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Liquidity, default, taxes, and yields on municipal bonds

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Abstract

We examine the effects of liquidity, default and personal taxes on the relative yields of Treasuries and municipals using a generalized model with liquidity risk. The municipal yield model includes liquidity as a state factor. Using a unique transaction dataset, we estimate the liquidity risk of municipals and its effect on bond yields. Empirical evidence shows that municipal bond yields are strongly affected by all three factors. The effects of default and liquidity risk on municipal yields increase with maturity and credit risk. Liquidity premium accounts for about 9–13% of municipal yields for AAA bonds, 9–15% for AA/A bonds and 8–19% for BBB bonds. A substantial portion of the maturity spread between long- and short-maturity municipal bonds is attributed to the liquidity premium. Ignoring the liquidity risk effect thus results in a severe underestimation of municipal bond yields. Conditional on the effects of default and liquidity risk, we obtain implicit tax rates very close to the statutory tax rates of high-income individuals and institutional investors. Furthermore, these implicit income tax rates are quite stable across bonds of different maturities. Results show that including liquidity risk in the municipal bond pricing model helps explain the muni puzzle.

JEL classification: G0; G12

Keywords: Liquidity risk; Tax-exemption; Default risk; Municipal bond yields; Maturity spreads

1. Introduction

The fixed-income securities market is an important segment in the US financial markets. This market has been particularly innovative and experienced considerable growth recently. Not surprisingly, there is an extensive literature attempting to explain yield differences between fixed-income securities. A subject that has long intrigued financial researchers is how the yield spreads between tax-exempt and taxable securities are determined. Are default and liquidity risk priced in municipal bonds? What portion of these yield spreads is attributed to taxes, default, and liquidity risk? These issues are fundamentally important from an investment perspective due to the sheer size of the municipal market, which now approaches 1.9 trillion dollars.

Bond returns are subject to different tax treatments, depending on both the issuer and the investor. Interest on municipal bonds is exempt from federal income taxes though not necessarily exempt from state taxes. By contrast, interest on Treasury and government agency bonds is subject to federal income taxes but exempt from state income taxes. In equilibrium, one expects the after-tax returns of taxable and tax-exempt bonds to be equal holding maturity and risk constant. Unfortunately, empirical evidence has not conformed very well to this expectation but instead indicates that municipal bond yields are often

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higher than expected relative to yields on US Treasury bonds. The relatively high yields of municipals imply a tax rate lower than expected for the marginal tax rates of high-income individuals and corporations. The implied marginal tax rate is much lower for long-maturity municipal bonds than for short-maturity bonds of similar quality and characteristics.

Several hypotheses have been advanced to explain this muni puzzle. The institutional demand hypothesis suggests that the marginal tax rate is determined by institutional trading activity (see Fortune, 1973; Galper and Peterson, 1971; Kimbal, 1977; Fama, 1977). Commercial banks can purchase municipals to shield their income from taxes. Since commercial banks prefer short-term bonds, the implicit tax rate would tend to be high for these bonds relative to long-term bonds. Other explanations for the yield curve anomaly include tax-timing options (Constantinides and Ingersoll, 1984), clientele effects (Mussa and Kormendi, 1979; Kidwell and Koch, 1983), time-varying risk premiums (Trzcinka, 1982), and changes in tax regimes (Poterba, 1989). However, studies have shown that none of these factors can satisfactorily explain the municipal puzzle (see Green, 1993; Liu et al., 2003).

In an important paper, Green (1993) argues that high-tax investors can avoid taxes on coupon income by constructing portfolios of taxable bonds that generate offsetting losses or investment interest expenses. If these investors are marginal across these portfolios and municipals bonds, they will apply the same discount factors to the after-tax cash flows from both positions. Using this relationship, Green derives an equilibrium model to explain the relative yields of taxable versus tax-exempt bonds. Empirical evidence shows that Green's model explains a considerable portion of the relative yield differences between taxable and tax-exempt bonds (see Green, 1993). However, the model continues to underestimate the long-term tax-exempt yields. This raises a question that there may still be missing factors in Green's model.

Municipal bonds are not risk-free due to the unique features of municipal assets and unpredictable political processes (see Hempel, 1972; Zimmerman, 1977; and Trzcinka, 1982; Litvack and Rizzo, 1999).[1] Thus, default risk may not be trivial. However, empirical evidence on the role of default risk in municipal bond pricing is inconclusive. Several studies (e.g. Trzcinka, 1982; Yawitz et al., 1985; Scholes and Wolfson, 1992; Kim et al., 1993; Stock, 1994; Wu, 1991; and Liu et al., 2003) show that credit risk differences explain the relative yields of taxable and tax-exempt bonds. Other studies (Gordon and Malkiel, 1981; Skelton, 1983; Ang et al., 1985; and Chalmers, 1998) find that differential default risk cannot explain the municipal bond puzzle.

A critical factor completely left out by previous municipal bond pricing models is liquidity risk. The municipal market is very illiquid compared to the US Treasury, equity, futures and foreign exchange markets, and trading cost is high (see Fabozzi, 1997; Downing and Zhang, 2004).[2] Although it has been long recognized that the municipal market is illiquid and liquidity risk is a potentially important determinant of municipal yields, few studies have provided a quantitative assessment of the size of liquidity risk premium. An exception is Neis (2006) who explores whether movements in high-rated municipal yields are driven by liquidity and finds a substantial liquidity premium for municipal bonds.

In this paper, we estimate the liquidity premium of municipal bonds using the transaction database recently made available by the municipal securities rulemaking board (MSRB). Our study accounts for the effects of liquidity risk, personal taxes, and default probability on the relative yields of taxable and uninsured tax-exempt bonds. We construct a broad liquidity measure for the municipal market along the line of Pastor and Stambaugh (2003), which captures temporary price fluctuations induced by order flow. By incorporating liquidity as an additional state factor, we find that the explanatory power of the pricing model of municipal bonds is greatly improved.

Empirical evidence shows that liquidity risk premium accounts for a significant portion of municipal bond yields. Results suggest that investors require a higher yield on those municipal bonds whose returns are more sensitive to aggregate market liquidity. Within a rating class, the sensitivity of municipal yields to market-wide liquidity increases with maturity. Controlling for maturity, we find that the sensitivity of municipal yields to market-wide liquidity increases monotonically as the bond rating drops from AAA to BBB. Overall, liquidity risk premium explains about 9–13% of the observed yields for AAA bonds, 9–15% for AA/A bonds and 8–19% for BBB bonds with different maturities. Ignoring liquidity risk results in a substantial underestimation of municipal bond yields.

Including liquidity risk in the pricing model also helps explain the municipal yield curve anomaly. Long-maturity municipal yields are high relative to the equivalent after-tax yields of Treasury bonds mainly because the effect of liquidity risk increases with maturity. Our results show that the liquidity premium accounts for a substantial portion of the maturity spread between long- and short-term municipal bonds. Conditional on default and liquidity risk effects, we obtain implicit income tax rates very close to the statutory rates of high-income individuals and corporations. More importantly, these implicit tax rates are quite stable over maturity.

[1] Municipal assets cannot be seized as easily as corporate assets in bankruptcy proceedings because they may be physically difficult to seize or legally protected. In addition, the incentive problem or moral hazard is perceived to be more severe for municipalities.

[2] Harris and Piwowar (2006) report that the effective spreads in muni bonds average almost 2% of price.

Empirical estimates of the liquidity premium are highly correlated with traditional liquidity variables. We find that municipal bonds with high volume and trading frequency and large issue size tend to have a low liquidity risk premium. Thus, the Pastor–Stambaugh method used in this study to construct the aggregate liquidity of the municipal bond market appears to be quite effective in abstracting the liquidity feature of municipal bonds.

The remainder of this paper is organized as follows. Section 2 proposes a generalized municipal bond pricing model to incorporate the effects of default and liquidity risk. Section 3 discusses the empirical methodology and Section 4 describes the data. Section 5 presents empirical results for municipal bonds of different ratings, maturities and trading characteristics. Finally, Section 6 summarizes major findings and concludes the paper.

2. A municipal bond pricing model

In this section, we propose a generalized model that jointly accounts for the effects of taxes, default and liquidity risk to examine the relationship between municipal and Treasury bond yields. We impose the following assumptions in this model: (A1) Municipal and Treasury bonds are priced at par (=\$1) and both are noncallable. (A2) Investors buy and hold the bonds until maturity or default, whichever comes first. (A3) Taxation of long and short positions is asymmetric for individuals. The interest deductions from borrowings and short positions cannot exceed investment income from the positions these borrowings support. As a consequence, investors cannot be "marginal" on all bonds simultaneously but a single representative investor can still be at margin across municipals and portfolios of taxable bonds. (A4) Municipal bonds are subject to default with a default probability λ_t at time t and a constant recovery rate δ, specified as a proportion of the face value. (A5) Investors of municipal bonds face liquidity risk for which they require a premium to compensate for bearing this risk. Assumptions 1 and 3 are similar to Green (1993) while the remaining assumptions are added to accommodate the risk features of municipal bonds.

Our model is a generalization of Green's (1993) model to account for default and liquidity risk of municipal bonds. Green (1993) shows that the pricing model for the taxable default-free bond is:

$$1 = \sum_{t=1}^{T} C_T \frac{(1-\tau)d_t}{1-\tau d_t} + \frac{(1-\tau)d_T}{1-\tau d_T}, \tag{1}$$

where τ is the tax rate, C_T is the coupon yield of the riskless taxable bond with maturity T, $d_t = P_t/(1-\tau(1-P_t))$ is the after-tax discount factor and P_t is the pretax discount factor.

