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Trend Inflation, Indexation, and Inflation Persistence in the New Keynesian Phillips Curve

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Purely forward-looking versions of the New Keynesian Phillips curve (NKPC) generate too little inflation persistence. Some authors add ad hoc backward-looking terms to address this shortcoming. We hypothesize that inflation persistence results mainly from variation in the long-run trend component of inflation, which we attribute to shifts in monetary policy. We derive a version of the NKPC that incorporates a time-varying inflation trend and examine whether it explains the dynamics of inflation. When drift in trend inflation is taken into account, a purely forward-looking version of the model fits the data well, and there is no need for backward-looking components. (*JEL* E12, E31, E52)

In this paper we consider the extent to which Guillermo Calvo's (1983) model of nominal price rigidities can explain inflation dynamics without relying on arbitrary backward-looking terms. In its baseline formulation, the Calvo model leads to a purely forward-looking New Keynesian Phillips curve (NKPC): inflation depends on the expected evolution of real marginal costs. However, purely forward-looking models are deemed inconsistent with empirical evidence of significant inflation persistence (e.g., see Jeff Fuhrer and George Moore 1995). Accordingly, a number of authors have added backward-looking elements to enhance the degree of inflation persistence in the model and to provide a better fit with aggregate data. Lags of inflation are typically introduced by postulating some form of price indexation (e.g., see Lawrence Christiano, Martin Eichenbaum, and Charles Evans 2005) or rule-of-thumb behavior (e.g., see Jordi Galí and Mark Gertler 1999). These mechanisms have been criticized because they lack a convincing microeconomic foundation. Indexation is further criticized because it is inconsistent with the observation that many prices do indeed remain constant in monetary terms for several periods (e.g., see Mark Bils and Peter J. Klenow 2004; Emi Nakamura and Jon Steinsson 2007).

Here we propose an alternative interpretation of the apparent need for a structural persistence term. We stress that to understand inflation persistence it is important to model variation in trend inflation. For the United States, a number of authors model trend inflation as a driftless random walk (e.g., see Cogley and Thomas J. Sargent 2005a; Peter N. Ireland 2007; James H. Stock and Mark W. Watson 2007). Thus trend inflation contributes a highly persistent component to

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actual inflation. But this persistence arises from a source that is quite different from any intrinsic persistence implied by the dynamics of price adjustment. We indeed hypothesize that apparent structural persistence is an artifact of the interaction between drift in trend inflation and nonlinearities in the Calvo model of price adjustment. This interaction gives rise to autocorrelation in inflation that might be mistakenly attributed to intrinsic inflation persistence.

In general equilibrium, trend inflation is determined by the long-run target in the central bank's policy rule, and drift in trend inflation should ultimately be attributed to shifts in that target. Many existing versions of the NKPC abstract from this source of variation and attempt to model inflation persistence purely as a consequence of intrinsic dynamics.

In this paper we extend the Calvo model to incorporate variation in trend inflation. We log-linearize the equilibrium conditions of the model around a shifting steady state associated with a time-varying inflation trend. The resulting representation is a log-linear NKPC with time-varying coefficients. To estimate the parameters of the pricing model, our econometric approach exploits the cross-equation restrictions that the model imposes on a vector autoregression (VAR) for inflation, unit labor costs, and other variables. Following Sbordone (2002, 2006), we adopt a two-step estimation procedure. In step one, we estimate a reduced-form VAR, characterized by drifting parameters and stochastic volatility, as in Cogley and Sargent (2005a). Then we estimate the structural parameters of the pricing model by trying to satisfy the cross-equation restrictions implied by the theoretical model.

Our estimates point to four conclusions. First, our estimates of the backward-looking indexation parameter concentrate on zero. Indexation appears to be unnecessary once drift in trend inflation is taken into account. Second, the model provides a good fit to the inflation gap, and there is little evidence against the model's cross-equation restrictions. Third, our estimates of the frequency of price adjustment are broadly consistent with those emerging from micro-level studies. Finally, variation in trend inflation alters the relative weights on current and future marginal cost in the NKPC. As trend inflation increases, the weight on forward-looking terms is enhanced, while that on current marginal cost is muted.

The rest of the paper is organized as follows. The next section extends the Calvo model. Section II describes the econometric approach and characterizes the cross-equation restrictions. Sections III and IV describe the first- and second-stage estimates, respectively, and Section V discusses the model's implications for NKPC coefficients. Section VI concludes with suggestions for future research.

I. A Calvo Model with Drifting Trend Inflation

The NKPC is typically obtained by approximating the equilibrium conditions of the Calvo pricing model around a steady state with zero inflation. The model therefore carries implications for small fluctuations of inflation around zero.

Our objective is to characterize the model dynamics across periods with different rates of trend inflation, which we associate with different policy regimes. Hence we depart from traditional derivations of the Calvo model by allowing for a shifting trend-inflation process, which we model as a driftless random walk. As a consequence, when we approximate the nonlinear equilibrium conditions of the model, we take the log-linear approximation, in each period, around a steady state associated with a time-varying rate of trend inflation.[1] This modification brings with it another important departure from the standard assumptions, which we discuss in more detail below. When trend inflation varies over time, we have to take a stand about the evolution

[1] As usual, this approximation is valid only for small deviations of the variables from their steady-state value.

of agents' expectations: we therefore replace the assumption of rational expectation with one of subjective expectations and make appropriate assumptions on how these expectations evolve over time.

The importance of nonzero trend inflation for the Calvo model was first demonstrated by Guido Ascari (2004) and has been further studied by Jean-Guillaume Sahuc (2006) and Hasan Bakhshi et al. (2007), among others. They show that the level of trend inflation affects the dynamics of the Phillips curve, unless a sufficient degree of indexation is allowed. They also demonstrate that a solution to the optimal pricing problem does not exist when trend inflation exceeds a certain threshold. In addition, Michael T. Kiley (2007) and Ascari and Tiziano Ropele (2007) analyze the normative implications of positive trend inflation for monetary policy. None of these contributions, however, investigates the nature of the movements in trend inflation, nor provides an empirical estimation. We instead take the model to the data and estimate both the evolution of trend inflation and the parameters of the Calvo model, which we take to be the primitives of the NKPC.

Trend inflation and Calvo parameters in turn control the evolution of the NKPC coefficients, which are ultimately those of interest to policymakers. Time-varying coefficients distinguish our specification of the NKPC from the relationships embedded in most DSGE models. Even when allowing for a time-varying inflation target, estimated models typically do not carry implications of trend inflation fluctuations for the NKPC specification because they assume full indexation either to past inflation, to current trend inflation, or to a weighted average of the two.[2]

The rest of this section summarizes the assumptions underlying the generalized Calvo model and derives an extended version of the NKPC. Details of the derivation are in Appendix A.

As in the standard Calvo model, our generalization features monopolistic competition and staggered price setting. At any time t, only a fraction $(1 - \alpha)$ of firms, with $0 < \alpha < 1$, can reset prices optimally, while the remaining firms index their price to lagged inflation. The optimal nominal price X_t maximizes expected discounted future profits,[3]

$$\max_{X_t} \tilde{E}_t \sum_j \alpha^j \{Q_{t,t+j} \mathcal{P}_{t+j}(i)\}, \tag{1}$$

where $\mathcal{P}_{t+j} = \mathcal{P}(X_t \Psi_{tj}, P_{t+j}, Y_{t+j}(i), Y_{t+j})$, subject to the demand constraint

$$Y_{t+j}(i) = Y_{t+j}\left(\frac{X_t \Psi_{tj}}{P_{t+j}}\right)^{-\theta}. \tag{2}$$

The operator $\tilde{E}_t$ denotes subjective expectations formed with time t information. The variable $P_t \equiv \left[\int_0^1 P_t(i)^{1-\theta}\, di\right]^{1/(1-\theta)}$ is the aggregate price level, $Y_t \equiv \left[\int_0^1 Y_t(i)^{(\theta-1)/\theta}\, di\right]^{\theta/(\theta-1)}$ measures aggregate real output, and $P_t(i)$ and $Y_t(i)$ represent firm i's nominal price and output, respectively. $Q_{t,t+j}$ is a nominal discount factor between time t and $t + j$, $\theta \in [1, \infty)$ is the Dixit-Stiglitz elasticity of substitution among differentiated goods, and $X_t \Psi_{tj}/P_{t+j}$ is the relative price at $t + j$ of the firms that set price at t. The variable Ψ_{tj}, defined as

[2] See, for example, the DSGE models of Malin Adolfson et al. (2007), Ireland (2007), Frank Schorfheide (2005), and Frank Smets and Rafael Wouters (2003).

[3] Since each firm that changes prices solves the same problem, this price is the same for all the firms and therefore need not be indexed by i.

(3)
$$\Psi_{tj} = \begin{cases} 1 & j = 0 \\ \prod_{k=0}^{j-1} \Pi_{t+k}^{\rho} & j \geq 1 \end{cases},$$

captures the fact that individual firm prices that are *not* set optimally evolve according to

(4)
$$P_t(i) = \Pi_{t-1}^{\rho} P_{t-1}(i),$$

where $\Pi_t = P_t/P_{t-1}$ is time t gross rate of inflation and $\rho \in [0,1]$ measures the degree of indexation.

The firm's first-order condition is

(5)
$$\tilde{E}_t \sum_{j=0}^{\infty} \alpha^j Q_{t,t+j} Y_{t+j} P_{t+j}^{\theta} \Psi_{tj}^{1-\theta} \left(X_t - \frac{\theta}{\theta - 1} MC_{t+j,t} \Psi_{tj}^{-1} \right) = 0,$$

where $MC_{t+j,t}$ is the nominal marginal cost at $t + j$ of the firm that last reoptimized its price at t. Since we assume immobile capital, this cost differs from average marginal cost at time $t + j$, MC_{t+j}, creating a form of strategic complementarity.[4] Our assumptions imply that aggregate prices evolve as

(6)
$$P_t = \left[(1 - \alpha) X_t^{1-\theta} + \alpha \left(\Pi_{t-1}^{\rho} P_{t-1} \right)^{1-\theta} \right]^{1/(1-\theta)}.$$

In what follows, we denote the optimizing firms' relative price by $x_t \equiv X_t/P_t$ and gross trend inflation by $\overline{\Pi}_t$. We also define the stationary variables $\tilde{\Pi}_t = \Pi_t/\overline{\Pi}_t$, $g_t^{\bar{\pi}} = \overline{\Pi}_t/\overline{\Pi}_{t-1}$, $g_t^y = Y_t/Y_{t-1}$, and $\tilde{x}_t = x_t/\bar{x}_t$, where a bar over a variable indicates its value in steady state. We appropriately transform conditions (5) and (6) to express them in terms of these stationary variables.

