University of Illinois, Urbana-Champaign *Professor Tim Johnson*
FIN 512 - Spring 2015 *tcj@illinois.edu*

Lecture Note 9.1: Multi-name Credit Risk

Introduction:

Having studied markets for risky debt, we are now in a position to begin to analyze credit basket products. We have techniques that we can apply to multidimensional problems, and we have markets for hedging individual default risk.

Nevertheless, understanding the pricing and the risks of multi-credit securities has proved highly challenging for even the most sophisticated institutions. We will see why over the rest of the semester. Today, we start out by asking how far our methodology can go towards tackling these problems, and where its limitations lie.

To accomplish this, we are just going to study some idealized (simplified) securities. Later we will go into the mechanics of actual basket products in detail.

Outline:

I. What are basket products?
II. Two Canonical Examples.
III. Quantifying Default Correlation.
IV. Jumps to Default.
V. Wrong-Way Risk.
VI. Summary

I. What are basket products?

- Suppose we have a security whose cash-flows (payoffs) depends on the survival of a set of reference entities $\{X^{(i)}\}_{i=1}^{N}$.
  - ▶ The simplest example is just a portfolio of loans (or bonds).
  - ▶ Or, we could have a *first-to-default* swap that pays off 1.0 if any entity does not survive until T.
  - ▶ More generally, let us assume the cash-flows from our basket security at time t can be written as some function C of the form

$$C(t) = C(1_{\{\tau^{(1)} \leq t\}}, 1_{\{\tau^{(2)} \leq t\}}, \ldots, 1_{\{\tau^{(N)} \leq t\}})$$

    where $\tau^{(i)}$ is the (random) default time of entity i and $1_{\{A\}}$ is the indicator function of event A.

- We have seen that the bankruptcy risk of the individual entities may be hedgeable if we can trade some security (the stock, a bond, a CDS) whose returns can off-set the losses on default.

- The question for today is whether we can also analyze credit basket products within the PDE framework we have developed, and price them as derivatives via their expectation under the risk-neutral measure.

- For example, consider the case in which each of the entities has stock price whose dynamics obeys a CEV model.
  - ▶ If there are N firms indexed by $i = 1, 2, \ldots N$, the risk-neutralized model for each price would be

$$dS_t^{(i)} = (r - d^{(i)})S_t^{(i)}\,dt + \sigma_0^{(i)}\sqrt{S_t^{(i)}}\sqrt{S_0^{(i)}}\,dW^{(i)}.$$

    where $d^{(i)}$ is the dividend rate.

  - ▶ Note that today I'm just going to assume the riskless rate r is constant for simplicity.

  - ▶ Now the innovation process is a multivariate Gaussian,

$$dW \sim \sqrt{dt} \cdot \mathsf{N}(0, \mathcal{R})$$

    where $\mathcal{R}$ is an N by N matrix of the individual pairwise correlations.

- We can then model bankruptcy as equivalent to the event that $S^{(i)} = 0$.

- As we discussed in our earlier treatment of structural models, this approach involves some important assumptions.

- Another approach might be to assume that we can trade in a whole term-structure $\widehat{B}_{t,T}^{(i)}$ of zero-coupon bonds (with zero recovery) for each entity i.
  - ▶ To connect to our earilier notation,

$$\widehat{B}_{t,T}^{(i)} = B_{t,T}\mathrm{E}^Q[1_{\left\{\tau^{(1)}>T\right\}}] = B_{t,T}\,[1 - H^{(i)}(T;t)]$$

    where $H^{(i)}$ is the risk-neutral default probability, i.e., $\mathsf{Prob}^Q(\tau^{(i)} < T)$

- This assumption is actually equivalent to assuming we can trade CDSs (with known recovery) to each date T.

- To think through the construction of hedging (or replication) of basket products, it is sometimes easier to model the risky bond prices instead of CDS fees.

- If we can trade in the $\widehat{B}$s then, intuitively, we should be able to hedge complex cashflow processes that depend on the bankruptcy indicator varibles.
  - Remember that, thus far, we have required the underlying risk factors to be *diffusion processes*.