Consider next the pricing of risky municipal bonds. In equilibrium, the municipal bond price should be equal to the expected future payoff of the bond itself. When there is no default, the present value of the payoff for the municipal bond is:

$$\left(\sum_{t=1}^{T} M_T d_t + d_T\right)\prod_{t=1}^{T}(1-\lambda_t), \tag{2}$$

where M_t is the coupon yield on the municipal bond. Conversely, when default occurs, the investor receives a residual amount δ per one-dollar face value and a tax rebate $(1-\delta)\tau_g$ where τ_g is the capital gain (loss) tax rate. The present value of the expected cash flow if default occurs at time i, is given by

$$\left(\sum_{t=1}^{i-1} M_T d_t + \delta + (1-\delta)\tau_g\right)\lambda_i \prod_{k=1}^{i-1}(1-\lambda_k), \tag{3}$$

where δ is expressed in terms of the present value of the recovery amount. The value of the municipal bond at time t is then equal to the sum of (2) and (3). Given that the municipal bond is priced at par, the pricing formula can be written as:

$$\begin{aligned}1 = {} & \left(\sum_{t=1}^{T} M_T d_t + d_T\right)\prod_{t=1}^{T}(1-\lambda_t) \\ & + \sum_{i=1}^{T}\left[\left(\sum_{t=1}^{i-1} M_T d_t + \delta + (1-\delta)\tau_g\right)\lambda_i \prod_{k=1}^{i-1}(1-\lambda_k)\right].\end{aligned} \tag{4}$$

Combining the pricing formula for Treasuries in (1) with (4), we obtain the following equilibrium relationship between municipal and Treasury yields:

$$M_T = \Lambda_T + \beta_T C_t, \tag{5}$$

where

$$\Lambda_T = \frac{\frac{d_T\left[1-\prod_{t=1}^{T}(1-\lambda_t)\right]-(\delta+(1-\delta)\tau_g)\sum_{i=1}^{T}\left[\lambda_i\prod_{k=1}^{i-1}(1-\lambda_k)\right]}{1-\tau d_T}}{\frac{1}{1-\tau d_T}\left\{\sum_{t=1}^{T} d_t \prod_{t=1}^{T}(1-\lambda_t) + \sum_{i=1}^{T}\left[\left(\sum_{t=1}^{i-1} d_t\right)\lambda_i \prod_{k=1}^{i-1}(1-\lambda_k)\right]\right\}};$$

$$\beta_T = \frac{\sum_{t=1}^{T}\frac{(1-\tau)d_t}{1-\tau d_t}}{\frac{1}{1-\tau d_T}\left\{\sum_{t=1}^{T} d_t \prod_{t=1}^{T}(1-\lambda_t) + \sum_{i=1}^{T}\left[\left(\sum_{t=1}^{i-1} d_t\right)\lambda_i \prod_{k=1}^{i-1}(1-\lambda_k)\right]\right\}}.$$

The pricing formula above is however incomplete because it omits the effect of liquidity risk, which is considered to be a critical factor for municipal bond pricing. This omission may explain why the default model does not fare well in describing municipal yield behavior. Liquidity is perceived as an important feature of the investment environment. Previous studies have shown that the *level* of liquidity affects expected asset returns (see Amihud and Mendelson, 1986, 1991; Brennan and Subrahmanyam, 1996; Brennan et al., 1998; and Amihud, 2002). That the level of liquidity can affect transaction cost and asset price is not surprising. What is more important is whether liquidity risk is a systematic risk that affects equilibrium asset returns. In an influential paper, Pastor and Stambaugh (2003) show that expected stock returns are significantly affected by systematic liquidity risk.[3]

To account for the effect of liquidity risk, we add a liquidity risk factor to (5):

$$M_T = \Lambda_T + \beta_T C_t + \beta_L L \tag{6}$$

where β_L is the sensitivity of the yield of an individual municipal bond or portfolio to the aggregate liquidity factor L of the municipal market. Eq. (6) states that in equilibrium the municipal bond yield equals the Treasury bond yield adjusted for both tax and default effects plus a systematic liquidity risk premium, which compensates for the low liquidity of municipal bonds relative to Treasuries of equal maturity. The model implies that municipal bonds whose returns have higher sensitivities (β_L) to the aggregate market liquidity will have higher yields.

Because the municipal market is relatively illiquid compared to other markets, liquidity risk could potentially be a more important pricing factor for municipal bonds. When there is a widespread deterioration in liquidity, it will be more difficult to liquidate municipal bonds than Treasuries securities. In anticipation of costly liquidation in a low liquidity environment, investors would require higher yields to compensate for bearing this risk. Furthermore, the trade size of municipals is typically larger than that of equity transactions. Liquidity is expected to be more valuable for investors trading large orders even in routine transactions. In an unusual situation when the aggregate market liquidity suddenly dries up, it will be even more difficult to trade large quantities. Taken together, liquidity risk should be of more serious concern for municipal investors.

The relevance of liquidity risk to municipal bond pricing is measured by the sensitivity coefficient β_L to the aggregate liquidity of the municipal bond market. Similar to the risk measure in traditional asset pricing models, β_L captures the systematic risk (liquidity beta) of individual municipal bonds to the market-wide liquidity. We next describe the empirical procedure for estimating the systematic liquidity risk of municipal bonds.

[3] Recent studies in fixed-income markets have found that traditional term structure models of defaultable bonds explain only a small portion of yield spreads (see Huang and Huang, 2003; Liu and Wu, 2004; Liu et al., 2006; Liu et al., 2007). A potential cause is that liquidity risk is not accounted for by these models. Longstaff et al. (2005) find that the liquidity premium accounts for a significant portion of corporate bond spreads. Since the municipal market is very illiquid by conventional standards, liquidity risk could also be very important for pricing municipal bonds.

3. Empirical methodology

3.1. Estimation of the aggregate liquidity index

To estimate systematic liquidity risk β_L, we need to construct a market-wide liquidity measure (L) for municipal bonds. We next outline the procedure for constructing a measure of aggregate liquidity for the municipal bond market in the spirit of Pastor and Stambaugh (2003). The Pastor–Stambaugh liquidity measure captures temporary price changes associated with order flow. The rationale is that lower liquidity tends to correspond to stronger price reversals on the next trading day, resulting from the order flow in a given direction on a particular day. For example, in the absence of private information, a large number of sell orders (or a high sell volume) on a particular day will cause a greater price rebound on the next trading day. This dimension of liquidity can be gauged by the response of municipal returns in the next trading day to the signed volume in the preceding trading day. Specifically, the liquidity measure for a bond in a month t is the estimate of $\pi_{i,t}$ associated with signed volume in the following regression:

$$r^e_{i,j+1,t} = \rho_0 + \rho_1 r_{i,j,t} + \pi_{i,t}\,\text{sign}\,(r^e_{i,j,t}) \cdot \text{Vol}_{i,j,t} + u_{i,j+1,t} \tag{7}$$

where $r^e_{i,j,t} = r_{i,j,t} - r_{b,j,t}$ is the return of municipal bond i on day j in month t, $r_{i,j,t}$, in excess of the equally weighted municipal market return, $r_{b,j,t}$; $\text{sign}(r^e_{i,j,t})$ is the signed indicator which is equal to 1 if $r^e_{i,j,t}$ is positive, and -1 if it is negative; and $\text{Vol}_{i,j,t}$ is the par volume (in 10,000 dollars) for bond i. In this model, the order flow is measured by volume signed by the contemporaneous excess return on a particular municipal security. The order flow should be accompanied by a return that is expected to be partially reversed if the municipal security is not perfectly liquid. Presumably, the greater the expected reversal for a given volume, the lower the liquidity of the municipal security; that is, $\pi_{i,t}$ is negative. The model in (7) is estimated for individual bonds each month t using daily return and volume data. In empirical investigation, in order to obtain a reliable estimate of π, we select in each month only those bonds which have more than 10 observations.

The liquidity measure of each individual bond is then aggregated over all municipal bonds (N) in the sample month by month: $\hat{\pi}_t = \frac{1}{N}\sum_{i=1}^{N}\pi_{i,t}$. Systematic liquidity risk, β_L, is measured as the sensitivity of bond returns to unexpected innovations in market-wide liquidity. To obtain the unexpected liquidity innovations ($\hat{e}_t$), we estimate the following autoregressive model for the differenced liquidity measure:

$$\Delta\hat{\pi}_t = a_0 + a_1\Delta\hat{\pi}_{t-1} + a_2\hat{\pi}_{t-1} + e_t. \tag{8}$$

The above regression is essentially a second-order autoregression in the level series of market-wide liquidity. Since the residual term for municipal bond liquidity is usually quite small, we multiply the estimated $\hat{e}_t$ by 100: $L_t = 100 \cdot \hat{e}_t$. After constructing the market-wide liquidity measure, we include it as a state factor for the municipal bond yield model to estimate the liquidity risk premium.

