

Lecture Note 8.2: Single-Name Credit Derivatives

Introduction:

Much attention has been paid recently to the role of *credit derivatives* in the financial system. Today we will begin to examine how credit derivatives work. In this note we'll focus on the simplest ones – credit default swaps – and think about how to value and hedge them.

Last time, we saw how no-arbitrage methods could be used to price risky debt itself. Now we are dealing with derivatives whose underlying is that debt.

While no-arbitrage theory will show us how to price CDSs, we can always turn the game around and view the CDSs as primary securities. Then we can turn back to last week's topic and use these tools to value and hedge risky bonds.

Outline:

- I. An Overview of the Market.
- II. Pricing Credit Default Swaps.
- III. About Risk-Neutral Default Probabilities.
- IV. Summary

I. Single-Name Credit Derivatives

- In our study of swaps, we have already briefly encountered credit default swaps.
 - ▶ Recall that this is basically to an insurance policy that allows the owner to deliver a defaulted bond of some entity X and receive the the face value (par or 100%) of that bond.
 - * Note that it is NOT identical to a put with strike price 100. You cannot receive payment unless X formally defaults. (With an ordinary put, you might want to exercise it even if the bonds just trade down in price.)
 - * Like an insurance policy, a *traditional* CDS is purchased with a fee, which is payable quarterly up until the nearer of the default event or some termination date T . The fee is quoted as an annualized percent of the face value.
 - Starting April, 2009 (June, 2009 in Europe) CDS markets began adopting a different convention: to fix the fees at standard amounts (25bp, 100bp, 500bp, 10000bp) and then to negotiate an up-front fee to clear the market.
 - We will discuss the reasons for this below.
 - Market makers are still mostly quoting fees under the zero-value convention. So for this note, I'll usually assume this.

- ▶ There are active CDS markets for thousands of reference entities, including private companies and countries.
 - * Typical trades are for face values of \$5 or \$10 million.
 - * Coverage is usually for $T = 5$ years. For the most active names, an entire *term structure* of maturities from 6 months to 30 years is traded.
 - * The market has also adopted the convention that – unless specifically requested – CDS maturity dates will be taken to be either March 20, June 20, September 20 or December 20.
 - So a “5-year” contract entered into on April 20 would actually give protection for 5 years and 2 months.
 - * Also the standard assumption is that these 4 dates are when the fee payments will be made each year.
- ▶ As of 2009, ISDA estimated that there were 31 *trillion* dollars in principal amount of CDS insurance outstanding.
 - * By comparison the capitalization of the U.S. stock market was about 15 trillion.
 - * The size of total mortgage debt was 7 trillion, and there were 5 trillion outstanding in government bonds.
 - * Of course these comparisons are somewhat misleading. As

we have discussed, the notional outstanding amounts are very different from market values.

- * Moreover, these numbers include many non-netted and redundant exposures that in theory could be “compressed”.
- In fact, participants are working hard to streamline contracts so that more netting and compression will be possible to lower the outstanding amount.
- This was part of the motivation behind the convention changes mentioned above.

► CDS contracts now all specify *cash settlement* in which a reliable post-default price, R , is solicited from the market via an auction, open to anyone, for the underlying deliverable debt.

- * Insurance sellers must then pay all buyers $100 - R$.
- * The auctions are run electronically on a platform called Creditex (see www.creditfixings.com).
- * R is referred to as the *recovery amount*.

- ▶ Another important feature to note about this type of insurance is that the contract typically allows the buyer to deliver any of several debt obligations of the insured amount and receive par.
 - * For example, it could specify any senior unsecured bond.
 - * This gives the buyer a cheapest-to-deliver option. He/she will obviously choose to deliver the *most junior* security allowed with the longest maturity and lowest coupon , since these will be the ones worth least.
 - Then the payoff is $\max_i(100 - R^{(i)})$, where i indexes all the firm's eligible debt claims.
 - * The contract does specify what the currency of the obligation must be.
- ▶ For a given reference entity, there could be both senior and subordinated CDS as well as CDS specifically covering *bank loans*.
 - * These are called LCDS.
- ▶ As with any insurance, it is very important to iron out in advance the precise details of what events trigger payment.
 - * The formal definition of a credit event can still differ from contract to contract.

