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Common Risk Factors in Currency Markets

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We identify a "slope" factor in exchange rates. High interest rate currencies load more on this slope factor than low interest rate currencies. This factor accounts for most of the cross-sectional variation in average excess returns between high and low interest rate currencies. A standard, no-arbitrage model of interest rates with two factors—a country-specific factor and a global factor—can replicate these findings, provided there is sufficient heterogeneity in exposure to global or common innovations. We show that our slope factor identifies these common shocks, and we provide empirical evidence that it is related to changes in global equity market volatility. By investing in high interest rate currencies and borrowing in low interest rate currencies, U.S. investors load up on global risk. (*JEL* G12, G15, F31)

We show that the large co-movement among exchange rates of different currencies supports a risk-based view of exchange rate determination. In order to do so, we start by identifying a slope factor in exchange rate changes: The exchange rates of high interest rate currencies load positively on this factor, while those of low interest rate currencies load negatively on it. The covariation with this slope factor accounts for most of the spread in average returns between baskets of high and low interest rate currencies—the returns on the currency carry trade. We show that a no-arbitrage model of interest rates and exchange rates with two state variables—country-specific and global risk factors—can match the data, provided there is sufficient heterogeneity in countries' exposures to the global risk factor. To support this global risk interpretation, we provide evidence that the global risk factor is closely related to changes in volatility of equity markets around the world.

We identify this common risk factor in the data by building monthly portfolios of currencies sorted by their forward discounts. The first portfolio contains

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the lowest interest rate currencies, while the last contains the highest. The first two principal components of the currency portfolio returns account for most of the time-series variation in currency returns. The first principal component is a level factor. It is essentially the average excess return on all foreign currency portfolios. We call this average excess return the dollar risk factor, *RX*. The second principal component is a slope factor whose weights decrease monotonically from positive to negative from high to low interest rate currency portfolios. Hence, average returns on the currency portfolios line up with portfolio loadings on this second component. This slope factor is very similar to the return on a zero-cost strategy that goes long in the last portfolio and short in the first portfolio. We label this excess return the carry trade risk factor, HML_{FX}, for high minus low interest rate currencies. We obtain the same results for exchange rate changes as for currency returns. Our article is the first to document the common factor in exchange rates sorted by interest rates, which is the key ingredient in a risk-based explanation of carry trade returns.

In international finance, there is a large literature that studies asset pricing in integrated capital markets.[1] In this class of integrated capital market models, risk refers invariably to exposure to some common or global factor. We show that the slope factor in exchange rates provides a direct measure of the global risk factor. This factor, which was constructed from currency portfolios, explains variation in the country-level returns as well, and the estimated risk prices are very similar to those obtained from the currency portfolios. We explain about two-thirds of the cross-sectional variation when we allow for time variation in the betas of individual currencies with our factors, which is captured by variation in relative interest rates.

Building on our empirical findings, we derive conditions that candidate stochastic discount factors need to satisfy in order to match our currency portfolio returns. Our results refine the conditions derived by Backus, Foresi, and Telmer (2001) for replicating the forward premium anomaly in a large class of exponentially affine asset pricing models.[2] Heterogeneity in exposure to country-specific risk can generate negative uncovered interest rate parity (UIP) slope coefficients for individual currency pairs, as pointed out by Backus, Foresi, and Telmer (2001), but it cannot explain the cross-section of carry trade returns. The intuition is simple. Investors earn the carry trade premium by shorting baskets of low interest rate currencies and going long in baskets of high interest rate currencies. Provided that they invest in large baskets of currencies, carry trade investors are not exposed to any country-specific risk.

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[1] This literature includes world arbitrage pricing theory, developed by Adler and Dumas (1983) and Solnik (1983); a world consumption–capital asset pricing model (CAPM), Wheatley (1988); a world CAPM, Harvey (1991); world latent factor models, Campbell and Hamao (1992), Bekaert and Hodrick (1992), and Harvey, Solnik, and Zhou (2002); world multi-beta models, Ferson and Harvey (1993); and more recently work on time-varying capital market integration by Bekaert and Harvey (1995) and Bekaert, Hodrick, and Zhang (2009).

[2] In earlier work, Bekaert (1996) and Bansal (1997) had pointed out the need for heteroscedastic pricing kernels in order to produce time-varying currency risk premiums.

We show that heterogeneity in exposure to common risk can both explain the carry trade returns and deliver the negative UIP slope coefficients.[3] First, we need a large common or global component in the pricing kernel, because this is the only source of cross-sectional variation in currency risk premiums. Second, we need sufficient heterogeneity in exposure to the common component: Currencies with currently (on average) lower interest rates need to be temporarily (permanently) more exposed to the common component. Affine asset pricing models automatically satisfy the second condition, provided that an increase in the conditional volatility of the pricing kernel lowers the short-term interest rate. These two conditions ensure the existence of currency risk premiums and carry trade excess returns from the perspective of *all* investors, regardless of the home currency. Currency risk premiums are determined by a home risk premium that compensates for home country risk (e.g., a dollar risk premium for the U.S. investor) and a carry trade risk premium that compensates for global or common risk.

Without exposure to common risk, the carry risk premium is zero, as shorting baskets of low interest rate currencies and going long in baskets of high interest rate currencies does not expose investors to any country-specific or currency-specific risk. Temporary heterogeneity in exposure to common risk matches the conditional deviations from UIP; currencies with currently high interest rates deliver higher returns. Permanent differences in exposure to common risk match the unconditional deviations from UIP; currencies with on-average high interest rates also deliver higher returns. These unconditional deviations from UIP in the cross-section account for 40% of the total carry trade risk premium. In the data, we find that a measure of global equity volatility accounts for the cross-section of carry trade returns, as predicted by the model. High (low) interest rate currencies tend to depreciate (appreciate) when global equity volatility is high.

Many papers have documented the failure of UIP in the time series, starting with the work of Hansen and Hodrick (1980) and Fama (1984): Higher than usual interest rates lead to further appreciation, and investors earn more by holding bonds in currencies with interest rates that are *higher than usual*.[4] By building portfolios of positions in currency forward contracts sorted by forward discounts, Lustig and Verdelhan (2005, 2007) have shown that UIP fails in the cross-section, even when including developing currencies: Investors earn large excess returns simply by holding bonds from currencies with interest rates that are *currently high*, i.e., currently higher than those of other currencies, not only *higher than usual*, i.e., higher than usual for that same currency. Lustig and Verdelhan (2007) adopt the perspective of a U.S. investor and test

3 In closely related work, Brandt et al. (2006) infer the need for a large common component in the pricing kernel from the high Sharpe ratios in equity markets and the low volatility of exchange rates. Colacito and Croce (forthcoming) deliver a general equilibrium dynamic asset pricing model with this feature.

4 Hodrick (1987) and Lewis (1995) have surveyed this literature. Bansal and Dahlquist (2000) show that UIP works better for exchange rates of countries that have experienced higher rates of inflation.

this investor's Euler equation. Our article enforces the Euler equation of all investors. Furthermore, we distinguish between unconditional deviations and conditional deviations from UIP.

An alternative explanation of our findings is that the interest rate is simply *one* of the characteristics that determine returns, as suggested by Bansal and Dahlquist (2000).[5] Ranaldo and Soderlind (2010), for example, pursue this further by arguing that some currencies are viewed simply as safe havens and therefore earn a lower risk premium than others that are perceived as more risky. Based on the empirical evidence, we cannot definitively rule out a characteristics-based explanation. Interest rates and slope factor betas are very highly correlated in the data. However, we replicate these findings in the data simulated from a version of our model that is calibrated to match exchange rate and interest rate moments in the actual data. In the model-generated data, we cannot rule out a characteristics-based explanation either, even though the true data-generating process has no priced characteristics.

Our article is organized as follows: We start by describing the data, the method used to build currency portfolios, and the main characteristics of these portfolios. Section 2 shows that a single factor, HML_{FX}, explains most of the cross-sectional variation in foreign currency excess returns. Section 3 considers several extensions. We look at beta-sorted portfolios and confirm the same pattern in excess returns. By randomly splitting the sample, we also show that risk factors constructed from currencies not used as test assets still explain the cross-section. Finally, we show that our results continue to hold at the country level. In Section 4, we use a no-arbitrage model of exchange rates to interpret these findings. A calibrated version of the model replicates the key moments of the data. Finally, we show that an equity-based volatility measure accounts for the cross-section of currency excess returns, as predicted by the model. Section 5 concludes. A separate appendix available online reports additional robustness checks. The portfolio data can be downloaded from our websites and are regularly updated.

1. Currency Portfolios

We focus on investments in forward and spot currency markets. Compared with Treasury bill markets, forward currency markets exist for only a limited set of currencies and for short time periods. But forward currency markets offer two distinct advantages. First, the carry trade is easy to implement in these markets, and the data on bid-ask spreads for forward currency markets are readily available. This is not the case for most foreign fixed income markets. Second, these forward contracts are subject to minimal default and counter-party risk. This section describes the properties of monthly foreign currency excess returns

[5] Bansal and Dahlquist (2000) were the first to examine the cross-sectional relation between interest rates and currency risk premiums. They document that the Hansen and Hodrick (1980) and Fama (1984) findings seem to apply mostly to developed economies.

from the perspective of a U.S. investor. We consider currency portfolios that include developed and emerging market countries for which forward contracts are traded. We find that currency markets offer Sharpe ratios comparable to the ones measured in equity markets, even after controlling for bid-ask spreads. As do Lustig and Verdelhan (2005, 2007), we sort currencies on their interest rates and allocate them to portfolios. Where, however, those authors used T-bill yields to compute annual currency excess returns, our current article focuses on monthly investment horizons and uses only spot and forward exchange rates to compute returns.

1.1 Building currency portfolios

Currency excess returns. We use s to denote the log of the spot exchange rate in units of foreign currency per U.S. dollar, and f for the log of the forward exchange rate, also in units of foreign currency per U.S. dollar. An increase in s means an appreciation of the home currency. The log excess return rx on buying a foreign currency in the forward market and then selling it in the spot market after one month is simply

$$rx_{t+1} = f_t - s_{t+1}.$$

This excess return can also be stated as the log forward discount minus the change in the spot rate: $rx_{t+1} = f_t - s_t - \Delta s_{t+1}$. In normal conditions, forward rates satisfy the covered interest rate parity condition; the forward discount is equal to the interest rate differential: $f_t - s_t \approx i_t^{\star} - i_t$, where $i^{\star}$ and i denote the foreign and domestic nominal risk-free rates over the maturity of the contract. Akram, Rime, and Sarno (2008) study high-frequency deviations from covered interest-rate parity (CIP). They conclude that CIP holds at daily and lower frequencies. Hence, the log currency excess return equals approximately the interest rate differential less the rate of depreciation:

$$rx_{t+1} \approx i_t^{\star} - i_t - \Delta s_{t+1}.$$

Transaction costs. Since we have bid-ask quotes for spot and forward contracts, we can compute the investor's actual realized excess return net of transaction costs. The *net* log currency excess return for an investor who goes long in foreign currency is

$$rx_{t+1}^{l} = f_t^{b} - s_{t+1}^{a}.$$

The investor buys the foreign currency or equivalently sells the dollar forward at the bid price (f^b) in period t, and sells the foreign currency or equivalently buys dollars at the ask price (s_{t+1}^{a}) in the spot market in period $t+1$. Similarly, for an investor who is long in the dollar (and thus short the foreign currency), the net log currency excess return is given by

$$rx_{t+1}^{s} = -f_t^{a} + s_{t+1}^{b}.$$

Data. We start from daily spot and forward exchange rates in U.S. dollars. We build end-of-month series from November 1983 to December 2009. These data are collected by Barclays and Reuters and are available on Datastream. Lyons (2001) reports that bid-ask spreads from Reuters are roughly twice the size of inter-dealer spreads. We assume that net excess returns take place at these quotes. As a result, our estimates of the transaction costs are conservative. Lyons (2001) also notes that these indicative quotes track inter-dealer quotes closely, only lagging the inter-dealer market slightly at very high intra-day frequencies. This is clearly not an issue here at monthly horizons. Our main dataset contains at most 35 different currencies: of Australia, Austria, Belgium, Canada, Hong Kong, Czech Republic, Denmark, euro area, Finland, France, Germany, Greece, Hungary, India, Indonesia, Ireland, Italy, Japan, Kuwait, Malaysia, Mexico, Netherlands, New Zealand, Norway, Philippines, Poland, Portugal, Saudi Arabia, Singapore, South Africa, South Korea, Spain, Sweden, Switzerland, Taiwan, Thailand, and the United Kingdom. Some of these currencies pegged their exchange rates partly or completely to the U.S. dollar over the course of the sample. We keep them in our sample because forward contracts were easily accessible to investors. The euro series starts in January 1999. We exclude the euro area countries after this date and keep only the euro series.

Based on large failures of covered interest rate parity, we chose to delete the following observations from our sample: South Africa from the end of July 1985 to the end of August 1985; Malaysia from the end of August 1998 to the end of June 2005; Indonesia from the end of December 2000 to the end of May 2007; Turkey from the end of October 2000 to the end of November 2001; and United Arab Emirates from the end of June 2006 to the end of November 2006. In addition, there were widespread deviations from CIP in the fall of 2008, as reported, for example, by Jones (2009). However, the implications for the magnitude of returns that we report are limited.[6]

As a robustness check, we also a study a smaller dataset that contains the currencies of only 15 developed countries: Australia, Belgium, Canada, Denmark, euro area, France, Germany, Italy, Japan, Netherlands, New Zealand, Norway, Sweden, Switzerland, and the United Kingdom. We first focus the description of our results on our large sample, but we present all of our results on both samples.