3.2. Estimation of the municipal bond model

The municipal yield model in (6) can be estimated by the nonlinear regression method.[4] Since municipal bond yields are serially correlated, we correct the effect of autocorrelation. The first step in empirical investigation is to abstract the liquidity measure from (7) to (8). The regression model in (7) is particularly suitable for our investigation of municipal market liquidity because it requires only the excess return and par volume. These data are readily available from the transaction records provided by MSRB. The second step of the empirical investigation involves forming the portfolios of muni bonds by rating and maturity. There are advantages of estimating the yield model with portfolios. First, forming portfolios allows us to construct more regular time-series of municipal yields for regression estimation, to mitigate the problem of infrequent trading. Second, portfolio formation reduces data noise associated with individual bonds due to recording errors and price discreteness. Because portfolios are grouped by rating and maturity, we retain control on these two important characteristics. We calculate the yield for each portfolio from our data sample month by month. We then estimate the nonlinear model in (6) by the Gauss–Newton method using the monthly yield data of each portfolio.

4. Data description

We use the municipal bond database provided by the Municipal Securities Rulemaking Board in our empirical estimation.[5] The MSRB transaction reports contain data fields including CUSIP, security description, issue date, coupon, maturity date, trade date, time of trade, par amount of trade, transaction price, and yield. Useful data features include whether a transaction is sale to customer, purchase from customer or inter-dealer trade, and an indicator showing whether the trade occurs before the syndicate settlement date. Additional information on the characteristics of each bond is collected from Bloomberg, which includes the rating of a bond when it was issued, the issue size and type (e.g. general obligation or revenue bonds), whether the bond is callable or contains a sinking fund provision, and whether the bond is insured.[6] Zero-coupon Treasury yields are obtained from the Federal Reserve Board (FRB), where spot rates are estimated from Treasury prices using the Svensson method (see Bolder and Streliski, 1999).

The sample period is from July 2000 to June 2004. The initial sample contains a total of 1,056,774 bonds, 27,330,633 transactions and dollar volume of 11.7 trillion. To construct the data sample for our empirical estimation of the municipal yield model in (6), we match the raw MSRB data with bond characteristic information from Bloomberg and impose the following screening criteria. We drop trades that occur on or before the underwriting syndicate settlement date, and keep only secondary market trades. We eliminate bonds with unknown credit ratings, embedded option features (i.e., bonds with call and sinking fund provisions), variable rates or irregular coupons, and less than six months away from maturity, and bonds which are insured. We exclude insured bonds to better control the sample because insured bonds have no default risk and so may have different liquidity characteristics. In addition, we throw away transactions with obvious errors in prices or with missing prices. After these screenings, our final sample contains 48,278 bonds with a total of 753,268 secondary market transactions.

We group the individual bond data according to their ratings and maturities. Because there are very few speculative bonds in our data sample, we include only AAA, AA/A and BBB bonds. AA and A bonds are grouped together. There are relatively few trade and transaction price data for bonds with long-maturity. We therefore lump (equally weighted) all bonds with maturities from 18 to 22 years to form long-term portfolios. This enables us to assemble a larger number of observations in portfolios to examine the empirical properties of long-term municipals. The average maturity for these long-term portfolios is very close to 20 years: 19.9 years for AAA, 19.5 years for AA/A and 19.9 years for BBB bonds. For convenience, these portfolios are placed under the 20-year category for the corresponding rating class.

Table 1 provides the summary statistics for Treasuries and three rating groups of municipals by maturity. Panel A shows that yields of AAA bonds are lower than those of AA/A bonds which, in turn, are lower than those of BBB bonds. Yields of Treasury bonds with the same maturity are generally greater than those of AAA and AA/A bonds. However, Treasury yields may be lower than BBB yields, indicating that default and liquidity premia may outweigh the tax-exempt benefit of municipal bonds in this rating class. Fig. 1 plots the time-series of municipal and Treasury yields for maturities from one to ten years.

Panel B of Table 1 shows monthly averages of the number of transactions, volume (in par amount), and the number of bonds for each rating-maturity portfolio. For each month we calculate the number of transactions, and trading volume for all bonds in each portfolio and then average them over all months in the entire sample period. As shown, the AA/A bonds have the largest number of

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[4] In empirical estimation, we set capital gains tax rate $\tau_g = 0.4\tau_i$ or $\tau_g = \min(\tau_i, 0.20)$ before January 1, 2003 and $\tau_g = \min(\tau_i, 0.15)$ on and after.

[5] See Downing and Zhang (2004) for the detailed discussion of this dataset.

[6] Ideally one would like to use the ratings at the point in time in which bonds are included in the sample. However, this rating information is not available to us. Nevertheless, using the original rating should not cause a material effect on our results because most transactions occur when municipal bonds are young. When we later form portfolios based on transaction data to perform empirical estimation, the yields of portfolios are dominated by young and actively traded bonds which are less likely to have rating changes.

Table 1
Descriptive statistics

Maturity	Mean				Standard deviation			
	Treasury	Municipals			Treasury	Municipals		
		AAA	AA/A	BBB		AAA	AA/A	BBB
Panel A. Yields of Treasuries and municipals								
1	2.62	2.04	2.23	2.76	1.63	1.05	1.09	1.08
2	3.03	2.38	2.47	2.99	1.47	1.04	1.03	0.97
3	3.38	2.67	2.75	3.26	1.31	0.95	0.92	0.85
5	3.98	3.21	3.27	3.78	1.02	0.75	0.71	0.56
7	4.39	3.65	3.70	4.28	0.86	0.58	0.54	0.39
10	4.67	3.96	4.02	4.54	0.66	0.48	0.44	0.31
20	5.39	5.02	5.11	5.57	0.44	0.24	0.33	0.31

Maturity	Number of transactions (per month)	Par volume (million)	Number of bonds
Panel B. Trading characteristics of municipal bond portfolios			
AAA Bonds			
1	860.88	1632.79	270
2	710.58	305.63	249
3	686.83	214.36	231
5	1389.31	293.65	415
7	1235.77	259.39	335
10	1153.60	277.05	263
20	264.29	109.23	73
AA/A Bonds			
1	2714.81	906.67	918
2	2667.77	493.15	919
3	2548.71	467.06	821
5	4673.90	908.80	1335
7	3527.35	622.72	941
10	2839.83	619.86	647
20	277.96	129.37	55
BBB Bonds			
1	183.67	19.92	56
2	245.83	31.44	67
3	286.40	31.94	72
5	689.42	85.68	136
7	651.25	85.21	106
10	623.40	109.49	78
20	153.06	56.60	16

Panel A of this table reports the mean and standard deviation of yields for the 21 portfolios formed by rating (AAA, AA/A, BBB) and maturity (1-, 2-, 3-, 5-, 7-, 10-, and 20-year) as well as the Treasury yields of corresponding maturity. We calculate the yield of each portfolio month by month and then report mean and standard deviation of these monthly yield series. Panel B reports the monthly average of the number of transactions, par amount traded, and the number of bonds for municipal bond portfolios grouped by ratings and maturity. The sample period is from July 2000 to June 2004. Data sources: the Federal Reserve Board, Bloomberg and the MSRB.

transactions, volume and number of bonds, followed by AAA and BBB bonds. The five-year bonds have the highest number of transactions whereas 20-year bonds have the lowest number of transactions.

Panel A of Table 2 summarizes the yield spreads between Treasuries and AAA bonds, between AAA and AA/A bonds and between AA/A and BBB bonds. Average differences between Treasury and AAA yields are around 70 basis points. Average yield differences between AA/A and AAA bonds, and between BBB and AA/A are about 10 and 50 basis points, respectively. The term structure of the yield spread between Treasuries and AAA bonds exhibits a hump shape. The spread declines beyond the five-year maturity, which confirms the previous finding that prime municipal bond yields rise relative to Treasury yields when maturity gets longer (see Green, 1993). Panel B reports term premiums for Treasuries and municipals. The term premium is obtained by subtracting from the yield for a given maturity the corresponding yield of one-year maturity. Average yields for Treasuries increase less rapidly with maturity than do those of municipals. The columns of the minimum values of term premium show negative numbers for Treasuries at all maturities whereas they are all positive for municipals. While the yield curves of Treasuries may revert, this has not occurred for municipal bonds over the sample period.

