

The Term Structure of Currency Carry Trade Risk Premia[†]

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Fixing the investment horizon, the returns to currency carry trades decrease as the maturity of the foreign bonds increases. Across developed countries, the local currency term premia, which increase with the maturity of the bonds, offset the currency risk premia. Similarly, in the time-series, the predictability of foreign bond returns in dollars declines with the bonds' maturities. Leading no-arbitrage models in international finance do not match the downward term structure of currency carry trade risk premia. We derive a simple preference-free condition that no-arbitrage models need to reproduce in the absence of carry trade risk premia on long-term bonds. (JEL E43, G12, G15)

Carry trades correspond to simple investment strategies that are funded by borrowing in low interest rate currencies and invest in high interest rate currencies. What are their expected returns? According to the uncovered interest rate parity (UIP) condition, which assumes that investors are risk-neutral, expected carry trade returns should be zero. Yet, empirically, borrowing in low interest rate currencies and investing in Treasury bills of high interest rate currencies delivers large excess returns. This is the UIP puzzle, and it gave rise to a large literature that studies the role of systematic risk and expectational errors in exchange rates. Our paper revisits the empirical evidence on carry trades and deepens the puzzle.

Our paper explores the properties of the same carry trade investment strategy implemented with long-term bonds, and compares it to the standard strategy that uses Treasury bills. We focus on the same set of G10 countries and consider the strongest predictors of bond and currency returns: the level and slope of the yield curve. The first strategy we consider goes long the bonds of high interest rate currencies and short the bonds of low interest rate currencies, whereas the second strategy goes long the bonds of flat yield curve currencies and short the bonds of steep yield curve currencies. Most importantly, the investment horizon is one month, as in the

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classic tests of the UIP condition, not ten years, as in the tests of the UIP condition over long horizons. We find that, as the maturity of the bonds increases, the average excess return declines to zero. In other words, whereas the carry trade implemented with Treasury bills is profitable, the carry trade implemented with long-term bonds is not. Similar results emerge in individual country time-series predictability tests: as the maturity of the bonds increases, the predictability of the cross-country differences in dollar bond returns disappears.

The downward-sloping term structure of carry trade risk premia that we uncover represents a challenge for the leading models in international finance. To illustrate this point, we simulate the multi-country model of Lustig, Roussanov, and Verdelhan (2011). This reduced-form model, derived from the term structure literature, offers a flexible and transparent account of the UIP puzzle. Yet, we show that it implies a counterfactual flat term structure of currency carry trade risk premia. Our paper explains why other recent no-arbitrage models that replicate the UIP puzzle fail to replicate the absence of carry trade risk premia at the long end of the yield curve.

To guide future work in international finance, we derive a simple preference-free characterization of carry trade risk premia on infinite-maturity bonds when financial markets are complete. We rely on Alvarez and Jermann's (2005) decomposition of the stand-investor's marginal utility or pricing kernel into a permanent and a transitory component. To be precise, we show that the difference between domestic and foreign long-term bond risk premia, expressed in domestic currency, is determined by the difference in the volatilities of the permanent components of the stochastic discount factors (SDFs, e.g., the growth rate of the stand-in investor's marginal utility). Our preference-free result is the bond equivalent of the usual expression for the carry trade risk premium in no-arbitrage models: when borrowing and investing in Treasury bills, the carry trade risk premium is equal to the differences in volatilities of the SDFs (see Bekaert 1996; Bansal 1997; and Backus, Foresi, and Telmer 2001). This condition is the basis of most explanations of the UIP puzzles. Our novel characterization similarly imposes additional restrictions on foreign and domestic SDFs.

To link our theoretical results to our empirical findings, two assumptions are required: (i) long-term bond returns are good proxy for infinite-maturity bond returns, and (ii) the level and the slope of the yield curve summarize all the relevant predictors of carry trade excess returns. Under these two assumptions, the significant downward-sloping term structure of carry trade risk premia implies that differences in SDFs' volatilities must be significantly larger than differences in their permanent components' volatilities. To obtain stricter implications, let us consider a benchmark case: the absence of carry trade risk premia on long-term bonds. In this case, the volatilities of the permanent components need to be equalized across currencies. This is a natural benchmark that is not rejected by the data, unlike its short-term bond equivalent. But it is only a benchmark: the large standard errors around average excess returns on long-term bonds in the data still leave room for cross-country differences in the volatilities of the SDF's permanent components. Those differences, however, are clearly bounded.

Armed with our preference-free results, we revisit a large class of dynamic asset pricing models that have been used to study UIP violations, ranging from the reduced-form term structure model of Vasicek (1977) and Cox, Ingersoll, and Ross (1985) to their more recent multi-factor versions, to the Campbell and Cochrane

(1999) external habit model, the Bansal and Yaron (2004) long-run risks model, the disaster risk model, and the reduced-form model of Lustig, Roussanov, and Verdelhan (2011). We focus on models of the real SDF, given that there is no evidence that inflation risk can account for UIP deviations (if anything, UIP works better in high inflation environments: see Bansal and Dahlquist 2000) or for cross-country variation in local currency term premia. None of the models we consider can replicate our empirical findings in their standard calibrations. But, when feasible, we show how to modify and calibrate these models to match the absence of carry trade risk premia on long-term bonds.

Our results are related to, but different from, the long-run UIP condition. The long-run UIP condition compares foreign and domestic long-term interest rates to long-term changes in exchange rates. Chinn and Meredith (2005) finds that long-run UIP is a potentially valid description of the data. However, empirical tests lack power in finite samples: intuitively, there are few non-overlapping observations of 10-year windows available so far. From no-arbitrage conditions, we show that long-run UIP always holds for temporary shocks and thus long-run UIP deviations have to come from permanent shocks to exchange rates. For long-run UIP to hold at all times, exchange rates must not have any permanent shocks and thus be stationary in levels (up to a deterministic time trend). Yet, exchange rate stationarity in levels is sufficient but not necessary to satisfy our preference-free condition on the volatilities of the permanent SDF components. As a result, the carry trade risk premia on long-term bonds could be zero without implying that long-run UIP always holds. Under some additional regularity conditions, in that case, long-run UIP would hold on average in no-arbitrage models.

Recent work has documented a downward-sloping term structure of risk premia in equity markets (van Binsbergen, Brandt, and Koijen 2012), real estate markets (Giglio, Maggiori, and Stroebe 2015), and volatility markets (Dew-Becker et al. 2017). Backus, Boyarchenko, and Chernov (2018) provide a general analysis of the term structure of asset returns. Our work confirms the same pattern in currency markets, and offers a preference-free interpretation. Our theoretical result, although straightforward, has not been derived or used in the literature. On the one hand, at the short end of the maturity curve, currency risk premia are high when there is less overall risk in foreign countries' pricing kernels than at home (Bekaert 1996; Bansal 1997; and Backus, Foresi, and Telmer 2001). On the other hand, at the long end of the maturity curve, local bond term premia compensate investors mostly for the risk associated with transitory innovations to the pricing kernel (Bansal and Lehmann 1997; Hansen and Scheinkman 2009; Alvarez and Jermann 2005; Hansen 2012; Hansen, Heaton, and Li 2008; Backus, Chernov, and Zin 2014; Borovička, Hansen, and Scheinkman 2016; Backus, Boyarchenko, and Chernov 2018). In this paper, we combine those two insights to derive general theoretical results under the assumption of complete financial markets. Foreign bond returns allow us to compare the permanent components of the SDFs, which, as Alvarez and Jermann (2005) show, are the main drivers of SDFs. Our preference-free condition does not apply to exchange rate models that allow for market segmentation (see, e.g., Gabaix and Maggiori 2015 and Bacchetta and van Wincoop 2005, for leading examples). It remains to be determined whether these models can fit our facts, so we leave this as an open question for future research.

The rest of the paper is organized as follows. Section I focuses on the time-series and cross section of foreign bond risk premia. Section II compares recent no-arbitrage models to the empirical term structure of currency carry trade risk premia. In Section III, we derive the no-arbitrage, preference-free theoretical restriction imposed on bond returns and SDFs. Section IV links long-term UIP to the properties of exchange rates. Section V concludes. An Appendix and an online Appendix contain supplementary material and all proofs not presented in the main body of the paper.

I. Foreign Bond Returns in the Time-Series and Cross Section

We first describe the data and the notation, and then turn to our empirical results on the time-series and cross-sectional properties of foreign government bond returns.

A. Data

Our benchmark sample, to which we refer as the G10 sample, consists of a small homogeneous panel of developed countries with liquid bond markets: Australia, Canada, Germany, Japan, New Zealand, Norway, Sweden, Switzerland, and the United Kingdom. The domestic country is the United States. We calculate the returns of both coupon and zero-coupon bonds for these countries.

Our data on total return bond indices were obtained from Global Financial Data. The dataset includes a 10-year government bond total return index, as well as a Treasury bill total return index, in US dollars and in local currency. The data are monthly, starting in 1951:1 and ending in 2015:12. We use the 10-year bond returns as a proxy for long maturity bond returns. While Global Financial Data offers, to the best of our knowledge, the longest time-series of government bond returns available, the series have two key limitations. First, they pertain to coupon bonds, while the theory presented in this paper pertains to zero-coupon bonds. Second, they only offer 10-year bond returns, not the entire term structure of bond returns. To address these issues, we also use zero-coupon bond prices. Our zero-coupon bond dataset covers the same benchmark sample of G10 countries, but from at most 1975:1 to 2015:12, with different countries entering the sample at different dates.

Finally, we collect data on inflation rates and sovereign credit ratings. Inflation rates are calculated using monthly Consumer Price Index (CPI) data from Global Financial Data, whereas sovereign credit ratings are from Standard and Poor's, available over the 1989:7 to 2015:12 period. To construct averages of credit ratings, we assign each rating to a number, with a smaller number corresponding to a higher rating. A detailed description of all the data is available in the Appendix.

B. Notation

We now introduce our notation for bond prices, exchange rates, and bond and currency returns. In all cases, foreign variables are denoted as the starred version of their US counterpart.

Bonds.—The term $P_t^{(k)}$ denotes the price at date t of a zero-coupon bond of maturity k , while $y_t^{(k)}$ denotes its continuously compounded yield: $\log P_t^{(k)} = -ky_t^{(k)}$. The one-period holding return on the zero-coupon bond is $R_{t+1}^{(k)} = P_{t+1}^{(k-1)}/P_t^{(k)}$. The log excess return on the domestic zero-coupon bond, denoted $rx_{t+1}^{(k)}$, is equal to

$$(1) \quad rx_{t+1}^{(k)} = \log [R_{t+1}^{(k)}/R_t^f],$$

where the risk-free rate is $R_t^f = R_{t+1}^{(1)} = 1/P_t^{(1)}$. Finally, r_t^f denotes the log risk-free rate: $r_t^f = \log R_t^f = y_t^{(1)}$.

Exchange Rates.—The nominal spot exchange rate in foreign currency per US dollar is denoted S_t . Thus, an increase in S_t implies an appreciation of the US dollar relative to the foreign currency. The log currency excess return, given by

$$(2) \quad rx_{t+1}^{FX} = \log \left[\frac{S_t}{S_{t+1}} \frac{R_t^{f,*}}{R_t^f} \right] = r_t^{f,*} - r_t^f - \Delta s_{t+1},$$

is the log excess return of a strategy in which the investor borrows at the domestic risk-free rate, R_t^f , invests at the foreign risk-free rate, $R_t^{f,*}$, and converts the proceeds into US dollars at the end of the period.

Bond Risk Premia.—The log return on a foreign bond position (expressed in US dollars) in excess of the domestic (i.e., US) risk-free rate is denoted $rx_{t+1}^{(k),\$}$. It can be expressed as the sum of the bond log excess return in local currency plus the log excess return on a long position in foreign currency,

$$(3) \quad rx_{t+1}^{(k),\$} = \log \left[\frac{R_{t+1}^{(k),*}}{R_t^f} \frac{S_t}{S_{t+1}} \right] = \log \left[\frac{R_{t+1}^{(k),*}}{R_t^{f,*}} \frac{R_t^{f,*}}{R_t^f} \frac{S_t}{S_{t+1}} \right] \\ = \log \left[\frac{R_{t+1}^{(k),*}}{R_t^{f,*}} \right] + \log \left[\frac{R_t^{f,*}}{R_t^f} \frac{S_t}{S_{t+1}} \right] = rx_{t+1}^{(k),*} + rx_{t+1}^{FX}.$$

Taking conditional expectations, the total term premium in dollars consists of a foreign bond risk premium, $E_t[rx_{t+1}^{(k),*}]$, plus a currency risk premium, $E[rx_{t+1}^{FX}] = r_t^{f,*} - r_t^f - E_t[\Delta s_{t+1}]$.

