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Common factors in international bond returns

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Abstract

In this paper, we estimate and interpret the factors that jointly determine bond returns of different maturities in the US, Germany and Japan. We analyze both currency-hedged and unhedged bond returns. For currency-hedged bond returns, we find that a linear factor model with five factors explains 96.5% of the variation of bond returns. Using regression analysis, we show that these factors can be associated with changes in the level and steepness of the term structures in (some of) these countries. We compare our multi-factor model with one-dimensional (international) duration measures. The five-factor model also explains 98.5% of the cross-sectional variation in the expected bond returns in all countries. We find similar results when we jointly analyze both currency-hedged bond returns and bond returns that are not hedged for currency risk.

JEL classification: F21; G12; G15; E43

Keywords: Bond returns; Multi-country model; Common factors; Term structure of interest rates

1. Introduction

Most large investors do not invest the fixed income part of their portfolio solely in government bonds that are issued by their home country, but usually they diversify risk by investing in bonds issued by different countries. For risk management of such international bond portfolios, it is essential to have a joint model for the bond returns of the relevant maturities and in the relevant countries. Also, for the pricing

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of cross-currency interest rate derivatives, such as differential swaps or international basket options, a joint model for term structure movements in different countries is required (Frachot, 1995). In this paper, we empirically analyze a multi-country factor model that can be used for risk management purposes as well as for the pricing of cross-currency interest rate derivatives. The focus of the paper is the estimation and interpretation of the common factors that determine international bond returns of different maturities.

Our work is related to Knez et al. (1994) and Litterman and Scheinkman (1991). In these papers, a linear factor model is estimated for short-term US money market returns and long-term US government bond returns, respectively. In Knez et al. (1994), a four-factor model is proposed. The first two factors correspond to movements in the level and the steepness of the term structure of money market rates, while the other two factors account for differences in credit risk of the different money market instruments. Litterman and Scheinkman (1991) find, on the basis of a principal component analysis, that US bond returns are mainly determined by three factors, which correspond to level, steepness, and curvature movements in the term structure.

We extend these papers by jointly analyzing bond returns of different maturities in three countries, namely the US, Germany and Japan. In the spirit of the arbitrage pricing theory (APT) developed by Ross (1976), we assume a linear factor model for these bond returns, and assess how many factors are required to explain most of the variation in bond returns of all countries. Similar to Litterman and Scheinkman (1991), we use principal component analysis on the unconditional covariance matrix of bond returns of the different maturities in all countries to estimate the factors that determine these bond returns. The estimated principal components or *factor loadings* indicate per factor how this factor influences bond returns of the different maturities in each country. We also estimate the prices of the risk associated with each factor. Confidence intervals for the estimated factor loadings and factor risk prices are constructed using bootstrap techniques.

In our empirical analysis, we analyze bond returns that are hedged for currency risk as well as unhedged bond returns. In the case of hedged bond returns, the returns are driven only by changes in the underlying term structure. We use weekly data from 1990 to 1999 on Merrill Lynch bond indices for the US, Germany, and Japan. For each country, bond index returns for five maturity classes are used, from 1–3 years to larger than 10 years. In line with results presented by Ilmanen (1995), bond returns are positively correlated across countries.

For the hedged bond returns, we find that a five-factor model explains 96.5% of the total variation of international bond returns. Adding more factors to this model only slightly increases the explained variation, and the factor loadings for the extra factors are small and statistically insignificant.

To interpret these multi-country factors, we regress the corresponding factor mimicking portfolios (FMP) on the single-country factor mimicking portfolios and on changes in the level and the steepness of the underlying term structures. This indicates that the first factor of the five-factor model can be interpreted as a world level factor, because this factor represents movements in the level of the term structures in all countries in the same direction. This factor explains 46.6% of the variation in bond returns. It is closely related to the one-dimensional Macaulay (1938) duration measure and the duration measure for international bond portfolios proposed by Thomas and Willner (1997), but similar to this measure, it captures only some part of all movements in international bond prices. The second factor represents parallel shifts in the term structures of Japan and the US in opposite directions, and explains 27.5% of bond return variation. Similarly, the third factor represents parallel shifts in opposite directions in the term structures of Germany and the US, explaining 17% of bond return variation. The fourth and fifth factors represent changes in the steepness of the term structure of Germany and Japan, respectively, explaining 3.1% and 2.3% of the bond return variation.

Thus, for the 1990–1999 period, we conclude that the positive correlation between bond returns across countries is only driven by the correlation between the level of the term structures in the several countries. Changes in the slope of the term structure are not correlated across countries. The 1990–1999 period includes the period immediately after the German reunification, in which German interest rates exhibited unusual behavior relative to the US and Japan. To analyze the impact of this event, we redo the analysis using data from October 1994 until October 1999. We find that only the results for the fourth and fifth factors change. For this subperiod, there is a positive relation between changes in the steepness of the German and Japanese term structures.

Next, we estimate the risk price of each factor, and analyze whether the model can explain the expected returns on bonds of different maturities and different countries. It turns out that the five-factor model provides a good fit of these expected returns: the cross-sectional R^2 is equal to 98.5%. In fact, only the first two factors have statistically significant risk prices and these two factors explain already a large part of the cross-sectional variation in expected returns.

Finally, we analyze a linear factor model that describes both currency-hedged bond returns and bond positions that are not hedged for currency risk, by including currency returns for the DM/$ and yen/$ exchange rates in the principal component analysis. We find that two additional factors are needed to explain the same amount of variation as the five-factor model for hedged bond returns. The interpretation of the first five factors in this seven-factor model is similar to the five-factor model, but, because currency returns are correlated with hedged bond returns, the two extra factors are not simply a DM/$ and yen/$ factor.

The remainder of this paper is organized as follows. The next section describes in detail the linear factor model and the estimation methodology that is used. In Section 3, we describe the data and replicate the analysis of Litterman and Scheinkman (1991) for each country in our data. In Section 4, we estimate and interpret the multi-country factor model for hedged bond returns. In Section 5, we extend the multi-country model by also including currency returns. Section 6 concludes.

2. Model setup

2.1. Model specification

The starting point of our analysis is the following factor model for excess bond returns R^e_t:

$$R^e_t \equiv R_t - \iota_N r^{US}_t = \Gamma\lambda + \Gamma F_t + \varepsilon_t \tag{1}$$

where R_t is an N-dimensional vector containing (weekly) returns on bonds of different maturities and different countries, ι_N is an N-dimensional vector of ones, and r^{US}_t is the 1-week US risk-free short rate. The model states that excess returns R^e_t are determined by K common factors F_t through the $N \times K$ matrix Γ (the factor loadings), and an N-dimensional vector ε_t containing bond-specific residuals, which can either be interpreted as measurement error in the bond returns or as idiosyncratic risk components. This model fits into the framework of the arbitrage pricing theory (Ross, 1976), and thus the elements of the K-dimensional vector λ can be interpreted as the market prices of factor risk.

The unobservable variables F_t and ε_t are assumed to be i.i.d. distributed[1] with

$$\begin{aligned} &E[\varepsilon_t] = 0, \quad \mathrm{Var}[\varepsilon_t] = \sigma^2 \mathrm{I}_N, \\ &E[F_t] = 0, \quad \mathrm{Var}[F_t] = \mathrm{I}_K, \\ &\mathrm{Cov}[F_t, \varepsilon_t] = 0 \end{aligned} \tag{2}$$

where I_n is an $n \times n$ identity matrix and the zeros indicate null vectors and matrices of the appropriate size. The last three assumptions in Eq. (2) are just one choice for the normalizations that are required to properly define the factor loadings. The assumption that the residual variances are all equal to each other is restrictive, but it allows us to estimate the model (1) with principal component analysis, which is a simple and frequently used technique in interest rate modeling (see, for example, Buhler et al., 1999; Golub and Tilman, 1997; Rebonato, 1996; Singh, 1997). Knez et al. (1994) allow these residual variances to be different from each other, at the cost of having to use a more complicated estimation technique, i.e. maximum likelihood, and the possibly restrictive assumption that the returns follow a multivariate normal distribution.

Given the assumptions in (1) and (2), the return covariance matrix Σ of the excess returns R^e_t can be decomposed into two parts

$$\sum = \Gamma\Gamma' + \sigma^2 I_{3M}. \tag{3}$$

In the next subsection, we indicate how Eq. (3) can be used to estimate the factor loadings.

We consider bond returns of $M = 5$ different maturities, $\tau_l,\ldots,\tau_M$, in three countries,

[1] An extension to this model would be to include time-varying expected bond returns and time-varying variances and covariances of bond returns.

US, Germany, and Japan. The τ_i-maturity log-return in terms of the country specific currency from time t to time $t+1$ is denoted by $R^{\mathrm{US}}_{t,\tau_i}$ for US bond returns, and, similarly, by $R^{\mathrm{GER}}_{t,\tau_i}$ for Germany, and by $R^{\mathrm{JAP}}_{t,\tau_i}$ for Japan. In this paper, we take the viewpoint of a US investor, so that the German and Japanese bond returns have to be converted to \$-returns. We will consider both bond positions that are hedged for currency risk as well as unhedged bond positions.