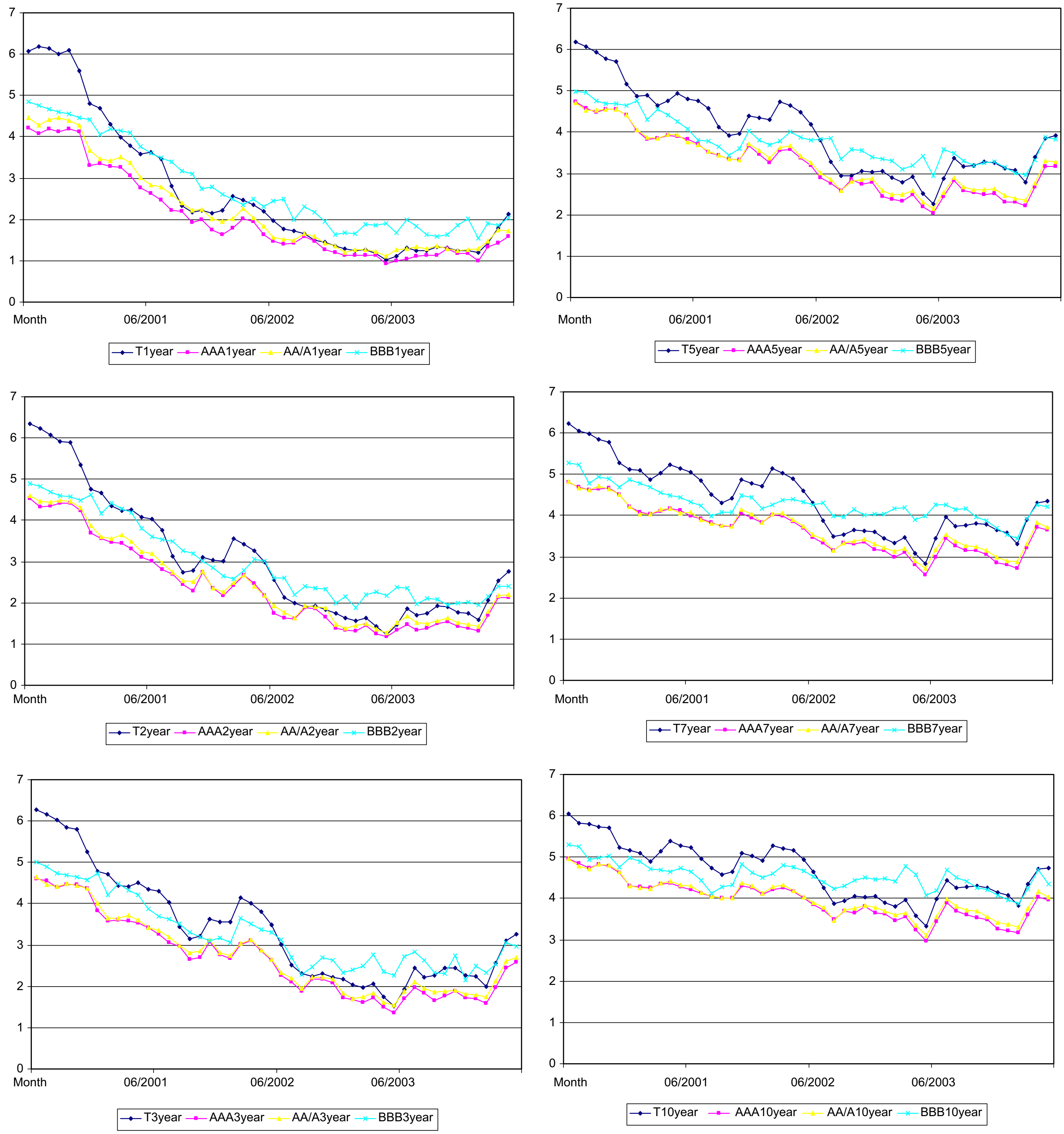

Fig. 1. Yields for different municipal rating/maturity portfolios and Treasuries. Monthly yield series are plotted from July 2000 to June 2004 for different rating portfolios and Treasuries with maturities of 1, 2, 3, 5, 7 and 10 years.

5. Empirical results

5.1. Estimation of the municipal yield model

We next estimate the municipal yield model using the monthly yield series for each maturity-rating group. Estimating the municipal bond yield model requires spot rates of pure risk-free discount bonds. The pretax discount factor P_t embedded in the after-tax discount factor d_t at time t is the inverse of one plus the spot rate of a pure discount bond to the power of t. By substituting the definition of the after-tax discount factor $d_t = P_t/(1 - \tau(1 - P_t))$ into the model and using the spot rate data obtained from the FRB, we can directly estimate the marginal income tax rate and default probability from the observed municipal and Treasury bond yields. The yield model is estimated for

Table 2
Summary statistics for the spreads of municipal bonds

Maturity	Mean			Minimum			Maximum		
	ΔTP	ΔPG	ΔGM	ΔTP	Δ PG	ΔGM	ΔTP	ΔPG	ΔGM
Panel A. Yield spreads between different bonds									
1	0.58***	0.19***	0.53***	0.02	0.01	0.10	2.11	0.39	0.99
2	0.66***	0.10***	0.52***	0.03	−0.07	0.08	1.89	0.23	0.92
3	0.71***	0.08***	0.51***	0.08	−0.08	−0.02	1.68	0.19	0.93
5	0.77***	0.06***	0.51***	0.10	−0.07	0.10	1.49	0.16	1.10
7	0.74***	0.06***	0.58***	0.21	−0.06	0.16	1.41	0.18	1.27
10	0.71***	0.06***	0.53***	0.23	−0.05	0.06	1.08	0.16	1.23
20	0.37***	0.09***	0.46***	0.01	−0.12	−0.07	0.86	0.16	0.84

Maturity	Mean				Minimum				Maximum			
	Treasury	Municipal			Treasury	Municipal			Treasury	Municipal		
		AAA	AA/A	BBB		AAA	AA/A	BBB		AAA	AA/A	BBB
Panel B. Maturity spreads												
2	0.41***	0.34***	0.24***	0.23***	−0.25	0.10	0.01	0.00	0.99	0.75	0.52	0.71
3	0.76***	0.64***	0.52***	0.50***	−0.34	0.20	0.00	0.06	1.57	1.23	1.03	1.20
5	1.36***	1.17***	1.03***	1.02***	−0.43	0.29	0.10	0.08	2.18	1.79	1.62	2.01
7	1.77***	1.61***	1.47***	1.52***	−0.32	0.38	0.20	0.12	2.65	2.40	2.26	2.58
10	2.04***	1.92***	1.78***	1.78***	−0.37	0.49	0.29	0.28	3.14	2.86	2.69	2.90
20	2.77***	2.98***	2.88***	2.81***	−0.16	1.11	1.42	1.07	4.08	4.01	3.92	3.99

All yield series are monthly and reported as annual percentages. Panel A presents the mean, minimum and maximum of the spreads of the municipal bond portfolios formed by rating and maturity. ΔTP is the yield spread between treasury and AAA municipal bonds, ΔPG is the yield spread between AAA and AA/A municipal bonds; ΔGM is the yield spread between AA/A and BBB municipal bonds. Panel B reports the maturity spreads for each rating class. Maturity spread is calculated by subtracting from the yield for a given maturity group the corresponding yield for one-year maturity. ***Represents significance at the 1% level.

bond groups with different ratings and maturities. In addition to spot rates, we need to construct the aggregate municipal bond market liquidity index before estimating the yield model. We estimate liquidity measures for individual bonds each month using the regression model in (7) and aggregate them over all bonds to obtain the monthly series of the liquidity index.

Panel A of Fig. 2 plots the estimates of the aggregate liquidity index for the muni market. By construction, negative liquidity index values imply positive liquidity costs. The aggregate liquidity measure shows occasional downward spikes. The largest downward spike occurs in September 2001 when the market was shut down due to the terrorist attack at the New York World Trade Center. The second largest spike occurs in the fall of 2002 when the market was jittered by the episode of WorldCom. The third largest spike occurs around July 2001 when the Enron scandal began to unravel. Another major downward spike occurs after the announcement of the tax cut by the Bush Administration on May 23, 2003, which triggered a sell-off in the municipal bond market as the tax-exempt advantage of municipals was eroded. Overall, the aggregate

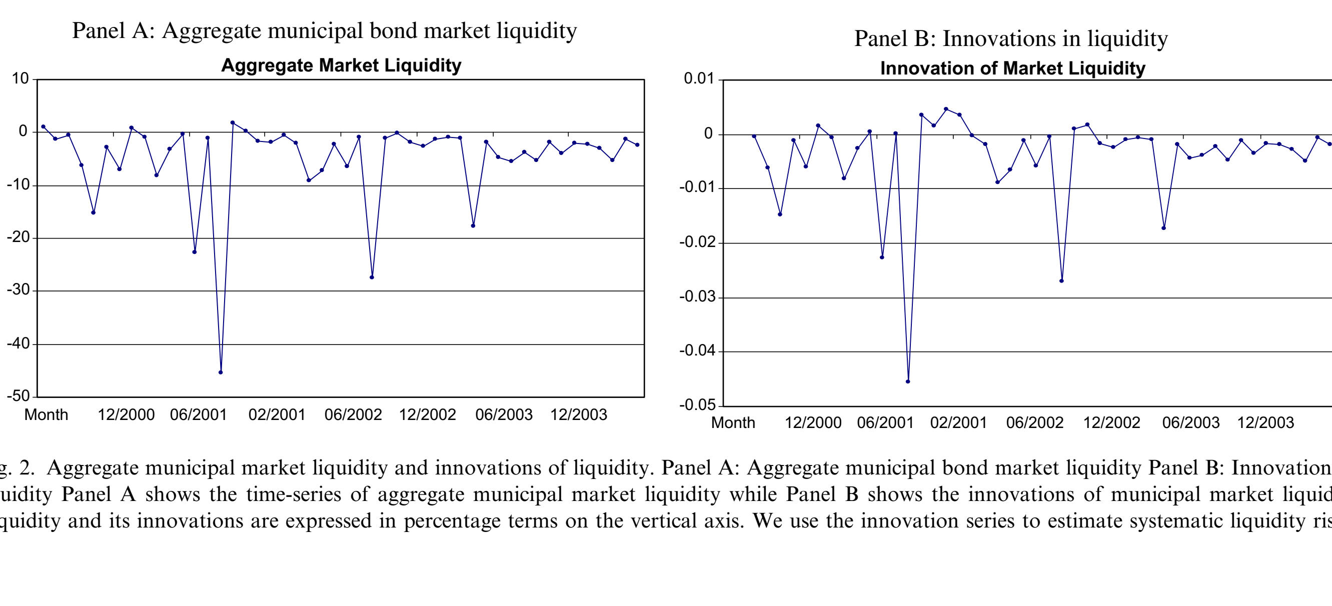

Fig. 2. Aggregate municipal market liquidity and innovations of liquidity. Panel A: Aggregate municipal bond market liquidity Panel B: Innovations in liquidity Panel A shows the time-series of aggregate municipal market liquidity while Panel B shows the innovations of municipal market liquidity. Liquidity and its innovations are expressed in percentage terms on the vertical axis. We use the innovation series to estimate systematic liquidity risk.

liquidity measure constructed from the Pastor–Stambaugh method appears to pick up significant events quite effectively.