Evaluating the resulting expressions in steady state, we derive a restriction between trend inflation and steady-state marginal cost,

(7)
$$\left(1 - \alpha \overline{\Pi}_t^{(1-\rho)(\theta-1)}\right)^{(1+\theta\omega)/(1-\theta)} \left[\frac{1 - \alpha \overline{qg}^y (\overline{\Pi}_t)^{\theta(1+\omega)(1-\rho)}}{1 - \alpha \overline{qg}^y (\overline{\Pi}_t)^{(\theta-1)(1-\rho)}} \right] = (1 - \alpha)^{(1+\theta\omega)/(1-\theta)} \frac{\theta}{\theta - 1} \overline{mc}_t,$$

where $q_{t,t+j} = Q_{t,t+j}(P_{t+j}/P_t)$ denotes a real discount factor. The parameter ω measures the extent of strategic complementarity. We then log-linearize the equilibrium conditions around a steady state characterized by a shifting trend inflation and, with usual manipulations, derive a version of the NKPC, which can be written in a familiar form as[5]

[4] The specific relation between firms and aggregate marginal costs is in equation (29) of Appendix A. Strategic complementarity reduces the aggregate price adjustment even when the fraction of sticky prices is small. For further discussion of strategic complementarities, see Michael Woodford (2003, ch. 3).

[5] A technical issue arises here, because multistep expectations are difficult to evaluate when parameters drift. We invoke an approximation that is standard in the macro learning literature (e.g., see George W. Evans and Seppo Honkapohja 2001): we assume that agents treat drifting parameters as if they would remain constant at the current level going forward in time. David M. Kreps (1998) refers to this assumption as an "anticipated-utility" model, and he recommends it as a way to model bounded rationality. Cogley and Sargent (2006) defend it as an approximation to Bayesian forecasting and decision making in high-dimensional state spaces. That approximation is very good in models that assume certainty equivalence. Our formulation implicitly assumes certainty equivalence because we log-linearize the firm's first-order conditions.

$$\hat{\pi}_t = \tilde{\rho}_t(\hat{\pi}_{t-1} - \hat{g}_t^{\bar{\pi}}) + \zeta_t \widehat{mc}_t + b_{1t}\tilde{E}_t\hat{\pi}_{t+1} + b_{2t}\tilde{E}_t\sum_{j=2}^{\infty}\varphi_{1t}^{j-1}\hat{\pi}_{t+j}$$
$$+ b_{3t}\tilde{E}_t\sum_{j=0}^{\infty}\varphi_{1t}^{j}\big(\hat{Q}_{t+j,t+j+1} + \hat{g}^{y}_{t+j+1}\big) + u_t. \quad (8)$$

Hatted variables denote log-deviations of stationary variables from their steady-state values.[6] An error term u_t is included to account for the fact that this equation is an approximation and to allow for other possible misspecifications. In what follows, we assume that u_t is a white noise process. We discuss later the validity of this assumption.

This equation differs from conventional versions of the NKPC in two respects. First, a number of additional variables appear on the right-hand side of (8). These include innovations to trend inflation $\hat{g}_t^{\bar{\pi}}$, higher-order leads of expected inflation, and terms involving the discount factor $\hat{Q}_t$ and real output growth $\hat{g}_t^y$. Excluding these variables when estimating traditional Calvo equations would result in omitted-variable bias on the coefficients on marginal cost and lagged inflation if the omitted terms are correlated with those variables.

Second, the coefficients $\tilde{\rho}_t$, ζ_t, b_{1t}, b_{2t}, b_{3t}, and φ_{1t} are nonlinear functions of trend inflation and the parameters of the pricing model α, ρ, θ, and ω (their exact expressions are given in equation (49) of Appendix A). When trend inflation drifts, the coefficients of equation (8) also drift (provided $\rho \neq 1$), even if the underlying Calvo parameters are constant. In other words, although α, ρ, θ, and ω might be invariant to shifts in trend inflation, the NKPC parameters $\tilde{\rho}_t$, ζ_t, b_{1t}, b_{2t}, b_{3t}, and φ_{1t} are not. In particular, higher trend inflation implies a lower weight on current marginal cost and a greater weight on expected future inflation.

The standard NKPC emerges as a special case when steady-state inflation is zero or when there is full indexation ($\rho = 1$). In those cases, $b_{2t} = b_{3t} = 0$, while the other coefficients collapse to those of the standard model. Another popular specification is also nested in equation (8). If one assumes that nonoptimized prices are fully indexed to a mixture of current trend inflation and one-period lagged inflation, the equation collapses to a form similar to the traditional NKPC, with constant coefficients and no extra forward-looking terms. In that case, a traditional NKPC formulation can be obtained simply by redefining the inflation gap as $\hat{\pi}_t = \pi_t - \rho\pi_{t-1} - (1-\rho)\bar{\pi}_t$.

II. Econometric Approach

Our objective is to estimate the underlying parameters of the Calvo model, α, ρ, and θ, which govern key behavioral attributes involving the frequency of price adjustment, the extent of indexation to past inflation, and the elasticity of demand. Combined with an evolving trend inflation, these parameters allow us to trace a time path for the drifting coefficients of the NKPC.

Our econometric approach exploits a set of cross-equation restrictions between the parameters of the Calvo model and those of a reduced-form vector autoregression with drifting parameters. That the reduced-form VAR has drifting parameters follows from our assumption that trend inflation drifts. In our model, the NKPC coefficients depend on $\bar{\Pi}_t$. Hence, drift in trend inflation induces drift in these coefficients. It follows that the reduced form of any structural model containing our version of the NKPC also has time-varying parameters. Among other things,

[6] Specifically, $\hat{\pi}_t = \ln(\Pi_t/\bar{\Pi}_t)$, $\widehat{mc}_t = \ln(mc_t/\overline{mc}_t)$, $\hat{Q}_{t,t+1} = \ln(Q_{t,t+1}/\bar{Q}_{t,t+1})$, $\hat{g}_t^{\pi} = \ln(\bar{\Pi}_t/\bar{\Pi}_{t-1})$, and $\hat{g}_t^y = \ln(g_t^y/\bar{g}^y)$. In the derivation we use the fact that the discount factor between time t and time $t+j$ is $Q_{t,t+j} = \Pi_{k=0}^{j-1}Q_{t+k,t+k+1}$.

we use this VAR to construct a measure of trend inflation and to represent agents' subjective beliefs.[7]

If inflation is indeed determined in accordance with the NKPC, the VAR should also satisfy a collection of nonlinear cross-equation restrictions. These are embedded in two relationships derived in the previous section, one involving the cyclical components of inflation and marginal cost (equation (8)) and the other connecting the evolution of steady-state values (equation (7)). These relations involve nonlinear combinations of the underlying parameters of the Calvo model (see definition (49) in Appendix A), which we collect in a vector $\boldsymbol{\psi} = [\alpha, \theta, \rho, \omega]'$.

To derive the cross-equation restrictions, we consider first the case where the VAR has constant parameters and then show its extension to the case of a VAR with random coefficients.

Suppose the joint representation of the vector time series $\mathbf{x}_t = (\pi_t, mc_t, Q_t, g_t^y)'$ is a VAR(p). Then, defining a vector $\mathbf{z}_t = (\mathbf{x}_t, \mathbf{x}_{t-1}, \ldots, \mathbf{x}_{t-p+1})'$, we can write the law of motion of $\mathbf{z}_t$ in companion form as

$$\mathbf{z}_t = \boldsymbol{\mu} + \mathbf{A}\mathbf{z}_{t-1} + \boldsymbol{\varepsilon}_{\mathbf{z}t}. \tag{9}$$

If the NKPC model is correct, reduced-form and structural forecasts of the inflation gap should coincide. The reduced-form conditional expectation of $\hat{\pi}_t$ is

$$\tilde{E}(\hat{\pi}_t | \hat{\mathbf{z}}_{t-1}) = \mathbf{e}_\pi' \mathbf{A} \hat{\mathbf{z}}_{t-1}, \tag{10}$$

where $\mathbf{e}_k$ represents a selection vector that picks up variable k in vector $\mathbf{z}_t$ and $\hat{\mathbf{z}}_t = \mathbf{z}_t - \boldsymbol{\mu}_\mathbf{z}$, where $\boldsymbol{\mu}_\mathbf{z} = (\mathbf{I} - \mathbf{A})^{-1}\boldsymbol{\mu}$. Similarly, the conditional expectation of the inflation gap from the NKPC is

$$\begin{aligned} \tilde{E}(\hat{\pi}_t | \hat{\mathbf{z}}_{t-1}) = {} & \tilde{\rho}\mathbf{e}_\pi' \hat{\mathbf{z}}_{t-1} + \zeta \mathbf{e}_{mc}' \mathbf{A}\hat{\mathbf{z}}_{t-1} + b_1 \mathbf{e}_\pi' \mathbf{A}^2 \hat{\mathbf{z}}_{t-1} + b_2 \mathbf{e}_\pi' \varphi_1 (\mathbf{I} - \varphi_1 \mathbf{A})^{-1} \mathbf{A}^3 \hat{\mathbf{z}}_{t-1} \\ & + b_3 (\mathbf{e}_Q' (\mathbf{I} - \varphi_1 \mathbf{A})^{-1} \mathbf{A} + \mathbf{e}_y' (\mathbf{I} - \varphi_1 \mathbf{A})^{-1} \mathbf{A}^2) \hat{\mathbf{z}}_{t-1}. \end{aligned} \tag{11}$$

After equating the two and imposing that they hold for all realizations of $\mathbf{z}_t$, we obtain a vector of nonlinear cross-equation restrictions involving the parameters of the Calvo model $\boldsymbol{\psi}$ and the VAR parameters $\boldsymbol{\mu}$ and $\mathbf{A}$:

$$\begin{aligned} \mathbf{e}_\pi' \mathbf{A} = {} & \tilde{\rho}\mathbf{e}_\pi' \mathbf{I} + \zeta \mathbf{e}_{mc}' \mathbf{A} + b_1 \mathbf{e}_\pi' \mathbf{A}^2 + b_2 \mathbf{e}_\pi' \varphi_1 (\mathbf{I} - \varphi_1 \mathbf{A})^{-1} \mathbf{A}^3 \\ & + b_3 (\mathbf{e}_Q' (\mathbf{I} - \varphi_1 \mathbf{A})^{-1} \mathbf{A} + \mathbf{e}_y' (\mathbf{I} - \varphi_1 \mathbf{A})^{-1} \mathbf{A}^2) \\ \equiv {} & g(\boldsymbol{\mu}, \mathbf{A}, \boldsymbol{\psi}), \end{aligned} \tag{12}$$

or

$$\mathsf{F}_1(\boldsymbol{\mu}, \mathbf{A}, \boldsymbol{\psi}) = \mathbf{e}_\pi' \mathbf{A} - g(\boldsymbol{\mu}, \mathbf{A}, \boldsymbol{\psi}) = 0. \tag{13}$$

The parameters must also satisfy the steady-state restriction (7), which we rewrite as

[7] The assumption that agents form expectations with a forecasting VAR is common in the learning literature. The forecast is a "perceived" law of motion (e.g., Fabio Milani 2006), and its coefficients, as ours, evolve over time. In the learning literature the time drift in the parameters is interpreted as updating of beliefs when more observations become available.

$$\mathsf{F}_2(\boldsymbol{\mu},\mathbf{A},\boldsymbol{\psi}) = \left(1 - \alpha\overline{\Pi}_t^{(1-\rho)(\theta-1)}\right)^{(1+\theta\omega)/(1-\theta)}\left[\frac{1 - \alpha\overline{qg}^{y}(\overline{\Pi}_t)^{\theta(1+\omega)(1-\rho)}}{1 - \alpha\overline{qg}^{y}(\overline{\Pi}_t)^{(\theta-1)(1-\rho)}}\right] - (1-\alpha)^{(1-\theta)/(1+\theta\omega)}\frac{\theta}{\theta-1}\overline{mc}_t = 0, \tag{14}$$

where $\overline{\Pi}_t$ and $\overline{mc}_t$ are the steady-state values of gross inflation and real marginal cost, respectively, implied by the VAR. We consolidate these two moment conditions by defining $\mathsf{F}(\boldsymbol{\mu},\mathbf{A},\boldsymbol{\psi}) = (\mathsf{F}_1' \, \mathsf{F}_2')'$. If the model is true, there exist values of $\boldsymbol{\mu},\mathbf{A},\boldsymbol{\psi}$ that set $\mathsf{F}(\boldsymbol{\mu},\mathbf{A},\boldsymbol{\psi}) = \mathbf{0}$.

