The Journal of FINANCE
The Journal of THE AMERICAN FINANCE ASSOCIATION

THE JOURNAL OF FINANCE • VOL. LXXIII, NO. 4 • AUGUST 2018

Estimating Private Equity Returns from Limited Partner Cash Flows

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ABSTRACT

We introduce a methodology to estimate the historical time series of returns to investment in private equity funds. The approach requires only an unbalanced panel of cash contributions and distributions accruing to limited partners and is robust to sparse data. We decompose private equity returns from 1994 to 2015 into a component due to traded factors and a time-varying private equity premium not spanned by publicly traded factors. We find cyclicality in private equity returns that differs according to fund type and is consistent with the conjecture that capital market segmentation contributes to private equity returns.

PRIVATE EQUITY (PE) IS A MAJOR institutional asset class and represents a significant fraction of investments by colleges, foundations, pension funds, and sovereign wealth funds, among others. A major drawback of PE for purposes of analysis is the lack of transactions-based performance measures. This greatly hampers portfolio allocation choice, which typically requires information about the risk, return, and covariance of asset classes. In liquid markets these estimates can be derived from statistical analysis of time series returns. Most PE time series, however, are based on nonmarket valuations or on multiyear internal rates of return broken down by fund vintage years.

The primary contribution of this paper is the introduction of a methodology based on Bayesian Markov Chain Monte Carlo (MCMC) to estimate a time series of PE returns using cash flows accruing to limited partners and factor returns from public capital markets. The identification strategy of this

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DOI: 10.1111/jofi.12688

methodology is similar to that of Cochrane (2005), Korteweg and Sorensen (2010), Driessen, Lin, and Phalippou (2012), Franzoni, Nowak, and Phalippou (2012), and Korteweg and Nagel (2016). Our contribution with respect to prior research is that, in addition to estimating factor loadings and αs, we are able to construct a quarterly time series of returns that is useful for understanding the intertemporal behavior of the asset class.

Our estimation approach decomposes returns into a component due to exposure to traded factors and a time-varying PE premium not spanned by traded factors. The factor exposures capture the systematic risks of various classes of PE and the time-varying PE premium can be interpreted as an α orthogonal to the traded factors.[1]

The estimation is based on a model of PE returns that identifies necessary assumptions and conditions for estimation. Because some of the assumptions required by the model may be violated in practice, we test its sensitivity with extensive simulations using both randomly generated data and pseudo-funds drawing on historical U.S. stock return data. We find that the estimation is robust to many violations of the assumptions but degrades when underlying asset returns are not significantly correlated with the traded factors and when idiosyncratic volatility is extremely high.

We apply the estimation procedure to quarterly cash flow data from institutional limited partnership investments obtained from Preqin covering the period 1996 to 2015. We construct return indices for PE as a whole as well as for subclasses (venture capital, buyout). We find that the estimated time series of PE returns is more volatile than those measured using standard industry indices. Moreover, it exhibits negligible serial dependence, in contrast to industry indices. This result is consistent with smoothing induced by a conservative appraisal process or by a delayed and partial adjustment to market prices, which often arises in illiquid asset markets (see, e.g., Geltner (1991), Ross and Zisler (1991)). We further find that the time series variation in returns differs widely across subclasses and is highly cyclical. The cycles correspond well with the time series variation in funding cycles and with anecdotal evidence about peaks and troughs in the performance of each of the subclasses. This result suggests that considerable diversification can be obtained within just the PE domain.

The second contribution of the paper is to test whether PE returns are spanned by portfolios constructed from publicly traded securities. This has an important bearing on whether low-cost PE replication strategies are feasible. We find that the PE specific factor is significant, which shows that the PE premium is not perfectly replicable by simple passive PE strategies. Our analysis of this factor suggests that part, but not all, of it is related to a proxy for illiquidity.

[1] We use the term "alpha" loosely here to denote a premium not captured by exposures to included factors. It may reflect actions under the control of the fund managers as well as other factors.

The third contribution of this paper is to test an economic theory about the source of PE returns. We use the estimated total return series for buyout funds to test the market segmentation theory that buyout funds add value when spreads between equity and fixed income yields are large (see, e.g., Kaplan and Strömberg (2009)). We find support for this hypothesis: buy-out fund returns are higher when the cross-market spread is larger.

Finally, although PE is unique in its cash flow structure and fee structure, our methodology may be useful in other market settings in which asset market values are infrequently observed and cash flow streams in between these market valuations are significant.

The paper is organized as follows. In Section I, we derive the model from first principles and use simulations to shed light on robustness, with a particular focus on when the methodology works well or poorly. In Section II, we discuss the data. Section III presents estimation results on the risk, return, and time series characteristics of PE, and tests the market segmentation hypothesis. Finally, in Section IV we conclude.

I. Methodology: Derivations and Tests

The intuition behind the methodology is that the present value of capital distributions is equal to the present value of capital investments when the discount rate is the time series of the average realized returns across the set of underlying illiquid investments, that is, the index of returns. Since the minimal aggregation level that we can work on empirically is a fund, we have one moment condition per fund. Further, since the number of funds is higher than the number of time periods, the system is overidentified and we can use maximum likelihood estimation to estimate which path the *latent* return index is more likely to have followed given the observed cash flow amounts and timing.[2]

In this section, we derive an approach to estimating a PE return index based on historical fund cash flows. As with all models developed for empirical application, it relies on assumptions that may be satisfied or violated depending on the underlying data-generating process. We list these assumptions and discuss how violations of them may affect estimation outcomes. This exercise offers a guideline for testing the robustness of the approach in later sections. In addition, an important benefit of the derivations below is that we derive from first principles the public market equivalent (PME) introduced by Kaplan and Schoar (2005).

A. Derivation

Consider a PE fund that makes N investments at times t_i, $i \in \{1, \ldots, N\}$, of amount I_i. Each investment pays a single terminal dividend D_{T_j} at times

[2] One basic requirement is that at least one capital distribution occurs in each quarter, but we find that the number of capital distributions is high enough, even during the financial crisis.

$T_j,\ j \in \{1, \dots, N\}$.[3] We then have the following equality for each investment i:

$$D_{T_j} = I_i g^i_{t_i+1} \dots g^i_{T_j}, \tag{1}$$

where g^i_t is one plus the rate of return on the investment during quarter t. Importantly, g^i_t is a realized return process that cannot be directly observed; it represents neither an ex-ante expected rate of return to a given investment nor a forward-looking discount rate.

ASSUMPTION 1:

$$\ln g^i_t = \ln g_t + \epsilon^i_t, \text{ where } \epsilon^i_t \text{ is i.i.d., normally distributed, and independent of } g_t. \tag{2}$$

This assumption is standard (see Cochrane (2005), among others).[4] $\ln g^i_t$ is decomposed into two components, one that is common across all investments (including a constant) and one that is idiosyncratic with a nonzero expectation. We denote $\text{Var}(\epsilon^i_t) = s^2$, and $\text{E}(\epsilon^i_t) = \mu$, and we set $\mu = -\phi.5s^2$; E(.) is the expectation operator across all investments during period t.

This assumption is violated when ϵ^i_t is not i.i.d. in the time series. For example, the fee structure could induce autocorrelation investments of funds that passed their hurdle rate at time t will have persistently lower realizations of ϵ^i_t. To investigate this issue, we will conduct extensive simulations and identify the situations that are most problematic.[5]

From assumption (1) it follows that

$$\text{E}\left(\exp \epsilon^i_t\right) = \exp\left(\text{E}\left(\epsilon^i_t\right) + .5\text{Var}\left(\epsilon^i_t\right)\right) = \exp(\mu + .5s^2) = 1. \tag{3}$$

[3] This is without loss of generality because an investment with multiple payoffs can be rewritten as separate investments made at the same time with one payoff each. Note also that the dividend paid at time T_i does not necessarily correspond to investment I_i. Because PE funds hold multiple underlying portfolio companies, cash flows received by an investor cannot be assigned to underlying investments. For example, a series of cash flows [−100, 200, −100, 400] cannot be decomposed into two transactions [−100, 200, 0, 0] and [0, 0, −100, 400] because part of the final $400 cash flow might be due to investments made with the first $100 paid into the fund. If it were observable, the separation into underlying investments would enable us to use standard techniques. Such a separation is not possible with fund cash flow data.

[4] Although standard, this assumption is restrictive and potentially violated. For example, if there are two types of investments (such as some with high average returns and some with low average returns due to risk differences), then ϵ^i_t will not be i.i.d, in the cross section. This is because there is a commonality among a subset of investments that is not captured by the econometrician. In a sense this is unavoidable (and probably why it is a standard assumption) but it is important to keep it mind, especially in a PE context where some investments could be junior debt, leveraged equity, real estate, or early-stage venture capital. Although these investment types are all quite different from one another, they all contain a common unmodeled component within each type. Empirically, we partially capture such commonalities by running analyses on subsamples of funds based on their type.

[5] Autocorrelation may also rise if different investments have different return persistence levels.

Using equations (2) and (3), we have:

$$\mathrm{E}\left(g_t^i\right) = \mathrm{E}\left(g_t \exp \epsilon_t^i\right) = \mathrm{E}\left(g_t\right) \mathrm{E}\left(\exp \epsilon_t^i\right) = g_t \mathrm{E}\left(\exp \epsilon_t^i\right) = g_t. \tag{4}$$

Hence, g_t is the expectation of gross returns across all investments by all funds during period t.

Econometricians cannot estimate equation (1) because the invested amount I_i needs to be compounded from time $t_i + 1$ to T_j and econometricians do not know whether the first investment is paid out at time T_j. However, we can bring each investment of a given fund to the same base date.[6] Combining equations (1) and (2), we can write

$$\frac{I_i}{g_{t_1} g_{t_2} \cdots g_{t_i}} \exp\left(\epsilon_{t_i+1}^i + \cdots + \epsilon_{T_j}^i\right) = \frac{D_{T_j}}{g_{t_1} \cdots g_{T_j}}. \tag{5}$$

Let us denote

$$U_{t_i,T_j}^i = \exp\left(\epsilon_{t_i+1}^i + \cdots + \epsilon_{T_j}^i\right). \tag{6}$$

Because ϵ_t^i are i.i.d. and normally distributed, $\ln(U_{t_i,T_j}^i)$ follows a normal distribution.[7]

When we sum equation (5) across the N investments made by a fund, and use the notation in equation (6), we can write

$$\sum_{i=1}^{N} \frac{I_i}{g_{t_1} \cdots g_{t_i}} U_{t_i,T_j}^i = \sum_{j=1}^{N} \frac{D_{T_j}}{g_{t_1} \cdots g_{T_j}}. \tag{7}$$

Let us define PV_Div, PV_Inv, and w_i as follows:

$$PV\_Div = \sum_{j=1}^{N} \frac{D_{T_j}}{g_{t_1} \cdots g_{T_j}}; \quad PV\_Inv = \sum_{i=1}^{N} \frac{I_i}{g_{t_1} \cdots g_{t_i}} U_{t_i,T_j}^i; \quad w_i = \frac{\frac{I_i}{g_{t_1} \cdots g_{t_i}}}{PV\_Inv}. \tag{8}$$

[6] Our data are observed by fund. Simply summing equation (1) across all investments in a given fund would still require econometricians to know the correspondence between cash flows. A solution is to discount each investment to the same base date as we do here.