We start with the case of hedged bond returns. The time t values of the DM/\$ and yen/\$ exchange rates are denoted by s^{DM}_t and s^{Yen}_t, and the one-period forward rates at time t are defined as F^{DM}_t and F^{Yen}_t. Then, the currency-hedged \$ returns on German bonds $R^{\mathrm{GER}}_{t,\tau_i}(\$, \text{Hedged})$ are given by[2,3]

$$R^{\mathrm{GER}}_{t,\tau_i}(\$,\text{Hedged}) = R^{\mathrm{GER}}_{t,\tau_i} + \ln(s^{\mathrm{DM}}_t) - \ln(F^{\mathrm{DM}}_t), \quad i = 1,\ldots,M. \tag{4}$$

For Japanese returns, an analogous relation holds. Since weekly forward currency rates are typically close to current spot exchange rates, the difference between the hedged bond returns and the local bond returns will be small relative to the total return. Therefore, hedged bond returns are primarily driven by changes in the underlying local term structure of interest rates. We let the vector R_t contain all US bond returns and the hedged bond returns for both Germany and Japan. Thus, the dimension of this return vector is equal to $3M$, so that N is equal to $3M$ in Eq. (1).

The model can easily be extended to include unhedged bond positions. If we define the currency return from week t to week $t+1$ as

$$\ln(F^{\mathrm{DM}}_t) - \ln(s^{\mathrm{DM}}_{t+1}), \tag{5}$$

for the DM/\$ currency, and, analogously, for the yen/\$, then it follows directly that an unhedged (excess) bond return is equal to the hedged (excess) bond return plus the currency return. Note that, because the currency return corresponds to a long position in an unhedged bond and a short position in a hedged bond, it is the result of a zero-investment strategy (similar to an excess return). Therefore, we should not subtract the risk-free rate from this currency return and we can directly add the two currency returns (DM/\$ and yen/\$) to the vector of *excess* hedged bond returns R^e_t, to obtain the $(3M+2)$-dimensional vector of hedged bond and currency returns $\tilde{R}^e_t$, and again assume a linear factor model for these excess returns as in (1) and (2). Note that, for every possible portfolio in international bonds, containing both hedged and/or unhedged positions, it is possible to map the portfolio weights of this portfolio into weights that correspond to the $(3M+2)$ bond and currency returns,

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[2] Because the time $t+1$ bond price is not known with certainty at time t, it is not possible to completely eliminate currency risk using forward currency contracts. To separate the currency return from the hedged bond return, we neglect this quantity risk, so that the return in Eq. (4) is an approximation to the feasible hedged bond return. We have calculated the feasible currency-hedged returns, and found that the differences with our approximation are very small. The correlation between the feasible and approximated hedged bond return is always at least 0.9995 and the difference in average returns and standard deviations is very small.

[3] We define returns in logarithms, to separate the currency return from the bond return in a convenient way.

so that, in this case, the linear factor model describes both hedged and unhedged bond returns.

For each factor, one can easily calculate how much of the average variation in excess bond returns is explained by this factor, namely

$$\frac{\sum_{i=1}^{N} (\Gamma_j \Gamma'_j)_{ii}}{\sum_{i=1}^{N} \Sigma_{ii}}, \quad j = 1,\ldots,K \tag{6}$$

where Γ_j is the jth column of Γ and Σ_{ii} is the ith diagonal element of Σ, the covariance matrix of R^e_t.

Because the factors F_t are unobserved, one would like to construct a portfolio that is sensitive to movements of a given factor, while it is insensitive to movements in all other factors. These factor mimicking portfolios are not uniquely determined (see Knez et al., 1994). In the case of the hedged bond returns model, a convenient choice is as follows. For factor j, the weights of this factor mimicking portfolio w_j are equal to the factor loadings Γ_j normalized to sum up to one, i.e. $\Gamma_j/(\iota'_{3M}\Gamma_j)$. The construction of a factor mimicking portfolio is slightly different if currency returns are included. Since the currency returns are the result of a zero-investment strategy, the factor loadings for these currency returns should be excluded when summing up to one. This leads to following formula for the factor mimicking portfolio: $w_j = \Gamma_j/([\iota'_{3M} 0\ 0]\Gamma_j)$ (note that the length of the vector Γ_j is $3M + 2$ in this case. As outlined below, we will normalize the sum of the factor loadings to be positive, so that a positive factor loading directly corresponds to a long position in the corresponding bond. One can easily check that this portfolio is only sensitive to factor j and not to the other factors. These factor mimicking portfolios can be used to analyze the properties of the factors.

2.2. Model estimation

Using data on the excess bond returns (and currency returns), we will perform a principal component analysis on the sample covariance matrix of the return vector R^e_t (or $\tilde{R}^e_t$), to estimate the factor loadings Γ. The principal components are given by the eigenvectors of this covariance matrix. Then, the first K principal components of the sample covariance matrix are consistent estimates[4] of the factor loadings Γ in Eq. (1).

Note that the factor loadings matrix Γ can be postmultiplied with an orthonormal

[4] The ordinary principal component estimates are biased upward given the model in (1), due to the residual variance terms. To correct for this bias, each principal component has to be multiplied with a scale factor, see Basilevsky (1995) and Appendix A. For our models, the residual variances turn out to be small, so that these scale factors are only slightly smaller than one. Litterman and Scheinkman (1991) make the assumption that the idiosyncratic shocks are negligible.

matrix Λ without changing the common covariance matrix part $\Gamma\Gamma'$ in Eq. (3). Hence, given the covariance matrix Σ, the factor loadings Γ cannot be distinguished from the loadings $\Gamma\Lambda$. Principal component analysis imposes that each subsequent factor explains as large as possible a part of the variation and co-variation of bond returns, by imposing that the factor loading is orthogonal to all the previous ones. Then, the factor loadings are uniquely determined, except for a sign change. We therefore impose that the sum of the factor loadings of a given factor is positive. As mentioned above, this sign restriction facilitates the interpretation of factor mimicking portfolios.

If all returns follow a multivariate normal distribution, the normal limit distribution of these estimators for the eigenvectors and eigenvalues is known explicitly, see Basilevsky (1995). For other bond return distributions, the asymptotic distribution will, in general, still be the normal distribution, but the expression for the asymptotic covariance matrix is complicated. Therefore, to obtain confidence intervals for the estimates of the principal components or factor loadings, we use a bootstrap technique, which is described shortly in Appendix A. As noted by Shao and Tun (1996), the bootstrap distribution converges to the asymptotic distribution under weak assumptions on the return distribution. Hence, by using the bootstrap technique, we are able to construct confidence intervals and standard errors for the factor loading estimates without having to make the normality assumption.

We also calculate the bootstrap distribution of the eigenvalues that correspond to the eigenvectors. In Appendix A, it is shown that the model in Eq. (1) implies that the last $3M$–K eigenvalues of Σ are equal to each other. Given the i.i.d. assumption on bond returns and weak assumptions on the bond return distribution (Shao and Tun, 1996), the eigenvalues are asymptotically normally distributed, and the restriction on the eigenvalues can be tested using a standard chi-square test statistic. We use the bootstrap distribution of the eigenvalues to estimate the covariance matrix of the eigenvalues and to calculate the test statistic. By performing this test for different numbers of factors K, one can test for the number of factors.

In a second step, we estimate the prices of factor risk, λ, given the estimated factor loadings. Note that the model implies that the expected excess bond returns (and currency returns) satisfy

$$E[R_t^e] = \Gamma\lambda. \tag{7}$$

We use the generalized methods of moments (Hansen, 1982) on the moment restrictions in (7) to estimate λ, using the estimated factor loadings for Γ. This is equivalent to a GLS regression of the average bond returns on the factor loadings matrix Γ. The covariance matrix of the estimated moment restrictions and standard errors of the estimates for λ are again calculated using bootstrap techniques, and we correct these standard errors for the estimation error in the factor loadings estimates. The factor price of risk of each factor can be interpreted as the expected excess return on the factor mimicking portfolio, divided by its standard deviation

$$\lambda_j = \frac{E[w'_j R_t^e]}{\sqrt{\mathrm{Var}[w'_j R_t^e]}}, \quad j = 1,\ldots,K \tag{8}$$

where w_j contains the weights of the factor mimicking portfolio of factor j.

2.3. Duration measures

The linear factor model, defined in Eqs. (1) and (2), is an extension of linear one-factor models that correspond to Macaulay's duration measure (Macaulay, 1938) and the international duration measure proposed by Thomas and Willner (1997). Macaulay's duration measure is based on a linear one-factor model, where the factor loading of a bond is equal to the duration of this bond. In this case, the factor represents a parallel shift in the entire term structure. If applied to international bond portfolios, this model implies that bonds from different countries, but with the same duration, have exactly the same factor loading. One problem of applying Macaulay's duration measure to international bond portfolios is that parallel shifts in term structures of different countries do not have the same variance and are not perfectly correlated. Therefore, to measure the sensitivity of international bond portfolios to parallel shifts in the local term structure (in our case the US term structure), Thomas and Willner (1997) propose a modification of Macaulay's duration measure, that is again based on a linear one-factor model. In this case, each bond has a factor loading that is equal to its duration times, a so-called country-beta. Analogous to the capital asset pricing model, this country-beta is defined as the covariance of the US (all-maturity) bond index return and the foreign country's all-maturity bond index return, divided by the variance of the US bond index return.[5] As we have taken the viewpoint of a US investor, the country-beta for the US is equal to one.