With drifting parameters, we modify the previous formulas by adding time subscripts to the companion form,

$$\mathbf{z}_t = \boldsymbol{\mu}_t + \mathbf{A}_t\mathbf{z}_{t-1} + \boldsymbol{\varepsilon}_{\mathbf{z}t}, \tag{15}$$

and appropriately redefining the function F as $\mathsf{F}_t(\boldsymbol{\mu}_t,\mathbf{A}_t,\boldsymbol{\psi})$ to represent the restrictions at a particular date. Stacking the residuals from each date into a long vector,

$$\mathcal{F}(\cdot) = [\mathsf{F}_1', \mathsf{F}_2', \ldots, \mathsf{F}_T']', \tag{16}$$

we seek values of $\boldsymbol{\mu}_t, \mathbf{A}_t,$ and $\boldsymbol{\psi}$ for which $\mathcal{F}(\cdot) = \mathbf{0}$.

Ideally, we would like to estimate the joint Bayesian posterior for VAR and parameters of the Calvo model, but that proved to be computationally intractable. In Cogley and Sbordone (2006), we outlined a Markov Chain Monte Carlo algorithm for simulating the joint posterior of this model, but upon further investigation we discovered that our algorithm did not converge. We tried to repair this defect but were unable to resolve the problem.[8] Since we are unable to simulate the posterior, we resort to a shortcut.

Following Sbordone (2002, 2006), we adopt a two-step estimation procedure. First, we fit to the data an unrestricted reduced-form VAR. Then, conditional on those estimates, we search for values of the parameters $\boldsymbol{\psi}$ that make $\mathcal{F}(\boldsymbol{\psi})$ close to zero, where "close" is defined in terms of an unweighted sum-of-squares

$$\hat{\boldsymbol{\psi}} = \arg\min \mathcal{F}(\hat{\boldsymbol{\mu}}_t, \hat{\mathbf{A}}_t, \boldsymbol{\psi})'\mathcal{F}(\hat{\boldsymbol{\mu}}_t, \hat{\mathbf{A}}_t, \boldsymbol{\psi}). \tag{17}$$

Estimation of the first-stage VAR follows Cogley and Sargent (2005a) and delivers a sample from the Bayesian posterior for $\boldsymbol{\mu}, \mathbf{A}$. The second stage defines an implicit function that maps the VAR parameters into best-fitting values for the parameters of the Calvo model. We view this as a change of variables $\boldsymbol{\psi} = h(\boldsymbol{\mu}, \mathbf{A})$ which transforms the posterior sample for the VAR parameters into a sample for the parameters of the pricing model. Thus, for each draw in the posterior for

[8] The convergence problem most likely follows from the existence of multiple solutions to (13) and (14). Roughly speaking, the algorithm fails to converge because it keeps switching across solutions, staying in the neighborhood of one for a long time, then switching to another. One branch—described in our earlier paper—makes economic sense, but the others do not. We experimented with a number of devices for eliminating the ill-behaved branches but failed to find one that made the algorithm converge. Among other things, we considered a particle filter, following Jesús Fernández-Villaverde and Juan Rubio-Ramírez (2007), but we decided against it because it is not promising for our model. The regularity conditions underlying the particle filter presume a unique solution to equations corresponding to (13) and (14) (assumption 2 of Fernández-Villaverde and Rubio-Ramírez 2008). In addition, our model is more complex than theirs in one key dimension. For computational reasons, they permit only one parameter to drift, while in our model the entire VAR parameter vector is free to drift.

$\boldsymbol{\mu}, \mathbf{A}$, we find the best-fitting value for $\boldsymbol{\psi}$ by solving (17). In this way, we induce a distribution for $\boldsymbol{\psi}$ from the distribution for $\boldsymbol{\mu}, \mathbf{A}$.

This procedure does not deliver a Bayesian posterior for $\boldsymbol{\psi}$, but it is logically coherent. Our inferences about $\boldsymbol{\psi}$ are suboptimal because they do not follow from Bayes's theorem. Essentially, we are using the likelihood function for an unrestricted VAR to learn about parameters of a restricted VAR. We would prefer to simulate the posterior for the restricted VAR, but since we cannot, we adopt this two-step estimator as a second-best approach.

Nevertheless, we conjecture that the Bayesian posterior for $\boldsymbol{\mu}, \mathbf{A}, \boldsymbol{\psi}$ would not differ greatly from the estimates reported below. This is based on the fact that the unrestricted VAR comes close to satisfying the cross-equation restrictions (evidence on this is reported below). That being the case, we suspect that the likelihoods for the restricted and unrestricted models are not terribly different. To verify this conjecture, we would have to simulate the posterior for the restricted VAR, which, as said, we are currently unable to do. We leave this for future research.

III. The First-Stage VAR

When trend inflation is nonzero, inflation depends not only on the evolution of marginal costs, but also on expectations of output growth and the discount rate. We therefore estimate a vector autoregression for inflation, log marginal cost, output growth, and a nominal discount factor. We allow for changes in the law of motion of these variables by estimating a VAR with drifting parameters and stochastic volatility. In this section, we describe the data, how the VAR is specified, and how the model is estimated.

A. *The Data*

Inflation is measured from the implicit GDP deflator, recorded in National Income and Product Account (NIPA) table 1.3.4. Output growth is calculated using chain-weighted real GDP, expressed in 2000 dollars and seasonally adjusted at an annual rate. This series is recorded in NIPA table 1.3.6. The nominal discount factor is constructed by expressing the federal funds rate on a discount basis. Federal funds data are monthly averages of daily figures and were converted to quarterly values by point-sampling the middle month of each quarter.

Marginal cost is approximated by unit labor cost. This is correct under the hypothesis of Cobb-Douglas technology: in this case the marginal product of labor is proportional to the average product, and real marginal cost (mc_t) is proportional to unit labor cost,

$$mc_t = w_t H_t / (1 - \delta) P_t Y_t = (1 - \delta)^{-1} ulc_t, \tag{18}$$

where $(1 - \delta)$ is the output elasticity to hours of work in the production function.

Because we exploit a restriction on trend marginal cost $\overline{mc}_t$ (equation (14)), we need a measure of unit labor costs in natural units rather than as an index number. To construct such a measure, we compute an index of total compensation in the nonfarm business sector from Bureau of Labor Statistics (BLS) indices of nominal compensation and total hours of work, then translate the result into dollars.[9] A (log) measure of real unit labor cost *ulc* is then obtained by subtracting (log of) nominal *GDP* from (log of) labor compensation. The new measure of *ulc* correlates almost

[9] Because we lack the right data for the nonfarm business sector, we perform the translation using data for private sector labor compensation, which we obtained from table B28 of the *Economic Report of the President* (2004). From that table, we calculated total labor compensation in dollars for 2002; the number for that year comes to \$4,978.61 billion. The BLS compensation index is then rescaled so that the new compensation series has that value in 2002.

perfectly with the BLS index number for unit labor cost in the nonfarm business sector, another measure commonly used in the literature (e.g., see Sbordone 2002, 2006). Finally, to transform the real unit labor cost (or labor share) into real marginal cost, we subtract the log exponent on labor, $(1-\delta)$, which we calibrate at 0.7.

The sample covers the period 1954:I to 2003:IV. Data from 1954:I to 1959:IV are used to initialize the prior, and the model is estimated using data from 1960:I through 2003:IV.

B. *The VAR Specification*

The reduced-form specification follows Cogley and Sargent (2005a). We write the VAR as

$$\mathbf{x}_t = \mathbf{X}_t' \boldsymbol{\vartheta}_t + \boldsymbol{\varepsilon}_{\mathbf{x}t}, \tag{19}$$

where $\mathbf{x}_t$ is a $N \times 1$ vector of endogenous variables ($N = 4$ in our case), $\mathbf{X}_t' = \mathbf{I}_N \otimes [1 \;\; \mathbf{x}_{t-l}']$, where $\mathbf{x}_{t-l}'$ represents lagged values of $\mathbf{x}_t$ and $\boldsymbol{\vartheta}_t$ denotes a vector of time-varying conditional mean parameters. In the companion-form notation used above, the matrix $\mathbf{A}_t$ refers to the autoregressive parameters in $\boldsymbol{\vartheta}_t$, and the vector $\boldsymbol{\mu}_t$ includes the intercepts.

As in Cogley and Sargent, $\boldsymbol{\vartheta}_t$ is assumed to evolve as a driftless random walk subject to reflecting barriers. Apart from the reflecting barrier, $\boldsymbol{\vartheta}_t$ evolves as

$$\boldsymbol{\vartheta}_t = \boldsymbol{\vartheta}_{t-1} + \mathbf{v}_t. \tag{20}$$

The innovation $\mathbf{v}_t$ is normally distributed, with mean $\mathbf{0}$ and variance $\boldsymbol{\Omega}$. Denoting by $\boldsymbol{\vartheta}^T$ the history of VAR parameters from date 1 to T, $\boldsymbol{\vartheta}^T = [\boldsymbol{\vartheta}_1', \ldots, \boldsymbol{\vartheta}_T']'$, the driftless random walk component is represented by a joint prior

$$f(\boldsymbol{\vartheta}^T, \boldsymbol{\Omega}) = f(\boldsymbol{\vartheta}^T | \boldsymbol{\Omega}) f(\boldsymbol{\Omega}) = f(\boldsymbol{\Omega}) \prod_{s=0}^{T-1} f(\boldsymbol{\vartheta}_{s+1} | \boldsymbol{\vartheta}_s, \boldsymbol{\Omega}). \tag{21}$$

Associated with this is a marginal prior $f(\boldsymbol{\Omega})$ that makes $\boldsymbol{\Omega}$ an inverse-Wishart variate.