[7] Lognormally distributed, U_{t_i,T_j}^i is, with the following two moments:

$$\mathrm{E}(U_{t_i,T_j}^i) = \exp(\mathrm{E}(\ln(U_{t_i,T_j}^i)) + \phi.5\mathrm{Var}(\ln(U_{t_i,T_j}^i))) \rightarrow \mathrm{E}(U_{t_i,T_j}^i) = 1.$$

and

$$\mathrm{Var}(U_{t_i,T_j}^i) = [\exp(\mathrm{Var}(\ln(\mathrm{U}_{\mathrm{t_i,T_j}}^{\mathrm{i}}))\ ) - 1]\exp(2\mathrm{E}(\ln(U_{t_i,T_j}^i)) + \mathrm{Var}(\ln(U_{t_i,T_j}^i)))$$

$$\rightarrow \mathrm{Var}(U_{t_i,T_j}^i) = \exp((T_j - t_i)s^2) - 1.$$

Dividing each side of equation (7) by PV_Inv, and using the above notation, we obtain[8]

$$\sum_{i=1}^{N} w_i \left(U^i_{t_i,T_j}\right) = \frac{PV\_Div}{PV\_Inv}. \tag{9}$$

The left side of equation (9) is a weighted sum of lognormal distributions, which has no closed-form expression. By the central limit theorem this sum will converge to a normal distribution, but Baker and Trietsch (2013) show that the lognormal distribution is a better approximation.[9]

ASSUMPTION 2: *As N goes to infinity, no single component should dominate. In our case, this is equivalent to*

(a) $w_i \to 0$, *when* $N \to \infty$ *(no dominantly large investment)*
(b) $\frac{(T_j - t_i)}{\sum_j^N (T_j - t_j)} \to 0$ *(no dominantly long investment).*

Under the two regularity conditions above, we can apply the lognormal central limit theorem of Baker and Trietsch (2013):[10]

$$\frac{PV\_Div}{PV\_Inv} = \sum_{i}^{N} w_i \left(U^i_{t_i,T_i}\right) \cong u \tag{10}$$

$$\text{where } \ln(u) \sim i.i.d.\ N\left(-0.5\sigma^2, \sigma^2\right), \quad \text{and} \quad \sigma^2 = \ln\left[\sum_i^N w_i^2 \exp\left((T_i - t_i)s^2\right)\right]. \tag{11}$$

[8] We work with the ratio rather than the difference because the ratio has the advantage that it is robust to different periods used to compute the present values. That is, if the valuation date is taken to be the first date of the sample, then present values of cash flows for funds formed at the end of the sample are smaller than present values of cash flows for funds started at the beginning of the sample. Taking a ratio removes these timing effects.

[9] The sum of independent random variables that have nonzero mean (such as lognormal distributions) converges very slowly to a normal distribution. In particular, an extensive literature shows that the sum of lognormal distributions remains close to a lognormal distribution (e.g. Fenton (1960), Barakat (1976)). This is why the lognormal distribution is sometimes said to be "quasi-permanent." Most recently, Baker and Trietsch (2013) formally introduce the "log-normal central limit theorem." They show that the lognormal distribution is a better approximation than the normal for the sum of a few strictly positive random variables, even if the summands are not i.i.d., provided that the summands satisfy two regularity conditions.

[10] We invoke an asymptotic result here. Given that each fund has at most about 20 investments, asymptotic results are unlikely to be accurate in our context. Using Monte Carlo simulations, we are able to compare the empirical distributions of $\ln u_h$ with the best-fitting normal distribution. Formal tests of the normal distribution reject the null that the empirical distribution is normal, but the violation is economically small (as shown in Internet Appendix, available in the online version of the article on *The Journal of Finance* website). Thus, this error will play a role in the precision of our estimation procedure. We quantify this effect in the next section (when we compare our estimated PE time series to the true one in various settings).

Let us denote variables associated with fund h with subscript h. We then have the following equality for each fund, which we refer to as the Present Value Ratio (PVR):[11]

$$\mathrm{PVR}_h = \ln \frac{PV\_Div_h}{PV\_Inv_h} \cong \ln u_h, \text{ where } \ln u_h \sim N\left(-0.5\sigma_h^2,\ \sigma_h^2\right). \tag{12}$$

Note that if the time series g_t equals the rate of return of the S&P 500 index, then the PVR is equal to the PME of Kaplan and Schoar (2005).[12]

Given that σ_h^2 is not a priori identifiable, we need to make a homoskedasticity assumption.

ASSUMPTION 3: *The volatility of* $\ln \frac{PV\_Div_h}{PV\_Inv_h}$ *is the same for all funds:* $\sigma_h^2 = \sigma^2$.[13]

Our strategy is to filter the PE returns $\{g_t\}$, such that the conditions in equation (12) are satisfied across funds and over time. Equation (12) represents an observation equation and the returns are latent parameters. As is standard in the literature, we assume that the state equation for the latent returns (or the equation of motion) is a function of a set of common factors plus an uncorrelated observation error in the observation equation.

ASSUMPTION 4: *The state equation dynamics of the filtering problem are given by:*

$$g_t = \alpha + \beta' F_t + f_t + r_t^f, \tag{13}$$

where $F_t = [F_{1,t}, \ldots, F_{J,t}]$ is a set of J common tradable factors that are observable in public markets, β contains the loadings on the common factors, F_t. α is the average PE return in excess of the systematic (and liquid) component of the PE return, and f_t is an asset-class-specific latent factor with mean zero that is orthogonal to the traded factors, F_t. This potentially makes PE nonredundant in the space of tradable assets.[14]

[11] We work with the log transformation to reduce the effect of outliers in the estimation.

[12] Our derivation can be seen as a formal proof of the proposition in Kaplan and Schoar (2005) that if a fund's PME is equal to unity when using realized S&P 500 returns as g_t, then investors are indifferent between investing in a PE fund or investing in the S&P 500 index.

[13] Assumption 3 is valid when funds have similarly concentrated portfolios, and when their investments have similar holding periods and idiosyncratic volatility. This can be violated by a variety of conditions (e.g. significant leverage differences across investments). Note also that if Assumption 1 is violated, then Assumption 3 is violated as well.

[14] Assumption 4 states that the cash flows associated with any investment are generated by a time-varying portfolio of assets that have unobserved but continuous latent values. We assume that returns are a linear function of an underlying systematic factor structure. Thus, if the latent asset values were observable, some portion of their return variance could be explained by common factors using standard regression methods. The latent factor process, f_t, can be viewed as the idiosyncratic component of PE returns. We can specify it as an AR(1) process to reflect the fact that the f_t process is not exposed to the forces of arbitrage because, by design, it is not tradable and is orthogonal to factors in the public markets. The f_t process may be persistent because of persistent aggregate manager skill, intertemporal variation in good investment opportunities, or trends in performance due to nonconstant returns to scale. However, in simulations we find that with time

We estimate the model using a Bayesian MCMC procedure as described in the Internet Appendix.[15] We treat the unobserved returns as parameters to be estimated (referred to as "data augmentation"), along with the other parameters of the data-generating process.

B. Further Discussion on the Methodology

We specify error terms as log ratios of summed discounted cash flows and minimize one large error term per fund in the estimation process. Problems may arise from this aggregation due to intertemporal compounding and cross-correlations. Specifying the price path of errors for individual funds (other than as i.i.d.) is intractable, however, which renders our simplification necessary for estimation. The degree to which this specification influences estimation outcomes is ultimately an empirical question, and one that motivates the extensive simulations detailed in Section I.C.

Several additional caveats to our approach are in order. First, a natural interpretation of the index is that it is the net return to investing in each of the PE funds in the database. This interpretation implicitly assumes that the returned capital D_t in any given period is immediately re-investable in all existing funds as opposed to only new funds. This is typically not the case. However, this assumption only affects interpretation of the premium factor—the latent factor component, f_t, of the total return index. The passive component due to $\beta' F_t$ comprises only marketable factors, in which investors can re-invest or rebalance.

Second, by presuming that the passive component is accessible to an investor, we are also implicitly assuming that leverage may be used to achieve a factor exposure greater than one. This caveat applies in any study using linear factor exposures. As we show below, some of the variation in the PE return series is explained by large exposures to public equity factors. PE may provide a means to relax borrowing constraints and this convenience may be priced (see Frazzini and Pedersen (2014)). We also use long-short factors, and implicitly assume that short-selling is feasible and costless in replicating the performance of such factors.

Third, our procedure solves for the best fit of the PE returns given fund cash flows. We are not solving for expected returns, but for estimates of realized PE returns. We take the cash flows as given to solve for the realized returns. To obtain estimates of forward-looking discount rates, we would need to embed an expectation process into a valuation model and tie the discount rates to

series shorter than 100 time periods (as is the case in our empirical section), we cannot estimate this autoregressive parameter with a reasonable degree of precision. As a result, we specify f_t as i.i.d. and normally distributed in the analysis and simulations below.

[15] The Internet Appendix may be found in the online version of this article.

estimates of our realized returns. While an interesting research topic, our current goals are more modest.[16]

C. Simulations

Before applying our methodology to real data, we investigate how it performs under known conditions. We generate cash flows whose true underlying returns are generated by standard processes, which we use to form hypothetical funds and then filter g_t using our methodology. These simulations provide useful guidance on accuracy under various scenarios. Through the appropriate use of priors and some parameterization of the return process, the procedure can handle sparse data and unbalanced panels of contributions and distributions. Yet the precision of our approach depends on the properties of the true data-generating process, especially in small samples.

C.1. Simulations When the Error Structure Is as Specified in Our Model

Using simulated panels of PE cash flow data, we determine whether we can recover unbiased estimates of the realized returns on a population of PE funds. We simulate our model with the error structure described in equation (12) and the following parameters:

(1) The gross return at date t is given by $g_t = 1 + \alpha + \beta \cdot R_t^M + f_t$, where $\alpha = 4\%$ per annum and $\beta = 1.5$ (the risk-free rate is zero).
(2) Factor returns R_t^M are i.i.d. and drawn from a normal distribution with an annual mean of 8% and an annual volatility of 20%.
(3) The time series of f_t is drawn from a normal distribution with mean zero, and volatility of either 1% or 10%. In the latter case, the resulting (true) net return ($g_t - 1$) has a mean of 16% and an annual volatility of 32% per annum.