We are not the first to study the relation between domestic and foreign bond returns. Prior work, from Campbell and Shiller (1991) to Bekaert and Hodrick (2001) and Bekaert, Wei, and Xing (2007), shows that investors earn higher returns on foreign bonds from a country in which the slope of the yield curve is currently higher than average for that country. Ang and Chen (2010) and Berge, Jordà, and Taylor (2011) show that yield curve variables can also be used to forecast currency excess returns. These authors, however, do not examine the returns on foreign bond portfolios expressed in domestic currency. The following papers consider foreign bond returns in US dollars: Dahlquist and Hasseltoft (2013) studies international bond

TABLE 1—TIME-SERIES PREDICTABILITY REGRESSIONS

	Bond dollar return diff.		Currency excess return		Bond local cur- rency return diff.		Slope diff. <i>p</i> -value	Observations
	$rx^{(10),\$} - rx^{(10)}$	$R^2(\%)$	rx^{FX}	$R^2(\%)$	$rx^{(10),*} - rx^{(10)}$	$R^2(\%)$		
	β		β		β			
<i>Panel A. Short-term interest rates</i>								
Australia	-0.15 (0.91)	-0.20	1.29 (0.55)	0.56	-1.44 (0.52)	1.51	0.20	492
Canada	-1.10 (0.69)	0.11	1.22 (0.58)	0.46	-2.32 (0.52)	3.64	0.02	492
Germany	1.52 (1.18)	0.37	2.49 (1.05)	1.71	-0.97 (0.40)	0.48	0.55	492
Japan	2.37 (0.71)	1.13	3.11 (0.70)	3.48	-0.74 (0.41)	0.13	0.47	492
New Zealand	0.69 (1.06)	-0.03	2.23 (0.44)	3.14	-1.54 (0.88)	1.62	0.20	492
Norway	0.72 (0.57)	0.08	1.74 (0.55)	2.26	-1.02 (0.34)	0.97	0.22	492
Sweden	-0.64 (0.86)	-0.02	0.89 (0.88)	0.25	-1.53 (0.52)	2.02	0.23	492
Switzerland	1.16 (0.90)	0.33	2.45 (0.79)	2.43	-1.29 (0.44)	1.69	0.30	492
United Kingdom	1.02 (1.03)	0.04	2.69 (1.24)	2.44	-1.67 (0.49)	1.39	0.32	492
Panel	0.65 (0.50)	-0.05	1.98 (0.49)	1.82	-1.34 (0.33)	1.37	0.00	4,428
Joint zero <i>p</i> -value	0.19		0.00		0.00		0.32	

(Continued)

risk premia in an affine asset pricing model and finds evidence for local and global risk factors, while Jotikasthira, Le, and Lundblad (2015) studies the co-movement of foreign bond yields through the lenses of an affine term structure model. Our paper revisits the empirical evidence on bond returns without committing to a specific term structure model.

C. Time-Series Predictability of Foreign Bond Returns

To study the properties of the cross-country differences in expected bond excess returns, we first run individual currency predictability regressions on variables that can be used to predict bond and currency returns. We focus on the level and the slope of the term structure, the two predictors that have been shown to forecast both bond and currency returns.

We regress the 10-year dollar bond log excess return differential ($rx_{t+1}^{(10),\$} - rx_{t+1}^{(10)}$) on the short-term interest rate differential ($r_t^{f,*} - r_t^f$, panel A of Table 1) and on the yield curve slope differential ($[y_t^{(10),*} - y_t^{(1,*)}] - [y_t^{(10)} - y_t^{(1)}]$, panel B of Table 1), focusing on the post-Bretton Woods sample period (1975:1 – 2015:12). Given that the 10-year dollar bond log excess return differential (left columns) can be decomposed into the sum of currency log excess returns (rx_{t+1}^{FX}) and local currency bond log excess return differentials ($rx_{t+1}^{(10),*} - rx_{t+1}^{(10)}$) as noted in equation (3), we also

TABLE 1—TIME-SERIES PREDICTABILITY REGRESSIONS (*Continued*)

	Bond dollar return diff.		Currency excess return		Bond local cur- rency return diff.		Slope diff. <i>p</i> -value	Observations
	$rx^{(10),\$} - rx^{(10)}$		rx^{FX}		$rx^{(10),*} - rx^{(10)}$			
	β	$R^2(\%)$	β	$R^2(\%)$	β	$R^2(\%)$		
<i>Panel B. Yield curve slopes</i>								
Australia	3.84 (1.69)	1.54	−1.00 (1.16)	−0.02	4.84 (0.96)	7.65	0.03	492
Canada	4.04 (1.23)	2.25	−0.72 (0.79)	−0.07	4.76 (0.81)	9.09	0.00	492
Germany	0.50 (1.57)	−0.18	−3.05 (1.37)	1.15	3.55 (0.82)	4.07	0.11	492
Japan	−0.32 (1.12)	−0.19	−4.18 (0.94)	2.91	3.85 (0.81)	3.96	0.02	492
New Zealand	2.94 (2.35)	1.26	−1.60 (1.28)	0.62	4.55 (1.41)	7.41	0.11	492
Norway	0.59 (0.98)	−0.12	−2.03 (0.97)	1.33	2.62 (0.52)	3.35	0.07	492
Sweden	3.12 (1.21)	2.12	−0.13 (1.02)	−0.20	3.25 (0.82)	5.29	0.06	492
Switzerland	0.97 (1.05)	−0.06	−3.59 (1.27)	1.97	4.55 (1.00)	8.82	0.01	492
United Kingdom	1.59 (1.28)	0.17	−3.17 (1.62)	2.11	4.75 (0.85)	7.95	0.03	492
Panel	1.94 (0.96)	0.42	−2.02 (0.82)	0.83	3.96 (0.66)	6.08	0.00	4,428
Joint zero <i>p</i> -value	0.08		0.01		0.00		0.00	

Notes: The table reports regression results of the bond dollar return difference ($rx_{t+1}^{(10),\$} - rx_{t+1}^{(10)}$, left panel) or the currency excess return (rx_{t+1}^{FX} , middle panel) or the bond local currency return difference ($rx_{t+1}^{(10),*} - rx_{t+1}^{(10)}$, right panel) on the difference between the foreign nominal interest rate and the US nominal interest rate ($r_t^{f,*} - r_t^f$, panel A) or the difference between the foreign nominal yield curve slope and the US nominal yield curve slope ($[y_t^{(10),*} - y_t^{(1,*)}] - [y_t^{(10)} - y_t^{(1)}]$, panel B). The column *Slope diff.* presents the p -value of the test for equality between the slope coefficient in the bond dollar return difference regression and the slope coefficient in the currency excess return regression for each country. The last line in each panel presents the p -value of the joint test that all individual-country regression coefficients in the respective column are zero. We use returns on 10-year coupon bonds. The holding period is one month and returns are sampled monthly. The log returns and the yield curve slope differentials are annualized. The sample period is 1975:1–2015:12. The balanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, and the United Kingdom. In individual country regressions, standard errors (in parentheses) are obtained with a Newey and West (1987) approximation of the spectral density matrix, with the lag truncation parameter (kernel bandwidth) equal to 29. Panel regressions include country fixed effects, and standard errors (in parentheses) are obtained using the Driscoll and Kraay (1998) methodology, with the lag truncation parameter (kernel bandwidth) equal to 29. All p -values are fixed- b p -values, calculated using the approximation of the corresponding fixed- b asymptotic distribution in Vogelsang (2011, 2012).

regress each of those two components on the same predictors (middle and right columns, respectively). By construction, the sum of the slope coefficients in the middle and right columns equals the slope coefficients in the corresponding left column. All regressions include a constant.

For each individual country regression, we report Newey and West (1987) standard errors, setting the value of the lag truncation parameter (kernel bandwidth) to $S = 29$, following the recommendation of Lazarus et al. (2018). Given the well-known potential issues with using asymptotic distributions for statistical inference in finite samples, we use the Kiefer and Vogelsang (2005) nonstandard fixed- b distributions for inference. In particular, we calculate fixed- b p -values for t -statistics

and Wald tests using the methodology discussed in Vogelsang (2012). The panel regressions include country fixed effects, and standard errors are calculated using the Driscoll and Kraay (1998) methodology, which corrects for heteroskedasticity, serial correlation, and cross-equation correlation; we also calculate fixed- b p -values for our panel regression coefficients. In the interest of space, the full results (which include the estimates of the regression constant terms and the p -values for the constant and slope terms) are reported in the online Appendix. Our results are not driven either by the choice of kernel bandwidth or by the use of fixed- b distributions for inference: the online Appendix reports similar regression results with Newey and West (1987) and Driscoll and Kraay (1998) standard errors calculated with a kernel bandwidth of $S = 6$.

When using interest rate differentials as predictors, there is no consistent evidence in support of predictability of 10-year bond return differentials in dollars: indeed, out of nine countries, there is evidence of return predictability only for Japan. The slopes and constants are insignificant for all the other countries in the sample. In a panel regression, the slope coefficient is small and not statistically different from zero. In addition, we cannot reject the null that all constants in the individual country regressions are zero and, similarly, that all slope coefficients are zero (p -values of 0.82 and 0.19, respectively). This evidence supports the view that there is no difference between expected dollar returns on long bond returns in these countries.

To better understand the lack of predictability, we decompose the dollar excess return differential into currency log excess returns and local currency bond log excess return differentials. As seen in the table, currency log excess returns are strongly forecastable by interest rate differentials (Hansen and Hodrick 1980, Fama 1984): as documented in the existing literature, higher than usual interest rate differentials in a given country pair predict higher than usual currency log excess returns. In a joint test of all slope coefficients, we can reject the null that interest rates do not predict currency excess returns. But, while Treasury bill return differentials in US dollars are forecastable, long-term bond return differentials in US dollars are not. The deterioration of return predictability for long-maturity bonds, compared to Treasury bills, appears to be due to the offsetting effect of local currency bond returns: higher than usual interest rate differences in a given country predict lower local currency bond return differences. Again, we can reject the null that interest rate differences do not predict local currency bond return differences at the 1 percent confidence level. In the panel regression, the local bond return slope coefficient is -1.34 , largely offsetting the 1.98 slope coefficient in the currency excess return regression. The net effect on dollar bond returns is only 65 basis points, the slope coefficient is not statistically significant, and the panel regression adjusted R^2 is -0.05% .¹ Thus, from

¹ Note that the local currency log bond excess return differential contains the interest rate differential with a negative sign: $rx_{t+1}^{(10),*} - rx_{t+1}^{(10)} = r_{t+1}^{(10),*} - r_{t+1}^{(10)} - (r_t^{f,*} - r_t^f)$, where $r_{t+1}^{(10),*}$ and $r_{t+1}^{(10)}$ are the foreign and domestic holding-period bond returns. Thus, the local currency log bond excess return differential is highly predictable by the interest rate differential (right column of panel A in Table 1), simply because the interest rate spread in effect predicts itself, as it is a component (with a negative sign) of the dependent variable. When we regress the local currency log return differential (instead of the excess returns) on the interest rate differential, there is no evidence of predictability (see online Appendix). Measurement error in the short rate could also give rise to a negative relation, even in the absence of true predictability. The slope of the yield curves predicts the local currency log return differential.

the perspective of a US investor, the time variation in the currency excess return is largely offset by the variation in the local term premium.

When using yield curve slope differentials as predictors, a similar finding emerges. On the one hand, currency log excess returns are forecastable by yield curve slope differentials: a steeper than usual slope in a given country predicts lower than usual currency log excess returns. On the other hand, a slope steeper than usual in a given country also predicts higher local currency bond excess returns. In the panel regression, the local bond excess return slope coefficient is 3.96, more than offsetting the -2.02 slope coefficient in the currency excess return regression. The local currency bond return predictability merely confirms the results for US bond excess returns documented by Fama and Bliss (1987), Campbell and Shiller (1991), and Cochrane and Piazzesi (2005). The net effect on dollar bond excess return differences is 194 basis points in a surprising direction: a steeper slope seems to weakly forecast *higher* dollar returns for foreign bonds, rather than lower dollar returns as for foreign Treasury Bills. The slope coefficient in the panel regression is statistically significant, albeit marginally (the p -value is 0.06). From the perspective of a US investor, the time variation in the currency excess return is more than offset by the variation in the local term premium. This reverses the usual carry trade logic: investors want to short the currencies with lower than average slopes to harvest the local bond term premium. We turn now to the economic significance of these results.

To do so, we explore the risk-return characteristics of a simple investment strategy that goes long the foreign bond and shorts the US bond when the foreign short-term interest rate is higher than the US interest rate (or the foreign yield curve slope is lower than the US yield curve slope), and goes long the US bond and shorts the foreign bond when the US interest rate is higher than the foreign country's interest rate (or the US yield curve slope is lower than the foreign yield curve slope). To assess the risk-return trade-off, investors commonly look at the corresponding Sharpe ratios of an investment strategy, defined as the expected return less the risk-free rate divided by its standard deviation. In the absence of arbitrage opportunities, there is a one-to-one mapping from the R^2 s in predictability regressions to the unconditional Sharpe ratios on investment strategies that exploit the predictability (see Cochrane 1999, pp. 75–76). Table 2 shows this mapping. Panel A reports the results of the interest rate-based strategy, whereas panel B focuses on the slope-based strategy. The very low R^2 s reported in Table 1 lead to low returns and Sharpe ratios in Table 2.