- We need to identify specifically what those risk factors are and how they are related to each other.

- To see why, we need to consider some specific examples.

II. Two Canonical Examples.

- Suppose we have a collateralized debt obligation (CDO) which is a collection of assets held in a trust and which pays off the cash-flows from those assets to holders of different classes of its securities (tranches) in strict order of seniority.

- To gain intuition, let us consider a very simple example of such a CDO:

  The asset side consisting of exactly two underlying bonds of risky entities, X and Y, each with notional value 100.

  The liability side consists of just two CDO tranches, J (for junior) and S (senior), whose "attachment point" is 100, i.e. the two tranches have equal face value.

- Assume all cash-flows are at the end (T) and that if either X or Y defaults its bonds have zero recovery.

- Then at T:
  - ▶ Everybody gets their principal back if nobody defaults.
  - ▶ J gets par unless *either* X or Y defaults.
  - ▶ S gets par unless *both* X and Y default.

- Clearly tranche J has more risk than S, and so will trade at a lower price. **But how much?**

- The key observation is that there is some extra risk here that *is not totally described by the individual default probabilities* of X and Y.

- Suppose the X and Y each individually have 10% chance of dying by T.

- Then we could describe this situation in (at least) two very different ways, according to the following tables.

Scenario One Probabilities:

	Y dead	Y alive	total prob:
X dead	1	9	10
X alive	9	81	90
total prob:	10	90	

Scenario Two Probabilities:

	Y dead	Y alive	total prob:
X dead	9	1	10
X alive	1	89	90
total prob:	10	90	

- J prefers the second scenario. But this case is much <u>worse</u> for S: the chance of losing are NINE TIMES higher.

- The single most important thing to understand about valuing CDOs is that the guys at the bottom (the lowest tranche) typically get wiped out *unless all the reference entities survive*. Whereas the guys at the top ONLY get hurt *if all the reference entities die*.

- In other words, investors in basket products are exposed to default correlation risk. But the sign of the exposure differs for different degrees of seniority.
  - Junior tranches are said to be "long" correlation, and seniors "short."
  - Remember: *correlated default equals correlated survival.* (Holding the marginals fixed, you can't increase the mass in one diagonal cell without also increasing the mass in the other one.)
- Each tranche holder is speculating the same way (they are long) on *overall* credit risk of the basket. But they may be exposed quite differently to the risk of correlated defaults.

- Now consider a second example.
- Suppose you are interested in buying default protection on X from Y. *How does the counterparty risk affect the price you are willing to pay?*
- Let's say we are just buying a simple binary credit put, except that now it has the additional exposure to Y.
- Then its payoffs at T are:

Risky Counterparty Payoffs:

	Y dead	Y alive
X dead	0	100
X alive	0	0

which looks very much like our junior tranche above,

Junior Tranche Payoffs:

	Y dead	Y alive
X dead	0	0
X alive	0	100

- Clearly the counterparty risk is going to cause this put to be priced at a discount to equivalent riskless ones.

- But again we can observe that the amount of this counterparty risk discount is going to depend on the probability of *joint* default.

- Note that, relative to the CDO example, the payoff of our put occurs in an off-diagonal cell of the matrix. Intuitively, this tells us we are going to <u>dislike</u> correlation.

- Now consider what information we need to value our two example securities.

III. Quantifying Default Correlation.

- Let us write the payoffs of both our example securities with the indicator functions introduced earlier.

$$P^{JuniorCDO}(T) = 1_{\{\tau^{(X)}>T\}} \cdot 1_{\{\tau^{(Y)}>T\}}$$

and

$$P^{RiskyCDS}(T) = 1_{\{\tau^{(X)}\leq T\}} \cdot 1_{\{\tau^{(Y)}>T\}}.$$

- To compute expectations that give us the prices, we need to have the risk-neutral joint distribution of the two survival variables. Define

$$H^{XY}(t_1, t_2; t) \equiv \text{Prob}_t^Q\{\tau^{(X)} \leq t_1 \text{ and } \tau^{(Y)} \leq t_2\}$$