Portfolios. At the end of each period t, we allocate all currencies in the sample to six portfolios on the basis of their forward discounts $f - s$ observed at

[6] Jones (2009) offers this example to illustrate the size of the implied returns at the peak of these CIP deviations: "Assuming that USD funds were available, the arbitrageur would attempt to borrow $1m dollars at 12-month USD LIBOR and enter into a foreign exchange swap to Euros to invest the funds for an identical term in Euro Libor. On completion of the swap and repayment of the loan, the arbitrageur will be left with approximately $12,600 (126bp) profit." We can safely regard a return of 126 basis points in one of the 26 years of our sample as measurement error. Taking this into account would change the average return by around 5 basis points.

the end of period t. Portfolios are rebalanced at the end of every month. They are ranked from low to high interest rate; portfolio 1 contains the currencies with the lowest interest rate or smallest forward discounts, and portfolio 6 contains the currencies with the highest interest rate or largest forward discounts. We compute the log currency excess return rx_{t+1}^{j} for portfolio j by taking the average of the log currency excess returns in each portfolio j. For the purpose of computing returns net of bid-ask spreads, we assume that investors *short* all the foreign currencies in the *first* portfolio and go *long* in all the other foreign currencies.

The total number of currencies in our portfolios varies over time. We had a total of 9 countries at the beginning of the sample in 1983 and 26 at the end in 2009. We include only currencies for which we have forward and spot rates in the current and subsequent period. The maximum number of currencies attained during the sample is 34; the launch of the euro accounts for the subsequent decrease in the sample size.

1.2 Returns to currency speculation for a U.S. investor

Table 1 provides an overview of the properties of the six currency portfolios from the perspective of a U.S. investor. For each portfolio j, we report average changes in the spot rate Δs^j, the forward discounts $f^j - s^j$, the log currency excess returns $rx^j = -\Delta s^j + f^j - s^j$, and the log currency excess returns net of bid-ask spreads rx_{net}^{j}. We report log returns because these are the sum of the forward discount and the change in spot rates. We also report log currency excess returns on carry trades or high-minus-low investment strategies that go long in portfolio $j = 2, 3, \ldots, 6$, and short in the first portfolio: $rx_{net}^{j} - rx_{net}^{1}$. All exchange rates and returns are reported in U.S. dollars, and the moments of returns are annualized: We multiply the mean of the monthly data by 12 and the standard deviation by $\sqrt{12}$. The Sharpe ratio is the ratio of the annualized mean to the annualized standard deviation.

The first panel reports the average rate of depreciation for all currencies in portfolio j. According to the standard UIP condition, the average rate of depreciation $E_T\left(\Delta s^j\right)$ of currencies in portfolio j should equal the average forward discount on these currencies $E_T\left(f^j - s^j\right)$, reported in the second panel. Instead, currencies in the first portfolio trade at an average forward discount of -297 basis points, but they appreciate on average by only 64 basis points over this sample. This adds up to a log currency excess return of -233 basis points on average, which is reported in the third panel. Currencies in the last portfolio trade at an average discount of 901 basis points but depreciate by only 282 basis points on average. This adds up to a log currency excess return of 620 basis points on average.

The fourth panel reports average log currency excess returns net of transaction costs. Since we rebalance portfolios monthly, and transaction costs are incurred each month, these estimates of net returns to currency speculation are

Table 1
Currency portfolios—U.S. investor

Portfolio	1	2	3	4	5	6	1	2	3	4	5
	Panel I: All Countries						Panel II: Developed Countries				
	Spot change: Δs^j						Δs^j				
Mean	−0.64	−0.92	−0.95	−2.57	−0.60	2.82	−1.81	−1.87	−3.28	−1.57	−0.82
Std	8.15	7.37	7.63	7.50	8.49	9.72	10.17	9.95	9.80	9.54	10.26
	Forward Discount: $f^j - s^j$						$f^j - s^j$				
Mean	−2.97	−1.23	−0.09	1.00	2.67	9.01	−2.95	−0.94	0.11	1.18	3.92
Std	0.54	0.48	0.47	0.52	0.64	1.89	0.77	0.62	0.63	0.66	0.74
	Excess Return: rx^j (without b–a)						rx^j (without b–a)				
Mean	−2.33	−0.31	0.86	3.57	3.27	6.20	−1.14	0.93	3.39	2.74	4.74
Std	8.23	7.44	7.66	7.59	8.56	9.73	10.24	9.98	9.89	9.62	10.33
SR	−0.28	−0.04	0.11	0.47	0.38	0.64	−0.11	0.09	0.34	0.29	0.46
	Net Excess Return: rx^j_{net} (with b–a)						rx^j_{net} (with b–a)				
Mean	−1.17	−1.27	−0.39	2.26	1.74	3.38	−0.02	−0.11	2.02	1.49	3.07
Std	8.24	7.44	7.63	7.55	8.58	9.72	10.24	9.98	9.87	9.63	10.32
SR	−0.14	−0.17	−0.05	0.30	0.20	0.35	−0.00	−0.01	0.21	0.15	0.30
	High-minus-Low: $rx^j - rx^1$ (without b–a)						$rx^j - rx^1$ (without b–a)				
Mean		2.02	3.19	5.90	5.60	8.53		2.07	4.53	3.88	5.88
Std		5.37	5.30	6.16	6.70	9.02		7.18	7.11	8.02	9.64
SR		0.38	0.60	0.96	0.84	0.95		0.29	0.64	0.48	0.61
	High-minus-Low: $rx^j_{net} - rx^1_{net}$ (with b–a)						$rx^j_{net} - rx^1_{net}$ (with b–a)				
Mean		−0.10	0.78	3.42	2.91	4.54		−0.09	2.04	1.51	3.09
		[0.30]	[0.30]	[0.35]	[0.38]	[0.51]		[0.41]	[0.40]	[0.45]	[0.54]
Std		5.40	5.32	6.15	6.75	9.05		7.20	7.11	8.04	9.66
SR		−0.02	0.15	0.56	0.43	0.50		−0.01	0.29	0.19	0.32

(*continued*)

Table 1
Continued

Portfolio	1	2	3	4	5	6	1	2	3	4	5
	Panel I: All Countries						Panel II: Developed Countries				
	Real Interest Rate Differential: $r^j - r$						$r^j - r$				
Mean	−1.81	−0.13	0.45	1.04	1.80	3.78	−1.11	0.20	0.76	1.27	3.01
Std	0.56	0.56	0.49	0.57	0.65	0.77	0.78	0.60	0.62	0.62	0.71
	Frequency										
Trades/currency	0.20	0.34	0.41	0.44	0.42	0.14	0.14	0.28	0.36	0.35	0.10

This table reports, for each portfolio j, the average change in log spot exchange rates Δs^j, the average log forward discount $f^j - s^j$, the average log excess return rx^j without bid-ask spreads, the average log excess return rx^j_{net} with bid-ask spreads, the average return on the long short strategy $rx^j_{net} - rx^1_{net}$ and $rx^j - rx^1$ (with and without bid-ask spreads), the real interest rate differential $r^j - r$, and the portfolio turnover. Log currency excess returns are computed as $rx^j_{t+1} = -\Delta s^j_{t+1} + f^j_t - s^j_t$. All moments are annualized and reported in percentage points. Standard errors are reported between brackets. For excess returns, the table also reports Sharpe ratios, computed as ratios of annualized means to annualized standard deviations. The portfolios are constructed by sorting currencies into six groups at time t based on the one-month-forward discount (i.e., nominal interest rate differential) at the end of period $t-1$. The first portfolio contains currencies with the lowest interest rates. The last portfolio contains currencies with the highest interest rates. Panel I uses all countries; panel II focuses on developed countries. Data are monthly, from Barclays and Reuters (Datastream). The sample period is 11/1983–12/2009.

conservative. After taking into account bid-ask spreads, the average return on the first portfolio drops to −117 basis points. Note that the first column reports *minus* the actual log excess return for the first portfolio, because the investor is short in these currencies. The corresponding Sharpe ratio on this first portfolio is −0.14. The return on the sixth portfolio drops to 338 basis points. The corresponding Sharpe ratio on the last portfolio is 0.35.

The fifth panel reports returns on zero-cost strategies that go long in the high interest rate portfolio and short in the low interest rate portfolio. The spread between the net returns on the first and the last portfolio is 454 basis points. This high-minus-low strategy delivers a Sharpe ratio of 0.50, after taking into account bid-ask spreads. We also report standard errors on these average returns between brackets. The average returns on the last four investment strategies are statistically significantly different from zero.

Currencies in portfolios with higher forward discounts tend to experience higher real interest rates. The ex post real interest rate differences are computed off the forward discounts.[7] There is a large spread of 559 basis points in (ex post) real interest rates between the first and the last portfolio. The spread is somewhat smaller (412 basis points) on the sample of developed currencies.

Finally, the last panel reports the frequency of currency portfolio switches. We define the average frequency as the time average of the ratio of the number of portfolio switches divided by the total number of currencies at each date. The average frequency is 29.84%, implying that currencies switch portfolios roughly every three months. When we break it down by portfolio, we get the following frequency of portfolio switches (in percentage points): 20% for the first, 34% for the second, 41% for the third, 44% for the fourth, 42% for the fifth, and 14% for the sixth. Overall, there is substantial variation in the composition of these portfolios, but there is more persistence in the composition of the corner portfolios.

We have documented that a U.S. investor with access to forward currency markets can generate large returns with annualized Sharpe ratios that are comparable to those in the U.S. stock market. Table 1 also reports results obtained on a smaller sample of developed countries. We obtain similar results. The Sharpe ratio on a long-short strategy is 0.32.

1.3 Average vs. current interest rate differences

What fraction of the return differences across currency portfolios are due to differences in average interest rates vs. differences in current interest rates between currencies? In other words, are we compensated for investing in high

[7] We compute real interest rates as nominal interest rates minus expected inflation. We use the lagged one-year change in log consumer price index as proxy for expected inflation. For some countries in the developing group, we have no consumer price index data. This is the case for Kuwait, Saudi Arabia, and United Arab Emirates. The data for Turkey start in May 1986. The data for South Africa start in January 2008.

interest rate currencies or for investing in currencies with currently high interest rates? We address this question by sorting currencies on average forward discounts in the first half of the sample and then computing the realized excess returns in the second part of the sample. Thus computed, these returns correspond to an implementable investment strategy.

The top panel in Table 2 reports the results from this sort on average forward discounts. The bottom panel reports the results from the standard sort on current forward discounts over the same sample. Even the sort on average interest differences produces a monotonic pattern in excess returns: Currencies with higher average interest rates tend to earn higher average returns. Before transaction costs, this sort produces a 5.34% "unconditional" carry trade premium compared with a 10.16% conditional carry trade premium. Hence, the unconditional premium accounts for 52% of the total carry trade premium. After transaction costs, the numbers change to 2.83% and 6.28%, respectively. After transaction costs, the conditional premium accounts for 45% of the total. However, the strategy of rebalancing by sorting on current interest rates delivers much higher Sharpe ratios than the unconditional strategy. The unconditional sort produces a Sharpe ratio of 0.23, compared with 0.70 for the conditional sort. Per unit of risk, the compensation for "conditional" carry trade risk is much higher.

These unconditional sorts of currencies seem to pick up mainly variation in average real interest rates across currencies: The countries in the first portfolio have average real interest rate differentials of −96 basis points in the second half of the sample, compared with 243 basis points in the last portfolio.

2. Common Factors in Currency Returns

This section shows that the sizable currency excess returns described in the previous section are matched by covariances with risk factors.

2.1 Methodology

Linear factor models predict that average returns on a cross-section of assets can be attributed to risk premiums associated with their exposure to a small number of risk factors. In the arbitrage pricing theory (APT) of Ross (1976), these factors capture common variation in individual asset returns. A principal component analysis of our currency portfolios reveals that two factors explain more than 80% of the variation in returns on these six portfolios. The top panel in Table 3 reports the loadings of our currency portfolios on each of the principal components as well as the fraction of the total variance of portfolio returns attributed to each principal component. The first principal component explains 70% of the common variation in portfolio returns, and can be interpreted as a *level* factor, since all portfolios load equally on it. The second principal component, which is responsible for close to 12% of common variation,

Table 2
Currency portfolios—sorts on mean forward discounts (half sample)

Portfolio	1	2	3	4	5	6	1	2	3	4	5
	Panel I: All Countries						Panel II: Developed Countries				
	Sorts on Mean Forward Discounts (Half Sample)										
	Excess Return: rx^j (without b–a)						rx^j (without b–a)				
Mean	−2.28	−0.69	0.09	1.14	1.74	3.06	−2.94	−0.61	2.01	1.44	1.86
SR	−0.24	−0.18	0.01	0.15	0.18	0.26	−0.28	−0.06	0.24	0.15	0.21
	Net Excess Return: rx^j_{net} (with b–a)						rx^j_{net} (with b–a)				
Mean	−1.52	−1.21	−0.67	0.45	0.67	1.31	−1.94	−1.42	1.18	0.26	0.48
Std	9.45	3.78	7.32	7.75	9.95	11.88	10.41	10.32	8.37	9.49	9.02
SR	−0.16	−0.32	−0.09	0.06	0.07	0.11	−0.19	−0.14	0.14	0.03	0.05
	High-minus-Low: $rx^j_{net} - rx^1_{net}$ (with b–a)						$rx^j_{net} - rx^1_{net}$ (with b–a)				
Mean		0.32	0.86	1.97	2.19	2.83		0.51	3.11	2.20	2.42
SR		0.04	0.10	0.23	0.25	0.23		0.05	0.25	0.20	0.21
	Real Interest Rate Differences: $r^j - r$						$r^j - r$				
Mean	−0.96	0.52	−0.23	0.61	0.92	2.43	−1.16	−0.68	0.48	0.27	1.56
Std	0.44	0.60	0.49	0.43	0.55	0.49	0.73	0.42	0.44	0.47	0.45
	Sorts on Current Forward Discounts (Half Sample)										
	Excess Return: rx^j (without b–a)						rx^j (without b–a)				
Mean	−3.83	−1.36	0.22	1.99	2.22	6.33	−2.25	−0.53	0.91	1.94	3.90
SR	−0.50	−0.20	0.03	0.32	0.29	0.67	−0.24	−0.06	0.10	0.22	0.37
	Net Excess Return: rx^j_{net} (with b–a)						rx^j_{net} (with b–a)				
Mean	−2.81	−2.23	−0.70	1.02	0.81	3.46	−1.26	−1.48	−0.15	0.84	2.50
SR	−0.37	−0.33	−0.10	0.16	0.11	0.37	−0.13	−0.16	−0.02	0.10	0.24

(*continued*)

Table 2
Continued

Portfolio	1	2	3	4	5	6	1	2	3	4	5
	Panel I: All Countries						Panel II: Developed Countries				
	High-minus-Low: $rx^j_{net} - rx^1_{net}$ (with b–a)						$rx^j_{net} - rx^1_{net}$ (with b–a)				
Mean		0.58	2.11	3.83	3.63	6.28		−0.22	1.11	2.10	3.76
SR		0.11	0.43	0.66	0.54	0.70		−0.03	0.14	0.24	0.35
	Real Interest Rate Differences: $r^j - r$						$r^j - r$				
Mean	−1.43	−0.12	0.30	0.81	1.31	3.65	−1.40	−0.26	0.25	0.75	2.69
Std	0.49	0.49	0.33	0.47	0.55	0.67	0.72	0.43	0.42	0.49	0.56

This table reports, for each portfolio j, the average log excess return rx^j without bid-ask spreads, the average log excess return rx^j_{net} with bid-ask spreads, the average net return on the long short strategy $rx^j_{net} - rx^1_{net}$ and $rx^j - rx^1$, and the real interest rate difference $r^j - r$. Log currency excess returns are computed as $rx^j_{t+1} = -\Delta s^j_{t+1} + f^j_t - s^j_t$. All moments are annualized and reported in percentage points. For excess returns, the table also reports Sharpe ratios, computed as ratios of annualized means to annualized standard deviations. In the top panel, the portfolios are constructed by sorting currencies into six groups at time t based on the *average* one-month-forward discount (i.e., nominal interest rate differential) *over the first half of the sample* (11/1983–12/1994). The first portfolio contains currencies with the lowest average interest rates. The last portfolio contains currencies with the highest average interest rates. In the bottom panel, the portfolios are constructed by sorting currencies on current one-month-forward discounts. Panel I uses all countries; panel II focuses on developed countries. Data are monthly, from Barclays and Reuters (Datastream). The sample period is 1/1995–12/2009.