In our sample, none of the individual country dollar returns on the interest rate level strategy are statistically significant. The equally weighted dollar return on the interest rate strategy is 0.70 percent per annum, and this return is not statistically significant. The equally weighted annualized Sharpe ratio is 0.11, not significantly different from zero. Furthermore, with the exception of Sweden, none of the individual country dollar bond returns on the slope strategy are statistically significant either, even though the currency excess returns and the local currency bond returns typically are. The equally weighted return on the slope strategy is -1.03 percent per annum, and this dollar return is not statistically significant. The equally weighted annualized Sharpe ratio is -0.13 , not significantly different from zero. In short, there is no evidence of economically significant dollar return predictability. This is not to say that no significant return predictability exists in our sample. To the contrary, there are large currency excess returns (with a Sharpe ratio of 0.47 for an

TABLE 2—DYNAMIC LONG-SHORT FOREIGN AND US BOND PORTFOLIOS

	Bond dollar return diff. $rx^{(10),\$} - rx^{(10)}$		Currency excess return rx^{FX}		Bond local currency return diff. $rx^{(10),*} - rx^{(10)}$	
	Mean	Sharpe ratio	Mean	Sharpe ratio	Mean	Sharpe ratio
<i>Panel A. Short-term interest rates</i>						
Australia	1.28 (2.23)	0.09 (0.16)	3.55 (1.75)	0.31 (0.16)	-2.27 (1.34)	-0.27 (0.15)
Canada	-0.46 (1.42)	-0.05 (0.16)	0.86 (1.13)	0.12 (0.16)	-1.31 (0.85)	-0.24 (0.15)
Germany	2.19 (1.93)	0.18 (0.16)	3.86 (1.72)	0.35 (0.16)	-1.67 (1.16)	-0.23 (0.16)
Japan	0.93 (2.19)	0.07 (0.16)	1.54 (1.73)	0.14 (0.16)	-0.61 (1.43)	-0.07 (0.16)
New Zealand	0.65 (2.56)	0.04 (0.16)	3.84 (1.87)	0.31 (0.17)	-3.19 (1.76)	-0.28 (0.16)
Norway	0.69 (1.98)	0.05 (0.16)	3.17 (1.60)	0.30 (0.16)	-2.48 (1.42)	-0.28 (0.16)
Sweden	-0.40 (2.03)	-0.03 (0.15)	2.30 (1.73)	0.21 (0.16)	-2.71 (1.38)	-0.31 (0.16)
Switzerland	0.57 (2.05)	0.04 (0.16)	1.14 (1.97)	0.09 (0.15)	-0.57 (1.13)	-0.07 (0.16)
United Kingdom	0.89 (2.00)	0.07 (0.15)	3.09 (1.59)	0.30 (0.16)	-2.20 (1.25)	-0.27 (0.15)
Equally weighted	0.70 (1.00)	0.11 (0.16)	2.59 (0.87)	0.47 (0.17)	-1.89 (0.60)	-0.51 (0.16)
<i>Panel B. Yield curve slopes</i>						
Australia	-1.88 (2.22)	-0.13 (0.16)	2.89 (1.80)	0.25 (0.17)	-4.76 (1.34)	-0.57 (0.14)
Canada	-2.07 (1.41)	-0.23 (0.15)	1.23 (1.10)	0.18 (0.16)	-3.30 (0.83)	-0.61 (0.16)
Germany	1.98 (1.92)	0.16 (0.16)	5.17 (1.71)	0.47 (0.16)	-3.19 (1.14)	-0.44 (0.15)
Japan	-0.71 (2.21)	-0.05 (0.16)	4.60 (1.73)	0.41 (0.16)	-5.31 (1.42)	-0.60 (0.16)
New Zealand	-0.18 (2.57)	-0.01 (0.16)	3.49 (1.90)	0.28 (0.17)	-3.67 (1.77)	-0.32 (0.15)
Norway	-0.56 (2.05)	-0.04 (0.15)	2.84 (1.73)	0.27 (0.16)	-3.40 (1.39)	-0.38 (0.15)
Sweden	-3.62 (1.99)	-0.28 (0.16)	1.32 (1.75)	0.12 (0.17)	-4.94 (1.38)	-0.57 (0.16)
Switzerland	0.47 (2.00)	0.04 (0.15)	4.80 (1.91)	0.40 (0.15)	-4.33 (1.17)	-0.58 (0.16)
United Kingdom	-2.73 (1.95)	-0.21 (0.16)	2.06 (1.61)	0.20 (0.16)	-4.79 (1.30)	-0.59 (0.16)
Equally weighted	-1.03 (1.24)	-0.13 (0.16)	3.16 (1.08)	0.47 (0.16)	-4.19 (0.80)	-0.83 (0.16)

Notes: For each country, the table presents summary return statistics of investment strategies that go long the foreign country bond and short the US bond when the foreign short-term interest rate is higher than the US interest rate (or the foreign yield curve slope is lower than the US yield curve slope), and go long the US bond and short the foreign country bond when the US interest rate is higher than the country's interest rate (or the US yield curve slope is lower than the foreign yield curve slope). Results based on interest rate levels are reported in panel A and results based on interest rate slopes are reported in panel B. The table reports the mean, standard deviation and Sharpe ratio (denoted SR) for the currency excess return (rx^{FX} , middle panel), for the foreign bond excess return on 10-year government bond indices in foreign currency ($rx^{(10),*} - rx^{(10)}$, right panel) and for the foreign bond excess return on 10-year government bond indices in US dollars ($rx^{(10),\$} - rx^{(10)}$, left panel). The holding period is one month. The table also presents summary return statistics for the equally weighted average of the individual country strategies. The slope of the yield curve is measured by the difference between the 10-year yield and the one-month interest rate. The standard errors (denoted SE and reported between parentheses) were generated by bootstrapping 10,000 samples of non-overlapping returns. The log returns are annualized. The data are monthly and the sample is 1975:1–2015:12.

equally weighted portfolio in the interest-rate strategy): these are the well-known deviations from UIP, i.e., the carry trade returns. And there are large local currency long-term bond returns (with a Sharpe ratio of 0.51 for an equally weighted portfolio in the interest-rate strategy): these are related to the well-known deviations from the expectation hypothesis, i.e., the local term premia. The two corresponding Sharpe ratios are larger than the one on the US aggregate stock market. However, the currency risk premia and the local term premia cancel out, so the Sharpe ratio on foreign bond returns in dollars is not significant. From the vantage point of a US investor, there is no evidence of economically significant return predictability in long-term foreign bonds.

D. Cross-Sectional Properties of Foreign Bond Returns

After focusing on time-series predictability, we turn now to cross-sectional evidence, as in Lustig and Verdelhan (2007). There is no mechanical link between the time-series and cross-sectional evidence. The time-series regressions test whether a predictor that is higher than its average implies higher returns, while the cross-sectional tests show whether a predictor that is higher in one country than in others implies higher returns in that country (see Hassan and Mano 2019). Our cross-sectional evidence echoes some papers that study the cross section of bond returns: Koijen et al. (2018) and Wu (2012) examine the currency-hedged returns on “carry” portfolios of international bonds, sorted by a proxy for the carry on long-term bonds. But these papers do not examine the interaction between currency and term risk premia, the topic of our paper.

We sort countries into three portfolios on the level of the short-term interest rates or the slope of their yield curves. Portfolios are rebalanced every month and those formed at date t only use information available at that date. Portfolio-level log excess returns are obtained by averaging country-level log excess returns across all countries in the portfolio. We first describe results obtained with the 10-year bond indices, reported in Table 3, and then turn to the zero-coupon bonds to study the whole term structure, presented in Figure 2.

Sorting by Interest Rates.—We start with the currency portfolios sorted by short-term interest rates. In order to focus on the conditional carry trade, we use interest rates in deviation from their past 10-year rolling mean as the sorting variable.² Thus, the first (third) portfolio includes the conditionally low (high) interest rate currencies. Clearly, the classic uncovered interest rate parity condition fails in the cross section: the currencies in the third portfolio only depreciate by 60 basis points per year on average, not enough to offset the 2.65 percent interest rate difference and thus delivering a 2.05 percent return. As a result, average currency excess returns increase from low- to high-interest-rate portfolios, ranging from -0.61 percent to 2.05 percent per year over the last 40 years. Thus, the long-short currency carry trade (invest in Portfolio 3, short Portfolio 1) implemented with Treasury bills delivers an average annual log excess return of 2.04 percent $- (-0.61 \text{ percent}) = 2.65$ percent and a

²The unconditional carry trade goes long in currencies with high average interest rates (Hassan and Mano 2019). That is not the focus of our paper.

TABLE 3—CROSS-SECTIONAL PREDICTABILITY: BOND PORTFOLIOS

Portfolio:	Sorted by short-term interest rates				Sorted by yield curve slopes			
	1	2	3	3 – 1	1	2	3	1 – 3
<i>Panel A. Portfolio characteristics</i>								
Inflation rate mean	2.90 (0.16)	3.45 (0.19)	4.81 (0.23)	1.91 (0.20)	4.89 (0.23)	3.41 (0.19)	2.87 (0.18)	2.02 (0.19)
Inflation rate standard deviation	1.03	1.23	1.48	1.30	1.39	1.16	1.20	1.26
Rating mean	1.45 (0.02)	1.25 (0.02)	1.49 (0.02)	0.04 (0.04)	1.54 (0.02)	1.38 (0.02)	1.28 (0.02)	0.25 (0.03)
Rating (adj. for outlook) mean	1.50 (0.03)	1.37 (0.02)	1.84 (0.02)	0.33 (0.04)	1.84 (0.02)	1.50 (0.02)	1.37 (0.02)	0.47 (0.03)
$y_t^{(10),*} - r_t^{*,f}$ mean	1.52	0.92	−0.44	−1.96	−0.81	0.85	1.96	−2.76
<i>Panel B. Currency excess returns</i>								
$-\Delta s_{t+1}$ mean	−0.44	0.11	−0.60	−0.16	−0.95	0.38	−0.36	−0.58
$r_t^{f,*} - r_t^f$ mean	−0.17	0.54	2.65	2.81	3.35	0.55	−0.88	4.23
rx_{t+1}^{FX} mean	−0.61 (1.35)	0.66 (1.44)	2.04 (1.36)	2.65 (1.14)	2.41 (1.48)	0.92 (1.38)	−1.24 (1.40)	3.65 (1.18)
rx_{t+1}^{FX} Sharpe ratio	−0.07	0.07	0.23	0.36	0.26	0.11	−0.14	0.49
<i>Panel C. Local currency bond excess returns</i>								
$rx_{t+1}^{(10),*}$ mean	3.53 (0.69)	2.60 (0.69)	−0.25 (0.73)	−3.78 (0.77)	−1.01 (0.76)	2.29 (0.69)	4.61 (0.70)	−5.61 (0.74)
$rx_{t+1}^{(10),*}$ Sharpe ratio	0.80	0.58	−0.05	−0.77	−0.21	0.53	1.00	−1.18
<i>Panel D. Dollar bond excess returns</i>								
$rx_{t+1}^{(10),\$}$ mean	2.92 (1.56)	3.26 (1.58)	1.80 (1.57)	−1.12 (1.33)	1.40 (1.64)	3.21 (1.57)	3.36 (1.62)	−1.96 (1.38)
$rx_{t+1}^{(10),\$}$ Sharpe ratio	0.29	0.32	0.18	−0.13	0.14	0.33	0.32	−0.22
$rx_{t+1}^{(10),\$} - rx_{t+1}^{(10)}$ mean	0.14 (1.64)	0.48 (1.64)	−0.98 (1.73)	−1.12 (1.33)	−1.38 (1.81)	0.43 (1.63)	0.59 (1.75)	−1.96 (1.38)

Notes: The countries are sorted by the level of their short-term interest rates in deviation from the 10-year mean into three portfolios (left section) or the slope of their yield curves (right section). The slope of the yield curve is measured by the difference between the 10-year yield and the one-month interest rate. The standard errors (denoted SE and reported in parentheses) were generated by bootstrapping 10,000 samples of non-overlapping returns. The table reports the average inflation rate, the standard deviation of the inflation rate, the average credit rating, the average credit rating adjusted for outlook, the average slope of the yield curve ($y_t^{(10),*} - r_t^{*,f}$), the average change in exchange rates (Δs), the average interest rate difference ($r_t^{f,*} - r_t^f$), the average currency excess return (rx_t^{FX}), the average foreign bond excess return on 10-year government bond indices in foreign currency ($rx_t^{(10),*}$) and in US dollars ($rx_t^{(10),\$}$), as well as the difference between the average foreign bond excess return in US dollars and the average US bond excess return ($rx_t^{(10),\$} - rx_t^{(10)}$). For the excess returns, the table also reports their Sharpe ratios (denoted SR). The holding period is one month. The log returns are annualized. The balanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, and the United Kingdom. The data are monthly and the sample is 1975:1–2015:12.

Sharpe ratio of 0.36 (panel B), higher than the Sharpe ratio on the US S&P500 equity index over the same period.

However, implementing the carry trade with long-term bonds does not yield a similar performance, as local currency bond premia decrease from low- to high-interest rate portfolios, from 3.53 percent to −0.25 percent, implying that the 10-year bond carry trade entails a local currency bond premium of −0.25 percent − 3.53 percent = −3.78 percent (panel C). Therefore, the long-short currency carry trade implemented with long-term government bonds, which is the sum of the two long-short returns above, delivers a negative average return of −3.78 percent + 2.66 percent = −1.12 percent (panel D) that is not statistically significant.

This contrasts with the equivalent trade using Treasury bills. The dollar bond risk premia (-1.12 percent) are statistically different from the carry trade risk premia (2.66 percent) because the local term premia (-3.78 percent) are statistically significant. Investors have no reason to favor the long-term bonds of a particular set of countries on the basis of average returns after converting the returns into the same currency (here, US dollars).

Inflation risk is not a natural explanation for local currency bond excess returns, because inflation is higher and more volatile in countries that are in the last portfolio than in the first portfolio (panel A of Table 3, left section). Similarly, sovereign default risk is not a natural explanation, given that the countries in the first portfolio tend to have slightly better credit ratings than the countries in the last portfolio. The relatively high term premium of 3.53 percent in the first portfolio thus corresponds to relatively low inflation and low default risk.