Median of the aggregate liquidity measure is −2.12% whereas mean is −4.99. Excluding the extreme value of September 2001, the mean liquidity index is −4.04%. These numbers suggest that the average liquidity cost is around 2.1% to 4%, which represents the cost for a $10 thousand trade in constant "municipal bond market" dollars of year 2000. The correlation between the liquidity measure and the aggregate municipal bond market return is 0.05, indicating that liquidity cost is somewhat lower when the market performance is better.

As noted earlier, we impose a restriction of minimal 10 return observations for each individual bond included in the estimation of liquidity series. In addition, we exclude those cases in which there are transactions on successive days but price was unchanged. After imposing these restrictions, the sample for estimating the market liquidity index contains 14,671 bonds over the entire sample period.[7] The mean π value (−4.99) is of economic significance. The aggregate T value of the liquidity measure π over the entire sample period is −79.45. Including the variable $\text{sign}(r^e_{i,j,t})\ \text{Vol}_{i,j,t}$ in (7) increases the R^2 value. On average, the R^2 value increases from 22.5% to 46.5% when signed volume is added to the regression. The increase in the goodness-of-fit is economically significant.

Panel B of Fig. 2 plots the innovations of the liquidity measure. As shown, there are high temporal variations in liquidity innovations. The innovation series appears to be serially independent. The autocorrelation of innovations of the liquidity measure is −0.11, which is relatively small. We employ this innovation series in the nonlinear regression model to capture the effects of unanticipated liquidity shocks on yields of municipal bonds.

Panel A of Table 3 reports the estimates of the generalized municipal bond model with liquidity risk. The recovery rate is set to 0.66 (see Moody's Investors Service, 2002; Neis, 2006; Litvack and Rizzo, 1999). Parameter estimates for tax rate and default probability are expressed in percentage terms, and standard errors are reported in parentheses. The estimates of the liquidity risk parameter are very significant and of a negative sign. Since the liquidity measure L also has a negative sign, the negative coefficient of the liquidity factor translates into a positive effect on municipal bond yields. The sensitivity of municipal bond yields to the liquidity factor increases significantly with maturity, suggesting a much higher liquidity risk for longer-term bonds. Moreover, the sensitivity of yields to market-wide liquidity increases as credit risk increases, indicating that lower quality bonds tend to have higher liquidity risk.

The estimates of marginal income tax rate and default parameter are all highly significant. The estimated marginal tax rates are around 32–33%. Two interesting patterns are observed. First, the estimates of implicit tax rates are very close to the maximum statutory income tax brackets of high-income individuals and corporations over the sample period. Second, these implicit tax rate estimates are very stable over maturity. These findings contrast sharply with previous findings that the implicit tax rates are substantially below the statutory tax rates of high-income individuals and corporations, and that the estimates of the implicit tax rate declines drastically with maturity.[8] Our empirical estimates do not exhibit these anomalies in implicit income tax rates. Results suggest that the so-called municipal yield puzzle may be attributable to missing factors in traditional models.

The estimates of default probabilities are consistent with ratings. These estimates are annualized default probabilities. The implied default probabilities are in line with previous estimates.[9] The results show that even for high quality municipal bonds with AAA and AA ratings, there is a significant amount of default premium. This result is consistent with the finding of Longstaff et al. (2005) who find that the default premium is over 50% of the yield spread even for AAA-rated corporate bonds. Within each rating class, the default probability estimates exhibit an upward trend with maturity. The upward-sloping term structure of the default probability is consistent with empirical evidence on investment-grade bonds.

Panel B of Table 3 reports the results when the liquidity variable is excluded. There are two major impacts on empirical estimates when liquidity risk is ignored. First, the estimate of the implicit tax rate declines with maturity. Second, the estimated default probability tends to be higher when the liquidity variable is ignored, especially for AA/A and BBB bonds. Results show that ignoring liquidity risk tends to bias the estimate of marginal income tax rate downward and that of default risk upward.

We next conduct the likelihood ratio test to see if liquidity risk adds significant explanatory power to the municipal bond model. The test statistic is $\text{LR} = \frac{(\text{SSR}^C - \text{SSR}^U)/k}{\text{SSR}^U/(n-K)}$ where SSR^C is the sum of the squared residuals of the constrained model, which imposes the condition that $\beta_L = 0$, SSR^U is the sum of the squared residuals of the unconstrained (full) model, K (=3) is the number of explanatory variables, k (=1) is the number of coefficients restricted to be zero, and n (=48) is the number of observations. The test statistic LR follows an F distribution with (k, $n - K$) degrees of freedom if the disturbance term is normal under the null hypothesis that the restricted coefficients are zero (see Gallant, 1987).

Panel C of Table 3 reports the results of likelihood ratio tests. The critical F value is 4.0 at the 5% level. Likelihood

[7] In all, there are 22,856 bonds/month observations over the entire sample period. This sample size is comparable to that used by the Trade Reporting and Compliance Engine (TRACE) to track the performance of corporate bonds.

[8] See Table 3 in Green (1993, p. 239).

[9] See Yawitz et al. (1985), and Wu (1991).

Table 3
Estimates of the municipal yield model

Maturity		AAA				AA/A				BBB			
		Tax rate	Default probability	Liquidity	R^2	Tax rate	Default probability	Liquidity	R^2	Tax rate	Default probability	Liquidity	R^2
Panel A. Nonlinear regression estimates of the model with the liquidity variable													
1	Value	32.87***	0.130***	−0.062***	99.73	32.38***	0.316***	−0.068***	99.67	32.96***	0.714***	−0.101***	99.31
	Std. Err	(1.47)	(0.03)	(0.02)		(1.55)	(0.03)	(0.02)		(2.91)	(0.06)	(0.03)	
2	Value	32.03***	0.171***	−0.072***	99.67	32.54***	0.244***	−0.087***	99.73	31.21***	0.721***	−0.109***	99.38
	Std. Err	(1.70)	(0.04)	(0.02)		(1.81)	(0.04)	(0.02)		(3.73)	(0.10)	(0.02)	
3	Value	32.29***	0.183***	−0.081***	99.71	32.81***	0.285***	−0.095***	99.74	33.73***	0.751***	−0.120***	99.40
	Std. Err	(1.79)	(0.05)	(0.02)		(1.92)	(0.05)	(0.02)		(3.29)	(0.10)	(0.05)	
5	Value	32.05***	0.268***	−0.082***	99.80	31.83***	0.446***	−0.099***	99.43	32.26***	0.840***	−0.137***	99.73
	Std. Err	(2.55)	(0.09)	(0.02)		(2.38)	(0.08)	(0.03)		(5.39)	(0.21)	(0.04)	
7	Value	33.42***	0.553***	−0.085***	99.63	33.77***	0.702***	−0.103***	99.61	33.18***	0.911***	−0.127***	99.90
	Std. Err	(2.54)	(0.10)	(0.02)		(2.62)	(0.11)	(0.02)		(7.22)	(0.24)	(0.03)	
10	Value	32.91***	0.618***	−0.108***	99.59	33.22***	0.742***	−0.136***	99.67	32.18***	1.090***	−0.172***	99.91
	Std. Err	(3.47)	(0.15)	(0.02)		(3.29)	(0.14)	(0.02)		(8.72)	(0.22)	(0.03)	
20	Value	32.65***	0.722***	−0.146***	99.75	32.38***	0.798***	−0.160***	99.65	33.05***	1.160***	−0.216***	99.63
	Std. Err	(4.96)	(0.24)	(0.03)		(4.68)	(0.23)	(0.04)		(4.95)	(0.24)	(0.04)	

Maturity		AAA			AA/A			BBB		
		Tax rate	Default probability	R^2	Tax rate	Default probability	R^2	Tax rate	Default probability	R^2
Panel B: Estimates of the municipal bond model without the liquidity variable										
1	Value	32.94***	0.248***	99.71	31.08***	0.349***	99.63	30.02***	0.792***	99.23
	Std. Err	(1.31)	(0.03)		(1.48)	(0.03)		(2.99)	(0.06)	
2	Value	31.47***	0.216***	99.63	31.76***	0.297***	99.65	30.92***	0.822***	99.32
	Std. Err	(1.57)	(0.04)		(1.65)	(0.04)		(3.28)	(0.09)	
3	Value	29.70***	0.217***	99.69	32.16***	0.353***	99.02	31.28***	1.039***	99.40
	Std. Err	(1.68)	(0.05)		(1.76)	(0.05)		(2.88)	(0.09)	
5	Value	31.04***	0.345***	99.24	30.37***	0.523***	99.33	29.82***	1.127***	99.70
	Std. Err	(2.26)	(0.08)		(2.10)	(0.07)		(4.70)	(0.19)	
7	Value	28.54***	0.519***	99.51	30.71***	0.766***	99.55	26.71***	1.324***	99.84
	Std. Err	(2.32)	(0.09)		(2.28)	(0.09)		(6.58)	(0.30)	