Table 3
Principal components

Panel I: All Countries						
Portfolio	1	2	3	4	5	6
1	0.42	0.43	0.18	−0.15	0.74	0.20
2	0.38	0.24	0.15	−0.27	−0.61	0.58
3	0.38	0.29	0.42	0.12	−0.28	−0.71
4	0.38	0.04	−0.35	0.83	−0.03	0.18
5	0.43	−0.08	−0.72	−0.44	−0.03	−0.30
6	0.45	−0.81	0.35	−0.03	0.11	0.06
% Var.	71.95	11.82	5.55	4.00	3.51	3.16

Panel II: Developed Countries					
Portfolio	1	2	3	4	5
1	0.44	0.66	−0.54	−0.25	0.12
2	0.45	0.25	0.75	0.01	0.41
3	0.46	0.02	0.19	0.04	−0.86
4	0.44	−0.27	−0.29	0.78	0.20
5	0.45	−0.66	−0.14	−0.57	0.17
% Var.	78.23	10.11	4.97	3.49	3.20

This table reports the principal component coefficients of the currency portfolios presented in Table 1. In each panel, the last row reports (in %) the share of the total variance explained by each common factor. Data are monthly, from Barclays and Reuters (Datastream). The sample period is 11/1983–12/2009.

can be interpreted as a *slope* factor, since portfolio loadings increase monotonically across portfolios. Since average excess returns increase monotonically across portfolios, the second principal component is the only plausible candidate risk factor that might explain the cross-section of portfolio excess returns, as none of the other principal components exhibit monotonic variation in loadings.

Motivated by the principal component analysis, we construct two candidate risk factors: the average currency excess return, denoted RX, and the difference between the return on the last portfolio and the one on the first portfolio, denoted HML_{FX}. The correlation of the first principal component with RX is 0.99. The correlation of the second principal component with HML_{FX} is 0.94. Both factors are computed from net returns, after taking into account bid-ask spreads. The bottom panel confirms that we obtain similar results even when we exclude developing countries from the sample. It is important to point out that these components capture common variation in exchange rates, not interest rates. When we redo our principal component analysis on the changes in spot exchange rates that correspond to the currency portfolios, we get essentially the same results.

The two currency factors have a natural interpretation. HML_{FX} is the return in dollars on a zero-cost strategy that goes long in the highest interest rate currencies and short in the lowest interest rate currencies. RX is the average portfolio return of a U.S. investor who buys all foreign currencies available in the forward market. This second factor is essentially the currency “market” return in dollars available to a U.S. investor, which is driven by the fluctuations of the U.S. dollar against a broad basket of currencies.

Cross-sectional asset pricing. We use Rx^j_{t+1} to denote the average excess return in levels on portfolio j in period $t+1$. All asset pricing tests are run on excess returns in levels, not log excess returns, to avoid having to assume joint log-normality of returns and the pricing kernel. In the absence of arbitrage opportunities, this excess return has a zero price and satisfies the following Euler equation:

$$E_t\left[M_{t+1}Rx^j_{t+1}\right] = 0.$$

We assume that the stochastic discount factor M is linear in the pricing factors Φ:

$$M_{t+1} = 1 - b(\Phi_{t+1} - \mu_\Phi),$$

where b is the vector of factor loadings and μ_Φ denotes the factor means. This linear factor model implies a beta pricing model: The expected excess return is equal to the factor price λ times the beta of each portfolio β^j:

$$E[Rx^j] = \lambda'\beta^j,$$

where $\lambda = \Sigma_{\Phi\Phi}b$, $\Sigma_{\Phi\Phi} = E(\Phi_t - \mu_\Phi)(\Phi_t - \mu_\Phi)'$ is the variance-covariance matrix of the factor, and β^j denotes the regression coefficients of the return Rx^j on the factors. To estimate the factor prices λ and the portfolio betas β, we use two different procedures: a generalized method of moments (GMM) estimation applied to linear factor models, following Hansen (1982), and a two-stage ordinary least squares (OLS) estimation following Fama and MacBeth (1973), henceforth FMB. In the first step, we run a time-series regression of returns on the factors. In the second step, we run a cross-sectional regression of average returns on the betas. We do not include a constant in the second step ($\lambda_0 = 0$).

2.2 Results

Table 4 reports the asset pricing results obtained using GMM and FMB on currency portfolios sorted by forward discounts. The left-hand side of the table corresponds to our large sample of developed and emerging countries, while the right-hand side focuses on developed countries. We describe first the results obtained on our large sample.

Cross-sectional regressions. The top panel of the table reports estimates of the market prices of risk λ and the stochastic discount factor (SDF) loadings b, the adjusted R^2, the square root of mean-squared errors $RMSE$, and the

Table 4
Asset pricing—U.S. investor

Panel I: Risk Prices

	All Countries							Developed Countries						
	$\lambda_{HML_{FX}}$	λ_{RX}	$b_{HML_{FX}}$	b_{RX}	R^2	RMSE	χ^2	$\lambda_{HML_{FX}}$	λ_{RX}	$b_{HML_{FX}}$	b_{RX}	R^2	RMSE	χ^2
GMM_1	5.50	1.34	0.56	0.20	70.11	0.96		3.29	1.90	0.29	0.20	64.78	0.64	
	[2.25]	[1.85]	[0.23]	[0.32]			14.39%	[2.59]	[2.20]	[0.23]	[0.23]			45.96%
GMM_2	5.51	0.40	0.57	0.04	41.25	1.34		3.91	3.07	0.35	0.32	−55.65	1.34	
	[2.14]	[1.77]	[0.22]	[0.31]			16.10%	[2.52]	[2.05]	[0.22]	[0.22]			52.22%
FMB	5.50	1.34	0.56	0.20	70.11	0.96		3.29	1.90	0.29	0.20	64.78	0.64	
	[1.79]	[1.35]	[0.19]	[0.24]			9.19%	[1.91]	[1.73]	[0.17]	[0.18]			43.64%
	(1.79)	(1.35)	(0.19)	(0.24)			10.20%	(1.91)	(1.73)	(0.17)	(0.18)			44.25%
Mean	**5.08**	**1.33**					**3.14**	**1.90**						

(*continued*)

Table 4
Continued

Panel II: Factor Betas

	All Countries						Developed Countries					
Portfolio	α_0^j	$\beta_{HML_{FX}}^j$	β_{RX}^j	R^2	$\chi^2(\alpha)$	*p–value*	α_0^j	$\beta_{HML_{FX}}^j$	β_{RX}^j	R^2	$\chi^2(\alpha)$	*p–value*
1	−0.10	−0.39	1.05	91.64			0.36	−0.51	0.99	94.31		
	[0.50]	[0.02]	[0.03]				[0.53]	[0.03]	[0.02]			
2	−1.55	−0.11	0.94	77.74			−1.17	−0.09	1.01	80.69		
	[0.73]	[0.03]	[0.04]				[0.85]	[0.04]	[0.04]			
3	−0.54	−0.14	0.96	76.72			0.62	−0.00	1.04	86.50		
	[0.74]	[0.03]	[0.04]				[0.79]	[0.03]	[0.03]			
4	1.51	−0.01	0.95	75.36			−0.17	0.12	0.97	82.84		
	[0.77]	[0.03]	[0.05]				[0.85]	[0.03]	[0.04]			
5	0.78	0.04	1.06	76.41			0.36	0.49	0.99	94.32		
	[0.82]	[0.03]	[0.05]				[0.53]	[0.03]	[0.02]			
6	−0.10	0.61	1.05	93.84								
	[0.50]	[0.02]	[0.03]									
All					6.79	34.05%					2.63	75.64%

The panel on the left reports results for all countries. The panel on the right reports results for developed countries. Panel I reports results from GMM and Fama–McBeth asset pricing procedures. Market prices of risk λ, the adjusted R^2, the square root of mean-squared errors $RMSE$, and the p-values of χ^2 tests on pricing errors are reported in percentage points. b denotes the vector of factor loadings. Excess returns used as test assets and risk factors take into account bid-ask spreads. All excess returns are multiplied by 12 (annualized). Shanken (1992)-corrected standard errors are reported in parentheses. We do not include a constant in the second step of the FMB procedure. Panel II reports OLS estimates of the factor betas. R^2s and p-values are reported in percentage points. The standard errors in brackets are Newey and West (1987) standard errors computed with the optimal number of lags according to Andrews (1991). The χ^2 test statistic $\alpha' V_\alpha^{-1} \alpha$ tests the null that all intercepts are jointly zero. This statistic is constructed from the Newey–West variance-covariance matrix (1 lag) for the system of equations (see Cochrane 2005, p. 234). Data are monthly, from Barclays and Reuters in Datastream. The sample period is 11/1983–12/2009. The alphas are annualized and in percentage points.

p-values of χ^2 tests (in percentage points).[8] The market price of HML_{FX} risk is 550 basis points *per annum*. This means that an asset with a beta of one earns a risk premium of 5.5% per annum. Since the factors are returns, no-arbitrage implies that the risk prices of these factors should equal their average excess returns. This condition stems from the fact that the Euler equation applies to the risk factor itself, which clearly has a regression coefficient β of one on itself. In our estimation, this no-arbitrage condition is satisfied. The average excess return on the high-minus-low strategy (last row of the top panel in Table 4) is 508 basis points. This value differs slightly from the previously reported mean excess return because we use excess returns in *levels* in the asset pricing exercise, but Table 1 reports *log* excess returns, defined as differences between the forward discount and the changes in the log of the exchange rates. So, the estimated risk price is only 42 basis points removed from the point estimate implied by linear factor pricing. The GMM standard error of the risk price is 225 basis points. The FMB standard error is 179 basis points. In both cases, the risk price is more than two standard errors from zero, and thus highly statistically significant.

The second risk factor RX, the average currency excess return, has an estimated risk price of 134 basis points, compared with a sample mean for the factor of 133 basis points. This is not surprising, because all the portfolios have a beta close to one with respect to this second factor. As a result, the second factor explains none of the cross-sectional variation in portfolio returns, and the standard errors on the risk price estimates are large: For example, the GMM standard error is 185 basis points. When we drop the dollar factor, the RMSE rises from 96 to 148 basis points, but the adjusted R^2 is still above 70%. The dollar factor does not explain any of the cross-sectional variation in expected returns, but it is important for the level of average returns. When we include a constant in the second step of the FMB procedure, the RMSE drops to 97 basis points, with only HML_{FX} as the pricing factor. Adding a constant to the dollar risk factor is redundant because the dollar factor acts like a constant in the cross-sectional regression (all of the portfolios' loadings on this factor are equal to one).

The lambdas indicate whether risk is priced, and HML_{FX} risk clearly is in the data. The loadings (b) have a natural interpretation as the regression coefficients in a multiple regression of the SDF on the factors. The t-statistics on b_{HML} consistently show that the carry trade risk factor helps explain the

[8] Our asset pricing tables report two p-values: In Panel I, the null hypothesis is that all the cross-sectional pricing errors are zero. These cross-sectional pricing errors correspond to the distance between the expected excess return and the 45-degree line in the classic asset pricing graph (expected excess return as a function of realized excess returns). In Panel II, the null hypothesis is that all intercepts in the time-series regressions of returns on risk factors are jointly zero. We report p-values computed as 1 minus the value of the chi-square cumulative distribution function (for a given chi-square statistic and a given degree of freedom). As a result, large pricing errors or large constants in the time series imply large chi-square statistics and low p-values. A p-value below 5% means that we can reject the null hypothesis that all pricing errors or constants in the time series are jointly zero.

cross-section of currency returns in a statistically significant way, while the dollar risk factor does not.

Overall, the pricing errors are small. The RMSE is 96 basis points, and the adjusted R^2 is 70%. The null that the pricing errors are zero cannot be rejected, regardless of the estimation procedure: All of the p-values (reported in percentage points in the column labeled χ^2) exceed 5%. These results are robust. They also hold in a smaller sample of developed countries, as shown in the right-hand side of Table 4.

