

# Sovereign Risk Premia and Global Macroeconomic Conditions\*

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## Resubmission

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# Sovereign Risk Premia and Global Macroeconomic Conditions

## Resubmission

### ABSTRACT

We study how shifting global macroeconomic conditions affect sovereign bond prices. Bondholders earn premia for two sources of systematic risk: exposure to low-frequency changes in the state of the economy, as captured by expected macroeconomic growth and volatility, and exposure to higher-frequency macroeconomic shocks. Our model predicts that the first source, labeled long-run macro risk, is the primary driver of the level and the cross-sectional variation in sovereign bond premia. We find support for this prediction using sovereign bond return data for 43 countries over the 1994-2018 period. A long-short portfolio based on long-run macro risk earns 8.11% per year in our sample.

JEL Codes: F34, G12, G13, G15, G32

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# 1 Introduction

The [Rubinstein-Lucas-Breeden \(1976, 1978, 1979\)](#) consumption-based paradigm is the natural starting point for understanding risk premia embedded in asset prices. It directly links expected asset returns to risk factors that are macroeconomic, and therefore systematic, in nature. While the canonical consumption-based model fails empirically, because aggregate quarterly consumption growth data is just too smooth, there has been substantial progress over the last 20 years in refining the paradigm to better fit the evidence ([Ludvigson, 2013](#)). However, one important asset class has not experienced much of the revival in consumption-based pricing: sovereign bonds. This is the challenge we address in the paper, both theoretically and empirically.

We study sovereign risk premia in a model in which global macroeconomic conditions shift abruptly between expansion and recession regimes. Recessions are associated with lower expected growth and higher uncertainty both for global consumption and for the sovereign bond issuer's output.<sup>1</sup> Sovereign bonds are priced by a global representative agent in an environment where security markets are internationally integrated. In comparison to an observationally equivalent model without regime changes, the presence of shifting macroeconomic conditions along with endogenous sovereign debt and default decisions results in countercyclical default risk and higher sovereign spreads.<sup>2</sup>

Part of the increase in sovereign spreads is due to bond prices embedding a larger risk premium. When investors prefer early resolution of uncertainty, the low-frequency covariation between sovereign bond returns and global consumption brought about by the regime shifts creates an additional risk premium, akin to the long-run risk model of [Bansal and Yaron \(2004\)](#). This premium arising from 'long-run macro risk' is an order of magnitude higher than the canonical one, associated with high-frequency covariation between bond returns and global consumption innovations, which we label 'short-run macro risk'. Specifically, we show that 97% of the total bond risk premium comes from long- as opposed to short-run macro risk in emerging markets.

We find empirical support for the model's novel cross-sectional predictions. The model predicts that

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<sup>1</sup>A long literature starting with [Hamilton \(1989\)](#) posits that business cycles involve sudden shifts between regimes of high expected growth and low uncertainty (expansions), and regimes of reduced expected growth and heightened uncertainty (recessions). Finance models with regime-shifts in macroeconomic conditions include [Hackbarth, Miao and Morellec \(2006\)](#), [Bhamra, Kuehn and Strebulaev \(2010a, 2010b\)](#), [Chen \(2010\)](#), [Arnold, Wagner and Westermann \(2013\)](#), [Chen and Strebulaev \(2019\)](#), and [Bhamra, Dorion, Jeanneret and Weber \(2021\)](#).

<sup>2</sup>The debt decision trades off the benefits of debt against the rise in default risk. The default decision trades off the advantage of smaller future debt payments against an immediate output drop associated with default. Our model builds on the contingent-claim framework approach developed by [Hayri \(2000\)](#), [Gibson and Sundaresan \(2001\)](#), [Andrade \(2009\)](#), and [Jeanneret \(2015, 2017\)](#).

sovereign bonds of countries that are more sensitive to shifts in the global business cycle, i.e., countries with higher long-run macro risk, deliver higher expected excess returns. In contrast, the premium for short-run macro risk is expected to be negligible, i.e., long-run macro risk dominates short-run macro risk in explaining cross-sectional variation in bond risk premia.

We test these predictions using quarterly sovereign bond data for 43 countries from 1994:Q1 to 2018:Q2. Because most emerging economies lack long time series of quarterly output data, we measure long-run macro risk using sovereign bond returns. Our primary measure of a country's exposure to shifts in the global business cycle uses regressions of bond excess returns on **changes in U.S. consumption growth moments** calculated from the Federal Reserve Bank of Philadelphia's Survey of Professional Forecasters (SPF). The first and second moments of consumption growth are respectively the quarterly mean and dispersion (75th-25th percentile) of U.S. consumption growth forecasts. Figure 1 shows substantial cross-sectional variation on the sensitivity of bond excess returns to changes in both expected consumption growth (Panel A) and uncertainty about consumption growth (Panel B).

In line with the model, the aforementioned cross-sectional variation in risk exposure leads to economically significant cross-sectional variation in average excess returns. Using independent double sorts, we classify countries into high, medium, and low long-run macro risk based on the sensitivity of their bond returns to shifts in the global business cycle. **Countries with high exposure have above the median return sensitivities to changes in both consumption growth moments. The sovereign bond prices of these countries decrease the most when recessions occur, i.e., when expected consumption decreases and consumption growth uncertainty increases.** Countries with low exposure have below the median sensitivities to changes in both consumption growth moments, i.e., their sovereign bond prices *decrease the least* when recessions occur. The remaining countries have a medium degree of long-run macro risk. An equally-weighted portfolio of high-exposure countries earns 14.55% per year above the risk-free rate, followed by 9.23% per year for medium-exposure countries and 6.44% for low-exposure ones. The average return of the high-minus-low long-short portfolio is 8.11% per year, with a  $t$ -statistics of 2.75.

Results based on single-sorts are also consistent with the theory. The long-short portfolio based on terciles of exposure to **expected consumption growth** is 4.17% per year, with a  $t$ -statistics of 1.79. The long-short portfolio based on terciles of exposure to **consumption growth uncertainty** has an average excess return of 9.25% per year, with a  $t$ -statistics of 3.74. Hence, both exposures matter for explaining the cross-section of bond excess returns, as predicted by our model. In contrast, sorting countries

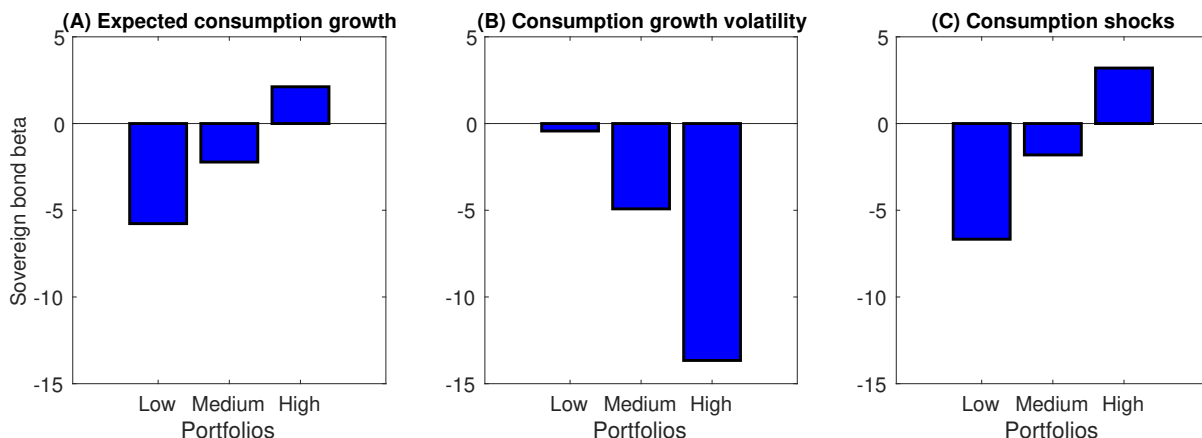


Figure 1: **Cross-sectional exposure of sovereign bonds to U.S. economic conditions.**

The figure illustrates cross-sectional variation in the exposure of sovereign bond excess returns to changes in global macroeconomic conditions. The sample has 43 emerging countries grouped into tercile portfolios. Panel A displays the exposure to quarterly changes in expected U.S. consumption growth, computed as the mean of consumption growth forecasts. Panel B displays the exposure to quarterly changes in U.S. consumption growth uncertainty, measured by the cross-sectional dispersion in consumption growth forecasts. Panel C displays the exposure to U.S. consumption growth. Consumption data is from the Bureau of Economic Analysis and consumption growth forecast series are from the Survey of Professional Forecasters at the Federal Reserve Bank of Philadelphia. Bond returns use JP Morgan EMBI Global indices. Bond portfolios are formed every quarter, and their construction is detailed in Section 5.3. Data span the 1994:Q1-2018:Q2 period.

ing the cross-section of bond excess returns, as predicted by our model. In contrast, sorting countries into long-short portfolios based on the exposure of bond excess returns to contemporaneous U.S. consumption shocks delivers statistically insignificant excess returns and with the opposite sign predicted by theory, despite considerable cross-sectional variation in short-run risk (Panel C). The data thus confirm the prediction that long-run macro risk is more important than, and not spanned by, short-run macro risk in explaining sovereign bond risk premia.

Our baseline empirical results are robust to several methodological changes. For example, very similar results obtain when the investment horizon is one quarter rather than one year. Moreover, it is irrelevant whether bond return sensitivities to changes in first and second consumption growth moments are estimated separately or jointly. Similarly, it does not matter whether we double sort on sensitivities to changes in expected consumption growth and consumption growth uncertainty or, alternatively, single sort on the sensitivities to their first principal component. Furthermore, results continue to hold using value- or GDP-weighted portfolios rather than equally-weighted ones, which indicates that our findings are not driven by small countries with potentially less liquid bonds. We also find that our measures of

long-run macro risk are not simply proxying for betas with respect to corporate bond or stock market index returns. Finally, we obtain similar results when measuring long-run macro risk using alternative data sources. For example, results remain robust when we respectively replace the first and second moments of expected consumption growth forecast data by the lagged slope of the U.S. yield curve (Harvey, 1989) and the macroeconomic uncertainty index of Jurado, Ludvigson and Ng (2015).

This paper contributes to a growing literature exploring time-variation in sovereign credit risk and bond valuation. Pan and Singleton (2008) and Longstaff et al. (2011) document strong comovement in sovereign credit spreads. These studies identify a factor structure that is strongly tied to U.S. financial conditions, a result confirmed by Ang and Longstaff (2013). Augustin and Tedongap (2016) highlight the role of macroeconomic risk in explaining the international comovement in the term structure of sovereign credit default swap (CDS) spreads. Hilscher and Nosbusch (2010) find that an improvement in a country's terms of trade reduces its sovereign credit spread. Doshi, Jacobs and Zurita (2017) show that the impact of economic and financial conditions on sovereign credit spreads varies substantially over time, while Augustin (2018) finds that the relative importance of global and country-specific macroeconomic risks varies with the slope of the CDS term structure. More recently, Chari, Dilts Stedman and Lundblad (2021) find that U.S. monetary policy shocks affect emerging market bond and stock valuations more strongly during monetary tightening than easing, and such effects appear to be largely driven by changes in risk premia.

Our analysis differs from the papers mentioned above across two critical dimensions. First, our primary goal is to study the cross-sectional pricing of sovereign bonds, rather than its time-series variation or comovement. Second, we focus on bond excess returns (risk premia) rather than credit spreads. This is not a mere terminology issue as such measures capture distinct information. While risk premia only capture compensation for systematic risk, spreads are also affected by default probabilities and expected losses upon default. Our contribution is to theoretically link sovereign bond risk premia to macroeconomic risk exposure in the consumption-based paradigm and to conduct cross-sectional tests to validate the theory.

We are aware of three papers that empirically study the cross-section of sovereign bond risk premia. Borri and Verdelhan (2012) show that average sovereign bond excess returns line up well with betas with respect to U.S. stock or BBB corporate bond returns, in conformity with their theoretical model. Borri and Verdelhan's pioneering paper also constructs six sovereign bond portfolios used in later work by Lettau, Maggiori and Weber (2014) and He, Kelly and Manela (2017). Lettau, Maggiori and Weber

(2014) find that the association between the returns of the six bond portfolios and their corresponding CAPM market betas is stronger when aggregate stock returns are one standard deviation below their average, consistent with the downside risk CAPM. He, Kelly and Manela (2017) find that the average returns of the six bond portfolios are better aligned with their betas relative to an intermediary capital factor, defined as shocks to the aggregate capital ratio of U.S. bond market primary dealers, than with their CAPM betas. In contrast to the existing literature, we show that consumption-based risk exposure can rationalize the dispersion of average excess returns across sovereign bond portfolios.

Finally, our work belongs to a growing literature exploring how global risk factors drive the cross-section of corporate bond returns (e.g., Bai, Bali and Wen, 2019; Chung, Wang and Wu, 2019; and Bali, Subrahmanyam and Wen 2021). We contribute to this line of work by showing, both theoretically and empirically, that cross-country variation in exposure to U.S. consumption moments leads to differences in average sovereign bond excess returns.

The remainder of the paper is organized as follows. Section 2 presents a model describing how a country's exposure to global macroeconomic conditions affects the risk premium in its sovereign bonds. Section 3 details the model calibration. Section 4 discusses the theoretical predictions and includes simulations aimed at quantitatively comparing moments in the model and the data. Section 5 has empirical tests of the model's cross-sectional predictions, while Section 6 concludes the paper.

## 2 The model

We develop a dynamic asset-pricing model for sovereign bond valuation in the presence of two sources of systematic risk. The first source of risk stems from the positive correlation between shocks to a sovereign issuer's output and shocks to global consumption. The second source of systematic risk captures exposure to sudden changes in global macroeconomic conditions: the sovereign issuer's output and global consumption face synchronous regime shifts. The two sources of risk are fundamentally different. While the former involves frequent shocks to output and consumption levels, the latter involves infrequent shocks to their first and second moments. In the model, the two sources of systematic risk endogenously generate risk premia in sovereign bonds. Sovereign default (and debt) policies are set optimally by the sovereign issuer's government, while a global representative agent determines the pricing of the bonds.

## 2.1 Economic environment

We first define the dynamics of global consumption. The global economy can be in expansion or recession, and the conditional moments of consumption growth characterize the global business cycle. We then describe the state-price density of the representative agent.

### 2.1.1 Consumption

Let  $C_t$  denote the instantaneous stream of global consumption, with dynamics given exogenously by

$$\frac{dC_t}{C_t} = \mu_{c,s_t} dt + \sigma_{c,s_t} dZ_{c,t}, \quad s_t = \{L, H\}, \quad (1)$$

where  $Z_{c,t}$  is a standard Brownian motion under the physical probability measure  $\mathbb{P}$ . The first and second conditional moments of consumption growth  $\mu_{c,s_t}$  and  $\sigma_{c,s_t}$  take different values depending on the current state of the economy, denoted by  $s_t$ . The economy switches unexpectedly between a recession state ( $s_t = L$ ) and an expansion state ( $s_t = H$ ) according to a two-state Markov chain. Expected consumption growth is procyclical, while consumption growth uncertainty is countercyclical, that is,  $\mu_{c,H} \geq \mu_{c,L}$  and  $\sigma_{c,H} \leq \sigma_{c,L}$ . The physical probability per unit of time of the economy leaving state  $s_t$  is  $\lambda_{s_t}$ , hence the expected duration of state  $s_t$  is  $1/\lambda_{s_t}$ . Recessions are shorter than expansions when  $1/\lambda_L < 1/\lambda_H$ .

### 2.1.2 State-price density

The representative agent has Epstein-Zin-Weil preferences. As we discuss below, this implies that her marginal utility moves not only with current consumption innovations but also when the economy suddenly shifts states, i.e., when consumption growth first and second moments change. The state-price density  $\pi_t$  is given by:

$$\pi_t = \left( \beta e^{-\beta t} \right)^{\frac{1-\gamma}{1-\frac{1}{\psi}}} C_t^{-\gamma} \left( p_{C,t} e^{\int_0^t p_{C,u}^{-1} du} \right)^{-\frac{\gamma-\frac{1}{\psi}}{1-\frac{1}{\psi}}}, \quad (2)$$

where  $\gamma$  is the coefficient of relative risk aversion,  $\psi$  is the elasticity of intertemporal consumption, and  $\beta$  is the subjective time discount factor. The price-consumption ratio  $p_{C,t}$  is the value of the claim to the perpetual stream of consumption (per unit of consumption today) and depends on the state of



the economy (see the Internet Appendix A.1). If  $\gamma \neq \frac{1}{\psi}$ , the price-consumption ratio jumps when the economy shifts states, and so does the state-price density. When  $\psi > 1$ ,  $p_{C,t}$  is procyclical, i.e., it jumps down when the economy transitions from expansion to recession and jumps up when it changes from recession to expansion.

Following the long-run risk literature (e.g., [Bansal and Yaron, 2004](#)), we assume the representative agent prefers early resolution of uncertainty ( $\gamma > 1/\psi$ ). Hence she dislikes bad news about future consumption, i.e., when expected consumption growth decreases or the volatility of consumption growth increases. The shift from expansion to recession is associated with a downward jump in the price-consumption ratio  $p_{C,t}$  that, being raised to a negative power when both  $\psi > 1$  and  $\gamma > 1/\psi$ , results in an upward jump in the state-price density. The magnitude of such an upward jump is time-invariant and denoted by the constant  $\Delta_H > 1$ .

For pricing purposes, it is useful to think in terms of the risk-neutral probability of switching states  $\hat{\lambda}_L$  and  $\hat{\lambda}_H$ . Because the representative agent dislikes bad news about consumption growth moments, the risk-neutral probability of switching from expansion to recession is higher than its corresponding physical probability ( $\hat{\lambda}_H > \lambda_H$ ), and, conversely, the risk-neutral probability of transitioning from recession to expansion is lower than its physical counterpart ( $\hat{\lambda}_L < \lambda_L$ ). Hence, securities are priced as if the probability of recessions is higher (and the probability of expansions is lower) than in reality. It turns out that the aforementioned constant  $\Delta_H > 1$  provides the conversion factors from physical to risk-neutral probabilities (see the Internet Appendix A):

$$\hat{\lambda}_H = \Delta_H \lambda_H \quad \text{and} \quad \hat{\lambda}_L = \frac{1}{\Delta_H} \lambda_L, \quad (3)$$

such that the risk-neutral rate of news arrival is  $\hat{p} = \hat{\lambda}_L + \hat{\lambda}_H$  and the long-run risk-neutral distribution of the states is  $(\hat{f}_L, \hat{f}_H) = (\frac{\hat{\lambda}_L}{\hat{p}}, \frac{\hat{\lambda}_H}{\hat{p}})$ .

Equation (3) provides the intuition behind a critical source of risk premium in the economy: securities whose prices jump down when a global recession hits provide a risk premium because they are priced as if recessions are more likely than in reality. Such risk premium ultimately arises from the representative agent's preference for early resolution of uncertainty and the associated disutility she experiences when there is bad news about consumption growth moments.

The following sections show that the price of sovereign bonds endogenously jumps down when the economy enters a recession. Therefore, sovereign bond prices must embed a risk premium to compensate investors for this exposure to the global business cycle. We first present the dynamics of output for a

small indebted country exposed to various sources of macroeconomic risk. We then derive sovereign bond valuation for a given set of default and debt decisions. Later we endogenize these policies.

## 2.2 Output and government revenue

There are several small countries in the economy. The level of country  $i$ 's output is denoted by  $X_{i,t}$  and evolves according to

$$\frac{dX_{i,t}}{X_{i,t}} = \mu_{X,i,s_t} dt + \sigma_{X,i,s_t} dZ_{i,t}, \quad s_t = \{L, H\}, \quad (4)$$

where the conditional expected growth rate is  $\mu_{X,i,s_t}$  and the conditional volatility is  $\sigma_{X,i,s_t}$ . The dynamics of output depends on the global business cycle. Specifically, global recessions imply lower expected output growth rate ( $\mu_{X,i,H} > \mu_{X,i,L}$ ) and higher output volatility ( $\sigma_{X,i,H} < \sigma_{X,i,L}$ ).<sup>3</sup> Such regime shift synchronicity drives 'long-run macro risk' in sovereign bonds. In addition, the instantaneous output shocks  $Z_{i,t}$  are correlated with consumption growth shocks  $Z_{c,t}$ , with correlation in state  $s_t$  denoted by  $\rho_{i,s_t}$ . Such correlation drives 'short-run macro risk' in the model. We assume that global consumption is not affected by a country's economic performance because each country is small relative to the global economy.

Country  $i$ 's government revenue  $Y_{i,t}$  is exposed to the same sources of variation as its output  $X_{i,t}$ . This type of modeling assumes, like [Arellano and Bai \(2017\)](#), that the government faces an unmodeled constraint such that it cannot raise taxes to prevent an imminent default. Specifically, we define government revenue as

$$Y_{i,t} = \exp(-\varphi_y t) X_{i,t}^\eta, \quad s_t = \{L, H\}, \quad (5)$$

where  $\eta$  is a leverage parameter that amplifies fluctuations in government revenue relative to output, and  $\varphi_y$  is a parameter that pins down the unconditional expected growth rate of government revenue. By Ito's Lemma, the dynamics of government revenue satisfies

$$\frac{dY_{i,t}}{Y_{i,t}} = \underbrace{\left( -\varphi_y + \eta \mu_{X,i,s_t} + \frac{1}{2} \eta (\eta - 1) \sigma_{X,i,s_t}^2 \right)}_{\mu_{Y,i,s_t}} dt + \underbrace{\eta \sigma_{X,i,s_t}}_{\sigma_{Y,i,s_t}} dZ_{i,t}, \quad s_t = \{L, H\}. \quad (6)$$

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<sup>3</sup>In Section 4.3, we let  $\mu_{X,i,H} - \mu_{X,i,L}$  and  $\sigma_{X,i,L} - \sigma_{X,i,H}$  vary across countries to explore the cross-sectional predictions of our model.

When  $\eta > 1$ , government revenue shocks are more volatile than output shocks, as in [Chen \(2013\)](#), and expected revenue growth displays higher sensitivity to the global business cycle. While we do not explicitly model the micro foundations of such amplification mechanism, its rationale is easy to explain. First, some particularly volatile sub-components of national output (e.g., commodities) may be more highly taxed than others. Second, the part of government revenue available for servicing debt may consist of taxed output minus relatively inflexible components of public spending such as public sector pensions. In the Internet Appendix B.3 we report empirical evidence for  $\eta > 1$  in emerging markets.

## 2.3 Sovereign bond valuation

Country  $i$  issues an infinite maturity sovereign bond characterized by a perpetual coupon  $c_i$ . In the absence of default, the bond value equals  $\frac{c_i}{r_{B,s_t}}$  when the current state is  $s_t$ , which is the present value of the continuous stream of coupons  $c_i$  discounted at a riskless perpetuity rate  $r_{B,s_t}$  given by

$$r_{B,s_t} = r_{s_t} + \frac{r_j - r_{s_t}}{\hat{p} + r_j} \hat{p} \hat{f}_j, \quad j \neq s_t, \quad (7)$$

where  $r_{s_t}$  is the equilibrium (instantaneous) risk-free interest rate in state  $s_t$ .<sup>4</sup> The discount rate  $r_{B,s_t}$  captures the expectation that the instantaneous risk-free rate changes with the global business cycle, and thus differs from the current instantaneous risk-free rate.

The government defaults on sovereign debt when revenue  $Y_{i,t}$  reaches a lower threshold whose level is contingent on the state of the economy. That is, there are state-dependent default thresholds  $Y_{D,i,s_D}$  for default occurring in state  $s_D = \{L, H\}$ . We assume that default may occur only once. It may occur smoothly when the government revenue falls to the threshold  $Y_{D,i,s_D}$  as a result of shocks  $Z_{i,t}$ , or discretely when the economy changes state, and the default threshold varies accordingly. When the government defaults, at a time denoted by  $t_{D,i}$ , the bond coupon  $c_i$  is reduced by a fraction  $\kappa \in (0, 1)$  thereafter due to debt restructuring. The sovereign bond value  $B_{i,s_t}$  in state  $s_t$  equals (see the Internet

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<sup>4</sup>The Internet Appendix A.1 presents the equation of the instantaneous risk-free interest rate, which is identical to that in [Bhamra, Kuehn and Strebulaev \(2010a, 2010b\)](#). Higher consumption uncertainty ( $\sigma_{c,H} < \sigma_{c,L}$ ) and lower expected growth ( $\mu_{c,H} > \mu_{c,L}$ ) in recession increases the demand for the risk-free asset, which implies that the instantaneous risk-free interest rate is procyclical in equilibrium ( $r_H > r_L$ ).

Appendix A.2)

$$B_{i,s_t} = \underbrace{\mathbb{E}_t \left[ \int_t^{t_{D,i}} c_i \frac{\pi_u}{\pi_t} du \middle| s_t \right]}_{\text{Before default}} + \underbrace{\mathbb{E}_t \left[ \int_{t_{D,i}}^{\infty} (1 - \kappa) c_i \frac{\pi_u}{\pi_t} du \middle| s_t \right]}_{\text{After default}} \quad (8)$$

$$= \frac{c_i}{r_{B,s_t}} \left[ 1 - \underbrace{\sum_{s_D} \kappa \frac{r_{B,s_t}}{r_{B,s_D}} q_{i,s_t s_D}}_{\text{Default risk discount}} \right], \quad s_t, s_D = \{L, H\}, \quad (9)$$

with

$$q_{i,s_t s_D} = \mathbb{E}_t \left[ \frac{\pi_{t_{D,i}}}{\pi_t} \text{Prob}(s_D | s_t) \middle| s_t \right], \quad (10)$$

where  $q_{i,s_t s_D}$  represents the value of the Arrow-Debreu default claim that pays one unit of consumption at default time  $t_{D,i}$  if the current state is  $s_t$  and the state at the moment of default is  $s_D$  (see the Internet Appendix A.3).

The bond value is equal to the risk-free bond value  $\frac{c_i}{r_{B,s_t}}$  minus a default risk discount. This default risk discount depends on the value of the Arrow-Debreu default claim, the bond discount rate in both states (at the time of issuance and at the time of default), and the fraction of the bond that is reduced in default. The corresponding bond credit spread in state  $s_t$  is  $CS_{i,s_t} = \frac{c_i}{B_{i,s_t}} - r_{B,s_t}$ .

When the economy switches from expansion to recession, the expected growth rate of a country's government revenue falls, and its volatility increases. As a result, the country faces a higher default probability, which increases the value of the Arrow-Debreu default claim and reduces bond valuation. Sovereign bond prices are thus strongly exposed to the business cycle.