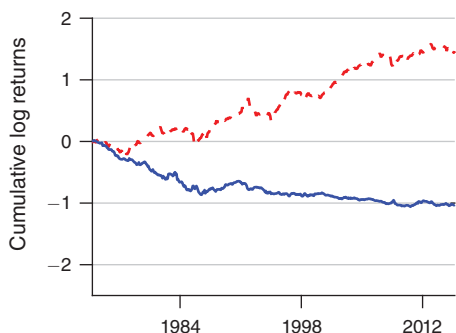
Sorting by Slopes.—Similar results emerge when we sort countries into portfolios by the slope of their yield curves. There is substantial turnover in these portfolios, more so than in the usual interest rate-sorted portfolios. On average, the flat slope currencies (first portfolio) tend to be high interest rate currencies, while the steep slope currencies (third portfolio) tend to be low interest rate currencies. As expected, average currency log excess returns decline from 2.41 percent per annum on the first portfolio (low slope, high short-term interest rates) to -1.24 percent per annum on the third portfolio (high slope, low short-term interest rates) over the last 40 years (panel B). Therefore, a long-short position of investing in flat-yield-curve currencies (Portfolio 1) and shorting steep-yield-curve currencies (Portfolio 3) delivers a currency excess return of 3.65 percent per annum and a Sharpe ratio of 0.49 . Our findings confirm those of Ang and Chen (2010): the slope of the yield curve predicts currency excess returns at the short end of the maturity spectrum.

However, those currency premia are offset by term premia, as local currency bond excess returns and currency excess returns move in opposite directions across portfolios. In particular, the first portfolio produces negative bond average excess returns of -1.01 percent per year, compared to 4.61 percent on the third portfolio (panel C), so the slope carry trade generates an average local currency bond excess return of -5.61 percent per year. This result is not mechanical: the spread in the slopes is about one-half of the spread in local currency excess returns. The corresponding average dollar bond excess returns range from 1.40 percent to 3.36 percent, so the slope carry trade implemented with 10-year bonds delivers an average annual dollar excess return of -1.96 percent (panel D), which is not statistically significant.

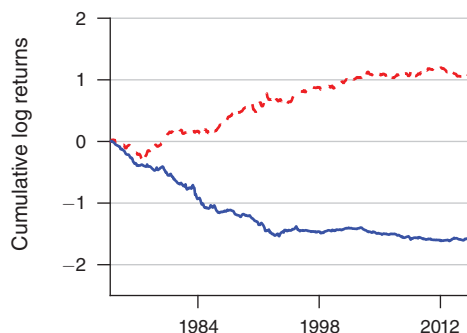
Importantly, this strategy involves long positions in bonds issued by countries with slightly lower average inflation, lower inflation volatility, and slightly better average sovereign credit rating, despite the higher term premium. Therefore, the offsetting effect of local currency term premia is unlikely to be due to inflation or credit risk.

Looking across Subsamples.—To summarize the portfolio results and test their robustness across subsamples, Figure 1 plots the cumulative returns on interest rate and slope carry strategies over the entire sample. The full line is the cumulative local currency bond excess return (in logs), while the dashed line is the cumulative

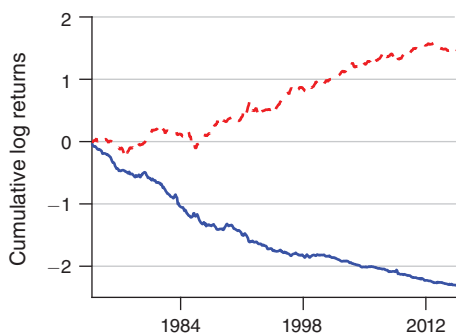
Panel A. Sorting on interest rate levels



Panel B. Sorting on interest rate deviations



Panel C. Sorting on yield curve slope levels



Panel D. Sorting on yield curve slope deviations

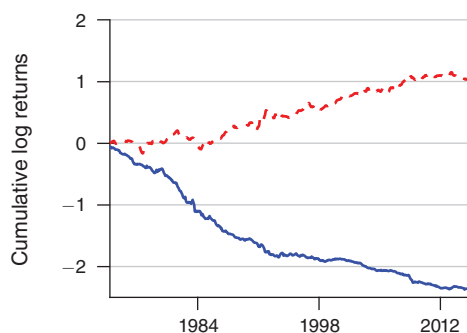


FIGURE 1. FX PREMIA AND LOCAL CURRENCY BOND PREMIA

Notes: The figure shows the cumulative log excess returns on interest rate and slope carry investment strategies that go long in high interest rate (at slope) currencies and short low interest rate (steep slope) currencies. The full line is the cumulative log currency excess return. The dashed line is the cumulative log local currency bond excess return. The cumulative log dollar excess return is represented by the sum of these two lines. At each date t , the countries are sorted by the interest rate (slope of the yield curves) into three portfolios. The slope of the yield curve is measured by the difference between the ten-year yield and the three-month interest rate at date t . The holding period is one month. The balanced panel consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, and the United Kingdom. The data are monthly and the sample is 1975:1–2015:12.

currency excess return (in logs). The cumulative dollar excess return is the sum of these two (not shown). Panel A sorts by interest rates and panel B sorts by interest rate deviations. Panels C and D sort by slope and slope deviations, respectively. Overall, the same patterns reappear in each of these plots, but there are three noticeable differences.

First, when we sort on interest rates, the offsetting effect of local currency bond excess returns is slightly weaker, and the currency excess returns are larger, than when we sort on interest rate deviations. The latter sort only captures the conditional carry premium, while the former captures the entire carry premium. The carry trade risk premium is somewhat larger when sorting on interest rates, as documented by Lustig, Roussanov, and Verdelhan (2011) and Hassan and Mano (2019), as these sorts capture both the conditional carry trade premium (long in currencies with currently high interest rates) and the unconditional carry trade premium (long in

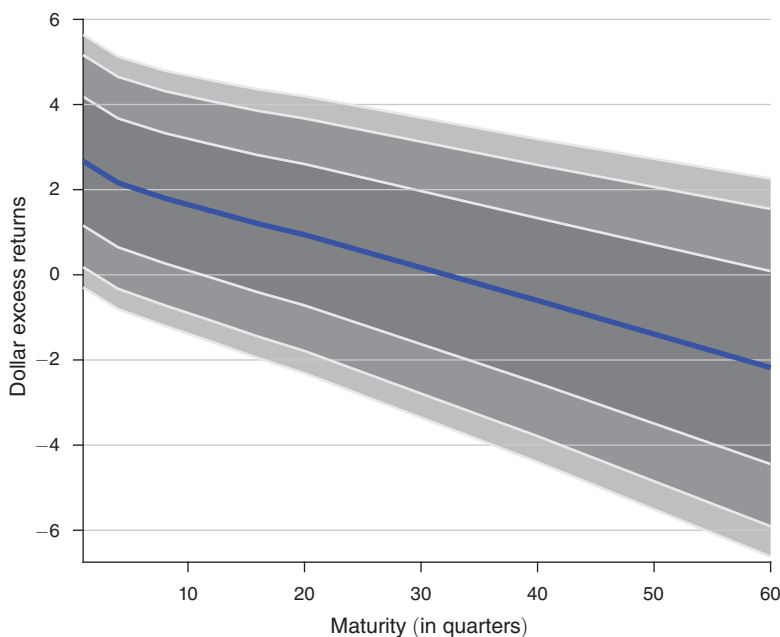


FIGURE 2. LONG-MINUS-SHORT FOREIGN BOND RISK PREMIA IN US DOLLARS

Notes: The figure shows the dollar log excess returns as a function of the bond maturities. Dollar excess returns correspond to the holding period returns expressed in US dollars of investment strategies that go long and short foreign bonds of different countries. The unbalanced panel of countries consists of Australia, Canada, Japan, Germany, Norway, New Zealand, Sweden, Switzerland, and the United Kingdom. At each date t , the countries are sorted by the slope of their yield curves into three portfolios. The first portfolio contains countries with flat yield curves while the last portfolio contains countries with steep yield curves. The slope of the yield curve is measured by the difference between the ten-year yield and the three-month interest rate at date t . The holding period is one quarter. The returns are annualized. The dark shaded area corresponds to one-standard-error bands around the point estimates. The gray and light gray shaded areas correspond to the 90 percent and 95 percent confidence intervals. Standard deviations are obtained by bootstrapping 10,000 samples of non-overlapping returns. Zero-coupon data are monthly, and the sample window is 1985:4–2015:12.

currencies with high average interest rates). Since our paper is about the conditional currency carry trade, we focus mainly on the demeaned sorts. Second, when we use interest rate (deviations) sorts, the negative contribution of local currency bond premia weakens starting in the mid-1980s (1990s), compared to the earlier part of the sample. This is not surprising given that interest rates started to converge across G10 countries in the second part of the sample (Wright 2011). In other words, when G10 countries have similar low interest rates, these rates do not predict large differences in term premia across countries. Third, the offsetting effect of local currency bond premia is larger when we sort on the slope of the yield curve, and it does not weaken in the second part of the sample.

Looking across Maturities.—The results we discussed previously focus on the 10-year maturity. We now turn to the full maturity spectrum, using the zero-coupon bond dataset. The panel is unbalanced, and because of data limitations, we can only examine three-month holding period returns over the 1985:4–2015:12 sample, which mitigates predictability (1985:4 is the first month for which we have data for at least three foreign bonds). As a result, the standard errors are larger than in

the benchmark sample. Building on the previous results, countries are sorted by the slope of their term structures because the slope appears as the strongest predictor of both currency and term premia over this sample. We sort only on the slope of the yield curve, not on the slope deviation from its 10-year mean, because of the shorter sample.

Our findings are presented graphically in Figure 2, which shows the dollar log excess returns as a function of the bond maturity, using the same set of funding and investment currencies. Investing in the short-maturity bills of countries with flat yield curves (mostly high short-term interest rate countries), while borrowing at the same horizon in countries with steep yield curves (mostly low short-term interest rate countries) leads to an average dollar excess return of 2.67 percent per year (with a standard error of 1.50 percent). This is the slope version of the standard currency carry trade. However, when we implement the same strategy using longer maturity bonds instead of short-term bills, the dollar excess return decreases monotonically as the maturity of the bonds increases. The zero-coupon findings confirm our previous results and seem to rule out measurement error in the 10-year coupon bond indices as an explanation. At the long end (15-year maturity), the bond term premium more than offsets the currency premium, so the slope carry trade yields a (nonsignificant) average annual dollar return of -2.18 percent (with a standard error of 2.28 percent). The average excess returns at the long-end of the yield curve are statistically different from those at the short-end: the difference between those returns correspond to the local term premia, which are equal to -4.85 percent with a standard error of 1.82 percent. Therefore, carry trade strategies that yield positive average excess returns when implemented with short-maturity bonds yield lower (or even negative) excess returns when implemented using long-maturity bonds.³

E. Robustness Checks

We consider many robustness checks, both regarding our time-series results and our cross-sectional results. The time-series predictability robustness checks are reported in online Appendix Section I, whereas the cross-sectional portfolio robustness checks can be found in online Appendix Section II.

As regards time-series predictability, we consider predictability regressions using inflation and sovereign credit as additional controls, we consider an alternative decomposition of dollar bond returns into an exchange rate component and a local currency bond return difference, we include predictability results with GBP as the base currency and we report predictability results using different time-windows (1983:10–2007:12, 1975:1–2007:12, 1983:10–2015:12) and investment horizons (three months). We find that our main results are robust to those alternative specifications.

As regards cross-sectional currency portfolios, we consider different lengths of the bond holding period (3 and 12 months), different time windows, different samples of countries, sorts by (non-demeaned) interest rate levels, and other

³When we use interest rate sorts, the term structure is flat: the carry premium is 3.71 percent per annum (with a standard error of 1.80 percent), while the local 15-year bond premium is only -0.21 percent per annum (with a standard error of 1.76 percent), so the dollar bond premium at the 15-year maturity is 3.50 percent (with a standard error of 2.32 percent). As noted in the previous subsection and apparent in panel A of Figure 1, interest rates in levels do not predict bond excess returns in the cross section over the 1985:4–2015:12 sample.

potential explanations of excess returns. Our results appear robust to the choice of the bond holding period and across time windows. Furthermore, our results appear robust across several samples of countries. Introducing more countries adds power to the experiment, but forces us to consider less liquid and more default-prone bond markets. In what may be of particular interest, we show that inflation risk or credit risk are unlikely explanations for differences in term premia even in larger sets of countries and different time windows. For both our benchmark sample (1975:1–2015:12) and a longer sample (1951:1–2015:12), term premia are higher in low inflation countries. Thus, assuming that there is a positive association between average inflation rates and exposure to inflation risk, inflation risk does not account for our findings. This is true not only for our benchmark set of countries, but also for the extended sets of countries. Similarly, the cross-sectional patterns in term premia we observe empirically are not likely to be due to sovereign default risk. As seen in Table 3, countries with high average local currency bond premia have average credit ratings (both unadjusted and adjusted for outlook) that are lower than or similar to the ratings of countries with low average local currency bond premia. That finding is robust to considering different sample periods: it holds both in the long sample period (1951:1–2015:12) and in the 1989:7–2015:12 period, during which full ratings are available. Our dataset does not include transaction costs. Adrian, Fleming, and Vogt (2017) show that liquidity measures on US government bonds (bid-ask spread, depth, and price impact) all worsen with the maturity. In a liquidity-as-characteristic model, investors would want to be compensated for that, and thus one should observe a bigger average excess return on long versus short-term bonds. Liquidity issues would thus go against our findings. Therefore, we find no empirical evidence in favor of an inflation-, credit-, or liquidity-based explanation of our findings in this sample of G10 currencies, and thus pursue a simple interest rate risk interpretation that seems the most relevant, especially over the last 30 years.

