ENVIRONMENTAL REPERCUSSIONS AND THE ECONOMIC STRUCTURE: AN INPUT-OUTPUT APPROACH

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I

POLLUTION is a by-product of regular economic activities. In each of its many forms it is related in a measurable way to some particular consumption or production process: The quantity of carbon monoxide released in the air bears, for example, a definite relationship to the amount of fuel burned by various types of automotive engines; the discharge of polluted water into our streams and lakes is linked directly to the level of output of the steel, the paper, the textile and all the other water-using industries and its amount depends, in each instance, on the technological characteristics of the particular industry.

Input-output analysis describes and explains the level of output of each sector of a given national economy in terms of its relationships to the corresponding levels of activities in all the other sectors. In its more complicated multi-regional and dynamic versions the input-output approach permits us to explain the spatial distribution of output and consumption of various goods and services and of their growth or decline — as the case may be — over time.

Frequently unnoticed and too often disregarded, undesirable by-products (as well as certain valuable, but unpaid-for natural inputs) are linked directly to the network of physical relationships that govern the day-to-day operations of our economic system. The technical interdependence between the levels of desirable and undesirable outputs can be described in terms of structural coefficients similar to those used to trace the structural interdependence between all the regular branches of production and consumption. As a matter of fact, it can be described and analyzed as an integral part of that network.

It is the purpose of this report first to explain how such "externalities" can be incorporated into the conventional input-output picture of a national economy and, second, to demonstrate that — once this has been done — conventional input-output computations can yield concrete replies to some of the fundamental factual questions that should be asked and answered before a practical solution can be found to problems raised by the undesirable environmental effects of modern technology and uncontrolled economic growth.

II

Proceeding on the assumption that the basic conceptual framework of a static input-output analysis is familiar to the reader, I will link up the following exposition to the numerical examples and elementary equations presented in chapter 7 of my book entitled *"Input Output Economics"* (Oxford University Press, N.Y. 1966).

Consider a simple economy consisting of two producing sectors, say, Agriculture and Manufacture, and Households. Each one of the two industries absorbs some of its annual output itself, supplies some to the other industry and delivers the rest to final consumers — in this case represented by the Households. These inter-sectoral flows can be conveniently entered in an input-output table. For example:

TABLE 1. — INPUT-OUTPUT TABLE OF A NATIONAL ECONOMY (IN PHYSICAL UNITS)

From \ Into	Sector 1 Agriculture	Sector 2 Manufacture	Final Demand Households	Total Output
Sector 1 Agriculture	25	20	55	100 bushels of wheat
Sector 2 Manufacture	14	6	30	50 yards of cloth

The magnitude of the total outputs of the two industries and of the two different kinds of

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inputs absorbed in each of them depends on, (1) the amounts of agricultural and manufactured goods that had to be delivered to the final consumers, i.e., the Households and, (2) the input requirements of the two industries determined by their specific technological structures. In this particular instance Agriculture is assumed to require 0.25 (= 25/100) units of agricultural and 0.14 (= 14/100) units of manufactured inputs to produce a bushel of wheat, while the manufacturing sector needs 0.40 (= 20/50) units of agricultural and 0.12 (= 6/50) units of manufactured product to make a yard of cloth.

The "cooking recipes" of the two producing sectors can also be presented in a compact tabular form:

TABLE 2. — INPUT REQUIREMENTS PER UNIT OF OUTPUT

Into / From	Sector 1 Agriculture	Sector 2 Manufacture
Sector 1 Agriculture	0.25	0.40
Sector 2 Manufacture	0.14	0.12

This is the "structural matrix" of the economy. The numbers entered in the first column are the technical input coefficients of the Agriculture sector and those shown in the second are the input coefficients of the Manufacture sector.

III

The technical coefficients determine how large the total annual outputs of agricultural and of manufactured goods must be if they are to satisfy not only the given direct demand (for each of the two kinds of goods) by the final users, i.e., the Households, but also the intermediate demand depending in its turn on the total level of output in each of the two productive sectors.

These somewhat circular relationships are described concisely by the following two equations:

$$X_1 - 0.25X_1 - 0.40X_2 = Y_1$$
$$X_2 - 0.12X_2 - 0.14X_1 = Y_2$$

or in a rearranged form,

$$\begin{aligned} 0.75X_1 - 0.40X_2 &= Y_1 \\ -0.14X_1 + 0.88X_2 &= Y_2 \end{aligned} \qquad (1)$$

X_1 and X_2 represent the unknown total outputs of agricultural and manufactured commodities respectively; Y_1 and Y_2 the given amounts of agricultural and manufactured products to be delivered to the final consumers.