## 2.4 Optimal policies

In our model, governments optimally choose their level of indebtedness and timing of default. The endogenous level of sovereign debt trades off the economic benefits of debt issuance against the economic cost of default. Issuing debt is beneficial to a country, but excessive debt raises the risk of default. Default is detrimental to economic performance because it instantaneously reduces the country's government revenue level by a fraction  $\alpha \in (0, 1)$ .<sup>5</sup> We define the benefits of debt issuance by  $r_g$  per unit of

<sup>5</sup>The presence of an economic cost provides governments the motivation for avoiding default, as in [Arellano \(2008\)](#), [Andrade \(2009\)](#), [Hatchondo and Martinez \(2009\)](#), [Yue \(2010\)](#), [Borri and Verdelhan \(2012\)](#), [Jeanneret \(2015\)](#), or [Rebelo, Wang and Yang \(2021\)](#). [Mendoza and Yue \(2012\)](#), [Hebert and Schreger \(2017\)](#), and [Andrade and Chhaochharia \(2018\)](#)

time. The total incentive for debt issuance, denoted by  $I_{i,s_t}$ , corresponds to (see the Internet Appendix A.4.1)

$$I_{i,s_t} = \mathbb{E}_t \left[ \int_t^\infty r_g B_{i,s_0} \frac{\pi_u}{\pi_t} du \middle| s_t \right] \quad (11)$$

$$= \frac{r_g}{r_{B,s_t}} B_{i,s_0}. \quad (12)$$

There are different interpretations for the debt issuance incentive  $I_{i,s_t}$ . For example, a government can use debt proceeds at issuance to finance long-term public investments yielding a return  $r_g$  per unit of time, as in [Jeanneret \(2015\)](#). In this case,  $I_{i,s_t}$  reflects the present value generated by public investments in infrastructure, for instance. Alternatively,  $r_g$  may represent the government's private benefits for issuing debt, possibly related to the ability to direct rents to favored counterparties executing public works or to transfer resources to base voters. While it is beyond the objective of the paper to micro-found the existence of the debt benefits, our specification is sufficiently general to encompass various reasons for issuing debt.<sup>6</sup>

When choosing the debt policy at time  $t = 0$ , the country  $i$ 's government maximizes the *ex ante* level of sovereign wealth, defined as the debt issuance incentives  $I_{i,s_t}$  plus the present value of the government revenue. The present value of government revenue, which we denote by  $G_{i,s_t}$  in state  $s_t$ , is given by (see the Internet Appendix A.4.2)

$$G_{i,s_t} = \underbrace{\mathbb{E}_t \left[ \int_t^{t_{D,i}} Y_{i,u} \frac{\pi_u}{\pi_t} du \middle| s_t \right]}_{\text{Before default}} + \underbrace{\mathbb{E}_t \left[ \int_{t_{D,i}}^\infty (1 - \alpha) Y_{i,u} \frac{\pi_u}{\pi_t} du \middle| s_t \right]}_{\text{After default}} \quad (13)$$

$$= \frac{Y_{i,t}}{r_{Y,i,s_t}} - \underbrace{\alpha \sum_{s_D} \frac{Y_{D,i,s_D}}{r_{Y,i,s_D}} q'_{i,s_t s_D}}_{\text{Costs of default}} \quad (14)$$

with

$$q'_{i,s_t s_D} = \mathbb{E}_t \left[ \frac{\pi_{t_{D,i}}}{\pi_t} \frac{Y_{i,t_D}}{Y_{D,i,s_D}} \text{Prob}(s_D | s_t) \middle| s_t \right], \quad s_t, s_D = \{L, H\}, \quad (15)$$

provide empirical evidence on the costs of sovereign defaults.

<sup>6</sup>It is also common to assume that risk-averse governments issue sovereign debt (purchased by risk-neutral agents) to smooth consumption/investment (e.g., [Eaton and Gersovitz, 1981](#)). However, in this paper we consider an environment with risk-averse lenders and a risk-neutral government, which precludes any smoothing motives.

and

$$r_{Y,i,s_t} = r_{s_t} - \hat{\mu}_{Y,i,s_t} + \frac{(r_j - \hat{\mu}_{Y,i,j}) - (r_{s_t} - \hat{\mu}_{Y,i,s_t})}{\hat{p} + r_j - \hat{\mu}_{Y,i,j}} \hat{p} f_j, \quad j \neq s_t, \quad (16)$$

where  $\hat{\mu}_{Y,i,s_t} = \mu_{Y,i,s_t} - \gamma \sigma_{c,s_t} \rho_{i,s_t} \sigma_{Y,i,s_t}$  is the expected growth rate under the risk-neutral measure.

The two terms of Equation (13) respectively represent the discounted government revenue before and after default, indicating that the government loses a fraction  $\alpha$  of its revenue at the time of default. When default risk increases, the present value of the government revenue declines because of a higher expected cost of default. Default risk is captured in Equation (14) with the Arrow-Debreu default claim  $q'_{i,s_t s_D}$ , which pays out  $\frac{Y_{i,t_D}}{Y_{D,i,s_D}}$  at default if default occurs in state  $s_D$  and the current state is  $s_t$ .<sup>7</sup>

The discount rate  $r_{Y,i,s_t}$  applying to government revenue in Equation (14), conditional on the current state being  $s_t$ , is the discount rate for a perpetuity with stochastic risk-free rate  $r_t$  and expected growth rate  $\hat{\mu}_{Y,i,t}$  under the risk-neutral measure, which are currently equal to  $r_{s_t}$  and  $\hat{\mu}_{Y,i,s_t}$ . If the economy stays in state  $s_t$  forever, the discount rate reduces to the standard expression  $r_{Y,i,s_t} = r_{s_t} - \hat{\mu}_{Y,i,s_t}$ . In general, however, the economy can change states, and thus we need to account for the time spent in recession and expansion at future times.

The optimal coupon maximizes sovereign wealth  $W_{i,s_t} = G_{i,s_t} + I_{i,s_t}$  at the time of debt issuance  $t = 0$ , and as such satisfies

$$c_{i,s_0}^* = \arg \max W_{i,s_0}, \quad (17)$$

where  $s_0$  is the state of the global economy when debt is contracted. The optimal coupon depends on the initial state  $s_0$ . Without loss of generality, we study model predictions assuming the global economy is in expansion when sovereign debt is issued.

The government maximizes net sovereign wealth by choosing the optimal state-contingent default thresholds  $Y_{D,i,s_D}$ , which are determined by solving the following two smooth-pasting conditions (see the Internet Appendix A.4.3):

$$\left. \frac{\partial (W_{i,s_t}(Y_{i,t}) - B_{i,s_t})}{\partial Y_{i,t}} \right|_{Y_{i,t}=Y_{D,i,s_t}} = \frac{1 - \alpha}{r_{Y,i,s_t}}, \quad s_t = \{L, H\}. \quad (18)$$

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<sup>7</sup>If default occurs as a result of a regime change, the level of government revenue immediately changes from  $Y_{i,t_D}$ , the level just before default, to  $Y_{D,i,s_D}$ , the level at default. Consequently, we must consider the modified set of Arrow-Debreu default claims  $q'_{i,s_t s_D}$  derived in the Internet Appendix A.3.2. See [Bhamra, Dorion, Jeanneret and Weber \(2021\)](#) for additional technical details.

The default thresholds determining the optimal timing of default in each state of the economy reflect the trade-off between the benefits and costs of default. On the one hand, defaulting is beneficial because it reduces future bond coupon payments through debt restructuring, increasing net sovereign wealth. On the other hand, defaulting is costly as it reduces future government revenue, decreasing net sovereign wealth. Similarly, greater debt issuance is beneficial, but too much debt becomes costly due to the increase in default risk. The problem of the government consists of solving Equation (17) subject to Equation (18). A closed-form solution to this optimization problem does not exist, and we use standard numerical procedures.

## 2.5 Sovereign risk premia

We now discuss how the two sources of macroeconomic risk generate risk premia in sovereign bond prices. Country  $i$ 's bond risk premium  $BP_{i,s_t}$  in state  $s_t$ , defined as the instantaneous expected return on the bond over the risk-free rate, is equal to

$$BP_{i,s_t} = \underbrace{\gamma \sigma_{c,s_t} \rho_{i,s_t} \sigma_{i,s_t}^B}_{\text{Short-run macro risk}} + \underbrace{\lambda_{s_t} \Theta_{s_t}^P R_{i,s_t}^B}_{\text{Long-run macro risk}}, \quad s_t = \{L, H\}. \quad (19)$$

The first term of Equation (19) captures the compensation for short-run macro risk, arising when sovereign bond prices covary with global consumption innovations. The market price of risk associated with such consumption shocks is  $\gamma \sigma_{c,s_t}$ , while  $\sigma_{i,s_t}^B$  is the instantaneous volatility of sovereign bond returns:

$$\sigma_{i,s_t}^B = \frac{\partial \ln B_{i,s_t}}{\partial \ln Y_{i,s_t}} \sigma_{Y,i,s_t} \quad (20)$$

with  $\frac{\partial \ln B_{i,s_t}}{\partial \ln Y_{i,s_t}} = -\frac{Y_{i,s_t} c_i}{B_{i,s_t} r_{B,s_t}} \sum_{s_D} \phi \frac{r_{B,s_t}}{r_{B,s_D}} q'_{i,s_t s_D}$ .

The representative agent dislikes bonds of countries with output growth shocks that correlate positively with consumption growth shocks, i.e.  $\rho_{i,s_t} > 0$ . An unexpected negative change in consumption increases sovereign default risk, which reduces bond valuation when the agent's marginal utility of consumption rises. Hence, a positive covariation between bond returns and consumption shocks commands a positive bond risk premium.

The second term of Equation (19) captures the compensation for long-run macro risk, arising from

the exposure of sovereign bond prices to changes in global macroeconomic conditions. This risk premium component is determined by the probability  $\lambda_{s_t}$  of leaving state  $s_t$ , the price of risk associated with this change of state  $\Theta_{s_t}^P = 1 - \Delta_{s_t}$ , and the change in bond valuation caused by the change of state, given by  $R_{i,s_t}^B = \frac{B_{i,j}}{B_{i,s_t}} - 1$ ,  $s_t \neq j = \{L, H\}$ . Note that the product  $\Theta_{s_t}^P R_{i,s_t}^B$  is always positive. In recessions,  $\Theta_L^P = 1 - \Delta_L > 0$  given that  $\Delta_L = \frac{1}{\Delta_H} < 1$ , and  $R_{i,L}^B = \frac{B_{i,H}}{B_{i,L}} - 1 > 0$ , as bond prices are higher in expansion than in recession. In contrast, in expansions we have  $\Theta_H^P = 1 - \Delta_H < 0$  (given that  $\Delta_H > 1$ ) and  $R_{i,H}^B = \frac{B_{i,L}}{B_{i,H}} - 1 < 0$ .

Because the representative agent prefers early resolution of uncertainty, she dislikes securities whose prices decline at the onset of recessions. This is the case of sovereign bonds, because recessions trigger an increase in default risk by lowering the issuer's expected output growth and raising its output growth volatility. We can also interpret the compensation for long-run macro risk in terms of risk-neutral probabilities. As the state of the economy follows a Markov process, recessions arrive at uncertain times. With preference for early resolution of uncertainty, the agent prices sovereign bonds using the risk-neutral transition probabilities, that is assuming that recessions are more likely than in reality, leading to  $\Theta_{s_t}^P > 0$ .

### 3 Model calibration

The conditional moments of consumption growth switch randomly across expansion and recession states according to a Markov chain. Based on Hamilton (1989), we estimate a two-state Markov-regime switching model on quarterly U.S. consumption data during the 1994Q1-2018Q2 period. Consumption data is from the Bureau of Economic Analysis and includes real non-durable goods plus services. The long-run physical probability of state  $s_t = L$  is set to the frequency of NBER recessions observed during the postwar period (i.e.,  $f_L = 13.53\%$ ).<sup>8</sup> The Internet Appendix B.1 reports additional details.

Figure 2 illustrates the series of consumption growth (Panel A), the filtered probability of being in the recession regime (Panel B), and the implied two-state business cycle (Panel C). The economy is in state  $s_t = L$  during the 2007-9 Global Financial Crisis and episodes of negative consumption growth in 2011 and 2013. The expected growth rate of global consumption is  $\mu_{c,L} = 0.06\%$  in recession and  $\mu_{c,H} = 2.05\%$  in expansion, while consumption growth uncertainty is  $\sigma_{c,L} = 1.10\%$  in recession and

<sup>8</sup>Following the literature (e.g., Bhamra, Kuehn and Strebulaev, 2010a, 2010b; Boguth and Kuehn, 2013; Lettau, Ludvigson and Wachter, 2008), we focus on postwar data because consumption data recorded during the prewar period contain significant measurement errors (Romer, 1989). Over the 1952Q1-2018Q2 period, the U.S. economy was in recessions and expansions during 36 and 230 quarters, respectively.



$\sigma_{c,H} = 0.96\%$  in expansion. The probabilities per unit of time of leaving the expansion and recession states are respectively  $\lambda_H = 6.32\%$  and  $\lambda_L = 40.39\%$ .

Figure 2 [about here]

The output dynamics of the sovereign bond issuing country is calibrated as follows. First, from Datastream we obtain quarterly real GDP data for the 40 emerging countries whose data cover at least one U.S. recession.<sup>9</sup> Then we compute each country's quarterly log growth GDP rate. Using the filtered U.S. business cycle to classify expansion and recession quarters, we compute the first and second moments of each country's output growth over the states  $s_t = H$  and  $s_t = L$ . The Internet Appendix B.2 provides more details on the procedure. Finally, we average across the 40 countries to find the conditional moments of output growth for the representative country. Such averaging mitigates concerns that most emerging markets in our sample have short time series of quarterly GDP data, and that some of the data may have quality issues. The representative country features an expected output growth rate equal to  $\mu_{X,L} = 1.68\%$  in recession and  $\mu_{X,H} = 4.21\%$  in expansion, while its output growth volatility is  $\sigma_{X,L} = 5.66\%$  in recession and  $\sigma_{X,H} = 3.62\%$  in expansion. The instantaneous correlation between output growth and consumption growth is  $\rho_L = 0.064$  in recession and  $\rho_H = 0.034$  in expansion.

The economic contraction at default is  $\alpha = 5\%$ , the average fraction of output lost reported in [Mendoza and Yue \(2012\)](#) across 23 sovereign default events in the 1977-2009 period. The debt haircut fraction is  $\phi = 75\%$ , the ISDA's market convention for pricing emerging market credit derivatives. The return on public investment  $r_g$  is equal to the unconditional output growth rate of the sovereign bond issuing country. The government revenue leverage factor is  $\eta = 4$ , consistent with emerging economies' data.<sup>10</sup> The value of  $\varphi_y$  is set such that government revenue has the same unconditional expected growth as output.

The representative agent's preferences are governed by risk aversion  $\gamma = 10$ , elasticity intertemporal substitution (EIS)  $\psi = 2$ , and time discount rate  $\beta = 4\%$ . We discuss the role of investor preferences on the bond risk premium and the price of risk in the Internet Appendix B.4. Our calibration implies an unconditional U.S. equity risk premium of 4.22% per year and generates model-implied price-dividend

<sup>9</sup>The countries are Argentina, Bahrain, Bolivia, Brazil, Bulgaria, Chile, Colombia, Croatia, Czechia, Dominican Republic, Ecuador, Estonia, Greece, Hungary, India, Kazakhstan, Latvia, Lithuania, Malta, Malaysia, Mexico, Morocco, Mozambique, Namibia, Peru, Philippines, Poland, Romania, Russia, Slovakia, Slovenia, South Africa, South Korea, Taiwan, Tanzania, Thailand, Turkey, Uganda, Venezuela, and Vietnam.

<sup>10</sup>We compute leverage for a set of emerging countries as the ratio of government revenue volatility to output growth volatility, as defined in the model. Leverage equals to 4.06 on average (3.80 when GDP-weighted) and ranges between 2.65 for Mexico and 6.73 for Colombia. See the Internet Appendix B.3 for computation details and country-level statistics.

ratio and level of stock return volatility that are consistent with the data (see the Internet Appendix C). Conditionally, the equity risk premium is 2.88% per year in expansions and 12.83% per year in recessions, consistent with the view that equity investors require a higher expected excess return during recessions. Finally, the instantaneous real risk-free interest rate is 3.74% per year in recession and 4.89% per year in expansion. Table 1 summarizes the parameter values on the baseline calibration.

Table 1 [about here]

## 4 Model predictions

This section discusses the role of macroeconomic risk for sovereign bond pricing. We first analyze how the two distinct sources of macroeconomic risk in the model – short-run and long-run macro risk – drive a country’s credit risk when debt and default policies are endogenous. Then we assess their impact on the risk premium embedded in sovereign bonds and derive implications for the variation in risk premia across countries. While our focus is mostly on model predictions for levels of government revenue observed at the time of financing ( $Y = 1$ ), we also explore a simulated economy that accounts for cross-sectional and time-series variability in default risk to adequately compare bond moments with their empirical counterparts.

In short, there are two key results. First, by accounting for long-run macro risk, the model increases the sovereign risk premium by tenfold and matches the level of default risk in the data, but cannot match the substantial average excess returns observed empirically. Second, the model generates the following empirically testable proposition: sovereign bonds of countries with greater long-run macro risk, i.e., whose first or second output growth moments are more sensitive to regime shifts in global macroeconomic conditions, contain a higher risk premium.

### 4.1 Macroeconomic risks, default risk, and optimal policies

We start by investigating how the different sources of macro risk affect the level of default risk, as well as a country’s optimal debt and default decisions. Table 2 compares and contrasts three observationally equivalent models. Column (A) displays results of the baseline model; a world in which there are both long- and short-run macro risks. Columns (B) and (C) each feature a restricted case nested within the baseline model. In Column (B) there is no long-run macro risk, that is, the sovereign issuer’s output process is not affected by shifts in global macroeconomic conditions, i.e.,  $\mu_{X,i,H} = \mu_{X,i,L}$  and

$\sigma_{X,i,H} = \sigma_{X,i,L}$ . In Column (C) there is no short-run macro risk, that is, innovations to the sovereign issuer's output process are conditionally uncorrelated with innovations in the global consumption process, i.e.,  $\rho_{i,s_t} = 0$ . The unconditional moments of the sovereign issuer's output process are identical across the three cases. Specifically, in Column (B)'s calibration, the expected growth rate and volatility of the issuer's output match the unconditional first and second moments of output growth of the baseline model.<sup>11</sup> All predictions reported in Table 2 are for levels of government revenue observed at issuance time ( $Y = 1$ ).

Table 2 [about here]

Comparing results in Columns (A) and (B) shows that long-run macro risk generates higher and countercyclical sovereign default risk. The unconditional credit spread increases from 68 bps per year in Column (B) to 154 bps in Column (A), although the total level of output risk remains observationally equivalent. Default risk is thus higher in a model in which output growth moments change across global regimes than in a model with constant output growth moments. The credit spread also becomes strongly countercyclical. In the full model, the bond credit spread fluctuates from 134 bps in expansions to 276 bps in recessions. In contrast, it remains stable at respectively 68 and 67 bps if the country's moments are unaffected by the global business cycle. The presence of long-run macro risk thus generates more realistic sovereign credit spread predictions in terms of level and variation over the global business cycle.

It is worth noting that we obtain these predictions under optimal debt and default policies. Table 2 shows that the optimal default threshold is higher in recessions than in expansions, as in [Bhamra et al. \(2010a\)](#), and such variation contributes to the countercyclicality in default risk with respect to the global business cycle. Table 2 also shows that the government optimally chooses lower debt levels in the presence of long-run macro risk to partially offset the increase in default risk. The optimal debt coupon decreases from 0.308 in Column (B) to 0.138 in Column (A). However, this debt policy response is not strong enough to reverse the positive impact on the level of default risk.

In contrast to the sharp differences between Columns (A) and (B), the results in Columns (A) and (C) are very similar. Hence, whether or not the issuer's output and global consumption shocks are contemporaneously correlated matters very little for the sovereign's optimal policies and the resulting sovereign default risk. Long-run macro risk, i.e., the issuer's exposure to shifts in global macro

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<sup>11</sup>The unconditional expected output growth is a weighted average using the long-term probabilities, given by  $\mu = f_L \mu_L + f_H \mu_H$ . Following [Timmermann \(2000\)](#), the unconditional variance of output growth equals  $\sigma^2 = f_L \sigma_L^2 + f_H \sigma_H^2 + f_L (1 - f_L) (\mu_H - \mu_L)^2$ .

conditions, is thus expected to be the economically most important source of macroeconomic risk for sovereign bond pricing.

Figure 3 displays unconditional credit spreads under different local economic conditions, that is, for different levels of government revenue  $Y$ . Panel A of the figure shows that credit spreads increase dramatically as local economic conditions deteriorate. When default risk is high, small shocks to output can lead to very large changes in spreads, and thus bond prices become very volatile.

Figure 3 [about here]

## 4.2 Analysis of the bond risk premium

This section discusses the predictions for the bond risk premium, which captures total compensation for macroeconomic risk exposure. We first determine how the risk premium varies over the business cycle and then decompose it into compensation for short- and long-run macro risks. We find that the bond risk premium is much higher in recessions and is primarily driven by long-run macro risk. Then, based on a model simulation, we show that the model can match default risk in the data but cannot generate the level of risk premia observed empirically.

### 4.2.1 Conditional bond risk premium

The bond risk premium is 117 bps per year unconditionally, as reported in Column (A) of Table 2. This compensation is higher during recessions (366 bps) than during expansions (78 bps). In Column (B), we switch off the exposure of a country's output moments to the business cycle. In this counterfactual case, the bond risk premium becomes negative (-14 bps), because bond prices now vary countercyclically over the business cycle due to variation in the equilibrium risk-free rate: in recessions, a lower risk-free rate increases the value of sovereign bonds, becoming hedge assets to investors. This countercyclicality in sovereign bond valuation is clearly at odds with the evidence that sovereign bond prices in emerging markets tend to fall during recessions. This analysis illustrates the critical role of a country's exposure to global macroeconomic conditions in sovereign bond pricing.

In Column (C), we consider the alternative case of independent shocks between sovereign output and global consumption growth. In this case, the bond risk premium is 112 bps per year, and both the level and the variation across states are similar to the full model (Column A). The correlation between shocks to consumption growth and sovereign output growth (short-run macro risk) thus has a negligible impact on the bond risk premium.

Panel B of Figure 3 displays the bond risk premium for different levels of government revenue  $Y$ . The figure shows that the bond risk premium generally increases as local economic conditions deteriorate. The relation is not entirely monotonic because bond prices become much less exposed to future shifts in the business cycle when default is imminent. The unconditional sovereign bond premium may be as high as 250 bps per year if local economic conditions worsen dramatically after the bond's issuance.

We now investigate in more detail how long- and short-run macro risks affect the bond risk premium, whose decomposition is given by Equation (19). Columns (A) and (B) of Table 6 break down the bond risk premium into its two individual components, while Column (C) reports the total risk premium. Below, we analyze each component of the bond risk premium separately.

Table 3 [about here]

#### 4.2.2 Long-run macro risk

Long-run macro risk reflects the impact of regime shifts in global macroeconomic conditions on the sovereign's output growth dynamics. When the global business cycle shifts from expansion to recession, the sovereign issuer's expected output growth rate decreases and its output growth volatility increases. Because the representative agent has a preference for early resolution of uncertainty ( $\gamma > \frac{1}{\psi}$ ), the change from expansion to recession is associated with an upward jump in marginal utility. At the same time, the change from expansion to recession is associated with higher default risk and a large decrease in sovereign bond prices. Therefore, sovereign bond investors experience negative returns in bad times and, as such, require compensation for such risk.

Column (A) of Table 3 shows that compensation for long-run macro risk is the main contributor to the bond risk premium. When revenue  $Y$  equals 1, this component amounts to 352 bps per year in recessions and 76 bps in expansions, representing 96% of the total bond risk premium in recessions and 97% in expansions. To understand the magnitude of the long-run macro risk compensation, recall that this risk premium is the product of three components:  $\lambda_{s_t} \Theta_{s_t}^P R_{i,s_t}^B$ , where  $\lambda_{s_t}$  is the probability per unit of time of leaving state  $s_t$ ,  $\Theta_{s_t}^P$  is the price of risk associated with a change of state, and  $R_{i,s_t}^B$  is the relative variation in bond prices caused by the change of state. In expansion, we have  $\lambda_H = 6.32\%$ ,  $\Theta_H^P = 1 - \Delta_H = -0.674$ , and  $R_{i,H}^B = -17.79\%$ , which implies a long-run macro risk premium of 76 bps. In recessions, we have  $\lambda_L = 40.39\%$ ,  $(1 - \Delta_L) = 0.403$ , and  $R_{i,L}^B = 21.63\%$ , which generates a long-run macro risk premium of 352 bps. The long-run macro risk premium is thus sizable because, first, the price of macro risk is high and, second, sovereign bonds are expected to decrease (increase)

severely when the global economy switches to a recession (expansion). Panel B of Figure 3 shows that long-run macro risk remains the primary driver of sovereign bond risk premia under different local economic conditions, that is, for different levels of revenue  $Y$ .

#### 4.2.3 Short-run macro risk

Short-run macro risk stems from the positive covariation between shocks to the sovereign's output and shocks to global consumption. Note that these innovations are frequent, i.e., every quarter in our calibration. But, fitting the data, output growth shocks are small and independent from the low-frequency changes in global macroeconomic conditions that characterize business cycle fluctuations. These frequent shocks are also transitory in the sense they do not affect expected growth rates and volatilities of global consumption or the sovereign's output. Default risk increases and sovereign bonds fall in value when there are negative shocks. If  $\rho_{i,s_t} > 0$ , risk averse investors demand compensation for holding bonds whose value depreciate when global consumption falls.

Column (B) of Table 3 shows that the compensation for short-run macro risk is tiny. When revenue equals  $Y = 1$ , this risk premium component equals 2 bps per year in expansion and 14 bps in recession. Thus, less than 5% of the unconditional bond risk premium originates from the high-frequency correlation between the sovereign's output and global consumption. To grasp why the risk premium associated with short-run macro risk is small, recall that it is the product of four components:  $\gamma \sigma_{c,s_t} \rho_{i,s_t} \sigma_{i,s_t}^B$ , where  $\gamma = 10$  is the risk aversion coefficient,  $\sigma_{c,s_t}$  is consumption growth volatility,  $\rho_{i,s_t}$  is the correlation between output and consumption shocks, and is  $\sigma_{i,s_t}^B$  the volatility of bond returns. In expansion, we have  $\sigma_{c,H} = 0.96\%$ ,  $\rho_{i,H} = 0.034$ , and  $\sigma_{i,H}^B = 6.63\%$ , which implies a short-run macro risk premium of 2 bps. In recession, we have  $\sigma_{c,L} = 1.10\%$ ,  $\rho_{i,L} = 0.064$ , and  $\sigma_{i,L}^B = 19.58\%$ , which implies a short-run macro risk premium of 14 bps. Despite the high risk aversion, the risk premium is negligible because global consumption volatility is low, and output innovations in emerging markets are only weakly related to global consumption shocks. Figure 3 shows that the short-run macro risk's contribution to bond risk premia remains tiny under different local economic conditions, i.e., for different  $Y$ .