We consider a population of 500 funds with five investments each and 80 quarters of data. Each quarter, several \$1 investments are started in each fund and the value of the investment grows at a rate of g_t in quarter t. Each investment is sold following a simple rule: each quarter, one in x investments is sold. In all but the first set of simulations (case 1) x is set to 28. This means that investments last no more than seven years and thus funds last no more than 12 years (since the last investment occurs in year 5). All investments not liquidated at the end of the sample (year 20) are terminated at that time.

Panel A of Table I reports results from the simulations. We first consider the simple case in which investments are held for two quarters on average and the standard deviation of the PVR across funds (referred to as idiosyncratic volatility) is $\sigma_h^2 = 1\%$. For each of the following cases, we incrementally add

[16] Recent work by Korteweg and Nagel (2016) explores the relation between PMEs and discount rates. The problem of correlated forward-looking discount rates and cash flows is also considered by Brennan (1997) and Ang and Liu (2004).

Table I
Monte Carlo Simulations When the Error Structure Is as Specified in Our Model

We generate a set of cash flows for 500 funds over 80 quarters. Each case is simulated 100 times. In Panel A, we report the mean and volatility of the estimated time series of returns and the correlation and mean squared error between the estimated and true time series of returns. In panel B estimated α and β quartiles are shown. There are six cases. In Case 1, investments are held for two quarters on average and the standard deviation of the PVR across funds is $\sigma_h^2 = 1\%$ (referred to as idiosyncratic volatility). Each of the following cases changes one parameter at a time and keeps the previous changes made. Case 2 increases the average holding period to 3.5 years. Case 3 increases idiosyncratic volatility to 10%. Case 4 increases the number of investments per fund from 5 to 20. Case 5 shows results when the true error term is uniformly distributed instead of lognormally distributed. Case 6 considers, in addition to all of the previous handicaps, the wrong priors (the prior on α is 0 and the prior on β is 1).

Panel A: Recovering the Full Time Series of Returns

	Mean	Volatility	Correlation	MSE (*100)
Truth	0.16	0.32	100.0%	0.00
1. Simple case	0.16	0.32	98.7%	0.06
2. Increase holding period	0.16	0.32	97.6%	0.12
3. Increase idiosyncratic volatility	0.17	0.32	96.0%	0.20
4. More investments per fund	0.17	0.32	95.3%	0.21
5. Error is not lognormally distributed	0.17	0.32	95.0%	0.24
6. Wrong priors	0.16	0.30	94.5%	0.26

Panel B: Recovering Model Parameters

	α			β		
Percentiles	25th	50th	75th	25th	50th	75th
Truth		0.04			1.50	
1. Simple case	0.02	0.04	0.05	1.47	1.50	1.53
2. Increase holding period	0.02	0.04	0.05	1.47	1.50	1.53
3. Increase idiosyncratic volatility	0.03	0.05	0.06	1.45	1.50	1.55
4. More investments per fund	0.03	0.05	0.06	1.45	1.50	1.55
5. Error is not lognormally distributed	0.03	0.05	0.06	1.45	1.50	1.55
6. Wrong priors	0.04	0.06	0.07	1.23	1.27	1.31

additional "handicaps" to our simple estimation. Case 2 increases the average holding period to 3.5 years. Case 3 increases idiosyncratic volatility to 10% (which matches the empirical distribution of PVRs in our data set). Case 4 increases the number of investments per fund from 5 to 20. Case 5 shows results when the error term is uniformly distributed instead of lognormally distributed. Finally, Case 6 has the wrong priors (the prior on α is 0 and the prior on β is 1), in addition to all of the previously considered "handicaps."

Table I shows that, in the first "simple" case, we retrieve the correct values of the parameters' mean and standard deviation, and the average correlation between the *true* time series and the estimated time series is high at 98.7%. In panel B of the table, which shows the 25th, 50th, and 75th percentiles of the simulated distributions for α and β, we find that for this simple case, while the median value is unbiased, there is significant variation in both the annual α and the market β. These results together suggest that, even in this simple case, it is difficult to retrieve a precise estimate of factor loadings.

As we add handicaps, we find that precision—as measured by the correlation with the true index—clearly declines. It decreases from 98.7% in the simple case to 94.5% in the case that includes all handicaps. We also note that the median α is no longer equal to the true value (although the difference is small). In Case 6, with diffuse priors, the estimated β is lower than the true β, which reduces the volatility of the systematic factor component, but the estimated mean performance is not far from the true one.[17] The correlation between the true time series of returns and the estimated time series of returns remains high.

In sum, Table I shows that if the data-generating process has the assumed final structure (equation (12)), then the methodology retrieves a reasonable correlate of the time series of true returns. However, the estimation of α and factor loadings is less precise and influenced by the prior.

C.2. Simulations When Investments Are Listed Stocks

To evaluate the robustness of our approach to the nature of the true (but unknown) equity return–generating process, we simulate panels of cash flows in which PE funds randomly buy and sell actual listed stocks. We begin with a simple setup. We take listed stocks from 2001 to 2010. During that time period the value-weighted stock market index (our factor) had overall returns close to zero. In contrast, the equally weighted stock index had high and more cyclical returns. If we simulate PE funds investing in listed equity, our g_t time series should look like the CRSP equally weighted index.

Specifically, we take all U.S. common stocks with 20 years of valid monthly data in CRSP (2001 to 2010). This simplification has the advantage of avoiding holes in the return series. We obtain 3,120 stocks from which we form 624 funds with five investments each. Each fund starts at the same time and is liquidated 10 years later. We apply our algorithm and use a single-factor model with the CRSP value-weighted (CRSP-VW) as the factor. We then compute a "true" index, that is the equal-weighted stock index constructed from the 3,120 stocks (C-EW). The CRSP equally weighted index (CRSP-EW) has a slightly higher average return and volatility than C-EW because our data restriction drops small and infrequently traded stocks. Nevertheless, the two indices are quite close.

The cash flows are generated as in the other simulation setups: Each fund makes one investment per year for five years. Each quarter, one in 24 investments across the funds is liquidated so that all funds are fully liquidated by

[17] The volatility of f_t mechanically increases to match the specified moment condition, but it does not increase enough. As a result, α is too high (since β is too low).

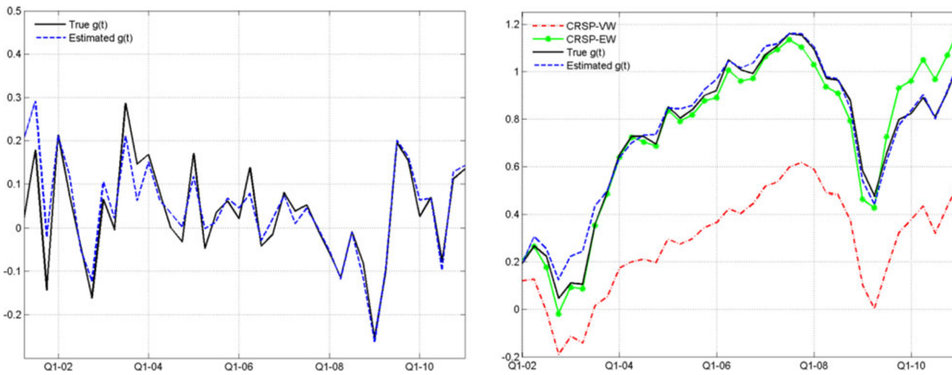


Figure 1. Monte Carlo simulations with listed stocks as the underlying investments. We generate a set of cash flows for 500 funds over 40 quarters. The simulation setup is described in detail in the text. Each case is simulated 100 times. Each investment is matched to one of the 3,120 stocks with 10 years of valid monthly data in CRSP (Q1-2001 to Q4-2010) and earns the same return as the matched stock during its holding period. The true return at time t is the average return across the 3,120 stocks at time t, which is close (by construction) to the CRSP equally weighted stock index. The left panel compares the time series of estimated returns (blue dashed line) and true returns (black solid line). The right panel shows the cumulative log return of these two return series as well as that of the CRSP value weighted index (dashed red line), and the CRSP equally weighted index (dotted green line). (Color figure can be viewed at wileyonlinelibrary.com)

the end of year 10. We have an unbalanced panel of cash flows similar to that observed in practice and we know the true underlying index return series.

Figure 1 summarizes the results. The left figure compares the average time series of estimated returns (blue dashed line) and true returns (black solid line). We retrieve the correct mean but we underestimate volatility by 3% per annum. Our estimated time series has a 92% correlation with the observed equally weighted stock market index on average. By construction, there are fewer observations, and thus the average series has a greater deviation at the beginning. The average correlation goes up to 95% if we remove the first year of our estimated time series.

The right panel of Figure 1 shows the cumulative log return of the true and mean estimated return series as well as that of the CRSP Value-Weighted index (dashed red line), and the CRSP Equally Weighted index (dotted green line) when the first year is removed. The factor (CRSP-VW) is flat throughout that decade, which reflects the well-known "lost decade" for value-weighted stock indices. In contrast, we see strong cycles in the equally weighted stock indices (CRSP-EW and C-EW). The estimated return series recovers these cycles well.

C.3. Simulations to Identify Problem Zones

Using actual stocks to simulate data has important benefits: it is reasonable to assume that underlying PE investment returns—particularly for buyout funds—resemble those of individual stocks, such that their underlying return dynamic is different from the one we assumed. The downside of this approach, however, is that we cannot vary certain key parameters. In this subsection we vary different parameters to identify zones in which our method performs less well.

Here we simulate data from equation (2) rather than equation (12). Accordingly, instead of having errors distributed around PVRs, we have errors added to each return each quarter for each investment. This allows us to change the idiosyncratic volatility of these errors (σ_ε) to assess whether the econometrician working with equation (12) out of necessity is still doing a good job when the assumptions we made in Section I.B are violated.

Empirically, equation (12) will hold less precisely as σ_ε increases because the convergence of the summation of the error terms will be slower, which should lead to a small-sample problem. Other variables that affect total volatility should also affect estimation precision. As in the simulations above, we work with a one-factor model. In such a setup, total volatility is affected by three elements: beta (β), PE-specific risk (σ_f), and idiosyncratic volatility (σ_ε). Our return time series (g) is decomposed into a systematic component ($\beta \cdot R^M$) and a PE-specific component (f). The question is how these three volatility drivers affect the two return components.

