DML

Infinity ω

$\emptyset \in \omega$

$A \in \omega \Rightarrow \{A, \{A\}\} \in \omega$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{j=0}^{n} ar^j = \begin{cases} \dfrac{ar^{n+1}-a}{r-1} & r \neq 1 \\ (n+1)\,a & r = 1 \end{cases}$$

Equivalence: reflexive, symmetric, transitive

Equality: equivalence satisfying Leibniz axioms: $x = y$ iff $\varphi(x) \Leftrightarrow \varphi(y)\ \forall \varphi$

Pre-order (quasi-order): reflexive, transitive

(Partial) order: anti-symmetric pre-order

Linear order: partial order where all element are comparible

$F: D \to R \qquad F \subseteq D \times R$

Domain, Range (co-domain)

support of F: $\{x \in D: F \text{ is defined on } x\}$

Image of F: $\{F(x),\ x \in D\}$

F is total: support = domain

onto (surjection): range = image

one-to-one (injection): one-to-one correspondence

into

bijection: total surjective injection

Reflexive $\forall x: xRx$

Anti-reflexive $\forall x: \neg(xRx)$

Symmetric $\forall x,y: xRy \to yRx$

Anti symmetric $\forall x,y: xRy \wedge yRx \to x = y$

Asymmetric $\forall x,y: xRy \to \neg(yRx)$

Transitive $\forall x,y,z: xRy \wedge yRz \to xRz$

Connex $\forall x,y\ \ xRy \vee yRx$

Transitive closure: of binary relation R on set X is smallest relation on X that contains R and is transitive

$R^{+} = \bigcup_{i \in \{1,2,\dots\}} R^{i}$ where

$R^{1} = R, \quad R^{i+1} = R.R^{i}$

λ-notation

Axiom of

Extensionality: 2 set equal if have same elements

Pairing: X & Y are sets, exists $\{X, Y\}$

Union: union over the elements of a set exists (is a set)

Separation any definable subclass of a set is a set

Choice: given any collection of bins, each containing at least one obj it is possible to make selection of exactly one obj from each bin

Order pair $(A,B) = \{A, \{A,B\}\}$

Cardinality $|A|$

same cardinality: bijection $A \to B$

$|A| < |B|$: injection $A \to B$ but not bijection $A \to B$

Probability space: (Ω, F, P) — Ω set.., $F \subset \Omega$.., P: probability...

$A_n^k = \dfrac{n!}{(n-k)!} \qquad C_n^k = \dfrac{n!}{k!(n-k)!}$

Permutation with repetitions $P_{n_1, n_2 \dots n_k} = \dfrac{n!}{n_1!\, n_2! \dots n_k!}$

Combinations with repetitions:

$$\tilde{C}_n^k = C_{n+k-1}^k = \frac{(n+k-1)!}{k!(n-1)!}$$

Digraphs isomorphism

$G_1 = (V_1, E_1) \quad G_2 = (V_2, E_2)$

Bijective func. $F: V_1 \to V_2$ is called isomorphism if $(v', v'') \in E_1$ $\Leftrightarrow (F(v'), F(v'')) \in E_2$

Connected / Disconnected graphs

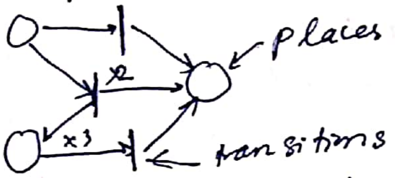
Multigraphs $\quad |V|$ order of G

Complete graphs – K_n $\quad$ Empty graphs – O_n

Simple graphs

Bipartite graphs (bigraphs)

Petrinet: bipartite directed (multi-)graphs



C_n – elementary loop $\quad P_n$ – elementary chain

Length of path $\quad$ Graphs diameter

Euler path: each edge exactly once
→ semi-Eulerian

Euler loop: → Eulerian

Euler theorem: ∃ Euler loop iff graphs is connected & degrees of all nodes are even

Graphs complement: $G = (V, E) \quad G^c = (V, E^c)$

Planar graphs: Euler's formular
$v - e + f = 2$
(connected plane graphs)

Forest → $e \leq 3v - 6 \quad (v \geq 3)$

Terminal node: degree = 1

Adjacency matrix

Kuratowski's theorem

subdivision: result of inserting vertices into edges

Finite graphs is planar iff doesn't contain subgraphs that is subdivision of K_5 or $K_{3,3}$

Forest: loop-free (acyclic)

Tree: 2 ve any vertices are connected by exactly 1 path: $v = e + 1$

Linear homogeneous recurrence relation

$$a_n = c_1 a_{n-1} + c_2 a_{n-2} + \dots + c_k a_{n-k}$$

Characteristic equation

$$r^k - c_1 r^{k-1} - c_2 r^{k-2} - \dots - c_{k-1} r - c_k = 0$$

has t distinct roots $r_1, r_2, \dots r_t$ with multiplicities $m_1, m_2, \dots m_t$ respectively

$m_1 + m_2 + \dots + m_t = k \quad (m_i \geq 1)$

$$\Rightarrow a_n = (\alpha_{1,0} + \alpha_{1,1} n + \dots + \alpha_{1,m_1-1} n^{m_1-1}) r_1^n + (\alpha_{2,0} + \alpha_{2,1} n + \dots + \alpha_{2,m_2-1} \cdot n^{m_2-1}) r_2^n + \dots + (\alpha_{t,0} + \alpha_{t,1} n + \dots + \alpha_{t,m_t-1} n^{m_t-1}) r_t^n$$

Linear non-homogeneous recurrence relation

$$a_n = c_1 a_{n-1} + \dots + c_k a_{n-k} + F(n)$$

$$\to \{a_n^{(p)} + a_n^{(h)}\}$$

$$F(n) = (b_t n^t + b_{t-1} n^{t-1} + \dots + b_1 n + b_0) s^n$$

- s is root of characteristic equation with multiplicity m:
$$a_n^{(p)} = n^m (p_t n^t + \dots + p_1 n + p_0) s^n$$
- s is not a root of …
$$a_n^{(p)} = (p_t n^t + p_{t-1} n^{t-1} + \dots + p_0) s^n$$

Boolean Func.

$B = \{0, 1\}$

$B^n = \{(x_1, x_2, \dots x_n) \mid x_i \in B \text{ for } 1 \leq i \leq n\}$

x_i : Boolean value

$f: B^n \to B$ Boolean func of degree n

Boolean Expressions

Equivalent Expressions

Dual of Boolean expression:
sum ↔ products
$0 \leftrightarrow 1$

Proposition: T or F

precedence

- negation of p: $\neg p$ (not)
- conjunction of p and q: $p \wedge q$ (and)
- disjunction of p and q: $p \vee q$ (or)
- exclusive or of p & q: $p \oplus q$
- implication: $p \to q$ (F when T → F)
- bi-implication: $p \leftrightarrow q$ (T when T↔T or F↔F)

$\neg \quad \wedge \quad \vee \quad \to \quad \leftrightarrow$ (precedence)

Tautology (always T) $\quad$ Satisfiable
$\equiv$

chapter 8, 10, 11 & 12 $\quad$ chapter 3 § 17