Column (D) of Table 3 shows that the conclusion is similar in an observationally equivalent model with short-run macro risk only. In this 'restricted model', the issuer's output moments are constant over time, but the output process has the same unconditional consumption-CAPM beta as the baseline model. Accordingly, the constant output volatility equals the unconditional output volatility of the baseline model, while the correlation between issuer's output shocks and global consumption shocks

increases to 0.15. The correlation is higher than the 0.034 in expansions and 0.064 in recessions of the baseline model because, for a fixed level of *conditional* correlation between output and consumption innovations, the joint fluctuation of output and consumption moments due to regime shifts increases the *unconditional* correlation between output and consumption. The bond risk premium in this restricted model rises to 10 bps per year, and thus remains just one tenth of the total unconditional risk premium in the baseline model (117 bps).

Overall, our results show that the low risk premium arising from the consumption-CAPM, which has been extensively studied in the equity market, echoes in the sovereign debt market. Long-run macro risk is the primary source of risk compensation as it increases the bond risk premium by a factor of 10.

#### 4.2.4 Model simulation and discussion

Results in Tables 2 and 3 show that a model with long-run macro risk generates significantly higher sovereign credit spreads and bond risk premia than a canonical model with short-run macro risk only. This section reports that such large increases are still insufficient to match sovereign bond data in emerging markets.

We simulate a multi-country economy to compare theoretical predictions with the data. In contrast to model results tabulated in the paper, which are analytically calculated for a single country with government revenue maintained at the level of bond issuance ( $Y = 1$ ), the simulated economy consists of a large set of countries endowed with different levels of government revenue  $Y$  and default risk. The model simulation thus generates an artificial panel of countries whose credit spreads and bond excess returns display cross-sectional and time variation. The Internet Appendix D describes the simulation procedure and presents full simulation results. We find that the level and the term structure of default probabilities in our simulated economy closely matches the empirical counterparts, as the 1-, 5-, and 10-year default probabilities are 2.23%, 10.63%, and 20.08% in the simulation and respectively 2.88%, 11.29%, and 19.11% in the data. The simulation's annualized volatility of sovereign bond excess returns is also very close to that in the data.

The average bond excess return in the simulations is 1.90% per year, while the average credit spread is 162 bps. These values are higher than their respective 1.17% and 154 bps reported in Table 2 because variations in default risk do not even out in the time series and the cross-section. Yet, both average excess returns and credit spreads are still significantly below their empirical counterparts in the 1994:Q1-2018:Q2 period, as measured using JP Morgan EMBI Global indices. The average sovereign

bond excess return in the data is 8.53% per year (2.13% per quarter) above one-month T-bills and 5.36% per year above AAA corporate bonds, while the average credit spread in the data is 471 bps above Treasuries and 376 bps above AAA corporates.

Various economic mechanisms absent in our paper could help close the gap between the model and data. A first plausible channel is compensation for illiquidity (see, e.g., [Duffie, Pedersen and Singleton, 2003](#); [Longstaff, Pan, Pedersen and Singleton, 2011](#); [Chen, Cui, He and Milbradt \(2018\)](#)). Emerging market bonds might be substantially more illiquid than Treasuries or AAA Corporates, particularly during times of distress when investors move out of emerging markets ([Jotikasthira, Lundblad and Ramadorai, 2012](#)). Investors would then require a countercyclical liquidity risk premium. A second channel would be to account for time-varying required compensation for risk. Such time variation could be driven by preference parameters as in [Campbell and Cochrane \(1999\)](#) or [Albuquerque, Eichenbaum, Luo and Rebelo \(2016\)](#). Alternatively, it could result from fluctuations on the total amount of risk absorbed by the public sector rather than by investors, as in U.S. unconventional monetary policy experiments or International Monetary Fund rescue programs ([Chari, Dilts Stedman and Lundblad, 2021](#)). The common feature of the time-varying liquidity and required risk compensation channels is to generate stronger sovereign bond price declines at the onset of global recessions, thereby increasing the unconditional risk premium required by bond investors.

### 4.3 Cross-sectional implications

While the model is unable to generate bond risk premia as large as observed in emerging markets, it provides novel testable predictions in the cross-section. We find that variation in long-run macro risk across countries leads to much more cross-sectional dispersion in sovereign bond risk premia than does short-run macro risk variation. The testable implications are that long-short investment strategies exploiting cross-sectional differences in long-run macro risk should deliver high average returns, whereas the average return of long-short strategies based on short-run macro risk should be negligible.

We first consider variation in short-run macro risk, as measured by the instantaneous correlation between the issuer's output and global consumption shocks. Investors dislike bonds of countries that perform poorly when the global economy experiences negative shocks and are thus averse to short-run macro risk when  $\rho_i > 0$ . In contrast, investors favor short-run macro risk when  $\rho_i < 0$ , that is when countries perform countercyclically, such that their bonds offer a hedge against adverse consumption shocks. The model can thus generate a positive or a negative short-run macro risk premium, depending



on the sign of output-consumption correlation. Panel A of Table 4 shows that the level of this risk premium remains below 30 bps per year in magnitude, even for relatively high levels of correlation. Cross-sectional variation in short-run macro risk is therefore associated with little cross-sectional variation in sovereign bond risk premia.

Table 4 [about here]

Now we explore how the bond risk premium varies as the degree of long-run macro risk varies across countries. To that end, we compute, for each country  $i$ , the degree of exposure to the business cycle as follows:

$$\phi_{\mu,i} = \frac{\mu_{Y,i,H} - \mu_{Y,i,L}}{\bar{\mu}_{Y,H} - \bar{\mu}_{Y,L}} \quad (21)$$

$$\phi_{\sigma,i} = \frac{\sigma_{Y,i,L} - \sigma_{Y,i,H}}{\bar{\sigma}_{Y,L} - \bar{\sigma}_{Y,H}}, \quad (22)$$

where  $\mu_{Y,i,s_t}$  and  $\sigma_{Y,i,s_t}$  are the conditional mean and volatility of country  $i$ 's revenue growth, while  $\bar{\mu}_{Y,s_t}$  and  $\bar{\sigma}_{Y,s_t}$  reflect the conditional moments for the average country (baseline calibration). When  $\phi_{\mu,i} > 1$  and  $\phi_{\sigma,i} > 1$ , both revenue growth moments are more exposed to global recessions for country  $i$  than for the baseline country, whereas country  $i$ 's exposures are relatively lower when  $\phi_{\mu,i} < 1$  and  $\phi_{\sigma,i} < 1$ .

Panel B of Table 4 shows that the unconditional bond risk premium associated with long-run macro risk can exceed 240 bps per year for reasonable calibration values. When revenue equals  $Y = 1$ , a country with high macro risk ( $\phi_{\mu,i} = \phi_{\sigma,i} = 2$ ) features a bond risk premium of 242 bps, whereas a country with low macro risk ( $\phi_{\mu,i} = \phi_{\sigma,i} = 0.5$ ) has a bond risk premium of 48 bps. The bond risk premium thus varies meaningfully in the cross-section with the degree of long-run macro risk, much more so than with variation in short-run macro risk. Overall, the theoretical results of Table 4 lead to the following empirically testable cross-sectional hypotheses:

**Hypothesis 1:** *Long-run macro risk is priced in the cross-section of sovereign bonds. Investors buying bonds with high macro risk and shorting bonds with low macro risk obtain significant average excess returns.*

**Hypothesis 2:** *Both sources of long-run macro risk are relevant for sovereign bond pricing. Long-short portfolios formed on either exposure to expected growth or growth volatility deliver significant average excess returns.*

**Hypothesis 3:** *Short-run macro risk only delivers a modest bond risk premium. Investors buying bonds with high short-run macro risk and shorting bonds with low short-run macro risk do not obtain significant average excess returns.*

## 5 Empirical analysis

Our model predicts that sovereign bonds of countries that are more sensitive to shifts in the global business cycle deliver higher expected excess returns. In the previous section, we organized this basic prediction into three hypotheses. In this section, we test these hypotheses using sovereign bond data for 43 countries from 1994 to 2018. First, we describe the data. Second, we discuss the measurement of long-run macro risk across countries and present preliminary results. Then we report our portfolio sorting results and discuss various robustness analyses.

### 5.1 Sovereign bond data

We use JP Morgan EMBI Global indices to compute country-level sovereign bond returns. Each country's EMBI index is a value-weighted portfolio of U.S. dollar-denominated government bonds meeting liquidity criteria. We obtain quarterly EMBI data from Datastream over the 1994:Q1 to 2018:Q2 period. After restricting the sample to countries with at least 20 quarters of data, the sample consists of 43 countries. We compute **quarterly excess returns** for each country using monthly risk-free return data from Kenneth French's website. Table 5 reports descriptive statistics for quarterly excess returns by country. The last rows present averages of country statistics and pooled averages.

Table 5 [about here]

The average excess return across all countries is 2.14% per quarter or 8.56% per year.<sup>12</sup> There is substantial variation in the average excess return across countries, for example it is 1.09% per quarter for China and 3.77% per quarter for Russia. Furthermore, the (untabulated) time series average of quarterly cross-sectional standard deviations in excess returns is 6.5% per quarter, also large compared to the 2.14% average quarterly excess return. This paper's goal is to shed light on such cross-sectional variation in returns, both theoretically and empirically.

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<sup>12</sup>Note that countries enter and exit the sample over time. For example, there are 14 countries at the beginning of the sample period and 39 at the end. Therefore, the average excess return across any two countries in Table 5 is not necessarily comparable because such returns may be computed over very different sample periods.

## 5.2 Measuring long-run macro risk

We must measure each country's degree of long- and short-run macro risk to test our hypotheses. In principle, measuring short-run macro risk is straightforward: it is defined in the model as the contemporaneous correlation between shocks to global consumption growth and shocks to the issuing country's output growth. But measuring long-run macro risk is challenging. With long and high-quality time series of country-level GDP data, one could directly measure the empirical counterparts of the  $\phi_{\mu,i}$  and  $\phi_{\sigma,i}$  parameters. In this case, one would observe several global expansion and recession cycles and could easily quantify how much each country's first and second GDP growth moments vary with shifts in the global business cycle. Unfortunately, quarterly GDP data for emerging markets span a limited period and are of questionable quality. Therefore, we must pursue alternative methodologies to measure long-run macro risk.

We propose two approaches to assess cross-sectional differences in long-run macro risk. Both approaches rely on sovereign bond returns, consistent with our model in which sovereign bond prices respond to shifts in the global business cycle. In a preliminary approach, we directly connect data with theory: we estimate a country's level of long-run macro risk as the sensitivity of its sovereign bond return to events triggering global regime changes. However, this approach has limited statistical power because, in addition to the challenge in precisely dating regime events, our sample contains relatively few of those events in the time series and relatively few countries in the cross-section. Our main approach circumvents these issues by combining survey data on U.S. consumption growth and conditional portfolio sorting.

### 5.2.1 Preliminary analysis

In our model, sovereign bonds of countries with higher long-run macro risk are more sensitive to global regime changes. While it is hard to date regime changes in global macroeconomic conditions precisely, the sudden worsening of the Global Financial Crisis in 2008:Q4 is by far the best example of a perceived regime shift synchronous with large asset price changes in our sample. Sovereign bond excess returns were -24% on average during that quarter, but with significant cross-country dispersion, as they ranged from -65% to 3%. Our model predicts that countries whose bond prices declined the most during such a regime change should have higher unconditional excess returns. That is, the relationship between returns observed in 2008:Q4 and their corresponding 'full sample' returns should be negative.

Figure 4 [about here]

Figure 4 contains four scatter plots, each corresponding to a different definition of the 'full sample'. As Table 5 reports, countries enter and exit the sample over time, and there are just 10 countries with complete data over the entire sample period (1994:Q1 to 2018:Q2). The number of countries increases from 10 (Panel A) to 15 (Panel B), 18 (Panel C), and 23 (Panel D) as the beginning of the sample period is delayed. Note that we cannot use all 43 countries for which there are some sovereign bond data because 7 countries were not part of the sample as of 2008:Q4, and 13 countries have time-series data that are too short (as little as 10 years) for interpreting average returns as expected returns. All four scatter plots suggest a negative relationship between average quarterly excess returns over the 'full sample' and excess returns in 2008:Q4. This preliminary exercise confirms the model prediction that countries whose bonds are more sensitive to changes in global macroeconomic conditions are riskier and offer a higher risk premium.<sup>13</sup>

### 5.2.2 Main approach

The preliminary approach of the previous section suffers from low statistical power due to limited data availability, compounded by the difficulty in identifying regime changes using realized consumption data. Using surveys of U.S. consumption growth forecasts helps alleviate this problem. Survey (and market) participants are not constrained to using realized consumption alone when making their forecasts. Thus, using survey participants' view about regime changes allows us to employ information that cannot be easily gleaned from historical consumption data. Moreover, to the extent that survey and market participants share the same information, the regime changes implied by such forecasts will be synchronous with large asset price movements. We can thus measure each country's exposure to regime changes by investigating the co-variation between its sovereign bond returns and changes in survey forecasts. We use survey data from the quarterly Survey of Professional Forecasters, maintained by the Federal Reserve Bank of Philadelphia.

Our measure of expected consumption growth is the mean of the one-quarter-ahead U.S. consumption growth forecasts. The interquartile range of these growth forecasts is our measure of consumption growth uncertainty. Figure 5 plots the time series of these two state variables, with NBER recession periods in shaded grey. In line with our model, during recessions, expected consumption growth declines severely while consumption growth uncertainty increases strongly.

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<sup>13</sup>In the Internet Appendix E.2, we report results using seven potential regime changes that we identify applying the [Harding and Pagan \(2006\)](#)'s procedure to realized U.S. consumption growth data. This analysis confirms that countries whose bonds are more sensitive to regime changes in consumption offer higher average excess returns over the full sample.

Figure 5 [about here]

For each country we compute two quarterly measures of long-run macro risk. Using the most recent 20 quarterly observations, we estimate the slope coefficient of a univariate regression of sovereign bond excess returns on the contemporaneous change in either expected consumption growth or consumption growth uncertainty.<sup>14</sup> We refer to such slope coefficients as the risk loadings  $\beta_i^\mu$  and  $\beta_i^\sigma$ , respectively. For robustness, we later consider the joint estimation of  $\beta_i^\mu$  and  $\beta_i^\sigma$  in a single regression rather than using two separate univariate regressions.

Using the first principal component (PC) of changes in expected consumption growth and changes in consumption growth uncertainty, we construct a third measure of long-run macro risk that aggregates information across first and second moments. The first PC is positively correlated with changes in expected consumption growth and negatively correlated with changes in consumption growth uncertainty. We find the corresponding risk loadings  $\beta_i^{pc}$  by regressing sovereign bond excess returns on the first PC each quarter using rolling 20-quarter windows.

For methodological consistency, we also measure a country's short-run macro risk using sovereign bond returns. Consistent with our model, sovereign bond prices should respond to high-frequency (i.e., quarterly) global consumption shocks, as these are correlated with local output shocks. Each quarter, using the most recent 20 observations, we run time-series regressions of sovereign bond excess returns on changes in (realized) consumption growth and collect the slope coefficients  $\beta_i^c$ . We measure consumption as real non-durables goods plus services expenditures from the Bureau of Economic Analysis.

Table 5 reports the risk loadings  $\beta_i^\mu$ ,  $\beta_i^\sigma$ ,  $\beta_i^{pc}$  and  $\beta_i^c$  by country computed using the full sample period, given each country's data availability. Averaging across countries, we find that bond excess returns tend to be positively associated with fluctuations in both expected consumption growth ( $\beta^\mu = 1.13$ ) and the first PC ( $\beta^{pc} = 1.96$ ), but negatively associated to variations in consumption uncertainty ( $\beta^\sigma = -8.14$ ), in line with our theory. However, the average risk loading associated with realized consumption growth is negative ( $\beta^c = -1.10$ ). Most importantly, risk loadings vary substantially in the cross-section, which is crucial to test our theoretical predictions.

Figure 6 illustrates how long-run macro risk varies over time and in the cross-section. Panel A displays the 20-quarter rolling-sample (rather than full-sample)  $\beta_i^{pc}$  averaged across all countries. Panel

<sup>14</sup>The Survey of Professional Forecasters data is collected roughly at the middle of each calendar quarter and made public shortly thereafter. Hence, we must ensure that our sovereign return data is properly aligned with survey data. For example, our data for the second quarter span sovereign bond returns over March, April, and May, which are matched to changes in survey forecasts from about February 15 to about May 15.

B depicts the time series of 20-quarter rolling-sample  $\beta_i^{pc}$  for the 10 countries with data available throughout the entire sample period. The latter plot shows strong cross-sectional variation in the degree of long-run macro risk and that the ranking of countries is relatively stable over time, indicating that the risk loadings capture intrinsic country-level characteristics.

Figure 6 [about here]

Finally, Figure 7 plots full sample average bond excess returns against full sample  $\beta_i^{pc}$ . As in Figure 4, because countries enter and exit the sample over time, there are four scatter plots with different 'full sample' lengths. In all cases, the relationship between bond excess returns and  $\beta_i^{pc}$  is clearly positive. Therefore, as the theory predicts, higher long-run macro risk measured by regression-based risk loadings is associated with higher average excess returns.

Figure 7 [about here]

### 5.3 Portfolio sorting and main results

We now test our theoretical predictions using a portfolio sorting approach. This approach allows us to use data for all individual countries without discarding any sovereign return data. This, along with the use of survey data, contributes to increasing the statistical power of our empirical analysis. Moreover, the portfolio sorting procedure, standard in empirical asset pricing, has the attractive interpretation of representing implementable trading strategies.

Each quarter we build equally-weighted bond portfolios by sorting countries according to their risk loadings  $\beta_i^\mu$ ,  $\beta_i^\sigma$ ,  $\beta_i^{pc}$  or  $\beta_i^c$ . First we use an independent double sorting approach to classify countries as 'High', 'Medium', or 'Low' long-run macro risk. Specifically, the 'High' long-run macro risk portfolio contains countries with  $\beta_i^\mu$  above the cross-sectional median *and*  $\beta_i^\sigma$  below the cross-sectional median. These are countries whose sovereign bond prices decrease the most when recessions occur, i.e., when expected consumption decreases and consumption growth uncertainty increases. The 'Low' portfolio contains countries whose sovereign bond prices decrease the least when recessions occur. That is, countries with  $\beta_i^\mu$  below the median and  $\beta_i^\sigma$  above the median. The 'Medium' portfolio has the remaining countries. In addition to independent double-sorting, we also form portfolios based on separate single sorts on  $\beta_i^\mu$ ,  $\beta_i^\sigma$ ,  $\beta_i^{pc}$ , and  $\beta_i^c$ . In all cases, portfolios are formed every quarter but held over a one-year investment period. As a result, our sample has 74 partially overlapping one-year returns in the 1999Q1:2018Q2 period. We compute *t*-statistics based on [Newey and West \(1987\)](#) standard errors to account for the overlap. Table 6 presents our main empirical results.

Table 6 [about here]

Panel A.I of Table 6 shows that sovereign bond excess returns increase monotonically with long-run macro risk. The 'High' portfolio has constituent countries with correspondingly high average values for  $\beta_i^\mu$  (1.39) and low average values for  $\beta_i^\sigma$  (-12.10).<sup>15</sup> Such large level of long-run macro risk is compensated with significant subsequent average excess returns, equal to 14.55% per year. The 'Medium' portfolio average excess returns equal to 9.23% per year. The 'Low' portfolio has the lowest average excess return, equal to 6.44% per year. The 'High' minus 'Low' (HML) long-short portfolio earns on average 8.11% per year, which is economically large and statistically significant at the 1% level. Figure 8 illustrates the cumulative performance of the three portfolios.

Figure 8 [about here]

In Panel A.II of Table 6, we report single-sorts using the risk loadings on the first PC of changes in expected consumption growth and consumption growth uncertainty ( $\beta_i^{pc}$ ), instead of an independent double sort on  $\beta_i^\mu$  and  $\beta_i^\sigma$ . Again, sovereign bond risk premia increase monotonically with long-run macro risk. The average return of the HML strategy is 7.22% per year, with a  $t$ -statistic of 3.02. These results are consistent with the positive relation between full-sample  $\beta_i^{pc}$  and average excess returns illustrated in Figure 7.

In Panel B, we single sort countries by each long-run macro risk component ( $\beta_i^\mu$  and  $\beta_i^\sigma$ ) separately and observe similar results. The HML strategy that goes long countries with relatively low  $\beta_i^\sigma$  and shorts countries with relatively high  $\beta_i^\sigma$  earns average subsequent returns of 9.25% per year, with a  $t$ -statistic of 3.74. Similar, but less strikingly, the strategy that buys countries with high  $\beta_i^\mu$  and shorts countries with low  $\beta_i^\mu$  earns 4.17% per year on average, with a marginally significant  $t$ -statistic of 1.79. These results indicate that both sources of long-run macro risk contribute to explaining cross-sectional differences in sovereign bond excess returns.

In contrast, Panel C of Table 6 shows that the bond risk premium associated with short-run risk is negligible. Thus, even though there exists some cross-sectional variation in the risk loadings  $\beta_i^c$ , such variation does not translate into consistent variation in excess returns across portfolios. To see that, the HML strategy produces returns that have the opposite sign predicted by theory and are economically small and statistically insignificant ( $t$ -statistic of -0.67).

<sup>15</sup>In Table 6, we report pre-sort betas. In untabulated regressions, we compute the post-sort betas of the three sovereign bond portfolios by regressing their excess returns on contemporaneous changes in expected consumption growth and consumption growth uncertainty. The post-sort betas of the 'Low', 'Medium', and 'High' portfolios are, respectively, 2.67, 3.65, and 6.40 for expected consumption growth and -2.70, -7.05, and -12.40 for consumption growth uncertainty. The Internet Appendix E.1 provides details on the portfolio allocation by country and the rebalancing frequency.

Overall, confirming our theoretical predictions, we find that sovereign bonds issued by countries with higher long-run macro risk deliver higher excess returns. Specifically, the premium for long-run macro risk is economically and statistically sizable (Hypothesis 1) and arises through a country's exposure to both expected consumption growth and consumption growth uncertainty (Hypothesis 2). In addition, we show that the premium for short-run macro risk is empirically non-existent, i.e., long-run macro risk dominates short-run macro risk in driving the bond risk premium (Hypothesis 3). Therefore, the data provide strong empirical support for the properties of the bond risk premium predicted by our theory.

## 5.4 Robustness analysis

In this section, we verify the robustness of our findings by changing the empirical approach, controlling for existing explanations, and using alternative measures of long-run macro risk.

### 5.4.1 Methodological changes

We first show that the positive relation between average excess returns and long-run macro risk is robust to various methodological changes. We change four different features of the methodology and find that in all cases: i) average sovereign bond excess returns continue to increase monotonically with long-run macro risk; ii) the average return of the long-short HML strategy remains statistically significant. Table 7 reports the results.

Table 7 [about here]

Panel I of Table 7 shows that our findings are not sensitive to reducing the investment holding period from one year to one quarter. The average return of the HML strategy is 2.08% per quarter, with a  $t$ -statistic of 2.75. In Panel II, the length of the rolling window for estimating risk loadings  $\beta_i^\mu$  and  $\beta_i^\sigma$  is reduced from 20 quarters to 12 quarters. Accordingly, the sample period increases from 1999:Q1-2018:Q2 to 1997:Q1-2018:Q2. The HML return remains economically large (10.01% per year), with a  $t$ -statistic of 2.23. In Panel III, we change the estimation of the risk loadings  $\beta_i^\mu$  and  $\beta_i^\sigma$ . Rather than running two separate univariate regressions, we run multivariate regressions and estimate  $\beta_i^\mu$  and  $\beta_i^\sigma$  jointly. The average return of the HML strategy is 6.59% per year, with a  $t$ -statistic of 2.77. In Panel IV, we use value- as opposed to equal-weighted portfolios. The value-weights data are from JP Morgan and are based on bonds' market capitalizations. The return of the value-weighted HML strategy is 5.27% per year, with a  $t$ -statistic equal to 1.96. This finding suggests that our results are not driven by countries that play an economically negligible role in investors' portfolios. In untabulated



results, we compute GDP-weighted returns and find that, while the magnitude of HML return decreases to 2.99% per year because larger countries tend to display lower average sovereign bond returns, the result remains statistically significant with a  $t$ -statistic equal to 2.12.

#### 5.4.2 Controlling for other factors

Our theory predicts that risk premia in sovereign bonds primarily stem from exposure to low-frequency changes in global macroeconomic conditions, i.e., long-run macro risk. In this section, we investigate whether our empirical results may be related to alternative explanations. Specifically, we evaluate the robustness of our results by adding controls to the rolling time-series regressions used to compute the risk loadings  $\beta_i^\mu$  and  $\beta_i^\sigma$ . Table 8 reports the results.

Table 8 [about here]

First, we check that the pricing implications of long-run macro risk do not depend on whether we account for short-run macro risk when estimating the risk loadings. In Panel I, we estimate both  $\beta_i^\mu$  and  $\beta_i^\sigma$  in regressions that control for contemporaneous consumption growth. Next, in Panels II and III, we separately control for two variables used by [Borri and Verdelhan \(2012\)](#): excess returns on the S&P 500 index and excess returns on the Bank of America Merrill Lynch BBB Corporate Bond Index. We ensure that the returns on such indices are contemporary to the returns on the sovereign bond portfolios in the rolling regressions. [Borri and Verdelhan \(2012\)](#) construct six sovereign bond portfolios and find that their betas with respect to these indices line up well with the portfolios' corresponding average excess returns. Results in Panels I through III show that our results are robust to such controls. In all cases, average excess returns increase with long-run macro risk. The return of the HML strategy is also economically large and statistically significant. Therefore, our long-run macro risk measures are not just proxying for short-run macro risk or CAPM-like betas.<sup>16</sup>

Finally, we check how our consumption-based findings relate to intermediary asset pricing theory. [He, Kelly and Manela \(2017\)](#) find that their betas with respect to shocks to the capital of financial market intermediaries can explain [Borri and Verdelhan \(2012\)](#)'s portfolio average excess returns. Using data from Asaf Manela's website, we re-estimate  $\beta_i^\mu$  and  $\beta_i^\sigma$  while controlling for shocks to the intermediary

<sup>16</sup>Using data from Adrien Verdelhan's website, we also compare our baseline long-short portfolio returns to one constructed from [Borri and Verdelhan \(2012\)](#). Over the same period (1999:Q1 to 2010:Q2), our average HML return was 12.0% per year, slightly below the 13.4% per year of the zero-cost strategy that goes long [Borri and Verdelhan \(2012\)](#)'s portfolio 6 and short portfolio 1. The standard deviations of returns are respectively 20.5% and 22.3% per year. The correlation between the two long-short returns is 0.44. This moderate correlation suggests that the underlying sources of risk premia in the two long-short strategies are neither fully independent nor fully overlapping.

capital risk factor. Results in Panel IV of Table 8 show that bond excess returns continue to increase with long-run macro risk monotonically. However, the distribution of returns is compressed: the average excess returns of the 'Low' portfolio increases, while the average excess return of the 'High' portfolio decreases relative to the baseline results in Panel I of Table 6. At the same time, the dispersion in pre-sort risk loadings is halved. Nonetheless, the HML return of 5.37% per year still is economically sizable and marginally statistically significant ( $t$ -statistic of 1.85). We can thus conclude that, even though there is some overlap between our consumption-based long-run macro risk explanation and [He, Kelly and Manela \(2017\)](#)'s intermediary capital story, our results are not subsumed by theirs.