Time-series regressions. The bottom panel of Table 4 reports the constants (denoted α^j) and the slope coefficients (denoted β^j) obtained by running time-series regressions of each portfolio's currency excess returns Rx^j on a constant and risk factors. The returns and alphas are in percentage points per annum. The first column reports alpha estimates. The second portfolio has a large negative alpha of −155 basis points per annum, significant at the 5% level. The fourth portfolio has a large alpha of 151 basis points per annum, significant at the 5% level. The other alpha estimates are much smaller and not significantly different from zero. The null that the alphas are jointly zero cannot be rejected at the 5% or 10% significance level. Using a linear combination of the portfolio returns as factors entails linear restrictions on alphas. When the two factors HML_{FX} and RX_{FX} are orthogonal, it is easy to check that $\alpha^1 = \alpha^6$, because $\beta^6_{HML_{FX}} - \beta^1_{HML_{FX}} = 1$ by construction and $\beta^6_{RX} = \beta^1_{RX} = 1$. In this case, the risk prices exactly equal the factor means. This is roughly what we find in the data.

The second column of the same panel reports the estimated betas for the HML_{FX} factor. These betas increase monotonically from −.39 for the first portfolio to .61 for the last currency portfolio, and they are estimated very precisely. The first three portfolios have betas that are negative and significantly different from zero. The last two have betas that are positive. The third column shows that betas for the dollar factor are essentially all equal to one. Obviously, this dollar factor does not explain any of the variation in average excess returns across portfolios, but it helps explain the average level of excess returns. These results are robust and comparable to the ones obtained on the sample of developed countries (reported on the right-hand side of the table).

A natural question is whether the unconditional betas of the bottom panel of Table 4 are driven by the covariance between exchange rate changes and risk factors, or between interest rate changes and risk factors. This is important because the conditional covariance between the log currency returns and the carry trade risk factor obviously depends on only the spot exchange rate changes:

$$cov_t\left[rx^j_{t+1}, HML_{FX,t+1}\right] = -cov_t\left[\Delta s^j_{t+1}, HML_{FX,t+1}\right].$$

The regression of the log changes in spot rates for each portfolio on the factors reveals that these betas are almost identical to the ones for portfolio returns (with a minus sign), as expected.[9] Low interest currencies offer a hedge against carry trade risk because they appreciate when the carry return is low, not because the interest rates on these currencies increase. High interest rate currencies expose investors to more carry risk, because they depreciate when the carry return is low, not because the interest rates on these currencies decline. This is exactly the pattern that our no-arbitrage model in Section 4 delivers. Our analysis within the context of the model focuses on conditional betas.[10]

Average vs. current interest rate differences. In Table 2, we showed that the sorts on mean forward discounts produce a spread in currency returns of about half of the total spread. These portfolios still load very differently on HML_{FX}, the factor that we construct from the sort on current interest rates. In the second part of the sample, starting in January 1995, the first portfolio's HML_{FX} loading is $-.49$ (with a standard error of 0.07), and the loading of the sixth portfolio is 0.39 (0.07). Hence, the spread in loadings is 0.88, only 12 basis points less than the spread in the betas of the portfolios sorted by current interest rates. The market price of risk at 3.3%, however, is lower than the mean of HML_{FX} (6.9%) and is not precisely estimated over this short sample.

3. Robustness

This section provides more evidence on the nature of currency risk premiums that directly supports a risk-based explanation of our findings.

3.1 Other test assets: Beta-sorted portfolios

First, in order to show that the sorting of currencies on forward discounts really measures the currency's exposure to the risk factor, we build portfolios based on each currency's exposure to aggregate currency risk as measured by HML_{FX}. For each date t, we first regress each currency i log excess return rx^i on a constant and HML_{FX} using a 36-month rolling window that ends in period $t-1$. This gives us currency i's exposure to HML_{FX}, and we denote it $\beta_t^{i,HML}$. Note that it uses information available at date t. We then sort currencies into six groups at time t based on these slope coefficients $\beta_t^{i,HML}$. Portfolio 1 contains currencies with the lowest betas. Portfolio 6 contains currencies with the highest betas. Table 5 reports summary statistics on these portfolios. The first panel reports average changes in exchange rates. The second panel shows that average forward discounts increase monotonically from portfolio 1 to portfolio 6. Thus, sorts based on forward discounts and sorts based on betas are clearly related, which implies that the forward discounts convey information about riskiness of individual currencies. The third panel reports the average

9 Results available in the separate appendix.

10 Unconditional betas at the level of portfolios approximate conditional betas for individual currencies to the extent that covariation between conditional means of exchange rate changes and factors is small.

Table 5
***HML_{FX}*-Beta-Sorted currency portfolios—U.S. investor**

Portfolio	1	2	3	4	5	6	1	2	3	4	5
	Panel I: All Countries						Panel II: Developed Countries				
	Spot change: Δs^j						Δs^j				
Mean	−1.29	−1.33	−1.15	−2.34	−0.40	0.53	−2.15	−0.43	−0.18	−1.11	−2.28
Std	8.82	7.85	8.18	7.67	8.67	8.44	9.61	9.44	10.47	10.21	9.96
	Discount: $f^j - s^j$						$f^j - s^j$				
Mean	−1.40	−0.34	0.70	1.01	1.58	3.73	−1.67	−0.67	0.68	1.01	2.53
Std	0.66	0.66	0.77	0.66	0.73	0.59	0.79	0.62	0.88	0.98	0.59
	Excess Return: rx^j (without b-a)						rx^j (without b-a)				
Mean	−0.11	0.99	1.85	3.35	1.98	3.20	0.48	−0.24	0.86	2.12	4.80
Std	8.92	7.88	8.20	7.69	8.63	8.43	9.70	9.48	10.47	10.22	9.96
SR	−0.01	0.13	0.23	0.44	0.23	0.38	0.05	−0.02	0.08	0.21	0.48
	High-minus-Low: $rx^j - rx^1$ (without b-a)						$rx^j - rx^1$ (without b-a)				
Mean		1.10	1.96	3.47	2.09	3.31		−0.72	0.38	1.64	4.32
		[0.33]	[0.38]	[0.45]	[0.55]	[0.57]		[0.40]	[0.52]	[0.54]	[0.60]
Std		5.41	6.28	7.48	9.15	9.56		6.64	8.67	9.08	10.02
SR		0.20	0.31	0.46	0.23	0.35		−0.11	0.04	0.18	0.43
	Pre-formation β						Pre-formation β				
Mean	−0.39	−0.24	−0.15	−0.01	0.21	0.56	−0.43	−0.24	−0.03	0.06	0.37
Std	0.28	0.25	0.27	0.28	0.44	0.45	0.28	0.31	0.54	0.52	0.47
	Post-formation β						Post-formation β				
Estimate	−0.34	−0.19	−0.19	−0.01	0.13	0.34	−0.38	−0.09	0.04	0.04	0.38
s.e	[0.04]	[0.04]	[0.04]	[0.05]	[0.06]	[0.04]	[0.05]	[0.05]	[0.04]	[0.05]	[0.03]

This table reports, for each portfolio j, the average change in the log spot exchange rate Δs^j, the average log forward discount $f^j - s^j$, the average log excess return rx^j without bid-ask spreads, and the average returns on the long short strategy $rx^j - rx^1$. The left panel uses our sample of developed and emerging countries. The right panel uses our sample of developed countries. Log currency excess returns are computed as $rx^j_{t+1} = -\Delta s^j_{t+1} + f^j_t - s^j_t$. All moments are annualized and reported in percentage points. Standard errors are reported between brackets. For excess returns, the table also reports Sharpe ratios, computed as ratios of annualized means to annualized standard deviations. Portfolios are constructed by sorting currencies into six groups at time t based on slope coefficients β^i_t. Each β^i_t is obtained by regressing currency i log excess return rx^i on HML_{FX} on a 36-period moving window that ends in period $t-1$. The first portfolio contains currencies with the lowest βs. The last portfolio contains currencies with the highest βs. We report the average pre-formation beta for each portfolio. The last panel reports the post-formation betas obtained by regressing realized log excess returns on portfolio j on HML_{FX} and RX_{FX}. We only report the HML_{FX} betas. The standard errors are reported in brackets. Data are monthly, from Barclays and Reuters (Datastream). The sample period is 11/1983–12/2009.

log excess returns. They are monotonically increasing from the first to the last portfolio, even though the spread is smaller than the one created by ranking directly on interest rates. Clearly, currencies that co-vary more with our risk factor—and are thus riskier—provide higher excess returns. The last panel reports the post-formation betas. They vary monotonically from −0.31 to 0.38. This finding is quite robust. When we estimate betas using a 12-month rolling window, we also obtain a 300-basis-point spread between the first and the last portfolio.[11]

3.2 Other factors: Splitting samples

Second, to guard against a mechanical relation between the returns and the factors, we randomly split our large sample of developed and emerging countries into two subsamples. To do so, we sorted countries alphabetically and consider two groups. We found that risk factors built using currencies that do not belong to the portfolios used as test assets can still explain currency excess returns.[12] However, the market price of risk appears higher and less precisely estimated than on the full sample, and thus further from its sample mean. This happens because, by splitting the sample, we introduce more measurement error in HML_{FX}. This shrinks the betas in absolute value (toward zero), lowers the spread in betas between high and low interest rate portfolios, and hence inflates the risk price estimates. However, portfolio betas increase monotonically from the first to the last portfolio, showing that common risk factors are at work on currency markets.

3.3 Country-level asset pricing

Third, we take our model to country-level data. We run country-level Fama and MacBeth (1973) tests. Creating portfolios of stocks could potentially lead to data-snooping biases (Lo and MacKinlay 1990) and destroy information by shrinking the dispersion of betas (e.g., as argued recently by Ang, Liu, and Schwarz, 2010). In order to address these concerns, we use country-level excess returns as test assets, but we continue to use the currency portfolios to extract our two currency risk factors, HML_{FX} and RX. We first study unconditional currency excess returns before turning to conditional currency excess returns.

Fama and MacBeth (1973). The Fama and MacBeth procedure has two steps. In the first step, we run time-series regressions of each country i's currency excess return on a constant, HML_{FX}, and RX:

$$Rx^i_{t+1} = c^i + \beta^i_{HML} HML_{FX,t+1} + \beta^i_{RX} RX_{t+1} + \epsilon_{i,t+1}, \text{ for a given } i, \forall t. \quad (1)$$

11 Finally, we also double-sorted by forward discounts (3 bins) and betas (2 bins), and we found that there was no significant spread in betas/returns to be generated. This is not surprising if as is the case in our model, interest rates measure the currency's exposure to the common risk factor, and the betas are measured with error.

12 The detailed results are reported in the separate appendix.

In the second step, we run cross-sectional regressions of all currency excess returns on betas:

$$Rx_t^i = \lambda_{HML,t}\beta_{HML}^i + \lambda_{RX,t}\beta_{RX}^i + \xi_t, \text{ for a given } t, \forall i.$$

We compute the market price of risk as the mean of all these slope coefficients: $\lambda_c = \frac{1}{T}\sum_{t=1}^{T}\lambda_{c,t}$ for c = *HML*, *RX*. This procedure is identical to the original Fama and MacBeth (1973) experiment.

The excess returns on individual currencies that are used as test assets do *not* take into account bid-ask spreads because we do not know *a priori* whether investors should take a short or a long position on each particular currency. In the interest of consistency, we use the same risk factors HML_{FX} and RX reported in Table 1; those risk factors take into account bid-ask spreads. We obtain similar results with risk factors that do not take into account bid-ask spreads, but the means of the risk factor are higher.

Unconditional country currency risk premiums. Table 6 reports our results on two samples. In both samples, the market prices of risk are positive and less than one standard error from the means of the risk factors. The RMSE and the mean absolute pricing error are larger than those obtained on currency portfolios, but we cannot reject the null hypothesis that all pricing errors are jointly zero. High beta countries tend to offer high unconditional currency excess returns.

Conditional country currency risk premiums. We now turn to conditional risk premiums. We start by reporting the results obtained with managed investments and then turn to time-varying factor betas. Investors can adjust their position in a given currency based on the interest rate at the start of each period to exploit the return predictability and increase the Sharpe ratio. We consider such managed investment strategies to capture the cross-section of *conditional* expected excess returns in addition to the raw currency excess returns. To construct these managed positions, we multiply each currency excess return by the appropriate beginning-of-month forward discount, normalized by subtracting the average forward discount across currencies and dividing by the cross-sectional standard deviation of forward discounts in the given period. We use the same procedure and the same risk factors as above on this augmented set of test assets; Table 6 reports these results as well. The market prices of risk are positive and significant and are in line with those obtained on the unconditional returns. The cross-sectional fit has improved. The carry and dollar risk factors are priced in the cross-section of currency excess returns and account for a large share of the cross-sectional differences in country excess returns in both samples.

