

Lecture Note 2.2: Interest Rate Swaps

A huge segment of the derivatives market consists of swap agreements where the periodic payments are tied to prevailing interest rates in the future. These claims are a crucial tool of financial engineering and risk management.

In this note, we'll study the most fundamental type of interest rate swap in our no-arbitrage framework. In fact, when we step back from the most idealized contracts and settings, it becomes impossible to exactly replicate/hedge the cash-flows. But that is part of the reason the swaps market is so big: they are not always redundant, but are themselves primary securities that help complete the market.

Outline:

- I.** Standard interest rate swaps
- II.** The underlying markets
- III.** Valuation when the floating rate is riskless
 - (A)** Dynamic replication
 - (B)** Static replication
- IV.** Risky floating rates
- V.** Speculating with swaps: an example
- VI.** Summary

I. Standard interest rate swaps.

- You may already be familiar with how basic – vanilla – interest rate swaps work.
 - ▶ One party agrees to make fixed periodic payments – which are quoted as an interest rate on a notional principal amount.
 - ▶ The other pays a *variable* rate on that amount.
 - ▶ Typically both exposures originate with bond issues.
 - ▶ Notice that since both of the payments are based on the same notional amount, the principals at maturity exactly cancel (unlike in currency swaps).
 - ▶ The swap is described by:
 - (A) The notional amount;
 - (B) The fixed rate;
 - (C) The variable rate fixing dates;
 - (D) The method of determination of the variable rate;
 - (E) The money exchange dates, and instructions.
 - ▶ As of March 2013, all ordinary IRSs in the U.S. are required to be cleared through a central counterparty.

► **Example of swap terms:**

- * 7-year \$100m swap of 3.1% fixed (quarterly compounded) against 90-day LIBOR.
- * Quarterly re-set; quarterly payment. Day count = actual/360.
- * LIBOR as per BBA Reuters screen at 12:00 GMT.

► Note that the variable cash-flow paid on any given date has been determined 90 days earlier (in this example).

► The 90-day period is called the *tenor* of the swap.

- Up until recently *all* IRS trading was OTC.
- Starting in late 2013 and continuing today, USD IRS trading has migrated to electronic platforms called *swap execution facilities* (SEFs).
 - European regulations mandating SEF trading will be phased in through 2016.
- According to CFTC rules, SEFs are supposed to promote transparent pricing – pre- and post-trade – and equal, open access to anybody who wants to participate.
- There are today some 15-20 platforms competing for business. Everyone expects that only a handful will survive the next couple of years.

- The SEFs are supposed to support (at least) one of two main types of trading.

RFQ – or Request for Quote. This is essentially the e-version of how OTC markets used to work. A client chooses the swap he is interested in (perhaps from a menu) and which side of the market he is on and how large a trade he wants to do. The SEF then broadcasts the client's interest to at least 3 other members (dealers) who can post responses. The client can then trade with one of them or walk away.

LOB – or Limit Order Book. This is how most public markets now work. They are open-access continuous markets where anybody can post binding limit orders for any swap at any price (and cancel or change it at every time) and everybody can see all posted prices and quantities (although not the identities of the people who submitted them). Trade then occurs when someone decides to submit a market order.

- Because SEF trading is easier when the menu of contracts is limited, the market has recently introduced the idea of only allowing the fixed-rate side of contracts to take on a limited number of values, for example 50bp, 100bp, 150bp, and so on.

► These are being called “market agreed coupon” or MAC swaps.

* They also fix the re-set/payment dates to be specific calendar days that don't change.

- ▶ In the usual OTC world – and still in most SEF trading – the fixed rates change every day to make the contracts have zero initial value.
 - * But that has the disadvantage that old contracts and new contracts do not exactly off-set and the market has to be able to trade old swaps with any possible fixed rate on any date.
 - * Likewise, the OTC market has to cope with swaps whose payment and expiration days can be arbitrary.
- MAC swaps are an example of features of a derivative contract evolving in response to changes in market infrastructure.
- Interest rate swaps play a huge role in the capital markets. They are used by thousands of companies, governments, banks, and pension funds as a way of managing mismatches that may occur between the interest rate exposure in their liabilities and that in their assets.
- Yet misunderstanding them, has been the cause of many of the largest derivatives disasters.
- In fact, basic interest rate swaps are not that hard to understand.
 - ▶ Don't get intimidated by the ugly notation!

II. The underlying market.

- For today, we are concerned with trades in the fixed-income market where the underlying securities are bonds of companies or governments.