### 5.4.3 Alternative measures of long-run macro risk

We now use different proxies for expected consumption growth and consumption growth uncertainty, thus far measured from the Survey of Professional Forecasters. As an alternative to expected consumption growth, we rely on [Harvey's \(1988; 1989\)](#) insight that the term structure of default-free interest rates contain information about future economic growth. We follow [Harvey \(1989\)](#) and consider the one-quarter lagged slope of the term structure (10- minus 2-year Treasury rates from FRED) as a measure of expected consumption growth. Besides, as an alternative to consumption growth uncertainty, we use the macroeconomic uncertainty index of [Jurado, Ludvigson and Ng \(2015\)](#).<sup>17</sup> We now measure a country's long-run macro risk with  $\beta_i^s$  (exposure to changes in the lagged yield curve slope) and  $\beta_i^u$  (exposure to changes in the macroeconomic uncertainty index) coefficients, which we obtain with rolling time series regressions of sovereign bond returns using 20-quarter windows. Table 9 contains the results.

Table 9 [about here]

Panel A.I of Table 9 displays results of double sorting based on the risk loadings  $\beta_i^s$  and  $\beta_i^u$ . As before, the 'Low' portfolio has countries whose sovereign bond returns decrease the least when global macroeconomic conditions deteriorate, that is, those with  $\beta_i^s$  below the median and  $\beta_i^u$  above the median; the 'High' portfolio contains countries whose sovereign bond returns decrease the most when global macroeconomic conditions deteriorate, i.e.,  $\beta_i^s$  above the median and  $\beta_i^u$  below the median. Average excess returns monotonically increase in long-run macro risk, and the average return of the HML long-short strategy is 7.07% per year, with a  $t$ -statistic of 3.34.

<sup>17</sup>The uncertainty index of [Jurado, Ludvigson and Ng \(2015\)](#) has been used in empirical asset pricing by [Bali, Brown and Tang \(2017\)](#) to study the cross-section of average stock returns and by [Bali, Subrahmanyam and Wen \(2021\)](#) to investigate the cross-section of average corporate bond returns. This index is available on Sydney Ludvigson's website.

Panel A.II single-sorts on an aggregate measure of long-run macro risk. We construct the first PC of four time series: changes in expected consumption growth and in consumption growth uncertainty, both measured from the Survey of Professional Forecasters, one-quarter lagged slope of the yield curve, and the macroeconomic uncertainty index. The correlations of the first PC with the four aforementioned time series are, respectively, 0.74, -0.75, 0.40, and -0.66. Each quarter we estimate the corresponding risk loading  $\beta_i^{pcall}$  based on this first PC using 20-quarter windows. Countries are sorted into tercile portfolios. Panel A.II of Table 9 shows that the average return of the HML strategy is 9.56% per year, with a  $t$ -statistic equal to 3.83.

Finally, Panel B reports single sorting results. When sorting countries on  $\beta_i^s$ , the risk loading based on lagged changes in the yield curve slope, the HML return is 4.17% per year with a  $t$ -statistic of 2.14 (Column I). When sorting on  $\beta_i^u$ , the sensitivity to changes in the macroeconomic uncertainty index, the HML long-short strategy has an average return equal to 8.41% per year, with a  $t$ -statistic of 3.96 (Column II). Confirming our previous findings, average excess returns increase monotonically with long-run macro risk, as our theory predicts.

Overall, the results in this section demonstrate that our empirical results are not sensitive to how we measure a country's exposure to expected consumption growth and consumption growth uncertainty. Replacing the baseline consumption series from forecast data by reasonable alternative measures, we obtain equally strong results. Thus, our empirical tests provide robust evidence that long-run macro risk plays a central role in explaining cross-sectional differences in average sovereign bond excess returns.

## 5.5 Fama-MacBeth regressions

In this section, we depart from the portfolio sorting approach and focus on Fama-MacBeth regressions. Each quarter we regress future one-year excess returns onto the different risk loadings and then compute the average slope coefficients across such quarterly regressions. We report  $t$ -statistics based on Newey and West (1987) standard errors with 3 lags to account for the time overlap.

Note that there are two concerns related to the Fama-MacBeth methodology in our analysis. First, the number of countries in our cross-section is sometimes small. Second, the risk loadings are estimated in a sample of 20 quarters only. Because of such small sample issues, our Fama-MacBeth estimates should be taken with a grain of salt, as they may be sensitive to outliers. Nonetheless, we present the results in Table 10 for completeness.

Table 10 [about here]

We run five Fama-MacBeth regressions. In Column I of Table 10, we show that the average coefficient on  $\beta_i^\mu$  in univariate regressions is positive, in line with our portfolio sorts, but the coefficient is statistically insignificant. By contrast, Column II reports an average coefficient on  $\beta_i^\sigma$  that is negative and statistically significant with a  $t$ -statistic of -3.85, in line with our previous findings. The conclusion remains similar when running regressions of excess returns on both  $\beta_i^\mu$  and  $\beta_i^\sigma$  simultaneously, as presented in Column III.

In Column IV, we run univariate regressions of bond excess returns on  $\beta_i^{pc}$ , the slope coefficient on the first PC of changes in expected consumption growth and consumption growth uncertainty. Recall that such component varies pro-cyclically. In line with our portfolio sorts, the average coefficient on  $\beta_i^{pc}$  is positive, with a  $t$ -statistic of 3.01. In Column V, we exploit  $\beta_i^{pc-all}$ , which is the risk loading to the first PC of changes in expected consumption growth, changes in consumption growth uncertainty, changes in the one-quarter lagged slope of the yield curve, and changes in the macroeconomic uncertainty index of [Jurado, Ludvigson and Ng \(2015\)](#). The coefficient on  $\beta_i^{pc-all}$  is 2.39, with a  $t$ -statistic equal to 3.32. Furthermore, we find that a one-standard-deviation increase in  $\beta_i^{pc}$  ( $\beta_i^{pc-all}$ ) translates into an increase in average excess returns of 7.4% (7.2%) per year, which is economically sizable. Thus, we conclude that, despite the potential shortcomings of the Fama-MacBeth methodology in our setting, results are broadly consistent with the portfolio sorts and the theory.

## 6 Conclusion

This paper links risk premia embedded in sovereign bonds to macroeconomic risk exposure, both theoretically and empirically. While the correlation with high-frequency consumption shocks is a well-established source of consumption-based risk premia, our model features a novel source of risk: a country's exposure to slowly-moving changes in consumption growth moments occurring when the global business cycle shifts between expansions and recessions. We label this exposure to global macroeconomic conditions 'long-run macro risk'. We show that regime changes in global macroeconomic conditions generate higher and countercyclical default risk and introduce an additional source of risk premium in sovereign bonds. Specifically, bonds of countries experiencing lower economic growth and higher uncertainty during global recessions are particularly risky and must offer a higher expected excess return. In our calibration, **long-run macro risk accounts for 97% of the sovereign bond risk premium.**

We test the model's cross-sectional predictions using sovereign bond data for 43 countries from 1994:Q1 to 2018:Q2. We determine the degree of long-run macro risk of a country by the sensitivity

of its bond excess returns to changes in the first and second moments of U.S. consumption growth, measured with data from the Survey of Professional Forecasters. After sorting countries into tercile portfolios based on such exposures, we find that average bond excess returns increase monotonically with long-run macro risk. A long-short strategy that buys sovereign bonds with high exposure and shorts sovereign bonds with low exposure earns an economically high and statistically significant return of 8.11% per year. This finding is robust to several methodological changes, including the consideration of alternative proxies for consumption growth moments such as the lagged slope of the yield curve ([Harvey, 1989](#)) and the macroeconomic uncertainty index of [Jurado, Ludvigson and Ng \(2015\)](#).

While the canonical consumption-based model fails empirically for emerging market sovereign bonds, as it does for U.S. equities, we identify a significant relationship between a country's expected bond excess return and its exposure to aggregate consumption growth moments. Macroeconomic risk thus plays a fundamental role in our understanding of the cross-section of sovereign bonds. Future research may investigate whether exposure to regime shifts in global macroeconomic conditions helps explain the cross-section of equity and currency returns in emerging markets.

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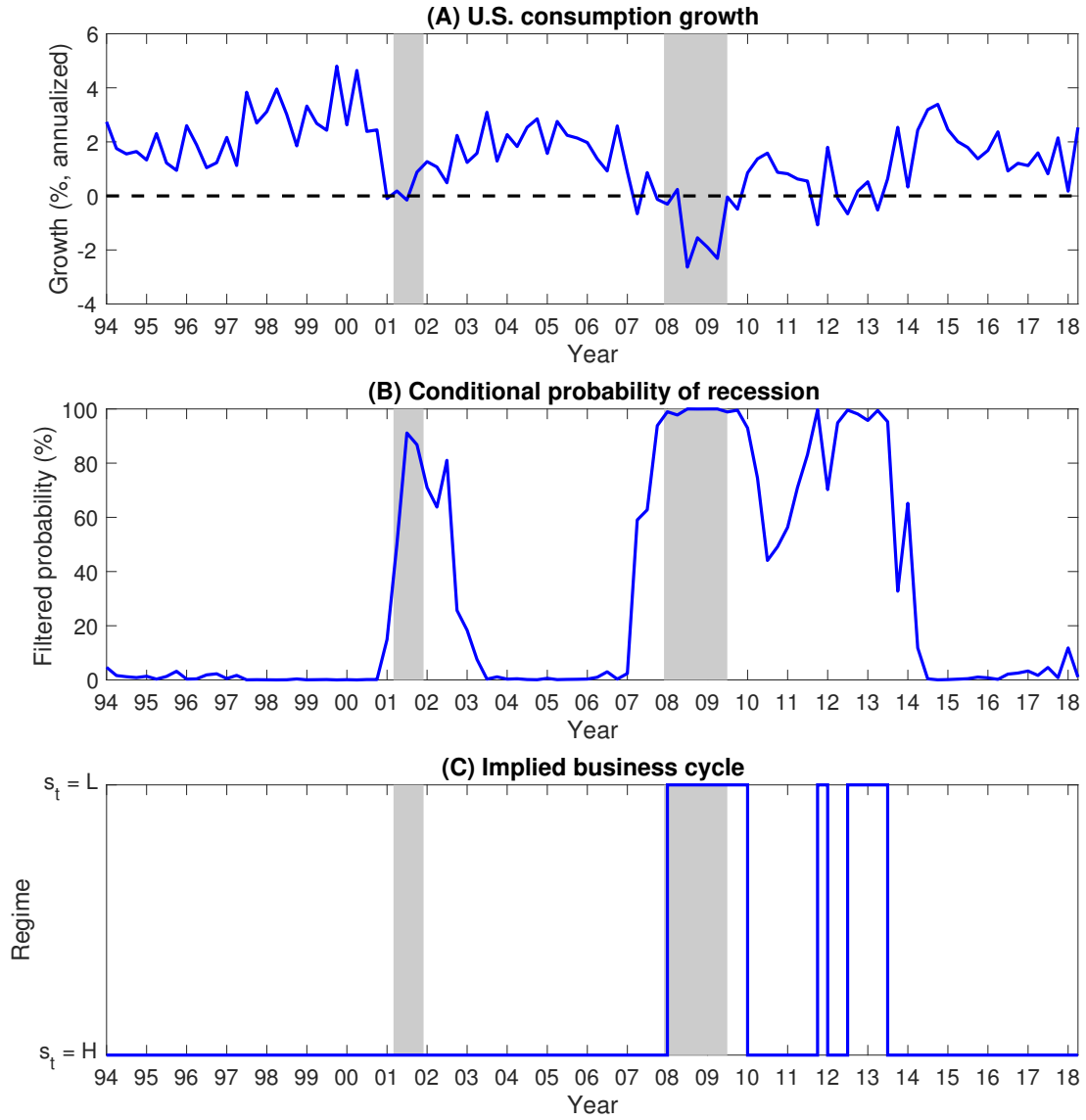


Figure 2: **Global consumption and the business cycle, 1994–2018.**

The figure illustrates the time variation of global consumption and the business cycle dates used in the model calibration. Panel A displays the annualized U.S. consumption growth rate. Panel B displays the probability of recession estimated from a two-state Markov-regime switching model on U.S. consumption data. Consumption is defined as real non-durables goods plus service consumption expenditures from the Bureau of Economic Analysis. Grey areas denote NBER recessions. Data span the 1994:Q1–2018:Q2 period.

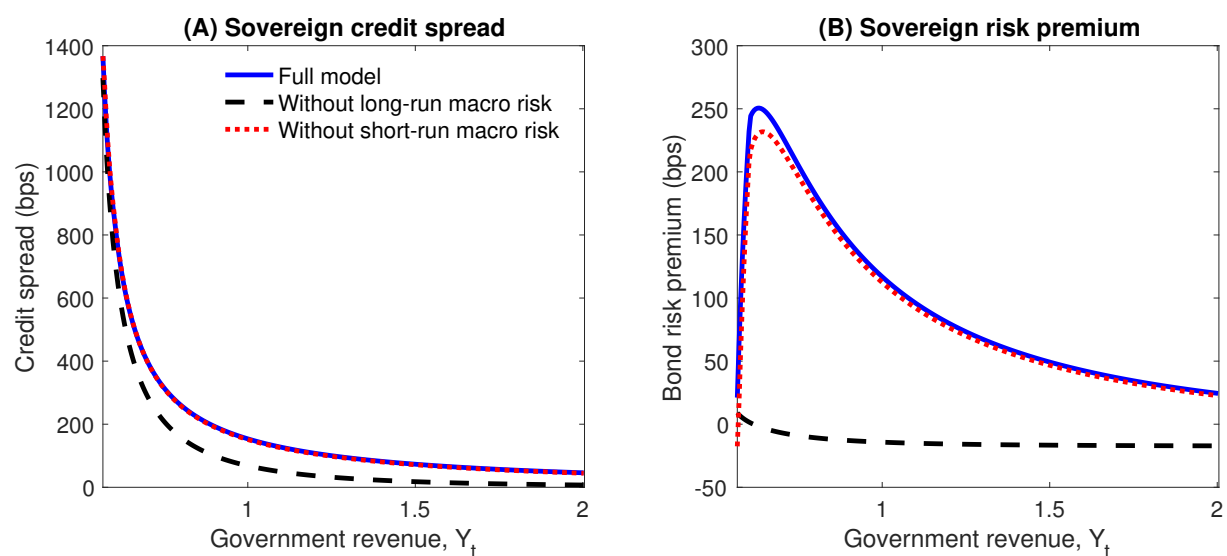


Figure 3: **Sovereign credit spread and risk premium by local economic conditions.**

The figure illustrates the impact of macroeconomic risk on the sovereign credit spread (Panel A) and the bond risk premium (Panel B). Results are reported for different levels of local economic conditions as measured by government revenue. The panels compare results of the full model with two nested cases: i) without long-run macro risk and ii) without short-run macro risk. The predictions without long-run macro risk are obtained by switching-off the country's exposure to the global business cycle (i.e., constant mean and volatility of output growth), while the model without short-run macro risk implies independent shocks between sovereign output and global consumption growth. All cases are observationally equivalent in terms of risk and imply identical unconditional output growth moments. Bond risk premium and credit spread are in basis points (bps) per annum. Unless otherwise specified the parameters are those of the baseline calibration in Table 1.

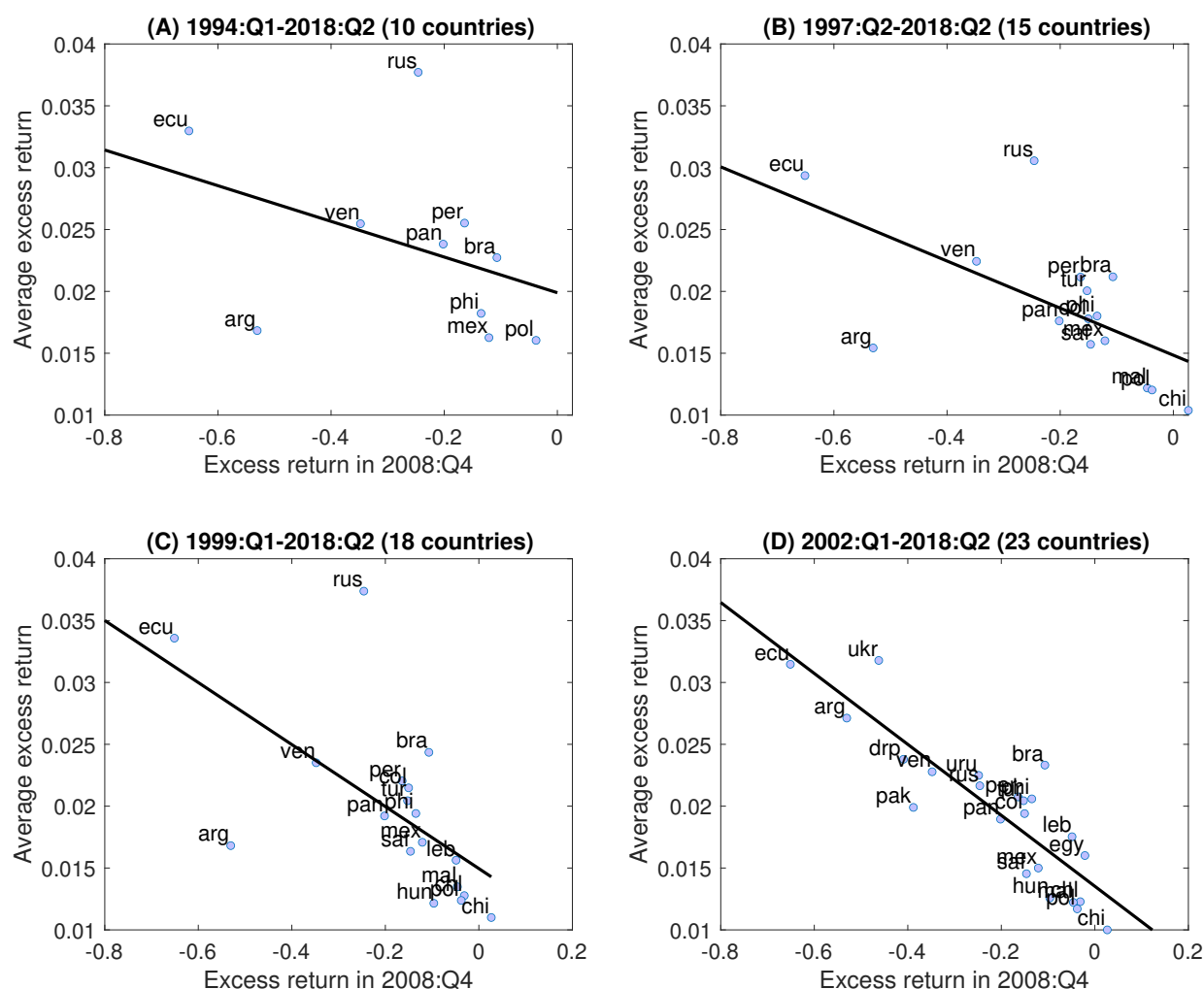


Figure 4: **Average bond excess returns and exposure to the Global Financial Crisis.**

The figure plots average quarterly bond excess returns versus bond excess returns in 2008:Q4 at the sudden worsening of the Global Financial Crisis. Average quarterly excess returns are calculated over four different samples, each of them with a different number of countries based on data availability. Best fit lines are displayed. Data span the 1999:Q1-2018:Q2 period.

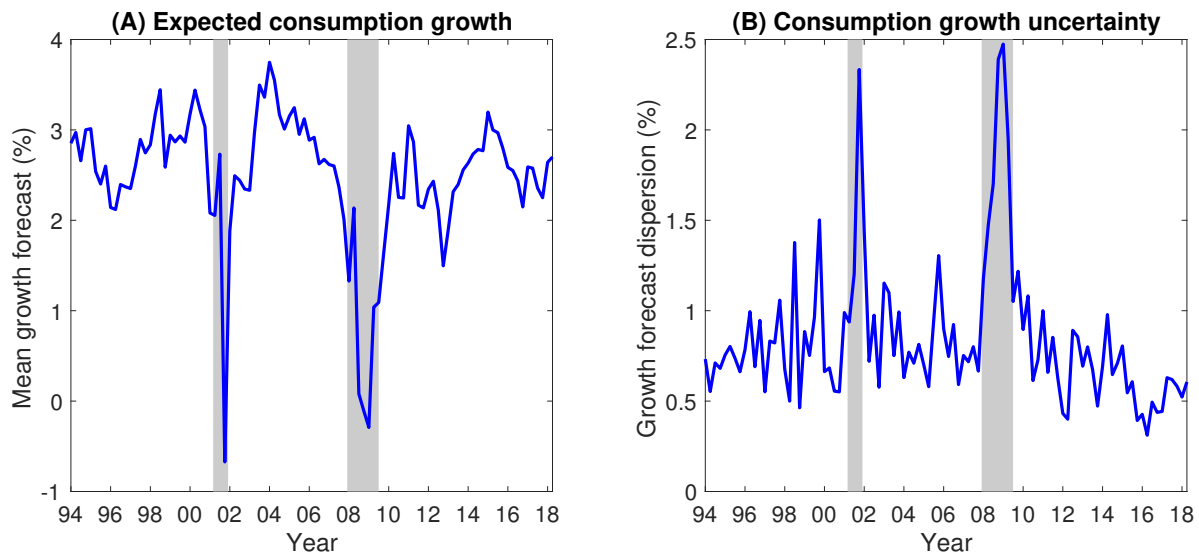


Figure 5: **Time series of consumption growth moments.**

The figure illustrates time variation in consumption growth moments. Panel A displays expected U.S. consumption growth computed as the quarterly mean of one-quarter-ahead consumption growth forecasts. Panel B displays U.S. consumption growth uncertainty defined as the quarterly cross-sectional dispersion in consumption growth forecasts. Both series are annualized and from the Survey of Professional Forecasters at the Federal Reserve Bank of Philadelphia. Grey areas denote NBER recessions. Data span the 1994:Q1-2018:Q2 period.

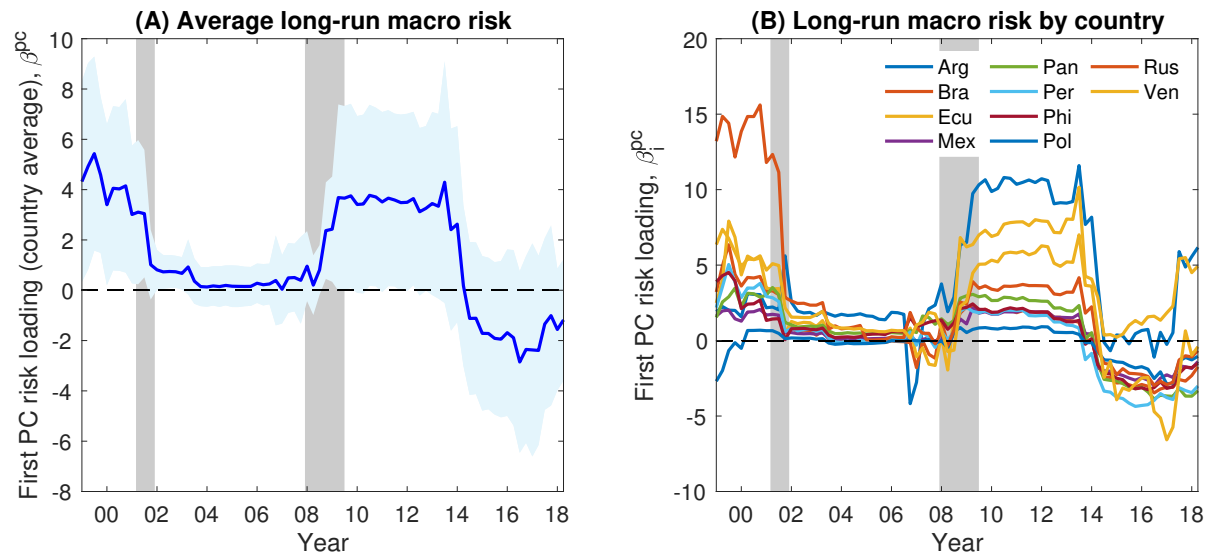


Figure 6: **Time series of long-run macro risk.**

The figure illustrates time variation in long-run macro risk as measured by the risk loading  $\beta_t^{pc}$ . Each country's  $\beta_t^{pc}$  is computed with rolling time series univariate regressions of past 20 quarterly bond excess returns on the contemporaneous first principal component of changes in expected consumption growth and consumption growth uncertainty. Panel A displays the average risk loading across all countries and one-standard-deviation bounds. Panel B displays the risk loadings for each of the 10 countries with data availability throughout the entire sample period. Grey areas denote NBER recessions. Data span the 1994:Q1-2018:Q2 period.