The reflecting barrier is encoded in an indicator function, $I(\boldsymbol{\vartheta}^T) = \prod_{s=1}^{T} I(\boldsymbol{\vartheta}_s)$. The function $I(\boldsymbol{\vartheta}_s)$ takes a value of zero when the roots of the associated VAR polynomial are inside the unit circle, and it is equal to one otherwise. This restriction truncates and renormalizes the random walk prior, $p(\boldsymbol{\vartheta}^T, \boldsymbol{\Omega}) \propto I(\boldsymbol{\vartheta}^T) f(\boldsymbol{\vartheta}^T, \boldsymbol{\Omega})$. This represents a stability condition for the VAR, which rules out explosive representations for the variables in question.[10]

To allow for stochastic volatility, we assume that the VAR innovations $\boldsymbol{\varepsilon}_{\mathbf{x}t}$ can be expressed as

$$\boldsymbol{\varepsilon}_{\mathbf{x}t} = \mathbf{V}_t^{1/2} \boldsymbol{\xi}_t,$$

where $\boldsymbol{\xi}_t$ is a standard normal vector, which we assume to be independent of parameters innovation $\mathbf{v}_t : E(\boldsymbol{\xi}_t \mathbf{v}_s) = 0,$ for all t, s. We model $\mathbf{V}_t$ as a multivariate stochastic volatility process,

$$\mathbf{V}_t = \mathbf{B}^{-1} \mathbf{H}_t \mathbf{B}^{-1\prime}, \tag{22}$$

[10] Explosive representations might be useful for modeling hyperinflationary economies, but we regard them as implausible for the post–World War II United States.

where $\mathbf{H}_t$ is diagonal and $\mathbf{B}$ is lower triangular. The diagonal elements of $\mathbf{H}_t$ are assumed to be independent, univariate stochastic volatilities that evolve as driftless geometric random walks:

$$\ln h_{it} = \ln h_{it-1} + \sigma_i \eta_{it}. \tag{23}$$

The innovations η_{it} have a standard normal distribution, are independently distributed, and are assumed independent of innovations $\mathbf{v}_t$ and $\boldsymbol{\xi}_t$. The random walk specification for h_{it} is chosen to represent permanent shifts in innovation variance, such as those emphasized in the literature on the Great Moderation (see, for example, Margaret M. McConnell and Gabriel Perez-Quiros 2000; James H. Stock and Mark W. Watson 2002). The factorization in (22) and the log specification in (23) guarantee that $\mathbf{V}_t$ is positive definite, while the free parameters in $\mathbf{B}$ allow for time-varying correlation among the VAR innovations $\varepsilon_{\mathbf{x}t}$.

We work with a VAR(2) representation, ordering the variables as log g_t^y, log mc_t, log Π_t, Q_t.[11] Details on the calibration of the priors for the VAR parameters are in Appendix B. The posterior distributions were simulated using a Markov Chain Monte Carlo simulation algorithm that is identical to the one used by Cogley and Sargent (2005a).

C. *Trend Inflation and Persistence of the Inflation Gap*

Two features of the VAR are relevant for the NKPC, namely, how trend inflation varies and how that variation alters inflation-gap persistence. Following Stephen Beveridge and Charles R. Nelson (1981), we define trend inflation as the level to which inflation is expected to settle after short-run fluctuations die out, $\bar{\pi}_t = \lim_{j\to\infty} E_t \pi_{t+j}$. We approximate this by calculating a local-to-date t estimate of mean inflation from the VAR,

$$\bar{\pi}_t = \mathbf{e}_\pi' (\mathbf{I} - \mathbf{A}_t)^{-1} \boldsymbol{\mu}_t. \tag{24}$$

In general equilibrium, mean inflation is usually pinned down by the target in the central bank's policy rule. Accordingly, we interpret movements in $\bar{\pi}_t$ as reflecting changes in this aspect of monetary policy.[12]

Figure 1 portrays the median estimate of trend inflation at each date, shown as a dotted line, and compares it with actual and mean inflation. The latter are recorded as solid and dashed lines, respectively, and all are expressed at annual rates. The estimates are conditioned on data through the end of the sample, so we denote them $\bar{\pi}_{t|T}$.

Two features of the graph are relevant for what comes later. The first, of course, is that trend inflation varies. We estimate that it rose from 2.3 percent in the early 1960s to roughly 4.75 percent in the 1970s, then fell to around 1.65 percent at the end of the sample. A time-varying inflation trend makes our inflation gap quite different from those in conventional Calvo models. We measure the inflation gap as deviation of inflation from the time-varying trend, $\pi_t - \bar{\pi}_{t|T}$. By contrast, in conventional versions of the Calvo model, the inflation gap is measured as the deviation from a constant mean, reflecting the assumption of a constant steady-state rate of inflation. The measurement of the inflation gap matters a great deal because it affects the degree of persistence. As the figure illustrates, the mean-based gap is more persistent than trend-based measures. Notice, for example, the long runs at the beginning, middle, and end of the sample when inflation

[11] Exploring the sensitivity of results to alternative lag lengths or variable orderings is left to future research.

[12] Notice that $\bar{\pi}_t$ is a driftless random walk to a first-order approximation: this follows from the fact that a first-order Taylor expansion makes $\bar{\pi}_t$ linear in the VAR parameter vector $\boldsymbol{\vartheta}_t$, which evolves as a driftless random walk when away from the reflecting barrier. In this respect, our specification agrees with that of the inflation target in Ireland (2007).

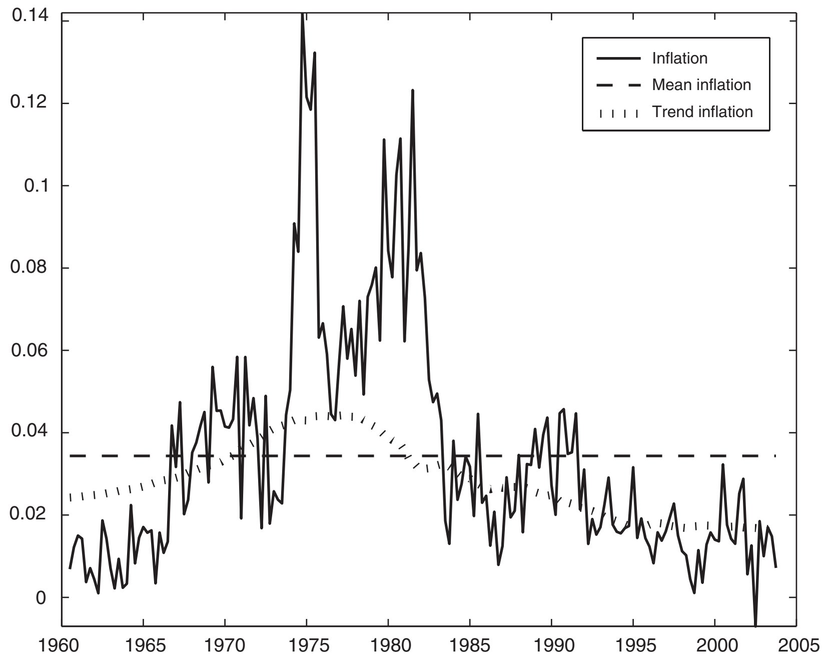

FIGURE 1. INFLATION, MEAN INFLATION, AND TREND INFLATION

does not cross the mean. In contrast, inflation crosses the trend line more often, especially after the Volcker disinflation.

Table 1 summarizes the autocorrelation of the inflation gap. The first row refers to actual inflation. For this measure, trend inflation is just the sample average, and the inflation gap is the deviation from the mean. The autocorrelation hovers around 0.8 both for the whole sample and for the two subsamples. In the second row, the inflation gap is measured by subtracting the median estimate of trend inflation from actual inflation. This matters only slightly for the period before the Volcker disinflation, but afterward the autocorrelation of the inflation gap is reduced substantially, to around 0.3.[13]

Purely forward-looking versions of the Calvo model are often criticized for generating too little persistence and, accordingly, the model is modified by introducing a backward-looking element. The table and figure make us wonder whether this "excess persistence" reflects an exaggeration of the persistence in mean-based measures of the gap rather than a deficiency of persistence in forward-looking models. In particular, if the trend-based measure is right, for the period after 1984 the NKPC needs to explain only a modest degree of persistence. A purely forward-looking version may be adequate after all.

TABLE 1—AUTOCORRELATION OF THE INFLATION GAP

	1960–2003	1960–1983	1984–2003
Inflation	0.834	0.843	0.784
Trend-based gap	0.769	0.801	0.305

While Figure 1 is suggestive, one should not attach too much significance to the particular path for $\bar{\pi}_{t|T}$ shown there. On the contrary, there is a lot of uncertainty about the level of trend

[13] Cogley, Giorgio Primiceri, and Sargent (2007) investigate this issue more rigorously and conclude that inflation-gap persistence was significantly lower after the Volcker disinflation.

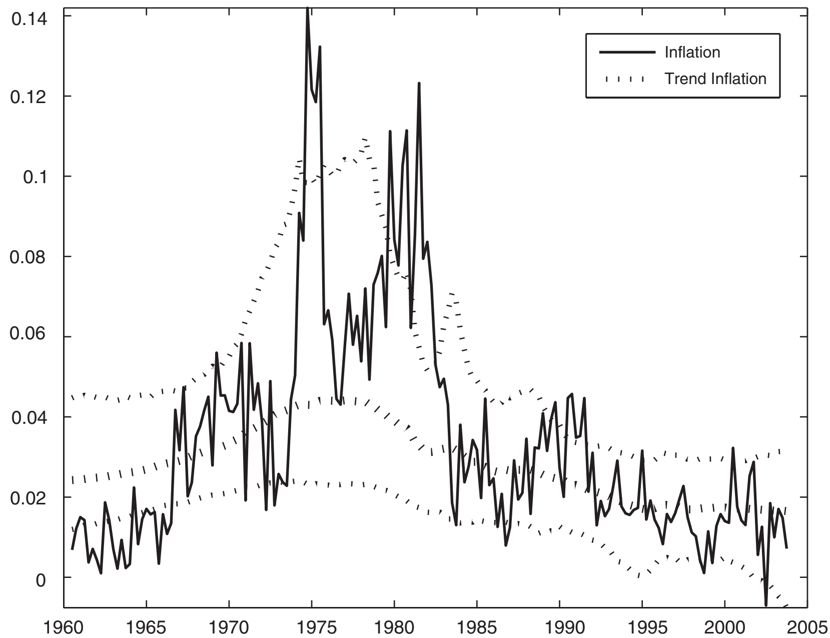

Figure 2. Ninety Percent Credible Sets for Trend Inflation

inflation at any given date. Figure 2 conveys a sense of this uncertainty by displaying marginal 90 percent credible sets at each date.[14] The credible sets are widest during the Great Inflation and are narrower when trend inflation is low. Our estimates of the structural parameters take this uncertainty into account because they are based on the entire posterior sample for trend inflation, not just on the mean or median path. Nevertheless, it is worth pointing out that other measures proposed in the literature are roughly in line with our estimates. For instance, Sharon Kozicki and Peter A. Tinsley (2002) estimate a multivariate VAR with shifting endpoints and derive an "anchor of long-horizon inflation expectations," while Ireland (2007) estimates the Federal Reserve's long-run inflation target in the context of a general equilibrium model. Both measures of underlying inflation have trajectories similar to our estimates of $\bar{\pi}_t$. Although these authors' estimates peak at slightly higher values than ours, they still fall within our uncertainty band. The sequence of ten-year-ahead inflation expectations from the Survey of Professional Forecasters, another widely reported measure of long-run inflation expectations, has a similar contour.[15]