► Let's review how these markets work and what we are really assuming about them.

In particular, we will confine ourselves to the U.S. dollar-denominated markets.

- First, for government debt, our discussion will assume the existence of zero coupon bonds – as usual denoted $B_{t,T}$ – to any given maturity T .
- In reality, the government only issues zero-coupon bonds out to maturities of one year.
 - However, they also issue coupon bonds out to 30 years maturity AND the government permits the coupons themselves to be detached and traded separately.
 - These detached coupons, then, are themselves little zero-coupon bonds.
 - These are called STRIPS. There are Treasury strips with maturities of every February, May, August, and November of every year. So, not literally every maturity is traded, but still a lot.
- We also assume that you can borrow these $B_{t,T}$ s at a known fixed rate in case you want to short-sell them.

- ▶ This is accurate. The market for borrowing/lending government bonds is called the **repo** market.
- ▶ Dealers will be happy to either lend you money, taking bonds as collateral – or do the reverse, which allows you to short the bonds after you borrow them.
 - * For arcane historical reasons, these loans are actually structured as legal sales of the underlying bond with a simultaneous agreement to buy it back (hence “repurchase agreement” or repo) later.
- ▶ Unless your particular government bond is in very short supply (called being “on special”) it is considered **general collateral** and the borrowing/lending rates are almost the same as the Federal Funds rate or the short term T-bill rate.
 - * Thus funding a general collateral short position is nearly costless.
- While you can generally lock-in **term repos** for periods of several weeks, it is not always possible to guarantee a fixed borrowing cost for longer periods (e.g. several months).
 - ▶ So this introduces some uncertainty in the cost of carrying short positions for extended durations.
- What about non-governmental rates?
- Since the major private sector players in the swaps markets are big banks, what are their primary funding vehicles?

- ▶ Answer: the major banks all publish continuous menus of 2-sided markets for their own deposits for virtually any maturity (usually out to 1-year).
- ▶ These deposits, once originated, are not redeemable early by either the borrower or the lender. (Nor are they guaranteed by the government.)
- ▶ So they are effectively little zero-coupon bonds.
 - * Lending to the bank (at the bid rate) is economically the same as buying one of their deposits.
 - * Borrowing from the bank (at the ask rate) is the same as shorting one of the deposits.
- ▶ But note that only other big banks (and pre-approved high-quality customers) can freely borrow at these interbank rates.
- Below when I am considering a bank interest rate, or equivalently the dollar price of the deposit, I will use the superscript *ED* (for eurodollar).
- Finally, for non-bank private sector entities, there are usually very few publically traded debt instruments that we can use to do arbitrages.
 - ▶ We will see later that we can use *credit derivatives* to synthetically construct little zero-coupon bonds for them too.
 - ▶ But for now let's just think about government and bank rates.

III. Valuation: riskless floating rate.

- Back to swaps.
- At first glance, interest rate swaps seem a lot more complicated than simple fixed price delivery contracts (like commodity swaps) since we don't know all the cash flows in advance.
- It turns out that replicating them isn't that tough.
- Let's first think about valuing one in which the floating rate is *the riskless interest rate* between payment periods.
- Do this by considering the values of the fixed and floating sides separately, and then subtracting.
- The fixed side is trivial (just like for interest-only currency swaps). If the notional amount of the swap is X and the fixed interest rate (the "swap rate") is s , this side is just worth

$$\text{fixed side value} = s X \sum_{t=1}^T B_{0,t}$$

- The floating side is a little trickier. At date t , this side gets an amount that I will denote $X R_{t-1,t}$ where R is the riskless simple, per-period interest rate (i.e. not annualized).
- Question: how can we replicate a payment stream of amounts that are *unknown today*?
- One answer: **dynamically!**

- Consider the cash-flows from a strategy that does the following transactions:
 - (A) Sell short X face-value of risk-free zero coupon bonds (treasury-bills) maturing at T .
 - (B) Buy X worth of one-period t-bills, i.e. maturing at t the next payment date.
 - (C) When we get to t , reinvest the same principle (X) in the *new* one-period t-bills, i.e. maturing at $t + 1$.
- Then at each swap date, $t < T$, the cash-flow is just the one-period interest earned, $X R_{t-1,t}$.
- And at T it is the same because we use our principle X to pay for the maturing T -period bills that we sold in step 1.
- What is the cost of this strategy?
- The cost is just the net cash we take in today, which is

$$\text{floating side value} = X - XB_{0,T}.$$

This is the cost of funding a dynamic strategy that exactly replicates the floating-side cash-flows. Hence if the floating side value is not this amount, we have an arbitrage.