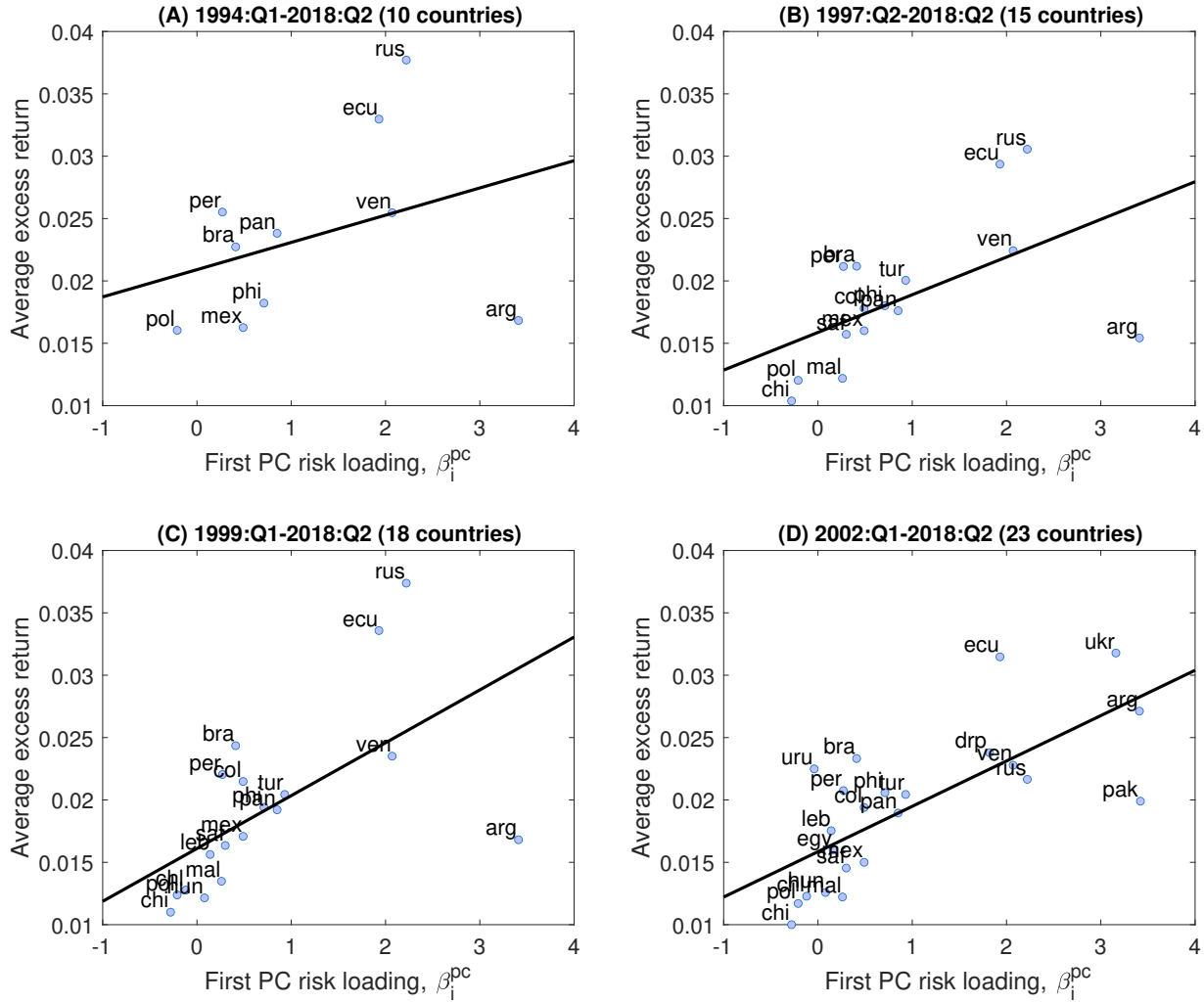


Figure 7: **Average bond excess returns and long-run macro risk.**

This figure plots average quarterly bond excess returns versus long-run macro risk as measured by the risk loading  $\beta_i^{pc}$ . Each country's  $\beta_i^{pc}$  is computed with a univariate time series regression of quarterly bond excess returns on the first principal component of changes in expected consumption growth and consumption growth uncertainty, using all data available. Average quarterly bond excess returns are calculated over four different samples, each of them with a different number of countries based on data availability. Best fit lines are displayed. Data span the 1994:Q1-2018:Q2 period.

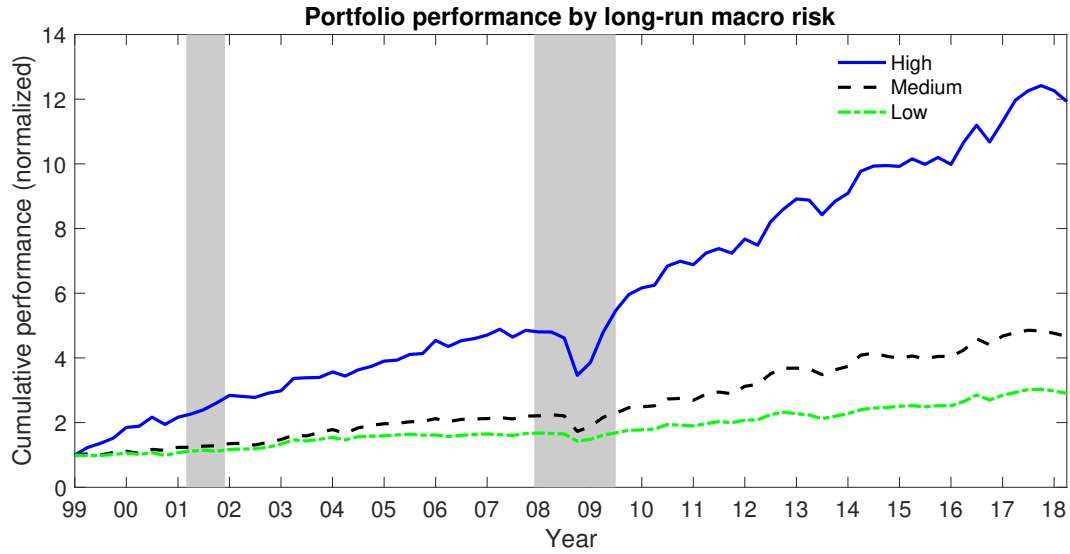


Figure 8: **Cumulative performance of trading strategies based on long-run macro risk.**

The figure shows the cumulative performance of portfolios sorted on long-run macro risk as measured by the risk loadings  $\beta_i^\mu$  and  $\beta_i^\sigma$ . These loadings are respectively computed using rolling univariate time series regressions of past 20 quarterly bond excess returns on contemporaneous changes in expected consumption growth and consumption growth uncertainty. The investment strategy is based on independent double sorts in which the 'High' ('Low') macro risk portfolio has countries with  $\beta_i^\mu$  above (below) the cross-sectional median and  $\beta_i^\sigma$  below (above) the cross-sectional median. The 'Medium' portfolio has the remaining countries. Portfolios are formed every quarter and held for one year. There are four vintages of one-year-horizon investments (one for each quarter) and the quarterly-rebalanced trading strategy displayed on the figure equally weights across the four vintages. Grey areas denote NBER recessions. Data span the 1999:Q1-2018:Q2 period.



Table 1 : **Model calibration.**

The table reports parameter values in the model calibration. The state of the global economy  $s_t = H$  refers to expansion while  $s_t = L$  corresponds to recession. The frequency of the data is quarterly and values are annualized when applicable.

Variable	Notation	Conditional values		Source
State of the global economy	$s_t$	L	H	
<i>Panel A: Global environment</i>				
Expected consumption growth (%)	$\mu_{c,s_t}$	0.059	2.048	Estimates of a Markov-switching model on U.S. consumption data (Bureau of Economic Analysis), 1994:Q1-2018:Q2
Consumption growth volatility (%)	$\sigma_{c,s_t}$	1.097	0.958	
Long-term probability (%)	$f_{s_t}$	13.53	86.47	
Convergence rate	$p$	0.467	0.467	
<i>Panel B: Agent preferences</i>				
Time preference	$\beta$	0.04	0.04	
Relative risk aversion	$\gamma$	10	10	
Elasticity of intertemporal substitution	$\psi$	2	2	
<i>Panel C: Country characteristics</i>				
Expected output growth (%)	$\mu_{X,s_t}$	1.677	4.212	Real GDP data conditioned on the Markov-switching model (Datastream), 1994:Q1-2018:Q2 Calibrated to the data
Output growth volatility (%)	$\sigma_{X,s_t}$	5.663	3.618	
Correlation with consumption	$\rho_{s_t}$	0.064	0.034	
Leverage	$\eta$	4	4	
Return on public investment (%)	$r_g$	3.870	3.870	Set to unconditional output growth
Default costs (fraction of output)	$\alpha$	0.050	0.050	<a href="#">Mendoza and Yue (2012)</a>
Debt reduction in default	$\kappa$	0.750	0.750	ISDA's CDS pricing convention

Table 2 : **Model predictions under optimal policies.**

The table reports the bond risk premium, credit spread, debt coupon value, and default thresholds in a model with optimal debt and default policies. Column A reports predictions for the full model, while Column B contains the predictions of a model that switches off a country's exposure to the global business cycle (i.e.,  $\mu_{X,L} = \mu_{X,H}, \sigma_{X,L} = \sigma_{X,H}$ ). Both cases imply identical unconditional output growth moments. Column C displays the predictions of a model in which output shocks are independent from consumption shocks (i.e.,  $\rho_{s_t} = 0$ ). Panel A has unconditional results, while Panels B and C have results conditional on the global economy being in recession or expansion. Bond risk premium and credit spread are in basis points (bps) per annum. We use baseline calibration parameters on Table 1 and report predictions for levels of government revenue observed at issuance time ( $Y = 1$ ).

	<b>Full model</b>	<b>No business cycle exposure</b>	<b>Independent output shocks</b>
	(A)	(B)	(C)
<i>Panel A: Unconditional</i>			
Bond risk premium (bps)	117	-14	112
Credit spread (bps)	154	68	151
Coupon	0.138	0.308	0.143
Default threshold	0.572	0.565	0.574
<i>Panel B: Recession</i>			
Bond risk premium (bps)	366	-41	349
Credit spread (bps)	276	67	272
Default threshold	0.603	0.562	0.604
<i>Panel C: Expansion</i>			
Bond risk premium (bps)	78	-10	75
Credit spread (bps)	134	68	132
Default threshold	0.568	0.565	0.569

Table 3 : **Sovereign risk premium decomposition.**

The table reports predictions for the bond risk premium and its decomposition by source of macroeconomic risk. Column A displays the risk premium associated with a country's exposure to low-frequency but severe changes in consumption growth (long-run macro risk). Column B displays the risk premium associated with a country's exposure to small but frequent consumption growth shocks (short-run macro risk). Column C shows the total bond risk premium of the full model. Column D reports the bond risk premium in a observationally-equivalent 'restricted model' without long-run macro risk. In such model the country's exposure to the global business cycle is switched-off (i.e.,  $\mu_{X,L} = \mu_{X,H}$ ,  $\sigma_{X,L} = \sigma_{X,H}$ ), but its output volatility and output-consumption correlation match their unconditional counterparts of the baseline model. Results in Panel A are unconditional while those in Panels B and C are conditioned on the global economy being in recession or expansion. Bond risk premium is in basis points (bps) per annum. We use baseline calibration parameters on Table 1 and report predictions for levels of government revenue observed at issuance time ( $Y = 1$ ).

	I. Full model			II. Restricted model
	Long-run macro risk (A)	Short-run macro risk (B)	Total (C)	Short-run macro risk (D)
<i>Panel A: Unconditional</i>				
Bond risk premium (bps)	113	4	117	10
Decomposition (%)	97	3		
<i>Panel B: Recession</i>				
Bond risk premium (bps)	352	14	366	12
Decomposition (%)	96	4		
<i>Panel C: Expansion</i>				
Bond risk premium (bps)	76	2	78	10
Decomposition (%)	97	3		

Table 4 : **Sovereign risk premium and cross-sectional predictions.**

The table reports cross-sectional predictions on the bond risk premium. Each value arises from a different combination of exposure to the various sources of macroeconomic risk. Panel A shows the bond risk premium for different levels of sovereign output-global consumption correlation ( $\rho_{st}$ ). In Panel B, the rows show the premium for different exposures of expected output growth rate to global expected consumption growth ( $\phi_\mu$ ), while the columns show the premium for different exposures of output growth volatility to global consumption volatility ( $\phi_\sigma$ ). Bond risk premium is in basis points (bps) per annum. We use baseline calibration parameters on Table 1 and report predictions for levels of government revenue observed at issuance time ( $Y = 1$ ).

*Panel A: Premium for short-run macro risk (bps)*

Output-consumption correlation, $\rho_{st}$				
-0.20	-0.10	0	0.10	0.20
-17	-8	0	8	17

*Panel B: Premium for long-run macro risk (bps)*

		Volatility exposure, $\phi_\sigma$			
		0.5	1	1.5	2
Expected growth exposure, $\phi_\mu$	0.5	48	65	78	95
	1	102	117	130	139
	1.5	160	172	183	193
	2	219	228	236	242

Table 5 : **Descriptive statistics.**

The table reports descriptive statistics for JP Morgan EMBI Global excess returns and estimated risk loadings by country. Excess returns are per quarter and expressed in percentage points. The risk loadings  $\beta_i^\mu$ ,  $\beta_i^\sigma$ ,  $\beta_i^{pc}$ , and  $\beta_i^c$  respectively represent the exposure of a country's bond excess returns to contemporaneous changes in expected consumption growth, consumption growth uncertainty, their first principal component, and realized consumption growth. These risk loadings are estimated using univariate regressions as discussed in Section 5.3. The full sample period is 1994:Q-2018:Q2, but countries enter and exit the sample at different times.

Country	Sample period	Excess returns (% per quarter)		Risk loadings (full sample)			
		Mean	Std	$\beta_i^\mu$	$\beta_i^\sigma$	$\beta_i^{pc}$	$\beta_i^c$
Argentina	94Q1:18Q2	1.68	14.05	3.65	-12.93	3.41	4.24
Belize	07Q2:18Q2	2.47	17.91	8.36	-26.15	8.38	8.41
Brazil	94Q1:18Q2	2.27	7.86	-0.75	-3.61	0.41	0.90
Bulgaria	94Q1:14Q1	2.70	8.81	-1.36	-7.70	0.91	0.83
Chile	99Q3:18Q2	1.28	2.91	-0.38	0.04	-0.12	-0.60
China	94Q2:18Q2	1.09	2.51	-0.99	-0.15	-0.28	-0.89
Colombia	97Q2:18Q2	1.78	5.97	-0.47	-3.63	0.49	-1.48
Cote d'Ivoire	98Q3:08Q2, 10Q3:18Q2	3.44	21.19	-1.70	-26.66	4.20	-15.33
Croatia	96Q3:04Q3, 09Q4:18Q2	1.31	4.99	-1.05	-3.32	0.26	-1.01
Dominican Rep.	02Q1:18Q2	2.38	9.22	1.54	-7.60	1.82	-0.14
Ecuador	94Q1:18Q2	3.30	15.29	-0.49	-11.76	1.93	0.24
Egypt	01Q3:18Q2	1.56	4.43	-0.03	-1.20	0.16	1.06
El Salvador	02Q3:18Q2	1.76	6.67	2.14	-6.37	2.12	0.69
Gabon	08Q1:18Q2	2.40	10.10	5.80	-19.04	6.20	-2.94
Georgia	08Q3:18Q2	2.65	7.92	8.18	-14.13	6.21	-2.96
Ghana	08Q1:18Q2	2.87	10.45	4.08	-18.27	5.17	-1.02
Hungary	99Q2:18Q2	1.18	3.69	0.18	-0.18	0.08	-0.47
Indonesia	04Q3:18Q2	2.12	7.48	2.25	-10.82	3.14	0.07
Iraq	06Q2:18Q2	3.01	11.32	4.80	-11.46	4.09	-4.35
Jamaica	08Q1:18Q2	2.72	9.55	6.79	-15.16	5.56	3.92
Kazakhstan	07Q3:18Q2	2.05	9.27	6.93	-14.43	5.51	-1.68
Korea	94Q1:04Q2	1.15	5.63	-1.49	-3.06	0.06	-4.46
Lebanon	98Q3:18Q2	1.55	3.25	0.19	-0.46	0.14	-1.13

Continued.

Country	Sample period	Excess returns (% per quarter)		Risk loadings (full sample)			
		Mean	Std	$\beta_i^\mu$	$\beta_i^\sigma$	$\beta_i^{pc}$	$\beta_i^c$
Lithuania	10Q1:17Q1	1.97	3.46	-1.67	-0.85	-0.46	1.01
Malaysia	97Q1:18Q2	1.17	4.38	-0.88	-2.99	0.26	-1.23
Mexico	94Q1:18Q2	1.63	5.32	-0.04	-2.82	0.49	-0.09
Morocco	94Q1:06Q4, 13Q1:18Q2	1.47	6.04	-0.13	-5.24	0.87	0.30
Nigeria	94Q1:07Q1, 11Q2:18Q2	3.25	9.79	-3.43	-5.13	-0.09	-4.77
Pakistan	01Q3:18Q2	2.01	11.89	5.07	-12.99	3.42	-4.84
Panama	94Q1:18Q2	2.38	7.60	-0.35	-5.39	0.85	-0.37
Peru	94Q1:18Q2	2.55	8.49	-1.42	-3.99	0.27	-0.17
Philippines	94Q1:18Q2	1.82	5.46	-0.51	-4.91	0.71	-1.58
Poland	94Q1:18Q2	1.60	5.14	-1.37	-1.21	-0.21	0.18
Russia	94Q1:18Q2	3.77	14.10	0.35	-11.97	2.22	0.51
Serbia	05Q3:18Q2	1.65	6.76	3.06	-9.65	3.20	-0.22
South Africa	95Q1:18Q2	1.59	4.59	-0.81	-3.13	0.30	-1.43
Sri Lanka	08Q1:18Q2	2.44	9.56	3.88	-14.14	4.27	-4.91
Thailand	97Q3:06Q2	1.43	9.07	-2.37	-8.09	0.63	-5.39
Turkey	96Q3:18Q2	2.00	7.36	-1.11	-7.18	0.93	-0.85
Ukraine	00Q3:18Q2	3.83	17.65	4.10	-13.33	3.16	-5.66
Uruguay	01Q3:18Q2	2.22	9.93	-0.15	-0.04	-0.04	1.75
Venezuela	94Q1:18Q2	2.55	11.80	0.09	-11.58	2.07	-0.53
Vietnam	06Q1:18Q2	1.48	4.89	0.02	-7.54	1.60	-0.83
<i>Country average</i>		<i>2.14</i>	<i>8.49</i>	<i>1.13</i>	<i>-8.14</i>	<i>1.96</i>	<i>-1.10</i>
<i>Pooled average</i>		<i>2.13</i>	<i>9.41</i>	<i>0.39</i>	<i>-6.95</i>	<i>1.33</i>	<i>-0.88</i>

Table 6 : **Main empirical results.**

The table reports statistics for equally-weighted sovereign bond portfolios sorted on macroeconomic risk, as measured by the risk loadings  $\beta_i^\mu$ ,  $\beta_i^\sigma$ ,  $\beta_i^{pc}$ , and  $\beta_i^c$ . These loadings are respectively computed using rolling univariate time series regressions of past 20 quarterly excess returns on contemporaneous changes in expected consumption growth and consumption growth uncertainty (Panels A.I and B), their first principal component (Panel A.II), and realized consumption growth (Panel C). Panel A.I reports results of independent double sorts in which the 'High' ('Low') macro risk portfolio has countries with  $\beta_i^\mu$  above (below) the cross-sectional median and  $\beta_i^\sigma$  below (above) the cross-sectional median. The 'Medium' portfolio has the remaining countries. Panels A.II, B, and C report results of single sorts based on  $\beta_i^{pc}$ ,  $\beta_i^\mu$ ,  $\beta_i^\sigma$ , or  $\beta_i^c$  terciles. Portfolios are formed every quarter and held for one year. The table shows each portfolio's average excess returns in percentage points per year and the average risk loadings of the portfolio's constituent countries. 'HML' denotes a zero-cost strategy that is long the 'High' risk portfolio and short the 'Low' risk one. The sample period is 1999:Q1-2018:Q2. We report *t*-statistics based on [Newey and West \(1987\)](#) standard errors with 3 lags. Statistical significance at the 10%, 5%, and 1% levels is denoted by \*, \*\*, and \*\*\* respectively.

Panel A: Sorting on both long-run macro risk components ( I:  $\beta^\mu$  and  $\beta^\sigma$ ; II:  $\beta^{pc}$ )

	I. Double sorting				II. Sorting on first principal component			
	Low	Medium	High	HML	Low	Medium	High	HML
$\beta^\mu$ (I) or $\beta^{pc}$ (II)	-3.68	-3.61	1.39	<b>5.07</b>	-0.44	0.94	3.98	<b>4.42</b>
$\beta^\sigma$	-1.57	-4.99	-12.10	<b>-10.53</b>				
Exc. return	6.44	9.23	14.55	<b>8.11***</b>	6.94	7.67	14.15	<b>7.22***</b>
( <i>t</i> -stat)				<b>(2.75)</b>				<b>(3.02)</b>

Panel B: Single sorting on each long-run macro risk component ( $\beta^\mu$  or  $\beta^\sigma$ )

	I. Expected consumption growth				II. Consumption growth uncertainty			
	Low	Medium	High	HML	Low	Medium	High	HML
$\beta^\mu$ (I) or $\beta^\sigma$ (II)	-5.77	-2.22	2.12	<b>7.89</b>	-0.43	-4.92	-13.66	<b>-13.16</b>
Exc. return	8.82	6.77	12.99	<b>4.17*</b>	6.24	7.13	15.49	<b>9.25***</b>
( <i>t</i> -stat)				<b>(1.79)</b>				<b>(3.74)</b>

Panel C: Single sorting on short-run macro risk ( $\beta^c$ )

	Low	Medium	High	HML
$\beta^c$	-6.67	-1.81	3.20	<b>9.87</b>
Exc. return	11.78	6.62	9.90	<b>-1.88</b>
( <i>t</i> -stat)				<b>(-0.67)</b>

Table 7 : **Robustness analysis - methodological changes.**

The table reports statistics for sovereign bond portfolios sorted on long-run macro risk under alternative specifications. The risk loadings  $\beta_i^\mu$  and  $\beta_i^\sigma$  are respectively computed using rolling time series regressions of past quarterly excess returns on contemporaneous changes in expected consumption growth and consumption growth uncertainty. The table reports results of quarterly independent double sorts in which the 'High' ('Low') risk portfolio has countries with  $\beta_i^\mu$  above (below) the cross-sectional median and  $\beta_i^\sigma$  below (above) the cross-sectional median. The 'Medium' portfolio has the remaining countries. Panel I uses a one-quarter rather than a one-year holding period. Panel II computes risk loadings using 12 quarters of data rather than 20. Panel III estimates the risk loadings  $\beta_i^\mu$  and  $\beta_i^\sigma$  jointly rather than individually. Panel IV reports results using value-weighted rather than equal-weighted portfolios. The table shows each portfolio's average excess returns in percentage points per year and the average risk loadings of the portfolio's constituent countries. 'HML' denotes a zero-cost strategy that is long the 'High' risk portfolio and short the 'Low' risk one. The sample period is 1997:Q1-2018:Q2 in Panel II and 1999:Q1-2018:Q2 in Panels I, III, IV and V. We report  $t$ -statistics based on [Newey and West \(1987\)](#) standard errors with 3 lags in Panels II to V. Statistical significance at the 10%, 5%, and 1% levels is denoted by \*, \*\*, and \*\*\* respectively.

	I. Quarterly holding period				II. Estimation over 12 quarters			
	Low	Medium	High	HML	Low	Medium	High	HML
$\beta^\mu$	-3.55	-3.41	1.57	<b>5.12</b>	-6.31	-4.76	0.74	<b>4.32</b>
$\beta^\sigma$	-1.17	-4.56	-11.62	<b>-10.45</b>	0.41	-3.66	-10.23	<b>-6.88</b>
Exc. return ( $t$ -stat)	1.22	2.26	3.30	<b>2.08*** (2.75)</b>	5.37	9.07	15.46	<b>10.01** (2.23)</b>
	III. Joint estimation of risk loadings				IV. Value-weighted portfolios			
	Low	Medium	High	HML	Low	Medium	High	HML
$\beta^\mu$	-3.65	-3.26	1.26	<b>7.91</b>	-3.68	-3.61	1.39	<b>5.07</b>
$\beta^\sigma$	-1.46	-5.05	-10.90	<b>-9.44</b>	-1.57	-4.99	-12.10	<b>-10.53</b>
Exc. return ( $t$ -stat)	6.55	8.97	13.13	<b>6.59*** (2.77)</b>	7.00	7.24	12.27	<b>5.27** (1.96)</b>



Table 8 : **Robustness analysis - adding controls.**

The table reports statistics for sovereign bond portfolios sorted on long-run macro risk using various controls. The risk loadings  $\beta_i^\mu$  and  $\beta_i^\sigma$  are respectively computed using rolling time series regressions of past 20 quarterly excess returns on contemporaneous changes in expected consumption growth and consumption growth uncertainty. Each panel reports results considering a different contemporaneous control variable. The controls are realized consumption growth (Panel I), S&P 500 excess returns (Panel II), Bank of America Merrill Lynch BBB Corporate Bond Index excess returns (Panel III), and [He, Kelly and Manela \(2017\)](#)'s intermediary capital risk factor (Panel IV). The table reports results of independent double sorts in which the 'High' ('Low') risk portfolio has countries with  $\beta_i^\mu$  above (below) the cross-sectional median and  $\beta_i^\sigma$  below (above) the cross-sectional median. The 'Medium' portfolio has the remaining countries. Portfolios are formed every quarter and held for one year. The table shows each portfolio's average excess returns in percentage points per year and the average risk loadings of the portfolio's constituent countries. 'HML' denotes a zero-cost strategy that is long the 'High' risk portfolio and short the 'Low' risk portfolio. The sample period is 1999:Q1-2018:Q2. We report *t*-statistics based on [Newey and West \(1987\)](#) standard errors with 3 lags. Statistical significance at the 10%, 5%, and 1% levels is denoted by \*, \*\*, and \*\*\* respectively.

	I. Control for consumption growth				II. Control for stock returns			
	Low	Medium	High	HML	Low	Medium	High	HML
$\beta^\mu$	-3.83	-3.50	1.75	<b>5.58</b>	-4.05	-4.01	1.28	<b>4.36</b>
$\beta^\sigma$	-1.33	-5.16	-12.02	<b>-10.69</b>	-1.18	-4.91	-11.68	<b>-10.50</b>
Exc. return	6.39	9.67	14.43	<b>8.04***</b>	7.33	8.83	14.44	<b>7.11**</b>
( <i>t</i> -stat)				<b>(2.88)</b>				<b>(2.34)</b>
	III. Control for corporate bond returns				IV. Control for intermediary capital factor			
	Low	Medium	High	HML	Low	Medium	High	HML
$\beta^\mu$	-3.67	-3.21	2.01	<b>5.68</b>	-3.46	-2.09	1.99	<b>5.07</b>
$\beta^\sigma$	-0.92	-4.47	-11.35	<b>-10.43</b>	1.21	-1.01	-5.93	<b>-4.72</b>
Exc. return	6.02	9.27	15.24	<b>9.22***</b>	7.71	8.91	13.09	<b>5.37*</b>
( <i>t</i> -stat)				<b>(3.07)</b>				<b>(1.85)</b>

Table 9 : **Alternative long-run macro risk measures.**

The table reports statistics for equally-weighted sovereign bond portfolios sorted on alternative measures of long-run macro risk, denoted by the risk loadings  $\beta_i^s$ ,  $\beta_i^u$ , and  $\beta_i^{pcall}$ . The loadings  $\beta_i^s$  and  $\beta_i^u$  are respectively computed using rolling univariate time series regressions of past 20 quarterly excess returns on contemporaneous changes in the one-quarter-lagged slope of the yield curve (10-year minus 2-year Treasury rates) and in the macroeconomic uncertainty index of [Jurado, Ludvigson and Ng \(2015\)](#). The loading  $\beta_i^{pcall}$  uses the first principal component of changes in expected consumption growth, consumption growth uncertainty, the lagged slope of the yield curve, and the macroeconomic uncertainty index of [Jurado, Ludvigson and Ng \(2015\)](#). Panel A.I reports results of independent double sorts in which the 'High' ('Low') long-run macro risk portfolio has countries with  $\beta_i^s$  above (below) the cross-sectional median and  $\beta_i^u$  below (above) the cross-sectional median. The 'Medium' portfolio has the remaining countries. Panel A.II and Panel B report results of single sorts based on  $\beta_i^{pcall}$ ,  $\beta_i^s$ , or  $\beta_i^u$  terciles. Portfolios are formed every quarter and held for one year. The table shows each portfolio's average excess returns in percentage points per year and the average risk loadings of the portfolio's constituent countries. 'HML' denotes a zero-cost strategy that is long the 'High' long-run macro risk portfolio and short the 'Low' long-run macro risk one. The sample period is 1999:Q1-2018:Q2. We report  $t$ -statistics based on [Newey and West \(1987\)](#) standard errors with 3 lags. Statistical significance at the 10%, 5%, and 1% levels is denoted by \*, \*\*, and \*\*\* respectively.