Figure 2 plots the correlations between the *true* time series of returns and the *estimated* returns plotted against different levels of idiosyncratic volatility. Panel A shows the precision of the estimation of the total return time series (g) as a function of both idiosyncratic volatility σ_ε and PE specific volatility σ_f.[18] When σ_ε increases from negligible to very large (150% per annum), the correlation between the estimated and true time series always decreases, but the magnitude of the reduction seems relatively modest.

Panel A also shows the effect of changing σ_f on the relationship between correlation and idiosyncratic volatility (σ_ε). Panel B shows the same effect but for changes β instead of σ_f. As can be seen, even though σ_f and β both increase total volatility, they have opposite effects on estimation precision (i.e., correlation): increasing σ_f decreases precision, whereas increasing β increases precision. When both (σ_ε) and σ_f increase, the correlation decreases but remains reasonable.

Figure 2, Panel B shows the impact of large changes in β by setting it equal to 0.75, 0.33, and 0. The latter case is equivalent to running the analysis without any factors. Here the correlation decreases quickly. When β is below 0.33 and (σ_ε) is above 20%, the correlation goes below 50%. We still recover part of the true time series (correlation is statistically different from zero), but the economic magnitude is much smaller.[19]

[18] Recall that σ_ε is the volatility (annualized) of the shock added each quarter to each investment return.

[19] In the Internet Appendix we also run simulations with varying priors on β. We find that changing priors has a limited impact on correlation. We further find that estimates of β. and volatility are biased when we combine variations of different parameters: level of volatility, true β., and wrong/right priors. When the prior on β. is too low, the estimated β. remains below the true

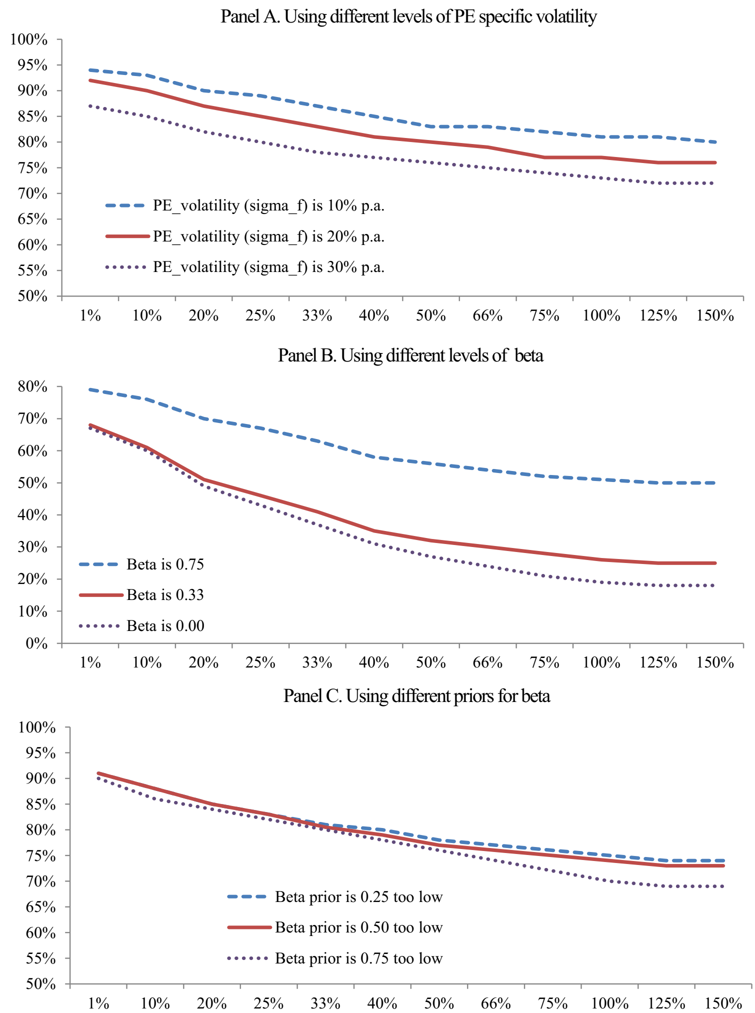


Figure 2. Estimation precision with quarterly return shocks. This figure shows the average correlation obtained across simulations, as a function of the volatility (annualized) of the shock added each quarter to each investment return (referred to as idiosyncratic volatility). This quarterly error term is not modeled by the econometrician; instead, the error term in the econometrics model is based on the net present value of funds. By default, the true time series of returns are generated using a one-factor model with a true β of 1.5, an α of 4% per annum, plus a mean-zero PE-specific return with 20% annual volatility and idiosyncratic volatility. There is no autocorrelation in residuals and no fee structure applied to the cash flows. The average holding period (duration) is 3.5 years, and the prior on β is correct. The sample contains 600 funds that make 20 investments each. Panel A plots results for different levels of annual volatility for the PE-specific return. Panel B plots results for different assumptions regarding the true β. Panel C plots results for different priors on β: the prior is lower than the true β by either 0.25, 0.50 or 0.75. Panel D plots results for a different number of funds in each economy. Panel E plots results for different holding periods. (Color figure can be viewed at wileyonlinelibrary.com)

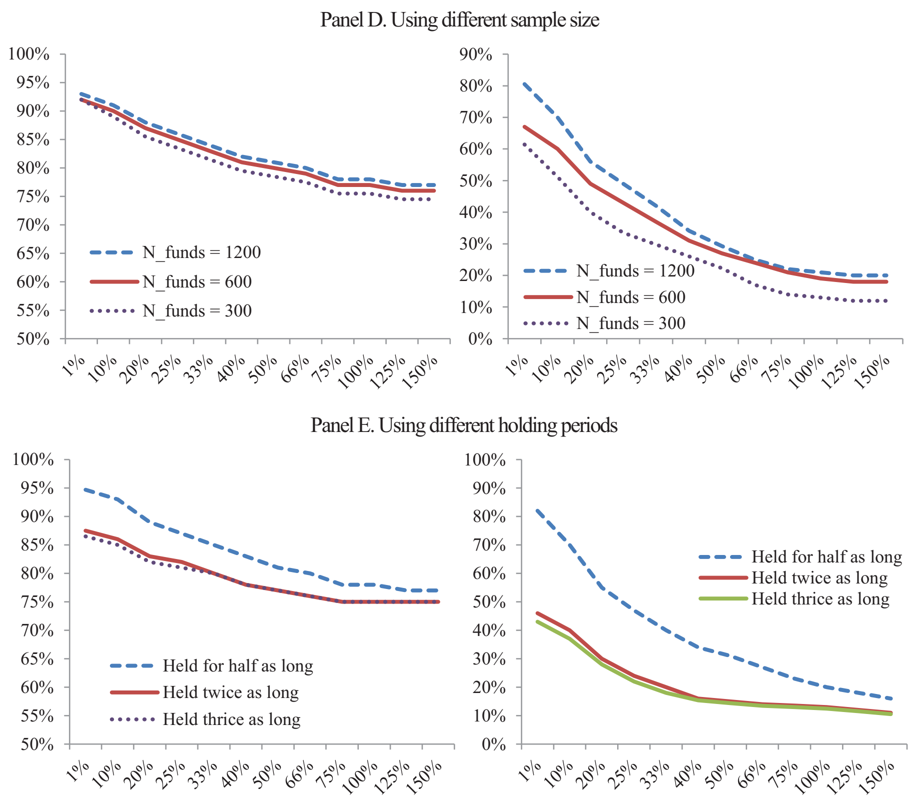


Figure 2. *Continued.*

The reason for this is that retrieving the systematic return component is easier than retrieving the PE-specific one. The PE-specific component is a pure random variable that the code needs to recover, while we impose a prior for the systematic part, which restricts the search space. Even if this prior is wrong (see Figure 2, panel C), the estimated time series is highly correlated with the true variation.[20]

If the factor loadings (i.e., the estimated β) are low, then returns largely comprise a pure PE component and idiosyncratic risk. Hence, it is more difficult to retrieve total returns and the correlation falls quickly even though total volatility is going down.

β. When incorrect priors are combined with high volatility and a low true β., we estimate a β. that is only half of the true β. Volatility is also significantly underestimated in this case.

[20] To elaborate, we generate *Beta*Rm*, where *Rm* is the same for the simulation and in the estimation. Therefore, even if our β. prior is wrong, given that the model is right, the correlation will still be high.

We can confirm this insight by computing the correlation between the true and estimated PE-specific return component only (*f*).[21] If we repeat the graphs in the first three panels, the three lines overlap one another, that is there is no change as a function of σ_f, β, and error in the prior on β. For this reason, we do not tabulate it.

The right-hand side of Figure 2, panel D shows the correlation between the true and estimated PE-specific return component only (*f*) for different sample sizes. One can clearly see that the impact of idiosyncratic volatility (σ_ε) on estimation precision is dramatic. Idiosyncratic volatility mainly affects the PE-specific return estimation: the higher it is, the more difficult it is to isolate and recover the PE-specific component. In contrast, idiosyncratic volatility does not have much of an effect on the estimation of the systematic component (left-hand side of Figure 2, panel D). It is for this reason that as idiosyncratic volatility rises, the correlation between total returns decreases only modestly.[22]

Panel E is similar to panel D but varies the sparsity of the cash flow matrix (i.e., by changing the average holding period) instead of the number of funds. We find that the effect on the total return correlation is minor, while the effect on the PE-specific return estimates is large.

Notice that in terms of statistical significance, which is commonly used to assess econometric success, all correlations are statistically different from zero. For example, p-values are less than 1% when the correlation is 10%. For the cases in which we choose extremes in parameters, the correlations are always positive and significant at the 1% level. When σ_f is 30% and β is 1.5, the volatility of the PE market index is $\sqrt{1.5^2\sigma_m^2 + \sigma_f^2} = 42.4\%$, which is more than twice the volatility of the S&P 500 index. Idiosyncratic volatility of 150% for a PE investment is also very high.

Overall, our results show that, we cannot recover the latent time series for an asset class that has small exposures to specified factors. This suggests that our methodology is less useful for estimation of an index built from unusual alternative asset classes such as collectibles, for example, unless the relevant factors can be identified.

In the Internet Appendix we report results from 11 different economies, where we vary the number of investments per fund, introduce a regular dividend, and consider other changes of interest. We find that our results are robust to most changes with the exception of introducing a "hold-on-to-losers" rule. As can be seen from the derivations above, intertemporal independence is an important assumption. If investments are more likely to be monetized when they have performed better, the independence assumption is violated. However, the correlation between the true and estimated time series remains economically reasonable at about 90%.

[21] Note that this is equivalent to setting beta to zero and looking at the correlation between true and estimated g_t as we do above.