- **Conclusion:** Net value of the swap to the fixed payor is

$$V = 1 - B_{0,T} - s \sum_{t=1}^T B_{0,t}$$

(times the notional amount X).

- Also note that

$$\begin{aligned} 1 - B_{0,T} &= \sum_{t=1}^T [B_{0,t-1} - B_{0,t}] \\ &= \sum_{t=1}^T B_{0,t} \left[\frac{B_{0,t-1}}{B_{0,t}} - 1 \right] \\ &= \sum_{t=1}^T B_{0,t} R_{0,t-1,t}^f. \end{aligned}$$

- The last line uses the definition of the one-period *forward rate*.
 - ▶ $R_{0,t-1,t}^f$ is then the (simple) riskless rate we could lock in today between dates $t - 1$ and t .
 - ▶ It is not hard to show that the no-arbitrage price of a forward contract to buy a bond at t_1 which matures at t_2 is $B_{0,t_1,t_2}^f = \frac{B_{0,t_2}}{B_{0,t_1}}$.
 - ▶ The forward interest rate is then related to the forward bond price by

$$B_{0,t_1,t_2}^f = \frac{1}{1 + R_{0,t_1,t_2}^f}$$

- Using the forward rate expression for the valuation of our floating side, the swap valuation formula is then

$$V = X \left[\sum_{t=1}^T B_{0,t} R_{0,t-1,t}^f - s \sum_{t=1}^T B_{0,t} \right] = X \left[\sum_{t=1}^T B_{0,t} (R_{0,t-1,t}^f - s) \right].$$

- **We can view interest rate swap values as weighted sum of the spreads of the fixed over the floating forward rates.**
- If we set $V = 0$ and solve for the fair swap rate s we get

$$s = \frac{\sum_{t=1}^T R_{0,t-1,t}^f B_{0,t}}{\sum_{t=1}^T B_{0,t}}.$$

- Just like for commodity swaps, **the fair rate of an interest rate swap is a weighted average of today's forward rates.**

- The above derivation was the first time we have used a *dynamic replication* strategy to price a derivative.
- Interestingly, we could have derived the same formulas by staying within the static replication framework we used to value commodity swaps.
- That is, we can still view them as an agreement by one side to deliver a fixed amount of a commodity in exchange for a fixed amount of cash.
- The trick is realize that the “commodity” is a bond!
 - Specifically, it is a risk-free zero-coupon bond which matures a fixed amount of time after the delivery date.
- Consider these two deals:

	Swap 1:		Swap 2:	
	side 1 pays	side 2 pays	side 1 pays	side 2 pays
t_0	Y bonds maturing at t_1	X \$	-	-
t_1	-	-	Y \$	$X(1 + R_{t_0,t_1})$ \$
t_2	Y bonds maturing at t_3	X \$	-	-
t_3	-	-	Y \$	$X(1 + R_{t_2,t_3})$ \$
\vdots	\vdots			\vdots

- Swap 1 is what I just described – same as a commodity swap.
- Swap 2 is nothing but the same goods exchanged in Swap 1, but held for one more period and then exchanged.

- Clearly, the swaps are worth the same. The time- t_1 value of the time- t_2 cash-flows in Swap 2 are exactly the values in Swap 1.
- Now rearrange the cash-flows in Swap 2:

	side 1 pays	side 2 pays
t_1	$Y - X \text{ or } X(\frac{Y}{X} - 1)$	$X R_{t_0, t_1}$
\vdots	\vdots	\vdots

- It is just a plain vanilla interest rate swap with notional amount X and fixed rate $s = \frac{Y}{X} - 1$.
- Now recognize that *we already learned how to value Swap 1* by no-arbitrage. Hence we know the no-arbitrage value for Swap 2.
- To value Swap 1 – just like the other commodity swaps – all we need are forward prices of the “commodity” – which is a one-period bond.
- So we can conclude that the value (to side 1) of our swap is

$$V = X \sum_{t=0}^{T-1} B_{0,t} - Y \sum_{t=0}^{T-1} (B_{0,t} B_{0,t,t+1}^f)$$

where, as above, B_{0,t_1,t_2}^f is the forward price of a contract to buy a t-bill at t_1 which matures at t_2 .

- If the notional amount is $X = 1$ and $Y = 1 + s$ then it has the same value as an interest rate swap with fixed rate s , i.e. Swap 2.