or, equivalently, the joint density $h^{XY}(t_1, t_2; t)$ via

$$H^{XY}(t_1, t_2; t) = \int_0^{t_1} \int_0^{t_2} h^{XY}(t, u, v) \, dudv.$$

- Then the CDO tranche value can be written

$$B_{t,T} \int_T^{\infty} \int_T^{\infty} h^{XY}(u, v; t) \, dudv.$$

and the credit put with risky counterparty is

$$B_{t,T} \int_T^{\infty} \int_0^{T} h^{XY}(u, v; t) \, dudv.$$

- The trouble is *where do we get this joint distribution?*

- We have seen in an earlier lecture that we can extract the individual – or marginal – risk-neutral default distributions from the prices of traded CDSs.
  - ▶ Recall, if we have an entire term structure of CDS fees for each entity, then there is a straightforward iterative system we can solve to get $h^{(i)}(T;t)$ for each T.
- But, as the example above makes clear, the two marginal distributions are <u>not</u> sufficient to determine the joint distribution.
- While there is a great deal of information in the traded prices of default insurance for each entity, the market alone cannot supply enough information to value our securities.
  - ▶ We need to supply a model of *default correlation*.
  - ▶ Building, estimating, and testing such a specification adds significant complexity to valuing credit basket products.
- But what do we really mean by default correlation?
- In fact, there are distinct ways of quantifying such a correlation and they do not always correspond to each other in obvious ways.
- Mathematically, by definition, for any two random variables, correlation is the covariance divided by the product of the two standard deviations.

- So for any two entities, we can define the correlation between their survival indicator variables.

$$r_{XY}(T) \equiv$$
$$\frac{\mathrm{E}[1_{\{\tau^{(X)}>T\}}\ 1_{\{\tau^{(Y)}>T\}}] - \mathrm{E}[1_{\{\tau^{(X)}>T\}}]\mathrm{E}[1_{\{\tau^{(Y)}>T\}}]}{\sqrt{\mathrm{E}[(1_{\{\tau^{(X)}>T\}})^2] - (\mathrm{E}[1_{\{\tau^{(X)}>T\}}])^2}\sqrt{\mathrm{E}[(1_{\{\tau^{(Y)}>T\}})^2] - (\mathrm{E}[1_{\{\tau^{(Y)}>T\}}])^2}}$$

  - Or, more simply

$$\frac{p_{XY} - p_X p_Y}{\sqrt{p_X(1-p_X)}\ \sqrt{p_Y(1-p_Y)}}$$

    where p_X stands for the risk-neutral survival probability of X, and p_{XY} stands for the risk-neutral probability of joint survival to T.

  - This formula gives one definition of default correlation.

  - It enables us to answer the question: "by how much does the survival probability of Y change if we observe that X has survived?"

$$p_{Y|X} = \frac{p_{XY}}{p_X} = p_Y + r_{XY}\ \sqrt{p_Y(1-p_Y)}\ \sqrt{\frac{1-p_X}{p_X}}$$
$$p_{Y|X^c} = \frac{p_Y - p_{Y|X}\ p_X}{1-p_X}$$

    where X^c stands for the complement of the survival event (i.e., $\tau^{(X)} < T$).

- ▶ You could use that last formula to simulate bivariate default events through (i) flipping a coin whose probability of head is p_X and then (ii) flipping a second coin for Y whose p is altered depending on the outcome of the first flip.

- A second type of correlation we could define is the correlation between the default times themselves.

$$\tilde{r}_{XY} \equiv$$
$$\frac{\mathrm{E}[\tau^{(X)}\ \tau^{(Y)}] - \mathrm{E}[\tau^{(X)}]\mathrm{E}[\tau^{(Y)}]}{\sqrt{\mathrm{E}[(\tau^{(X)})^2] - (\mathrm{E}[\tau^{(X)}])^2}\sqrt{\mathrm{E}[(\tau^{(Y)})^2] - (\mathrm{E}[\tau^{(Y)}])^2}}$$

- This quantity is nice in that it is not defined with respect to a particular time horizon.
  - ▶ Depending on the nature of the model, it might have a different value from $r_{XY}(T)$.