The factor model in (1) can also be used to calculate duration-type risk measures for a given bond portfolio, as shown by Golub and Tilman (1997). If the bond portfolio has a weight vector w and corresponding return $w'R_t$, the 'PCA-duration' for factor j is given by $w'\Gamma_j$. Recalling that the variance of the factors F_t is equal to one, this PCA-duration measures the percentage price change of the portfolio when there is a positive shock of one (i.e. one standard deviation) to the underlying factor in the next period. As argued by Willner (1996) and Golub and Tilman (1997), using multiple PCA-durations to assess the risk of a bond portfolio typically gives a more accurate description of interest rate risk than using Macaulay's one-dimensional duration measure. We will make a similar argument for international bond portfolios, by comparing the models corresponding to Macaulay's duration measure and the duration measure of Thomas and Willner (1997), with the multi-factor multi-country model in Eq. (1).

3. Data description and results for single-country models

The data we use are total returns on Merrill Lynch Government Bond Indices for the US, Germany, and Japan, which are available through Datastream. We have chosen these bond indices because they are available at a relatively high frequency, namely weekly. These weekly data start at January 8, 1990; we use data until October

[5] Thomas and Willner (1997) define the country-beta in terms of yield changes instead of bond returns.

11, 1999, which renders 510 time-series observations on weekly bond index returns. This period includes the German reunification. We will return to this issue in the next section. For each country, five maturity classes are available: 1–3, 3–5, 5–7, 7–10, and more than 10 years. We construct excess bond returns, using Datastream data on the 1-week Eurodollar interest rate.

For all Merrill Lynch bond indices, including the all-maturity bond index, we also collect the (average) yield-to-maturity of bonds in a given index for all weeks in the sample period. As outlined in the next section, these yield-to-maturities will be used to quantitatively support the interpretation of the factor loading estimates.

The bond indices are all denominated in US $, and are not hedged for currency risk. To construct returns on bond positions that are hedged for currency risk, we use data on spot and forward exchange rates for the DM/$ and yen/$ exchange rates.[6] These data are also from Datastream. Thus, we have data on 15 returns on (currency-hedged) bond indices, as defined in Eq. (4), for US, Germany and Japan, and we have data on two currency returns, as defined in Eq. (5), for the DM/$ and the yen/$ exchange rates.

In Table 1, we provide statistics on the bond and currency returns. In almost all cases, both the average returns and the standard deviations increase with the maturity of the bond index, as expected. Average bond returns are highest for Japan, which corresponds to the decrease of Japanese interest rates over the last 10 years. We also calculate the skewness and kurtosis of the bond returns. It turns out that the return distributions are not very asymmetric. But, especially for Japanese bond returns, the tails of the return distribution are fatter than the normal distribution. Given this evidence against normally distributed bond returns, we will in the sequel calculate confidence intervals and standard errors using bootstrap techniques rather than the explicit expressions for these, which only hold under the normality assumption (see Section 2).

Recall that we assume that returns are i.i.d. distributed. To investigate whether this assumption makes sense, we report the autocorrelations of bond returns. There seems to be some negative autocorrelation in the (short-maturity) bond returns, which is consistent with mean-reverting behavior of interest rates. From the statistics on currency returns, it follows that hedging currency risk would have led to slightly higher average total returns, namely 0.34% per year for German bonds, and 0.19% for Japanese bonds. Currency returns are more volatile than hedged bond returns. The correlation between the yen/$ and DM/$ currency returns is quite high, namely 0.44, which implies that unhedged German and Japanese bond returns are more strongly correlated than hedged returns.

In Table 2, we present the average correlations between hedged bond returns and currency returns, and in Fig. 1, we plot the correlations between hedged bond returns of different maturities and different countries. In general, the cross-country bond

[6] Because 1-week forward exchange rates are not available for the entire data period, we transform 1-month forward rates to 1-week forward rates, assuming that 1-week and 1-month interest rates are equal. Because of the short forward maturity, the error caused by this assumption will be small.

Table 1
Descriptive statistics hedged bond and currency returns

	Average return	Standard deviation	Skewness	Kurtosis	Autocorrelation
US 1–3 years	6.43%	1.70%	−0.12	3.71	−0.03
US 3–5 years	7.26%	3.62%	−0.17	3.52	−0.09
US 5–7 years	7.65%	4.87%	−0.30	3.92	−0.12
US 7–10 years	7.82%	6.24%	−0.37	4.21	−0.14
US >10 years	8.49%	8.84%	−0.32	4.19	−0.14
Germany 1–3 years	6.12%	2.84%	0.06	5.98	−0.34
Germany 3–5 years	6.95%	3.56%	0.12	4.10	−0.18
Germany 5–7 years	7.45%	4.29%	−0.17	6.14	−0.14
Germany 7–10 years	7.34%	5.44%	−0.70	5.80	−0.05
Germany >10 years	8.26%	8.31%	−0.43	5.13	−0.07
Japan 1–3 years	7.50%	3.06%	0.73	9.23	−0.36
Japan 3–5 years	9.80%	4.34%	0.62	7.01	−0.16
Japan 5–7 years	10.07%	5.05%	0.25	5.88	−0.12
Japan 7–10 years	10.36%	6.19%	0.16	5.74	−0.06
Japan >10 years	10.96%	7.79%	−0.11	5.50	0.08
DM/$ currency return	−0.34%	11.18%	−0.08	4.11	−0.08
Yen/$ currency return	−0.19%	12.81%	0.98	9.19	−0.03

Summary statistics are calculated from 510 weekly observations from January 1990 until October 1999, for hedged bond returns and DM/$ and yen/$ currency returns. All returns are in US dollars, and defined as in Eqs. (4) and (5). The results are presented on a weekly basis, except for the average and standard deviation, which have been annualized assuming zero autocorrelation of weekly returns.

Table 2
Average correlations hedged bond returns and currency returns

	US	Germany	Japan	DM/$	Yen/$
US bond returns	1				
German hedged bond returns	0.29	1			
Japanese hedged bond returns	0.11	0.28	1		
DM/$ currency return	0.14	−0.08	−0.08	1	
Yen/$ currency return	−0.01	0.06	−0.05	0.44	1

Correlations are calculated from 510 weekly observations from January 1990 until October 1999, for hedged bond returns and DM/$ and yen/$ currency returns, and averaged over the five bond maturity classes for each country.

return correlations lie between 0 and 0.5, indicating that interest rate movements across countries are positively correlated. The graph also shows that, within each country, bond returns are highly correlated, and that there is no clear maturity pattern in the cross-country correlations of bond returns. Table 2 shows that the average cross-country correlations are between 0.11 and 0.29. These numbers are smaller than reported by Ilmanen (1995), who reports correlations between 0.40 and 0.55,

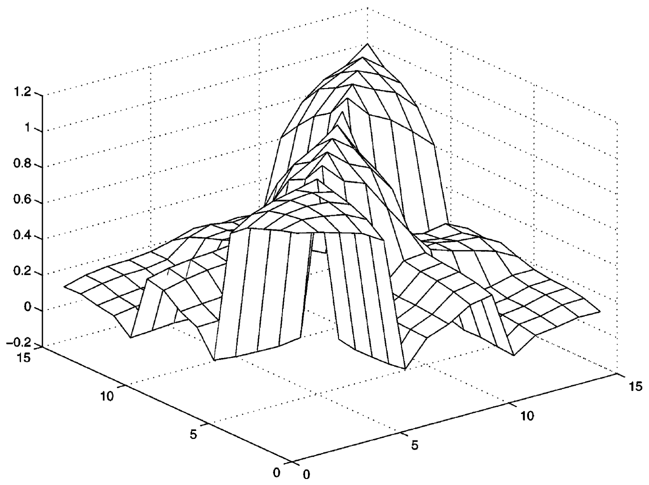


Fig. 1. Correlation matrix hedged bond returns: The graph contains correlations between (currency-hedged) bond returns for weekly data from 1990 to 1999. On the x- and y-axis, the numbers 1–5 correspond to US returns, with maturities from 1–3 to >10 years, the numbers 6–10 correspond to German bond returns (in US \$), and the numbers 11–15 correspond to Japanese bond returns (in US \$).

for the period 1978–1993 using monthly data. The correlation between hedged bond returns and currency returns is close to zero in almost all cases. Only for the DM/\$ currency return and US bond returns, there seems to be some small positive correlation.