- So plug those values in. Also use our earlier expression $B_{0,t_1,t_2}^f = \frac{B_{0,t_2}}{B_{0,t_1}}$. And now our valuation formula says

$$\begin{aligned}
 V &= \sum_{t=0}^{T-1} B_{0,t} - (1+s) \sum_{t=0}^{T-1} \frac{B_{0,t+1}}{B_{0,t}} B_{0,t} \\
 &= \sum_{t=0}^{T-1} B_{0,t} - (1+s) \sum_{t=0}^{T-1} B_{0,t+1} \\
 &= \sum_{t=0}^{T-1} B_{0,t} - (1+s) \sum_{t=1}^T B_{0,t} \\
 &= \sum_{t=1}^T [B_{0,t-1} - B_{0,t}] - s \sum_{t=1}^T B_{0,t} \\
 &= [1 - B_{0,T}] - s \sum_{t=1}^T B_{0,t}
 \end{aligned}$$

- And this is the same as the formula we had on page 6 based on our dynamic replication.

IV. Floating rates with credit risk.

- In practice, almost all interest rate swaps use LIBOR, or commercial paper, or other risky rates for their floating side.
 - ▶ This is because the swaps originate via risky corporations (especially banks) adjusting their funding profiles.
 - ▶ So the floating rate that they are concerned about is their *own* borrowing rate, which reflects their riskiness.
- The most popular swaps are tied to either the 90-day interbank rate (LIBOR, EURIBOR, etc) or the 1-day interbank rate.
- The latter type is called an *overnight indexed swap* or OIS.
 - ▶ They generally have a short horizon (e.g. $T = 1$ month) and stipulate just a single payment at T of the difference between the fixed rate and the compound interest that would have been earned by rolling over 1-day deposits at the interbank rate.
 - ▶ Specifically, the fixed receiver gets

$$s = [\Pi_{t=0}^{T-1}(1 + R_{t,t+1}^{IB}) - 1].$$

where R^{IB} is the one-day (not annualized) interbank rate and s is the T -period simple rate (also not annualized).

- How does the credit risk in the floating rate affect the valuation?

- Suppose you have promised to pay the floating side of a 1yr-LIBOR swap for T years.
- One way to replicate these payments (and to hedge your obligation) would be the same dynamic strategy we used above: simply invest the notional amount in 1-year LIBOR deposits and roll them over, while also borrowing against the terminal repayment of principal.
 - ▶ Again, the replication cost would be (notional times) $1 - B_{0,T}^{ED}$, where the superscript (stands for Eurodollar) which is the same as the interbank rate (LIBOR).
- But notice some assumptions have crept in here.
 - ▶ I'm investing in bank deposits at the LIBOR rate. What if the bank I'm investing in (or whose certificates I'm buying) goes bankrupt? (There's a reason why they can't borrow at the riskless rate!)
 - * Notice this is distinct from the risk of my *counterparty* going bankrupt....
 - * ...unless the bank I invest with is my counterparty.
 - * So the implication is **The counterparty must be a LIBOR credit.**
 - ▶ I also assumed that I could sell short a T -period bank deposit. I would want this to be with the same bank I'm investing in so that I was hedged against the loss of my principal in the event they default.

- Conversely, suppose a counterparty promises you an annual stream of 1yr-LIBOR payments for T years. How could you hedge this? And how much cash would the hedging transactions produce today?
- You could do the opposite of the above: borrow each period at the LIBOR rate, and invest $B_{0,T}^{ED}$ to fund your terminal principal repayment.
 - ▶ Again, think of this as selling short a one-period risky bond each period and holding on to a T -period bond.
 - ▶ Again, you have the risk that the T -period bond defaults, which is ok only if the one-period bond you are short is of the same entity (so that your principal is hedged).
 - ▶ And, once more, those one-period bonds only truly offset your counterparty's promise if they are issued by him.
- Summarizing: exact replication of the floating side only works under some strict conditions.
 - ▶ But if it does then it also takes care of counterparty risk.
 - ▶ And the value is $(1 - B_{0,T}^{ED})$.

- If we have to assume the floating payor is a LIBOR-rated entity (e.g., a AA-rated bank) then what if we make the same assumption about the fixed payor?
- Then the promise of a LIBOR credit entity to pay an amount s at each future date clearly has value

$$s \cdot \sum_{t=1}^T B_{0,t}^{ED}$$

since, if we can transact in risky zero-coupon bonds of the counterparty, this is exactly what it would cost us to replicate the payments.