II. The Term Structure of Currency Carry Trade Risk Premia: A Challenge

In this section, we show that the downward-sloping term structure of currency carry trade risk premia is a challenge even for a reduced-form model.

A. The Necessary Condition for Replicating the UIP Puzzle

We start with a review of a key necessary condition for replicating the UIP puzzle, established by Bekaert (1996) and Bansal (1997) and generalized by Backus, Foresi, and Telmer (2001). To do so, we first introduce some additional notation.

Pricing Kernels and Stochastic Discount Factors.—The nominal pricing kernel is denoted by $\Lambda_t(\varpi)$; it corresponds to the marginal value of a currency unit delivered at time t in the state of the world ϖ . The nominal SDF M is the growth rate of the pricing kernel: $M_{t+1} = \Lambda_{t+1}/\Lambda_t$. Therefore, the price of a zero-coupon bond that promises one currency unit k periods into the future is given by

$$(4) \quad P_t^{(k)} = E_t \left(\frac{\Lambda_{t+k}}{\Lambda_t} \right).$$

SDF Entropy.—SDFs are volatile, but not necessarily normally distributed. In order to measure the time-variation in their volatility, it is convenient to use entropy, rather than variance (Backus, Chernov, and Zin 2014). The conditional entropy L_t of any random variable X_{t+1} is defined as

$$(5) \quad L_t(X_{t+1}) = \log E_t(X_{t+1}) - E_t(\log X_{t+1}).$$

If X_{t+1} is conditionally lognormally distributed, then the conditional entropy is equal to one-half of the conditional variance of the log of X_{t+1} : $L_t(X_{t+1}) = (1/2)\text{var}_t(\log X_{t+1})$. If X_{t+1} is not conditionally lognormal, the entropy also depends on the higher moments: $L_t(X_{t+1}) = \kappa_{2t}/2! + \kappa_{3t}/3! + \kappa_{4t}/4! + \dots$, where $\{\kappa_{it}\}_{i=2}^{\infty}$ are the cumulants of $\log X_{t+1}$.⁴

Exchange Rates.—When markets are complete, the change in the nominal exchange rate corresponds to the ratio of the domestic to foreign nominal SDFs,

$$(6) \quad \frac{S_{t+1}}{S_t} = \frac{\Lambda_{t+1}}{\Lambda_t} \frac{\Lambda_t^*}{\Lambda_{t+1}^*}.$$

The no-arbitrage definition of the exchange rate comes directly from the Euler equations of the domestic and foreign investors, for any asset return R^* expressed in foreign currency terms: $E_t[M_{t+1} R_{t+1}^* S_t / S_{t+1}] = 1$ and $E_t[M_{t+1}^* R_{t+1}^*] = 1$.

Currency Risk Premia.—As Bekaert (1996) and Bansal (1997) show, in models with lognormally distributed SDFs the conditional log currency risk premium $E_t(rx^{FX})$ equals the half difference between the conditional variance of the log domestic and foreign SDFs. This result can be generalized to non-Gaussian economies. When higher moments matter and markets are complete, the currency risk premium is equal to the difference between the conditional entropy of two SDFs (Backus, Foresi, and Telmer 2001),

$$(7) \quad E_t(rx_{t+1}^{FX}) = r_t^{f,*} - r_t^f - E_t(\Delta s_{t+1}) = L_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) - L_t\left(\frac{\Lambda_{t+1}^*}{\Lambda_t^*}\right).$$

According to the UIP condition, expected changes in exchange rates should be equal to the difference between the home and foreign interest rates and, thus, the currency risk premium should be zero. In the data, the currency risk premium is as large as the equity risk premium. Any complete market model that addresses the UIP puzzle must thus satisfy a simple necessary condition: high interest rate countries must exhibit relatively less volatile SDFs. In the absence of differences in conditional volatility, complete market models are unable to generate a currency risk premium and the UIP counterfactually holds in the model economy.

Why is the downward term structure of currency carry trade risk premia a challenge for arbitrage-free models? Intuitively, the models need to depart from risk neutrality in order to account for the UIP deviations at the short end of the yield

⁴The literature on disaster risk in currency markets shows that higher order moments are critical for understanding currency returns (see Brunnermeier, Nagel, and Pedersen 2009; Gourio, Siemer, and Verdelhan 2013; Farhi et al. 2013; Chernov, Graveline, and Zviadadze 2011).

curve and large carry trade risk premia. Yet, for the exact same investment horizon, the models need to deliver zero risk premia at the long end of the yield curve, thus behaving as if investors are risk-neutral. In the rest of the paper, we highlight this tension and describe necessary conditions for arbitrage-free models to replicate our empirical evidence.

B. An Example: A Reduced-Form Factor Model

We start by showing that even a flexible, N -country reduced-form model calibrated to match the currency carry trade risk premia does not replicate the evidence on long-term bonds. Several two-country models satisfy the condition described in equation (7) and thus replicate the failure of the UIP condition, but they cannot replicate the portfolio evidence on carry trade risk premia. The reason is simple: when those models are extended to multiple countries, investors in the models can diversify away the country-specific exchange rate risk and there are no cross-sectional differences in carry trade returns across portfolios. To the best of our knowledge, only two models can so far replicate the portfolio evidence on carry trades: the multi-country long-run risk model of Colacito et al. (2018) and the multi-country reduced-form factor model of Lustig, Roussanov, and Verdelhan (2011). We focus on the latter because of its flexibility and close forms, and revisit the long-run risk model in online Appendix Section V, along with other explanations of the UIP puzzle. Moreover, in Section IV of the online Appendix, we cover a wide range of term structure models, from the seminal Vasicek (1977) model to the classic Cox, Ingersoll, and Ross (1985) model and to the most recent, multi-factor dynamic term structure models. To save space, we focus here on their most recent international finance version, illustrated in Lustig, Roussanov, and Verdelhan (2014). To replicate the portfolio evidence on carry trades, Lustig, Roussanov, and Verdelhan (2011, 2014) show that no-arbitrage models need to incorporate global shocks to the SDFs along with country heterogeneity in the exposure to those shocks. Following Lustig, Roussanov, and Verdelhan (2014), we consider a world with N countries and currencies in a setup inspired by classic term structure models.

Using their benchmark calibration, we calculate the model-implied term structure of currency risk premia when implementing the slope carry trade strategy (invest in low yield slope currencies, short the high yield slope interest rate currencies). This is very similar to investing in high interest rate countries while borrowing in low interest rate countries. The simulation details are provided in the online Appendix. Figure 3, obtained with simulated data, is the model counterpart to Figure 2, obtained with actual data. A clear message emerges: while this model produces UIP deviations (and thus currency risk premia) at the short end of the yield curve, the model produces a flat term structure of currency carry trade risk premia. We turn now to a novel necessary condition that dynamic asset pricing models need to satisfy in order to generate a downward-sloping term structure.

III. Foreign Long-Term Bond Returns and the Properties of SDFs

In this section, we derive a novel, preference-free necessary condition that complete market models need to satisfy in order to reproduce the downward-sloping

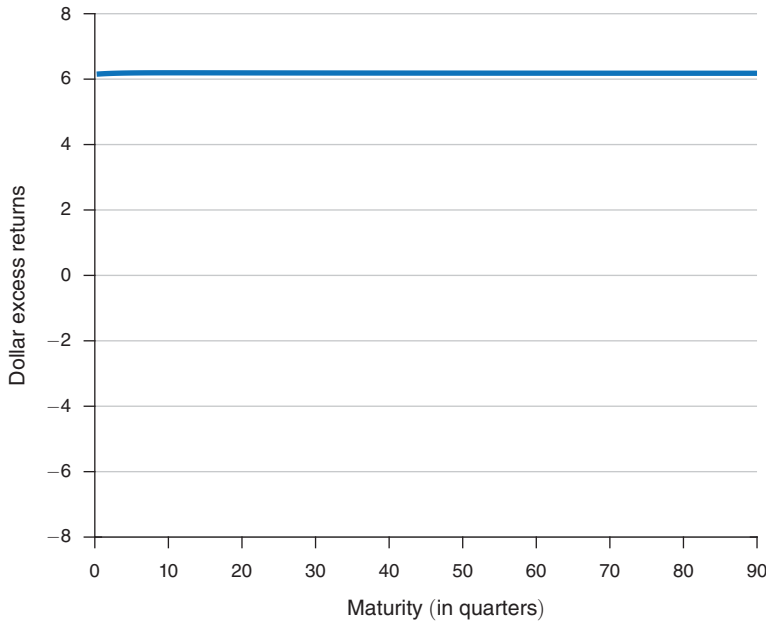


FIGURE 3. SIMULATED LONG-MINUS-SHORT FOREIGN BOND RISK PREMIA IN US DOLLARS

Notes: The figure shows the simulated average dollar log excess return of the slope carry trade strategy as a function of the bond maturities in the reduced-form model of Lustig, Roussanov, and Verdelhan (2014). At each date t , currencies are sorted into three portfolios by the slope of their yield curve (measured as the difference between the ten-year and the three-month yields). The first portfolio contains the currencies of countries with low yield slopes, while the third portfolio contains the currencies of countries with high yield slope. The slope carry trade strategy invests in the first portfolio and shorts the third portfolio. The model is simulated at the monthly frequency. The holding period is one month and returns are annualized.

term structure of currency carry trade risk premia. To do so, we first review a useful decomposition of the pricing kernel.

A. Pricing Kernel Decomposition

Our results build on the Alvarez and Jermann (2005) decomposition of the pricing kernel Λ_t into a permanent component Λ_t^P and a transitory component Λ_t^T using the price of the long-term bond:

$$(8) \quad \Lambda_t = \Lambda_t^P \Lambda_t^T, \quad \text{where } \Lambda_t^T = \lim_{k \rightarrow \infty} \frac{\delta^{t+k}}{P_t^{(k)}},$$

where the constant δ is chosen to satisfy the following regularity condition: $0 < \lim_{k \rightarrow \infty} P_t^{(k)} / \delta^k < \infty$ for all t . Note that Λ_t^P is equal to

$$(9) \quad \Lambda_t^P = \lim_{k \rightarrow \infty} \frac{P_t^{(k)}}{\delta^{t+k}} \Lambda_t = \lim_{k \rightarrow \infty} \frac{E_t(\Lambda_{t+k})}{\delta^{t+k}}.$$

The second regularity condition ensures that the expression above is a martingale. Alvarez and Jermann (2005) assume that, for each $t + 1$, there exists a random

variable x_{t+1} with finite expected value $E_t(x_{t+1})$ such that almost surely $(\Lambda_{t+1}/\delta^{t+1})(P_{t+1}^{(k)}/\delta^k) \leq x_{t+1}$ for all k . Under those regularity conditions, the infinite-maturity bond return is

$$(10) \quad R_{t+1}^{(\infty)} = \lim_{k \rightarrow \infty} R_{t+1}^{(k)} = \lim_{k \rightarrow \infty} P_{t+1}^{(k-1)}/P_t^{(k)} = \frac{\Lambda_t^T}{\Lambda_{t+1}^T}.$$

The permanent component, Λ_t^P , is a martingale and is an important part of the pricing kernel: Alvarez and Jermann (2005) derive a lower bound of its volatility and, given the size of the equity premium relative to the term premium, conclude that it accounts for most of the SDF volatility.⁵ In other words, a lot of persistence in the pricing kernel is needed to jointly deliver a low term premium and a high equity premium. Throughout this paper we assume that stochastic discount factors Λ_{t+1}/Λ_t and returns R_{t+1} are jointly stationary.

B. Main Preference-Free Result on Long-Term Bond Returns

We now use this pricing kernel decomposition to understand the properties of the dollar returns of long-term bonds. Recall that the dollar term premium on a foreign bond position, denoted by $E_t[rx_{t+1}^{(k),\$}]$, can be expressed as the sum of foreign term premium in local currency terms, $E_t[rx_{t+1}^{(k),*}]$, plus a currency risk premium, $E_t[rx_{t+1}^{FX}] = r_t^{f,*} - r_t^f - E_t[\Delta s_{t+1}]$. Here, we consider the dollar term premium of an infinite-maturity foreign bond, so we let $k \rightarrow \infty$.

PROPOSITION 1: *If financial markets are complete, the foreign term premium on the long-term bond in dollars is equal to the domestic term premium plus the difference between the domestic and foreign entropies of the permanent components of the pricing kernels:*

$$(11) \quad E_t[rx_{t+1}^{(\infty),\$}] = E_t[rx_{t+1}^{(\infty)}] + L_t\left(\frac{\Lambda_{t+1}^P}{\Lambda_t^P}\right) - L_t\left(\frac{\Lambda_{t+1}^{P,*}}{\Lambda_t^{P,*}}\right).$$

To intuitively link the long-run properties of pricing kernels to foreign bond returns and exchange rates, let us consider the simple benchmark of countries represented by stand-in agents with power utility and i.i.d. consumption growth rates. In that case, all SDF shocks are permanent ($\Lambda_t = \Lambda_t^P$ for all t). As we shall see, such model is counterfactual. In this model, the risk-free rate is constant, so bonds of different maturities offer the same returns. Foreign bond investments differ from domestic bond investments only because of the presence of exchange rate risk

⁵ Proposition 2 in Alvarez and Jermann (2005) establishes that $L_t(\Lambda_{t+1}^P/\Lambda_t^P) \geq E_t \log R_{t+1} - \log R_{t+1}^\infty$ for any return R_{t+1} and that $L(\Lambda_{t+1}^P/\Lambda_t^P)/L(\Lambda_{t+1}/\Lambda_t) \geq \min\{1, (E \log R_{t+1}/R_t^f - E \log R_{t+1}^\infty/R_t^f)/(E \log R_{t+1}/R_t^f + L(1/R_t^f))\}$ for any positive return R_{t+1} such that $E \log R_{t+1}/R_t^f + L(1/R_t^f) > 0$. Alvarez and Jermann (2005, Table 2, p. 1989) takes the latter expression to the data and reports several lower bounds for the relative variance of the permanent component. These lower bounds, obtained with either yields or holding-period returns on long-term bonds, range from 0.76 to 1.11. Thus, the variance of the permanent component is at least 76 percent of the total variance of the SDF.

and, since consumption growth rates are i.i.d, exchange rates are stationary in changes but not in levels. Finally, carry trade excess returns are the same at the short end (see equation (7)) and at the long end (see equation (11)) of the yield curve, so the term structure of currency carry trade risk premia is flat. A power utility model with only permanent shocks cannot match the facts.