- * Usually it is straightforward: if X fails to perform any of the required covenants in any of its debt contracts, that's a default – on all of them.
- * This includes failure to pay or repudiation or moratorium.
- * Things can get tricky, however, if X undertakes a *debt restructuring*. This is often quasi-voluntary, i.e., it is the result of negotiation between X and its bondholders. But it has the result of harming those bondholders. They ultimately receive less principle and interest than they were originally due.
- * Under current ISDA documentation, restructuring may or may not count as a credit event. There are 4 main types of contract.

XR Restructuring does not count as a default.

CR Any restructuring counts as default.

MR Any restructuring counts as default BUT in a restructuring only debts that mature within 2.5 years are protected.

MM Any restructuring counts as default BUT in a restructuring only debts that mature within 5 years are protected.

- * For North American reference companies, XR is now the standard.

- * For sovereign entities it is usually CR. But there is still some ambiguity about which actions exactly constitute a restructuring (c.f. Greece 2011).
- If X is acquired, the acquiror becomes the new reference entity.
 - * Usually default by a “downstream” subsidiary of X is a credit event for X , but not default by an affiliate of common parent.
 - * As far as I know, there is no clear way of determining what happens if a company splits apart into several successor companies!
- The 2009 agreements established 5 global “Determination Committees” (for separate geographical regions) who decide exactly when a credit event occurred for a given reference entity.
 - * Sometimes, it takes several weeks after the fact to fix the precise date.
 - * Under the new market standards, if you have bought credit protection, you will receive payment on any default within a 60 day “look-back period.”
 - So even if your contract expires at T , if it is determined at $T + 30$ that a default happened at $T - 10$, you are covered.

- ▶ Like most OTC contracts, CDS are not assignable without the permission of the original parties. However it is fairly common for traders to ask for such permission, which is called a *novation*.
 - * If I have sold insurance on Ford to Citibank, and later I buy the same insurance from Deutsche Bank, I can send an electronic request to Citibank for a novation and they must respond within 2 hours.
 - * If they agree, my trades are cancelled and Deutsche Bank becomes the seller of Ford insurance to Citibank. Otherwise I have two outstanding trades on my books.
 - * When a novation occurs, I receive (or pay, if negative) the mark-to-market difference in values between the two swaps and Citi continues to pay the original rate to DB.
- ▶ Along with the possibility of netting out redundant positions, U.S. and European regulators have required use of a central counterparty (CCP) starting in 2013 for many CDS contracts.

- ▶ In addition, starting in 2014 CFTC rules require index CDSs to trade on SEFs just like interest rate swaps.
 - * Trades are supposed to be reported immediately to one of the so-called “swap data repositories”.
- ▶ Curiously, single-name CDS contracts are not regulated by the CFTC under Dodd-Frank. So they will only migrate to SEFs if the market wants to go that way.
- ▶ Meanwhile, non-US entities are working hard to make sure the U.S. rules don't apply to them.
- ▶ You may have already wondered *who insures the insurers?* What would be the point in buying insurance on a AAA company if it were written by a CCC company?
 - * The market convention is to quote contracts assuming the seller is a AA-type credit. Lesser rated players cannot sell unless some one else is guaranteeing their performance (or unless both sides agree on to use a CCP).
 - * As with any other OTC derivative, the contracting parties can also stipulate some marking-to-market and collateralization provisions. And the issue is obviously less important for centrally cleared contracts.