- ▶ Again, the formula is only valid if we can do the replication.
- ▶ If we can't literally trade counterparty deposits, we could realize the same value if we could purchase default insurance on each payment, since the cost of this insurance would be the same as counterparty's credit spread.

* More about that later.

- ▶ Notice that in this course we don't simply assume we can "discount at the risky rate" to value risky cash-flows
- Now let's put the two sides of the swap together.
 - ▶ First recall that

$$1 - B_{0,T}^{ED} = \sum_{t=1}^T [B_{0,t-1}^{ED} - B_{0,t}^{ED}]$$

- We can re-write the floating side in terms of forward rates:

$$\begin{aligned}
 \sum_{t=1}^T [B_{0,t-1}^{ED} - B_{0,t}^{ED}] &= \sum_{t=1}^T \frac{B_{0,t-1}^{ED} - B_{0,t}^{ED}}{B_{0,t}^{ED}} \cdot B_{0,t}^{ED} \\
 &= \sum_{t=1}^T \left[\frac{B_{0,t-1}^{ED}}{B_{0,t}^{ED}} - 1 \right] \cdot B_{0,t}^{ED} \\
 &= \sum_{t=1}^T R_{0,t-1,t}^{f,ED} \cdot B_{0,t}^{ED}
 \end{aligned}$$

- In the last equality, I am defining $R_{0,t-1,t}^{f,ED}$ – the LIBOR forward rates – in direct analogy with the riskless case.
- As with t-bills, the forward rates are the LIBOR rates you could lock-in today, and the ratio of zero-coupon bond prices is the no-arbitrage value of a forward contract to deliver a LIBOR deposit in the future.
- Now subtract the value of the fixed side from this last expression.
- The difference between the fixed and floating values is

$$V_0(s) = \sum_{t=1}^T (R_{0,t-1,t}^{f,ED} - s) B_{0,t}^{ED}$$

just like the formula for t-bill swaps.

- The zero-value swap rate today likewise is

$$s_0^{ED} = \frac{\sum_{t=1}^T R_{0,t-1,t}^{f,ED} B_{0,t}^{ED}}{\sum_{t=1}^T B_{0,t}^{ED}}.$$

The risky swap rate is still an average of risky forward rates.

- One last formula. Using the fair swap rate today, s_0 , we can re-write the value (to the fixed payor) of a swap entered into at rate s_{old}

$$V(s_0; s_{old}) = \sum_{t=1}^T (R_{0,t-1,t}^{f,ED} - s_{old}) B_{0,t}^{ED} = \sum_{t=1}^T (s_0 - s_{old}) B_{0,t}^{ED}$$

- When we compare interest rate swap rates based on LIBOR with the riskless yield curve, we can actually learn something quite interesting about the economy.
- “Swap spreads” are defined to be the difference between risky swap rates and the yield-to-maturity on a riskless (par) bond with the same maturity.
- Using the formulas we derived, we can express this difference in a neat way.
- The first step is to realize that our weighted-average-spread formula for LIBOR swaps hardly changes at all if we change the weightings slightly:

$$s^{ED} = \frac{\sum_{t=1}^T R_{0,t-1,t}^{f,ED} B_{0,t}^{ED}}{\sum_{t=1}^T B_{0,t}^{ED}} \approx \frac{\sum_{t=1}^T R_{0,t-1,t}^{f,ED} B_{0,t}^{TB}}{\sum_{t=1}^T B_{0,t}^{TB}}.$$

- ▶ Here TB denotes risk-free zero-coupon bond (t-bill) prices.
- ▶ We are NOT saying these are the same as the risky zero prices, just that the relative weightings will be extremely close for both types of bonds.
- Next, the yield-to-maturity on, say, the “current” 10-year Treasury, y_{10}^{TB} , is also the coupon a *new* 10-year treasury would pay if it were to have a price of par (i.e. 100% of face value).
- This means

$$1 = y_T^{TB} \sum_{t=1}^T B_{0,t}^{TB} + B_{0,T}^{TB}$$

which says

$$y_T^{TB} = \frac{1 - B_{0,T}^{TB}}{\sum_{t=1}^T B_{0,t}^{TB}}.$$

- But if we play our game of writing the numerator in terms of forward rates again, this is

$$y_T^{TB} = \frac{\sum_{t=1}^T R_{0,t-1,t}^{f,TB} B_{0,t}^{TB}}{\sum_{t=1}^T B_{0,t}^{TB}}.$$

- ▶ This is the exact same as our formula for the riskless swap rate!