An alternative approach for testing asset pricing models with time-varying risk exposures is to estimate the factor loadings using rolling windows instead

Table 6
Country-level asset pricing

$\lambda_{HML_{FX}}$	λ_{RX}	$b_{HML_{FX}}$	b_{RX}	R^2	$RMSE$	$MAPE$	χ^2
Panel I: Developed Countries							
Unconditional Betas							
3.69	2.93	3.90	3.70	59.55	1.11	0.86	
[2.31]	[1.76]	[2.47]	[2.25]				36.10
Unconditional and Conditional Betas using Managed Currency Excess Returns							
3.65	2.94	3.85	3.71	75.53	1.17	0.88	
[2.20]	[1.76]	[2.34]	[2.25]				39.85
Conditional Betas using Rolling Windows							
3.30	2.43	3.49	3.06	84.19	0.69	0.57	
[2.05]	[1.76]	[2.19]	[2.24]				36.84
Conditional Betas using Forward Discounts							
3.96	2.92	4.18	3.68	31.70	1.45	1.08	
[2.50]	[1.71]	[2.66]	[2.18]				34.73
Panel II: All Countries							
Unconditional Betas							
3.40	2.54	4.04	5.04	48.29	2.85	1.88	
[2.53]	[1.38]	[3.15]	[2.89]				42.36
Unconditional and Conditional Betas using Managed Currency Excess Returns							
4.78	2.69	5.74	5.25	51.51	2.67	1.62	
[2.44]	[1.38]	[3.03]	[2.89]				41.77
Conditional Betas using Rolling Windows							
4.64	2.34	5.59	4.54	65.76	2.21	1.48	
[1.99]	[1.35]	[2.47]	[2.82]				37.84
Conditional Betas using Forward Discounts							
4.43	2.23	5.33	4.33	66.22	2.36	1.82	
[1.59]	[1.28]	[1.95]	[2.63]				33.62

The table reports results from the Fama–MacBeth asset pricing procedure using individual currency excess returns. Market prices of risk λ, the adjusted R^2, the square root of mean-squared errors $RMSE$, the mean absolute pricing error $MAPE$, and the p-values of χ^2 tests on pricing errors are reported in percentage points. b denotes the vector of factor loadings. Excess returns used as test assets do *not* take into account bid-ask spreads. Risk factors HML and RX come from portfolios of currency excess returns that take into account bid-ask spreads. HML correspond to a carry trade strategy, long high interest rate currencies, and short low interest rate currencies. RX corresponds to the average currency return across all portfolios. All excess returns are multiplied by 12 (annualized). We do not include a constant in the second step of the FMB procedure. The standard errors in brackets are Newey and West (1987) standard errors computed with the optimal number of lags according to Andrews (1991). Data are monthly, from Barclays (Panel I) and Barclays and Reuters (Panel II) in Datastream. The sample period is 11/1983–12/2009.

of incorporating conditioning information explicitly. To estimate the risk prices, we run a set of cross-sectional regressions:

$$Rx^i_{t+1} = \lambda_{HML,t}\beta^i_{HML,t} + \lambda_{RX,t}\beta^i_{RX,t} + \xi_{t+1}, \text{ for a given } t, \forall i, \tag{2}$$

where $\beta^i_{HML,t}$ and $\beta^i_{RX,t}$ are estimated by running time-series regressions similar to Equation (1) but over the subsample of T_{window} periods up to period t. We report results obtained with rolling windows of length $T_{window} = 36$ months (therefore, we exclude currencies for which less than three years of observations are available—there are six such currencies in our sample). The model's cross-sectional fit is evaluated by comparing the true unconditional average returns with their predicted values:

$$E(Rx^i_{t+1}) = E\left(\lambda_{HML,t}\beta^i_{HML,t} + \lambda_{RX,t}\beta^i_{RX,t}\right), \forall i. \tag{3}$$

The results of tests based on this procedure are also reported in Table 6. The estimated prices of carry risk are very close (within half of a standard error) to the sample means of the HML_{FX} factor, at 4.6% in the full sample and 3.3% in the smaller sample of developed countries (compared with sample means of 5.1 and 3.1, respectively). The market price of carry risk is statistically significant in the full sample, but not in the smaller one. The estimated prices of dollar risk are similar to those reported previously. The cross-sectional fit of the model is also similar to that with other methods, with high cross-sectional R^2-values of 65.8% and 84.2% in the full and small samples, respectively.

Another standard approach for estimating dynamic factor loadings that allows us to use conditioning information without enlarging the asset space to include managed returns is to explicitly model betas as linear functions of the currency-specific forward discounts.[13] In particular, assume that $\beta^i_{HML,t} = h^i_0 + h^i_1 z^i_t$ and $\beta^i_{RX,t} = d^i_0 + d^i_1 z^i_t$, where z^i_t is the country-specific forward discount, standardized as described above. The parameters h^i_0, h^i_1, d^i_0, and d^i_1 can be estimated from the linear regression

$$\begin{aligned} Rx^i_{t+1} = c^i + h^i_0 HML_{FX,t+1} + h^i_1 z^i_t HML_{FX,t+1} + d^i_0 RX_{t+1} + d^i_1 z^i_t RX_{t+1} \\ + \epsilon_{i,t+1}, \text{ for a given } i. \end{aligned} \tag{4}$$

The factor risk prices $\lambda_{HML,t}$ and $\lambda_{RX,t}$ can then be estimated by running the second-stage cross-sectional regressions in Equation (2) on the fitted conditional betas. The pricing errors and cross-sectional tests then can be used to evaluate the unconditional restriction in Equation (3) as before. The results of this estimation are in the bottom rows in Table 6. This method produces very similar results to the rolling-window approach, which provides further evidence for the role of forward discounts in capturing the currencies' dynamic exposures to common sources of risk.

[13] For example, Ferson and Harvey (1999) use both the rolling window and the linear instrumental variable approaches to estimate dynamic factor loadings; see numerous references therein.

The country-level results are consistent with our portfolio-level results. We focus on portfolios in the rest of the article because they allow us to extract the slope factor. They also offer a simple nonparametric way of estimating conditional covariances, which are key for our analysis.

4. A No-arbitrage Model of Exchange Rates

We derive new restrictions on the stochastic discount factors (at home and abroad) that need to be satisfied in order to reproduce the carry trade risk premium that we have documented in the data. These restrictions are different from the restrictions that need to be satisfied to reproduce the negative UIP slope coefficients. We impose minimal structure by considering a no-arbitrage model for interest rates and exchange rates.

Our model has an exponentially affine pricing kernel and therefore shares some features with other models in this class, such as those proposed by Frachot (1996), Brennan and Xia (2006), and, in particular, Backus, Foresi, and Telmer (2001). However, unlike these authors, we do not focus on currency pairs but consider a world with N different countries and currencies, where N is large. This allows us to distinguish between common and country-specific factors.[14]

In each country i, the logarithm of the SDF m^i follows

$$-m^i_{t+1} = \alpha^i + \chi^i z^i_t + \sqrt{\gamma^i z^i_t}u^i_{t+1} + \chi^i z^w_t + \sqrt{\delta^i z^w_t + \kappa^i z^i_t}u^w_{t+1}.$$

There is a common global state variable z^w_t and a country-specific state variable z^i_t. The common state variable enters the pricing kernel of all investors in N different countries. The country-specific state variable obviously does not. This distinction between idiosyncratic (country-specific) and common (global) risk is very natural in a setting with a large number of countries and currencies.

The currency-specific innovations u^i_{t+1} and global innovations u^w_{t+1} are independent and identically distributed gaussian, with zero mean and unit variance; u^w_{t+1} is a world shock, common across countries, while u^i_{t+1} is country-specific (and thus uncorrelated across countries). The same innovations that drive the pricing kernel variation will govern the dynamics of the country-specific and world volatility processes. The country-specific volatility component is governed by an autoregressive square root process:

$$z^i_{t+1} = (1-\phi)\theta + \phi z^i_t + \sigma\sqrt{z^i_t}u^i_{t+1}.$$

The world volatility component is also governed by a square root process:

$$z^w_{t+1} = (1-\phi)\theta + \phi z^w_t + \sigma\sqrt{z^w_t}u^w_{t+1}.$$

[14] Papers that attribute the failure of UIP to systematic risk exposures include recent papers by Backus et al. (2001), Harvey et al. (2002), Brennan and Xia (2006), Lustig and Verdelhan (2007), Bansal and Shaliastovich (2010), Farhi and Gabaix (2010), Colacito (2008), Alvarez, Atkeson, and Kehoe (2009), and Verdelhan (2010). Earlier works include Korajczyk (1985), Cumby (1988), Bekaert and Hodrick (1992), Bekaert (1996), and Bansal (1997).

We assume that the standard deviation of innovations to the common and country-specific factors is identical; we refer to this volatility as σ. We also assume that the price of local risk depends on only local risk aversion but that the price of global risk is allowed to depend on both local and global risk aversion. As a result, the conditional market price of risk has a domestic component given by $\sqrt{\gamma^i z_t^i}$ and a global component given by $\sqrt{\delta^i z_t^w + \kappa^i z_t^i}$.

We assume that financial markets are complete but that some frictions in the goods markets prevent perfect risk-sharing across countries. As a result, the change in the real exchange rate Δq^i between the home country and country i is

$$\Delta q_{t+1}^i = m_{t+1} - m_{t+1}^i,$$

where q^i is measured in country i goods per home country good. An increase in q^i means a real appreciation of the home currency. For the home country (the United States), we drop the superscript.

Assumption 4.1. All countries share the same parameters $(\alpha, \chi, \gamma, \kappa)$, but not δ. The home country has the average δ loading on the global component.

Hence, we can drop the superscript i for all parameters except δ^i. All of the parameters are assumed to be nonnegative. With this notation, the real risk-free interest rate (in logarithms) is given by

$$r_t^i = \alpha + \left(\chi - \frac{1}{2}(\gamma + \kappa)\right) z_t^i + \left(\chi - \frac{1}{2}\delta^i\right) z_t^w.$$

The standard object of interest is the slope coefficient from a UIP regression of exchange rate changes on the interest rate differential. For an "average" country with the same exposure to global innovations as the United States ($\delta^i = \delta$), this is given by $Cov\left(\Delta q_{t+1}^i, r_{t+1}^i - r_{t+1}\right) / Var\left(r_{t+1}^i - r_{t+1}\right) = \chi / \left(\chi - \frac{1}{2}(\gamma + \kappa)\right)$.[15] Large values of γ and κ deliver negative UIP slope coefficients.

[15] When δ is identical at home and abroad, the change in the exchange rate is

$$\Delta q_{t+1}^i = \chi(z_t^i - z^i) + \left(\sqrt{\gamma z_t^i} u_{t+1}^i - \sqrt{\gamma z_t} u_{t+1}^i\right).$$

The real interest rate differential is given by

$$r_t^i - r_t = \left(\chi - \frac{1}{2}(\gamma + \kappa)\right)(z_t^i - z_t).$$

Hence, the real UIP slope coefficient for a country with the same δ as the domestic one is given by

$$\frac{\chi\left(\chi - \frac{1}{2}(\gamma + \kappa)\right) var(z_t^i - z^i)}{\left(\chi - \frac{1}{2}(\gamma + \kappa)\right)^2 var(z_t^i - z^i)} = \frac{\chi}{\left(\chi - \frac{1}{2}(\gamma + \kappa)\right)}.$$

Our focus is on the *cross-sectional* variation in conditional expected excess returns. Since the log pricing kernel m_{t+1} and the log excess returns $rx_{t+1} = r_t^i - r_t - \Delta q_{t+1}^i$ are jointly normally distributed, the Euler equation $E[MR^i] = 1$ implies that the expected excess return in levels (i.e., corrected for the Jensen term) is the conditional covariance between the log pricing kernel and returns:

$$E_t[rx_{t+1}^i] + \frac{1}{2}Var_t[rx_{t+1}^i] = -Cov_t[m_{t+1}, rx_{t+1}^i] = Var_t[m_{t+1}] - Cov_t[m_{t+1}^i, m_{t+1}], \qquad (5)$$

where lower letters denote logs. The second equality follows because only the exchange rate component $\Delta q_{t+1}^i = m_{t+1} - m_{t+1}^i$ of the log currency returns rx_{t+1}^i matters for the conditional covariance.

4.1 Restricted model

In order to explore the role of heterogeneity in the global risk exposures across currencies captured by δ^i on the cross-section of expected currency returns, we first focus on a restricted version of the model in which the time variation in the global component of the conditional price of risk depends on only the global factor: $\kappa = 0$. In this restricted version, the logarithm of the SDF m^i reduces to a more familiar two-factor Cox, Ingersoll, and Ross (1985) type process such as the one exploited by Backus, Foresi, and Telmer (2001), with the key difference being the heterogeneity in δ^i.

Using the expression for the pricing kernels, Equation (5) simplifies to

$$E_t[rx_{t+1}^i] + \frac{1}{2}Var_t[rx_{t+1}^i] = \gamma z_t + \sqrt{\delta z_t^w}\left(\sqrt{\delta z_t^w} - \sqrt{\delta^i z_t^w}\right).$$

We can express this risk premium in terms of quantities and prices of risk. The loading on the domestic (dollar) shock is equal to one for returns on any currency, and γz_t is the price of dollar-specific risk. The risk price for global shocks demanded by the domestic investor is δz_t^w, and the quantity of global risk in currency i depends on the relative exposures of the two SDFs to the global shock; since higher δ^i implies lower interest rates, *ceteris paribus*, this loading can be interpreted as carry beta:

$$\beta_t^{Carry} = \frac{\sqrt{\delta z_t^w} - \sqrt{\delta^i z_t^w}}{\sqrt{\delta z_t^w}} = 1 - \sqrt{\frac{\delta^i}{\delta}}. \qquad (6)$$

The currency risk premium is *independent* of the foreign country-specific factor z_t^i. That is why we need asymmetric loadings on the common component as a source of variation in currency risk premiums across currencies. The currency risk premium is also independent of the foreign country-specific loading γ^i. We have thus set γ^i equal to γ to keep the model parsimonious. In the absence of asymmetries in the exposure to global shocks, all currency risk premiums are identical and equal to γz_t, an implication that is clearly at odds with the data. Our sorts of currencies by current interest rates have shown a large amount of cross-sectional variation in currency risk premiums.

Building currency portfolios to extract factors. We sort currencies into portfolios based on their forward discounts, as we have done in the data. We use H to denote the set of currencies in the last portfolio and L to denote the currencies in the first portfolio. The carry trade risk factor hml and the dollar risk factor $\overline{rx}$ are defined as follows:

$$hml_{t+1} = \frac{1}{N_H} \sum_{i \in H} rx^i_{t+1} - \frac{1}{N_L} \sum_{i \in L} rx^i_{t+1},$$

$$\overline{rx}_{t+1} = \frac{1}{N} \sum_{i} rx^i_{t+1},$$

where N_H and N_L denote the number of currencies in each portfolio. We let $\overline{\sqrt{x_t}}$ denote the average of $\sqrt{x^j_t}$ across all currencies in portfolio j. The portfolio composition changes over time, and in particular it depends on the global state variable z^w_t.

In this setting, the carry trade and dollar risk factors have a very natural interpretation. The first one measures the common innovation, while the second one measures the domestic country-specific innovation. In order to show this result, we appeal to the law of large numbers and assume that the country-specific shocks average out within each portfolio.

Proposition 1. The innovation to the hml risk factor measures exposure to only the common factor u^w_{t+1}, and the innovation to the dollar risk factor measures exposure to only the country-specific factor u_{t+1}:

$$hml_{t+1} - E_t[hml_{t+1}] = \left(\overline{\sqrt{\delta^L_t}} - \overline{\sqrt{\delta^H_t}} \right) \sqrt{z^w_t} u^w_{t+1},$$

$$\overline{rx}_{t+1} - E_t[\overline{rx}_{t+1}] = \sqrt{\gamma} \overline{\sqrt{z_t}} u_{t+1}.$$

The role of heterogeneity. When currencies share the same loading on the common component, there is no hml risk factor. However, if lower interest rate currencies have different exposure to the common volatility factor—$\overline{\sqrt{\delta^L}} \neq \overline{\sqrt{\delta^H}}$—then the innovation to hml measures the common innovation to the SDF. As a result, the return on the zero-cost strategy hml measures the stochastic discount factors' relative exposure to the common shock u^w_{t+1}.