- * Notice that, even without collateral or clearing, the buyer is *somewhat* protected against the seller's credit deteriorating: the buyer will stop paying the fee if seller defaults.
 - They will suffer no loss if they can replace their CDS with another one with a healthy counterparty.
 - The risk, then, is of a *simultaneous* impairment of both the seller and the reference entity.
 - This extra risk – which will be reflected in the price – is sometimes called “wrong way risk”.
 - * Of course, if an insurance writer has written so many contracts that it is deemed *too big to fail*, then ultimately the government may fulfill its obligations.
 - This happened for AIG.
 - * Moreover, during the height of the financial crisis, major governments themselves began providing credit insurance on the debt of banks and other finance companies.
- The CDS market continues to evolve rapidly. It has proven extremely valuable to many participants. We are in a period of uncertainty about regulatory requirements right now. But when this is resolved, the market will likely resume its rapid growth.

- A second important class of credit derivatives was designed specifically to address the issue of counterparty risk. These are credit-linked notes.
 - ▶ These are bonds issued by entity Y which don't pay back their principal if the reference entity X defaults before maturity.
 - * Or, more directly, Y may have the right to deliver bonds of X instead of money at time T .
 - ▶ Notice that the buyer, Z, of such a bond is a *seller* of credit insurance. The compensation will be reflected in a high coupon rate.
 - ▶ But NOW there is no reason for Y to worry about Z's credit. Y got Z's cash up front.
 - ▶ Of course, Z may now demand a slightly higher coupon because of the risk that Y will not perform.
 - ▶ This is the same trick re-insurers use when trying to buy catastrophe coverage from many small investors.
 - * They sell bonds (with high coupon rates) that they do not re-pay if a covered event occurs.

- Besides CDS and CLN, there are several other single-name credit derivatives that are less common, but also interesting.

Binary Event Contracts. These pay out either 0 or 1 depending upon whether a credit event occurs. Recently, the CBOE has listed contracts of this type.

Recovery Swaps. These are like forward contracts, not swaps. We agree today on a “fair” forward recovery amount, F , and then *if* a credit event happens, the short-side pays the long-side $R - F$. Together, these last two types of contracts split a CDS into its two component types of risk.

Ratings-Trigger Swaps. These contracts allow the deliverable bonds of the reference entity to be delivered (for par) on any downgrade by a rating agency. Similarly, equity-trigger swaps define the relevant event to be a fall (to some prespecified level) in the stock price. These could have been useful to, e.g., Bear Stearns employees.

Dividend Swaps. These are like return swaps: one side pays a fixed fee and receives the total payouts to shareholders – including payments for tender offers – from a specific company or basket of stocks. They can be used to protect against the risk of an LBO or special dividend that would affect a company’s leverage.

Credit-Spread Swaps. Here one side agrees to pay the difference between the yield on two bonds (or indices) such as the Moody’s Aaa - Baa yield, or the the spread between particular Brazilian and U.S. Government bonds.

II. Pricing Credit Default Swaps

- Last time, we studied structural models of credit risk, which led us to the insight that *that bankruptcy risk is hedgeable*, at least under certain models.
- Let's think about what this implies about the valuation of credit derivatives.
- First, it means a risky zero coupon bond must be priced as its expected discounted value, using the risk-neutral probabilities of all possible payoffs.
- Suppose the future values of the firm are V_T and the risk-neutral probability of each outcome is $q(V_T)$ and the firm is bankrupt if $V_T \leq V_B$.

► Then, because the possible payoffs are so simple, the pricing formula has an easy interpretation:

$$\hat{B}_t(V_t) = B_{t,T} \cdot \sum_{V_T} F_T(V_T) \cdot q(V_T) = B_{t,T} \cdot \sum_{V_T > V_B} 1 \cdot q(V_T)$$

► Here $B_{t,T}$ is the risk-free bond, and I'm assuming you get nothing for the bond if the firm has gone bankrupt.

- This says something remarkable. If $R = 0$,
The risky bond price is just the risk-neutral probability that the firm survives (times B)

- Likewise a security that pays off one at T **only** when the firm dies before T is worth

$$B_{t,T} \cdot \sum_{V_T \leq V_B} 1 \cdot q(V)$$

i.e. *the risk-neutral probability that the firm dies (times B)*.