To provide further insight into the bond return data, a linear factor model for hedged excess bond returns, as in Eq. (1), is estimated for each country separately. In line with Litterman and Scheinkman (1991) and Singh (1997), we estimate a three-factor model for the hedged excess bond returns. Estimation is performed using principal component analysis. The results for these single-country models will be used as a benchmark for the multi-country model.

In Fig. 2, the factor loadings of the single-country three-factor models are graphed. For all three countries, the estimates for the three factors are in line with the existing literature on linear factor models for bond returns and principal component analysis (see Litterman and Scheinkman, 1991; Rebonato, 1996; Singh, 1997). For the first factor, the factor loadings for bond returns all have the same sign and increase with the maturity of the bond. Note that a change in the level of the term structure will more strongly influence long-maturity bond returns. Thus, this factor can be interpreted as a factor that influences the *level* of the term structure of interest rates. The second factor can be called a *steepness* factor, as movements in this factor imply a steepening or flattening of the yield curve. The third factor changes the *curvature* of the yield curve, because it influences bond returns of intermediate maturities in the opposite direction of short-maturity and long-maturity bond returns. The confidence intervals indicate that the factor loadings are estimated quite accurately.[7]

[7] In all cases, we use 1000 bootstrap simulations to calculate confidence intervals and standard errors.

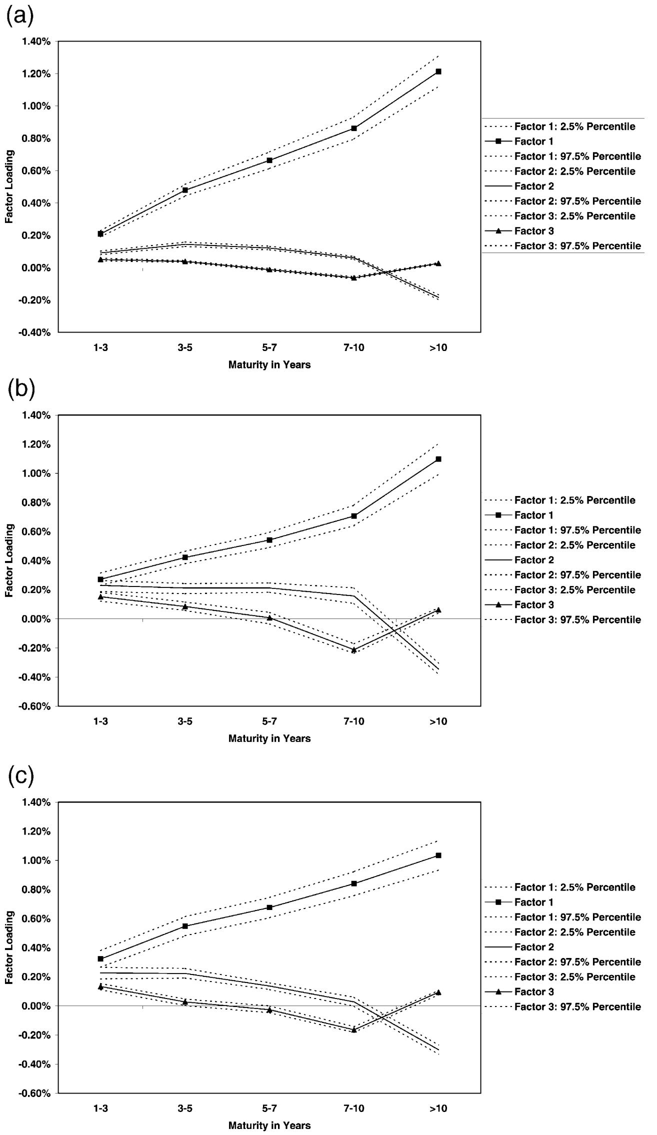


Fig. 2. (a–c) Results single-country models: For each country, a three-factor model is estimated for (hedged) bond returns. The graphs contain estimates and 95% confidence intervals for the factor loadings of the three factors. The factor loadings represent the weekly return that would be caused by a one standard deviation shock to the factor. The confidence intervals are obtained applying the bootstrap technique.

Although the shape of the factor loadings is the same for all three countries, the explained variance per factor is quite different across countries, as shown in Table 3. Using Eq. (6), it follows that, for US bonds, the first factor explains on average 96.9% of the variance of excess bond returns, and the explained variance for the

Table 3
Explained variance single-country three-factor models

	Factor 1	Factor 2	Factor 3	Total
US bond returns	96.91%	2.68%	0.24%	99.83%
	(0.27%)	(0.25%)	(0.03%)	(0.17%)
German hedged bond returns	84.91%	10.73%	2.98%	98.62%
	(2.01%)	(1.71%)	(0.32%)	(1.24%)
Japanese hedged bond returns	89.64%	7.22%	1.83%	98.69%
	(1.03%)	(0.81%)	(0.19%)	(0.75%)

For hedged bond returns in each country, a separate three-factor model is estimated using principal component analysis. The table reports the average explained variance for each factor as a fraction of the total variance of bond returns, calculated using Eq. (3), as well as the standard error of this ratio in brackets. This standard error is calculated applying the bootstrap technique.

second and third factors is quite low. In Litterman and Scheinkman (1991), the explained variance is lower for the first factor, and higher for the second and third factors. The differences with their study are due to the use of a different data period, but also due to the fact that we use returns on portfolios of bonds within a certain maturity class, whereas Litterman and Scheinkman (1991) use individual bond price data, which might contain more idiosyncratic risk. For German and Japanese bonds, the explained variance is lower compared to the US for the first factor, and higher for the second and third factors. This indicates that, over the last 10 years, large steepness and curvature movements of the yield curve have occurred more often in Germany and Japan than in the US. As shown in Table 3, for each country, three factors explain on average at least 98.5% of the variation in excess bond returns.

Finally, for every country, we test the appropriateness of one-, two-, and three-factor models. For the US, the hypothesis that the remaining eigenvalues are equal to each other is rejected for all numbers of factors. For Germany and Japan, this restriction is rejected for the one- and two-factor models, but it is not rejected in the case of the three-factor model. We will return to the issue of the number of factors when we analyze the multi-country models.

4. Empirical results of multi-country model

In this section, we will restrict attention to the 15 currency-hedged excess bond returns, and present results for the multi-country linear factor model for these bond returns. We find that a five-factor model explains on average 96.4% of the bond return variation, and 98.5% of the cross-sectional variation in average bond returns. Furthermore, the factor loadings of additional factors are always individually insignificant, as well as the risk prices of these additional factors. Therefore, we choose to restrict attention to a five-factor model. Note also that the factor loadings of these first five factors remain exactly the same if we would estimate a model with a larger number of factors, because we do not rotate the estimated factor loadings.

4.1. Regression analysis for factor mimicking portfolios

Besides a graphical inspection of the shape of the factor loadings, we will use two regressions to interpret the factor loadings of the five factors in the multi-country model. First, we regress the FMP return of *multi-country* (MC) factor i (i = 1,…,5), denoted $R_{t,i}^{\mathrm{FMP,MC}}$, on the returns of the factor mimicking portfolios of all factors of the three-factor *single-country* models for the US, Germany, and Japan. The latter returns are denoted $R_{t,j}^{\mathrm{FMP,US}}$, $J = 1,\ldots,3$, for the US. Similar notation is used for the two other countries. The factor mimicking portfolios are constructed as described in Section 2.1. This leads to the following regression equations

$$R_{t,i}^{\mathrm{FMP,MC}} = \alpha_i + \sum_{j=1}^{3} \beta_{ij} R_{t,j}^{\mathrm{FMP,US}} + \sum_{j=1}^{3} \delta_{ij} R_{t,j}^{\mathrm{FMP,GER}} + \sum_{j=1}^{3} \gamma_{ij} R_{t,j}^{\mathrm{FMP,JAP}} + \varepsilon_{t,i}, \quad i = 1,\ldots,5. \tag{9}$$

Given standard OLS assumptions, the parameters in Eq. (9) can be estimated consistently using OLS. Below, we will use these coefficient estimates to interpret the multi-country factors in terms of the single-country factors.