IV. Estimates of the Structural Parameters

Next we turn to estimates of the structural parameters $\boldsymbol{\psi} = [\alpha, \theta, \rho, \omega]$. The strategic complementarity parameter ω has already been pinned down. That parameter is defined as $\omega = \delta/(1 - \delta)$, where $1 - \delta$ is the Cobb-Douglas labor elasticity. We calibrated $\delta = 0.3$ when constructing data

[14] A credible set is a Bayesian analog to a confidence interval. The marginal credible sets portray uncertainty about the location of $\bar{\pi}_t$ at a given date. However, the figure does not address whether changes in $\bar{\pi}_t$ across time are statistically significant. Cogley, Primiceri, and Sargent (2007) elaborate on this point.

[15] A strict comparison with the Survey of Professional Forecasters (SPF) would be inappropriate because the SPF measures expectations of consumer price inflation, while we measure inflation by the GDP deflator. Furthermore, we measure infinite-horizon expectations rather than ten-year-ahead forecasts.

on real marginal cost,[16] and that fixes $\omega = 0.429$. That leaves three free parameters—α, θ, and ρ—to estimate.

When solving the minimum-distance problem (17), we constrain α, ρ, and θ to lie in the economically meaningful ranges listed in Table 2. We also check whether the estimates satisfy the conditions for existence of a steady state (the inequalities (39) and (40) in Appendix A). Those conditions are satisfied for roughly 99 percent of the draws in the Monte Carlo sample.

Table 3 summarizes the second-stage estimates. Because the distributions are non-normal, the table records the median and 90 percent confidence intervals. The estimates are economically sensible; they accord well with microeconomic evidence, and they are reasonably precise.

Since our main interest is to assess the importance of a backward-looking component, an especially interesting outcome concerns the indexation parameter, whose median estimate is zero. Approximately 78 percent of the estimates lie exactly on the lower bound of zero, and 90 percent are less than 0.15. This contrasts with much of the empirical literature based on time-invariant models in which the indexation parameter is estimated as low as 0.2 and as high as one, and is statistically significant. For instance, when estimating a wage-price system, Sbordone (2006) estimates values for ρ ranging from 0.15 to 0.22, depending on the VAR specification. In a general equilibrium model, Smets and Wouters (2007) estimate a value of approximately 0.25. Marc P. Giannoni and Woodford (2003) estimate a value close to one, and Christiano, Eichenbaum, and Evans (2005) set ρ exactly equal to one. Marco Del Negro and Schorfheide (2006) also find the parameter ρ concentrating near zero, but they need highly autocorrelated mark-up shocks to obtain this result. In contrast, we reconcile $\rho = 0$ with a white noise mark-up shock. Other authors, following Galí and Gertler (1999), introduce a role for past inflation assuming the presence of rule-of-thumb firms, instead of assuming indexation, and similarly estimate significant coefficients on lagged inflation. In those models, an important backward-looking component is needed to fit inflation persistence, but that is not the case here.

Table 2—Admissible Range for Estimates

α	ρ	θ
$(0,1)$	$[0,1]$	$(1,\infty)$

Table 3—Estimates of the Structural Parameters

	α	ρ	θ
Median	0.588	0	9.8
90 percent confidence interval	(0.44, 0.70)	(0, 0.15)	(7.4, 12.1)

In our model the persistence of trend inflation explains most of the persistence in inflation, which makes it easier to fit data on the inflation gap $\pi_t - \bar{\pi}_t$ with a purely forward-looking model. Secondly, there is no need for a backward-looking term because we appropriately account for time-variation in $\bar{\pi}_t$. Our approximation implies that the NKPC includes additional leads of inflation, rather than lags, and these have more weight the higher is trend inflation. Since these forward-looking terms are positively correlated with past inflation (as must be true when inflation predicts future inflation more than one quarter ahead and Granger-causes output growth and the nominal interest rate), their omission could spuriously generate a positive coefficient on lagged inflation in estimates of the hybrid NKPC. This artificial inflation persistence creates a "persistence puzzle" for forward-looking models of inflation gaps relative to a constant long-run average inflation rate.[17]

[16] See Section IIIA. The results we report below about ρ are robust to other plausible values of δ, and the estimates of α and θ are only marginally affected (an increase in δ tends to lower α and raise θ).

[17] Ireland (2007) also finds no role for indexation to past inflation when trend inflation is allowed to drift. His hypothesis for why trend inflation alters the NKPC, however, is different from ours, since it is due to indexation of

At this point, we must temper our conclusion about ρ by acknowledging an identification problem. Andreas Beyer and Roger E. A. Farmer (2007) demonstrate that identification of forward- and backward-looking terms in the NKPC depends on assumptions about other structural equations in a general equilibrium model. When those equations are unspecified—as they are here—identification hinges on auxiliary assumptions about features such as VAR lag length and/or the autocorrelation properties of the cost-push shock. Thus, in a fundamental sense, whether our estimates are supported by convincing economic restrictions is not clear.

This issue is difficult to resolve using macro data alone, but micro data provide some help. Bils and Klenow (2004) report a median duration of prices of 4.4 months for a sample period covering 1995–1997, and Nakamura and Steinsson (2007) obtain similar results for two longer samples, covering 1988–1997 and 1998–2005, respectively. Specifications of the Calvo model involving an indexation component are hard to reconcile with their evidence. When $\rho > 0$, every firm changes price every quarter, some optimally rebalancing marginal benefit and marginal cost, others mechanically marking up prices in accordance with the indexation rule. Unless the optimal rebalancing happened to result in a zero price change, or lagged inflation were exactly zero, conditions that are very unlikely, no firm would fail to adjust its nominal price. In a world such as that, Bils-Klenow and Nakamura-Steinsson would not have found that a large fraction of prices remains unchanged each month. We interpret this as additional evidence in support of a purely forward-looking model.[18]

Turning to the degree of nominal rigidity, our median estimate for the fraction of sticky-price firms is $\alpha = 0.588$ per quarter, with a 90 percent confidence interval of $(0.44, 0.70)$. In conjunction with the estimate of $\rho = 0$, our point estimate implies a median duration[19] of prices of 1.31 quarters, or 3.9 months, with a 90 percent confidence interval ranging from 2.5 to 5.8 months. Thus, our estimates are not far from the unadjusted estimates of Bils and Klenow (2004) and Nakamura and Steinsson (2007). Results from micro studies are sensitive, however, to the treatment of sales and product substitution. For instance, Bils and Klenow report that the median duration increases to 5.5 months after removing sales price changes. In contrast, Nakamura and Steinsson (2007) find a longer median duration of about eight months after excluding sales and product substitutions. Bils and Klenow's adjusted estimate is close to the upper end of our confidence interval, but the Nakamura-Steinsson number lies outside.

Finally, the median estimate of θ implies a steady-state markup of about 11 percent, which is in line with other estimates in the literature. For example, this is the same order of magnitude as markups estimated by Susanto Basu (1996) and Basu and Miles Kimball (1997) using sectoral data. With economy-wide data, in the context of general equilibrium models, estimates range from around 6 to 23 percent, depending on the type of frictions in the model. Julio J. Rotemberg and Woodford (1997) estimate a steady-state markup of 15 percent ($\theta \approx 7.8$). Jeffrey D. Amato and Thomas Laubach (2003), in an extended model which also includes wage rigidity, estimate a steady-state markup of 19 percent. Rochelle M. Edge, Laubach, and John C. Williams (2003) find a slightly higher value, 22.7 percent ($\theta = 5.41$). The estimates in Christiano, Eichenbaum, and Evans (2005) span a larger range, varying from around 6.35 to 20 percent, depending on details of the model specification. We conclude that, although obtained through a different estimation strategy, our markup estimate falls within the range found by others.

nonoptimally reset prices to trend inflation.

[18] It should be noted, however, that the introduction of a backward-looking component through rule-of-thumb behavior, as in Galí and Gertler (1999), does not suffer from this problem. In those models, a fraction α of firms does maintain prices unchanged in each quarter.

[19] For a purely forward-looking Calvo model, the waiting time to the next price change can be approximated as α^t, and from that, one can calculate that the median waiting time is $-\ln(2)/\ln(\alpha)$. The median waiting time is less than the mean because the distribution of waiting times has a long upper tail.

We should at this point comment on another identification issue. As pointed out earlier in the literature and recently emphasized by Eichenbaum and Jonas D. M. Fisher (2007), in standard Calvo models the contributions of real and nominal rigidities to the coefficient on marginal cost may not be separately identified. But in our generalized Calvo model we are indeed able to identify all three parameters of interest: ρ, α, and θ.[20] The question is, then, which features of this model allow the identification.

In order to explore this issue, we estimated nested versions of the model, progressively turning off various of its features one at the time. First, we estimated the model omitting terms involving the discount factor and output growth. As expected, these terms do not matter, and the estimates are unaffected (see Table C.1 in Appendix C). Next, we also omitted terms involving higher-order leads of inflation. Again, the estimates are unaffected (see Table C.2). We then deactivated the dependence of the NKPC coefficients upon trend inflation, obtaining a representation resembling a model involving a mixed form of indexation, as discussed earlier. In this case, the estimates of α and θ are marginally affected, but ρ remains concentrated on zero (see Table C.3). All these estimates were obtained using both sets of cross-equation restrictions, i.e., those linking the steady-state values of inflation and marginal cost (equation (14)), as well as those involving deviations from the steady state (equation (12)). If, in addition, we deactivate either set of restrictions, the model seems to be underidentified, and the numerical optimizer frequently fails to find a minimum. We conclude that two features matter for identification, namely, time variation in $\bar{\pi}_t$ and $\mathbf{A}_t$ and the use of additional cross-equation restrictions coming from the steady-state relationship.