10	Value	28.43***	0.614***	99.53	27.94***	0.772***	99.59	23.39***	1.329***	99.83
	Std. Err	(3.01)	(0.13)		(2.83)	(0.12)		(7.31)	(0.34)	
20	Value	27.46***	0.667***	99.71	25.29***	0.889***	99.64	22.62***	1.414***	99.47
	Std. Err	(4.06)	(0.18)		(4.26)	(0.20)		(4.87)	(0.22)	

Maturity	AAA Bonds	AA/A Bonds	BBB Bonds
	F (1,$n-3$)	F (1,$n-3$)	F (1,$n-3$)
Panel C. Likelihood ratio tests			
1	13.62	16.56	21.72
2	15.56	15.64	15.61
3	17.90	15.41	14.38
5	14.32	13.60	11.37
7	17.09	13.53	9.25
10	13.10	13.14	6.81
20	8.44	12.02	17.19

Panel A reports parameter estimates of default probability, tax rate and liquidity risk for the municipal yield model in (6), $M_T = \Lambda_T + \beta_T C_t + \beta_L L$, where M_T is the yield on the municipal and C_T is the yield on the Treasury of maturity T, and L is the innovation of municipal market-wide liquidity. The sample period is from July 2000 to June 2004. All yield series are monthly. Municipal bonds are separated into three different rating categories: AAA, AA/A and BBB, and seven maturities. Standard errors are reported in parentheses. *, ** and *** represent significance at the 10%, 5% and 1% levels, respectively. Panel B reports the estimates of the municipal bond yield model when the liquidity risk variable is excluded. Panel C reports the likelihood ratio test statistics under the restriction that the effect of liquidity risk is equal to zero (i.e., $\beta_L = 0$) in the nonlinear municipal yield model in (6). The test statistics follow an F distribution with (1, $n-3$) degrees of freedom. The critical F value is 4.0 at the 5% level.

ratio tests reject the null hypothesis of $\beta_L = 0$ for all maturity-rating groups. Consistent with the t-tests in Panel A, results show that liquidity risk adds significant explanatory power to the municipal model. Thus, the effect of liquidity risk should be incorporated into the model to better explain the behavior of the relative municipal-Treasury yield curves.

In summary, results show that personal taxes, liquidity and default risk are important determinants of municipal bond yields. Default and liquidity risk estimates increase as bond quality decreases and maturity increases. Controlling for default and liquidity risk effects, the estimated marginal investor's income tax rate becomes quite stable across bonds of different maturities.

5.2. Decomposition of municipal bond yields

The results above show significant effects of liquidity risk on municipal bonds of different ratings and maturities. A question of particular interest is how much municipal bond yield can be attributed to liquidity risk. Table 4 reports the results of yield decomposition, where the estimate of each spread component is expressed in magnitude (%) and in the proportion of the observed yield (% yield) for each yield component. We report the yield components for each rating and maturity group. In addition, both observed yields and yield estimates (the sum of liquidity and non-liquidity components) are reported in the leading column of each rating class.

As shown, the liquidity component of municipal yields generally increases with maturity and credit risk in terms of both the magnitude and the proportion of observed yields. Intuitively, the liquidity risk premium should increase with maturity. When liquidity deteriorates, the discount rate of municipal bond increases and bond price decreases. The magnitude of this price drop increases with duration (or maturity). Thus, the liquidity risk premium would tend to be higher for bonds with longer maturity. The liquidity spread in basis points increases from 18 to 67 for AAA bonds as maturity rises from 1 to 20 years. Correspondingly, the liquidity spread in the proportion of the observed yield increases from 9% to 13%. For AA/A bonds, the liquidity spread increases from 20 to 75 bps, which accounts for about 9–15% of the observed yields of different maturities. For BBB bonds, it increases from 21 to 107 bps, or about 8–19% of the observed yield. Results show that the amount of liquidity premium is sizable for most bonds and particularly large for lower-quality long-term municipal bonds.

The non-liquidity yield component also increases with maturity and credit risk. This increase is associated more with the change in default risk than with taxes since implied marginal tax rates are not materially different across ratings and maturities. However, the non-liquidity yield component in *percentage* terms declines over maturity. Although the non-liquidity component increases in magnitude with maturity, observed yields appear to increase

Table 4
Components of municipal bond yields

Maturity	AAA Bonds						AA/A Bonds						BBB Bonds					
	Yield		Non-liquidity component		Liquidity component		Yield		Non-liquidity component		Liquidity component		Yield		Non-liquidity component		Liquidity component	
	Obs.	Est.	%	% Yield	%	% Yield	Obs.	Est.	%	% Yield	%	% Yield	Obs.	Est.	%	% Yield	%	%Yield
1	2.04	2.00	1.82	89.11	0.18	8.87	2.23	2.18	1.98	88.96	0.20	8.77	2.76	2.71	2.51	90.80	0.21	7.45
2	2.38	2.33	2.13	89.33	0.21	8.77	2.47	2.45	2.22	90.03	0.23	9.16	2.99	2.96	2.69	90.08	0.26	8.85
3	2.67	2.64	2.40	89.91	0.24	8.96	2.75	2.72	2.47	89.81	0.25	9.15	3.26	3.21	2.88	88.39	0.33	9.99
5	3.21	3.19	2.89	90.18	0.30	9.29	3.27	3.26	2.95	90.22	0.31	9.47	3.78	3.77	3.31	87.46	0.46	12.18
7	3.65	3.64	3.30	90.32	0.35	9.50	3.70	3.67	3.32	89.67	0.35	9.59	4.28	4.25	3.61	84.25	0.65	15.15
10	3.96	3.93	3.55	89.63	0.38	9.59	4.02	4.00	3.52	87.66	0.47	11.74	4.54	4.53	3.84	84.47	0.69	15.25
20	5.02	4.99	4.32	85.96	0.67	13.40	5.11	5.10	4.35	85.07	0.75	14.77	5.10	5.54	4.47	80.23	1.07	19.30

Reported above are observed yields, estimated yields, the yield component associated with tax, default and risk-free rates (non-liquidity component), and the component associated with liquidity risk for portfolios formed by ratings and maturity. The liquidity and non-liquidity components are expressed both in magnitude (in percentage) and in the proportion of the observed spread.

faster, due to an increase in liquidity risk as maturity increases. As a result, the proportion of the non-liquidity component declines with maturity whereas that of the liquidity component increases with maturity.

This finding suggests that liquidity risk is a more important factor contributing to the rising maturity premium of municipals relative to Treasuries. The liquidity premium accounts for a substantial portion of the maturity premium. For example, in Panel B of Table 2, the maturity premium for AAA bonds is 34 bps for two-year bonds and 298 bps for 20-year bonds, a difference of 264 bps. The corresponding maturity premium for Treasuries increases from 41 to 277, a 236 bps difference. It appears that the maturity premium increases faster for long-maturity municipal bonds. But if we adjust the maturity premium of 20-year municipals by the incremental liquidity premium of 46 bps (67–21 bps), we come up with an adjusted maturity premium of only 218 bps, which is lower than the corresponding maturity premium increase of Treasuries. Thus, liquidity risk appears to be the driving force behind the high yields of municipals relative to Treasuries at long-maturity.

5.3. Liquidity and bond characteristics

Previous studies have shown that liquidity may vary with securities with different characteristics and trading activities such as coupon, issuance size, volume, and trading frequency. In this section, we examine whether bonds with certain traditional liquidity characteristics have different liquidity spreads. We first rank individual municipals month by month based on coupon rate, size of the bond issue, volume, and trading frequency and sort them into three portfolios: high, medium and low. We then estimate the municipal yield model for the high and low portfolios ranked in terms of these characteristics. Since the estimates of default probability and marginal income tax rates are similar to those in Table 3, we only report estimates of liquidity risk parameters for the interest of brevity. Table 5 reports estimates of the liquidity risk parameter for different rating classes and maturities. All parameter estimates are significant at least at the 5% level. The results show that the liquidity beta (β_L) is lower in absolute value for bonds with high issuance size and volume. This pattern is consistent across ratings. A similar pattern is found for high and low trading frequency groups. Although liquidity beta also tends to be lower for high-coupon portfolios, the relationship is somewhat weaker. In general, liquidity beta increases as rating decreases and maturity increases.

