$$(\vec{e_1}', \vec{e_2}', \vec{e_3}') = (\vec{e_1}, \vec{e_2}, \vec{e_3})\, A_{old\text{-}new}$$

$$\underset{M(O)}{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}} = \underset{O'(O)}{\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}} + A_{old\text{-}new} \underset{M(O')}{\begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix}}$$

Symmetric matrix $A = A^T$

skew symmetric matrix: $a_{ij} = -a_{ji}$

$A = -A^T$

$(AB)^T = B^T . A^T$

$(AB)^{-1} = B^{-1} . A^{-1}$

$(A^m)^{-1} = (A^{-1})^m$, $m \in N^+$

$\det(A) = 0 \rightarrow A$ is singular

$\det(A) = \sum a_{ij} (-1)^{i+j} M_{ij}$

Cramer's rule: $x_j = \dfrac{\det(A_j)}{\det(A)}$ ($\det A \neq 0$) replace col j with the right

Hệ pt tuyến tính thuần nhất có nghiệm k tầm thường $\Leftrightarrow \det(A) = 0$

$\sum_{i=1}^{n} \alpha_i \vec{a_i} = \vec{0}$, $\sum_{i=0}^{n} \alpha_i^2 \neq 0$

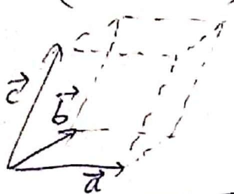
$\Rightarrow \vec{a_1}, \dots, \vec{a_n}$ are linearly dependent

orthonormal

Triple (scalar) product

$$\boxed{V = |\vec{a} . (\vec{b} \times \vec{c})| = (\vec{a}, \vec{b}, \vec{c})}$$

$$(\vec{a}, \vec{b}, \vec{c}) = \begin{vmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{vmatrix} (\vec{e_1}, \vec{e_2}, \vec{e_3})$$



$$\boxed{\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b} . (\vec{a} . \vec{c}) - \vec{c} (\vec{a} . \vec{b})}$$

$$(\vec{a} \times \vec{b}) . (\vec{c} \times \vec{d}) = \begin{vmatrix} \vec{a}\vec{c} & \vec{a}\vec{d} \\ \vec{b}\vec{c} & \vec{b}\vec{d} \end{vmatrix} = (\vec{a}, \vec{b}, \vec{c} \times \vec{d})$$

$(\vec{a}, \vec{b}, \vec{a} \times \vec{b})$: basis

$S_{ABC} = \frac{1}{2} |\vec{a} \times \vec{b}|$

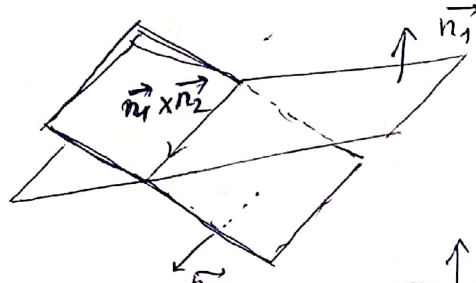
$\vec{a}, \vec{b}, \vec{c}$ form a basis

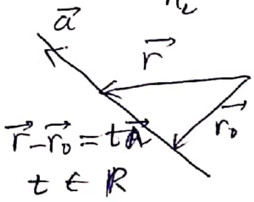
$\Rightarrow \vec{b} \times \vec{c}, \vec{c} \times \vec{a}, \vec{a} \times \vec{b}$: basis

prove: $\alpha \vec{b} \times \vec{c} + \beta \vec{c} \times \vec{a} + \gamma \vec{a} \times \vec{b} = \vec{0} \;|.\vec{a}$

$\Rightarrow \alpha = 0 \dots$ ⊠

$$(\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}) = (\vec{a}, \vec{b}, \vec{c})^2$$

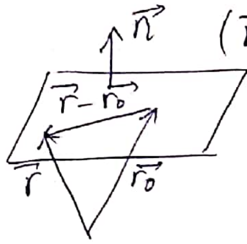




$\vec{r} - \vec{r_0} = t\vec{a}$

$t \in R$

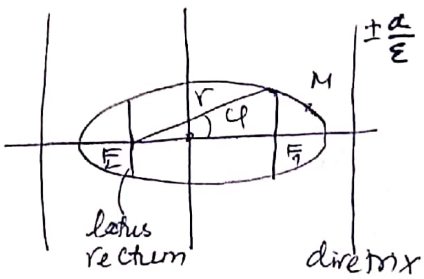
$(\vec{r} - \vec{r_0}) . \vec{n} = 0$



Skew lines: $\vec{r} = \vec{r_1} + \vec{a_1} t$

$\vec{r} = \vec{r_2} + \vec{a_2} t$

$$\rho = \frac{|(\vec{r_2} - \vec{r_1}, \vec{a_1}, \vec{a_2})|}{|\vec{a_1} \times \vec{a_2}|}$$



canonical form

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a > b > 1$

$c^2 = a^2 - b^2$, $\varepsilon = \frac{c}{a} < 1$

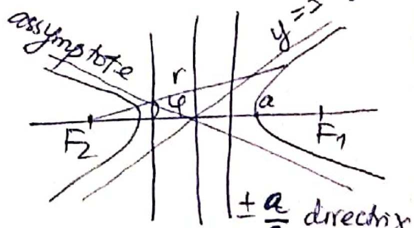
$MF = a \pm \varepsilon x$

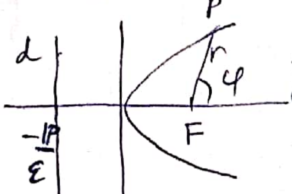
$\frac{dis(M, F_1)}{dis(M, d_1)} = \varepsilon$