*Panel A: Sorting on multiple long-run macro risk proxies (I:  $\beta^s$  and  $\beta^u$ , II:  $\beta^{pcall}$ )*

	I. Double sorting				II. Sorting on first principal component			
	Low	Medium	High	HML	Low	Medium	High	HML
$\beta^s$	-0.24	3.24	8.26	<b>8.50</b>				
$\beta^u$	-0.35	-1.24	-2.04	<b>-1.69</b>				
$\beta^{pcall}$					0.17	1.48	4.08	<b>3.91</b>
Exc. return	5.62	10.14	12.69	<b>7.07***</b>	5.77	7.83	15.33	<b>9.56***</b>
( $t$ -stat)				<b>(3.34)</b>				<b>(3.83)</b>

*Panel B: Single sorting on each long-run macro risk proxy ( $\beta^s$  or  $\beta^u$ )*

	I. Lagged yield curve slope				II. Macroeconomic uncertainty index			
	Low	Medium	High	HML	Low	Medium	High	HML
$\beta^s$	-1.01	3.50	9.24	<b>8.23</b>				
$\beta^u$					-0.27	-1.02	-2.43	<b>-2.16</b>
Exc. return	8.44	7.56	12.61	<b>4.17**</b>	6.49	7.50	14.91	<b>8.41***</b>
( $t$ -stat)				<b>(2.14)</b>				<b>(3.96)</b>

Table 10 : **Fama-MacBeth regressions.**

The table reports results of Fama-MacBeth regressions. Each quarter we run regressions of future one-year excess returns on sovereign bonds onto risk loadings  $\beta_i^\mu$ ,  $\beta_i^\sigma$ ,  $\beta_i^{pc}$ , or  $\beta_i^{pc_{all}}$ . These loadings are respectively computed using rolling time series regressions of past 20 quarterly excess returns on changes in expected consumption growth (I) or consumption growth uncertainty (II), on changes in both moments (III), on the first principal component of the changes in these two moments (IV), and on the first principal component of changes in these two moments, the lagged slope of the yield curve (10-year minus 2-year Treasury rates), and the macroeconomic uncertainty index of [Jurado, Ludvigson and Ng \(2015\)](#). The sample period is 1999:Q1-2018:Q2. We report  $t$ -statistics based on [Newey and West \(1987\)](#) standard errors with 3 lags. Statistical significance at the 10%, 5%, and 1% levels is denoted by \*, \*\*, and \*\*\* respectively.

	I. Expected growth	II. Consumption uncertainty	III. Expected growth & consumption uncertainty	IV. Principal component	V. Principal component (all series)
$\beta^\mu$ ( $t$ -stat)	0.45 (0.74)		-0.21 (-0.46)		
$\beta^\sigma$ ( $t$ -stat)		-0.70*** (-3.85)	-0.77*** (-3.63)		
$\beta^{pc}$ ( $t$ -stat)				2.03*** (3.01)	
$\beta^{pc_{all}}$ ( $t$ -stat)					2.39*** (3.32)

Internet Appendix to

**“Sovereign Risk Premia and Global Macroeconomic  
Conditions”**

[Not for publication]

This Internet Appendix presents supplementary material and results not included in the main body of the paper.

## A Theory

This Appendix provides details on the model derivation. We determine all claims and endogenous variables according to the state of the global economy, which can be in expansion ( $s_t = H$ ) or in recession ( $s_t = L$ ). The country  $i$  subscripts are dropped for convenience.

### A.1 State-price density and equilibrium risk-free rate

This section closely follows [Bhamra, Kuehn and Strebulaev \(2010b\)](#) and describes the state-price density and the equilibrium risk-free rate. The state-price density is initially derived by [Duffie and Skiadas \(1994\)](#) for the general class of stochastic differential utility function proposed by [Duffie and Epstein \(1992\)](#). This utility function decouples the agent's risk aversion from her preference for intertemporal resolution of the uncertainty. The coefficient of relative risk aversion is  $\gamma$ , the elasticity of intertemporal substitution is  $\psi$ , and the subjective time discount factor is  $\beta$ .

The representative agent's state-price density  $\pi_t$  when  $\psi \neq 1$  is given by

$$\pi_t = \left( \beta e^{-\beta t} \right)^{\frac{1-\gamma}{1-\frac{1}{\psi}}} C_t^{-\gamma} \left( p_{C,s_t} e^{\int_0^t p_{C,s_u}^{-1} du} \right)^{-\frac{\gamma-\frac{1}{\psi}}{1-\frac{1}{\psi}}}, \quad (\text{A.1})$$

where  $C_t$  is the agent's consumption and  $p_{C,s_t}$  is the price-consumption ratio in state  $s_t$ . The latter satisfies the following system of implicit non-linear equations:

$$p_{C,s_t}^{-1} = \bar{r}_{s_t} - \mu_{c,s_t} + \gamma \sigma_{c,s_t}^2 - \left( 1 - \frac{1}{\psi} \right) \lambda_{s_t} \left( \frac{\left( \frac{p_{C,j}}{p_{C,s_t}} \right)^{\frac{1-\gamma}{1-\frac{1}{\psi}}} - 1}{1-\gamma} \right), \quad s_t, j \in \{L, H\}, j \neq s_t \quad (\text{A.2})$$

with

$$\bar{r}_{s_t} = \beta + \frac{1}{\psi} \mu_{c,s_t} - \frac{1}{2} \gamma \left( 1 + \frac{1}{\psi} \right) \sigma_{c,s_t}^2, \quad (\text{A.3})$$

where  $\mu_{c,s_t}$  and  $\sigma_{c,s_t}$  denote the first and second conditional moments of consumption growth.

The dynamics of the state-price density  $\pi_t$  follows the stochastic differential equation:

$$\frac{d\pi_t}{\pi_t} = -r_{s_t}dt - \frac{dM_t}{M_t} \quad (\text{A.4})$$

$$= -r_{s_t}dt - \Theta_{s_t}^B dB_t - \Theta_{s_t}^P dN_{s_t,t}, \quad (\text{A.5})$$

where  $M$  is a martingale under the physical measure,  $N_{s_t,t}$  a Poisson process which jumps upwards by one whenever the state of the global economy  $s_t = \{L, H\}$  switches,  $\Theta_{s_t}^P = 1 - \Delta_{s_t}$  is the market price of risk due to Poisson shocks when the global economy switches out of state  $s_t$ , and  $\Theta_{s_t}^B = \gamma\sigma_{c,s_t}$  is the market price of risk due to Brownian shocks in state  $s_t$ . The risk distortion factors are such that  $\Delta_H = \Delta_L^{-1}$ , where  $\Delta_H$  is the solution to  $G(\Delta_H) = 0$  with

$$G(x) = x^{-\frac{1-\frac{1}{\psi}}{\gamma-1}} - \frac{\bar{r}_H + \gamma\sigma_{c,H}^2 - \mu_{c,H} + \lambda_H \frac{1-\frac{1}{\psi}}{\gamma-1} \left( x^{\frac{\gamma-1}{\gamma-1-\psi}} - 1 \right)}{\bar{r}_L + \gamma\sigma_{c,L}^2 - \mu_{c,L} + \lambda_L \frac{1-\frac{1}{\psi}}{\gamma-1} \left( x^{-\frac{\gamma-1}{\gamma-1-\psi}} - 1 \right)}, \quad \psi \neq 1. \quad (\text{A.6})$$

Finally, the equilibrium instantaneous risk-free rate  $r_{s_t}$  is given by

$$r_{s_t} = \begin{cases} \bar{r}_L + \lambda_L \left[ \frac{\gamma-\frac{1}{\psi}}{1-\gamma} \left( \Delta_H^{-\frac{\gamma-1}{\gamma-1-\psi}} - 1 \right) - (\Delta_H^{-1} - 1) \right], & s_t = L \\ \bar{r}_H + \lambda_H \left[ \frac{\gamma-\frac{1}{\psi}}{1-\gamma} \left( \Delta_H^{\frac{\gamma-1}{\gamma-1-\psi}} - 1 \right) - (\Delta_H - 1) \right], & s_t = H. \end{cases} \quad (\text{A.7})$$

## A.2 Sovereign bond valuation and credit spread

The sovereign bond value, denoted by  $B_{s_t}(Y_t)$  when the current state is  $s_t$ , is determined by

$$B_{s_t}(Y_t) = \mathbb{E}_t \left[ \int_t^{t_D} c \frac{\pi_u}{\pi_t} du \middle| s_t \right] + \mathbb{E}_t \left[ \int_{t_D}^{\infty} (1 - \kappa) c \frac{\pi_u}{\pi_t} du \middle| s_t \right] \quad (\text{A.8})$$

$$= \mathbb{E}_t \left[ \int_t^{\infty} c \frac{\pi_u}{\pi_t} du \middle| s_t \right] - \mathbb{E}_t \left[ \frac{\pi_{t_D}}{\pi_t} \int_{t_D}^{\infty} \kappa c \frac{\pi_u}{\pi_{t_D}} du \middle| s_t \right], \quad (\text{A.9})$$

where  $c$  is the perpetual debt coupon,  $\kappa$  is the debt haircut in default, and  $t_D$  is the unknown default time. The first term of Equation (A.9) represents a risk-free claim that delivers  $c$  in every period. It corresponds to the value of a perpetual risk-free bond which equals

$$\mathbb{E}_t \left[ \int_t^{\infty} c \frac{\pi_u}{\pi_t} du \middle| s_t \right] = \frac{c}{r_{B,s_t}}, \quad (\text{A.10})$$

where  $r_{B,s_t}$  is the discount rate for a riskless perpetuity when the current state is  $s_t$  and is given by

$$r_{B,s_t} = r_{s_t} + \frac{r_j - r_{s_t}}{\hat{p} + r_j} \hat{p} \hat{f}_j, \quad j \neq s_t; \quad j, s_t = \{L, H\}. \quad (\text{A.11})$$

The discount rate  $r_{B,H}$  is lower than the corresponding instantaneous risk-free rate  $r_H$  (and analogously  $r_{B,L}$  is higher than  $r_L$ ) because the risk-free rate is expected to change in the future with the state of the global economy.

The second part of Equation (A.9) is given by

$$\mathbb{E}_t \left[ \frac{\pi_{t_D}}{\pi_t} \int_{t_D}^{\infty} \kappa c \frac{\pi_u}{\pi_{t_D}} du \middle| s_t \right] = \sum_{s_D} \mathbb{E}_t \left[ \text{Prob}(s_D | s_t) \frac{\pi_{t_D}}{\pi_t} \int_{t_D}^{\infty} \kappa c \frac{\pi_u}{\pi_{t_D}} du \middle| s_t \right] \quad (\text{A.12})$$

$$= \sum_{s_D} \mathbb{E}_t \left[ \text{Prob}(s_D | s_t) \frac{\pi_{t_D}}{\pi_t} \middle| s_t \right] \mathbb{E}_t \left[ \int_{t_D}^{\infty} \kappa c \frac{\pi_u}{\pi_{t_D}} du \middle| s_{t_D} \right] \quad (\text{A.13})$$

$$= \sum_{s_D} \frac{\kappa c}{r_{B,s_D}} q_{s_t s_D}(Y_t), \quad (\text{A.14})$$

where  $s_D \in \{L, H\}$  is the state at the time of default and the summation over  $s_D$  indicates that a default can occur in state  $s_D = L$  or state  $s_D = H$ . Given the state-price density is Markovian, Equation (A.12) can be separated into two parts, as shown in Equation (A.13). The first term of Equation (A.13) is equal to

$$\mathbb{E}_t \left[ \text{Prob}(s_D | s_t) \frac{\pi_{t_D}}{\pi_t} \middle| s_t \right] = q_{s_t s_D}(Y_t), \quad (\text{A.15})$$

which is the Arrow-Debreu claim paying one unit of consumption at the default time  $t_D$  when the current state is  $s_t$ , and denoted by  $q_{s_t s_D}(Y_t)$ . The second term of Equation (A.13) is the value, at default time, of a claim which pays  $\kappa c$  in perpetuity and whose discount rate is  $r_{B,s_D}$ . It is thus equal to  $\frac{\kappa c}{r_{B,s_D}}$ .

Combining Equations (A.9), (A.10), and (A.14), the sovereign bond value is equal to

$$B_{s_t}(Y_t) = \frac{c}{r_{B,s_t}} - \sum_{s_D} \frac{\kappa c}{r_{B,s_D}} q_{s_t s_D}(Y_t), \quad s_t, s_D = \{L, H\}. \quad (\text{A.16})$$

The sovereign credit spread that the agent requires for holding the country's government bond when

the current state is  $s_t$  is determined as follows:

$$CS_{s_t}(Y_t) = \frac{c}{B_{s_t}(Y_t)} - r_{B,s_t} \quad (\text{A.17})$$

$$= \frac{1}{\left[ \frac{1}{r_{B,s_t}} - \sum_{s_D} \frac{\kappa}{r_{B,s_D}} q_{s_t s_D}(Y_t) \right]} - r_{B,s_t} \quad (\text{A.18})$$

$$= r_{B,s_t} \left[ \frac{1}{1 - \sum_{s_D} \kappa \frac{r_{B,s_t}}{r_{B,s_D}} q_{s_t s_D}(Y_t)} - 1 \right], \quad s_t, s_D = \{L, H\}. \quad (\text{A.19})$$

The probability of sovereign default, over an horizon  $T$  and within a given state  $s_t$ , is given by:

$$P \left( \inf_{0 \leq t \leq T} Y_t \leq Y_{D,s_t} \middle| Y_0 > Y_{D,s_t} \right) = \Phi \left( \frac{\ln \left( \frac{Y_{D,s_t}}{Y_0} \right) - \left( \mu_{Y,s_t} - \frac{\sigma_{Y,s_t}^2}{2} \right) T}{\sigma_{Y,s_t} \sqrt{T}} \right) \quad (\text{A.20})$$

$$+ \left( \frac{Y_{D,s_t}}{Y_0} \right)^{\frac{2\mu_{Y,s_t}}{\sigma_{Y,s_t}^2} - 1} \Phi \left( \frac{\ln \left( \frac{Y_{D,s_t}}{Y_0} \right) + \left( \mu_{Y,s_t} - \frac{\sigma_{Y,s_t}^2}{2} \right) T}{\sigma_{Y,s_t} \sqrt{T}} \right),$$

where the conditional expected growth rate of government revenue is  $\mu_{Y,s_t}$ , the conditional volatility is  $\sigma_{Y,s_t}$ , and  $\Phi(\cdot)$  is the cumulative density function of a standard normal distribution.

### A.3 Arrow-Debreu default claims

This section derives the two kinds of Arrow-Debreu default claims used to price securities. The first kind of Arrow-Debreu claims captures a default triggered by the country's government revenue continuously falling below a default threshold within a given state. It is given by

$$q_{s_t s_D} = \mathbb{E}_t \left[ \frac{\pi_{tD}}{\pi_t} \text{Prob}(s_D | s_t) \middle| s_t \right]. \quad (\text{A.21})$$

The second kind of Arrow-Debreu claims accounts for a default arising from a sudden change in the state of the global economy, even if the level of the country's government revenue remains unchanged. This situation can occur when the global economy is in the economic state with the lower default threshold and switches to the other state, such that the default threshold instantaneously increases to a higher level. If the level of the country's government revenue was above the initial default threshold, but below the new default threshold, there is a sudden default. This second kind of Arrow-Debreu claims is



given by

$$q'_{s_t s_D} = \mathbb{E}_t \left[ \frac{\pi_{t_D}}{\pi_t} \frac{Y_{t_D}}{Y_{D,s_D}} \text{Prob}(s_D | s_t) \middle| s_t \right]. \quad (\text{A.22})$$

### A.3.1 First kind

The Arrow-Debreu default claim  $q_{s_t s_D}$  is the time- $t$  value of a security that pays one unit of consumption at the moment of default  $t_D$ , where  $s_t$  represents the current state of the global economy, and  $s_D$  the state at the default time. The time of default is the first time that the government revenue of the country falls to the threshold  $Y_{D,s_D}$ . By definition, this Arrow-Debreu claim is given by

$$q_{s_t s_D} = \mathbb{E}_t \left[ \frac{\pi_{t_D}}{\pi_t} \text{Prob}(s_D | s_t) \middle| s_t \right], \quad (\text{A.23})$$

which solves the pair of ordinary differential equations (ODE):

$$\frac{1}{2} \sigma_{Y,s_t}^2 Y^2 \frac{d^2 q_{s_t s_D}}{dY^2} + \mu_{Y,s_t} Y \frac{dq_{s_t s_D}}{dY} + \hat{\lambda}_{s_t} (q_{j s_D} - q_{s_t s_D}) - r_{s_t} q_{s_t s_D} = 0, \quad (\text{A.24})$$

where  $\mu_{Y,s_t}$  and  $\sigma_{Y,s_t}$  denote the expected growth rate and the volatility of government revenue in state  $s_t$  and  $\hat{\lambda}_{s_t}$  is the risk-neutral probability of leaving state  $s_t$ , with  $j \neq s_t$  and  $j, s_t = \{L, H\}$ .

The above ODEs are obtained by applying Ito's Lemma to the classical non-arbitrage condition

$$\mathbb{E}_t^{\mathbb{Q}} [dq_{s_t s_D} - r_{s_t} q_{s_t s_D}] = 0. \quad (\text{A.25})$$

The Arrow-Debreu claim payoffs are such that:

$$q_{s_t s_D}(Y) = \begin{cases} 1, & s_t = s_D, \quad Y \leq Y_{D,s_t} \\ 0, & s_t \neq s_D, \quad Y \leq Y_{D,s_t} \end{cases}, \quad s_t, s_D = \{L, H\} \quad (\text{A.26})$$

Therefore, each state of the global economy is characterized by a specific default threshold. The Arrow-Debreu claims are derived in two distinct cases:  $Y_{D,H} < Y_{D,L}$  or  $Y_{D,H} > Y_{D,L}$ .

In the first case,  $Y_{D,H} < Y_{D,L}$ , the default barriers are higher in recession and lower in expansion and each of the four Arrow-Debreu claims is determined over three separate intervals:  $Y \geq Y_{D,L}$ ,  $Y_{D,L} \geq Y \geq Y_{D,H}$ , and  $Y \leq Y_{D,H}$ .

From the payoff equations we can infer the values of the four Arrow-Debreu claims in the interval

$Y \leq Y_{D,H}$ . For the interval  $Y \geq Y_{D,L}$ , we are looking for a solution of the following general form:

$$q_{sts_D}(Y) = h_{sts_D} Y^k, \quad (\text{A.27})$$

which implies that  $k$  must be a root of the quartic equation

$$\left[ \frac{1}{2} \sigma_{Y,L}^2 k(k-1) + \mu_{Y,L} k + (-\hat{\lambda}_L - r_L) \right] \left[ \frac{1}{2} \sigma_{Y,H}^2 k(k-1) + \mu_{Y,H} k + (-\hat{\lambda}_H - r_H) \right] - \hat{\lambda}_L \hat{\lambda}_H = 0. \quad (\text{A.28})$$

The Arrow-debreu claims can be written as

$$q_{sts_D}(Y) = \sum_{m=1}^4 h_{sts_D,m} Y^{k_m} \quad (\text{A.29})$$

with  $k_1, k_2 < 0$  and  $k_3, k_4 > 0$ . However, when  $Y$  goes to infinity the Arrow-Debreu claims must be null, which indicates that we should have  $h_{sts_D,3} = h_{sts_D,4} = 0$ . We then obtain

$$q_{Ls_D}(Y) = \sum_{m=1}^2 h_{Ls_D,m} Y^{k_m} \quad (\text{A.30})$$

$$q_{Hs_D}(Y) = \sum_{m=1}^2 h_{Hs_D,m} \varepsilon(k_m) Y^{k_m}, \quad (\text{A.31})$$

where

$$\varepsilon(k_m) = -\frac{\hat{\lambda}_H}{\frac{1}{2} \sigma_{Y,H}^2 k(k-1) + \mu_{Y,H} k - (\hat{\lambda}_H + r_H)} = -\frac{\frac{1}{2} \sigma_{Y,L}^2 k(k-1) + \mu_{Y,L} k - (\hat{\lambda}_L + r_L)}{\hat{\lambda}_L}. \quad (\text{A.32})$$

Finally, over the interval  $Y_{D,L} \geq Y \geq Y_{D,H}$ , both  $q_{D,LL}$  and  $q_{D,LH}$  are known from the payoffs equations and are respectively equal to 1 and 0. Then,

$$q_{HL}(Y) = \frac{\hat{\lambda}_H}{r_H + \hat{\lambda}_H} + \sum_{m=1}^2 s_{L,m} Y^{j_m} \quad (\text{A.33})$$

$$q_{HH}(Y) = \sum_{m=1}^2 s_{H,m} Y^{j_m}, \quad (\text{A.34})$$

where

$$\frac{1}{2}\sigma_{Y,H}^2 j(j-1) + \mu_{Y,H} j - (\hat{\lambda}_H + r_H) = 0 \quad (\text{A.35})$$

with  $j_1 < j_2$ .

To summarize, the four Arrow-Debreu claims can be written as follows

$$q_{LL} = \begin{cases} \sum_{m=1}^2 h_{LL,m} Y^{k_m}, & Y \geq Y_{D,L} \\ 1, & Y_{D,L} \geq Y \geq Y_{D,H} \\ 1, & Y \leq Y_{D,H} \end{cases} \quad (\text{A.36})$$

$$q_{LH} = \begin{cases} \sum_{m=1}^2 h_{LH,m} Y^{k_m}, & Y \geq Y_{D,L} \\ 0, & Y_{D,L} \geq Y \geq Y_{D,H} \\ 0, & Y \leq Y_{D,H} \end{cases} \quad (\text{A.37})$$

$$q_{HL} = \begin{cases} \sum_{m=1}^2 h_{LL,m} \varepsilon(k_m) Y^{k_m}, & Y \geq Y_{D,L} \\ \frac{\hat{\lambda}_H}{r_H + \hat{\lambda}_H} + \sum_{m=1}^2 s_{L,m} Y^{j_m}, & Y_{D,L} \geq Y \geq Y_{D,H} \\ 0, & Y \leq Y_{D,H} \end{cases} \quad (\text{A.38})$$

$$q_{HH} = \begin{cases} \sum_{m=1}^2 h_{LH,m} \varepsilon(k_m) Y^{k_m}, & Y \geq Y_{D,L} \\ \sum_{m=1}^2 s_{H,m} Y^{j_m}, & Y_{D,L} \geq Y \geq Y_{D,H} \\ 1, & Y \leq Y_{D,H}. \end{cases} \quad (\text{A.39})$$

The eight constants ( $h_{LL,1}, h_{LL,2}, h_{LH,1}, h_{LH,2}, s_{L,1}, s_{L,2}, s_{H,1}, s_{H,2}$ ) are determined by eight threshold conditions, which are

$$\begin{aligned} \lim_{Y \rightarrow Y_{D,L}} q_{LL} &= 1, \quad \lim_{Y \rightarrow Y_{D,L}} q_{LH} = 0 \\ \lim_{Y \rightarrow Y_{D,L}^+} q_{HL} &= \lim_{Y \rightarrow Y_{D,L}^-} q_{HL}, \quad \lim_{Y \rightarrow Y_{D,L}^+} q_{HH} = \lim_{Y \rightarrow Y_{D,L}^-} q_{HH} \\ \lim_{Y \rightarrow Y_{D,L}^+} \dot{q}_{HL} &= \lim_{Y \rightarrow Y_{D,L}^-} \dot{q}_{HL}, \quad \lim_{Y \rightarrow Y_{D,L}^+} \dot{q}_{HH} = \lim_{Y \rightarrow Y_{D,L}^-} \dot{q}_{HH} \\ \lim_{Y \rightarrow Y_{D,H}} q_{HL} &= 0, \quad \lim_{Y \rightarrow Y_{D,H}} q_{HH} = 1, \end{aligned}$$

where  $\dot{q}_{s_t s_D}$  denotes the derivative of  $q_{s_t s_D}$  with respect to  $Y$ .

