**, and a wonderful manual for the TikZ/PGF graphics system by **tantau2015tikz** that will help you draw high-quality diagrams (like the one in figure 2.2).

3.6 What is a discussion?

After you present the results and show that your contributions work, it is important to *interpret* them, showing what they mean in the wider context of the thesis topic, for the researchers who work in the area, and for the more general public, such as for the users.

Separate discussion sections are therefore common in life sciences where some ambiguity in result interpretation is common, and the carefully developed

²<https://colorbrewer2.org>

³<https://sjmgarnier.github.io/viridisLite/>

intuition about the wider context is sometimes the only thing that the authors have. Exact sciences and mathematicians do not need to use the discussion sections as often. Despite of that, it is nice to position your output into the previously existing environment, answering:

- What is the potential application of the result?
- Does the result solve a problem that other people encountered?
- Did the results point to any new (surprising) facts?
- How (and why) is the approach you chose different from what the others have done previously?
- Why is the result important for your future work (or work of anyone other)?
- Can the results be used to replace (and improve) anything that is used currently?

If you do not know the answers, you may want to ask the supervisor. Also, do not worry if the discussion section is half-empty or thoroughly pointless; you may remove it completely without much consequence. It is just a bachelor thesis, not a world-saving avenger thesis.

Conclusion

In the conclusion, you should summarize what was achieved by the thesis. In a few paragraphs, try to answer the following:

- Was the problem stated in the introduction solved? (Ideally include a list of successfully achieved goals.)
- What is the quality of the result? Is the problem solved for good and the mankind does not need to ever think about it again, or just partially improved upon? (Is the incompleteness caused by overwhelming problem complexity that would be out of thesis scope, or any theoretical reasons, such as computational hardness?)
- Does the result have any practical applications that improve upon something realistic?
- Is there any good future development or research direction that could further improve the results of this thesis? (This is often summarized in a separate subsection called 'Future work'.)

This is quite common.

Bibliography

Appendix A

Using CoolThesisSoftware

Use this appendix to tell the readers (specifically the reviewer) how to use your software. A very reduced example follows; expand as necessary. Description of the program usage (e.g., how to process some example data) should be included as well.

To compile and run the software, you need dependencies XXX and YYY and a C compiler. On Debian-based Linux systems (such as Ubuntu), you may install these dependencies with APT:

```
apt-get install \  
  libsuperdependency-dev \  
  libanotherdependency-dev \  
  build-essential
```

To unpack and compile the software, proceed as follows:

```
unzip coolsoft.zip  
cd coolsoft  
./configure  
make
```

The program can be used as a C++ library, the simplest use is demonstrated in listing 1. A demonstration program that processes demonstration data is available in directory `demo/`, you can run the program on a demonstration dataset as follows:

```
cd demo/  
./bin/cool_process_data data/demo1
```

After the program starts, control the data avenger with standard WSAD controls.

Listing 1 Example program.

```
#include <CoolSoft.h>
#include <iostream>

int main() {
    int i;
    if(i = cool::ProcessAllData()) // returns 0 on error
        std::cout << i << std::endl;
    else
        std::cerr << "error!" << std::endl;
    return 0;
}
```