- That last security is like a binary-event CDS.
- Now consider how a CDS fee, φ , is related to probabilities.
 - ▶ We already saw that, if you were to pay a one-time fee for a one-year contract, then, assuming $R = 0$,

$$\varphi_1 = \text{Risk-Neutral Probability}(\text{bankruptcy next year})$$

is the no-arbitrage value of that year's fee. (if the fee is payable at the end of the year.)

- ▶ Similarly, if you agree today to buy one-year protection starting one year from now, the fair fee would be

$$\varphi_2 = \text{R-N Probability}(\text{bankruptcy year-after-next } \underline{\text{given survival this year}})$$

- ▶ In general, φ_n will be the *conditional* probability of death in the n th year, given survival through $n - 1$ years.

* If each year's survival is independent, each conditional death probability is the same as the first one: $\varphi_n = \varphi_1$.

- ▶ But now recognize that an N -year CDS is the same as buying a basket of N of these sequential one-year contracts, but with a single, average, fee for all N years.

- ▶ This tells us that φ must be roughly *the average risk-neutral bankruptcy probability per year over the next N years*.
- An important lesson from this discussion is that, in a sense, pricing credit derivatives is easier than a lot of other problems we've come across:
 - ▶ *We don't need the entire risk-neutral distribution. of V_T .*
 - ▶ All we need are the bankruptcy probabilities.
- Mathematically, we can summarize the information we need in one function: *the risk-neutral default density*.
 - ▶ Let me call this object $h(s; t)$. It is defined to be the (RN) probability (as of time t) that the firm dies in the interval $[s, s + ds]$
 - ▶ So the total (RN) probability of death by time T is just $\int_t^T h(s; t) ds$, which I'll call $H(T; t)$, the cumulative default distribution.
 - ▶ In terms of h and H we can also define the conditional risk-neutral default density, f , via

$$f(s; t) \equiv h(s; t) / (1 - H(s; t)) \quad \text{or} \quad h(s; t) = f(s; t) (1 - H(s; t)).$$

That is, the probability that the firm dies in the interval $[s, s + ds]$ is the probability that it survives up to s multiplied by the conditional probability of dying, given that it has survived that long.

► f is sometimes also called the **forward default intensity**, in analogy with forward interest rates.

• With these functions, we can be a little more precise in our pricing formula for a CDS.

► To start, with this notation, the value of a risky zero-coupon, zero-recovery bond maturing at s , must be

$$\hat{B}_{t,s} \equiv e^{-r(s-t)} (1 - H(s; t)) = B_{t,s} (1 - H(s; t))$$

where I'm assuming r is constant for simplicity.

► Next, what's the value of the insurance payoff to a CDS? If it pays off 1 at the instant of default (if that happens before T), its price must be

$$\int_t^T 1 e^{-r(s-t)} h(s; t) ds.$$

* This just sums (discounted-payoff-times-probability) over all possible default times until T .

* Replace 1 by $1 - R$ if you don't want to assume the recovery is zero.

► Now think about the price of the fee payments: what's the value of the constant stream φ to be paid until default?

- * The instantaneous fee paid from $[s, s+ds]$ is worth $\varphi e^{-r(s-t)} ds$ – if it happens.
- * We need to multiply that times the survival probability $1 - \int_t^s h(u; t) du = 1 - H(s; t)$ to get its value today (at t).
- * Hence, adding up all the fee payments, the total fee stream to T is worth

$$\int_t^T \varphi e^{-r(s-t)} (1 - H(s; t)) ds.$$

- So if we come across a CDS whose fee has been set to some number Φ , its net value (to the fee payor) is just the difference:

$$V(\Phi; h) = \int_t^T e^{-r(s-t)} h(s; t) ds - \Phi \int_t^T e^{-r(s-t)} (1 - H(s; t)) ds.$$

- * You could use this formula to figure out the fair up-front payment on a CDS whose “coupon” level was set arbitrarily say to $\Phi = 100\text{bp}$.