Proposition 2. The *hml* betas and the $\overline{rx}$ betas of the returns on currency portfolio j are

$$\beta^j_{hml,t} = \frac{\sqrt{\delta} - \sqrt{\delta^j_t}}{\sqrt{\delta^L_t} - \sqrt{\delta^H_t}},$$

$$\beta^j_{rx,t} = 1.$$

The betas for the dollar factor are all one. Not so for the carry trade risk factor. If the sorting of currencies on interest rate produces a monotonically decreasing ranking of δ on average, then the *hml* betas will increase monotonically as we go from low to high interest rate portfolios. As it turns out, the model with asymmetric loadings automatically delivers this if interest rates decrease when global risk increases. This case is summarized in the following condition:

Condition 4.2. The precautionary effect of global volatility on the real short rate dominates if

$$0 < \chi < \frac{1}{2}\delta^i. \tag{7}$$

This condition is intuitive and has a natural counterpart in most consumption-based asset pricing models: When precautionary saving demand is strong enough, an increase in the volatility of consumption growth (and, consequently, of marginal utility growth) lowers interest rates.

There is empirical evidence to support this assumption. The detrended short-term interest rate predicts U.S. stock returns with a negative sign (see Fama and French 1989 for the original evidence and Lettau and Ludvigson 2001 for a recent survey of the evidence), consistent with higher Sharpe ratios in low interest rate countries. To check this, we sort the same set of countries into six portfolios by their forward discounts, and we compute local currency equity returns in each portfolio. The Sharpe ratio is 0.61 in the lowest interest rate portfolio vs. 0.26 in the highest interest rate portfolio. Verdelhan (2010) reports similar findings on developed countries. Sorting by real interest rates delivers similar results: 0.62 in portfolio 1 compared with 0.32 in portfolio 6.

The real short rate depends both on country-specific factors and on a global factor. The only sources of cross-sectional variation in interest rates are the shocks to the country-specific factor z^i_t and the heterogeneity in the SDF loadings δ^i on the world factor z^w. As a result, as z^w increases, on average, the currencies with the high loadings δ will tend to end up in the lowest interest rate portfolios, and the gap $\left(\sqrt{\delta^L_t} - \sqrt{\delta^H_t}\right)$ increases. This implies that in bad times the spread in the loadings increases. Hence, the restricted model can generate variation in currency portfolio betas, even though the individual currencies' carry betas in Equation (6) are constant.

4.2 Full model

The restricted version of the model analyzed above implies that currencies with high δ loadings will have low interest rates on average and earn low average excess returns, while the opposite holds for currencies with low δ. As we show in Section 1.3, such permanent heterogeneity across currencies explains at most half of the cross-sectional variation in expected currency returns. The full model imputes variation in excess returns to dynamic evolution in individual currency betas, as well as to the permanent differences in these betas.

The expected excess return in levels (i.e., corrected for the Jensen term) in the full model is given by

$$E_t[rx_{t+1}^i] + \frac{1}{2}Var_t[rx_{t+1}^i] = \gamma z_t + (\delta z_t^w + \kappa z_t) - \sqrt{\delta z_t^w + \kappa z_t}\sqrt{\delta^i z_t^w + \kappa z_t^i}.$$

Relative to the restricted model, the foreign part of the currency risk premium $Cov_t[m_{t+1}^i, m_{t+1}]$ now has an additional country-specific component that depends on z_t^i/z_t. This new component captures transitory variation in the exposure of currencies to the global innovation, in addition to the permanent differences in exposure to the common innovation governed by δ^i. As foreign volatility increases z_t^i, the foreign SDF becomes more exposed to global innovations and, as a result, its currency beta with regard to the global shock decreases. The full model generates variation in individual currency betas in addition to currency portfolio betas. Again, cross-sectional variation in γ^i (exposure to country-specific shocks) does not help generate cross-sectional variation in currency risk premiums.

As before, two portfolios allow us to recover the innovations to the domestic pricing kernel:

$$hml_{t+1} - E_t[hml_{t+1}] = \left(\frac{1}{N_L}\sum_{i\in L}\sqrt{\delta^i z_t^w + \kappa z_t^i} - \frac{1}{N_H}\sum_{i\in H}\sqrt{\delta^i z_t^w + \kappa z_t^i}\right)u_{t+1}^w,$$

$$\overline{rx}_{t+1} - E_t[\overline{rx}_{t+1}] = \sqrt{\gamma z_t}u_{t+1} + \left(\sqrt{\delta z_t^w + \kappa z_t} - \frac{1}{N}\sum_i\sqrt{\delta^i z_t^w + \kappa z_t^i}\right)u_{t+1}^w.$$

The *hml* portfolio will have positive average returns if the pricing kernels of low interest rate currencies are more exposed to the global innovation:

$$\frac{1}{N_L}\sum_{i\in L}\sqrt{\delta^i z_t^w + \kappa z_t^i} > \frac{1}{N_H}\sum_{i\in H}\sqrt{\delta^i z_t^w + \kappa z_t^i}.$$

This will happen in equilibrium if the following conditions are satisfied:

Condition 4.3. The precautionary effect of domestic and global volatility on the real short rate dominates if

$$0 < \chi < \frac{1}{2}\delta^i, \; 0 < \chi < \frac{1}{2}(\gamma + \kappa).$$

If these conditions are satisfied, then an increase in domestic volatility lowers the real risk-free rate and temporarily implies higher exposure of the pricing kernel to the global innovation and hence lower betas for that particular currency. Hence, the unrestricted model contributes a second mechanism through which lower interest rate currencies earn lower risk premiums than higher interest rate currencies: variation in individual currency betas tied to interest rates in that currency.

4.3 Inflation

Finally, we specify a process for the nominal pricing kernel, in order to match moments of nominal interest rates and exchange rates. The log of the nominal pricing kernel in country i is simply given by the real pricing kernel less the rate of inflation π^i:

$$m_{t+1}^{i,\$} = m_{t+1}^i - \pi_{t+1}^i.$$

Inflation is composed of a country-specific component and a global component. We simply assume that the same factors driving the real pricing kernel also drive expected inflation. In addition, inflation innovations in our model are not priced. Thus, country i's inflation process is given by

$$\pi_{t+1}^i = \pi_0 + \eta^w z_t^w + \sigma_\pi \epsilon_{t+1}^i,$$

where the inflation innovations ϵ_{t+1}^i are independent and identically distributed gaussian. It follows that the nominal risk-free interest rate (in logarithms) is given by

$$r_t^i = \pi_0 + \alpha + \left(\chi - \frac{1}{2}(\gamma + \kappa)\right) z_t^i + \left(\chi + \eta^w - \frac{1}{2}\delta^i\right) z_t^w - \frac{1}{2}\sigma_\pi^2.$$

Importantly, the currency risk premiums on the one-period contracts that we consider in the data do not depend on the correlation between the innovations to the pricing kernel and the volatility processes, which we set to minus one, following the convention in the term structure literature. This correlation governs the slope of the term structure. For example, if we set this correlation to zero, eliminating conditional bond risk premiums altogether, we still get the exact same expressions for the one-period currency risk premiums in Equation (8). Of course, the theoretical currency risk premiums on contracts with longer maturity do depend on this correlation. Yet, empirically, Bekaert, Wei, and Xing (2007) find that term premiums play only a minor role in explaining currency risk premiums.

We now turn to the calibration of this no-arbitrage model. We show that it can match the key moments of currency returns in the data, while also matching the usual moments of nominal interest rates, exchange rates, and inflation.

4.4 Calibration

We calibrate the model by targeting annualized moments of monthly data. A version of our model that is calibrated to match the key moments of interest rates and exchange rates can match the properties of carry trade returns.

4.4.1 Moments. The calibration proceeds in two steps. In the first step, we calibrate a symmetric version of the model: All countries have the same parameters, including δ. All of the target moments of interest rates, exchange rates, and inflation have closed-form expressions in this symmetric version of the model, assuming the moments of the square root processes exist. In the second stage, we introduce enough heterogeneity in δ to match the carry trade risk premium.

Symmetric model. Let us start with a symmetric version of the full model. We first focus on real moments. There are eight parameters in the real part of the model: Five parameters govern the dynamics of the real stochastic discount factors (α, χ, γ, κ, and δ), and three parameters (ϕ, θ, and σ) describe the evolution of the country-specific and global factors (z and z^w).

We choose these parameters to match the following eight moments in the data: the mean, standard deviation, and autocorrelation of the U.S. real short-term interest rates; the standard deviation of changes in real exchange rates; the cross-sectional mean of the real UIP slope coefficients; the cross-country correlation of real interest rates; the maximum Sharpe ratio (the standard deviation of the log SDF); and a Feller parameter (equal to $2(1-\phi)\theta/\sigma^2$), which helps ensure that the z and z^w processes remain positive.[16] These eight moments, as well as the targets in the data that we match, are listed in Panel A of Table 7.

The data for this calibration exercise come from Barclays and Reuters (Datastream). Because of data availability constraints, we focus on the subset of developed countries. The sample runs from November 1983 to December 2009. However, for U.S. real interest rate data, we use the real zero-coupon yield curve data for the United States provided by J. Huston McCulloch on his website; the sample covers January 1997 to October 2009. For other countries, we use the past 12-month changes in the log Consumer Price Index (CPI) to

16 If the Feller condition $2(1-\phi)\theta/\sigma^2 > 1$ is satisfied, then there exists a unique positive solution to the equation defining the volatility process z in the continuous-time limit (Feller 1951).

Table 7
Calibrating the symmetric model

	Moment	Target (Monthly)	Target (Annualized)
	Panel A: 8 Targets – Moments of Real Variables		
β_{UIP}	$\frac{\chi}{\left(\chi-\frac{1}{2}(\gamma+\kappa)\right)}$	−0.50	−0.50
$E(r^{US})$	$\theta\left[\alpha+\left(\chi-\frac{1}{2}(\gamma+\kappa)\right)+\left(\tau-\frac{1}{2}\delta^i\right)\right]$	0.11%	1.37%
$Std(r^{US})$	$\sqrt{\left(\chi-\frac{1}{2}(\gamma+\kappa)\right)^2 var(z^i)+\left(\tau-\frac{1}{2}\delta^i\right)^2 var(z^w)}$	0.15%	0.51%
$\rho(r_t^{US})$	ϕ	0.95	0.95
$E_{cross}\left[Std(\Delta q)\right]$	$\sqrt{2\gamma\theta+2\chi^2 var(z^i)+o}$	3.13%	10.85%
$Std(m)$	$\sqrt{(\gamma+\delta+\kappa)\theta+\chi^2 var(z^i)+\tau^2 r(z^w)}$	14.43%	50.00%
$E_{cross}\left[Corr(r_t^{US},r_t^{\star})\right]$	$\left(\tau-\frac{1}{2}\delta^i\right)^2\frac{Var(z^w)}{Var(r)}$	0.19	0.19
$E(rx_t)$	$\gamma\theta$	0.04%	0.50%
$Feller$	$2(1-\phi)\frac{\theta}{Var(z^w)}$	20.00	20.00
	Panel B: 3 Targets – Moments of Inflation		
$E(\pi^{US})$	$\pi_0+\eta^w\theta$	0.24%	2.92%
$Std(\pi^{US})$	$\sqrt{(\eta^w)^2 var(z^w)+\sigma_\pi^2}$	0.32%	1.10%
$E_{cross}\left[R^2\right]$	$\frac{(\eta^w)^2 var(z^w)}{var(inflation)}$	0.26	0.26
	Panel C: Moments of Nominal Variables		
$E(i^{US})$	$\theta\left[\alpha+\left(\chi-\frac{1}{2}(\gamma+\kappa)\right)+\left(\tau+\eta^w-\frac{1}{2}\delta^i\right)\right]-\frac{1}{2}\sigma_\pi^2$	0.36%	4.29%
$Std(i^{US})$	$\sqrt{\left(\chi-\frac{1}{2}(\gamma+\kappa)\right)^2 var(z^i)+\left(\tau+\eta^w-\frac{1}{2}\delta^i\right)^2 var(z^w)}$	0.18%	0.63%
$E_{cross}\left[Std(\Delta s)\right]$	$\sqrt{2\gamma\theta+2\chi^2 var(z^i)+2\sigma_\pi^2+o}$	2.96%	10.25%
$E_{cross}\left[Corr(i_t^{US},i_t^{\star})\right]$	$\left(\tau+\eta^w-\frac{1}{2}\delta^i\right)^2\frac{Var(z^w)}{Var(r)}$	0.39	0.46

This table first reports the moments used in the calibration. The first column defines each moment, the second column presents its closed-form expression in the symmetric version of our model, while the last two columns report the monthly and annual empirical values of each moment in our data. The first panel reports moments of real variables: the UIP slope coefficient β_{UIP}; the mean, standard deviation, and autocorrelation of the U.S. real interest rate r^{US}; the average standard deviation of changes in real exchange rates Δq; the standard deviation of the log SDF m; the average cross-country correlation of real interest rates; the average return rx of a U.S. investor on currency markets, as well as the Feller coefficient. The second panel reports the mean and standard deviation of U.S. inflation, along with the average R^2 in regressions of each country's inflation on world inflation. The third panel presents moments that are not used in the calibration but implied by the moments described in the first two panels. The third panel thus reports the mean and standard deviation of U.S. nominal interest rates, the average standard deviation of nominal exchange rates, and the average cross-country correlation of nominal interest rates. Note that $var(z^w)=\frac{\sigma_w^2\theta}{1-\phi^2}$ and $var(z^i)=\frac{\sigma_i^2\theta}{1-\phi^2}$. $o=2(\delta+\kappa)\theta-2E\left(\sqrt{\delta^i z_t^w+\kappa^i z_t}\right)\left(\sqrt{\delta^i z_t^w+\kappa^i z_t^i}\right)$ is an order of magnitude smaller than the other terms. Data are monthly, from Barclays (Datastream). The sample runs from 11/1983 to 12/2009. For means and standard deviations, we report annualized values by multiplying their monthly counterparts by 12 and $\sqrt{12}$, respectively. The other moments are not annualized.

proxy for expected inflation when computing real interest rates. Inflation itself is computed as the one-month change in the log CPI. The average UIP slope coefficient in our sample is −0.53 on nominal series. However, the average real UIP slope coefficient is smaller, (−0.9).