[22] Note that the decay in precision is not overly sensitive to the number of funds in the sample.

C.4. Simulations on Post Fee Return Series

The existence of a complex, nonlinear fee structure (see Metrick and Yasuda (2010)) for managed assets such as PE funds affects the precision of our estimation approach. More subtly, when a complex fee structure is to be modeled, "true" fund returns for a given investment by a fund can be defined in three ways:

(1) Cash. The modeler may treat fees on a cash basis. Under this approach, the return on an investment is the realized difference between the purchase and sale price net of intermediate cash flows minus paid-out carried interest upon exit and the management fees paid to the fund (calculated pro-rata across ongoing fund investments). This approach does not capture unrealized fees for past performance or the contractual option value of future fees.

(2) Accrual. This method accrues fees based on paper gains in the portfolio's value up to the current date. This approach is common in the hedge fund industry where 20% incentive fees and high-water mark provisions are common. Hedge fund databases typically report returns on a monthly basis, while incentive fees are paid on a quarterly or annual basis, conditional on meeting a pre-specified hurdle. Incentive fees are accrued on an interim monthly basis until they are realized at the end of the fee determination period. This approach captures unrealized fees but ignores the contractual option value of future fees. For hedge funds, the 20% incentive fee clearly has value prior to the end of the fee determination period, even when it is out of the money, that is, when no fee has been accrued (see Goetzmann, Ingersoll, and Ross (2003)).

(3) Mark to Market. This method treats the management fees and carried interest as a call option on each investment, accrues the fees and interest in the period the investment is made, and revalues each investment each period as conditions change. Metrick and Yasuda (2010) pioneered this approach to PE valuation by taking the discounted expected future fees with respect to the risk-neutral pricing kernel. Although not commonly used in practice, the utility of marking manager fees to market is obvious. General partner (GP) actions that increase the value of the fees at the expense of the limited partners (LPs) may affect other variables of interest such as covariance with the market.

Because we have ex-post realized cash flows from LPs, we are unable to estimate PE indexes based on either an accrual or a mark-to-market basis. It is therefore important to study the effects of this limitation on outcomes. Throughout our derivations, R_t is the return per quarter net of fees irrespective of the fee structure. However, if true returns are defined so that they continuously account for changes in the expected carried interest to be paid at exit (the mark-to-market case), this affects the true β of the LPs investment and therefore our index. Furthermore, any fee structure will generate some autocorrelation in residuals and thus affect "true" unobservable returns in a predictable fashion. In particular, at the beginning of our return series, the true return will be lower due to higher management fees—often referred to as the J-curve effect.

Because fees are nonlinear returns to the underlying asset, when the market goes up, the LP payoff will be concave. It follows that the β of LP returns will be lower when the market is up and higher when the market is down because fees are not being earned.[23] Finally, if a fund has strong past performance, then all future payoffs will be subject to carried interest, which will also generate autocorrelation in the error term, as well as a nonlinear market exposure.[24] To address these potentially problematic issues, we perform two types of simulation.

Case 1: We simulate a whole-fund waterfall (also called European waterfall) following Sorensen, Wang, and Yang (2014) and Metrick and Yasuda (2010). In this case a carried interest is charged on all exited investments once the fund passes the 8% hurdle rate and we incorporate a 100% catch-up provision.

We allocate the management fees each quarter to each investment that is alive within a fund (equal allocation across investments) and the carried interest that is retained from the distribution of each investment (as done in practice). We then have a J-curve effect: at the beginning of the fund's life, post-fee returns are much lower than pre-fee returns because of the management fees.

This quarterly error term is not modeled by the econometrician; instead, the error term in the econometric model is on the net present value of funds (equation (12)). As above, returns are generated using a one-factor model with a true β of 1.5, α of 4% per annum, plus a mean-zero PE-specific return with 20% annual volatility and idiosyncratic volatility. There is no autocorrelation is residuals, the average holding period (duration) is 3.5 years, and the prior on β is correct. The sample contains 600 funds that make 20 investments each.

We plot the results in Figure 3, panel A. The format is the same as the previous figure. Adding only the carried interest does not greatly affect the

[23] We examine this nonlinearity by regressing our buyout index g_t on stock market returns (see Internet Appendix), and we find that our estimated index captures at least part of the nonlinearity predicted by Metrick and Yasuda (2010). As we are interested in constructing an after-fee index of PE performance, the fact that this nonlinearity is captured in the estimate is reassuring as it suggests that the nonlinear fee structures and implicit timing behavior about underlying payouts of PE are indeed reflected in our estimated PE return series.

[24] Another consideration is that β can differ across funds with some time-indexed commonality–for example, a dramatic increase in market values or a shift in market volatility. In particular, funds with similar age and past performance could have a similar β due to correlated market or industry-specific investment opportunities. Since we do not model this, possibly precision will be affected. It is roughly equivalent to running an OLS regression when errors are heteroskedastic. But, as with heteroskedasticity in a regression setup, adjustments pay off only to the extent that we know something about the underlying source of heteroskedasticity. Fully deriving and modeling the PE fee structure–induced heteroskedasticity is a promising area for future research that may require a different kind of database to study it thoroughly. In this paper we simply assess the loss in precision due to a mark-to-market fee return perspective.

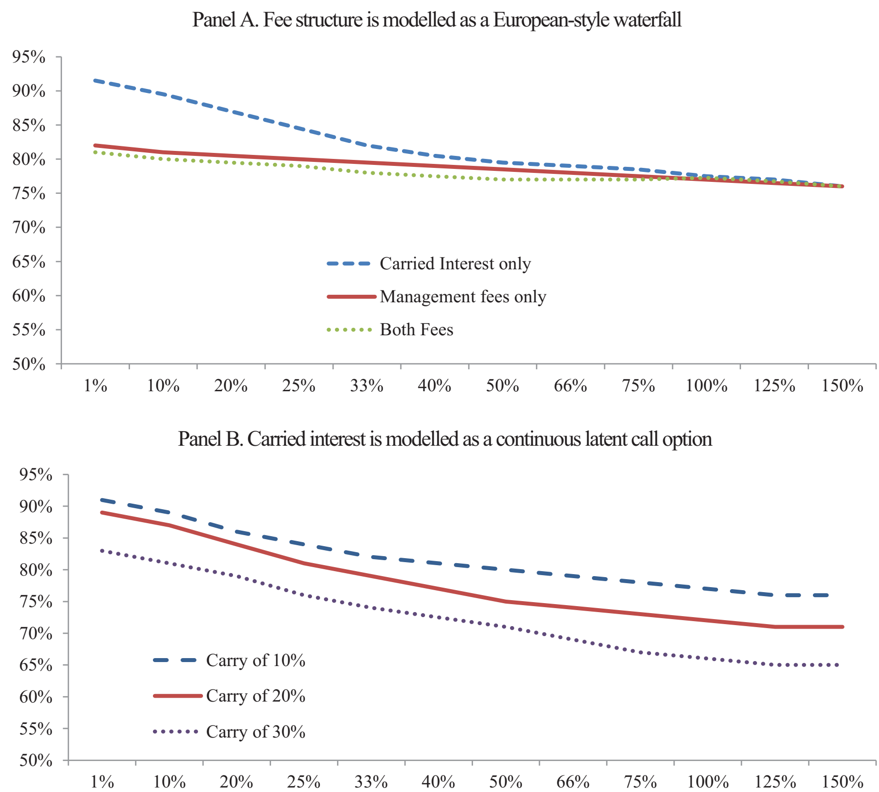


Figure 3. The impact of unmodeled fee structure on estimation precision. This figure shows the average correlation obtained across simulations, as a function of the volatility (annualized) of the shock added each quarter to each investment return (referred to as idiosyncratic volatility). This quarterly error term is not modeled by the econometrician; instead, the error term in the econometrics model is based on the net present value of funds. Returns are generated using a one-factor model with a true β of 1.5, an α of 4% per annum, plus a mean-zero PE-specific return with 20% annual volatility, and idiosyncratic volatility. There is no autocorrelation in residuals. The average holding period (duration) is 3.5 years, and the prior on β is correct. The sample contains 600 funds that make 20 investments each. Panel A shows results with a standard fee structure, whereby carried interest is paid when investments are exited and management fees are charged on capital committed (and thus generate a so-called J-curve). Panel B shows results with carried interest modeled as a continuous latent call option; the value of the investment each quarter is reduced by the change in the value of this option. (Color figure can be viewed at wileyonlinelibrary.com)

correlations. The J-curve effect, in contrast, reduces the average correlation significantly. However, idiosyncratic volatility has little impact on the average correlation when idiosyncratic volatility is above 33% per annum.[25]

[25] The Internet Appendix shows biases in estimated parameters. Management fees bias β downwards. When both fees are present, the bias in β is more pronounced. Biases in volatility follow a similar pattern.

Case 2: In this case we compute for each investment in each quarter the value of the carried interest and deduct it from the market value of the investment. That is, we mark to market the investment every quarter net of latent fees. As detailed in Metrick and Yasuda (2010), this option-based fee calculation is rather complex and requires numerical solutions. Rerunning their one-time calculation for each of our investments each quarter in each simulation is not feasible. However, given that our objective is to see how robust our method is to incorporating embedded call option values, not to obtain a realistic number, we adopt a simpler approach.[26]

For each investment in each quarter, we calculate the Black-Scholes value of a call option given investment return volatility, the investment value at that point in time, and the remaining time until exit. Twenty percent of the value of this call option (with a strike price set to investment cost) is the "unrealized" carried interest. We track the quarterly changes in "unrealized" carried interest, which is what we then subtract from the investment value each quarter. When the investment is exited we compute the carried interest due and then subtract the balance of the difference between the unrealized and the realized carried interest from the final dividend. We set management fees to zero here because, as shown above, management fees have a distinct effect on estimation precision.

These simulations imply a lower true β: it decreases from 1.50 to 1.37, 1.23, and 1.08 when carried interest is 10%, 20%, and 30%, respectively. This is because the true β is computed from the market value of the fund each quarter net of changes in the value of the call option. The cash flows, however, do not reflect this smooth process. We therefore overestimate *beta* in this setup. For example, with carried interest of 20%, we overestimate β by 9%, but volatility remains well estimated (non-tabulated).

Figure 3, panel B plots the correlation between the true and estimated return time series. We show results for different levels of carried interest and, as in the other figures, plot correlations as a function of idiosyncratic volatility. Correlations are lower as carried interest increases, but the decrease in precision appears to be economically small.

To summarize, if we set the true return process net of fees so that fees affect an investment return only during the quarter in which they are charged, we underestimate β. By contrast, if we set the true return process net of fees, so that fees are continuously deducted from the true investment value, we overestimate β. In both cases, the correlation between true and estimated returns falls moderately.