In the second case ( $Y_{D,H} > Y_{D,L}$ ), the default barriers are higher in expansion and lower in recession and each of the four Arrow-Debreu claims is determined over three separate intervals:  $Y \geq Y_{D,H}$ ,

$Y_{D,H} \geq Y \geq Y_{D,L}$ , and  $Y \leq Y_{D,L}$ . We then obtain

$$q_{LL} = \begin{cases} \sum_{m=1}^2 h_{LL,m} Y^{k_m}, & Y \geq Y_{D,H} \\ \sum_{m=1}^2 s_{L,m} Y^{j_m}, & Y_{D,H} \geq Y \geq Y_{D,L} \\ 1, & Y \leq Y_{D,L} \end{cases} \quad (\text{A.40})$$

$$q_{LH} = \begin{cases} \sum_{m=1}^2 h_{LH,m} Y^{k_m}, & Y \geq Y_{D,H} \\ \frac{\hat{\lambda}_L}{r_L + \hat{\lambda}_L} + \sum_{m=1}^2 s_{H,m} Y^{j_m}, & Y_{D,H} \geq Y \geq Y_{D,L} \\ 0, & Y \leq Y_{D,L} \end{cases} \quad (\text{A.41})$$

$$q_{HL} = \begin{cases} \sum_{m=1}^2 h_{LL,m} \varepsilon(k_m) Y^{k_m}, & Y \geq Y_{D,H} \\ 0, & Y_{D,H} \geq Y \geq Y_{D,L} \\ 0, & Y \leq Y_{D,L} \end{cases} \quad (\text{A.42})$$

$$q_{HH} = \begin{cases} \sum_{m=1}^2 h_{LH,m} \varepsilon(k_m) Y^{k_m}, & Y \geq Y_{D,H} \\ 1, & Y_{D,H} \geq Y \geq Y_{D,L} \\ 1, & Y \leq Y_{D,L}. \end{cases} \quad (\text{A.43})$$

The eight constants  $(h_{LL,1}, h_{LL,2}, h_{LH,1}, h_{LH,2}, s_{L,1}, s_{L,2}, s_{H,1}, s_{H,2})$  are determined by the following eight threshold conditions:

$$\begin{aligned} \lim_{Y \rightarrow Y_{D,L}} q_{LL} &= 1, \quad \lim_{Y \rightarrow Y_{D,L}} q_{LH} = 0 \\ \lim_{Y \rightarrow Y_{D,H}^+} q_{LL} &= \lim_{Y \rightarrow Y_{D,H}^-} q_{LL}, \quad \lim_{Y \rightarrow Y_{D,H}^+} q_{LH} = \lim_{Y \rightarrow Y_{D,H}^-} q_{LH} \\ \lim_{Y \rightarrow Y_{D,H}^+} \dot{q}_{LL} &= \lim_{Y \rightarrow Y_{D,H}^-} \dot{q}_{LL}, \quad \lim_{Y \rightarrow Y_{D,H}^+} \dot{q}_{LH} = \lim_{Y \rightarrow Y_{D,H}^-} \dot{q}_{LH} \\ \lim_{Y \rightarrow Y_{D,H}} q_{HL} &= 0, \quad \lim_{Y \rightarrow Y_{D,H}} q_{HH} = 1. \end{aligned}$$

### A.3.2 Second kind

We use the same approach to derive the second kind of Arrow-Debreu default claims, which accounts for the possibility that a default happens when the state of the global economy suddenly switches. When

$Y_{D,H} < Y_{D,L}$  the only claim that is different from that of the first kind is  $q_{HL}$  given by

$$q'_{HL} = \begin{cases} \sum_{m=1}^2 h_{LL,m} \varepsilon(k_m) Y^{k_m}, & Y \geq Y_{D,L} \\ \frac{\hat{\lambda}_H}{r_H + \hat{\lambda}_H - \mu_{Y,H}} \frac{Y}{Y_{D,L}} + \sum_{m=1}^2 s_{L,m} Y^{j_m}, & Y_{D,L} \geq Y \geq Y_{D,H} \\ 0, & Y \leq Y_{D,H}. \end{cases} \quad (\text{A.44})$$

When  $Y_{D,H} > Y_{D,L}$  the only claim that is different from that of the first kind is  $q_{LH}$  given by

$$q'_{LH} = \begin{cases} \sum_{m=1}^2 h_{LH,m} Y^{k_m}, & Y \geq Y_{D,H} \\ \frac{\hat{\lambda}_L}{r_L + \hat{\lambda}_L - \mu_{Y,L}} \frac{Y}{Y_{D,H}} + \sum_{m=1}^2 s_{H,m} Y^{j_m}, & Y_{D,H} \geq Y \geq Y_{D,L} \\ 0, & Y \leq Y_{D,L}. \end{cases} \quad (\text{A.45})$$

## A.4 Government

This section derives the debt issuance benefits, the present value of the country's government revenue, and the country's sovereign wealth.

### A.4.1 Debt issuance benefits

The government's motivation for issuing debt is to invest internally the amount of capital raised at the time of debt issuance ( $t = 0$ ). Financing public investments yields a return  $r_g$ . The government's incentives for issuing debt, denoted by  $I_{s_t}(Y_t)$  when the state is  $s_t$  at time  $t$ , equals

$$I_{s_t}(Y_t) = \mathbb{E}_t \left[ \int_t^\infty r_g \frac{\pi_u}{\pi_t} du \middle| s_t \right] B_{s_0}(Y_0) \quad (\text{A.46})$$

$$= r_g \mathbb{E}_t \left[ \int_t^\infty \frac{\pi_u}{\pi_t} du \middle| s_t \right] B_{s_0}(Y_0) \quad (\text{A.47})$$

$$= \frac{r_g}{r_{B,s_t}} B_{s_0}(Y_0). \quad (\text{A.48})$$

#### A.4.2 Discounted government revenue

The present value of the country's government revenue, denoted by  $G_{s_t}(Y_t)$  when the current state is  $s_t$ , can be written as

$$G_{s_t}(Y_t) = \mathbb{E}_t \left[ \int_t^{t_D} Y_u \frac{\pi_u}{\pi_t} du \middle| s_t \right] + \mathbb{E}_t \left[ \int_{t_D}^{\infty} (1 - \alpha) Y_u \frac{\pi_u}{\pi_t} du \middle| s_t \right] \quad (\text{A.49})$$

$$= \mathbb{E}_t \left[ \int_t^{\infty} Y_u \frac{\pi_u}{\pi_t} du \middle| s_t \right] - \alpha \mathbb{E}_t \left[ \int_{t_D}^{\infty} Y_u \frac{\pi_u}{\pi_t} du \middle| s_t \right]. \quad (\text{A.50})$$

The first term of Equation (A.50) can be written as

$$\mathbb{E}_t \left[ \int_t^{\infty} Y_u \frac{\pi_u}{\pi_t} du \middle| s_t \right] = Y_t \mathbb{E}_t \left[ \int_t^{\infty} \frac{\pi_u}{\pi_t} \frac{Y_u}{Y_t} du \middle| s_t \right] \quad (\text{A.51})$$

$$= Y_t \frac{1}{r_{Y,s_t}}, \quad (\text{A.52})$$

where  $r_{Y,s_t}$  is the discount rate applicable to risky government revenue, given by

$$r_{Y,s_t} = r_{s_t} - \hat{\mu}_{Y,s_t} + \frac{(r_j - \hat{\mu}_{Y,j}) - (r_{s_t} - \hat{\mu}_{Y,s_t})}{\hat{p} + r_j - \hat{\mu}_{Y,j}} \hat{p} f_j, \quad j \neq s_t; \quad j, s_t = \{L, H\}, \quad (\text{A.53})$$

with  $\hat{\mu}_{Y,s_t} = \mu_{Y,s_t} - \gamma \sigma_{c,s_t} \rho_{s_t} \sigma_{Y,s_t}$  denoting the expected growth rate under the risk-neutral measure.

From the strong Markov property, we can solve for the second term of Equation (A.50), which yields

$$\mathbb{E}_t \left[ \int_{t_D}^{\infty} Y_u \frac{\pi_u}{\pi_t} du \middle| s_t \right] = \sum_{s_D} q'_{s_t s_D} (Y_t) \frac{Y_{D,s_D}}{r_{Y,s_D}}. \quad (\text{A.54})$$

Combining Equations (A.50), (A.52), and (A.54), the present value of the country's government revenue is given by

$$G_{s_t}(Y_t) = \frac{Y_t}{r_{Y,s_t}} - \alpha \sum_{s_D} \frac{Y_{D,s_D}}{r_{Y,s_D}} q'_{s_t s_D} (Y_t). \quad (\text{A.55})$$

### A.4.3 Sovereign wealth and smooth pasting conditions

Sovereign wealth is defined as the present value of government revenue plus the benefits of issuing debt.

From the derivation above, sovereign wealth  $W_{s_t}(Y_t)$  at time  $t$  and for current state  $s_t$  is given by

$$W_{s_t}(Y_t) = G_{s_t}(Y_t) + I_{s_t}(Y_t) \quad (\text{A.56})$$

$$= \frac{Y_t}{r_{Y,s_t}} - \alpha \sum_{s_D} \frac{Y_{D,s_D}}{r_{Y,s_D}} q'_{s_t s_D}(Y_t) + \frac{r_g}{r_{B,s_t}} B_{s_0}(Y_0). \quad (\text{A.57})$$

We now derive the smooth-pasting conditions that ensure continuity in the objective function at the time of default (see [Merton, 1973](#); [Dumas, 1991](#)). For convenience, we denote the net value of sovereign wealth by  $\bar{W}_{s_t}(Y_t) \equiv W_{s_t}(Y_t) - B_{s_t}(Y_t)$ . Combining Equations (A.16) and (A.57),  $\bar{W}_{s_t}(Y_t)$  is given by

$$\begin{aligned} \bar{W}_{s_t}(Y_t) &= \frac{Y_t}{r_{Y,s_t}} - \alpha \sum_{s_D} \frac{Y_{D,s_D}}{r_{Y,s_D}} q'_{s_t s_D}(Y_t) + \frac{r_g}{r_{B,s_t}} B_{s_0}(Y_0) \\ &\quad - \left[ \frac{c}{r_{B,s_t}} - \sum_{s_D} \frac{c\kappa}{r_{B,s_D}} q_{s_t s_D}(Y_t) \right]. \end{aligned} \quad (\text{A.58})$$

The smooth-pasting conditions must satisfy the following equations:

$$\left. \frac{\partial \bar{W}_{s_t}(Y_t)}{\partial Y_t} \right|_{Y_t=Y_{D,s_t}} = \frac{\partial}{\partial Y_{D,s_t}} \left( \bar{W}_{s_t}(Y_t) \Big|_{Y_t=Y_{D,s_t}} \right), \quad s_t = \{L, H\}. \quad (\text{A.59})$$

From the definition of the Arrow-Debreu claims (A.26),  $\bar{W}_{s_t}(Y_t)$  at default time is

$$\bar{W}_{s_t}(Y_t) \Big|_{Y_t=Y_{D,s_t}} = Y_{D,s_t} \frac{1-\alpha}{r_{Y,s_t}} + \frac{r_g}{r_{B,s_t}} B_{s_0}(Y_0) - \frac{(1-\kappa)c}{r_{B,s_t}} \quad (\text{A.60})$$

and the right-hand side of Equation (A.59) is thus determined by

$$\frac{\partial}{\partial Y_{D,s_t}} \left( \bar{W}_{s_t}(Y_t) \Big|_{Y_t=Y_{D,s_t}} \right) = \frac{1-\alpha}{r_{Y,s_t}}. \quad (\text{A.61})$$

Hence, the smooth-pasting conditions satisfy the pair of equations given by

$$\left. \frac{\partial \bar{W}_{s_t}(Y_t)}{\partial Y_t} \right|_{Y_t=Y_{D,s_t}} = \frac{(1-\alpha)}{r_{Y,s_t}}, \quad s_t = \{L, H\}. \quad (\text{A.62})$$

## B Model calibration

This Appendix provides details about the model calibration.

### B.1 Transition probabilities

This section describes the estimation of transition probabilities. Following [Hamilton \(1989\)](#), we estimate a two-state Markov regime-switching model for U.S. consumption growth over the 1994:Q1-2018:Q2 period. We constrain the long-run frequency of the state  $s_t = L$  to correspond to the frequency of NBER recessions observed during the postwar period (1952Q1-2018Q2), that is,  $f_L = 0.1353$  and  $f_H = 1 - f_L$ .

We denote the probability of switching from state  $i$  to state  $j$  by  $T_{ij}$ , such that the transition probability matrix is  $T = \begin{bmatrix} T_{HH} & T_{HL} \\ T_{LH} & T_{LL} \end{bmatrix}$ , with  $T_{HH} = 1 - T_{HL}$  and  $T_{LL} = 1 - T_{LH}$ . As in [Bhamra, Kuehn and Strebulaev \(2010a, 2010b\)](#), the relation between the physical long-run frequency  $f_{s_t}$  and the transition probability matrix  $T_{ij}$  is  $f_H = \left(1 + \frac{T_{HL}}{T_{LH}}\right)^{-1}$ . Hence, for a given long-run frequency  $f_H$ , we must have  $\frac{T_{HL}}{T_{LH}} = f_H^{-1} - 1$ . From the constrained maximum likelihood estimation, we obtain the following transition probability matrix:

$$T = \begin{bmatrix} 0.9851 & 0.0149 \\ 0.0953 & 0.9047 \end{bmatrix}. \quad (\text{A.63})$$

While the constraint  $f_L = 0.1353$  ensures a reasonable long-run frequency of the state  $s_t = L$ , we verify that the constrained and the unconstrained estimations are not statistically different from each other: the Likelihood-ratio test has a  $p$ -value of 0.76. Finally, the probability  $\lambda_{s_t}$  that the global economy leaves the state  $s_t \in \{L, H\}$  is given by  $\lambda_L = pf_H$  and  $\lambda_H = pf_L$ , with  $p = -4\ln\left(1 - \frac{T_{HL}}{1-f_H}\right)$ .

### B.2 Conditional output growth moments

This section estimates the conditional output growth moments of the representative sovereign bond issuer. First, we determine business cycle dates based on the filtered probability of being in recession. Figure 2 in the paper displays the time series of U.S. consumption growth used in the estimation (Panel A) and the filtered conditional probability of being in the state  $s_t = L$  (Panel B). Panel C displays the quarters when the economy is in the recession state ( $s_t = L$ ). Based on this regime categorization, we



compute the conditional moments of output growth ( $\mu_{X,i,s_t}$  and  $\sigma_{X,i,s_t}$ ) and the correlation with U.S. consumption growth ( $\rho_{i,s_t}$ ) using GDP in constant U.S. dollars. Table A.1 reports the equally-weighted and GDP-weighted moments, as well as their median, standard deviation, and the interdecile range. The GDP-weights are computed using each country's average GDP in constant U.S. dollars. Both the median and the average country (using either equal or GDP weights) display higher output growth rate in expansion than in recession ( $\mu_{X,i,H} > \mu_{X,i,L}$ ) and lower output growth volatility in expansion than in recession ( $\sigma_{X,i,L} > \sigma_{X,i,H}$ ). The correlation between output growth and U.S. consumption growth ( $\rho_{i,s_t}$ ) is small in all cases.

Table A.1 [about here]

### B.3 Estimation of the leverage parameter

This section empirically assesses the leverage parameter  $\eta$  for a set of emerging economies. Based on data availability, we consider 10 countries with different sizes and levels of economic development. The countries are Bolivia, Brazil, Bulgaria, Chile, Colombia, India, Mexico, Philippines, Russia, and South Africa. We first construct each country's time series of government revenue ( $Y_{i,t}$ ) by multiplying GDP in constant U.S. dollars (the same concept used in the output calibration) by Revenue Excluding Grants as a percentage of GDP. The latter data is from the World Bank's website. Then we compute country-level leverage proxies as the ratio of unconditional government revenue growth volatility ( $\sigma_{Y,i}$ ) to unconditional output growth volatility ( $\sigma_{X,i}$ ), as defined in Equation (6) of the model.

Table A.2 reports the results. The average volatility of government revenue growth is 8.57% and the average volatility of output growth is 2.18%. This implies a GDP-weighted (equally-weighted) average leverage of 3.80 (4.06) and a standard deviation of 1.07. The GDP-weights are computed using each country's average GDP in constant U.S. dollars.

Table A.2 [about here]

### B.4 Investor preferences and the bond risk premium

This section discusses the role of investor preferences in determining the bond risk premium. Figure A.1 presents the bond risk premium for different levels of relative risk aversion ( $\gamma$ ) and time preference ( $\beta$ ), while Table A.3 reports corresponding predictions on the price of risk.

Figure A.1 and Table A.3 [about here]

The bond risk premium increases when the representative agent is more risk-averse (higher  $\gamma$ ). Both the short- and long-run macro risk premium components increase. The short-run macro risk premium, given by  $\gamma \sigma_{c,s_t} \rho_{i,s_t} \sigma_{i,s_t}^B$ , directly increases with the risk aversion coefficient  $\gamma$  through the price of risk. The long-run macro risk component also increases with risk aversion because investors display a stronger preference for early resolution of uncertainty when the difference between  $\gamma$  and  $\frac{1}{\psi}$  increases. In addition to these direct effects, higher risk aversion increases the precautionary motives and thus reduces the equilibrium risk-free rate. A lower risk-free rate increases the present value of the debt coupons that the government must service, thereby increasing default risk and the risk premium. Similarly, the bond risk premium decreases with time preference: less impatience (lower  $\beta$ ) translates into a lower risk-free rate and higher default risk.

Table A.3 indicates that the price of long-run macro risk is  $\Delta_H = \hat{\lambda}_H / \lambda_H = 1.674$ , which implies that investors overweight the probability of switching from expansion to recession by 67.4%. Investors thus price bonds as if recessions are more likely than in reality. The ratio of the unconditional risk-neutral default probability ( $\mathbb{Q}$ ) over the unconditional physical default probability ( $\mathbb{P}$ ), both computed at a 5-year horizon, reflects how much investors overweight the increase in default probability during recession. We compute the unconditional risk-neutral default probability ( $\mathbb{Q}$ ) using the long-term risk-neutral distribution ( $\hat{f}_L = 0.3048, \hat{f}_H = 0.6952$ ) to weight the default probability in each state  $s_t = \{L, H\}$ . Correspondingly, we use the real-world distribution ( $f_L = 0.1353, f_H = 0.8647$ ) to compute the physical default probability ( $\mathbb{P}$ ). The ratio of probabilities equals 1.94 in the baseline calibration, which indicates that investors price sovereign bonds as if the unconditional level of default risk were 94% greater than in reality, mostly because of long-run macro risk. Macroeconomic risk thus entails a substantial price of risk.

## C Equity risk premium

This Appendix derives the equity risk premium in the economy. As in [Abel \(1999\)](#), among others, we assume that dividends  $D_t$  lever up consumption such that

$$D_t = \exp(-\beta_d t) C_t^{\eta_d}, \quad (\text{A.64})$$

where  $\eta_d \geq 1$  is the leverage parameter and  $\beta_d > 0$  is an adjustment parameter determining the unconditional expected growth rate of dividends ([Andrei, Hasler and Jeanneret, 2019](#)). Applying Ito's

Lemma, the dividend process is:

$$\frac{dD_t}{D_t} = \underbrace{\left( -\beta_d + \eta_d \mu_{c,s_t} + \frac{1}{2} \eta_d (\eta_d - 1) \sigma_{c,s_t}^2 \right)}_{\mu_{d,s_t}} dt + \eta_d \sigma_{c,s_t} dZ_{c,t}. \quad (\text{A.65})$$

The stock price, denoted by  $S_{s_t}(D_t)$  when the current state is  $s_t$ , is the claim to the dividend process and given by

$$S_{s_t}(D_t) = \mathbb{E}_t \left[ \int_t^\infty D_u \frac{\pi_u}{\pi_t} du \middle| s_t \right] \quad (\text{A.66})$$

$$= \frac{D_t}{r_{d,s_t}}, \quad (\text{A.67})$$

where  $r_{d,s_t}$  is the discount rate applicable to the dividend dynamics, which is given by

$$r_{d,s_t} = r_{s_t} - \hat{\mu}_{d,s_t} + \frac{(r_j - \hat{\mu}_{d,j}) - (r_{s_t} - \hat{\mu}_{d,s_t})}{\hat{p} + r_j - \hat{\mu}_{d,j}} \hat{p} \hat{f}_j, \quad j \neq s_t; \quad j, s_t = \{L, H\}, \quad (\text{A.68})$$

where  $\hat{\mu}_{d,s_t} = \mu_{d,s_t} - \gamma \eta_d \sigma_{c,s_t}^2$  is the expected growth rate of dividend under the risk-neutral measure.

The equity risk premium  $EP_{s_t}$  in state  $s_t$ , defined as the instantaneous expected return on the stock in excess of the risk-free rate, is equal to

$$EP_{s_t} = \gamma \eta_d \sigma_{c,s_t}^2 + \lambda_{s_t} \Theta_{s_t}^P R_{i,s_t}^S, \quad s_t = \{L, H\}. \quad (\text{A.69})$$

The first term of Equation (A.69) captures compensation for instantaneous dividend innovations. The second term captures compensation for changes in global macroeconomic conditions. The latter risk premium component is determined by the probability  $\lambda_{s_t}$  of leaving state  $s_t$ , the price of risk associated with this change of state  $\Theta_{s_t}^P = 1 - \Delta_{s_t}$ , and the change in stock valuation caused by the change of state, given by  $R_{i,s_t}^S = \frac{S_{i,j}}{S_{i,s_t}} - 1$ ,  $s_t \neq j = \{L, H\}$ . The conditional stock return volatility is given by  $\sigma_{s_t}^S = \sqrt{(\sigma_{c,s_t})^2 + \lambda_{s_t} (R_{i,s_t}^S)^2}$ .

We calibrate the  $\eta_d$  and  $\beta_d$  parameters to generate theoretical price-dividend ratio and stock return volatility matching their empirical counterparts. Our calibration reproduces the log price-dividend ratio of 6.4 and stock return volatility of 19.8% per year reported in [Schorfheide, Song and Yaron \(2018\)](#).<sup>18</sup> Furthermore, we obtain a dividend growth volatility of 12% per year, in line with the 11.1% over the

<sup>18</sup>Similarly, [Beeler and Campbell \(2012\)](#) report a log price-dividend ratio of 6.36, a stock return volatility of 20.2%.

1930-2008 period reported by [Beeler and Campbell \(2012\)](#). Using such parameters and those on Table 1, Equation (A.69) implies conditional equity premia equal to 12.83% per year in recession and 2.88% in expansion, leading to an unconditional equity risk premium equal to 4.22% per year.

## D Model simulation

This Appendix describes the simulation procedure discussed in Section 4.2.4. We generate 500 artificial datasets consisting of 40 countries over 98 quarters, corresponding to the 1994:Q1-2018:Q2 period. Each sovereign experiences two types of common shocks: a systematic component in output growth shocks ( $dZ_{i,t}$ ) and synchronous changes in the state of the global economy  $s_t$ . To model such dependencies, we draw quarterly shocks to a sovereign's output from a distribution with conditional correlation  $\rho_{i,s_t}$  with quarterly shocks on U.S. consumption growth ( $dZ_{c,t}$ ). In each period, the state of the global economy may also switch, and we use the business cycle estimated over the 1994:Q1-2018:Q2 period to determine whether the economy is in expansion or recession. The state of the global economy determines not only the correlation between output and global consumption shocks ( $\rho_{i,s_t}$ ) but also the first and second moments of global consumption and output growth, as well as the equilibrium risk-free rate. While the business cycle is common across all countries, the exposure of their bond prices to a change in the state of the global economy can differ based on their current default risk level. The model simulation is thus useful to study how bond excess return and credit spreads vary over time and across countries in the model in a way that can be compared to the data.

At the start of the simulation, all sovereigns choose a debt coupon and default policies that correspond to those discussed in Section 4.1. To generate an initial cross-section of countries, each sovereign starts at date  $t = 0$  with a level of government revenue drawn randomly between 0.6 and 1, such that the initial distance-to-default varies across sovereigns. The initial cross-country distribution of government revenue is calibrated so that simulated default frequencies match those observed empirically (more details below). At the start of every period, the state of the global economy over the previous quarter is determined. Then, each sovereign observes its own state-dependent revenue dynamics over the quarter. If the level of government revenue crosses a state-dependent boundary ( $Y_{i,t} \leq Y_{D,i,s_t}$  in state  $s_t$ ), the sovereign defaults. We replace a defaulted sovereign with another sovereign at optimal indebtedness level and government revenue equal to  $Y_{i,t} = 1$ , such that the number of countries in the economy remains constant over time. Similarly, if government revenue reaches an upper threshold ( $Y_{i,t} \geq 3$ ), we replace the sovereign by a new one with government revenue equal to  $Y_{i,t} = 1$ , thus resetting its

indebtedness ratio to the optimal level. This adjustment prevents that the economy becomes dominated by a few disproportionately large countries, and prevents government indebtedness (debt value to output) from vanishing over time. Next, we extract the quarterly value of the bond and its credit spread. We compute the annualized bond excess return of country  $i$  in quarter  $t$  when the current state is  $s_t$  as  $R_{B,i,t,s_t}^e = \frac{1}{\Delta t} \frac{B_{i,t,s_t} + c\Delta t}{B_{i,t-1,s_{t-1}}} - r_{B,s_t}$ , where  $B_{i,t,s_t}$  is the value of the bond,  $c$  is the debt coupon (common across countries),  $r_{B,s_t}$  is the risk-free discount rate, and  $\Delta t = \frac{1}{4}$  is the discretized time increment.

We repeat each simulation 500 times. In total, the simulation thus consists of 500 economies of 40 countries each, which implies almost 2 million quarterly observations. We first compute the statistics for bond excess returns and credit spreads for each simulated economy exactly as in the data. We then average these statistics across economies and compute their 5th and 95th percentiles. This approach allows us to study the sampling distribution for statistics of interest produced for each economy.

Table A.4 presents the simulation results. Before comparing credit spreads and sovereign risk premia in the simulation to their empirical counterparts, we must check that the model generates reasonable default frequencies at different horizons. Panel A of Table A.4 shows that the level and the term structure of default probability in the simulation closely match their empirical counterparts. The average cumulative 1-, 5-, and 10-year default probabilities are 2.23%, 10.63%, and 20.08% across the simulated economies, while they are respectively 2.88%, 11.29%, and 19.11% for speculative-grade foreign currency sovereign bonds, as reported by [Standard and Poor's \(2020\)](#). These empirical default rates are well within the confidence intervals of our simulations. The model parameters and the initial cross-country distribution of government revenue are therefore properly calibrated.

Table A.4 [about here]

Panel B of Table A.4 compares bond excess returns and credit spreads in the simulation to those in the data. We report two sets of numbers for the empirical counterparts: 'raw' and 'AAA-adjusted', depending on the proxy for the risk-free rate. We compute the raw excess returns using the one-month T-bill from Ken French's website as the risk-free return, while the raw credit spreads as reported directly by JP Morgan for EMBI Global indices and thus based off the U.S. Treasury yield curve. The AAA-adjusted calculations use the ICE Bank of America AAA U.S. Corporate Index as a proxy for the risk-free asset, addressing the potential critique that Treasury bonds may not be appropriate proxies for the default-free borrowing rate because they are valued at a premium due to their extreme safety and liquidity (e.g., [Krishnamurthy and Vissing-Jorgensen, 2012](#)). Accordingly, we use the AAA Corporate Index return as the risk-free return for calculating bond excess returns and subtract the AAA Index credit

spreads (relative to the U.S. Treasury curve) from the EMBI credit spreads to compute the sovereign bond credit spreads.

Panel B of Table A.4 shows that the average (annualized) bond excess return in the simulations is 1.90%, while the average credit spread is 162 bps per year. Both the average risk premium and average credit spread are significantly below their empirical counterparts in the 1994Q1-2018Q2 period. The average excess return of sovereign bonds over one-month T-bills is 2.13% per quarter in the data, which corresponds to 8.53% per year. It is 5.37% per year above AAA Corporate bonds. The average EMBI credit spread is 471 bps per year above U.S. Treasuries and 376 bps above AAA-rated corporates, also significantly larger than the 162 bps of the simulations.<sup>19</sup> The medians of excess returns and credit spreads in the data are also larger, and their distributions more positively skewed, than those in the simulations. However, the standard deviation of bond excess returns in the model closely matches the data.