We target a UIP slope coefficient of −0.5, an average real interest rate of 1.4% per annum, an annualized standard deviation of the real interest rate of .5% per annum, and an autocorrelation (in monthly data) of 0.95. The annual standard deviation of real exchange rate changes is 10.8%. We target a maximum Sharpe ratio of 0.5. This is the average Sharpe ratio on equity returns (in local currency) in our sample for the lowest interest rate currencies with the highest Sharpe ratios. The average pairwise correlation of real interest rates is .2. The annual dollar risk premium is 0.5% per annum. A Feller coefficient of 20 ensures that all of the state variables following square-root processes are positive (this is exact in the continuous-time approximation, and implies a negligible probability of crossing the zero bound in discrete time).

We obtain the three inflation parameters (η^w, σ^π, and π_0) by targeting the mean, standard deviation, and fraction of inflation that are explained by the common component. In Panel B of Table 7, we list the expression for the variance of inflation and the fraction explained by the common component. We target an annualized standard deviation for inflation of 1.1% and an average inflation rate of 2.9%. Twenty-six percent of inflation is accounted for by the common component. Finally, for completeness, Panel C also shows the implied moments of nominal interest rates and exchange rates in this symmetric version of the model. The implied correlation of nominal interest rates is too high. Introducing heterogeneity in δ will address this problem.

Then, we solve a system of eleven equations to recover these eleven parameter values. The parameter values that we obtain are listed in Table 8. Recall that in the symmetric version of the model all countries share the same δ; we chose a value of 12.84 to match the moments described above. In the next step, we introduce heterogeneity in the δ-values.

Heterogeneity. In the second stage of the calibration, we introduce enough heterogeneity in the SDF loadings δ on the global shock across countries to match an empirical carry trade risk premium of 5.88% for the subset of developed countries—this is the carry risk premium before bid-ask spreads; the model obviously does not have transaction costs. The home country keeps the δ-value of 12.84. Table 8 shows the range of δ for the other countries. The δ^i-values are linearly spaced on the interval $[\underline{\delta}, \overline{\delta}]$ for all thirty currencies in our simulation. The moments reported were generated by drawing 100,000 observations from a model with thirty currencies.

Table 9 presents the simulation results. We list the moments for the nominal and real interest rates, exchange rates, and inflation in the data, as well as the moments implied by the model. Panel I reports the moments for the United States, i.e., the home country in the model. The model's home country interest rates match the U.S. interest rates in the data relatively well.

Table 8
Parameter values

Pricing Kernel Parameters						
α (%)	χ	γ	κ	$\delta*$	$\underline{\delta}$	$\overline{\delta}$
0.86	2.78	0.65	16.04	12.84	8.35	17.34
Factor and Inflation Dynamics						
ϕ	θ (*in bp*)	σ (%)	η^w	σ^π	π_0 (%)	
0.92	7.81	0.25	9.41	0.27	−0.49	

This table reports the parameter values for the calibrated version of the full model. All countries share the same parameter values except for δ. $\delta*$ is the parameter for the home country. These 11 parameters were chosen to match the 11 moments in Table 7. The parameters δ^i are linearly spaced on the interval $[\underline{\delta}, \overline{\delta}]$. α, σ, and π_0 are reported in percentages. θ is reported in basis points.

The average nominal interest rate is 4.7% in the model and 4.3% in the U.S. data. The model slightly underpredicts the volatility of U.S. nominal interest rates (0.6% vs. 0.5%), because the only variation in expected inflation is the common factor z^w. Finally, the model underpredicts the persistence of nominal interest rates (0.98 in the data vs. 0.92 in the model).

The model produces an average domestic real interest rate of 1.8% with a standard deviation of 0.4%, compared with 1.7% and 0.2%, respectively, in the U.S. Treasury Inflation-Protected Securities data and 1.4% and 0.5% using past annual inflation to proxy for expected inflation. The autocorrelation is 0.92, close to the data. These values are also close to the ones reported by Ang, Bekaert, and Wei (2008). The model matches the mean and standard deviation of U.S. inflation, but the model underpredicts the persistence of inflation: The first-order autocorrelation at monthly frequencies is 0.46 in the data, compared with 0.27 in the model.

Panel II reports the moments for the cross-section of countries. The model delivers real interest rates that are as correlated to the U.S. ones as their actual counterparts. The simulated real interest rates are on average lower, less volatile, and more persistent than the ex post real interest rates in the data, but these are subject to caution. We do not have time series of real interest rates for the countries in our sample (except for the United States) and, as already noted, we use a proxy for expected inflation.

The nominal interest rates produced by the model are somewhat lower and less volatile than those in the data. The model roughly matches the average pairwise correlation of foreign with U.S. interest rates: 0.5 in the model and in the data. The correlation in interest rates is driven by the common factor z^w. The model matches the mean and persistence of the inflation rates but slightly underestimates their volatilities. The model also matches the fraction of inflation rates' variations that are explained by the common component in inflation (26% vs. 31%). Recall that there is no inflation risk premium in the model. As a result, we could choose a richer process for (expected) common inflation that better matches the nominal interest rate and inflation

Table 9
Simulated moments

Moment	*Nominal Values*		*Real Values*	
	Data	Model	Data	Model
Panel I: Time Series Moments – Home Country				
Interest Rates				
$E\left[r^{US}\right]$	4.29%	4.74%	1.37%	1.84%
$Std\left[r^{US}\right]$	0.63%	0.50%	0.51%	0.41%
$\rho\left[r^{US}\right]$	0.98	0.92	0.95	0.92
Inflation				
$E\left[\pi^{US}\right]$	2.92%	2.89%		
$Std\left[\pi^{US}\right]$	1.10%	1.10%		
$\rho\left[\pi^{US}\right]$	0.46	0.27		
Panel II: Cross-Sectional Moments – All Countries				
Interest Rates				
$E_{cross}\left(E\left[r\right]\right)$	5.89%	4.62%	2.53%	1.72%
$E_{cross}\left(Std\left[r\right]\right)$	1.27%	0.50%	0.82%	0.42%
$E_{cross}\left(\rho\left[r\right]\right)$	0.69	0.92	0.54	0.92
$E_{cross}\left(corr\left[r^{US}, r^{\star}\right]\right)$	0.46	0.53	0.19	0.31
Exchange Rates				
$E_{cross}\left(Std\left[\Delta q_{t+1}\right]\right)$	10.25%	12.26%	10.85%	12.19%
$E_{cross}\left(\beta_{UIP}\right)$	-0.53	-0.46	-0.09	-0.46
$Std_{cross}\left(\beta_{UIP}\right)$	0.84	0.07	0.90	0.07
Inflation				
$E_{cross}\left(E\left[\pi\right]\right)$	2.90%	2.90%		
$E_{cross}\left(Std\left[\pi\right]\right)$	1.34%	1.10%		
$E_{cross}\left(\rho\left[\pi\right]\right)$	0.22	0.26		
$E_{cross}\left(R^2\right)$	0.26	0.31		
Stochastic Discount Factor				
$E_{cross}\left[Std\left(m_{t+1}\right)\right]$		0.53		0.53
$E_{cross}\left[corr\left(m^{US}_{t+1}, m_{t+1}\right)\right]$		0.97		0.97

This table reports the annualized means and standard deviations of nominal and real variables in the data and in the model. The autocorrelations (ρ) reported are monthly. In the first section of Panel I, the table presents the mean, standard deviation, and autocorrelation of the risk-free rate in the home country (the U.S.). In the second section of Panel I, the table presents the mean, standard deviation, and autocorrelation of the inflation rate in the home country. In the first section of Panel II, the table reports the cross-sectional average of the mean, standard deviation, autocorrelation, and cross-country correlation of the risk-free rates in all countries. In the second section of Panel II, the table reports the cross-sectional average of exchange rates' volatilities and the cross-sectional average and volatility of the UIP slope coefficients. In the third section of Panel II, the table reports the cross-sectional average of the mean, standard deviation, autocorrelation, and R^2 of inflation rates. The R^2 corresponds to the share of each country's inflation variance explained by the average inflation rate. In the fourth section of Panel II, the table reports the cross-sectional average of the SDF volatility and of the cross-country correlation of all SDFs.

data without changing any of our asset pricing results. However, to keep the model parsimonious, we chose not to, since inflation does not play a role in our mechanism.

Finally, we turn to exchange rates. In the model, the cross-sectional average of the standard deviation of changes in the log spot rates is 12.3%; the corresponding number in the data is 10.2%. Given that the standard deviation of the log pricing kernel is 53%, this implies that the pricing kernels have to be highly correlated across countries (see Brandt, Cochrane, and Santa-Clara 2006): The average pairwise correlation of the pricing kernels is 0.97. The cross-sectional average of the UIP slope coefficient is −.53 in the data, compared with −.46 in the model. However, the model substantially underpredicts the amount of cross-sectional variation in the UIP slope coefficient in the data because we have shut down all sources of heterogeneity except in the δ-values.

4.4.2 Simulated portfolios. Using the simulated data, we build currency portfolios in the same way as we did in the actual data. Table 10 reports the realized returns on these currency portfolios in the model. Panel I reports the results obtained when sorting on current forward discounts. These moments should be compared against the same moments in the data reported in Table 1. In the model, the volatility of changes in the exchange rates varies from 11.6% for portfolio 1 to 9.3% for portfolio 5. In the data, this volatility ranges from 9.5% to 10.3% in our small sample (7.4% to 9.7% in the large sample). As a result, the model overpredicts the volatility of changes in spot rates portfolio by portfolio by at most 200 basis points.

In the model, the volatility of the forward discounts is around 105 basis points for all portfolios. As a result, the model overstates the volatility of interest rates in portfolios 1–5 in both samples and understates the volatility of interest rates in portfolio 6 in our large sample. The model also underpredicts the average interest rates in portfolio 6. In the data, portfolio 6 comprises countries that temporarily experience unusually high and volatile inflation. Our parsimonious specification of a single inflation process for all countries and currencies is not rich enough to match this. However, real, not nominal, interest rates matter for currency excess returns. The model does a much better job matching the moments of average real interest differences.

When sorting currencies by current interest rates, the model produces a carry trade risk premium of 5.91% per annum (4.54% in the data in our large sample). The annualized Sharpe ratio is 0.48 (.50 in the data).

Panel II reports the results obtained when sorting by the average forward discounts. The long-short excess return drops to 3.48%, about 60% of the total carry trade risk premium. In the data, permanent differences in exposure to global innovations account for half of the total carry trade premium; in the model, they account for 60%.[17]

[17] In the restricted model ($\kappa = 0$), the entire carry trade premium is due to permanent differences in that version of the model. However, in the full model, part of the carry trade premium is due to transitory differences; κ governs the "transitory" fraction of the carry trade risk premium, because it measures the sensitivity of the price of global risk to local risk aversion.

Table 10
Currency portfolios—simulated data

Portfolio	1	2	3	4	5	6
			Panel I: Sorting on Current Forward Discounts			
			Spot change: Δs^j			
Mean	0.52	0.03	−0.20	−0.24	−0.38	−0.66
Std	11.60	10.03	9.49	9.31	9.29	9.82
			Forward Discount: $f^j - s^j$			
Mean	−2.87	−1.62	−0.86	−0.18	0.49	1.86
Std	1.11	1.06	1.05	1.04	1.05	1.08
			Excess Return: rx^j			
Mean	−3.39	−1.65	−0.66	0.06	0.87	2.52
Std	10.86	9.47	8.97	8.82	8.80	9.34
SR	−0.31	−0.17	−0.07	0.01	0.10	0.27
			High-minus-Low: $rx^j - rx^1$			
Mean		1.74	2.73	3.45	4.26	5.91
Std		6.64	7.48	8.48	9.53	12.27
SR		0.26	0.36	0.41	0.45	0.48
			Average real interest rate difference: $r^j - r$			
Mean	−2.87	−1.62	−0.86	−0.18	0.49	1.86
] *Std*	1.11	1.06	1.05	1.04	1.05	1.08
			Turnover			
Trades/currency	0.21	0.45	0.52	0.53	0.51	0.13
			Panel II: Sorting on Average Forward Discounts			
			Forward Discount: $f^j - s^j$			
Mean	−1.99	−1.44	−0.85	−0.21	0.46	1.41
Std	1.18	1.16	1.14	1.14	1.15	1.12
			High-minus-Low: $rx^j - rx^1$			
Mean		0.68	1.24	1.82	2.54	3.48
Std		5.57	5.82	6.34	7.12	7.89
SR		0.12	0.21	0.29	0.36	0.44
			Average real interest rate difference: $r^j - r$			
Mean	−1.99	−1.44	−0.85	−0.21	0.46	1.41
Std	1.18	1.16	1.14	1.14	1.15	1.12

Panel I of this table reports, for each portfolio j, the average change in log spot exchange rates Δs^j, the average log forward discount $f^j - s^j$, the average log excess return rx^j, and the average return on the long-short strategy $rx^j - rx^1$. All these moments are defined as in Table I. The portfolios are constructed by sorting currencies into six groups at time t based on the one-year-forward discount (i.e., nominal interest rate differential) at the end of period $t-1$. The first portfolio contains currencies with the lowest interest rates. The last portfolio contains currencies with the highest interest rates. All data are simulated from the model. Panel II of this table reports, for each portfolio j, the average return on the long-short strategy $rx^j - rx^1$, the average log forward discount $f^j - s^j$, and the average real interest rate difference: $r^j - r$. The portfolios are constructed by sorting currencies into six groups at time t based on the average one-year-forward discount (i.e., nominal interest rate differential) over the entire period. As a result, there is no rebalancing in this case.

By sorting on average forward discounts, we really are sorting by real interest rates. In the model, there is a 158-basis-point spread between real interest rates in the first and the last portfolio. In the data, we found a similar pattern (see Table 1), but the variation in real interest rates that we documented was much bigger.