[26] In practice, the value of the option is complex because one needs to assume a correlation between the investments made by a given fund. In addition, the payoff has different values as a function of investment value and fund value due to the catch-up provision (which is a combination of options). Further, the exit timing of each investment in the fund is endogenous and interdependent. Solving this problem at each point in time for each investment in each simulation is nearly infeasible given current computer capacities. Future work may incorporate these option values into our framework.

II. Data

Preqin collects the quarterly aggregated investments, distributions, and net asset values (NAVs) of PE funds as recorded by U.S. pension funds and obtained via routine Freedom of Information Act requests. We use the cash flow data set of Preqin as of October 2015 with data stopping in June 2015 (there is a reporting time lag). As in Harris, Jenkinson, and Kaplan (2014), we limit attention to focus on the subset of funds that are U.S.-focused.

The Preqin cash flow data set is increasingly used in academia. A recent example is a first study on the secondary market for PE fund stakes by Nadauld et al. (2017). The attraction of Preqin data is that they are publicly available (at a cost) and cash flows should be accurately reported and without a performance bias—pension funds would face serious sanctions if they deliberately misreported or selectively reported returns.

Data from Burgiss, used in Harris, Jenkinson, and Kaplan (2014), are also increasingly used and are generally perceived as more comprehensive and perhaps more accurate. We compare average performance derived from each of these data sets as we want to make sure that our performance figures are consistent with what would be derived in a more comprehensive data set. Coverage in both data sets jumps up in 1994 (e.g., fewer than 10 buyout funds are raised in any year before 1994 in both the Burgiss and the Preqin data sets). To assess funds' risk profile, we need to observe the cash flows of a sufficient number of funds at any point in time. We therefore start with vintage 1994. We stop with vintage 2008 because fewer funds are raised after 2008 and these funds have significant unrealized value as of 2015.

Table V in Harris, Jenkinson, and Kaplan (2014) shows PMEs by vintage year for buyout and venture capital funds. In our Table II, we copy the Harris, Jenkinson, and Kaplan (2014) figures and compute the corresponding statistics for the Preqin data set. The output statistics are extremely close.

Funds serving fiduciaries such as pension funds report their audited calculations of portfolio value (NAV) every quarter. FASB 157 now requires fund assets to be fair market–valued. However, the private nature of these investments and varying methodologies for evaluation leaves significant uncertainty. Ultimately, reported fund NAVs represent each fund manager's opinion about the assets in his or her portfolio.[27] As we select funds that are at least seven years old, this issue has a moderate impact on our results but we should bear this in mind when trying to assess the underlying "true" returns.

III. Empirical Results

A. Factor Exposures and Private Equity Premium

In Table III, we report parameter estimates of the factor loadings, β, and the α coefficients with different asset pricing factor models. The table reports

[27] Brown, Gredil, and Kaplan (2017) find that fund valuations are conservative except when follow-on funds are raised. In times of fundraising, Barber and Yasuda (2017) estimate that NAV is exaggerated by about 3% for buyout funds and about 5% for venture capital funds.

Table II

Descriptive Statistics: Preqin versus Burgiss Data Sets

This table shows the median and average fund PMEs, which compare private equity returns to equivalent-timed investments in the S&P 500 index. Statistics are displayed by vintage year (from 1994 to 2008). The average is weighted using the capital committed to the funds as weights. Statistics derived from the Burgiss data set are as reported by Harris, Jenkinson, and Kaplan (2014). Preqin is our working data set. Panel A focuses on venture capital funds while Panel B focuses on buyout funds. Only funds with a U.S. focus are included in this table.

Panel A: Venture Capital Fund PMEs

	Burgiss			Preqin		
	Funds	Median	Average	Funds	Median	Average
1994	23	1.40	3.42	11	1.41	3.92
1995	28	1.49	3.14	16	1.62	2.76
1996	23	2.16	4.34	15	1.33	2.17
1997	42	1.43	2.68	21	0.96	1.66
1998	58	0.99	1.74	30	1.00	1.51
1999	88	0.67	0.89	37	0.60	0.71
2000	109	0.66	0.78	71	0.68	0.80
2001	58	0.83	0.91	43	0.79	1.04
2002	21	0.76	0.79	24	0.63	0.82
2003	30	0.81	1.09	13	0.99	0.86
2004	49	0.75	1.23	29	0.72	0.92
2005	59	0.80	0.98	27	0.94	1.24
2006	70	0.80	0.95	41	0.80	0.86
2007	84	0.93	1.08	42	1.07	1.02
2008	58	0.85	1.05	33	0.95	0.99
Average		1.02	1.67		0.97	1.42
All funds	800			453		

Panel B: Buyout Fund PMEs

	Burgiss			Preqin		
	Funds	Median	Average	Funds	Median	Average
1994	20	1.09	1.46	18	1.09	1.21
1995	23	1.01	1.17	11	0.84	1.05
1996	18	1.13	1.05	13	1.13	1.17
1997	31	1.03	1.27	21	1.43	1.45
1998	46	1.40	1.31	31	1.17	1.15
1999	34	1.21	1.13	24	1.40	1.23
2000	60	1.38	1.48	30	1.52	1.60
2001	31	1.49	1.48	18	1.40	1.51
2002	23	1.34	1.51	17	1.29	1.51
2003	23	1.40	1.55	12	1.48	1.71
2004	50	1.29	1.45	30	1.35	1.41
2005	66	1.12	1.26	32	1.06	1.36
2006	80	1.03	1.02	44	1.09	1.03
2007	86	0.97	0.99	43	1.02	1.05

(*Continued*)

Table II—*Continued*

Panel B: Buyout Fund PMEs						
	Burgiss			Preqin		
	Funds	Median	Average	Funds	Median	Average
2008	64	0.96	1.03	39	0.99	1.02
Average		1.19	1.28		1.21	1.29
All funds	655			423		

posterior means and standard deviations of the parameters. The Internet Appendix offers details on the methodology, the choice of priors, and robustness checks.

We begin with models that rely on one, three, and four systematic factors. The one-factor model is the CAPM, the three-factor model is that of Fama and French (1993), which adds SMB (small minus big) and HML (high minus low) factors, and the four-factor model is that of Pástor and Stambaugh (2003), which adds a liquidity factor. Next, we report results for a one-factor model in which we use the CRSP equally weighted index instead of the CRSP value-weighted index as a measure of market returns. This is equivalent to the assumption that PE funds acquire companies that are drawn from a pool resembling the CRSP sample. That is, they are as likely to acquire a firm from the bottom decile as from the top decile of capitalization. This assumption is useful because the typical company purchased by a PE fund is relatively small compared to the universe of listed companies. The drawback is that the equal-weighted CRSP index is not investable.

We next create three models with only traded factors. The four factors are the Vanguard S&P 500 index minus the risk-free rate, Dimensional Fund Advisors (DFA) microcap mutual fund minus the Vanguard S&P 500, the DFA value mutual fund minus the Vanguard S&P 500, and the T. Rowe High Yield index minus the Vanguard S&P 500 index. The DFA microcap fund captures the small stock premium, the DFA value find captures the value premium, and the T. Rowe High Yield index captures the liquidity premium. For these factors to be treated as premiums, the S&P 500 index is shorted from all but the first factor. In practice, shorting the S&P 500 is feasible and bears a negligible cost.

Finally, we report results using the new Fama and French (2015) five-factor model, which includes "profitability" and "investment" as defined by Kenneth French's website.

Table III shows that the CAPM estimate of the β for PE is 1.4, which is unchanged using an equally weighted market index. The estimates for the four-factor loadings on the market, size, value, and liquidity factors are 1.5 for the market excess return, 0.8 for SMB, -0.1 for HML, and 0.5 for the liquidity factor. In the four-factor model, the posterior means of the market and SMB loadings are statistically significant, but this is not the case for the value and

Table III
Private Equity Factor Exposures

This table shows the estimated risk loadings and abnormal returns α using eight different asset pricing factor models. Panel A includes all U.S. private equity funds (except fund-of-funds) in the Preqin data set (N = 1,089). Panel B includes all U.S. venture capital funds (includes funds classified as balanced and growth). Panel C includes all U.S. buyout funds. Funds are raised between 1994 and 2008, and cash flows start in the first quarter of 1994 and end in the second quarter of 2015. α is annualized and defined as the constant that makes the average value-weighted PME equal to one, given the estimated risk loadings. Beneath each coefficient, in italics, we report the posterior standard deviation of the estimated parameters. The factor models that we use are: the CAPM, the three-factor model of Fama and French (1993), and the four-factor model of Pástor and Stambaugh (2003). The next model replaces the CRSP value weighted index as a measure of market returns by (i) the CRSP equally weighted index (panel A), (ii) equally weighted Nasdaq index (panel B), and (iii) equally weighted AMEX/NYSE index (panel C). The next three models use the following four traded factors: the Vanguard S&P 500 minus the risk-free rate, the DFA microcap mutual fund minus the Vanguard S&P 500, the DFA value mutual fund minus the Vanguard S&P 500, and the T. Rowe High Yield index minus the Vanguard S&P 500. Finally, the Fama-French five factors model include "profitability" and "investment" as defined on Kenneth French's website. Priors for the factor loadings are given in the internet Appendix Table IA.I [a], [b], and [c] indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Panel A: All Private Equity Funds

Model	β_{market}	β_{size}	β_{value}	$\beta_{illiquidity}$		α	Likelihood
CAPM	1.43[a]					0.02[a]	−68
	0.21					0.00	
3 factors (FF)	1.51[a]	0.81[c]	−0.07			0.01[c]	−63
	0.26	*0.45*	*0.28*			*0.01*	
4 factors (PS)	1.54[a]	0.82[c]	−0.12	0.51		−0.01[c]	−64
	0.28	*0.46*	*0.26*	*0.33*		*0.01*	
1 factor (EW)	1.40[a]					−0.01	−72
	0.20					*0.01*	
Traded 1 factor	1.63[a]					−0.02[a]	−70
	0.28					*0.01*	
Traded 3 factors	1.57[a]	0.83[b]	−0.12			−0.02[a]	−65
	0.26	*0.40*	*0.26*			*0.01*	
Traded 4 factors	1.81[a]	0.81[b]	−0.12	0.44		−0.05[a]	−65
	0.36	*0.40*	*0.27*	*0.43*		*0.01*	
	β_{market}	β_{size}	β_{value}	$\beta_{profitability}$	$\beta_{investment}$	α	Likelihood
5 factors (FF)	1.88[a]	1.04[b]	−0.19	0.49	0.16	−0.02[b]	−62
	0.36	*0.46*	*0.20*	*0.37*	*0.42*	*0.01*	