- Or, setting $V = 0$, the fair zero-cost fee must be

$$\varphi = \frac{\int_t^T e^{-r(s-t)} h(s; t) ds}{\int_t^T e^{-r(s-t)} (1 - H(s; t)) ds}$$

or

$$\frac{\int_t^T e^{-r(s-t)} (1 - H(s; t)) f(s; t) ds}{\int_t^T e^{-r(s-t)} (1 - H(s; t)) ds}$$

or

$$\frac{\int_t^T \hat{B}_{t,s} f(s; t) ds}{\int_t^T \hat{B}_{t,s} ds}$$

which, in discrete time, would be

$$\frac{\sum_{s=t}^T \hat{B}_{t,s} f(s; t)}{\sum_{s=t}^T \hat{B}_{t,s}}$$

which I hope begins to look familiar!

- Just like the swaps we analyzed in week 3, *the fair fee for a default swap is a weighted average of forward default probabilities* where the weights are the risky zero-coupon bond prices to each date.
- We can also use this to re-express the value of an existing CDS with fee Φ in terms of the current fair fee, φ :

$$V(\Phi; \varphi) = (\varphi - \Phi) \int_t^T e^{-r(s-t)} (1 - H(s; t)) ds.$$

- * This is the mark-to-market (MtM) formula for CDSs.
- * If you observed a post-Big Bang up-front (price, coupon) quote (V, Φ) , you could also use this equation to solve for the no-cost “conventional quote”, φ .

- One more observation: note that we can also re-write the integral term in the above expression as

$$\begin{aligned}
 & (\varphi - \Phi) \int_t^T e^{-r(s-t)} (1 - H(s; t)) ds \\
 &= \frac{(\varphi - \Phi)}{r} [1 - e^{-r(T-t)}] - (\varphi - \Phi) \int_t^T e^{-r(s-t)} H(s; t) ds
 \end{aligned}$$

where the first term is the value of a certain unit payment stream until T .

- * The second term subtracts off the value of *the remaining fees* $\int_s^T (\varphi - \Phi) e^{-r(u-s)} du$ that you don't get in case of default at s .

- * That is,

$$\int_t^T e^{-r(s-t)} H(s; t) ds = \int_t^T e^{-r(s-t)} h(s; t) \left[\int_s^T e^{-r(u-s)} du \right] ds$$

by an application of Fubini's theorem.

- * All this math has a very simple meaning. Going long a CDS at φ and short at Φ is NOT worth the present value of the difference in fees until T .
 - If the company defaults at s , the fees from s to T will never be paid.
 - (Notice: this is not about counterparty risk!)

Example:

- ▶ How much money do you make if you go long a 5-year CDS on Delta Airlines for 50 BP and then bad news comes out so the rate jumps to 550 BP?
 - * NOT 500 BP compounded for 5 years.
 - * If you off-set your position immediately, you only make the spread until default.
 - * That's what the MtM expression quantifies.
- If we go back and put in a payment of $(1 - R)$ for the insurance rate then the MtM formula doesn't change. The formula for the fair fee becomes

$$\varphi = \frac{(1 - R) \int_t^T e^{-r(s-t)} h(s; t) ds}{\int_t^T e^{-r(s-t)} (1 - H(s; t)) ds}$$

- ▶ But we do have to be careful about substituting in the \hat{B} s.
- ▶ A zero coupon bond with non-zero recovery has price

$$\tilde{B}_{t,T} = B_{t,T} (1 - H(T; t)) + R \int_t^T e^{-r(s-t)} h(s; t) ds.$$

- A useful application of these formulas is to take the market fees as given and *impute* the risk-neutral density $h(s; t)$ from them.
- To see how, write our last formula in discrete time, and with explicit reference to the time horizon.

$$\varphi_{t,T} = \frac{(1 - R) \sum_{s=t+1}^T B_{t,s} h(s; t)}{\sum_{s=t+1}^T B_{t,s} (1 - [\sum_{u=t+1}^s h(u; t)])}$$

- ▶ Now if I have a one-period-ahead (say one quarter) CDS, then $T = t + 1$ and this just says

$$\varphi_{t,t+1} = \frac{(1 - R) B_{t,t+1} h(t + 1; t)}{B_{t,t+1} (1 - h(t + 1; t))}$$

which is easy to solve for $h(t + 1; t)$.