Let us now turn to the opposite case: a model without permanent shocks in the SDF ($\Lambda_t = \Lambda_t^T$ for all t). In case of an adverse temporary innovation to the foreign pricing kernel, the foreign currency appreciates, so a domestic position in the foreign bond experiences a capital gain. However, this capital gain is exactly offset by the capital loss suffered on the long-term bond as a result of the increase in foreign interest rates. Hence, interest rate exposure completely hedges the temporary component of the currency risk exposure. In this case, as equation (11) shows, the long-term bond risk premium in dollars should be equal to the domestic term premium.

Beyond these two polar cases, Proposition 1 shows that in order to have differences across countries in bond risk premia, once converted in the same currency, no-arbitrage models need conditional entropy differences in the permanent component of their pricing kernels. If the domestic and foreign pricing kernels have identical conditional entropy, then high local currency term premia are always associated with low currency risk premia and vice-versa, so dollar term premia are identical across currencies.

Proposition 1 is thus the bond equivalent to the usual currency carry trade condition. We gather them below to emphasize their similarities:

$$(12) \quad E_t(rx_{t+1}^{FX}) = L_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) - L_t\left(\frac{\Lambda_{t+1}^*}{\Lambda_t^*}\right),$$

$$(13) \quad E_t[rx_{t+1}^{(\infty),\$}] - E_t[rx_{t+1}^{(\infty)}] = L_t\left(\frac{\Lambda_{t+1}^{\mathcal{P}}}{\Lambda_t^{\mathcal{P}}}\right) - L_t\left(\frac{\Lambda_{t+1}^{\mathcal{P},*}}{\Lambda_t^{\mathcal{P},*}}\right).$$

To reproduce large currency carry trade risk premia, no-arbitrage models need large differences in the volatilities of their SDFs. To reproduce the absence of dollar bond risk premia, no-arbitrage models need to feature the same volatilities of the martingale components of their SDFs. As we shall see, this condition is a key tool to assess existing international finance models.

C. Additional Assumptions and Interpretation

Dynamic asset pricing models that generate small amounts of dollar return differential predictability may produce moments in small samples that fall within the confidence intervals, but it is useful to have a clear benchmark: ours is no predictability in dollar bond return differentials for long bonds. This has been the null hypothesis in this literature. Our paper shows that this null cannot be rejected at longer maturities.

In order to use Proposition 1 to interpret our empirical findings, two additional assumptions are required.

First, since very long-term bonds are rarely available or liquid, we assume that infinite-maturity bond returns can be approximated in practice by 10- and 15-year bond returns. The same assumption is also present in Alvarez and Jermann (2005); Hansen, Heaton, and Li (2008); Hansen and Scheinkman (2009); and Hansen (2012). It is supported by the simulation of the state-of-the-art Joslin, Singleton and Zhu (2011) term structure model (see online Appendix Section VIII).

Second, we assume that the level and slope of the yield curve summarize all the relevant information that investors use to forecast dollar bond excess returns. Proposition 1 pertains to conditional risk premia and is, thus, relevant for interpreting our empirical time series predictability results and the average excess returns of currency portfolios sorted by conditioning information (the level of the short-term interest rate or the slope of the yield curve). Building portfolios sorted by conditioning variables is a flexible, nonparametric approach to bringing in conditioning information. We cannot definitively rule out the possibility that there are other predictors. In all of the models that we consider, no other predictors exist (except in pathological, knife-edge cases), but this may not be true in other models. In particular, there is a lively empirical debate on whether there are unspanned macro variables that have incremental out-of-sample forecasting power for bond returns (see Bauer and Hamilton 2018, for a thorough evaluation of the empirical evidence).

Under those two assumptions, a simple condition illustrates our empirical findings:

Condition 1: In order for the conditional dollar term premia on infinite-maturity bonds to be identical across countries, when financial markets are complete, the conditional entropy of the permanent SDF component has to be identical across countries: $L_t(\Lambda_{t+1}^{\mathcal{P}}/\Lambda_t^{\mathcal{P}}) = L_t(\Lambda_{t+1}^{\mathcal{P},*}/\Lambda_t^{\mathcal{P},*})$, for all t .

If this condition fails, under Assumptions 1 and 2, portfolios sorted on conditioning variables produce nonzero currency carry trade risk premia at the long end of the term structure, as the conditional dollar term premia of long-maturity bonds differ across countries.⁶ Condition 1 is satisfied when permanent shocks are common across countries ($\Lambda_{t+1}^{\mathcal{P}} = \Lambda_{t+1}^{\mathcal{P},*}$ for all t) and thus, in the absence of permanent shocks, when exchange rates are stationary in levels (up to a deterministic time trend). But note that the stationarity is sufficient but not necessary to satisfy Condition 1.

To develop some intuition for this condition, we rely on an example from Alvarez and Jermann (2005), who consider a model with conditionally log-normally distributed pricing kernels driven by both permanent and transitory shocks.

⁶For some countries (Australia, Canada, and Sweden), time-series regressions show that the yield slopes predict dollar bond returns with the “wrong” sign: while an increase in the yield slope decreases short-bond carry trade excess returns, it increases the dollar long-bond excess returns (cf. Table 1). To match this particular evidence, one may replace Condition 1 with the following condition: if $E_t(rx_{t+1}^{FX}) = L_t(\Lambda_{t+1}/\Lambda_t) - L_t(\Lambda_{t+1}^*/\Lambda_t^*) > 0$, then $E_t[rx_{t+1}^{(\infty),\$}] - E_t[rx_{t+1}^{(\infty)}] = L_t(\Lambda_{t+1}^{\mathcal{P}}/\Lambda_t^{\mathcal{P}}) - L_t(\Lambda_{t+1}^{\mathcal{P},*}/\Lambda_t^{\mathcal{P},*}) < 0$. We do not study this stricter condition as the amount of predictability on long-bond dollar excess returns is not economically significant, as can be seen in Table 2.

Example 1: Consider the following pricing kernel (Alvarez and Jermann 2005):

$$(14) \quad \log \Lambda_{t+1}^P = -\frac{1}{2}\sigma_P^2 + \log \Lambda_t^P + \varepsilon_{t+1}^P,$$

$$(15) \quad \log \Lambda_{t+1}^T = \log \beta^{t+1} + \sum_{i=0}^{\infty} \alpha_i \varepsilon_{t+1-i}^T,$$

where α is a square summable sequence, and ε^P and ε^T are serially independent and normally distributed random variables with mean zero, variance σ_P^2 and σ_T^2 , respectively, and covariance σ_{TP} . A similar decomposition applies to the foreign pricing kernel.

In this economy, Alvarez and Jermann (2005) show that the domestic term premium is given by the following expression: $E_t[rx_{t+1}^{(\infty)}] = (1/2)\alpha_0^2\sigma_T^2 + \alpha_0\sigma_{TP}$. Only transitory risk is priced in the market for long-maturity bonds: when marginal utility is transitorily high, interest rates increase because the representative agent wants to borrow, so long-term bonds suffer a capital loss. Permanent shocks to marginal utility do not affect the prices of long-term bonds at all. Similarly, the foreign term premium, in local currency terms, is $E_t[rx_{t+1}^{(\infty),*}] = (1/2)(\alpha_0^*\sigma_T^*)^2 + \alpha_0^*\sigma_{TP}^*$. The currency risk premium is the difference in the two countries' conditional SDF entropy:

$$(16) \quad E_t[rx_{t+1}^{FX}] = r_t^{f,*} - r_t^f - E_t[\Delta s_{t+1}]$$

$$= \frac{1}{2}(\alpha_0^2\sigma_T^2 + 2\alpha_0\sigma_{TP} + \sigma_P^2) - \frac{1}{2}((\alpha_0^*\sigma_T^*)^2 + 2\alpha_0^*\sigma_{TP}^* + (\sigma_P^*)^2).$$

As a result, the foreign term premium in dollars, given by equation (7), is

$$(17) \quad E_t[rx_{t+1}^{(\infty),\$}] = E_t[rx_{t+1}^{(\infty),*}] + E_t[rx_t^{FX}]$$

$$= \frac{1}{2}\alpha_0^2\sigma_T^2 + \alpha_0\sigma_{TP} + \frac{1}{2}(\sigma_P^2 - (\sigma_P^*)^2).$$

In the Alvarez and Jermann (2005) example, Condition 1 is satisfied, provided that $\sigma_P^2 = (\sigma_P^*)^2$. Then the foreign term premium in dollars equals the domestic term premium:

$$(18) \quad E_t[rx_{t+1}^{(\infty),\$}] = \frac{1}{2}\alpha_0^2\sigma_T^2 + \alpha_0\sigma_{TP} = E_t[rx_{t+1}^{(\infty)}].$$

This example illustrates our theoretical and empirical findings. It shows how SDFs can deliver carry trade risk premia with Treasury bills, when $\alpha_0 \neq \alpha_0^*$, $\sigma_T \neq \sigma_T^*$ or $\sigma_{TP} \neq \sigma_{TP}^*$, while producing no carry trade risk premia with long-term bonds when $\sigma_P = \sigma_P^*$. It also shows that exchange rate stationarity (i.e., when $\Lambda_t^P = \Lambda_t^{P,*}$ and) is a sufficient but not a necessary condition to produce no carry trade risk premia with long-term bonds. This example, however, lacks the time variation in risk

premia that has been extensively documented in equity, bond, and currency markets. We turn now to a second example, with time-varying risk premia.

In order to illustrate the SDF decomposition and Condition 1 in a very transparent setting, we focus here on a simple two-country version of the one-factor Cox, Ingersoll, and Ross (1985) model with only one kind of shock per country.

Example 2: The two-country Cox, Ingersoll and Ross (1985) model is defined by the following law of motions for the SDFs:

$$(19) \quad -\log \frac{\Lambda_{t+1}}{\Lambda_t} = \alpha + \chi z_t + \sqrt{\gamma z_t} u_{t+1},$$

$$(20) \quad z_{t+1} = (1 - \phi)\theta + \phi z_t - \sigma \sqrt{z_t} u_{t+1},$$

where a similar law of motion applies to the foreign SDF (with foreign variables and parameters denoted with an $*$), and where z_t and z_t^* are the two state variables that govern the volatilities of the normal shocks u_{t+1} and u_{t+1}^* .

The domestic risk-free rate is given by $r_t^f = \alpha + (\chi - (1/2)\gamma)z_t$. The log bond prices are affine in the state variable z : $p_t^{(n)} = -B_0^n - B_1^n z_t$, where the bond price coefficients evolve according to the second-order difference equations:

$$(21) \quad B_0^n = \alpha + B_0^{n-1} + B_1^{n-1}(1 - \phi)\theta,$$

$$(22) \quad B_1^n = \chi - \frac{1}{2}\gamma + B_1^{n-1}\phi - \frac{1}{2}(B_1^{n-1})^2\sigma^2 + \sigma\sqrt{\gamma}B_1^{n-1}.$$

Therefore, the transitory component of the pricing kernel is $\Lambda_t^T = \lim_{n \rightarrow \infty} \delta^{t+n} / P_t^{(n)} = \lim_{n \rightarrow \infty} (\delta^{t+n} e^{B_0^n + B_1^n z_t})$, so the temporary and martingale components of the SDF are

$$(23) \quad \frac{\Lambda_{t+1}^T}{\Lambda_t^T} = \delta e^{B_1^\infty[(\phi-1)(z_t-\theta) - \sigma\sqrt{z_t}u_{t+1}]},$$

$$(24) \quad \begin{aligned} \frac{\Lambda_{t+1}^P}{\Lambda_t^P} &= \frac{\Lambda_{t+1}}{\Lambda_t} \left(\frac{\Lambda_{t+1}^T}{\Lambda_t^T} \right)^{-1} \\ &= \frac{1}{\delta} e^{-\alpha - \chi z_t - B_1^\infty(\phi-1)(z_t-\theta) - \sqrt{\gamma} z_t u_{t+1} + B_1^\infty \sigma \sqrt{z_t} u_{t+1}}, \end{aligned}$$

where the constant δ is chosen in order to satisfy: $0 < \lim_{n \rightarrow \infty} P_t^{(n)} / \delta^n < \infty$. The limit of $B_0^n - B_0^{n-1}$ is finite: $\lim_{n \rightarrow \infty} (B_0^n - B_0^{n-1}) = \alpha + B_1^\infty(1 - \phi)\theta$, where B_1^∞ is defined implicitly in the second-order equation $B_1^\infty = \chi - (1/2)\gamma + B_1^\infty\phi - (1/2)(B_1^\infty)^2\sigma^2 + \sigma\sqrt{\gamma}B_1^\infty$. As a result, B_0^n grows at a linear rate in the limit. We choose the constant δ to offset the growth in B_0^n as n becomes very large. Setting $\delta = e^{-\alpha - B_1^\infty(1-\phi)\theta}$ guarantees that Assumption 1 in

Alvarez and Jermann (2005) is satisfied. The temporary component of the pricing kernel is thus equal to $\Lambda_t^T = \delta^t e^{B_1^\infty z_t}$.