Panel B: Venture Capital Funds

Model	β_{market}	β_{size}	β_{value}	$\beta_{illiquidity}$	α	Likelihood
CAPM	1.80[a]				0.00	−115
	0.30				*0.01*	
3 factors (FF)	1.72[a]	0.81[b]	−0.61		0.01	−102
	0.31	*0.39*	*0.41*		*0.01*	
4 factors (PS)	1.73[a]	0.89[b]	−0.64	0.49	−0.03[a]	−105
	0.30	*0.41*	*0.40*	*0.33*	*0.01*	
1 factor (EW)	1.46[a]				−0.03[a]	−123
	0.17				*0.01*	

(*Continued*)

Table III—*Continued*

Panel B: Venture Capital Funds							
Model	β_{market}	β_{size}	β_{value}	$\beta_{illiquidity}$		α	Likelihood
Traded 1 factor	1.82[a]					−0.04[a]	−109
	0.31					*0.01*	
Traded 3 factors	1.85[a]	1.03[b]	−0.82[b]			−0.04[a]	−102
	0.33	*0.43*	*0.37*			*0.01*	
Traded 4 factors	2.09[a]	0.91[b]	−0.90[b]	0.60		−0.05[a]	−101
	0.41	*0.41*	*0.37*	*0.42*		*0.01*	
	β_{market}	β_{size}	β_{value}	$\beta_{profitability}$	$\beta_{investment}$	α	Likelihood
5 factors (FF)	1.99[a]	0.93[b]	−0.80[b]	0.70	0.43	−0.05[a]	−109
	0.39	*0.44*	*0.41*	*0.49*	*0.53*	*0.01*	

Panel C: Buyout Funds							
Model	β_{market}	β_{size}	β_{value}	$\beta_{illiquidity}$		α	Likelihood
CAPM	*1.25* [a]					0.04[a]	120
	0.25					0.01	
3 factors (FF)	*1.22*[a]	*0.47*	*0.60*[c]			*0.01*[b]	119
	0.27	0.47	0.33			0.01	
4 factors (PS)	*1.32*[a]	*0.63*	*0.66*[c]	*0.56*[b]		*−0.03*[a]	118
	0.29	0.48	0.37	0.26		0.01	
1 factor (EW)	*1.18*[a]					*0.02*[a]	116
	0.24					0.01	
Traded 1 factor	*1.22*[a]					*0.02*[a]	116
	0.26					0.01	
Traded 3 factors	*1.31*[a]	*0.33*	*0.70*[c]			*−0.02*[b]	115
	0.29	0.45	0.40			0.01	
Traded 4 factors	*1.77*[a]	*0.02*	*0.48*[c]	*1.08*[b]		*−0.04*[a]	118
	0.41	0.31	0.29	0.52		0.01	
	β_{market}	β_{size}	β_{value}	$\beta_{profitability}$	$\beta_{investment}$	α	Likelihood
5 factors (FF)	1.60[a]	0.44	0.67[c]	1.06[b]	0.25	−0.06[a]	116
	0.34	*0.48*	*0.41*	*0.52*	*0.50*	*0.01*	

liquidity factor loadings. Nevertheless, the economic magnitude of 0.5 for the liquidity factor β is relatively large.

α is annualized and defined as the constant that makes the average value-weighted PVR equal to one, given the estimated risk loadings. Across all models, α is close to 0.

For the subset of venture capital funds (panel B), α is mostly negative across the different models. We note that venture capital has a significant negative loading on the Fama-French value factor, which is what we would expect from a strategy that buys high-growth companies. Venture capital strategies appear to load positively on growth stocks, which had low average returns. Also, the value-weighted stock-market index used in the CAPM had low returns over our sample period, which sets a low bar in terms of performance. When the index is the equally weighted Nasdaq index, venture capital exhibits a significant negative α.

For buyout funds (panel C) α is 4% annually and β is close to unity. This means that the performance metrics reported in prior literature are validated here. Our α using the (value-weighted) one-factor model is 4%, also consistent with the literature. However, α goes to 0 when we control for other risk factors. These results are in line with those of Stafford (2016) and Phalippou (2014), among others.

The coefficients on the value and liquidity factors are positive and significant. The inclusion of the Pástor-Stambaugh liquidity factor seems to have the greatest effect on α, reducing it from 1% to –3%. This can be interpreted as buyout funds harvesting a liquidity risk premium in the Pástor-Stambaugh sense that buyout funds have exposure to a liquidity factor constructed from publicly traded equities (see Franzoni, Nowak, and Phalippou (2012)).

In several specifications we reject the null that PE assets are redundant with respect to the standard Fama-French and Pástor-Stambaugh equity factors. These systematic factors capture a large portion of the total returns to investing in PE. However this does not necessarily imply that there is no value to PE, because none of these equity factor returns are available without incurring transaction costs. An open question is whether an investor can cheaply access the premiums of the tradable factors passively, or whether PE investments are a more efficient way to access these factor premiums. Addressing this question would involve an analysis of transaction costs (and investor size) that is beyond the scope of this paper.

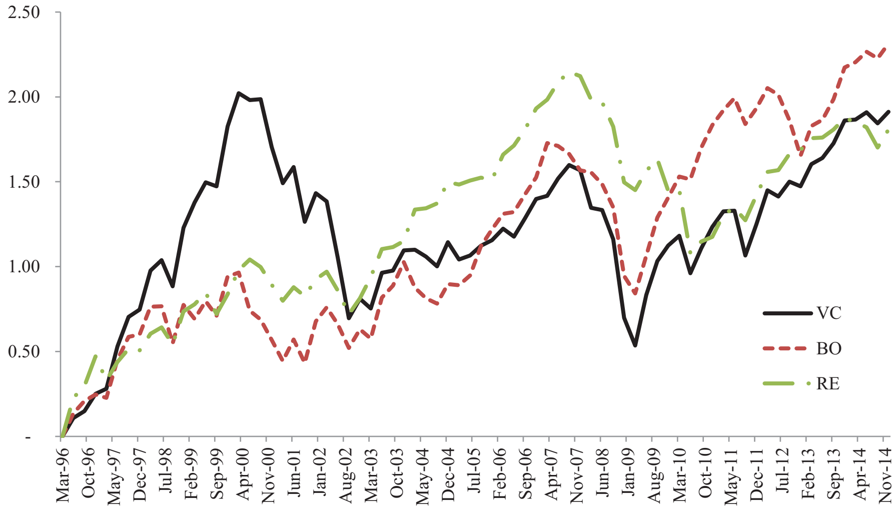
The different factor models have different level of goodness of fit. For each subset of PE funds, we select the model with the highest likelihood. We then obtain our best-fitted index for each type of PE funds. Figure 4, panel A plots the cumulated log total return index, g_t, for venture capital, buyout and real estate.

We find that venture capital observed a sharp increase in returns in the late 1990s, peaked mid-2000 and then decreased sharply. Venture capital returns then remained flat until 2012, when they started to increase sharply. Buyout returns show a different pattern. In particular they were relatively flat until 2003. They then increased sharply until early 2007, before collapsing with the 2008 crisis. They bounced back quickly, however, coincident with quantitative easing (QE).[28]

Real estate returns increased steadily up until the crisis, with clear acceleration as of 2003, then experienced a collapse, but returns started to recover from 2012. Remarkably, real estate started to decline in the third quarter of 2007, well ahead of available appraisal-based commercial property indexes. This is

[28] Many buyout funds refinanced their investments after mid-2010, when QE started, and could pay large dividends as a result. Also, IPO activity resumed at about the same time and many leveraged buy-out investments were exited that way. But again, our estimation method does not rely on observing an exit. Instead identification comes from observing higher NPV for funds that held more investments in their portfolio from mid-2010. This is what identifies the turning point.

Panel A. Cumulated log returns per sub-type of U.S. private equity funds



Panel B. Cumulated log returns of U.S. buyout funds, the S&P 500 index and the DFA value fund

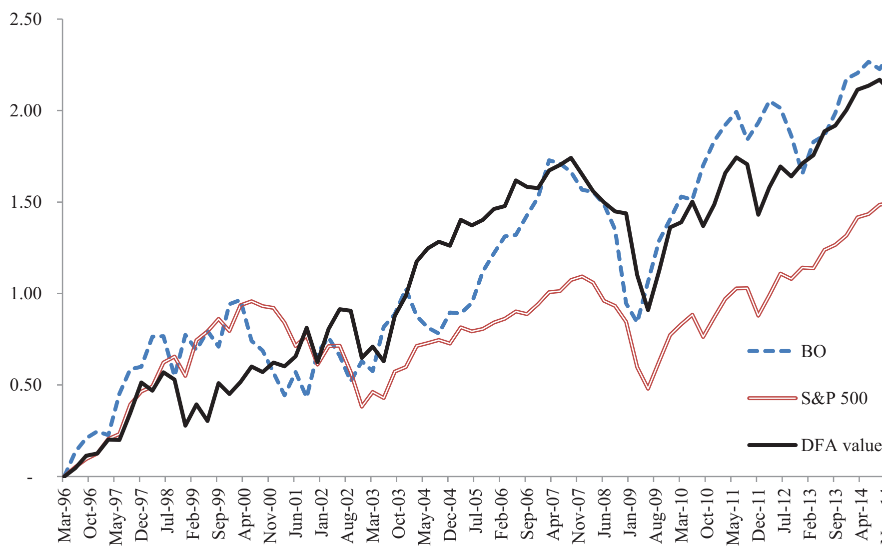


Figure 4. Cumulative private equity returns. This figure shows cumulated log returns of several indices. Panel A plots it for our index of buyout funds (BO), venture capital funds (VC), and real estate funds (RE). Panel B plots it for our index of buyout funds (BO), the S&P 500 index and the DFA value mutual fund (DFA value). (Color figure can be viewed at wileyonlinelibrary.com)

consistent with anecdotal evidence that the U.S. real estate market started to decline in the third quarter of 2007.

The fact that we obtain such marked and coherent cycles while using only factor returns and the cash flows of funds as classified by Preqin suggests that the estimation recovers more than the levered market index or a combination of factors. It is remarkable that we get such diverse cycles across fund types while all of the estimations used the same set of factors. These cycles are also consistent with conventional beliefs as reflected in industry reports and press coverage.

In addition, the imperfect co-movement across return indexes suggests that venture capital, real estate, and buyout fund cycles differ from one another. Hence, there are benefits to diversifying across PE investment classes. More generally, the evidence suggests that, even conditional on differing exposures to systematic factors, PE returns in different asset classes are influenced by other trends unrelated to publicly traded securities.