- ▶ Then, write down the analogous equation for $\varphi_{t,t+2}$. Then it is likewise just a linear equation for $h(t + 2; t)$, given that you already got $h(t + 1; t)$.
- ▶ And so on.

- In general, we will not find a constant default intensity per unit time.
 - ▶ That intensity is the quantity we defined earlier: $h/(1 - H)$.

III. Risk-Neutral Default Probabilities

- As that last application suggests, a very common use of the CDS market is to view it as providing primary pricing information – i.e., not view them as “derivatives” at all.
- Using their information, one can apply no-arbitrage pricing to all the pieces of a firm’s capital structure (bonds, loans, etc) without need of an explicit structural model.
 - ▶ This can be an advantage since it avoids the necessity of making the assumptions required by the structural models, and avoids some of the tricky issues of estimation and calibration they require.
 - ▶ Effectively, the term structure of CDS spreads identifies the risk-neutral default density, which is sufficient for a lot of valuation problems.
- While taking the credit derivatives prices as given can be very helpful for a lot of problems, one might also view them as another security that we could price if we did have a structural model that we believed.
 - ▶ Or, we might believe there were potential mispricings in the CDS themselves.
- The distinction here is the same as the distinction between using call options to extract the risk-neutral density (via butterflies) in order to price other derivatives *versus* pricing options “from scratch” using a model of underlying dynamics.

- Like butterfly prices, the extracted H density is “model-free”.
- But remember, there are some important limitations to relying on extracted risk-neutral probabilities.
 - ▶ It requires access to a lot of high quality data all of the time.
 - ▶ And today’s H tells you nothing about tomorrow’s H .
- So this is where structural models may be useful too.
 - ▶ As an example, recall that there is an exact formula for the distribution of default times under Longstaff-Schwartz type models (where V_B is constant and V_t has constant σ)
 - ▶ From this H , you can get h and f by differentiation with respect to s . Then the formulas above give the implied CDS fees.
 - * Moreover, we have a direct parameterization for how these densities will vary with the state variable V .

Here we are using on the structural model to *provide* extra information about where the risk-neutral probabilities come from.

IV. Summary

- This was a quick introduction to what is likely to remain one of the largest segments of derivatives markets.
 - ▶ The contracts and institutions will continue to evolve rapidly.
- We covered the pricing and hedging of single-name credit default swaps.
 - ▶ We can view these as derivatives which depend only on the risk-neutral default intensity.
 - ▶ Or we can view them as primary securities which allow us to price and hedge risky corporate bonds.
- Intuitively, credit default swaps fees are closely linked to the credit spread on a risky entity's debt.
 - ▶ More precisely, they correspond to a weighted average of the risk-neutral forward default probabilities, times $(1 - R)$, the loss-given-default.
- If the contracts are quoted in terms of fair-value fees, then the mark-to-market formula for an existing contract is not just a function of the change in fees.
 - ▶ It also depends on the remaining future default probabilities over the life of the contract.

Lecture Note 8.2: Summary of Notation

SYMBOL	PAGE	MEANING
R	p4	post-default price of bonds of entity X
$\hat{B}_t(V_t)$	p13	value of a 0-coupon, 0-recovery bond when firm's value is V_t
$q(V_T)$	p13	R-N probability (density) that firm value at T is V_T
$B_{t,T}$	p13	price of riskless 0-coupon bond maturing at T
V_B	p13	default boundary
φ_n	p14	fee for 1-year contract insuring against default during year n (to be paid at $n - 1$)
φ	p14	fee for multi-year contract insuring against default until T
r	p15	instantaneous riskless rate
$h(s; t)$	p15	R-N default density conditional on survival from t to s
$H(s; t)$	p15	cumulative R-N default distribution
$f(s; t)$	p15	$h/[1 - H]$
$V(\Phi, h)$	p17	value of a CDS with fee Φ when R-N density is h
$V(\Phi, \varphi)$	p18	value of a CDS with fee Φ when fair fee is φ
\tilde{B}	p20	price of risky zero-coupon bond w/ recovery R