- Statistically, however, how could one estimate a default correlation like this for two companies – risk-neutral or true – given that they're both still alive!?

- One possibility is to connect the correlation of the survival events to the correlation of some traded securities of the reference entities.

- In the structural approach, we could simply estimate the return correlation $\rho^{X,Y}$ from historical stock returns. That would be sufficient to compute the joint default density.

- We could then simulate successive steps of the multivariate process by drawing correlated normal innovations.
- Of course, the structural approach is no help at all if the reference entities are not firms.
  - ▶ But they do have to have some debt outstanding – or else no one would be interested in their credit risk!
- So we could numerically estimate correlations between their bond prices or credit spreads.
- However, it is important to understand that *these correlations are all different numbers.*
- In other words, there is no single unique meaning to "default correlation". And we cannot directly use estimates of one correlation in a situation where we need another one.
- The object that we ultimately need to find is the full joint distribution of the survival times.
  - ▶ To build that, we need to re-connect to our theory of the underlying risk factors.

IV. Jump-to-default models.

- We now have to confront a major gap in our modeling arsenal.

- If we want to view credit products as derivatives whose underlying risk factors are risky bond prices or CDS fees, we need to acknowledge that these processes usually have <u>discontinuous</u> returns opon the occurrence of credit events.
  - ▶ They jump.

- This suggests the need for a model of default events where at any instant for any entity there is some probability of such a jump.

- Suppose, at every date t there is an <u>independent</u> coin flip, I_t, with probability p_t that determines whether or not bankruptcy occurs for the ith name.
  - ▶ Then the probability of survival to time t_K is
  $$\mathrm{E_t}\left[\Pi_{k=1}^{K}(1-I_{t_k})\right] = \Pi_{k=1}^{K}\ \mathrm{E_t}\left[(1-I_{t_k})\right] = \Pi_{k=1}^{K}(1-p_{t_k})$$
  - ▶ In continuous time, we can describe the evolution of the probabilities by an intensity process λ_t such that $p_t \approx \lambda_t\ dt$ or $(1-p_t) = e^{-\lambda_t\ dt}$. Then
  $$\Pi_{u=t}^{T}\ (1-p_{t_k}) \rightarrow \mathrm{E_t}\left[e^{-\int_t^T\ \lambda_u\ du}\right].$$
  (Beware: λ doesn't stand for a market price of risk now.)

- ▶ If λ is a deterministic function, the expectation here is just $\exp(-\bar{\lambda}(T-t))$ where

$$\bar{\lambda} = \frac{1}{T-t} \int_t^T \lambda(u)\, du.$$

- An important special case is that of constant intensity.

- This default intensity model is consistent with a continuous-time model of the risky bond price which has jumps:

$$d\widehat{B}/\widehat{B} = (\mu + \lambda)\, dt - dJ_t.$$

- Here J_t is a *Poisson process* with intensity λ (true, not risk-neutral).
  - ▶ The Poisson process starts at 0 and jumps up to 1 in time interval dt with probability $\lambda\, dt$. Such a jump then drives the bond price to zero. And it stays there.
  - ▶ The risk premium $\mathrm{E_t}\left[d\widehat{B}/\widehat{B}\right] - r$ on the bond is $\pi = \mu - r$.

- This is the first model we have seen that explicitly incorporates jumps.

- Yes, jumps can be incorporated into the no-arbitrage pricing framework!
  - ▶ If we have two securities, $c(J)$ and $p(J)$, exposed only to jump risk, then we can create an instantaneously hedged portfolio that holds one unit of c and is short $\frac{c(1,t)-c(0,t)}{p(1,t)-p(0,t)}$ units of p. Then it will not change in value when J jumps from 0 to 1. So it must earn the riskless rate.

▶ This logic – plus the version of Ito's lemma for jump processes – leads to a no-arbitrage differential equation.