Figure 4, panel B plots our (log) index of buyout funds and compares it to the cumulated (log) returns of the Vanguard S&P 500 index fund and of the DFA value mutual fund (a passive low-cost mutual fund with a long track record investing in value stocks). Consistent with the findings of Harris, Jenkinson, and Kaplan (2014) among others, and with results in Table III, buyout funds outperform the S&P 500 index. However, our results above show that part of this performance is replicable using our estimated passive factor exposures. In addition, DFA value mutual fund returns exhibit a similar pattern to those of the buyout index. Value stocks experienced high returns from 2003, which is when buyout funds also posted high returns and started to raise record amounts of capital. As buyout funds tend to invest in value companies, this finding has implications for benchmarking and asset allocation decisions. For example, relatively small investors would have apparently been similarly well off with value stocks as with buyout funds.

B. Comparison with Industry Indexes

One practical advantage of our cash flow-based index is that it seeks to attribute returns to the time period in which they occur. In practice, some industry indices have the same objective but they use either self-reported NAVs or listed stocks of companies that are in the PE industry.

NAVs are potentially subject to inertia, for example, anchoring on prior appraisal values. The econometrics of appraisal-based indices have been well studied for commercial real estate (see Geltner (1991)). Among other things, they have volatilities that underestimate true volatilities and lag market values. In this section, we examine the relationship between our cash flow-based index and industry indices.

In Table IV, we report the annualized mean, standard deviation, interquartile range, and autocorrelation coefficient for some standard industry indices and for our cash flow-based indices (CF index). For the industry NAV-based indices, we use those of Cambridge Associates. For the listed equity-based

Table IV

Comparison of Private Equity Index with Industry Indices

Columns 2 to 6 report the following descriptive statistics for each index return: the annualized mean, volatility, 25^{th} and 75^{th} percentiles, and autocorrelation coefficient (computed at the quarterly frequency). Three indices are shown for four types of private equity funds: buyouts, venture capital, real estate, and all private equity. The three indices are our index, the Cambridge Associates index, and an index based on publicly listed companies. Data are from Q1-1995 to Q4-2014.

			Percentiles		
	Mean	Volatility	25^{th}	75^{th}	Autocorrelation
CF buyout index	0.17	0.26	−0.21	0.52	0.09
Cambridge Associates buyout index	0.15	0.11	0.02	0.28	0.40
LPX listed buyout index	0.14	0.29	−0.04	0.36	0.19
CF venture capital index	0.17	0.31	−0.15	0.53	0.14
Cambridge Associates venture index	0.18	0.26	0.00	0.26	0.60
LPX listed venture capital index	0.11	0.37	−0.37	0.45	0.12
CF real estate index	0.13	0.21	−0.08	0.42	0.11
Cambridge Associates real estate index	0.11	0.10	0.06	0.17	0.64
FTSE listed real estate index (REITS)	0.13	0.21	−0.09	0.38	0.14

indices, we use those of LPX. All of the mean and volatility estimates in Table IV are annualized.

Table IV shows that our cash flow-based indices are more volatile than the industry indices. The difference is particularly dramatic for real estate and buyouts. For buyouts, the volatility of our cash flow-based return time series is more than twice as high as that of Cambridge Associates (25% compared to 11%). It is smaller, however, than the volatility of the LPX buyout index, which is used in recent capital requirement regulation (e.g., Solvency II).[29]

For real estate, we estimate a volatility of 21% per annum, compared to 10% for Cambridge Associates.[30] Interestingly, 21% is exactly the volatility of the FTSE listed real estate index based on returns of listed real estate funds (REITS). This suggests that our estimated index may provide a more realistic estimate of real estate portfolio risk for investment managers.

There is a smaller difference in volatilities for venture capital, at 30% for our index and 26% for the venture capital index produced by Cambridge Associates. Note that the latter is driven solely by a sharp spike in volatility in 1999.

The results above indicate that existing PE return time series exhibit smoothing biases likely due to the appraisal process and the fact that valuations of illiquid assets may only partially adjust to market prices. In addition, we find

[29] Europe's key insurance regulator, the Solvency II Committee, has been criticized for using LPX indices rather than less-volatile appraisal-based indices as a basis for value-at-risk parameters in their calculations of PE capital requirements. Our estimates derived from PE funds cash flows lie between these two and are generally closer to those of the LPX indices.

[30] The NCREIF index, a commonly used index, has a volatility of only 5%. It is also appraisal-based.

that our PE return time series exhibit much less serial dependence, if any, in contrast to industry indices.[31]

C. Test of the Market Segmentation Hypothesis

The cyclicality of PE represents a challenge to PE investors who face the decision of how to time their investments, or how to maintain a continuous commitment to the asset class and manage expectations about short-term performance. This pattern is also difficult to explain in a standard economic framework. Kaplan and Strömberg (2009) introduce a novel theory of boom and bust cycles in PE. They propose that funds exploit segmentation between debt and equity markets. Kaplan and Strömberg (2009) extend the insights of the behavioral corporate finance literature to explain this correlation. In particular, Baker, Greenwood, and Wurgler (2003) present evidence that corporations choose financing channels based on relative capital market demand for equity versus debt. Kaplan and Strömberg (2009) argue that the ultimate source of the variation in relative demand for debt versus equity is market sentiment, and they find supportive evidence in a graph of EBITDA/enterprise value minus the high yield spread. When this variable is high, PE should be relatively profitable because the cost of debt financing is low compared to the return on assets.

Our cash flow-based buyout index allows us to empirically test the behavioral market segmentation hypothesis. In particular, we test whether PE is profitable when the Kaplan-Stromberg asset-debt yield spread is higher. Table V reports the results of regressions in which our PE cash flow index is the dependent variable and the independent variables include the asset-debt yield spread, the expected risk premium, the volume of buyout transactions (scaled by stock market capitalization), the Baker-Wurgler sentiment index, and a set of macroeconomic variables that capture credit conditions (the default spread, which is the difference in yields on AAA and BAA rated debt) and the health of the economy (growth in industrial production, inflation, and the change in the VIX index).

Our specification jointly tests the theory that market sentiment provides the opportunity for PE managers to create value, and that the source of that value is the asset-debt yield spread. If market sentiment is a significant determinant of PE returns, we should find a positive sign on the sentiment index and a negative sign on the change in the VIX. In our specification the sign on the default spread may go either way since, by construction, it is negatively correlated with the asset-debt spread.

The results are reported in Table V. We find that our index is significantly positively related to the asset-debt spread, consistent with the Kaplan-Strömberg hypothesis. Moreover, it is negatively related to the change in the VIX, and

[31] We also unsmooth appraisal-based return as in Getmansky, Lo, and Makarov (2004) and find that the volatility of the buyout series increases to 16%, that of venture capital increases to about 40%, and that of real estate increases to 16% (see Internet Appendix).

Table V

Private Equity Returns over the Business Cycle

This table shows how our cash flow-based buyout return index relates to macroeconomic variables. We compute t-statistics using Newey-West standard errors with four lags, which are shown beneath each coefficient in italics. The time period is from the first quarter of 1996 to the last quarter of 2014. The risk premium and scaled volume are from Haddad, Loualiche, and Plosser (2017) and are only available until Q4-2011.

Constant	0.03	0.03	−0.01	−0.01	−0.04	−0.13	−0.12	−0.14
	2.28	*2.01*	*−0.15*	*−0.18*	*−0.59*	*−1.40*	*−1.57*	*−1.90*
Ebitda/EV − High yield spread	0.02	0.02	0.02	0.02	0.02	0.02	0.03	0.03
	2.62	*2.54*	*2.20*	*2.19*	*2.03*	*1.68*	*2.86*	*3.01*
Change in VIX index		0.07	0.07	0.07	0.07	0.07	0.08	0.08
		1.53	*1.65*	*1.67*	*1.60*	*1.85*	*1.71*	*1.82*
Default spread (BAA-AAA)			3.59	3.78	6.27	12.92	12.89	12.34
			0.80	*0.72*	*0.98*	*1.69*	*2.17*	*2.09*
Inflation				0.33	0.37	0.20	0.71	0.78
				0.18	*0.21*	*0.12*	*0.45*	*0.51*
Sentiment index					0.03	0.04	0.05	0.05
					0.91	*1.34*	*1.73*	*1.74*
Industrial production growth						3.42	1.78	1.55
						1.73	*0.82*	*0.70*
Risk premium							0.01	0.01
							2.19	*2.10*
Scaled volume								0.00
								0.71
Adjusted R^2	0.10	0.13	0.14	0.14	0.15	0.22	0.37	0.37
Number of observations	76	76	76	76	76	76	64	64

positively related to both transaction volume and the expected risk premium. These results are consistent with the hypothesis that PE does well when the economy does well, and with the results in Haddad, Loualiche, and Plosser (2017).

One qualification of these findings is that we are measuring the contemporaneous effects of the asset-debt yield spread. The proposed channel through which this adds value is via the purchase of a higher yielding asset financed by issuing cheap debt. The fund cash flows we observe are deployment or realization of capital and are thus conditional on such a transaction occurring. Nevertheless, our index assumes that all firms in operation at a given date experience the same shocks. If we could separate transacting firms from firms that were not exploiting the spread, we might find a larger effect.

IV. Conclusion

Researchers and practitioners interested in understanding PE investment have been limited by the structure and nature of the data. This has made it particularly difficult to evaluate its time series characteristics. We develop a methodology for extracting a latent performance measure from nonperiodic cash flow information, and demonstrate how it may be further decomposed into passive and active components. We find that PE returns are only partially spanned by investable passive indices. Our estimates suggest that to a first approximation, PE is a levered investment in small and mid-cap equities.

We estimate the PE return for separate classes and show that their cycles are not highly correlated. This suggests that a diversified strategy across sub-asset classes of PE may be beneficial. Our cash flow-based PE indices also allow us to test current theories about the cyclical nature of PE returns. We find evidence in favor of the hypothesis of Kaplan and Strömberg (2009) that relative yields on corporate assets compared to high-yield debt explain the returns of PE investments.

Our methodology and results also have potential regulatory implications. Volatility measures for PE that came from our cash flow-based return series are at least as volatile as standard aggregate equity market indices. In contrast, estimates of PE volatility constructed from appraisal-based indices are much lower. The Solvency II Committee, the European Union's flagship project to harmonize European insurance supervision and set capital requirements (similar to Basel II), has chosen to use a publicly traded proxy for PE returns. Our results suggest that the volatility estimates derived from such an index are close to the volatility of true PE returns. Investors and regulators all benefit from more accurate estimates of returns and risk from illiquid PE.

Initial submission: June 23, 2014; Accepted: April 17, 2017
Editors: Bruno Biais, Michael R. Roberts, and Kenneth J. Singleton

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