This reduced-form model shows the difference between unconditional and conditional risk premia. In the case of a symmetric model ($\sigma = \sigma^*, \phi = \phi^*, \theta = \theta^*$), the unconditional risk currency risk premium is zero: $E[rx_{t+1}^{FX}] = (1/2)\gamma(E[z_t] - E[z_t^*]) = 0$, while the conditional currency risk premium is not: $E_t[rx_{t+1}^{FX}] = (1/2)\gamma(z_t - z_t^*) \neq 0$. The conditional risk premium moves with the two state variables z_t and z_t^* . These variables take different values across countries because countries experience different shocks. Our work is about the conditional risk premium: the portfolios are built by sorting on the level and slope of interest rates, which in this model are driven by the state variables z_t and z_t^* . When taking the average of all returns in the high interest rate portfolio for example, we are averaging over low values of the state variables z_t , not over all possible values of z_t . In this symmetric model, the average currency risk premium obtained by simply averaging all returns would be zero. Yet, as its simulation shows in the previous section, the average currency risk premium obtained on the high interest rate portfolio (for example) is as large as in the data. Our portfolio and predictability tests thus are about conditional risk premia, as is our preference-free condition.

In this two-country model, Condition 1 requires that

$$(25) \quad (\sqrt{\gamma} - B_1^\infty \sigma) z_t = (\sqrt{\gamma^*} - B_1^{\infty*} \sigma^*) z_t^*.$$

We leave the study of these restrictions for the online Appendix because this one-factor model is too simple. As noted in Campbell (2017), the Cox, Ingersoll, and Ross (1985) model implies that large yield spreads predict low term premia, contrary to the data. Condition 1, however, is a diagnostic tool that can be applied to richer models that are not subject to this criticism. In the online Appendix, we derive parametric restrictions to implement Condition 1 in four classes of dynamic term structure models, from the simple one-factor Vasicek (1977) and Cox, Ingersoll, and Ross (1985) models to their multi-factor versions. We also sketch there a multi-factor version of the Cox, Ingersoll, and Ross (1985) model with “temporary” and “permanent” factors that can reproduce the downward term structure of carry risk premia.

In the online Appendix, we also consider the current leading structural macro-finance models. We focus on models of the real SDF, because (i) there is no evidence to suggest that inflation risk can account for UIP deviations, and (ii) inflation risk does not seem to account for the cross-country variation in local currency bond excess returns, as shown in Section I. We consider two-country versions of the habit, long-run risk, and disaster risk models. In a nutshell, among the reduced-form term structure models we consider, Condition 1 implies novel parameter restrictions for all models (and in some cases, it rules out all permanent shocks or the time-variation in the price of risk). In order to save space, we summarize the implications of Condition 1 in Table 4. All the intermediary steps to determine the two components of the pricing kernels are reported in the online Appendix. In the habit model with common shocks, the carry trade risk premia and Condition 1 requires that countries exhibit the same product of the risk-aversion and

TABLE 4—CONDITION 1: DYNAMIC ASSET PRICING MODEL SCORECARD

	Symmetric models with country-specific shocks	Asymmetric models with common shocks
External habit model	Condition 1 is satisfied.	Condition 1 is satisfied if $\gamma^2\sigma^2$ is the same across countries.
Long-run risks model	Condition 1 is satisfied only if investors are risk-neutral.	Condition 1 is satisfied only in knife-edge cases.
Disaster model	Condition 1 is never satisfied.	Condition 1 is satisfied if the only heterogeneity across countries is in g_ω .

Notes: This table summarizes whether each class of models can satisfy Condition 1 and what are the potential sources of heterogeneity across countries. The left section of the table focuses on models with only country-specific shocks in which all countries have the same parameters. The right section focuses on models with only common shocks and heterogeneity in the parameters. In the habit model of Campbell and Cochrane (1999), γ denotes the risk aversion coefficient and σ the volatility of consumption growth shocks. In the long-run risk model (Bansal and Yaron 2004, Colacito and Croce 2011, Bansal and Shaliastovich 2013, Engel 2016), Condition 1 is always violated except in knife-edge cases. In the disaster model (Farhi and Gabaix 2016, Gabaix 2012, Wachter 2013), the parameter g_ω governs the country-specific productivity growth rate. The details of these models are in the online Appendix.

the consumption growth volatility parameter, but they can differ in the other parameter values. The long-run risk model satisfies Condition 1 only with common shocks and for knife-edge parameterizations. For the disaster risk model, common shocks are also necessary for Condition 1 to hold and heterogeneity has to be restricted to the country-specific productivity growth rate. Overall, the term structure of carry trade raises the bar for international finance models.

IV. UIP in the Long Run

Examining the conditional moments of one-period returns on long-maturity bonds, the focus of our paper, is not equivalent to studying the moments of long-maturity bond yields in tests of the long-horizon UIP condition. In this section, we show the links and differences between these moments. To do so, we use again the decomposition of the pricing kernel proposed by Alvarez and Jermann (2005). Exchange rate changes can be represented as the product of two components, defined below:

$$(26) \quad \frac{S_{t+1}}{S_t} = \left(\frac{\Lambda_{t+1}^{\mathcal{P}} \Lambda_t^{\mathcal{P},*}}{\Lambda_t^{\mathcal{P}} \Lambda_{t+1}^{\mathcal{P},*}} \right) \left(\frac{\Lambda_{t+1}^{\mathcal{T}} \Lambda_t^{\mathcal{T},*}}{\Lambda_t^{\mathcal{T}} \Lambda_{t+1}^{\mathcal{T},*}} \right) = \frac{S_{t+1}^{\mathcal{P}} S_{t+1}^{\mathcal{T}}}{S_t^{\mathcal{P}} S_t^{\mathcal{T}}}.$$

Exchange rate changes reflect differences in both the transitory and the permanent component of the two countries' pricing kernels.⁷ If two countries share the same martingale component of the pricing kernel, then the resulting exchange rate is stationary (up to a deterministic time trend) and Condition 1 is trivially satisfied. However, as already mentioned, exchange rate stationarity is obviously not necessary for Condition 1 to hold. This exchange rate decomposition implies a lower bound on the cross-country correlation of the permanent components of the

⁷ Note that $S_{t+1}^{\mathcal{P}}$, the ratio of two martingales, is itself not a martingale in general. However, in the class of affine term structure models, this exchange rate component is indeed a martingale.

SDFs. In the interest of space, we present it in Section VII of the online Appendix and focus in the main text on its implications for long-run UIP.

The definition of the transitory component of exchange rate changes, given by $\Delta s_{t+1}^T = (\lambda_{t+1}^T - \lambda_t^T) - (\lambda_{t+1}^{T,*} - \lambda_t^{T,*})$, where $\lambda_t^T \equiv \log \Lambda_t^T = \lim_{k \rightarrow \infty} (t+k) \log \delta + \lim_{k \rightarrow \infty} k y_t^{(k)}$, implies that a currency experiences a temporary appreciation when its long-term interest rates increase more than the foreign ones:

$$(27) \quad \Delta s_{t+1}^T = \log \delta - \log \delta^* + \lim_{k \rightarrow \infty} k (\Delta y_{t+1}^{(k)} - \Delta y_{t+1}^{(k),*}).$$

By backward substitution, it follows that the transitory component of the exchange rate in levels is given by the spread in long-term yields

$$(28) \quad s_t^T = s_0 + t(\log \delta - \log \delta^*) + \lim_{k \rightarrow \infty} k (y_t^{(k)} - y_t^{(k),*}) - \lim_{k \rightarrow \infty} k (y_0^{(k)} - y_0^{(k),*}).$$

This decomposition of exchange rates implies that deviations from the long-run UIP are due to the permanent component of exchange rates. We use $rx_{t \rightarrow t+k}^{FX} = k(y_t^{(k),*} - y_t^{(k)}) - \Delta s_{t \rightarrow t+k}$ to denote the currency excess return over a holding period of k years.

PROPOSITION 2: *When financial markets are complete, the expected rate of transitory depreciation per period is equal to the spread in the long-term interest rates:*

$$(29) \quad \lim_{k \rightarrow \infty} \frac{1}{k} E_t [\Delta s_{t \rightarrow t+k}^T] = - \lim_{k \rightarrow \infty} (y_t^{(k)} - y_t^{(k),*}).$$

Thus, the (per period) deviation from long-run UIP is the (per period) permanent component of the exchange rate change:

$$(30) \quad \begin{aligned} \lim_{k \rightarrow \infty} \frac{1}{k} E_t [rx_{t \rightarrow t+k}^{FX}] &= \lim_{k \rightarrow \infty} (y_t^{(k),*} - y_t^{(k)}) - \lim_{k \rightarrow \infty} \frac{1}{k} E_t [\Delta s_{t \rightarrow t+k}] \\ &= - \lim_{k \rightarrow \infty} \frac{1}{k} E_t [\Delta s_{t \rightarrow t+k}^P]. \end{aligned}$$

If exchange rates are stationary in levels, in which case the permanent component of exchange rate changes is zero, then per period long-run UIP deviations converge to zero. In this case, the slope coefficient in the regression of per period long-run exchange rate changes on yield differences converges to 1 and the intercept converges to 0. This result is previewed in Backus, Boyarchenko, and Chernov (2018), who show that claims to stationary cash flows earn a zero per period log risk premium over long holding periods. It follows that long-run deviations from UIP are consistent with no arbitrage only if the exchange rate is not stationary in levels.

To illustrate Proposition 2, let us study two examples where exchange rates are stationary in levels. We first go back to Example 1 and assume that each country's pricing kernel has no permanent innovations (i.e., $\varepsilon_t^P = 0$ for all t , so $\Lambda_t^P = 1$ for all t , and similarly for the foreign pricing kernel). In that case, the two pricing kernels and the exchange rate are stationary, due to the

square summability of sequences α and α^* . Bond yields satisfy $y_t^{(k)} = -\log \beta - (1/k)(\sigma_T^2/2) \sum_{i=0}^{k-1} \alpha_i^2 - (1/k) \sum_{i=0}^{\infty} (\alpha_{k+i} - \alpha_i) \varepsilon_{t-i}$ and the expected rate of per period transitory depreciation is $(1/k) E_t [\Delta s_{t \rightarrow t+k}^T] = (1/k) E_t [\Delta s_{t \rightarrow t+k}] = \log \beta - \log \beta^* + (1/k) \sum_{i=0}^{\infty} (\alpha_{k+i} - \alpha_i) \varepsilon_{t-i}^T - (1/k) \sum_{i=0}^{\infty} (\alpha_{k+i}^* - \alpha_i^*) \varepsilon_{t-i}^{T,*}$. In the limit of $k \rightarrow \infty$, Proposition 2 holds as both sides converge to zero, using the property $\lim_{k \rightarrow \infty} \alpha_k = \lim_{k \rightarrow \infty} \alpha_k^* = 0$ that arises from the square summability of sequences α and α^* .

We can also consider the symmetric two-country CIR model with country-specific factors presented in Example 2. In that model, the transitory component of the exchange rate is given by $s_t^T = s_0 + B_1^\infty ((z_t - z_0) - (z_t^* - z_0^*))$. As already noted, the pricing kernel is not subject to permanent shocks when $B_1^\infty = \sqrt{\gamma}/\sigma = \chi/(1-\phi)$.⁸ In that case, the exchange rate is stationary and hence $s_t = s_t^T$. The expected rate of depreciation is then equal to

$$(31) \quad \lim_{k \rightarrow \infty} \frac{1}{k} E_t [\Delta s_{t \rightarrow t+k}] = -\frac{\chi}{1-\phi} (z_t - z_t^*) = -\lim_{k \rightarrow \infty} (y_t^{(k)} - y_t^{(k),*}).$$

Even if exchange rates are not stationary in levels, as long as the unconditional entropies of the permanent components of the SDFs are the same across countries, then (under some additional regularity conditions), long-run UIP holds on average, as the following proposition shows.

PROPOSITION 3: *If financial markets are complete, the stochastic discount factors Λ_{t+1}/Λ_t and $\Lambda_{t+1}^*/\Lambda_t^*$ are strictly stationary, and $\lim_{k \rightarrow \infty} (1/k) L(E_t[\Lambda_{t+k}/\Lambda_t]) = 0$ and $\lim_{k \rightarrow \infty} (1/k) L(E_t[\Lambda_{t+k}^*/\Lambda_t^*]) = 0$, then the per period long-run currency risk premium is given by*

$$(32) \quad \lim_{k \rightarrow \infty} \frac{1}{k} E[rx_{t \rightarrow t+k}^{FX}] = \lim_{k \rightarrow \infty} E(y_t^{(k),*} - y_t^{(k)}) - \lim_{k \rightarrow \infty} \frac{1}{k} E[\Delta s_{t \rightarrow t+k}]$$

$$= \left[L\left(\frac{\Lambda_{t+1}^{\mathcal{P}}}{\Lambda_t^{\mathcal{P}}}\right) - L\left(\frac{\Lambda_{t+1}^{\mathcal{P},*}}{\Lambda_t^{\mathcal{P},*}}\right) \right].$$

This immediately implies that the per period currency risk premium converges to zero on average, and therefore long-run UIP holds on average, if Condition 1 is satisfied. Importantly, unconditional long-run UIP does not necessarily require stationary exchange rates, as Condition 1 is a weaker condition than exchange rate stationarity. Moreover, Condition 1 may hold while deviations from long-run UIP still exist. Proposition 3 only implies that long-run UIP holds on average, not at any date. As a result, the study of holding period returns of long-term bonds, the topic of this paper, is not the same as the study of UIP in the long run.