The model matches the turnover rather well. For the currency portfolios in the middle range, the turnover is about 2.5 trades per portfolio (1.2 in the data). This translates into a turnover rate (turnover per portfolio per currency) of about 45%; the rate is similar to that in the data (40% to 45%), mainly because in the early part of the sample we had very few currencies in each portfolio.

The simulated market price of carry risk varies for two reasons. First, it is high when the world risk factor z^w is high. Second, this effect is amplified by changes in portfolio composition: Higher world risk price drives the selection of low-global risk countries into high interest rate portfolios, and vice versa. Thus, in "bad times," when z^w is high, the spread between the average δ in the first and the last portfolio increases.

Despite the low unconditional market beta of the carry trade in the data, the carry risk factor HML_{FX} is very highly correlated with the stock market during periods of increased market volatility. The recent subprime mortgage crisis offers a good example. Between July 2007 and March 2008, the correlation between U.S. stock returns and HML_{FX} was .78. This pattern is consistent with the model. In the two-factor affine model, the conditional correlation of HML_{FX} and the SDF in the home country is

$$corr_t\left(HML_{t+1}, m_{t+1}\right) = -\sqrt{\frac{\delta z_t^w + \kappa z_t}{\delta z_t^w + (\gamma + \kappa)\, z_t}}.$$

In the restricted model, this expression collapses to

$$corr_t\left(HML_{t+1}, m_{t+1}\right) = -\sqrt{\frac{\delta z_t^w}{\delta z_t^w + \gamma\, z_t}}.$$

As the global component of the conditional market price of risk z_t^w increases, the conditional correlation between the stochastic discount factor at home and the carry trade returns HML_{FX} increases.

4.5 Testing the model

Finally, we subject our model to some "out-of-sample tests." We start by checking whether the model accurately describes the time variation in currency betas.

4.5.1 Time-varying betas. A statistically powerful way to capture time variation in the individual currencies conditional betas with respect to the two factors is to impose a functional relationship between betas and the conditioning variables (the forward discounts) that is the same across currencies. If this relationship is linear, as assumed in Equation (4), it can be estimated by running a pooled regression (e.g., as suggested by Cochrane 2011) for the entire panel of currencies:

$$\begin{aligned} Rx_{t+1}^i = c^i &+ b_{HML} HML_{FX,t+1} + b_{z_t^i \times HML} z_t^i HML_{FX,t+1} + b_{RX} RX_{t+1} \\ &+ b_{z_t^i \times RX} z_t^i RX_{t+1} + \epsilon_{i,t+1}, \end{aligned} \tag{8}$$

where the fixed effects c^i represent country-specific pricing errors.

Table 11 presents the results of this estimation (omitting the fixed effects), for the subsample of the developed countries and for the whole sample. The coefficient b_{HML} captures an average country's unconditional loading on the HML_{FX} factor; not surprisingly, it is small (0.07) and not significantly different from zero. Also not surprisingly, the coefficient b_{RX} that measures the average loading on the dollar risk factor is equal to one for the entire sample, and is somewhat lower at 0.85 in the developed countries sample. The variation in the conditional HML_{FX} betas is measured by the coefficient $b_{z\times HML}$. It reaches a value of 0.3 (for the entire sample) and, jointly with the unconditional loading, implies that a currency whose forward discount is one cross-sectional standard deviation higher than the average at that time has a conditional beta of 0.37. This coefficient is highly statistically significant. This confirms that the forward discounts contain conditioning information that is important for understanding the dynamics of carry beta. The conditional variation in the dollar factor loading captured by $b_{z\times RX}$ is essentially zero. The last panel reports the results obtained on model-generated data. The model does a remarkable job in replicating the time variation in the betas. In the model, the coefficient b_{HML} is equal to 0.08, b_{RX} is 1, and $b_{z\times HML}$ is equal to 0.35.

Finally, we also replicate on simulated data the asset pricing tests obtained on individual currencies. To save space, results are reported in the separate appendix. The price of carry risk estimated by the cross-sectional Fama–MacBeth regressions using both unconditional and conditional betas is close to the sample mean of the factor, and the model is able to explain roughly 60%–70% of sample variation in average currency returns.

4.5.2 Characteristics vs. covariances. Currency-specific attributes other than interest rates could explain some of our findings; maybe some currencies earn high returns merely because they have high interest rates, not because their returns co-vary positively with HML_{FX}. By sorting currencies on interest rate characteristics and using HML_{FX} as a factor, are we simply measuring the effects of interest rate characteristics on currency returns? We address this concern in two ways. First, we run tests to discriminate between these two explanations on actual and model-generated data. Second, we test other implications of the model.

The results in the top panel of Table 12 suggest that we are simply picking up the effects of characteristics. This panel reports the cross-sectional asset pricing results obtained after adding the average interest rate difference for each currency portfolio, which we can call the characteristic, as a factor. The carry trade risk factor is no longer statistically significant. On the basis of this "horse race" between the risk factor and the characteristic, one would conclude that the characteristic wins. However, in the bottom panel, we run the same estimation on simulated data from our calibrated no-arbitrage model in which only the risk is priced, not the characteristic. We use a small sample of 300 periods from the same simulation with thirty currencies that we used in Section 4.4.

Table 11
Time-varying betas: Data and model

	b_{HML}	b_{RX}	$b_{z\times HML}$	$b_{z\times RX}$
Panel I: Developed Countries				
	0.07	0.85	0.26	0.06
Robust	[0.05]	[0.09]	[0.08]	[0.04]
NW	[0.02]	[0.02]	[0.02]	[0.02]
Panel II: All Countries				
	0.07	1.01	0.31	0.02
Robust	[0.04]	[0.09]	[0.06]	[0.06]
NW	[0.02]	[0.02]	[0.03]	[0.03]
Panel III: Simulated data				
	0.08	1.00	0.35	−0.01
Robust	[0.01]	[0.00]	[0.01]	[0.00]
NW	[0.00]	[0.01]	[0.00]	[0.01]

The table reports results from the panel regressions of excess returns on individual currencies on the risk factors scaled with the currency-specific forward discounts. The excess returns used as test assets do *not* take into account bid-ask spreads. Risk factors HML and RX come from portfolios of currency excess returns that do take into account bid-ask spreads. HML correspond to a carry trade strategy, long high interest rate currencies, and short low interest rate currencies. RX corresponds to the average currency return across all portfolios. All excess returns are multiplied by 12 (annualized). z_t^i is the country-specific forward discount rescaled to have a cross-sectional mean of 0 and standard deviation of 1 at any time t. The standard errors in brackets are robust with clustering by month and currency (*Robust*) or Newey and West (1987) with 2 lags (*NW*). Data are monthly, from Barclays (Panel I) and Barclays and Reuters (Panel II) in Datastream. The sample period is 11/1983–12/2009.

The simulation-based estimates are essentially the same as the actual estimates from the data; the characteristic drives out the risk factor. The estimated risk price for HML_{FX} has the wrong sign.

This result is not surprising. In the model, as in the data, there is no variation in exposure to HML_{FX} across different currencies that is independent of interest rates. Furthermore, interest rates are computed from market prices that are recorded without measurement error; factor loadings are not. So, the outcome of this horse race, in which the risk factor is at a serious disadvantage, does not help distinguish between these competing explanations.

4.5.3 Volatility as a risk factor. As a final test of our model, we consider a measure of global financial market volatility as another proxy for the common risk factor. We expect global volatility to increase in bad times for global investors. If innovations to the common component of marginal utility growth u^w are indeed correlated with innovations to global volatility z^w, then volatility innovations could proxy for HML_{FX} innovations. In our model, these innovations are perfectly negatively correlated, so that volatility should command a negative price of risk.

In the data, our volatility measure is the average volatility of stock returns in local currency across all currencies in our sample. To build our volatility factor,

Table 12
Asset pricing—with characteristics

Panel I: Data

	All Countries						Developed Countries					
	$\lambda_{HML_{FX}}$	λ_{RX}	λ_{FD}	R^2	RMSE	*p–val*	$\lambda_{HML_{FX}}$	λ_{RX}	λ_{FD}	R^2	RMSE	*p–val*
FMB	−4.87	1.82	0.88	70.55	0.82		−7.44	−2.28	1.47	47.07	0.64	
	[5.22]	[1.64]	[0.54]			10.15	[10.97]	[3.73]	[1.52]			16.92

Panel II: Simulation

	$\lambda_{HML_{FX}}$	λ_{RX}	λ_{FD}	R^2	RMSE	*p–val*
FMB	−1.28	1.24	1.48	97.15	0.27	
	[8.26]	[2.54]	[1.79]			90.02

This table reports results from a Fama–McBeth asset pricing procedure with characteristics: The average interest rate differential in each portfolio is added to the second stage of the Fama–McBeth estimation. Market prices of risk λ, the adjusted R^2, the square root of mean-squared errors $RMSE$, and the p-values of χ^2 tests on pricing errors are reported in percentage points. The first panel uses actual data. Excess returns used as test assets and risk factors take into account bid-ask spreads. Data are monthly, from Barclays and Reuters in Datastream. The sample period is 11/1983–12/2009. The second panel uses the last 300 periods of simulated data. All excess returns are multiplied by 1200 (i.e., in percent, annualized). We do not include a constant in the second step of the FMB procedure.

we first compute the standard deviation over one month of daily MSCI price index changes for each currency, and then the cross-sectional mean of these volatility series. Our risk factor corresponds to volatility innovations, obtained as log differences of our global volatility series.

The top panel in Table 13 reports the loadings of different portfolio returns on the equity volatility factor. These loadings confirm our intuition: They decrease monotonically from the first to the last portfolio from 0.37 to −0.81 in the full sample (reported in the left panel), and from .58 to −.59 in the case of developed countries (reported in the right panel). High interest rate countries tend to offer low returns when equity volatility increases. Low interest rate countries, on the contrary, offer high returns when volatility goes up. As a result, the estimated price of volatility is negative (and statistically significant), as predicted by the model. Building on our work, Menkhoff et al. (2010) find that a measure of global volatility obtained from currency markets also explains the cross-section of our currency portfolios. Those results are also consistent with our model.

While the equity volatility risk factor does not use any information on exchange rates, it has explanatory power for the cross-section of currency excess returns. This is consistent with our model. However, it cannot replace HML_{FX} as the pricing factor. In a horse race between these two risk factors, HML_{FX} drives out innovations to the volatility factor. We have shown that HML_{FX} extracts the common component of the stochastic discount factors directly from currency returns; since the global volatility factor is not observed directly but has to be estimated, it is not surprising that HML_{FX} has superior explanatory power for returns. As a robustness check, we sort countries on their global

Table 13
Asset pricing–equity volatility risk factor (innovations)

Panel I: Factor Betas

	All Countries			Developed Countries		
Portfolio	$\beta^j_{Vol_{Equity}}$	β^j_{RX}	R^2	$\beta^j_{Vol_{Equity}}$	β^j_{RX}	R^2
1	0.37	1.04	74.78	0.58	0.99	72.55
	[0.12]	[0.05]		[0.25]	[0.06]	
2	0.22	0.94	76.21	0.16	1.01	80.01
	[0.10]	[0.04]		[0.14]	[0.04]	
3	0.19	0.95	74.34	0.20	1.04	86.67
	[0.10]	[0.04]		[0.13]	[0.03]	
4	0.13	0.95	75.44	–0.35	0.97	82.02
	[0.08]	[0.05]		[0.18]	[0.04]	
5	–0.10	1.06	76.30	–0.59	0.99	74.50
	[0.13]	[0.05]		[0.16]	[0.05]	
6	–0.81	1.07	63.84			
	[0.16]	[0.06]				

Panel II: Risk Prices

	All Countries			Developed Countries		
	$\lambda_{Vol_{Equity}}$	λ_{RX}	R^2	$\lambda_{Vol_{Equity}}$	λ_{RX}	R^2
FMB	–4.20	1.33	66.10	–2.31	1.91	48.12
	[1.41]	[1.35]		[1.46]	[1.73]	
	(1.65)	(1.35)		(1.53)	(1.73)	

The panel on the left reports empirical results using actual data for all countries. The panel on the right reports results for the simulated data from the calibrated model. Panel I reports OLS estimates of the factor betas. Panel II reports risk prices from the Fama–MacBeth cross-sectional regression. Market prices of risk λ and adjusted R^2s are reported in percentage points. Excess returns used as test assets and risk factors take into account bid-ask spreads. All excess returns are multiplied by 12 (annualized). To build our volatility factor, we first compute the standard deviation over one month of daily MSCI price index changes for each country in our sample. We then compute the cross-sectional mean of these volatility series. Our risk factor corresponds to volatility innovations, obtained as log differences of our global volatility series. We do not include a constant in the second step of the FMB procedure. The sample period is 11/1983–12/2009. The standard errors in brackets are Newey and West (1987) standard errors computed with the optimal number of lags according to Andrews (1991). Shanken (1992)-corrected standard errors are reported in parentheses.

equity volatility betas (as we did for HML_{FX} betas). Again, we obtain a clear cross-section of interest rates and currency excess returns. Countries that load more on global volatility offer higher excess returns because they bear more HML_{FX} risk.

5. Conclusion

By sorting currencies by their interest rates, we identify a slope factor in currency returns, driven entirely by common exchange rate variation among different currencies. The higher the currency's interest rate, the more the currency is exposed to this slope factor. This suggests a standard APT approach to explaining carry trade returns. The loadings on this slope factor line up with the average returns on the currency portfolios.

Furthermore, we derive conditions under which a standard affine model can replicate these carry trade returns. Heterogeneity in the loadings on a common component in each country's SDF is critical. In times of heightened volatility of the common innovations to the SDF, lower interest rate currencies endogenously become more exposed to the common innovations and hence they offer insurance, because their exchange rate appreciates in case of an adverse global shock. In addition, we can recover similar patterns in interest rates and currency returns by sorting currencies into portfolios based on their exposure to the carry trade risk factor and to a measure of global volatility in equity markets, not using any interest rate information whatsoever. This suggests that the common variation in exchange rates that we have uncovered after sorting currencies by their interest rates is not a statistical artifact produced by sorting the currencies by their interest rates but instead truly measures differences in exposure to global risk. While we cannot conclusively disprove them, our work raises the bar for other candidate explanations.

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