We next estimate the yields for each portfolio and decompose them into liquidity and non-liquidity components. Table 6 reports mean liquidity spreads for rating group both in magnitude (%) and proportion of the observed spread (% yield). The results show that liquidity spread is lower when issue size, volume and trading frequency are higher. The difference in liquidity spreads between high and low portfolios tends to increase with

Table 5
Liquidity risk parameter estimates for portfolios ranked by characteristics

Maturity	AAA Bonds		AA/A Bonds		BBB Bonds	
	Low	High	Low	High	Low	High
Coupon						
1	−0.0791	−0.0764	−0.0832	−0.0893	−0.1565	−0.1065
2	−0.0879	−0.0866	−0.1103	−0.1024	−0.1600	−0.1295
3	−0.0942	−0.0966	−0.1163	−0.1040	−0.1540	−0.1466
5	−0.1531	−0.1669	−0.2217	−0.1791	−0.2274	−0.2246
7	−0.1642	−0.1875	−0.2541	−0.2088	−0.3766	−0.2819
10	−0.2588	−0.2388	−0.2972	−0.2678	−0.4197	−0.3643
20	−0.2626	−0.2642	−0.3265	−0.3094	−0.4238	−0.4041
Issue size						
1	−0.0643	−0.0539	−0.0848	−0.0696	−0.1309	−0.0902
2	−0.0806	−0.0589	−0.1138	−0.0786	−0.1477	−0.0967
3	−0.0958	−0.0625	−0.1264	−0.0878	−0.2624	−0.1098
5	−0.1328	−0.1076	−0.2060	−0.1183	−0.3011	−0.1772
7	−0.1631	−0.1300	−0.2199	−0.1447	−0.3187	−0.2329
10	−0.2240	−0.1672	−0.2708	−0.1721	−0.3607	−0.3205
20	−0.3577	−0.1998	−0.4313	−0.2994	−0.6853	−0.4188
Volume						
1	−0.0682	−0.0535	−0.0794	−0.0687	−0.1412	−0.0896
2	−0.0805	−0.0619	−0.1242	−0.0754	−0.1567	−0.1158
3	−0.0964	−0.0758	−0.1403	−0.0878	−0.1571	−0.1319
5	−0.1226	−0.0977	−0.1576	−0.1195	−0.2598	−0.1842
7	−0.1642	−0.1165	−0.2004	−0.1325	−0.3018	−0.2622
10	−0.2331	−0.1646	−0.2617	−0.1828	−0.4303	−0.2628
20	−0.2995	−0.2557	−0.4915	−0.2945	−0.6930	−0.4338
Trading frequency						
1	−0.0598	−0.0821	−0.0833	−0.0445	−0.1566	−0.1141
2	−0.0679	−0.0677	−0.1209	−0.0813	−0.1611	−0.1422
3	−0.0850	−0.0873	−0.1057	−0.0861	−0.1673	−0.1715
5	−0.1491	−0.1355	−0.1470	−0.1240	−0.2601	−0.2733
7	−0.1884	−0.1728	−0.2338	−0.2014	−0.2939	−0.3293
10	−0.2592	−0.2238	−0.2942	−0.1991	−0.4008	−0.3482
20	−0.3036	−0.2432	−0.4446	−0.2127	−0.4648	−0.3688

This table reports the estimates of the liquidity risk parameter (β_L) for portfolios ranked by coupon, issue size, volume, and trading frequency, respectively, for each rating and maturity class. All parameter estimates are significant at least at the 5% level.

maturity and credit risk. On average, the differences in the liquidity risk premiums between the portfolios of high- and low-issue size are 11 bps (2.8%) for AAA bonds, 16 bps (4.1%) for AA/A bonds and 24 bps (5.4%) for BBB bonds. The differences between high- and low-volume portfolios are 7 bps (1.9%), 16 bps (3.8%) and 21 bps (4.3%), for AAA, AA/A and BBB bonds, respectively. The differences between the portfolios of high and low trading frequency also show a similar pattern, with 5 bps (1.1%), 15 bps (3.9%) and 17 bps (3.6%) for the three rating groups, respectively. For the coupon portfolios, we also find that higher coupon bonds tend to have lower liquidity spreads for AA/A and BBB bonds. The differences in the liquidity spread between high- and low-coupon portfolios are 5 bps (1.5%) and 9 bps (3.3%) for these two rating groups. However, there is no clear pattern for the spread differences for AAA bonds.

In summary, liquidity risk of municipal bonds is highly correlated with traditional liquidity variables. In general, those municipals with higher volume and trading frequency, and larger issue size have lower liquidity risk and liquidity spreads. Results indicate that the aggregate liquidity measure and estimated sensitivities to market-wide liquidity capture the liquidity features of muni very well.

5.4. Robustness check

The results above show a significant liquidity premium for municipal bonds and the existence of this premium helps explain the municipal bond puzzle. In this section, we perform robustness check to see if this inference is sensitive to different model specifications. An important question is whether there is a threshold effect such that bonds with trading activity above a certain level are considered liquid and hence carry little premium. To investigate this potential effect, we divide the sample into deciles based on volume and trading frequency. If there is indeed a threshold effect, we will observe that bonds with volume or trading frequency above a certain level will carry an insignificant liquidity premium. Table 7 shows the average yields of the bonds ranked by volume and trading frequency in deciles. Results show that municipal bond yields decline monotonically as bonds' trading intensity gets higher. Most interestingly, the yield appears to taper off

Table 6
Estimates of liquidity spreads for portfolios of different characteristics

All Maturity	AAA Bonds				AA/A Bonds				BBB Bonds			
	Low		High		Low		High		Low		High	
	%	% Yield	%	% Yield	%	% Yield	%	% Yield	%	% Yield	%	% Yield
Coupon												
Mean	0.36	10.38	0.36	10.60	0.46	12.99	0.41	11.50	0.62	16.09	0.53	12.78
Issue size												
Mean	0.36	10.11	0.25	7.36	0.47	12.88	0.31	8.82	0.71	16.99	0.47	11.60
Volume												
Mean	0.34	9.63	0.27	7.74	0.47	12.66	0.31	8.83	0.69	16.18	0.48	11.85
Trading frequency												
Mean	0.37	10.28	0.32	9.17	0.46	12.62	0.31	8.71	0.67	16.04	0.50	12.40

This table reports the estimates of liquidity risk premium in magnitude (%) and proportion of the observed spread (% yield) for each high and low portfolio ranked by coupon rates, issue size, volume, and trading frequency, respectively.

Table 7
Yields for portfolios ranked by volume and trading frequency

All Maturity	AAA Bonds									
	Low	2	3	4	5	6	7	8	9	High
Panel A: Volume										
Median	3.26	3.24	3.24	3.23	3.23	3.20	3.20	3.17	3.16	3.15
	AA/A Bonds									
Median	3.33	3.30	3.30	3.29	3.27	3.25	3.23	3.23	3.23	3.22
	BBB Bonds									
Median	3.90	3.86	3.85	3.83	3.82	3.81	3.80	3.75	3.73	3.64
Panel B: Trading frequency										
Median	3.24	3.24	3.24	3.23	3.21	3.21	3.20	3.20	3.18	3.14
	AA/A Bonds									
Median	3.29	3.29	3.29	3.27	3.27	3.27	3.26	3.25	3.24	3.23
	BBB Bonds									
Median	3.88	3.87	3.86	3.80	3.79	3.76	3.77	3.74	3.72	3.65

Table 8
Liquidity risk parameter estimates for portfolios ranked by trading activity

All maturity	AAA Bonds									
	Low	2	3	4	5	6	7	8	9	High
Panel A: Volume										
Median	−0.1949***	−0.1840***	−0.1653***	−0.154**	−0.1487**	−0.1459**	−0.1335**	−0.1235**	−0.1043	−0.0808
	(−2.72)	(−2.81)	(−2.54)	(−2.28)	(−2.03)	(−2.03)	(−1.93)	(−1.93)	(−1.87)	(−1.80)
	AA/A Bonds									
Median	−0.1988***	−0.1932***	−0.1825***	−0.1582***	−0.1559***	−0.1399**	−0.1286**	−0.1236**	−0.1269	−0.0855
	(−2.74)	(−2.45)	(−2.60)	(−2.65)	(−2.48)	(−2.31)	(−2.06)	(−2.09)	(−1.91)	(−1.88)
	BBB Bonds									
Median	−0.4336***	−0.3939***	−0.3519***	−0.3152***	−0.3059***	−0.2873***	−0.2363**	−0.2170**	−0.1655	−0.1418
	(−2.94)	(−3.21)	(−2.50)	(−2.60)	(−2.63)	(−2.52)	(−2.22)	(−2.12)	(−1.81)	(−1.70)
Panel B: Trading frequency										
Median	−0.2124***	−0.1768***	−0.1415***	−0.1414***	−0.1450***	−0.1329***	−0.1234**	−0.1138**	−0.1212	−0.1072
	(−3.01)	(−2.55)	(−2.66)	(−2.78)	(−2.87)	(−2.55)	(−2.13)	(−2.06)	(−1.81)	(−1.76)
	AA/A Bonds									
Median	−0.2914***	−0.2673***	−0.2349***	−0.2451***	−0.2066***	−0.1501**	−0.1393**	−0.1207**	−0.1261	−0.1087
	(−2.78)	(−2.70)	(−2.45)	(−2.73)	(−2.57)	(−2.31)	(−2.12)	(−2.01)	(−1.90)	(−1.81)
	BBB Bonds									
Median	−0.4346***	−0.3751***	−0.3477***	−0.3076***	−0.2693***	−0.2718***	−0.1971**	−0.1725**	−0.1497	−0.1128
	(−3.52)	(−3.74)	(−2.95)	(−2.82)	(−2.48)	(−2.54)	(−2.16)	(−1.97)	(−1.85)	(−1.76)

Panel A reports the median estimates of liquidity risk parameter when municipal bonds are further divided into deciles by volume. Volume is monthly volume averaged across all bonds in a portfolio. Panel B reports the median estimates of liquidity risk parameters when municipal bonds are divided into deciles by trading frequency. Trading frequency is the monthly number of transactions averaged across all bonds in a portfolio. T-values are reported in parentheses. *** and ** indicate significance at the one and 5%, respectively.