▶ Now the generalized Feynman-Kac theorem tells us that we can solve the equation to find $c = c(0, t)$ by *adjusting the jump intensity from λ to $\lambda + \pi$*, and computing c's expected discounted cash-flows under this altered model.

▶ In this case, $\pi = \lambda^Q - \lambda$ is the market price of jump risk.

* $*$ If it is positive, then the pricing measure is pessimistic: risk-neutral jumps happen more often than true ones.

* $*$ Note that since $\widehat{B}$ is a traded risk factor its expected return under the risk neutral measure is r, as usual.

  - However, unlike the diffusion case, we <u>do</u> need to know the market price of jump risk in order to do any calculations with the risk-neutral measure.

  - By adjusting the jump-intensity we are also altering the model's second moments, which is also not done in for diffusions.

• We have also seen that we can infer a risk-neutral default distribution from market prices of single-name credit protection.

▶ We can likewise extract a forward λ^Q for every date up to T from that inferred distribution via solving

$$1 - H(T; t) = \text{Prob}^Q\{\text{survival to } T\} = e^{-\int_t^T \lambda_u^Q \, du}.$$

- Market data – credit spreads – provide us with the *expected* risk-neutral default intensities to each date.
  - ▶ However in general these will not be constant: λ^Q will change over time.
  - ▶ But that is natural if the true intensity is also changing.
- We can handle this by letting λ itself obey its own random process. Such *stochastic intensity* models are a key building block in credit analysis.
- Notice that once we introduce stochastic lambdas, bonds (or any claim subject to default risk) are functions of <u>two</u> separate random processes: the lambda itself AND the jump process.
  - ▶ Each of these is subject to its own risk-neutralization.
- Returning to today's topic, if we have two risky entities, how can we model default correlation?
- Perhaps surprisingly, there is no simple definition of correlated Poisson process analogous to, say, a correlated normal distribution.
  - ▶ Intuitively, if the probability of $J^{(1)}$ jumping in the next instant is $\lambda^{(1)}\,dt$ and the probability of $J^{(2)}$ jumping is $\lambda^{(2)}\,dt$, then the probability of a *simultaneous* jump would be of order dt^2 which is effectively zero.
- Two different approaches are commonly used to overcome this inconvenience.

Common jump processes. Default could be triggered by several *distinct* jump events. We could, for example, have a bond process like

$$d\widehat{B}/\widehat{B} = (\mu + \lambda)\, dt - dJ_t^{(1)} - dJ_t^{(2)}.$$

Then, If two entities, X and Y each have a common dJ as well as their own idiosyncratic one, the common one will induce simultaneous default.

Correlated intensities. If we prefer for each entity to just have its own jump process, then we can have their λ processes driven by correlated Brownian motions. Then, although their two Poissons will not be able to literally jump at the exact same time, in any finite time interval the probability of joint default will increase with the correlation.

- These two approaches are not mutually exclusive. There could be a common jump term that also has stochastic intensity.
- Going a step further, one can also model the possibility of *default contaigon* by making the intensity processes themselves depend on the jumps of the other Poissons.
  - ▶ For example, if one process jumps (one firm defaults), that could raise the intensity of all the other jumps in the same industry.

V. Wrong-way risk in CDS.

- Now let's return to the problem we described last week of valuing a CDS written on entity X by entity Y when (1) Y can go bankrupt, and (2) Y and X can go bankrupt together.

- With our default intensity models we can quantify the effects of (1) and (2) precisely.

- I will do the calculations for the case that the default intensities of each entity are constant.

- Then, using the same notation as above, the price of security paying off 1 at T if X is dead and Y is alive is

  $$B_{t,T}\, H^X(T;t)\,(1-H^Y(T;t)) = B_{t,T}\,(1-e^{-\lambda^X(T-t)})\, e^{-\lambda^Y(T-t)}$$

  - ▶ Quite naturally, the second term represents the up-front discount we would demand to compensate for Y's credit.

- Or, in terms of the CDS fee, as we did before, we can equate the value of the protection leg and the fee leg, recognizing that both payments are contingent on Y being alive.