Hence, while the simulated economy successfully captures the level of default risk in the data, as measured by the term structure of default probability, and the standard deviation of bond returns, it cannot generate a risk compensation as large as what is observed in emerging bond markets. While long-run macro risk significantly increases risk-premia and credit spreads compared to a canonical model with short-run risk only, additional mechanisms generate excess returns in the data, as discussed in Section 4.2.4.

## **E Additional empirical results**

This Appendix presents additional empirical results.

### **E.1 Portfolio allocation and rebalancing**

This section reports the country allocation and rebalancing frequency for the baseline double-sort procedure used in Panel A.I of Table 6. We refer to 'portfolio allocation' as the fraction of time a country belongs to each of the three portfolios conditional on being on the sample. 'Rebalancing frequency' is the fraction of time a country switches portfolios conditional on being on the sample. Results in Table A.5 show that countries have a dominant allocation, suggesting there are intrinsic, long-lasting

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<sup>19</sup>Credit spreads are lower in the model than in the data because model-implied spreads are for perpetual bonds while the observed spreads are for finite maturity bonds. The term structure of spreads is severely negatively sloped when default risk is high (Augustin, 2018). Yet, the difference in average credit spreads is probably too significant to be attributed to such maturity mismatch effect only.

cross-country differences in terms of long-run macro risk. However, countries do shift portfolios over the sample period, i.e., the portfolio constituents vary over time. On average, the average rebalancing frequency across countries is 15%, which means that conditional on being on the sample, a country switches portfolios once every 7 quarters. Each of the three portfolios of Panel A.I in Table 6 contains 9 countries on average, such that the HML portfolio has 18 countries at a time, and its composition changes by approximately 3 countries every quarter.

Table A.5 [about here]

## E.2 Regime identification based on economic turning points

This section provides additional evidence that bond excess returns vary in the cross-section with exposure to shifts in macroeconomic regimes. The results here complement those in Section 5.3. While we previously focused on a single event (the sudden worsening of the Global Financial Crisis in 2008:Q4), we now expand the analysis to additional regime change events. We use the [Harding and Pagan \(2006\)](#) procedure to define 'turning points' in U.S. consumption growth and, as such, identify regime changes based on realized consumption data.

The starting point of the [Harding and Pagan \(2006\)](#) methodology is a plot of the time series of annual consumption growth, as illustrated in Figure A.2. The procedure seeks local maxima and minima on the plot. During our sample period – starting in 1994:Q1 – there are seven turning points, identified by the Peaks and Troughs on the plot. There are four Peaks and three Troughs in our sample, corresponding to a total of seven regime changes.

Figure A.2 [about here]

If market participants could recognize in real-time that Peaks are peaks, that is, consumption growth will be declining in the following quarters, then sovereign bond excess returns should be negative during Peak quarters, on average. Analogously, they should be positive during Trough quarters. Indeed, the average bond excess return is -0.3% during the Peak quarters, but 7.4% during the Trough quarters. Using the same rationale as in Section 5.3, countries whose bonds fall more when consumption switches from High to Low growth (i.e., at Peaks), or increase more when consumption switches from Low to High growth (i.e., at Troughs), are highly sensitive to regime shifts in the global economy. Our theory predicts these countries are riskier and should have average excess returns over the entire sample.

To pool data from Peaks and Troughs, we define the signed returns variable  $R_{Signed} = R \times \mathbf{1}$ , where  $R$  is the sovereign bond return at a turning point quarter and  $\mathbf{1}$  is an indicator function that equals  $-1$  if

it is a Trough and +1 if it is a Peak. Under this definition, countries that are highly sensitive to regime shifts will have relative low signed returns. For each country, we average  $R_{Signed}$  across the 7 turning points in our sample. Figure A.3 shows the results.

Figure A.3 [about here]

As in Figure 4 in the paper, there are four scatter plots, each of them corresponding to a different time subsample and number of countries dictated by data availability. All four scatter plots display a negative relationship between average quarterly excess returns and average signed returns. This result confirms the model prediction that countries whose bonds are more sensitive to regime changes in global consumption are riskier and thus offer a higher risk premium.



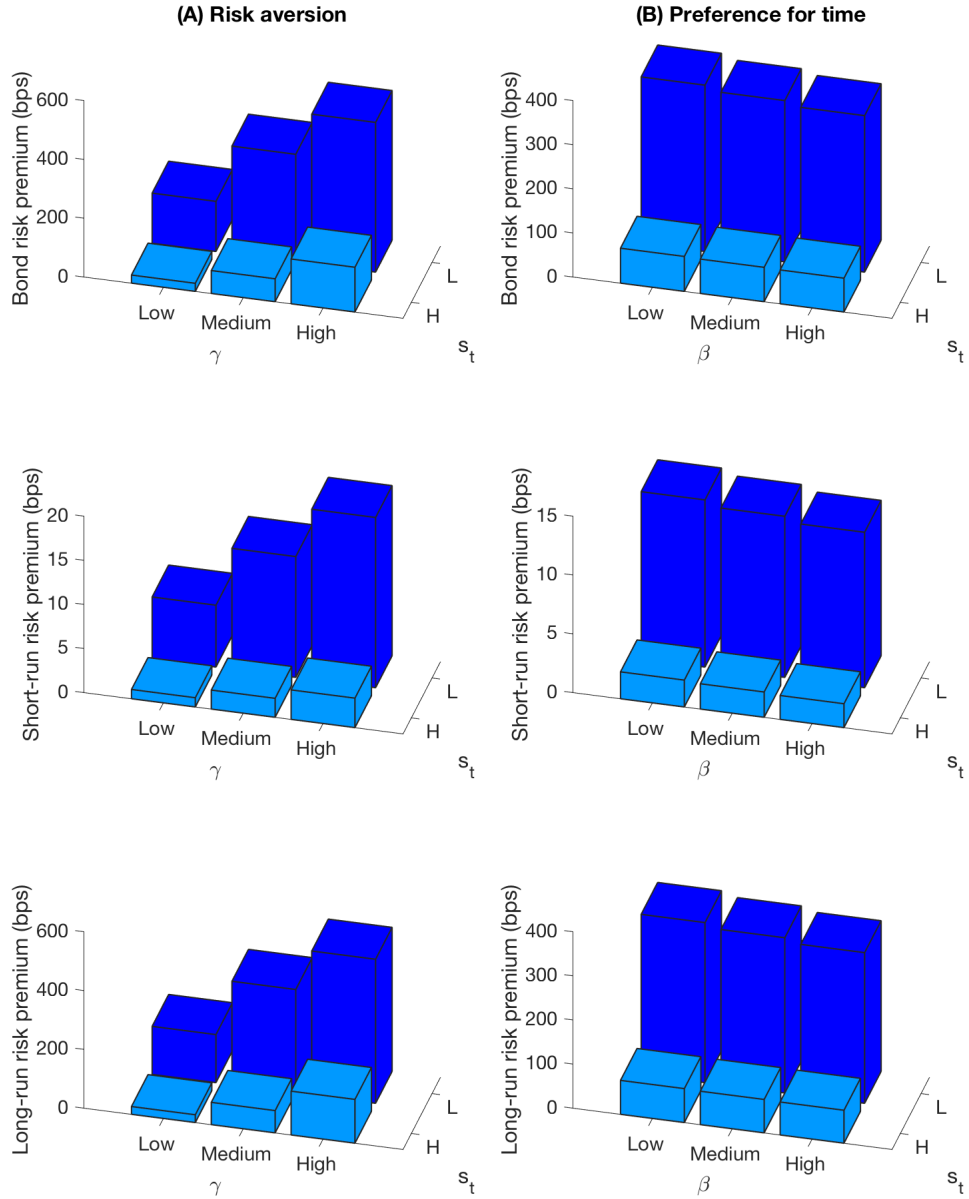


Figure A.1: **Sovereign risk premium by investor preferences.**

This figure illustrates how the bond risk premium varies with investor preferences. Panel A reports predictions for different levels of relative risk aversion, while Panel B reports predictions for different levels of preference for time. Low, medium, and high risk aversion corresponds to  $\gamma = 5$ ,  $\gamma = 10$ , and  $\gamma = 15$ , while low, medium, and high preference for time corresponds to  $\beta = 0.03$ ,  $\beta = 0.04$ , and  $\beta = 0.05$ , respectively. The figure compares predictions when the current state  $s_t$  is in recession ( $L$ ) or expansion ( $H$ ). Bond risk premium is in basis points (bps) per annum. Unless otherwise specified, we use the parameters of the baseline calibration (see Table 1) and report predictions for levels of government revenue observed at issuance time ( $Y = 1$ ).

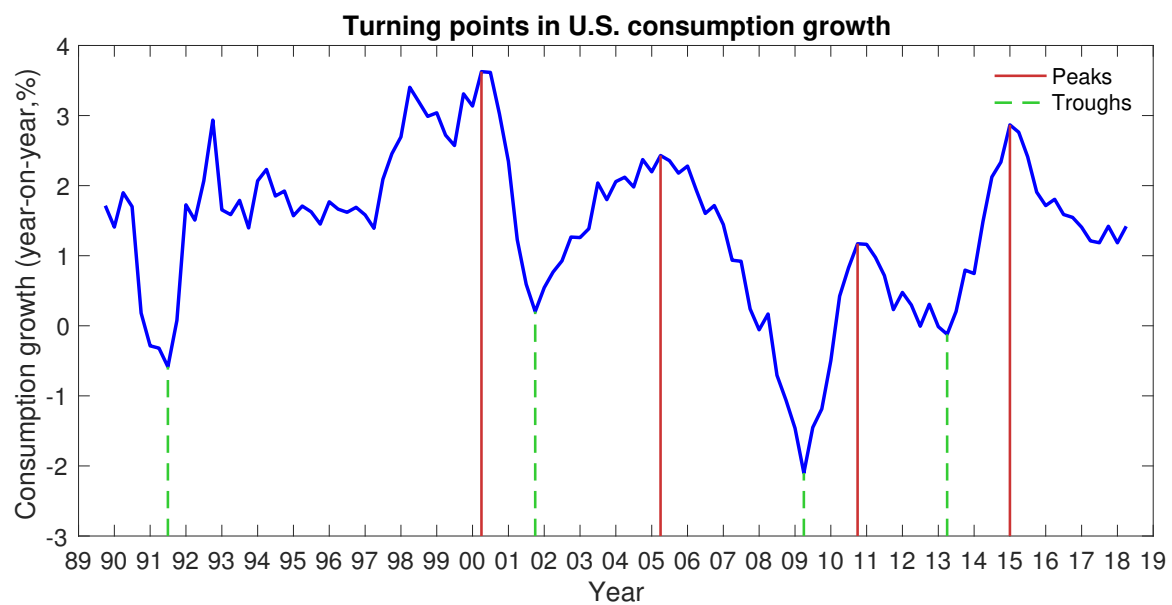


Figure A.2: **Regime changes based on economic turning points.**

This figure plots economic turning points, following the [Harding and Pagan \(2006\)](#)'s procedure to determine regime changes in realized consumption data. The plot illustrates the identified local maxima (peaks) and minima (troughs) of year-over-year U.S. consumption growth, expressed in percentage. Data span the 1990:Q1-2018:Q2 period.

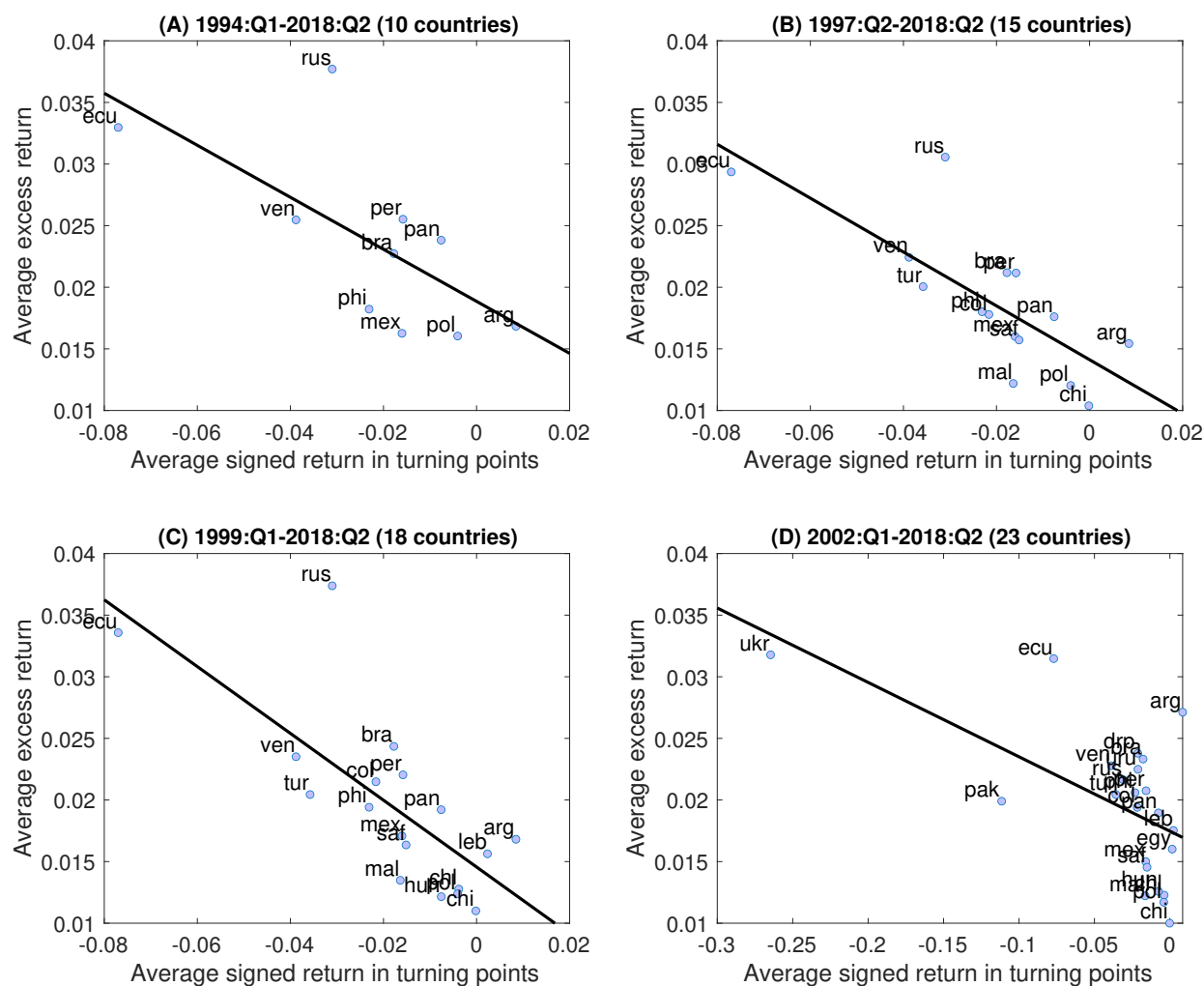


Figure A.3: **Average bond excess returns and economic turning points.**

This figure plots average quarterly bond excess returns versus the exposure to economic turning points. The exposure to turning points is computed, for each country, as the average signed return in the turning points identified in Figure A.2. Average quarterly excess returns are calculated over four different samples, each of them with a different number of countries with data available throughout the entire the sample. Best fit lines are displayed. Data span the 1994:Q1-2018:Q2 period.

Table A.1 : **Conditional output growth moments by country.**

The table displays the annualized mean and volatility of output growth for the emerging countries used in the model calibration. Conditional output growth moments are computed with quarterly real GDP data and use business cycle dating estimated with U.S. consumption data. The last rows report aggregate equally and GDP-weighted descriptive statistics using each country's average GDP in constant U.S. dollars, as well as details on the distribution. The Internet Appendix B.2 details the construction of the conditional moments. Data are from Datastream and span the 1994:Q1-2018:Q2 period.

Country	Output growth rate (%)		Output growth volatility (%)		Correlation with consumption	
	Recession	Expansion	Recession	Expansion	Recession	Expansion
Argentina	0.11	4.12	5.55	3.01	-0.03	0.05
Bahrain	5.17	2.89	5.35	4.32	0.09	0.02
Bolivia	5.41	3.90	1.56	2.14	0.55	-0.15
Brazil	2.46	2.25	3.49	2.23	0.00	0.10
Bulgaria	0.30	4.23	3.81	1.70	-0.49	0.22
Chile	2.46	4.00	2.34	2.14	-0.12	0.02
Colombia	2.59	4.41	2.51	1.56	-0.18	0.09
Croatia	-2.45	2.78	3.50	2.32	-0.19	0.12
Czech Republic	-1.50	3.11	2.45	1.27	-0.15	0.10
Dominican Republic	2.13	5.66	3.59	3.85	0.05	-0.04
Ecuador	3.50	3.75	2.13	2.16	-0.25	0.39
Estonia	-5.80	5.49	7.12	3.29	-0.12	-0.09
Greece	-3.80	1.59	3.50	2.66	0.13	0.41
Hungary	-1.71	2.99	3.05	1.29	0.16	0.13
India	5.16	6.82	3.64	2.32	0.57	-0.16
Kazakhstan	6.06	5.11	14.46	9.87	-0.06	0.08
Latvia	-5.86	5.39	5.49	3.13	0.34	0.07
Lithuania	-3.31	4.82	8.24	1.92	-0.23	-0.14
Malta	1.83	3.91	2.90	3.52	-0.11	-0.18
Malaysia	2.97	5.40	3.89	1.00	-0.20	0.04

Continued.

Country	Output growth rate (%)		Output growth volatility (%)		Correlation with consumption	
	Recession	Expansion	Recession	Expansion	Recession	Expansion
Mexico	0.21	2.73	4.05	2.42	0.63	0.10
Morocco	4.78	3.34	2.87	3.58	0.02	0.16
Mozambique	7.52	5.72	4.28	3.60	0.23	-0.02
Namibia	-0.81	4.54	7.64	7.06	-0.30	0.09
Peru	4.26	4.81	2.43	2.75	0.48	-0.22
Philippines	4.36	4.91	2.62	1.61	0.29	-0.16
Poland	2.20	4.27	1.44	1.86	0.01	0.01
Romania	2.04	3.10	5.53	3.12	0.34	0.21
Russia	-0.11	3.64	4.96	2.21	-0.26	0.16
Slovakia	0.58	4.39	6.88	2.33	0.02	0.03
Slovenia	-3.24	3.59	3.49	1.48	0.30	0.20
South Africa	1.24	2.94	1.56	1.12	0.67	0.09
South Korea	2.33	4.75	2.81	2.67	0.24	-0.06
Taiwan	-0.35	4.82	3.61	3.09	0.43	0.21
Tanzania	5.62	6.75	4.25	3.16	-0.28	-0.22
Thailand	2.70	3.62	3.68	3.97	0.41	-0.04
Turkey	2.87	4.73	6.50	4.29	-0.39	-0.01
Uganda	6.23	5.24	3.64	2.82	0.04	-0.15
Venezuela	0.47	2.18	2.72	8.34	-0.08	-0.12
Vietnam	8.48	5.80	19.54	6.62	-0.05	0.01
Average (equal-weighted)	1.68	4.21	5.66	3.62	0.06	0.03
Average (GDP-weighted)	1.87	3.92	4.45	3.05	0.16	0.04
Median	2.17	4.25	3.62	2.66	0.01	0.03
Standard deviation	3.44	1.21	8.13	4.40	0.29	0.15
10th percentile	-3.27	2.76	2.23	1.39	-0.27	-0.16
90th percentile	5.84	5.69	7.39	5.59	0.52	0.21

Table A.2 : **Leverage estimates for emerging economies.**

The table reports estimates of the leverage parameter  $\eta$  for 10 countries. Leverage is computed as the ratio of government revenue growth volatility to output growth volatility. The last rows report aggregate equally and GDP-weighted descriptive statistics using each country's average GDP in constant U.S. dollars, as well as the median. The Internet Appendix B.3 details the construction of the leverage estimates. Data are from Datastream and the World Bank and span the 1994:Q1-2018:Q2 period.

Country	Output growth volatility (%) (A)	Government revenue growth volatility (%) (B)	Leverage (B) / (A)
Bolivia	1.95	7.89	4.03
Brazil	2.42	8.14	3.37
Bulgaria	2.31	9.34	4.05
Chile	2.09	9.20	4.41
Colombia	1.32	8.89	6.73
India	2.54	8.45	3.33
Mexico	3.10	8.22	2.65
Philippines	1.76	6.46	3.68
Russia	3.05	14.82	4.86
South Africa	1.23	4.29	3.50
<i>Average (equal-weighted)</i>	2.18	8.57	4.06
<i>Average (GDP-weighted)</i>	2.55	9.61	3.80
<i>Median</i>	2.22	8.33	3.86

Table A.3 : **Sovereign risk premium, price of risk, and investor preferences.**

The table presents the model predictions for different preference parameters. Column A reports the bond risk premium, Column B the credit spread, Column C the ratio of the risk-neutral default probability ( $\mathbb{Q}$ ) over the physical default probability ( $\mathbb{P}$ ), and Column D the price of macro risk  $\Delta_H$ . Panel A reports predictions for the baseline calibration, Panel B reports predictions for different levels of relative risk aversion  $\gamma$ , while Panel C reports predictions for different levels of preference for time  $\beta$ . Bond risk premium and credit spread are in basis points (bps) per annum. Unless otherwise specified, we use the baseline calibration parameters on Table 1 and report predictions for levels of government revenue observed at issuance time ( $Y = 1$ ).

	Bond risk premium (bps)	Credit spread (bps)	Risk-neutral over physical default probability ( $\mathbb{Q}/\mathbb{P}$ )	Price of macro risk ( $\Delta_H$ )
	(A)	(B)	(C)	(D)
<i>Panel A: Baseline case</i>				
	117	154	1.94	1.67
<i>Panel B: Relative risk aversion</i>				
High ( $\gamma = 15$ )	201	209	2.94	2.35
Low ( $\gamma = 5$ )	47	117	1.31	1.25
<i>Panel C: Preference for time</i>				
High ( $\beta = 0.05$ )	112	158	2.02	1.70
Low ( $\beta = 0.03$ )	122	146	1.86	1.65

Table A.4 : **Model simulation vs. data.**

The table reports results of models simulations which are compared to the data over the 1994:Q1-2018:Q2 period. We generate 500 artificial datasets consisting of 40 countries and 98 quarters each using the parameters of the baseline calibration on Table 1. Panel A has the sovereign default probability computed at the 1-, 5-, and 10-year horizons. The empirical counterparts are the cumulative default rates for foreign-currency speculative-grade sovereign bonds from [Standard and Poor's \(2020\)](#). Panel B has bond pricing moments, averaged across simulations and the data. Bond excess returns are in percentage points per year and credit spreads are basis points (bps) per year. The empirical sovereign bond returns and credit spreads are from country-level JP Morgan EMBI Global indices for the countries in Table 5. 'Raw' denotes excess returns over one-month T-bills and credit spreads based off the U.S. Treasury curve, while 'AAA-adjusted' denotes excess returns and credit spreads over the ICE BofA AAA Corporate Bond index. Values in squared parentheses denote the 5th and 95th percentiles across the 500 artificial datasets. The Internet Appendix D explains the simulation procedure.

Panel A: Default probability at different horizons					
	1 year (A)	5 years (B)	10 years (C)		
Default rate (%)					
Simulation	2.23 [1.53, 2.96]	10.63 [7.42, 13.95]	20.08 [14.29, 25.95]		
Data	2.88	11.29	19.11		
Panel B: Bond price moments					
	(A) Mean	(B) Median	(C) Std. dev.	(D) 5th	(E) 95th
Bond excess return (%)					
Simulation	1.90 [1.28, 2.48]	1.70 [1.45, 1.97]	17.97 [16.24, 19.92]	-48.03 [-56.48, -40.35]	46.52 [40.09, 54.15]
Data (raw)	8.53	7.11	18.82	-36.35	61.12
Data (AAA-adjusted)	5.36	3.20	18.24	-36.35	54.00
Credit spread (bps)					
Simulation	162 [146, 181]	112 [101, 125]	175 [155, 194]	36 [32, 40]	459 [387, 535]
Data (raw)	471	323	535	90	1271
Data (AAA-adjusted)	376	234	518	16	1144



Table A.5 : **Country allocation by portfolios and rebalancing frequency.**

The table reports the country allocation and rebalancing frequency for the baseline double-sort procedure in Panel A.I of Table 6. Allocation is the fraction of time a country belongs to each of the three portfolios conditional on being on the sample. Rebalancing frequency is the fraction of time a country switches portfolios conditional on being on the sample. The sample period is 1994:Q1 to 2018:Q2 but countries enter and exit the sample at different times as reported in Table 5.

Country	Portfolio allocation (%)			Rebalancing frequency (%)	Country	Portfolio allocation (%)			Rebalancing frequency (%)
	Low	Medium	High			Low	Medium	High	
Argentina	10.81	9.46	79.73	12.16	Lebanon	52.63	24.56	22.81	21.05
Belize	0.00	0.00	100.00	0.00	Lithuania	0.00	0.00	100.00	0.00
Brazil	40.54	45.95	13.51	24.32	Malaysia	66.67	33.33	0.00	14.29
Bulgaria	21.43	76.79	1.79	25.00	Mexico	40.54	41.89	17.57	14.86
Chile	75.47	9.43	15.09	16.98	Morocco	8.89	26.67	64.44	11.11
China	65.75	20.55	13.70	9.59	Nigeria	43.40	56.60	0.00	11.32
Colombia	29.03	58.06	12.90	11.29	Pakistan	6.82	20.45	72.73	11.36
Cote d'Ivoire	11.11	46.67	42.22	8.89	Panama	28.38	21.62	50.00	17.57
Croatia	63.16	36.84	0.00	5.26	Peru	45.95	45.95	8.11	21.62
Dominican Rep.	4.65	46.51	48.84	18.60	Philippines	51.35	17.57	31.08	8.11
Ecuador	2.70	51.35	45.95	13.51	Poland	48.65	43.24	8.11	24.32
Egypt	65.91	31.82	2.27	13.64	Russia	2.70	28.38	68.92	24.32
El Salvador	0.00	39.02	60.98	14.63	Serbia	0.00	39.29	60.71	10.71
Gabon	0.00	33.33	66.67	22.22	South Africa	71.43	28.57	0.00	14.29
Georgia	0.00	68.75	31.25	18.75	Sri Lanka	63.16	5.26	31.58	10.53
Ghana	0.00	10.53	89.47	10.53	Thailand	50.00	50.00	0.00	8.33
Hungary	66.67	33.33	0.00	25.93	Turkey	32.81	57.81	9.38	15.63
Indonesia	27.27	12.12	60.61	6.06	Ukraine	22.45	28.57	48.98	24.49
Iraq	36.00	16.00	48.00	8.00	Uruguay	33.33	17.78	48.89	24.44
Jamaica	0.00	0.00	100.00	0.00	Venezuela	2.70	37.84	59.46	12.16
Kazakhstan	0.00	40.00	60.00	25.00	Vietnam	3.70	51.85	44.44	22.22
Korea	47.06	52.94	0.00	17.65					