Table 9
Estimates of the municipal yield model using trading frequency and volume as liquidity variables

Maturity		AAA				AA/A				BBB			
		Tax rate	Default probability	Liquidity	R^2	Tax rate	Default probability	Liquidity	R^2	Tax rate	Default probability	Liquidity	R^2
Panel A. Estimates of the municipal yield model using trading frequency (10^{-4}) as the liquidity proxy													
1	Value	33.40***	0.228***	0.355	99.71	31.71***	0.379***	−0.329	99.63	29.94***	0.888***	−0.504	99.15
	Std. Err	(1.36)	(0.03)	(0.27)		(1.65)	(0.04)	(0.38)		(2.89)	(0.07)	(0.91)	
2	Value	31.50***	0.222***	−0.169	99.63	32.26***	0.342***	−0.592	99.65	31.92***	0.938***	−0.180	99.25
	Std. Err	(1.62)	(0.07)	(0.94)		(1.87)	(0.09)	(1.01)		(3.67)	(0.14)	(1.02)	
3	Value	28.43***	0.247**	−0.358**	99.77	31.57***	0.308***	0.405	99.02	31.32***	1.040***	0.083	99.41
	Std. Err	(1.76)	(0.08)	(0.178)		(2.05)	(0.09)	(0.72)		(3.03)	(0.12)	(0.23)	
5	Value	30.97***	0.338***	−0.159	99.24	28.77***	0.398***	−0.168	99.36	27.65***	0.918***	−0.147**	99.70
	Std. Err	(2.65)	(0.14)	(0.80)		(2.40)	(0.12)	(0.12)		(4.70)	(0.19)	(0.07)	
7	Value	26.34***	0.505***	−0.283	99.55	28.05***	0.750***	−0.256***	99.59	20.45***	1.273***	−0.282***	99.84
	Std. Err	(2.75)	(0.16)	(0.19)		(2.50)	(0.12)	(0.11)		(6.87)	(0.33)	(0.11)	
10	Value	26.96***	0.563***	−0.196	99.58	24.16***	0.775***	−0.312**	99.59	19.95***	1.305***	−0.324***	99.83
	Std. Err	(3.24)	(0.18)	(0.17)		(3.19)	(0.18)	(0.14)		(7.00)	(0.32)	(0.08)	
20	Value	24.95***	0.655***	−0.539	99.72	27.06***	0.879***	−0.421	99.66	20.78***	1.394***	−0.051	99.73
	Std. Err	(4.42)	(0.21)	(0.56)		(4.16)	(0.20)	(0.63)		(4.86)	(0.23)	(0.04)	
Panel B. Estimates of the municipal yield model using volume (10^{-13}) as the liquidity proxy													
1	Value	32.58***	0.252***	−0.254	99.71	31.19***	0.354***	−0.470	99.63	29.82***	0.798***	−1.038	99.15
	Std. Err	(1.40)	(0.03)	(0.34)		(1.58)	(0.04)	(2.14)		(3.04)	(0.06)	(0.96)	
2	Value	30.97***	0.208***	−0.494	99.63	30.96***	0.264***	−0.531	99.65	31.02***	0.952***	−4.325	99.25
	Std. Err	(2.25)	(0.05)	(0.56)		(1.73)	(0.08)	(0.38)		(3.29)	(0.10)	(4.75)	
3	Value	29.64***	0.217****	−0.396	99.69	31.79***	0.317***	−0.632	99.02	31.23***	1.023***	−0.540	99.41
	Std. Err	(1.74)	(0.05)	(1.22)		(1.85)	(0.07)	(0.90)		(2.94)	(0.10)	(2.22)	
5	Value	32.90***	0.513***	−2.056	99.24	28.95***	0.413***	−0.759	99.34	29.22***	1.094***	−0.547	99.70
	Std. Err	(2.51)	(0.13)	(1.30)		(2.30)	(0.11)	(0.52)		(4.76)	(0.20)	(1.63)	
7	Value	30.11***	0.539***	−1.721**	99.55	29.60***	0.835***	−1.305	99.58	25.33***	1.312***	−0.718	99.84
	Std. Err	(2.55)	(0.13)	(0.83)		(2.33)	(0.12)	(0.78)		(6.73)	(0.32)	(0.75)	
10	Value	30.66***	0.581***	−1.970	99.57	27.52***	0.803***	−0.591	99.59	23.07***	1.405***	−0.557*	99.83
	Std. Err	(3.29)	(0.17)	(1.28)		(2.89)	(0.15)	(0.76)		(7.17)	(0.33)	(0.33)	
20	Value	24.25***	0.668***	−3.813	99.72	27.50***	0.891***	−2.599	99.64	22.69***	1.511***	−1.164	99.73
	Std. Err	(4.70)	(0.21)	(2.72)		(4.08)	(0.20)	(2.86)		(5.06)	(0.24)	(1.95)	

Panel A reports the results when trading frequency of the portfolio is used as the liquidity variable. Trading frequency is the monthly number of transactions averaged across all bonds in a portfolio. Panel B reports the results when volume is used as the liquidity variable. Volume is monthly volume averaged across all bonds in a portfolio.

after the 8th decile, indicating that the effect of liquidity declines when bond trading intensity reaches a certain level. Consistent with this argument, Table 8 reports the median estimates of the liquidity risk parameter and t-values for all maturity groups. Results show that the effect of liquidity becomes insignificant for bonds with trading activity in the last two deciles. Thus, for certain highly liquid bonds, the liquidity premium is inconsequential.

The Pastor–Stambaugh method captures the effect of systematic liquidity risk on bond yields. This approach differs from the traditional approach in that it focuses on the dimension of the undiversifiable liquidity risk effect. The Pastor–Stambaugh measure has several advantages over the traditional microstructure-based liquidity measures such as bid-ask spreads, volume, and trading frequency. First, these traditional liquidity measures are security specific. Thus, the liquidity premium in the traditional sense contains the idiosyncratic component. By contrast, the Pastor–Stambaugh measure provides a more standard representation of systematic liquidity risk that contributes to the expected bond return (yield). Second, the conventional measures may not capture the true liquidity. For example, bid-ask spreads may be affected by factors such as inventory concerns, adverse selection and minimum tick size. It is not clear whether this variable represents information risk or liquidity. Similarly, volume or trading frequency is not a perfect measure of liquidity. Volume and trading frequency are strongly associated with volatility which is traditionally thought to impair liquidity. In fact, liquidity is often low when volume is high. For example, the volume on the NYSE set its historical record on October 19, 1987 when the market was highly illiquid. Unlike these measures, the Pastor–Stambaugh method offers a standard approach to measure systematic liquidity risk consistent with asset pricing theory.

To see how the Pastor–Stambaugh measure fares against other liquidity variables, we estimate the yield model using simpler liquidity measures such as volume and trading frequency. The results of using these liquidity proxies are reported in Table 9. Panel A reports the results when trading frequency is used as the liquidity variable. The results show that the coefficient of trading frequency is generally negative, suggesting that bonds with higher trading frequency tend to have a lower liquidity premium or yield. However, the coefficients are insignificant for most bonds. Similar results are found in Panel B where we use volume as the liquidity variable. The effect of volume on municipal bond yield is negative but the coefficient estimates are mostly insignificant. The results show that neither trading frequency nor volume explains the equilibrium municipal bond yield better than the Pastor–Stambaugh liquidity measure. Pastor and Stambaugh (2003) indicate that while volume and trading frequency appear to be useful in explaining cross-sectional differences in liquidity, they do not capture time variation in liquidity very well. Our results are consistent with this view.

In summary, we find that the Pastor–Stambaugh method is quite robust against other methods based on alternative liquidity measures. Results suggest that liquidity risk is an important determinant of municipal bond yields. Ignoring the liquidity premium of municipal bonds thus results in erroneous inference for the marginal investor's income tax rate.

6. Conclusions

In this paper, we propose a generalized model that incorporates the effects of liquidity risk, default risk and personal taxes on municipal bond yields. The model explains the yields of uninsured municipal bonds relative to those of Treasury bonds very well. We find that long-term municipal bond yields are higher than the equivalent after-tax Treasury yields largely because both liquidity risk and default risk are higher.

Empirical evidence shows that liquidity risk is an important determinant of municipal bond yields. The sensitivity of municipal yields to market-wide liquidity increases with credit risk and maturity. Liquidity premiums are higher for bonds with lower ratings. A large portion of the maturity premium for long-term municipal bonds is attributable to the liquidity premium. In addition, default risk significantly affects yields of uninsured municipal bonds. This default effect partially explains why yields of municipal bonds tend to be high relative to yields of Treasuries of equal maturity, especially for long-maturity bonds. Tax rates are another important determinant of the relative municipal yield. The anomalies of declining implicit income tax rates over maturity documented in previous studies disappear after we control for the effects of liquidity and default risk.

Finally, we find that liquidity spreads estimated from the yield model are highly correlated with traditional liquidity variables. The Pastor–Stambaugh liquidity measure fares quite well against other simpler liquidity measures. Our findings suggest that liquidity risk should be accounted for in order to explain more satisfactorily the relative yields of tax-exempt and taxable bonds.

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