A disadvantage of the regression in Eq. (9) is that it shifts the interpretation of the multi-country factor loadings to the interpretation of the single-country factor loadings. Therefore, we also perform a second regression in which we directly relate the multi-country factor mimicking portfolio returns to the underlying interest rate term structures. As a measure for the level of each country's term structure, we take the yield-to-maturity of the all-maturity bond index for each country (denoted for the US and similarly for the other countries), and as a measure for the steepness of each term structure, we take the difference between the yield-to-maturity of the >10-year bond index minus the yield-to-maturity of the 1–3 years bond index (denoted for the US and similarly for the other countries). We then regress the factor mimicking portfolio return on the contemporaneous changes in the level and steepness of the term structures

$$R_{t,j}^{\mathrm{FMP,MC}} = \alpha_i + \beta_{i,1}\Delta y_t^{\mathrm{US}} + \beta_{i,2}\Delta s_t^{\mathrm{US}} + \delta_{i,2}\Delta y_t^{\mathrm{GER}} + \delta_{i,2}\Delta s_t^{\mathrm{GER}} + \gamma_{i,1}\Delta y_t^{\mathrm{JAP}} + \gamma_{i,2}\Delta s_t^{\mathrm{JAP}} + \varepsilon_{t,i}, \quad i = 1,\ldots,5 \tag{10}$$

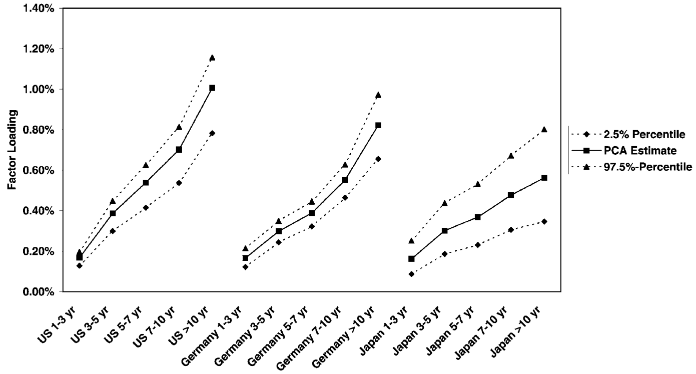
for each multi-country factor.

4.2. Interpretation of factor loadings

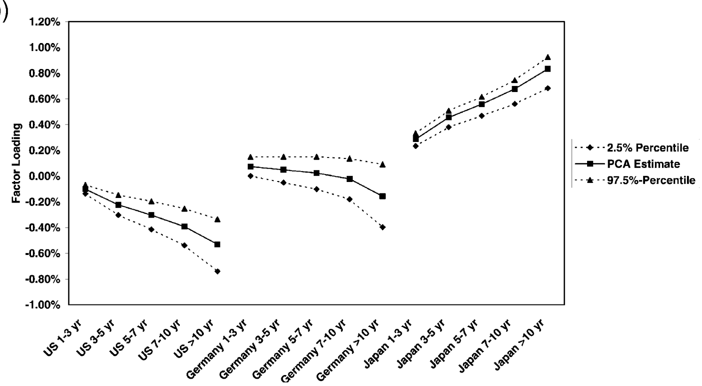
We now turn to the estimation results and the interpretation of the factor loadings. In Fig. 3a–e, we graph the estimated factor loadings together with 95% confidence intervals for the factor loadings, and in Table 4, we give for each factor the explained variance, relative to the total variance of bond returns. In Tables 5 and 6, the results of the regressions (9) and (10) are presented.

We interpret the first factor as a *world level factor*. This factor shifts the entire term structure in all countries in the same direction. This is confirmed by the

(a)



(b)



(c)

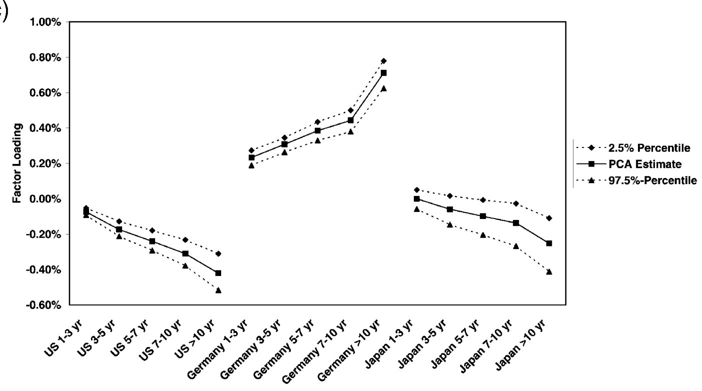
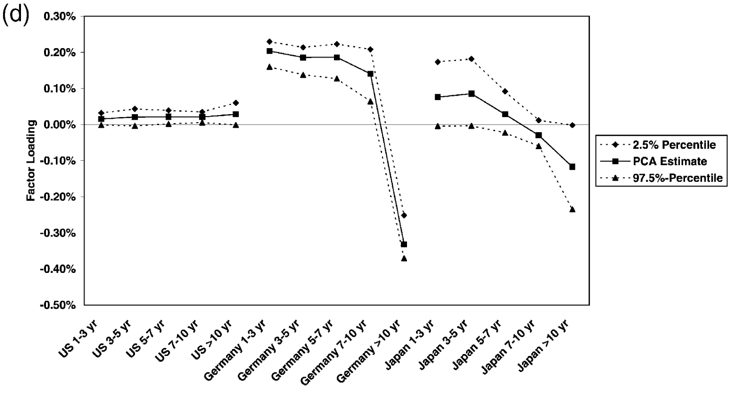


Fig. 3. (a–e) Results multi-country model for hedged bond returns. A five-factor model is estimated for (hedged) bond returns in US, Germany, and Japan. The graphs contain estimates and confidence intervals for the factor loadings of the five factors. The factor loadings represent the weekly return that would be caused by a one standard deviation shock to the factor. The confidence intervals are obtained applying the bootstrap technique.



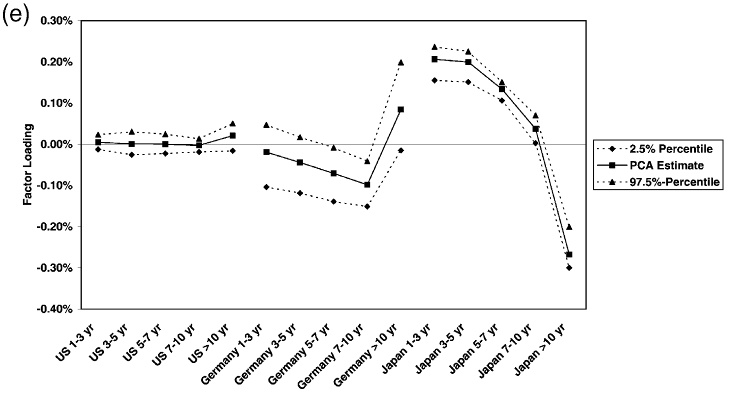


Fig. 3. *Continued*

Table 4
Explained variance multi-country models

	Multi-country model: hedged bond returns	Multi-country model: hedged bond returns and currency returns
Factor 1	46.62% (2.32%)	27.48% (0.79%)
Factor 2	27.49% (1.97%)	16.77% (1.36%)
Factor 3	16.94% (1.28%)	10.21% (0.49%)
Factor 4	3.13% (0.23%)	8.10% (0.17%)
Factor 5	2.31% (0.24%)	6.21% (0.11%)
Factor 6	–	14.94% (0.42%)
Factor 7	–	13.72% (0.34%)
Total	96.49% (1.25%)	97.41% (1.12%)

For hedged bond returns in all countries, a five-factor model is estimated using principal component analysis. For hedged bond returns and currency returns, a seven-factor model is estimated. The table reports the average explained variance for each factor as a fraction of the total variance, as well as the standard error of this ratio in brackets. This standard error is calculated applying the bootstrap technique.

Table 5
Factor mimicking portfolio regressions

	US			Ger			Japan			R^2
	FMP 1	FMP 2	FMP 3	FMP 1	FMP 2	FMP 3	FMP 1	FMP 2	FMP 3	
FMP 1	0.41	0.00	0.00	0.33	0.00	0.00	0.27	0.00	0.00	99.99%
FMP 2	−1.28	−0.01	0.00	−0.15	0.11	0.01	2.30	0.01	0.00	99.99%
FMP 3	−3.84	−0.03	0.01	6.41	0.13	0.06	−1.66	0.21	0.00	99.99%
FMP 4	0.18	0.01	0.01	−0.08	0.81	−0.01	−0.14	0.20	0.01	99.99%
FMP 5	0.16	−0.07	0.02	−0.29	−0.43	0.12	0.14	1.55	−0.01	99.99%

For each factor of the five-factor multi-country model for hedged bond returns, a factor mimicking portfolio (FMP) is constructed as described in Section 2.1. Similarly, for the three factors of each single-country model, FMPs are constructed. For each factor of the multi-country model, the corresponding FMP return is regressed on a constant and all single-country FMP returns. The table reports coefficient estimates for these regressions. Since the R^2 is almost 100% for all regressions, standard errors are very small and are, therefore, not reported.