The model is overidentified, with three free parameters to fit $9 \cdot T$ elements in $\mathcal{F}(\cdot)$. However, in this environment we cannot justify the conventional J-test for overidentifying restrictions. To provide an overall measure of fit, we follow John Y. Campbell and Robert J. Shiller (1987) by informally comparing the expected inflation gap implied by the NKPC and the unconstrained VAR. The VAR inflation forecast is given by equation (10), while the NKPC forecast is implicitly defined by the right-hand side of equation (11). The distance between the two forecasts measures the extent to which the cross-equation restrictions are violated.

The top panel of Figure 3 plots the two series, showing VAR and NKPC forecasts as solid and dotted lines, respectively. To calculate the two forecasts, we condition on median estimates of the VAR and Calvo parameters. As the figure shows, NKPC forecasts closely track those of the unrestricted VAR. The two series have a correlation of 0.978, and the deviations between them are small in magnitude and represent high-frequency twists and turns.

The bottom panel looks more closely at the distance between (10) and (11). For that panel, we calculate cross-equation errors for every draw in the Monte Carlo sample and then plot a 90 percent marginal confidence interval for each date. Except for a handful of dates, those confidence intervals include zero. Hence, there is little evidence against the model's cross-equation restrictions.

Finally, we revisit the assumption that the cost-push shock u_t is unforecastable. Remember that we assumed that $E(u_t | \hat{\mathbf{z}}_{t-1}) = 0$. If we measure u_t as the residual in (8) and take its expectation with respect to the right-hand-side variables in the VAR, $\hat{\mathbf{z}}_{t-1}$, we obtain equation (11).[21] Therefore, the question of predictability in u_t is closely connected to the validity of the cross-equation restrictions. Intuitively, predictable movements in u_t would drive a wedge between the left- and right-hand sides of (12), so if u_t were forecastable, the VAR would not satisfy the NKPC

[20] As we pointed out before, these estimates are conditional on the calibrated value of ω.

[21] As before, that expectation is taken with respect to the time-varying law of motion $(\boldsymbol{\mu}_t, \mathbf{A}_t)$, and we use the "anticipated-utility" approximation for nonlinear terms.

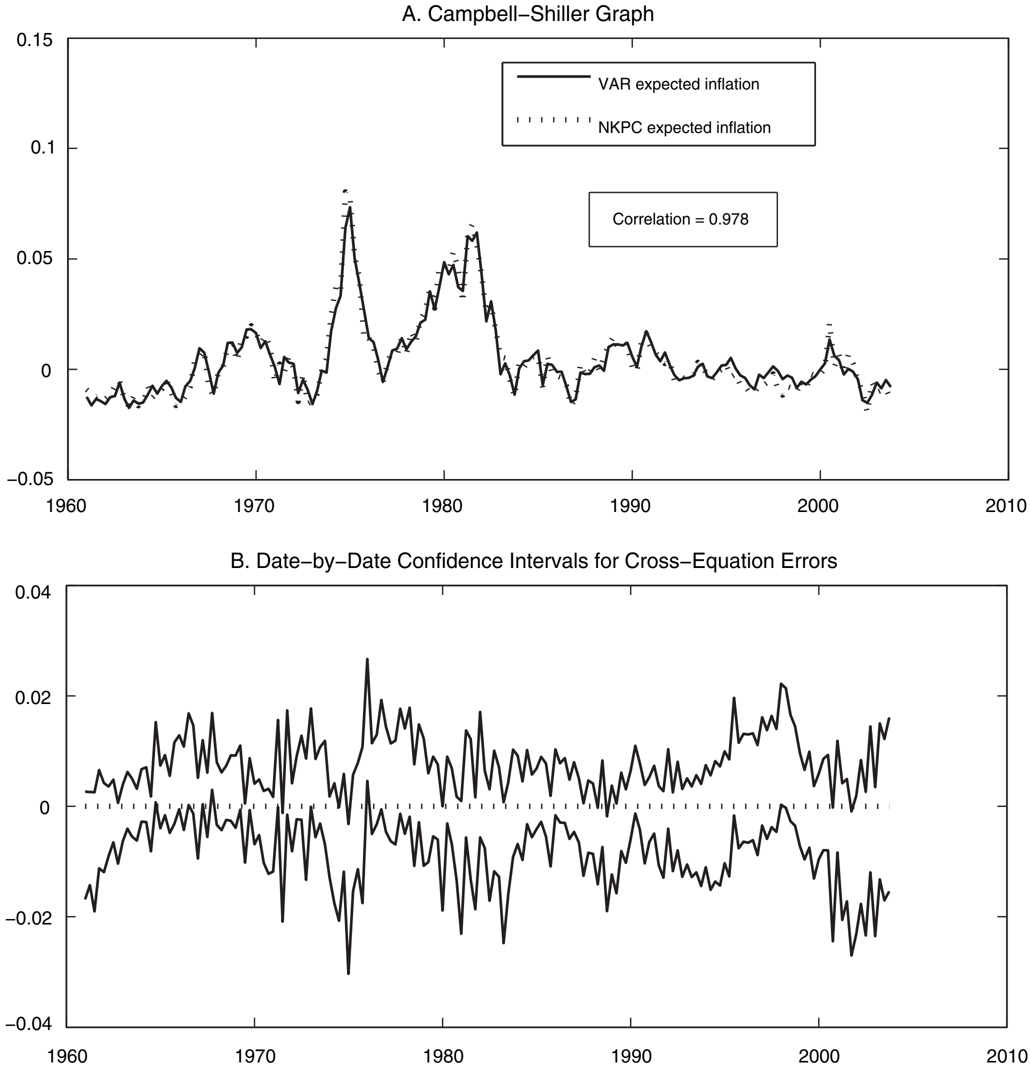

FIGURE 3. ASSESSING THE CROSS-EQUATION RESTRICTIONS

cross-equation restrictions. The deviations shown in Figure 3 can indeed be interpreted as a measure of this wedge. Because those discrepancies are small, it follows that predictable movements in u_t must also be small. Hence, there is little evidence against the assumption that u_t is white noise.

V. NKPC Coefficients

Next, we address how trend inflation affects the NKPC parameters $\zeta_t, b_{1t}, b_{2t},$ and b_{3t}. Again conditioning on median estimates of VAR and Calvo parameters, Figure 4 portrays the NKPC coefficients, computed as in (49). Dashed lines represent the conventional approximation, which assumes zero trend inflation, and solid lines represent our approximation, which depends on VAR estimates of $\bar{\pi}_t$.

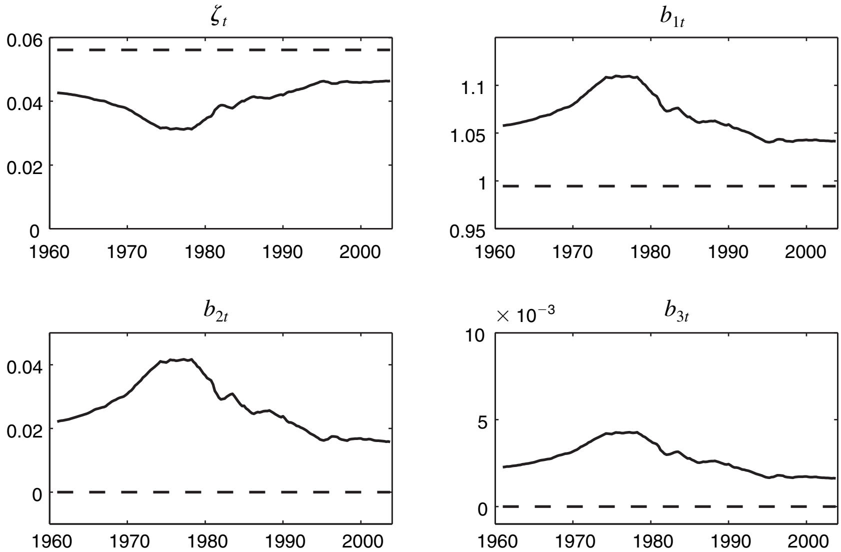

FIGURE 4. NKPC COEFFICIENTS

The shape of the time path for the NKPC coefficients is clearly dictated by $\bar{\pi}_t$. The parameter ζ_t, which represents the weight on current marginal cost, varies inversely with trend inflation, while the three forward-looking coefficients in (8) vary directly with it. Thus, as trend inflation rises, the link between inflation and current marginal cost is weakened, and the influence of forward-looking terms is enhanced. This shift in price-setting behavior follows from the fact that positive trend inflation accelerates the rate at which a firm's relative price is eroded when it lacks an opportunity to reoptimize. This makes firms more sensitive to contingencies that may prevail far in the future if their price remains stuck for some time. Thus, relative to the conventional approximation, current costs matter less and anticipations matter more.

The path for ζ_t echoes a point emphasized by Cogley and Sargent (2005b) and Primiceri (2006). They argue that the Fed's reluctance to disinflate during the Great Inflation was due in part to beliefs that the sacrifice ratio had increased. In traditional Keynesian models, the sacrifice ratio depends inversely on the coefficient on real activity. The less sensitive inflation is to current unemployment or the output gap, the more slack will be needed to disinflate. Cogley-Sargent and Primiceri recursively estimate backward-looking Phillips curves and find that inflation indeed became less sensitive to real activity during the Great Inflation. It is interesting that estimates of our forward-looking model also point toward a decline in the coefficient on real activity (i.e., real marginal cost) during the 1970s. In that respect, our estimates are consistent with theirs, while providing a structural interpretation for the declining coefficient.

Our model suggests that concerns about the slope of the short-run Phillips curve might have been exaggerated because parameters like ζ_t are not structural. In our model, a credible policy reform that reduced $\bar{\pi}_t$ would increase ζ_t, thus making inflation more sensitive to current marginal cost. By assuming that parameters like ζ_t were invariant to shifts in trend inflation, policy analysts in the 1970s probably overstated the cost of a disinflationary policy.

Focusing on the forward-looking coefficients, notice that the coefficient b_{3t} on the terms in (8), which involves forecasts of output growth and the discount factor, is always close to zero. Hence the terms involving expectations of output growth and the discount factor make a negligible contribution to inflation. What matters more is how trend inflation alters the coefficients on expected inflation, b_1 and b_2. Figure 4 shows that b_1 flips from slightly below one when trend inflation is zero to between 1.05 and 1.1 for our estimates of $\bar{\pi}_t$. Similarly, when trend inflation is zero, b_2 is also zero, and multistep expectations of inflation drop out of equation (8). When, instead, trend inflation is positive, those higher-order expectations enter the equation with coefficients of 0.02–0.04.

VI. Conclusion

Inflation is highly persistent, but much of that persistence is due to shifts in trend inflation. The inflation gap—i.e., actual minus trend inflation—is less persistent than inflation itself. Many previous papers on the NKPC neglect variation in trend inflation and attribute *all* the persistence of inflation to the inflation gap. Matching that exaggerated degree of persistence requires a backward-looking component which is typically motivated either as reflecting indexation or rule-of-thumb behavior. Many New Keynesian economists are uncomfortable about the backward-looking component because its microfoundations are less well developed than those of the forward-looking element.