  - ▶ The insurance leg is then worth

$$(1-R)\int_t^T e^{-r(s-t)}\; h^X(s;t)\;(1-H^Y(s;t))\,ds$$

$$= \lambda^X(1-R)\int_t^T e^{-r(s-t)}\;(1-H^X(s;t))\;(1-H^Y(s;t))\,ds$$

▶ But the fee leg is worth

$$\varphi \int_t^T e^{-r(s-t)} \left(1 - H^X(s;t)\right) \left(1 - H^Y(s;t)\right) ds$$

▶ So the fair fee is just $\lambda^X(1-R)$ – which is the same as if Y were riskless.

▶ Interestingly, the CDS is is actually hedged against counterparty risk if the counterparty's default risk is independent of the reference entity.

  * This is because you have the ability to stop paying the fee if Y dies.

- Moreover, I also neglected to value any further cash-flow that would result from Y's death.
  ▶ Recall that in default we have a claim on Y equal to the replacement value of the CDS.

  ▶ But if that value is negative, the bankruptcy court has a claim on us.

  ▶ If the replacement value is determined without regard to further counterparty risk, then we computed previously that, in terms of the time-s CDS fee, our contract's value at time s would be

$$V(\varphi_s;\varphi) = (\varphi_s - \varphi) \int_s^T e^{-r(u-s)} \left(1 - H^X(u;s)\right) du.$$

- ▶ So if Y is bankrupt and they are net winners, we lose the swap but have to pay its value. So the net economic effect on us is zero.
- ▶ But if we are winners we lose something worth V but only gain a claim worth $R^Y V$.
- ▶ Hence our valuation should include another term like

$$\int_t^T e^{-r(s-t)}\ h^Y(s;t)\ (R^Y - 1)\max[V(\varphi_s;\varphi), 0]\, ds$$

- In my example calcuation, the assumption I made about constant default intensities means that in the future the cost of protection on X will also be $\lambda^X(1-R)$.
  - ▶ So in this case, I'm justified in ignoring the extra term: V will be zero.
  - ▶ But this will NOT be the case in most problems.

- Now let's put in default correlation.

- Suppose that there are three separate Poissons: idiosyncratic ones for X and Y (with intensities λ^{X^i} and λ^{Y^i}) and a common systematic one with intensity λ^Z.
  - ▶ So the probability of X defaulting in dt is $(\lambda^{X^i} + \lambda^Z)\ dt$.
  - ▶ Thus each marginal default event is still exponentially distributed.

- Then the up-front price of my simple time-T credit put with a riskless counterparty is worth

$$B_{t,T}\ H^X(T;t) = B_{t,T}\ (1 - e^{-(\lambda^{X^i}+\lambda^Z)(T-t)})$$

- But with Y as my counterparty, it's only worth

$$B_{t,T}\, H^{X^i}\, (1-H^{Y^i})\, (1-H^Z) = B_{t,T}\, (1-e^{-\lambda^{X^i}(T-t)})e^{-\lambda^{Y^i}(T-t)}e^{-\lambda^{Z^i}(T-t)}$$

- That last expression is the (discounted) risk-neutral probability that there was no common jump, or idiosyncratic Y jump, but there was an idiosyncratic X jump. We can also write this

$$B_{t,T}\ (1 - e^{-\lambda^{X^i}(T-t)})e^{-\lambda^Y(T-t)}$$

  where $\lambda^Y = \lambda^{Y^i} + \lambda^Z$.

- So, relative to the uncorrelated case, we are still penalizing the contract for the TOTAL default risk of Y but only paying for the idiosyncratic component of X's risk

- What about the CDS fee?