- Now subtract this from our (approximate) expression for the LIBOR swap rate:

$$s^{ED} - y_T^{TB} = \frac{\sum_{t=1}^T \left[R_{0,t-1,t}^{f,ED} - R_{0,t-1,t}^{f,TB} \right] B_{0,t}^{TB}}{\sum_{t=1}^T B_{0,t}^{TB}}.$$

- **The swap spread is a weighted average of LIBOR forward credit spreads.**

V. Swap Trading: An Example

- In 1998 LTCM – a large hedge fund – lost over a billion dollars with this position:

long (=receiving fixed) 5-year-forward sterling-LIBOR swaps.

short gilts.

short (= paying fixed) 5-year-forward deutsche-mark-FIBOR swaps.

long bunds.

- **Question:** What bet were they making?
- To answer this, let's examine the pieces. Take just the sterling side first.
- We now know that the value of a LIBOR swap is determined by an average of risky forward rates.
- So there are two ways paying floating can win.
 1. All rates could go down, or
 2. the risk-premium in LIBOR could decline.
- The fund managers were paying floating in sterling AND short gilts. The short gilt position was calibrated precisely to off-set the first exposure – i.e. to the level of rates.
- So we can conclude: they were long swap spreads.

The sterling position was a bet that U.K. credit spreads would narrow.

- Since the swaps involved were forward, the bet was that this would happen *or be expected to happen* five years in the future.
- The mark position was exactly the reverse bet – for Germany.
- This meant that the trade did not necessarily require either countrys' credit spreads to go up or down. Instead, the whole thing boils down to...

speculating that U.K. and German credit spreads would converge.

- The firm believed that Britain would eventually join Euro, but didn't think there was much of an edge in just betting that rates themselves would converge.
 - ▶ However, once in the Euro, sterling LIBOR and deutsche mark LIBOR would be the *same rate*, i.e. there would only be one interbank rate.
 - ▶ So only one credit spread for bank risk could prevail.
- As a footnote, the short gilt/bund position involved in this trade was actually accomplished via a separate swap: a *return swap*.
 - ▶ We'll talk about them next time.

VI. Lecture Summary.

- Even though interest rate swaps seem very different from commodities swaps, the valuation formula *looks the exact same*.
- Replicating the floating side of an interest rate swap can either be done *dynamically* by rolling over one-period investments at the floating rate, or *statically* by trading in forward contracts (or synthesizing forwards) for one-period bonds.
- The main assumption behind the formula: no counterparty risk.
- There are very active liquid markets for standard swaps based on interbank floating rates for every major currency for maturities as long as 30 years.
 - ▶ These are extremely valuable tools for speculation and hedging. However, they may not themselves be pure derivatives.
 - ▶ When the floating rate is not the riskless rate replication of the swap is only possible if (a) the floating rate is the borrowing cost of the floating rate payor in the swap; and (b) one can trade (short-sell) bonds of that party or otherwise hedge his default risk.

Lecture Note 2.2: Summary of Notation

SYMBOL	PAGE	MEANING
s	p9	<i>fixed rate for an interest rate swap</i>
X	p9	<i>Notional amount of interest rate swap</i>
R_{t_0,t_1}	p9	<i>simple interest rate at t_0 for a payment at t_1</i>
R_{t_0,t_1,t_2}^f	p11	<i>simple (uncompounded) forward rate at t_0 for an investment at t_1 paying off at t_2</i>
B_{t_0,t_1,t_2}^f	p11	<i>forward price at t_0 of a zero-coupon bond deliverable at t_1 maturing at t_2</i>
V	p11	<i>value of a swap with fixed rate s</i>
R^{IB}	p16	<i>interbank (risky) interest rate</i>
Π	p16	<i>product operator</i>
$R^{ED}, R^{f,ED}, B^{ED}$	p20	<i>interest rates, forward rates, and zero-coupon bond prices for a company of LIBOR (or Eurodollar) credit</i>
s_0^{ED}	p21	<i>fair rate for a new LIBOR swap at $t = 0$</i>
$R^{TB}, R^{f,TB}, B^{TB}$	p21	<i>interest rates, forward rates, and zero-coupon bond prices for a risk-free credit (eg treasury-bills)</i>
$V(s_0; s_{old})$	p21	<i>value to fixed-payor of swap entered into at rate s_{old} when current rate is s_0</i>
y_T^{TB}	p22	<i>yield-to-maturity on a treasury bond maturing at T = coupon rate for a treasury which is worth its face value</i>