Table 6
Regressions of factor mimicking portfolio returns on term structure changes

	US		Germany		Japan		R^2
	Level change	Steepness change	Level change	Steepness change	Level change	Steepness change	
FMP 1	−2.80	−0.98	−2.34	−0.83	−1.67	−0.27	87.32%
FMP 2	8.85	1.31	1.05	0.47	−14.09	−1.35	72.99%
FMP 3	26.09	9.54	−42.61	−8.04	9.65	3.25	74.42%
FMP 4	−1.07	−0.57	1.85	5.42	0.92	1.59	55.49%
FMP 5	−0.71	−1.76	1.53	−3.38	1.21	9.50	53.47%

For each factor of the five-factor multi-country model for hedged bond returns, a factor mimicking portfolio (FMP) is constructed as described in Section 2.1. These FMP returns are regressed on a constant, the change in the level of each country's term structure, as measured by the yield-to-maturity of the all-maturity bond index, and on the change in the steepness of each country's term structure, as measured by the difference between the yield-to-maturity of the longest-maturity bond index (>10 years), and the yield-to-maturity of the shortest-maturity bond index (1–3 years). The table reports coefficient estimates for these regressions.

regression results in Tables 5 and 6. These tables show that the first single-country factor of each country and the change in the level of the term structure of each country, respectively, have by far the largest coefficients (in absolute value) in regressions (9) and (10). This indicates that the factor mimicking portfolio return for the first multi-country factor is mostly determined by level movements of the local term structures. As shown in Table 4, this factor accounts for 46.6% of all

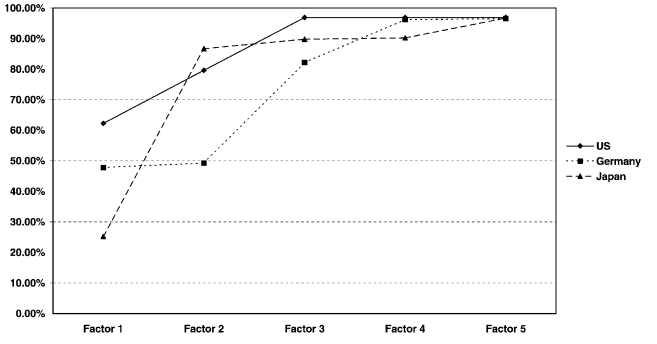


Fig. 4. Explained variance multi-country model for hedged bond returns. The graph shows the explained variance relative to the total variance in bond returns for each country, as a function of the number of factors.

variation in the international excess bond returns. In Fig. 4, we give the explained variance per country. This graph shows that the first factor explains around 60% of the variation in US bond returns, around 50% of the variation of hedged German bond returns, and 25% of the variation of hedged Japanese bond returns. As described in Section 2, the factor loadings in Fig. 3a directly describe the weights of a factor mimicking portfolio, which has a weight of 41% in US bonds, 32% in German bonds, and 27% in Japanese bonds.

As described in Section 2, this first factor is also related to Macaulay's duration and the multi-country duration of Thomas and Willner (1997). In Fig. 5, we graph the factor loadings of the one-factor models that correspond to these two duration

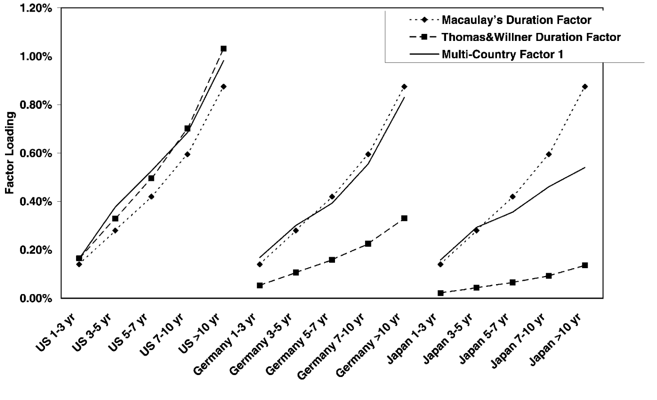


Fig. 5. Duration factors: The graph shows the factor loadings associated with Macaulay's duration and the duration of Thomas and Willner (1997). The graph also contains the factor loadings of the first factor of the multi-country model. The factor loadings represent the weekly return that would be caused by a one standard deviation shock to the factor.

measures.[8,9] This graph shows that the first factor of our multi-country model is closely related to the Macaulay-duration factor, which confirms our interpretation of this factor as a world level factor. This also implies that the factor mimicking portfolios that correspond to factors 2–5, (analyzed below), have a Macaulay duration that is close to zero. The graph also shows that, since the duration measure of Thomas and Willner (1997) aims at measuring the international bond portfolio sensitivity to shifts in the US term structure, and because the country-beta's are equal to 0.32 for Germany and 0.13 for Japan, the factor loadings that correspond to this duration measure are low for German and Japanese bonds. Given that the linear one-factor models that correspond to Macaulay's duration and the duration of Thomas and Willner (1997) are restricted versions of the general linear one-factor model, the fact that this general first factor captures around 47% of total bond return variation, indicates that these two duration measures only capture some part of all movements in the term structures of several countries. On the other hand, as long as one invests in internationally diversified portfolios, with country weights close to the weights of the factor mimicking portfolio, the one-factor model can be used to calculate accurate risk measures.

We interpret the second factor, whose factor loadings are plotted in Fig. 3b, as a *Japan minus US level factor*. Again, this factor primarily influences the level of the term structure of interest rates, as the factor loadings have the same sign within each country and because the factor loadings are increasing with maturity. This factor influences Japanese bond returns in the opposite direction of US bond returns, while German bond returns are hardly influenced by this factor. This interpretation is confirmed by the regression results in Tables 5 and 6. In particular, Table 6 shows that the FMP return of the second factor is strongly related to opposite level movements of the Japan and US term structures. This factor mimicking portfolio consists of long positions in Japanese bonds, and short positions in US bonds. Thus, for a given bond portfolio, the PCA-duration associated with this factor measures how sensitive the portfolio is to a change in the difference between the US and Japanese term structures. This factor explains 27.5% of the average bond return variation. In particular, it explains around 60% of Japanese bond return variation and 20% of US bond return variation.

Similarly, we interpret the third factor as a *Germany minus US level factor*; this factor explains 16.9% of the international bond return variation. We interpret the fourth and fifth factors as a Germany steepness factor and a Japan steepness factor;

[8] For the Macaulay-duration one-factor model, the variance of the factor is chosen such that the factor loadings are close, in a least-squares sense, to the factor loadings for the first factor of the multi-country model. As the one-factor model of Thomas and Willner (1997) aims at measuring the sensitivity to US term structure movements, we choose the factor variance such that, for US bonds, the factor loadings are close to the factor loadings of the first factor of the multi-country model.

[9] To calculate these factor loadings, we assume that the durations of the bond indices are equal to 2, 4, 6.5, 8.5 and 12.5 years, respectively. The country-beta's of Thomas and Willner (1997) are estimated using US, German and Japanese all-maturity bond index returns, that are constructed by equally weighting the five maturity classes that are available for each country.

these factors explain 3.1% and 2.3% of the bond return variation, respectively. The regression results in Table 6 show that each of these two factors is strongly related to changes in the steepness of the yield curves in Germany and Japan, respectively, while these two factors are not or only weakly related to the other term structure variables. In fact, these steepness factors are almost the same as the steepness factors in the single-country models for Germany and Japan, which is confirmed by the regression results in Table 5.

The fact that the steepness factors are almost completely country-specific, whereas the level movements in international yield curves are correlated, implies that the positive correlation between international bond returns seems to be caused by correlation between the levels of the international yield curves. This result is, however, sensitive to the data period chosen. The 1990–1999 period includes the period immediately after the German reunification in 1990, in which German interest rates exhibited unusual behavior relative to the US and Japan. In particular, during the 1990–1994 period, the German term structure was mostly inverted, so that especially the steepness of the German term structure was different from ‘normal’. To analyze the impact of the German reunification, we reestimate the linear factor model using data from October 1994 until October 1999.[10] Essentially, we find that only the results for the fourth and fifth factors change. The factor loadings for these two factors are graphed in Fig. 6a–b. The fourth and fifth factors now explain 2.8% and 2.0% of the bond return variation, respectively. The fourth factor corresponds to steepness movements of the German and Japanese term structures in the same direction, while the fifth factor corresponds to steepness movements of the German and Japanese term structures in the opposite direction. Since the factor loadings on the fourth factor are somewhat larger than on the fifth factor, there is a net positive relation between changes in the steepness of the German and Japanese term structures for the 1994–1999 period. We thus conclude that during the 1990–1994 period, movements in the steepness of the German term structure were uncorrelated with term structure movements in other countries, due to the purely country-specific reunification event. Afterwards, i.e. from 1994 to 1999, there has been a positive relation between steepness movements in Germany and Japan.

As mentioned above, for additional factors, the factor loadings are always individually insignificant. In particular, this implies that steepness movements in the US yield curve are not included in the five-factor model. In the loadings of additional factors, we do find steepness shapes for the US, but the explained variance of these factors is very low, which is in line with the results for the single-country model for the US. We also test for the number of factors, by testing for each K-factor model whether the remaining (15-K) eigenvalues are equal to each other, as described in Section 2 and Appendix A. It turns out that even the model with 13 factors is rejected, as the hypothesis that the 14th and 15th eigenvalues are equal to each other is rejected. As noted by Basilevsky (1995), these formal statistical tests typically

[10] In 1994, German interest rates returned to normal levels and the shape of the term structure changed from inverted to increasing.