⁸The restrictions $B_1^\infty = \sqrt{\gamma}/\sigma = \chi/(1-\phi)$ have a natural interpretation as restrictions on the long-run loadings of the exchange rate on the risk factors: $\sum_{i=1}^{\infty} E_t[\Delta s_{t+i}] = \sum_{i=1}^{\infty} E_t[m_{t+i} - m_{t+i}^*] = \sum_{i=1}^{\infty} \phi^{i-1} \chi(z_t^* - z_t)$.

V. Conclusion

While holding period bond returns, expressed in a common currency, differ across G10 countries at the short end of the yield curve (the UIP puzzle), they are rather similar at the long end. In other words, the term structure of currency carry trade risk premia is downward-sloping. Replicating such a term structure is nontrivial for most models: recent no-arbitrage models of international finance that are able to address the UIP puzzle fail to replicate the downward-sloping term structure of carry trade risk premia.

We derive a preference-free result that helps assess existing models and guides future theoretical and empirical work. In order to exhibit similar long-term bond returns when expressed in the same units, complete market models need to exhibit the same volatility of the permanent components of their pricing kernels. This condition implies novel parameter restrictions in the workhorse no-arbitrage models of international finance.

Our results show that exchange rate risk is different from equity and bond risk. In order to account for the high equity premium and the low term premium, most of the variation in the marginal utility of wealth in no-arbitrage models must come from *permanent* shocks. Yet, differences across countries in how *temporary* shocks affect the marginal utility of wealth, and thus exchange rates, appear as natural way to align no-arbitrage models with the downward-sloping term structure of carry trade risk premia.

APPENDIX

Data

Our zero-coupon bond dataset runs from at most 1975:1 to 2015:12. We use the entirety of the dataset in Wright (2011) and complement the sample, as needed, with sovereign zero-coupon curve data sourced from Bloomberg, estimated from government notes and bonds as well as interest rate swaps of different maturities. The panel is unbalanced: for each currency, the sample starts with the beginning of the Wright (2011) dataset. The starting dates for each country are as follows: 1987:2 for Australia, 1986:1 for Canada, 1973:1 for Germany, 1985:1 for Japan, 1990:1 for New Zealand, 1998:1 for Norway, 1992:12 for Sweden, 1988:1 for Switzerland, 1979:1 for the United Kingdom, and 1971:12 for the United States. For New Zealand, the data for maturities above 10 years start in 1994:12. Yields are available for bond maturities ranging from 3 months to 15 years, in 3-month increments.

To construct averages of credit ratings, we assign each rating to a number, with a smaller number corresponding to a higher rating. In particular, a credit rating of AAA corresponds to a numerical value of 1, with each immediately lower rating getting assigned the immediately higher numerical value: AA+ corresponds to a numerical value of 2 and AA to 3, all the way down to CC- (22) and SD (23). We also construct rating series adjusted for outlook, as follows: a “Negative” outlook corresponds to an upward adjustment of 0.5 in the numerical value of the rating, a “Watch Negative” outlook to an upward adjustment of 0.25, a “Stable” or “Satisfactory” outlook to no adjustment, and a “Positive,” “Strong,” or “Very Strong” outlook to

a downward adjustment of 0.5. For example, a credit rating of BB (coded as 12) receives a numerical value of 12.5 with a “Negative” outlook and a value of 11.5 with a “Positive” outlook. In order to construct credit rating averages for portfolios formed before 1989:7, we backfill each country’s credit rating by assuming that the country’s rating before 1989:7 is equal to its rating at the first available observation.

Proofs

PROOF OF PROPOSITION 1:

The proof builds on some results in Backus, Foresi, and Telmer (2001) and Alvarez and Jermann (2005). Specifically, Backus, Foresi, and Telmer (2001) shows that the foreign currency risk premium is equal to the difference between domestic and foreign total SDF entropy:

$$(A1) \quad (f_t - s_t) - E_t[\Delta s_{t+1}] = L_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) - L_t\left(\frac{\Lambda_{t+1}^*}{\Lambda_t^*}\right).$$

Furthermore, Alvarez and Jermann (2005) establishes that total SDF entropy equals the sum of the entropy of the permanent pricing kernel component and the expected log term premium:

$$(A2) \quad L_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) = L_t\left(\frac{\Lambda_{t+1}^p}{\Lambda_t^p}\right) + E_t\left(\log \frac{R_{t+1}^{(\infty)}}{R_t^f}\right).$$

Applying the Alvarez and Jermann (2005) decomposition to the Backus, Foresi, and Telmer (2001) expression yields the desired result.

To derive the Backus, Foresi, and Telmer (2001) expression, consider a foreign investor who enters a forward position in the currency market with payoff $S_{t+1} - F_t$. The investor’s Euler equation is

$$(A3) \quad E_t\left(\frac{\Lambda_{t+1}^*}{\Lambda_t^*}(S_{t+1} - F_t)\right) = 0.$$

In the presence of complete, arbitrage-free international financial markets, exchange rate changes equal the ratio of the domestic and foreign pricing kernels:

$$(A4) \quad \frac{S_{t+1}}{S_t} = \frac{\Lambda_{t+1}}{\Lambda_t} \frac{\Lambda_t^*}{\Lambda_{t+1}^*}.$$

Dividing the investor’s Euler equation by S_t and applying the no-arbitrage condition, the forward discount is

$$(A5) \quad f_t - s_t = \log E_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) - \log E_t\left(\frac{\Lambda_{t+1}^*}{\Lambda_t^*}\right).$$

The second component of the currency risk premium is expected foreign appreciation; applying logs and conditional expectations to the no-arbitrage condition above leads to

$$(A6) \quad E_t[\Delta s_{t+1}] = E_t\left(\log \frac{\Lambda_{t+1}}{\Lambda_t}\right) - E_t\left(\log \frac{\Lambda_{t+1}^*}{\Lambda_t^*}\right).$$

Combining the two terms of the currency risk premium leads to

$$(A7) \quad (f_t - s_t) - E_t[\Delta s_{t+1}] = \log E_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) - E_t\left(\log \frac{\Lambda_{t+1}}{\Lambda_t}\right) \\ - \log E_t\left(\frac{\Lambda_{t+1}^*}{\Lambda_t^*}\right) + E_t\left(\log \frac{\Lambda_{t+1}^*}{\Lambda_t^*}\right).$$

Applying the definition of conditional entropy in the equation above yields the Backus, Foresi, and Telmer (2001) expression.

To derive the Alvarez and Jermann (2005) result, first note that since the permanent component of the pricing kernel is a martingale, its conditional entropy can be expressed as follows:

$$(A8) \quad L_t\left(\frac{\Lambda_{t+1}^P}{\Lambda_t^P}\right) = -E_t\left(\log \frac{\Lambda_{t+1}^P}{\Lambda_t^P}\right).$$

The definition of conditional entropy implies the following decomposition of total pricing kernel entropy:

$$(A9) \quad L_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) = \log E_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) - E_t\left(\log \frac{\Lambda_{t+1}^T}{\Lambda_t^T} \frac{\Lambda_{t+1}^P}{\Lambda_t^P}\right),$$

or, using the expression above for the conditional entropy of the permanent pricing kernel component:

$$(A10) \quad L_t\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right) = -\log R_t^f - E_t\left(\log \frac{\Lambda_{t+1}^T}{\Lambda_t^T}\right) + L_t\left(\frac{\Lambda_{t+1}^P}{\Lambda_t^P}\right).$$

The Alvarez and Jermann (2005) result hinges on

$$(A11) \quad \lim_{k \rightarrow \infty} R_{t+1}^{(k)} = \Lambda_t^T / \Lambda_{t+1}^T.$$

Under the assumption that $0 < \lim_{k \rightarrow \infty} P_t^{(k)} / \delta^k < \infty$ for all t , one can write

$$(A12) \quad \lim_{k \rightarrow \infty} R_{t+1}^{(k)} = \lim_{k \rightarrow \infty} \frac{E_{t+1}\left(\frac{\Lambda_{t+k}}{\Lambda_{t+1}}\right)}{E_t\left(\frac{\Lambda_{t+k}}{\Lambda_t}\right)} = \frac{\lim_{k \rightarrow \infty} \frac{E_{t+1}(\Lambda_{t+k} / \delta^{t+k})}{\Lambda_{t+1}}}{\lim_{k \rightarrow \infty} \frac{E_t(\Lambda_{t+k} / \delta^{t+k})}{\Lambda_t}} = \frac{\frac{\Lambda_{t+1}^P}{\Lambda_{t+1}}}{\frac{\Lambda_t^P}{\Lambda_t}} = \Lambda_t^T / \Lambda_{t+1}^T.$$

Thus, the infinite-maturity bond is exposed only to transitory pricing kernel risk. ■

PROOF OF PROPOSITION 2:

The transitory component of the exchange rate in levels is given by the spread in long-term yields:

$$(A13) \quad s_t^T = s_0 + t(\log \delta - \log \delta^*) + \lim_{k \rightarrow \infty} k(y_t^{(k)} - y_t^{(k),*}) - \lim_{k \rightarrow \infty} k(y_0^{(k)} - y_0^{(k),*}).$$

This follows directly from the definition of the transitory component of the pricing kernel. Note that $\lim_{k \rightarrow \infty} k(y_t^{(k)} - y_t^{(k),*})$ is not a constant. If it was, the transitory component of the exchange rate would always be a constant.

This, in turn, implies that the rate of appreciation is given by

$$(A14) \quad \Delta s_{t \rightarrow t+k}^T = k(\log \delta - \log \delta^*) + \lim_{k \rightarrow \infty} k(y_{t+k}^{(k)} - y_{t+k}^{(k),*}) - \lim_{k \rightarrow \infty} k(y_t^{(k)} - y_t^{(k),*}).$$

After expressing everything on a per period basis, taking conditional expectations and then taking the limit as the holding period goes to infinity, this expression yields

$$(A15) \quad \lim_{k \rightarrow \infty} \frac{1}{k} E_t[\Delta s_{t \rightarrow t+k}^T] = -\lim_{k \rightarrow \infty} (y_t^{(k)} - y_t^{(k),*}),$$

where we have used $\log \delta = -\lim_{k \rightarrow \infty} E_t(y_{t+k}^{(k)}) = -\lim_{k \rightarrow \infty} E(y_t^{(k)})$; the average long yield in logs is equal to $\log \delta$ under regularity conditions given in Borovička, Hansen, and Scheinkman (2016, pp. 2515–16). ■

PROOF OF PROPOSITION 3:

We start from the following equation:

$$(A16) \quad \begin{aligned} \lim_{k \rightarrow \infty} \frac{1}{k} E[rx_{t \rightarrow t+k}^{FX}] &= \lim_{k \rightarrow \infty} E(y_t^{(k),*} - y_t^{(k)}) - \lim_{k \rightarrow \infty} \frac{1}{k} E[\Delta s_{t \rightarrow t+k}] \\ &= \lim_{k \rightarrow \infty} (1/k) E \left[L_t \left(\frac{\Lambda_{t+k}}{\Lambda_t} \right) - L_t \left(\frac{\Lambda_{t+k}^*}{\Lambda_t^*} \right) \right]. \end{aligned}$$

This follows from the definition of the currency risk premium $E_t[rx_{t \rightarrow t+k}] = L_t(\Lambda_{t+k}/\Lambda_t) - L_t(\Lambda_{t+k}^*/\Lambda_t^*)$ in equation (7), extended to longer horizons. Next, we note that $L(x_{t+1}) = EL_t(x_{t+1}) + L(E_t(x_{t+1}))$. Given the stationarity of the stochastic discount factor, we know that $\lim_{k \rightarrow \infty} (1/k)L(E_t(\Lambda_{t+k}/\Lambda_t)) = 0$. Hence, $\lim_{k \rightarrow \infty} (1/k)L(\Lambda_{t+k}/\Lambda_t) = \lim_{k \rightarrow \infty} (1/k)EL_t(\Lambda_{t+k}/\Lambda_t)$. It then follows that

$$(A17) \quad \begin{aligned} \lim_{k \rightarrow \infty} (1/k) E \left[L_t \left(\frac{\Lambda_{t+k}}{\Lambda_t} \right) - L_t \left(\frac{\Lambda_{t+k}^*}{\Lambda_t^*} \right) \right] \\ = \lim_{k \rightarrow \infty} (1/k) \left[L \left(\frac{\Lambda_{t+k}}{\Lambda_t} \right) - L \left(\frac{\Lambda_{t+k}^*}{\Lambda_t^*} \right) \right] = \left[L \left(\frac{\Lambda_{t+1}^{\mathcal{P}}}{\Lambda_t^{\mathcal{P}}} \right) - L \left(\frac{\Lambda_{t+1}^{\mathcal{P},*}}{\Lambda_t^{\mathcal{P},*}} \right) \right]. \end{aligned}$$

The last equality follows directly from the Alvarez-Jermann decomposition of the pricing kernel (see Proposition 6 in Alvarez and Jermann 2005). ■

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