- Now the protection leg is worth

$$(1-R)\int_t^T e^{-r(s-t)}\ h^{X^i}(s;t)\ (1-H^Y(s;t))\, ds$$

  i.e., for each s, the probability of an idiosyncratic X jump when Y is still alive. Or, here,

$$\lambda^{X^i}(1-R)\int_t^T e^{-r(s-t)}\ e^{-(\lambda^{X^i}+\lambda^{Y^i}+\lambda^Z)(s-t)}\, ds$$

- But the fee leg is also worth

$$\varphi \int_t^T e^{-r(s-t)} \quad e^{-(\lambda^{X^i}+\lambda^{Y^i}+\lambda^Z)(s-t)}\, ds$$

because the exponential term is the probability that none of the processes have jumped by time s

- So again the integrals cancel and the fair fee is $\lambda^{X^i}(1-R)$.
  - Now the default correlation has lowered the fee by the amount λ^Z
  - Interestingly the idiosyncratic component of Y's default risk still drops out.

- The risk of a swap counterparty going backrupt precisely when you need him to be healthy is called *wrong-way* risk in the markets.
  - It would be especially relevent, for example, in buying protection against Russian sovereign default from a Russian bank.

VI. Summary.

- Modeling credit *correlation* is crucial for pricing and hedging of multi-name credit products.

- Since there is a huge volume of such products outstanding, this is an important challenge for financial engineering.

- The ability to hedge the credit of each individual name in the reference basket is not sufficient to hedge their joint risks.

- Marginal default probabilities (credit spreads) do not nail down joint probabilities.

- Historical data can tell us correlations between, e.g., stock prices or bond credit spreads.
  - ▶ Under some modelling assumptions, these can be sufficient to define (risk-neutral) joint default densities.
  - ▶ This was the case in our two-entity examples, which showed the explicit correlation adjustment.

- In practice, default correlations can be time-varying and may be driven by systematic factors, geographical factors, and industry factors.

- Ultimately there must be a feasible hedging strategy for joint-default risk in order for the logic of no-arbitrage theory to apply.

Lecture Note 9.1: Summary of Notation

SYMBOL	PAGE	MEANING
$C(t)$	p2	date t cashflow for credit basket product
$\tau^{(i)}$	p2	default date for entity i
$1_{\{A\}}$	p2	indicator function =1 if A happens, 0 otherwise
r	p3	riskless rate (assumed constant throughout)
$S^{(i)}$	p3	ith firm's stock price
$\sigma_0^{(i)}$	p3	ith firm's stock initial volatility in CEV model
$d^{(i)}$	p3	ith firm's dividend yield
$\sim$	p3	"random with distribution... "
$\mathsf{N}(0, \mathcal{R})$	p3	normal with mean zero and correlation matrix $\mathcal{R}$
$\widehat{B}_{t,T}^{(i)}$	p4	zero-coupon, zero-recovery bond of entity i
$\mathsf{Prob}^Q(), \mathsf{E}^Q()$	p4	risk-neutral probability or expectation operators
$H^{(i)}(T;t)$	p4	risk-neutral cumulative default probability of entity i
$h^{(i)}(T;t)$	p4	risk-neutral default density (or discrete probability)
R	p5	recovery value of defaulted bond
φ	p5	CDS fee
X, Y	p6	individual entities in 2-name CDO basket
S, J	p6	senior and junior CDO tranches
$P^{JuniorCDO}$	p10	no-arbitrage price of example security 1
$P^{RiskyCDS}$	p10	no-arbitrage price of example security 2

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Lecture Note 9.1: Notation – continued

SYMBOL	PAGE	MEANING
$H^{X,Y}(t_1, t_2; t)$	p10	risk-neutral cumulative joint distribution of X, Y default
h^X, h^Y	p10	marginal risk-neutral default densities of X, Y
$r_{X,Y}$	p11	correlation of default outcome indicators for X, Y
$p_{X,Y}$	p11	prob that X, Y both surviveto some T
$p_{Y\|X}$	p12	prob that Y survives given that X does
$p_{Y\|X^c}$	p12	prob that Y survives given that X does not
I_{t_k}	p14	0-or-1 random variable denoting death at time t_k
p_{t_k}	p14	prob that $I_{t_k} = 1$
λ	p14	death prob per unit time
J	p15	Poisson process starting at 0 and jumping to 1 with exponentially intensity λ
π	p15	risk premium on asset exposed only to J-risk
λ^Q	p16	risk neutral default intensity
$V()$	p20	replacement value of a CDS contract