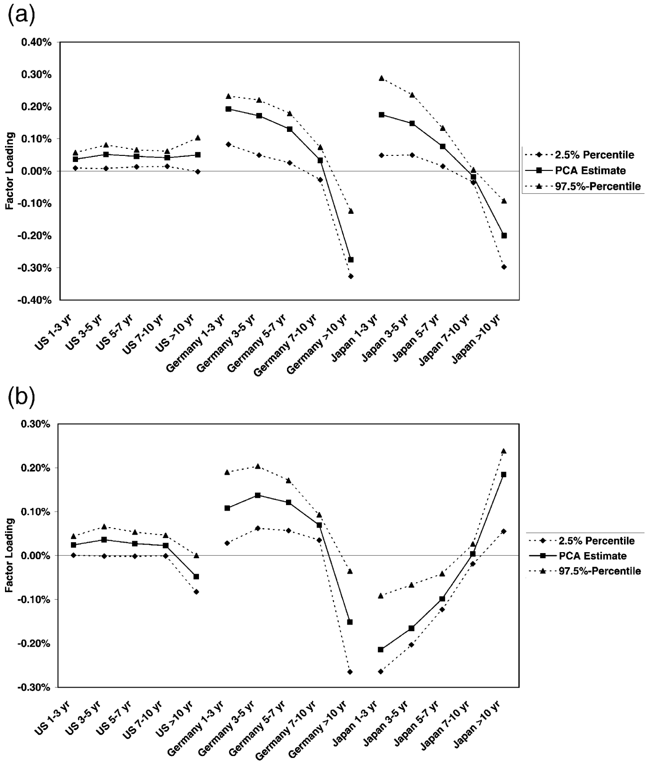


Fig. 6. (a–b) Subsample Oct 1994–Oct 1999. Results multi-country model for hedged bond returns: Using data for the period October 1994–October 1999, a five-factor model is estimated for (hedged) bond returns in US, Germany, and Japan. The graphs contain estimates and confidence intervals for the factor loadings of the fourth and fifth factors. The factor loadings represent the weekly return that would be caused by a one standard deviation shock to the factor. The confidence intervals are obtained applying the bootstrap technique.

tend to overestimate the number of factors in small samples, so that other considerations, such as the explained variance relative to the total variance and the interpretation of the factor loadings are also important when choosing the number of factors. Therefore, we do not attempt to extend the five-factor model with more factors.

4.3. Estimation of factor risk prices

As described in Section 2, in a second step, the prices of factor risk for this multi-country model can be estimated using GMM or, equivalently, a GLS regression of average bond returns on the factor loadings matrix Γ. As shown in Eq. (8), each factor price of risk equals the Sharpe-ratio of the factor mimicking portfolio. The estimates and corresponding standard errors are given in Table 7. This table shows that only the first two factors, the *world level factor* and the *Japan minus US level factor*, have a risk price that is significant at a 10% significance level, and the size

Table 7
Factor risk prices multi-country model for hedged bond returns

	Factor price of risk (standard error)	J-statistic (p-value)	Cross-sectional R^2
Factor 1	0.81 (0.33)	28.32 (0.013)	75.5%
Factor 2	0.56 (0.34)	25.27 (0.021)	96.5%
Factor 3	−0.13 (0.34)	25.12 (0.014)	97.2%
Factor 4	0.23 (0.32)	24.57 (0.010)	97.6%
Factor 5	0.43 (0.33)	22.88 (0.011)	98.5%

For each factor of the multi-country model, the market price of factor risk is estimated using GMM as described in the text. The table reports annualized market prices of risk, which can thus be interpreted as the expected excess return on the factor mimicking portfolio, divided by the standard deviation of this excess return. Standard errors are in brackets, and calculated applying the bootstrap technique. As a test of the overidentifying restrictions in Eq. (7), we also present the GMM J-statistic and the associated p-value, for one-factor to five-factor models. The number of overidentifying restrictions is equal to 15 minus the number of factors. The last column contains the R^2, which measures the fit on expected returns for one-factor to five-factor models.

of the factor risk prices is also largest for the first two factors. Setting all insignificant risk prices to zero, the results on the risk price estimation imply that the mean–variance efficient frontier is spanned by the two factor mimicking portfolios that correspond to the first two factors.

Table 7 also contains the GMM J-statistic, a test-statistic for testing the overidentifying restrictions in Eq. (7), and the corresponding p-value. The hypothesis that the five-factor model correctly describes expected returns of all bonds is statistically rejected. However, as discussed above, the individual factor risk prices are all insignificant except for the first two factors. Furthermore, the five-factor model provides a good fit of the average bond returns, as measured by the R^2 of the GLS regression, which is equal to 98.5%. This is confirmed by Fig. 7, where we graph the expected

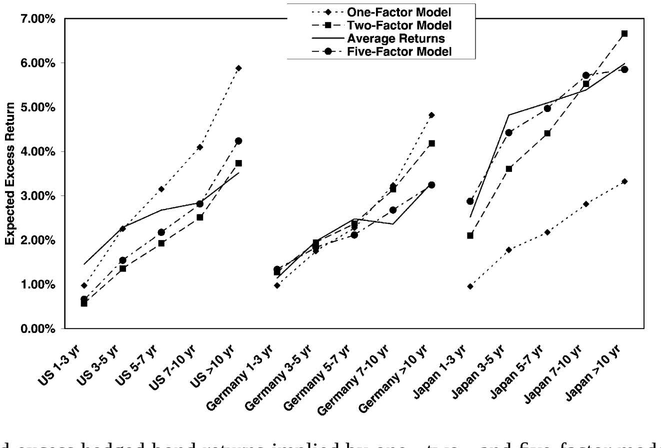


Fig. 7. Expected excess hedged bond returns implied by one-, two-, and five-factor models, and average returns. The figure contains the expected excess hedged bond returns, as implied by one-, two-, and five-factor models (Eq. (7)). The sample averages for the excess hedged bond returns are also graphed.

excess hedged bond returns implied by several models, and unrestricted estimates, the sample averages, of these expected returns. The figure shows that the two-factor model already gives a reasonable fit of the average bond returns, which is in line with the fact that only the first two factors have a statistically and economically significant risk price. The fact that the GMM *J*-test still leads to a rejection of the five-factor model, is due to the fact that bond returns are highly correlated within countries. This implies that even small expected return errors of two near-maturity bonds are enough to statistically reject the model.

5. An extension to unhedged bond returns

In this section, we analyze a multi-country model that jointly describes both hedged bond returns and DM/\$ and yen/\$ currency returns. This way, the risk of international bond portfolios that are not (or partially) hedged for currency risk can also be analyzed with a multi-country model. Therefore, we re-estimate the linear factor model in Eq. (1), but now for the 17-dimensional excess return vector $\tilde{R}_t^e$. Again, we choose to estimate a low-dimensional factor model, because almost all variation in the bond returns can be explained by a low number of factors. More specifically, we estimate a seven-factor model. As shown in Table 4, this model explains 97.4% of the bond and currency return variation.

To facilitate the interpretation of the factors, we perform an orthonormal rotation of the estimated factor loadings. We rotate the seven factors, by choosing the orthonormal rotation matrix that minimizes the sum of squared differences between the rotated factor loadings for hedged bond returns and the factor loadings of the five factors of the multi-country model of Section 4. It turns out that it is possible to obtain almost exactly the same factor loadings for hedged bond returns for the first five factors as for the multi-country model of Section 4. Therefore, we only report the loadings on the hedged bond returns for factors 6 and 7, as well as the loadings on the currency returns for all seven factors (see Fig. 8a–c. In Fig. 8a we see that, of the first five factors, the German and Japan steepness factors are correlated with movements in the yen/\$ and DM/\$ exchange rates, respectively. The sixth and seventh factors are essentially currency factors. The sixth factor mostly influences the yen/\$ exchange rate, and hardly influences hedged bond returns. The seventh factor primarily influences the DM/\$ exchange rate. This factor also causes some movements in the term structures of the US and Japan. We again test for the number of factors, and find that, even for the 15-factor model, the hypothesis of equality of the 16th and 17th eigenvalues is rejected. However, the explained variation of these additional factors is low, and the factor loadings are always individually insignificant.

Summarizing, including the currency returns requires two extra factors to explain the same amount of variation in hedged bond returns, but these two additional factors are not simply a DM/\$ factor and a yen/\$ factor. Instead, to account for the correlation between the two currencies and the correlation between bond and currency returns, all factors influence both bond and currency returns.