In this paper, we address whether a more exact version of the Calvo model can explain inflation dynamics without the introduction of ad hoc backward-looking terms. We derive a version of the NKPC as an approximate equilibrium condition around a time-varying inflation trend with coefficients that are nonlinear combinations of the parameters describing market structure, the pricing mechanism, *and* trend inflation. We estimate the model in two steps, first estimating an unrestricted VAR and then estimating the parameters of the pricing model by exploiting cross-equation restrictions on the VAR parameters.

We find that no indexation or backward-looking component is needed to model inflation dynamics once shifts in trend inflation are taken into account. The absence of indexation is consistent with microeconomic evidence that some nominal prices remain fixed for months at a time. Our estimate of the frequency of price adjustment is also in line with estimates from micro data.

Nevertheless, our analysis could be improved in a number of ways. In particular, we assume that the Calvo pricing parameters are invariant to shifts in trend inflation, which cannot literally be true. In a companion paper (Cogley and Sbordone 2005), we explore whether that assumption is a reasonable approximation for the kind of variation in $\bar{\pi}_t$ seen in postwar US data.

Another important extension concerns the origins of shifts in the Fed's long-run inflation target. Following much of the rest of the literature, we treat $\bar{\pi}_t$ as an exogenous random process.[22] Since this accounts for most of the persistence in inflation, explaining why it drifts is important. One plausible story is that the Fed updates its policy rule as it learns about the structure of the economy, and that shifts in $\bar{\pi}_t$ are an outcome of this process (Cogley and Sargent 2005b; Primiceri 2006; Sargent, Noah Williams, and Tao Zha 2006; Giacomo Carboni and Martin Ellison 2007). More work is needed to understand how this occurs.

Finally, because of computational limitations, we were forced to take econometric shortcuts. In future research, we hope to devise efficient algorithms for simulating the Bayesian posterior for models like this.

[22] Ireland (2007) explores a model where target inflation responds to exogenous supply shocks, but finds such a model statistically indistinguishable from one with an exogenous target.

Appendix A: NKPC with Nonstationary Trend Inflation

In this Appendix, we derive a log-linear approximation of the evolution of aggregate prices and the firms' first-order conditions, and explain how to combine them to obtain the NKPC in the text.

A. *Log-Linear Approximation of the Evolution of Aggregate Prices*

We first divide (6) by P_t to get

$$1 = (1-\alpha)x_t^{1-\theta} + \alpha(\Pi_{t-1}^{\rho}\Pi_t^{-1})^{1-\theta}. \tag{25}$$

Then we transform (25) to express it in terms of the stationary variables defined in the text:

$$1 = (1-\alpha)\bar{x}_t^{1-\theta}\tilde{x}_t^{1-\theta} + \alpha\big[\overline{\Pi}_t^{(1-\rho)(\theta-1)}\big](g_t^{\bar{\pi}})^{-\rho(1-\theta)}\tilde{\Pi}_{t-1}^{\rho(1-\theta)}\tilde{\Pi}_t^{-(1-\theta)}. \tag{26}$$

In steady state, $\tilde{\Pi}_t = 1$ and $\tilde{x}_t = 1$, and (26) defines a function $\bar{x}_t = \bar{x}(\overline{\Pi}_t)$:

$$\bar{x}_t = \left[\frac{1-\alpha\overline{\Pi}_t^{(1-\rho)(\theta-1)}}{1-\alpha}\right]^{1/(1-\theta)}. \tag{27}$$

Defining hat variables $\hat{x}_t \equiv \ln\tilde{x}_t$ and $\hat{\pi}_t \equiv \ln\tilde{\Pi}_t \equiv \ln(\Pi_t/\overline{\Pi}_t) \equiv \pi_t - \bar{\pi}_t$, the log-linear approximation of (26) around its steady state is

$$0 \simeq (1-\alpha)\bar{x}_t^{1-\theta}\hat{x}_t - \alpha\big[\overline{\Pi}_t^{(1-\rho)(\theta-1)}\big](\hat{\pi}_t - \rho(\hat{\pi}_{t-1} - \hat{g}_t^{\bar{\pi}})),$$

which, substituting $(1-\alpha)\bar{x}_t^{1-\theta}$ from (27), becomes

$$0 \simeq \big[1-\alpha\overline{\Pi}_t^{(1-\rho)(\theta-1)}\big]\hat{x}_t - \alpha\big[\overline{\Pi}_t^{(1-\rho)(\theta-1)}\big](\hat{\pi}_t - \rho(\hat{\pi}_{t-1} - \hat{g}_t^{\bar{\pi}})).$$

This expression gives a solution for $\hat{x}_t$ as a function of $\hat{\pi}_t$, $\hat{\pi}_{t-1}$, and $\hat{g}_t^{\bar{\pi}}$:

$$\hat{x}_t = \frac{\alpha\overline{\Pi}_t^{(1-\rho)(\theta-1)}}{1-\alpha\overline{\Pi}_t^{(1-\rho)(\theta-1)}}[\hat{\pi}_t - \rho(\hat{\pi}_{t-1} - \hat{g}_t^{\bar{\pi}})] = \frac{1}{\varphi_{0t}}[\hat{\pi}_t - \rho(\hat{\pi}_{t-1} - \hat{g}_t^{\bar{\pi}})], \tag{28}$$

where we set $\varphi_{0t} = \big[1-\alpha\overline{\Pi}_t^{(1-\rho)(\theta-1)}\big]/\big[\alpha\overline{\Pi}_t^{(1-\rho)(\theta-1)}\big]$.

B. *Log-Linear Approximation of Firm's FOC*

Marginal cost at $t+j$ of the firm that changed price at t relates to the average marginal cost at $t+j$ as

$$MC_{t+j,t} = MC_{t+j}\left(\frac{X\Psi_{tj}}{P_{t+j}}\right)^{-\theta\omega} = MC_{t+j}X_t^{-\theta\omega}\Psi_{tj}^{-\theta\omega}P_{t+j}^{\theta\omega}, \tag{29}$$

where ω is the elasticity of firm's marginal cost to its own output. Substituting this expression in equation (5), we have

$$\tilde{E}_t\sum_{j=0}^{\infty}\alpha^j Q_{t,t+j}Y_{t+j}P_{t+j}^{\theta}\Psi_{tj}^{1-\theta}\left(X_t^{(1+\omega\theta)} - \frac{\theta}{\theta-1}MC_{t+j}\Psi_{tj}^{-(1+\theta\omega)}P_{t+j}^{\theta\omega}\right) = 0,$$

which implies that

$$(30)\qquad X_t^{1+\omega\theta} = \frac{\dfrac{\theta}{\theta-1}\tilde{E}_t \sum_{j=0}^{\infty} \alpha^j q_{t,t+j} Y_{t+j} P_{t+j}^{\theta(1+\omega)-1} \Psi_{tj}^{-\theta(1+\omega)} MC_{t+j}}{\tilde{E}_t \sum_{j=0}^{\infty} \alpha^j q_{t,t+j} Y_{t+j} P_{t+j}^{\theta-1} \Psi_{tj}^{1-\theta}} \equiv \frac{C_t}{D_t},$$

where we have expressed the discount factor in real terms $(q_{t,t+j} = Q_{t,t+j}(P_{t+j}/P_t))$ and $q_{t,t+j} = \prod_{k=0}^{j-1} q_{t+k,t+k+1}$. Using the definition of Ψ_{tj} in (3), we can express the functions C and D in recursive form, respectively:

$$(31)\qquad C_t = \frac{\theta}{\theta-1} Y_t P_t^{\theta(1+\omega)-1} MC_t + \tilde{E}_t\left[\alpha q_{t,t+1}\, \Pi_t^{-\rho\theta(1+\omega)} C_{t+1}\right]$$

and

$$(32)\qquad D_t = Y_t P_t^{\theta-1} + \tilde{E}_t\left[\alpha q_{t,t+1}\, \Pi_t^{\rho(1-\theta)} D_{t+1}\right].$$

Deflating appropriately (31) and (32), we obtain

$$(33)\qquad \tilde{C}_t \equiv \frac{C_t}{Y_t P_t^{\theta(1+\omega)}} = \frac{\theta}{\theta-1} mc_t + \tilde{E}_t\left[\alpha q_{t,t+1} g_{t+1}^{y} (\Pi_{t+1})^{\theta(1+\omega)}\, \Pi_t^{-\rho\theta(1+\omega)}\, \tilde{C}_{t+1}\right];$$

$$(34)\qquad \tilde{D}_t \equiv \frac{D_t}{Y_t P_t^{\theta-1}} = 1 + \tilde{E}_t\left[\alpha q_{t,t+1} g_{t+1}^{y} (\Pi_{t+1})^{\theta-1}\, \Pi_t^{\rho(1-\theta)}\, \tilde{D}_{t+1}\right],$$

where $mc_t \equiv MC_t/P_t$ and $g_{t+1}^{y} = Y_{t+1}/Y_t$. Note that

$$(35)\qquad \frac{\tilde{C}_t}{\tilde{D}_t} = \frac{\dfrac{C_t}{Y_t P_t^{\theta(1+\omega)}}}{\dfrac{D_t}{Y_t P_t^{\theta-1}}} = \frac{C_t}{D_t} \frac{Y_t P_t^{\theta-1}}{Y_t P_t^{\theta(1+\omega)}} = \frac{C_t}{D_t} \frac{1}{P_t^{(1+\theta\omega)}} = \left(\frac{X_t}{P_t}\right)^{1+\theta\omega} \equiv x_t^{1+\theta\omega}.$$

From (33) and (34) evaluated at steady state we can solve for

$$(36)\qquad \bar{C}_t = \frac{\dfrac{\theta}{\theta-1}\overline{mc}_t}{1 - \alpha \overline{q g}^{y} (\overline{\Pi}_t)^{\theta(1+\omega)(1-\rho)}},$$

$$(37)\qquad \bar{D}_t = \frac{1}{1 - \alpha \overline{q g}^{y} (\overline{\Pi}_t)^{(\theta-1)(1-\rho)}},$$

and

$$(38)\qquad \bar{x}_t^{1+\theta\omega} = \frac{\bar{C}_t}{\bar{D}_t} = \left[\frac{1 - \alpha \overline{q g}^{y} (\overline{\Pi}_t)^{(\theta-1)(1-\rho)}}{1 - \alpha \overline{q g}^{y} (\overline{\Pi}_t)^{\theta(1+\omega)(1-\rho)}}\right] \frac{\theta}{\theta-1}\, \overline{mc}_t.$$

Note that we must assume that the following inequalities hold:[23]

$$\alpha \overline{qg}^{y} (\overline{\Pi}_t)^{\theta(1+\omega)(1-\rho)} < 1 \tag{39}$$

and

$$\alpha \overline{qg}^{y} (\overline{\Pi}_t)^{(\theta-1)(1-\rho)} < 1. \tag{40}$$

Combining (27) and (38), we obtain the restriction across the steady-state values of inflation and marginal costs reported in (7) in the main text.