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$c^2 = a^2 + b^2$

$\varepsilon = \frac{c}{a} > 1$





$y^2 = 2px$

$\frac{dis(P, F)}{dis(P, d)} = 1$

Conics: $r = \frac{p}{1 - \varepsilon \cos\varphi}$

$Ax^2 + 2Bxy + Cy^2 + 2Dx + 2Ey + F = 0$

$A^2 + B^2 + C^2 \neq 0$

$T_{old\text{-}new} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$

$C = A \Rightarrow \alpha = \frac{\pi}{4}$

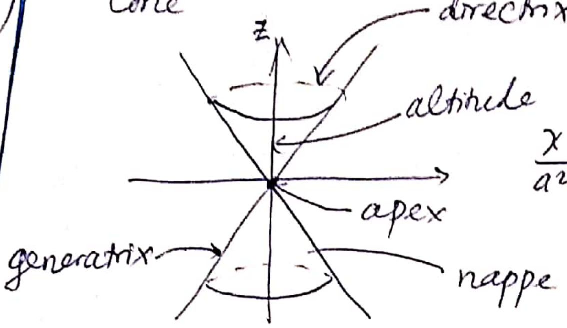
$C \neq A \Rightarrow \tan 2\alpha = \frac{2B}{A-C}$

$B^2 - AC$
- < 0: Ellipse
- > 0: Hyperbola
- $= 0$: Parabola

$P(x,y,z)$ is homogeneous if $\exists \alpha$

$P(tx, ty, tz) = t^\alpha P(x,y,z)$

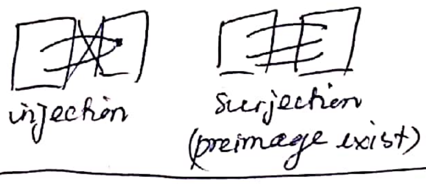
Cone

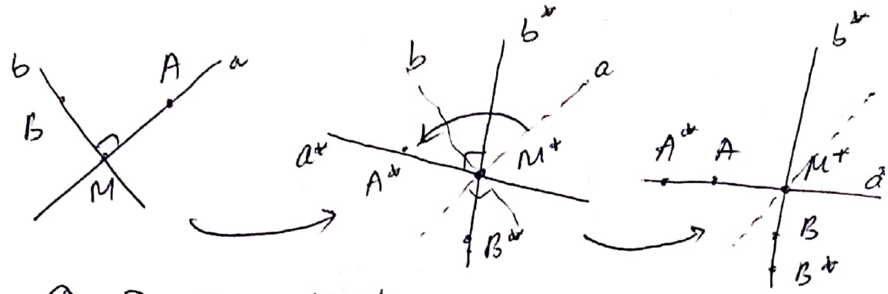


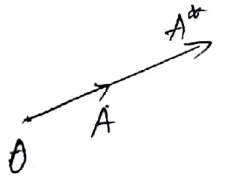
$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$

nappe (one half of a cone)

Bijection (one-to-one)



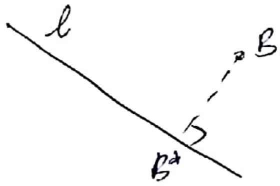




Homothety:
$\vec{OA^*} = \lambda \vec{OA}$

1. Transform MM^*
2. Rotate around M^* (colinear)
3. Additional reflection if needed
4. Dilation from a^* by factor $\frac{B^*M^*}{BM^*}$
5. Dilation from b^* by $\frac{A^*M^*}{AM^*}$

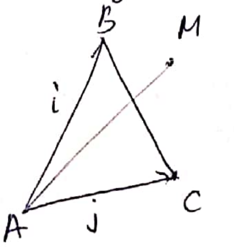
orthogonal projection



Rigid transformation

$f(A) f(B) = AB \ \forall A, B$

ex. $f(A) = B$, $f(B) = C$, $f(C) = A$



$A, \vec{i}, \vec{j}$

$f(A) = B = i$

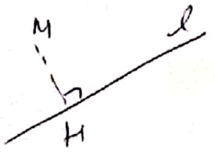
$f(\vec{i}) = f(\vec{AB}) = \vec{BC}$
$= \vec{j} - \vec{i}$

$f(\vec{j}) = f(\vec{AC}) = \vec{BA} = -\vec{i}$

$\vec{AM} = x\vec{i} + y\vec{j}$

$\vec{BM^*} = f(\vec{AM}) = x f(\vec{i}) + y f(\vec{j})$

$\vec{AM^*} = \vec{AB} + \vec{BM^*} = \dots$



dilation from l of factor λ (> 0)

$\vec{HM^*} = \lambda \vec{HM}$

$\lambda < 1$: Contraction

Quadrics

Cylinder
- Elliptic $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
- Hyperbolic $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
- Parabolic $y^2 = 2px$

Ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

Elliptic paraboloid $z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$

Hyperbolic paraboloid $z = \frac{x^2}{a^2} - \frac{y^2}{b^2}$

Hyperboloid
- 2 sheets $\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$
- 1 sheet $\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

Cone $z^2 = \frac{x^2}{a^2} + \frac{y^2}{b^2}$
(homogeneous equation)

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