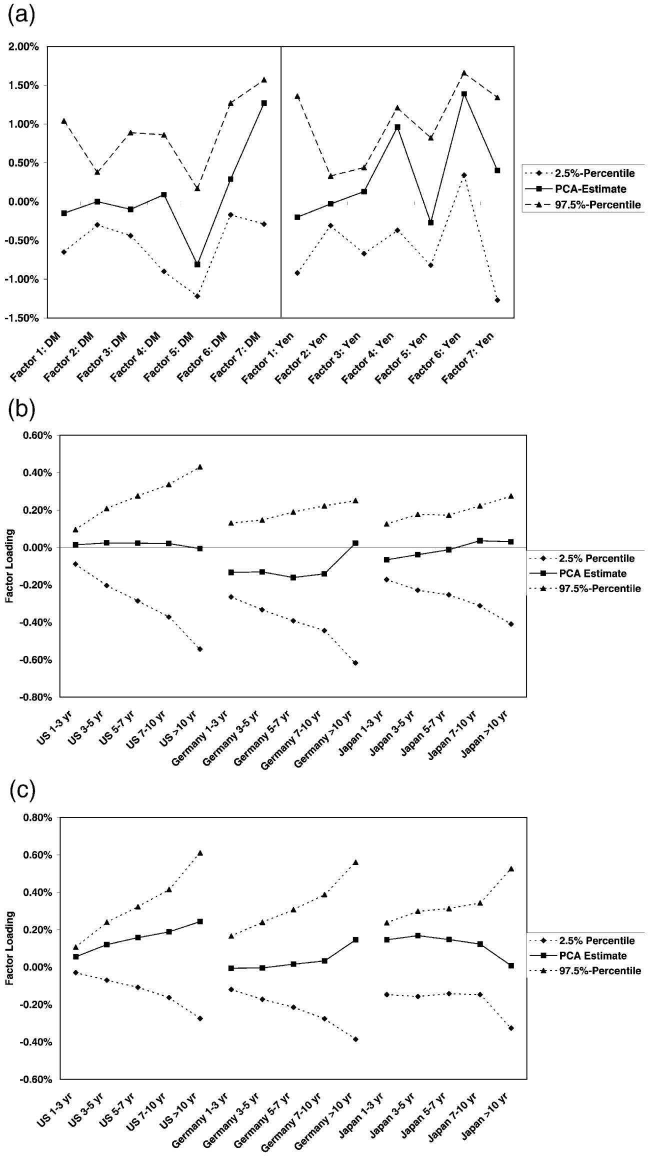


Fig. 8. (a–c) Results multi-country model for hedged bond returns and currency returns: A seven-factor model is estimated for (hedged) bond returns in US, Germany, and Japan, and DM/$ and yen/$ currency returns. The factor loadings are rotated as described in the text. Fig. 8a contains estimates and confidence intervals for the factor loadings of currency returns for the seven factors. The factor loadings represent the weekly return that would be caused by a one standard deviation shock to the factor. The confidence intervals are obtained applying the bootstrap technique. (b) and (c) contain the factor loadings of hedged bond returns for factors 6 and 7.

Note that the confidence intervals of the factor loadings for the currency returns are much larger than for the hedged bond returns. Apparently, the correlations between hedged bond returns and currency returns are less accurately estimated than the correlations between hedged bond returns in different countries.

Next, we again estimate the market prices of factor risk for the seven factors of this model. The results, not reported here, are very similar to the case of hedged bond returns: the first two factors have the largest prices of risk, and these two factor risk prices are significantly different from zero at the 10% significance level. For all other factors, the prices of risk are insignificant. In particular, the sixth and seventh factors, that represent primarily currency movements, have insignificant prices of risk.

Finally, we note that, if one is only interested in unhedged bond returns, it suffices to estimate a linear factor model for the 15 unhedged bond returns only. We have performed this estimation and the results show that the largest difference between this model and the model for hedged bond returns only is given by the factor loadings of the first factor. Therefore, in Fig. 9, we graph the factor loadings of the first factor of the model for unhedged bond returns only. This figure shows that, in this case, the first factor still influences bond returns of all maturities and all countries in the same direction, but the loadings for Germany and Japan are much larger than for the US. This is due to the currency risk associated with the unhedged bond returns for Germany and Japan.

6. Conclusions

In this paper, we jointly analyze bond returns of different maturities in the US, Germany and Japan. In particular, by specifying and estimating a linear factor model

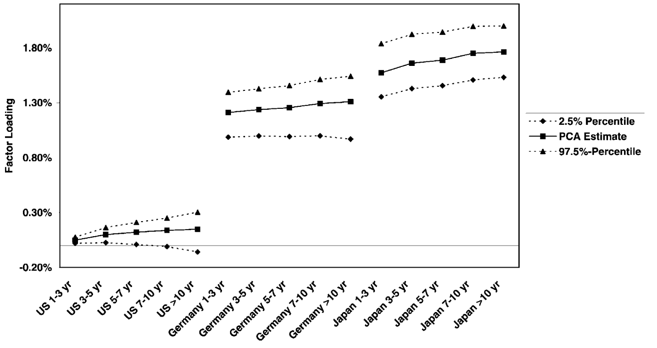


Fig. 9. First factor of model for 15 unhedged bond returns: A linear factor model is estimated for 15 *unhedged* bond returns in US, Germany, and Japan. (a) contains estimates and confidence intervals for the factor loadings on bond returns for the first factor. The factor loadings represent the weekly return that would be caused by a one standard deviation shock to the factor. The confidence intervals are obtained applying the bootstrap technique.

for these bond returns, we attempt to identify the common factors that determine these international bond returns.

We find that a five-factor model explains almost all variation in international bond returns that are hedged for currency risk. Using a regression analysis for the factor mimicking portfolios, we show that all these factors either influence the level or the steepness of the term structure of interest rates in different countries. Changes in the level of the term structure turn out to be positively correlated across countries, while changes in the steepness of the term structures are country specific. The latter result is partly caused by the German reunification, that led to unusual behavior of German interest rates.

The five-factor model also provides a good fit of the expected returns on the bonds of different maturities and different countries. Estimation of the factor risk prices reveals that only the first two factors have significant risk prices.

Finally, we extend the model by adding currency returns, so that the model describes returns on bond positions that are not hedged for currency risk as well. In this case, a seven-factor model explains almost all variation in bond and currency returns. The first five factors of this model are very similar to the five factors of the model for hedged bond returns only, and the additional two factors mainly describe currency returns.

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Appendix A. Bootstrapping of principal components analysis

In this appendix, we show how the bootstrap technique can be used to calculate standard errors and confidence intervals for the PCA estimates.

Given the linear factor model defined in Eqs. (1) and (2), the covariance matrix of bond returns can be decomposed as in Eq. (3). The first K principal components $x_1,\ldots, x_K$ of the covariance matrix Σ are defined by:

$$\begin{aligned} &\sum x_i = \delta_i x_i, \\ &x'_i x_i = \delta_i, \quad i = 1,\ldots,K, \\ &\delta_1 \geq \delta_2 \geq \ldots \geq \delta_K \geq \sigma^2. \end{aligned} \tag{A.1}$$

As shown by Basilevsky (1995), for the linear factor model in Eq. (1), the solution to (A.1) is given by:

$$\begin{aligned} &x_i = \Gamma_i \sqrt{\frac{\Gamma'_i \Gamma_i + \sigma^2}{\Gamma'_i \Gamma_i}}, \\ &\delta_i = \Gamma'_i \Gamma_i + \sigma^2, \quad i = 1,\ldots,K \\ &\text{with } \Gamma'_1 \Gamma_1 \geq \Gamma'_2 \Gamma_2 \geq \ldots \geq \Gamma'_K \Gamma_K. \end{aligned} \tag{A.2}$$

Furthermore, the remaining $3M–K$ eigenvalues are all equal to σ^2.

Hence, given a sample estimate $\hat{\Sigma}$ of the covariance matrix Σ and corresponding eigenvectors or principal components $\hat{x}_1,\ldots,\hat{x}_K$ and eigenvalues $\hat{\delta}_1,\ldots,\hat{\delta}_K$ of $\hat{\Sigma}$, estimates of the factor loadings σ^2 and the residual variance can be obtained as follows:

$$\hat{\Gamma}_i = \hat{x}_i\sqrt{\frac{\hat{\delta}_i-\hat{\sigma}^2}{\hat{\delta}_i}}, \quad i = 1,\ldots,K$$

$$\hat{\sigma}^2 = \frac{1}{3M-K}\sum_{i=K+1}^{3M}\hat{\delta}_i. \tag{A.3}$$

To approximate the bootstrap distribution of these estimates, suppose one has T time-series observations on the $3M$ bond returns. We assume that these bond returns are i.i.d. distributed over time. Then, draw T times with replacement from these T time-series observations, and calculate eigenvectors, eigenvalues, and the factor loadings from Eq. (A.3). By repeating this procedure sufficiently many times, the bootstrap distribution of the eigenvectors, eigenvalues and factor loadings can be approximated, which can be used to construct confidence intervals and test statistics as described in the text. For example, if we define as the vector that contains the pairwise differences between the smallest $3M–K$ eigenvalue estimates, $(\hat{\delta}_{K+2}-\hat{\delta}_{K+1}),\ldots,(\hat{\delta}_{3M}-\hat{\delta}_{3M-1})$, and if we define $\hat{V}$ as the bootstrap covariance matrix of this vector, the test statistic for testing whether the smallest $3M–K$ eigenvalues are equal to each other, is given by:

$$\hat{v}'\hat{V}^{-1}\hat{v}. \tag{A.4}$$

Under the assumption that bond returns are i.i.d. distributed, the vector is, under the null hypothesis, asymptotically normally distributed, with mean zero and covariance matrix V, so that the test statistic is asymptotically χ^2_{3M-K-1} distributed.

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