To derive a log-linear approximation of (35), we first define $\hat{C}_t = \ln(\tilde{C}_t/\bar{C}_t)$, $\hat{D}_t = \ln(\tilde{D}_t/\bar{D}_t)$, and $\widehat{mc}_t = \ln(mc_t/\overline{mc}_t)$ and then derive

$$\hat{C}_t = \varphi_{3t}\widehat{mc}_t + \varphi_{2t}\tilde{E}_t\big[\hat{q}_{t,t+1} + \hat{g}^{y}_{t+1} + \theta(1+\omega)(\hat{\pi}_{t+1} - \rho\hat{\pi}_t)\big] + \varphi_{2t}\tilde{E}_t\hat{C}_{t+1} \tag{41}$$

and

$$\hat{D}_t = \varphi_{1t}\tilde{E}_t\big[\hat{q}_{t,t+1} + \hat{g}^{y}_{t+1} + (\theta-1)(\hat{\pi}_{t+1} - \rho\hat{\pi}_t)\big] + \varphi_{1t}\tilde{E}_t\hat{D}_{t+1}. \tag{42}$$

For ease of notation, we have introduced the following symbols:[24]

$$\varphi_{1t} = \alpha\overline{qg}^{y}\overline{\Pi}_t^{(1-\rho)(\theta-1)}, \tag{43}$$

$$\varphi_{2t} = \alpha\overline{qg}^{y}\overline{\Pi}_t^{\theta(1+\omega)(1-\rho)},$$

$$\varphi_{3t} = 1 - \varphi_{2t}.$$

The log-linearization of (35) is then

$$(1+\theta\omega)\hat{x}_t = \hat{C}_t - \hat{D}_t, \tag{44}$$

from which we can solve for $\hat{\pi}_t$ using (28):

$$\hat{\pi}_t = \rho(\hat{\pi}_{t-1} - \hat{g}^{\bar{\pi}}_t) + \frac{\varphi_{0t}}{1+\theta\omega}(\hat{C}_t - \hat{D}_t). \tag{45}$$

C. *Inflation Dynamics*

Expressions (45), (41), and (42) represent a generalization of the Calvo model, expressed in a recursive form. By some simple manipulations, this representation can be further simplified to the following two equations:[25]

[23] For any value of $\overline{\Pi}_t$, $\bar{q}$, and $\bar{g}^y$, there exist values of the pricing parameters for which these inequalities hold. For example, if trend inflation were very high, then $\alpha \doteq 0$ might be needed to satisfy these inequalities. But that makes good economic sense, for the higher is trend inflation the more flexible prices are likely to be. Our estimates always satisfy these bounds.

[24] We have also suppressed the terms in expectations of $\hat{g}^{\bar{\pi}}_{t+1} \equiv \ln(\overline{\Pi}_{t+1}/\overline{\Pi}_t)$, $\hat{g}^{\bar{C}}_{t+1} \equiv \ln(\bar{C}_{t+1}/\bar{C}_t)$, and $\hat{g}^{\bar{D}}_{t+1} \equiv \ln(\bar{D}_{t+1}/\bar{D}_t)$, which are zero, since these are innovation processes.

[25] First, get an expression for $\hat{C}_t - \hat{D}_t$ by subtracting (42) from (41). Second, obtain an expression for $\hat{C}_t - \hat{D}_t$ in terms of inflation from (45), forward it one period, and take expectations. Substitute the last two expressions in the one

(46) $$\hat{\pi}_t - \rho\hat{\pi}_{t-1} = -\rho\hat{g}_t^{\bar{\pi}} + \tilde{\zeta}_t\widehat{mc}_t + \lambda_t(\tilde{E}_t\hat{\pi}_{t+1} - \rho\hat{\pi}_t) + \gamma_t\hat{D}_t,$$

(47) $$\hat{D}_t = \varphi_{1t}\tilde{E}_t(\hat{q}_{t,t+1} + \hat{g}_{t+1}^{y}) + \varphi_{1t}(\theta - 1)\tilde{E}_t(\hat{\pi}_{t+1} - \rho\hat{\pi}_t) + \varphi_{1t}\tilde{E}_t\hat{D}_{t+1},$$

where the coefficients are defined as

$$\tilde{\zeta}_t = \chi_t\varphi_{3t},$$

$$\lambda_t = \varphi_{2t}(1 + \varphi_{0t}),$$

$$\gamma_t = \chi_t\frac{\varphi_{2t} - \varphi_{1t}}{\varphi_{1t}},$$

$$\chi_t = \frac{\varphi_{0t}}{1 + \theta\omega}.$$

By the definitions in (43), if trend inflation were 0 ($\bar{\Pi} = 1$), the second equation would be irrelevant, since γ_t would be 0.

As a final step, we expand forward the second equation, substitute it into the first, and compact terms to obtain

(48) $$\hat{\pi}_t = \tilde{\rho}_t(\hat{\pi}_{t-1} - \hat{g}_t^{\bar{\pi}}) + \zeta_t\widehat{mc}_t + \tilde{b}_{1t}\tilde{E}_t\hat{\pi}_{t+1} + \tilde{b}_{2t}\tilde{E}_t\sum_{j=2}^{\infty}\varphi_{1t}^{j}\hat{\pi}_{t+j} + b_{3t}\tilde{E}_t\sum_{j=0}^{\infty}\varphi_{1t}^{j}(\hat{q}_{t+j,t+j+1} + \hat{g}_{t+1+j}^{y}) + u_t,$$

whose coefficients are defined as follows:

(49) $$\Delta_t = 1 + \rho\lambda_t + \gamma_t(\theta - 1)\rho\varphi_{1t},$$

$$\tilde{\rho}_t = \rho/\Delta_t,$$

$$\zeta_t = \tilde{\zeta}_t/\Delta_t,$$

$$\tilde{b}_{1t} = \frac{\lambda_t + \gamma_t(\theta - 1)(1 - \rho\varphi_{1t})\varphi_{1t}}{\Delta_t},$$

$$\tilde{b}_{2t} = \frac{\gamma_t(\theta - 1)(1 - \rho\varphi_{1t})}{\Delta_t},$$

$$b_{3t} = \frac{\gamma_t\varphi_{1t}}{\Delta_t}.$$

Note that to obtain this result we use the "anticipated utility" assumption, by which $\tilde{E}_t\Pi_{k=0}^{j}\varphi_{1t+k}\,x_{t+j} = \varphi_{1t}^{j+1}\tilde{E}_t x_{t+j}$, for any variable x_{t+j} and any $j > 0$.

Expression (8) in the text is obtained from (48) by transforming the real discount factor $\hat{q}_{t+j,t+j+1}$ in nominal terms via the following relationship:

$$\hat{q}_{t+j,t+j+1} = \hat{Q}_{t+j,t+j+1} + \hat{\pi}_{t+j+1}.$$

obtained at the start, and rearrange. Ascari and Ropele (2007) obtain a similar representation for a model with constant inflation trend, and no indexation.

The coefficients b_{1t} and b_{2t} of (8) in the text are related to the corresponding $\tilde{b}_{1t}$ and $\tilde{b}_{2t}$ of (48) here by

$$b_{1t} = \tilde{b}_{1t} + b_{3t},$$

$$b_{2t} = \tilde{b}_{2t} + b_{3t}.$$

Working with the expression in terms of a nominal discount factor allows us to use data on the nominal interest rate in the estimation, as explained in the text.

Appendix B: Priors for the VAR Parameters

We assume that VAR parameters and initial states are independent across blocks, so that the joint prior can be expressed as the product of marginal priors. Then we separately calibrate each of the marginal priors. Our choices closely follow those of Cogley and Sargent (2005a). The prior for the initial state $\boldsymbol{\vartheta}_0$ is assumed to be $N(\bar{\boldsymbol{\vartheta}}, \bar{\mathbf{P}})$. The mean and variance are set by estimating a time-invariant VAR using data from the training sample 1954:I–1959:IV. The initial VAR was estimated by OLS, and $\bar{\boldsymbol{\vartheta}}$ and $\bar{\mathbf{P}}$ were set equal to the resulting point estimate and asymptotic variance, respectively. Because $\bar{\boldsymbol{\vartheta}}$ is estimated from a short training sample, $\bar{\mathbf{P}}$ is quite large, making this prior weakly informative for $\boldsymbol{\vartheta}_0$.

For the state innovation variance $\boldsymbol{\Omega}$, we adopt an inverse-Wishart prior, $f(\boldsymbol{\Omega}) = IW(\bar{\boldsymbol{\Omega}}^{-1}, T_0)$. In order to minimize the weight of the prior, the degree-of-freedom parameter T_0 is set to the minimum for which the prior is proper, namely, 1 plus the dimension of $\boldsymbol{\vartheta}_t$. To calibrate the scale matrix $\bar{\boldsymbol{\Omega}}$, we assume $\bar{\boldsymbol{\Omega}} = \gamma^2\bar{\mathbf{P}}$ and set $\gamma^2 = 1.25\text{e-}04$. This makes $\bar{\boldsymbol{\Omega}}$ comparable to the value used in Cogley and Sargent (2005a), adjusting for the increased dimension of this model.

The parameters governing stochastic-volatility priors are set as follows. The prior for h_{i0} is log-normal, $f(\ln h_{i0}) = N(\ln \bar{h}_i, 10)$, where $\bar{h}_i$ is the estimate of the residual variance of variable i in the initial VAR. A variance of 10 on a natural-log scale makes this weakly informative for h_{i0}. The prior for $\mathbf{b}$—the free parameters in $\mathbf{B}$—is also normal with a large variance,

$$f(\mathbf{b}) = N(0, 10000 \cdot \mathbf{I}_6). \tag{50}$$

Finally, the prior for σ_i^2 is inverse gamma with a single degree of freedom, $f(\sigma_i^2) = IG(0.01^2/2, 1/2)$. This also puts a heavy weight on sample information. It is worth emphasizing that the priors for $\boldsymbol{\Omega}$ and σ_i^2—the parameters that govern the rate of drift in $\boldsymbol{\vartheta}_t$ and h_{it}—are very weak. In both cases, although the prior densities are proper, the tails are so fat that they do not possess finite moments. Thus, our priors about rates of drift are almost entirely agnostic.

Appendix C: Alternative Estimates of Structural Parameters

Table C.1—Omitting Terms Involving Q, g_y

	α	ρ	θ
Median	0.588	0	9.8
90 percent confidence interval	(0.45, 0.69)	(0, 0.14)	(7.4, 12.1)

Table C.2—Also Omitting Terms Involving Extra Leads of π

	α	ρ	θ
Median	0.581	0	10.4
90 percent confidence interval	(0.44, 0.69)	(0, 0.097)	(7.3, 13.6)

Table C.3—Also Shutting Off Time Variation in NKPC Coefficients

	α	ρ	θ
Median	0.568	0	12.1
90 percent confidence interval	(0.42, 0.67)	(0, 0.072)	(7.3, 16.2)

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