These two linear equations with two unknowns can obviously be solved, for X_1 and X_2 in terms of any given Y_1 and Y_2.

Their "general" solution can be written in form of the following two equations:

$$\begin{aligned} X_1 &= 1.457Y_1 + 0.662Y_2 \\ X_2 &= 0.232Y_1 + 1.242Y_2. \end{aligned} \qquad (2)$$

By inserting on the right-hand side the given magnitudes of Y_1 and Y_2 we can compute the magnitudes of X_1 and X_2. In the particular case described in table 1, $Y_1 = 50$ and $Y_2 = 30$. Performing the necessary multiplications and additions one finds the corresponding magnitudes of X_1 and X_2 to be, indeed, equal to the total outputs of agricultural (50 bushels) and manufactured (100 yards) goods, as shown in table 1.

The matrix, i.e., the square set table of numbers appearing on the right-hand side of (2),

$$\begin{bmatrix} 1.457 & 0.662 \\ 0.232 & 1.242 \end{bmatrix} \qquad (3)$$

is called the "inverse" of matrix,

$$\begin{bmatrix} 0.75 & -0.40 \\ -0.14 & 0.88 \end{bmatrix} \qquad (4)$$

describing the set constants appearing on the left-hand side of the original equations in (1).

Any change in the technology of either Manufacture or Agriculture, i.e., in any one of the four input coefficients entered in table 2, would entail a corresponding change in the structural matrix (4) and, consequently, of its inverse (3). Even if the final demand for agricultural (Y_1) and manufactured (Y_2) goods remained the same, their total outputs, X_1 and X_2, would have to change, if the balance between the total outputs and inputs of both kinds of goods were to be maintained. On the other hand, if the level of the final demands Y_1 and Y_2 had changed, but the technology remained the same, the corresponding changes in the total outputs X_1 and X_2 could be determined from the same general solution (2).

In dealing with real economic problems one takes, of course, into account simultaneously the effect both of technological changes and of anticipated shifts in the levels of final deliveries. The structural matrices used in such computations contain not two but several hundred sectors, but the analytical approach remains the same. In order to keep the following verbal argument and the numerical examples illustrating it quite simple, pollution produced directly by Households and other final users is not considered in it. A concise description of the way in which pollution generated by the final demand sectors can be introduced — along with pollution originating in the producing sectors — into the quantitative description and numerical solution of the input-output system is relegated to the Mathematical Appendix.

IV

As has been said before, pollution and other undesirable — or desirable — external effects of productive or consumptive activities should for all practical purposes be considered part of the economic system.

The quantitative dependence of each kind of external output (or input) on the level of one or more conventional economic activities to which it is known to be related must be described by an appropriate technical coefficient and all these coefficients have to be incorporated in the structural matrix of economy in question.

Let it be assumed, for example, that the technology employed by the Manufacture sector leads to a release into the air of 0.50 grams of a solid pollutant per yard of cloth produced by it, while agricultural technology adds 0.20 grams per unit (i.e., each bushel of wheat) of its total output.

Using $\bar{X}_3$ to represent the yet unknown total quantity of this external output, we can add to the two original equations of output system (1) a third,

$$\begin{aligned} 0.75X_1 - 0.40X_2 \qquad &= Y_1 \\ -0.14X_1 + 0.88X_2 \qquad &= Y_2 \\ 0.50X_1 + 0.20X_2 - \bar{X}_3 &= 0 \end{aligned} \tag{5}$$

In the last equation the first term describes the amount of pollution produced by Manufacture as depending on that sector's total output, X_1, while the second represents, in the same way, the pollution originating in Agriculture as a function of X_2; the equation as a whole simply states that X_3, i.e., the total amount of that particular type pollution generated by the economic system as a whole, equals the sum total of the amounts produced by all its separate sectors.

Given the final demands Y_1 and Y_2 for agricultural and manufactured products, this set of three equations can be solved not only for their total outputs X_1 and X_2 but also for the unknown total output $\bar{X}_3$ of the undesirable pollutant.

The coefficients of the left-hand side of augmented input-output system (5) form the matrix,

$$\begin{Bmatrix} 0.75 & -0.40 & 0 \\ -0.14 & 0.88 & 0 \\ 0.50 & 0.20 & -1 \end{Bmatrix} \tag{5a}$$

A "general solution" of system (5) would in its form be similar to the general solution (2) of system (1); only it would consist of three rather than two equations and the "inverse" of the structural matrix (4) appearing on the right-hand side would have three rows and columns.

Instead of inverting the enlarged structural matrix one can obtain the same result in two steps. First, use the inverse (4) of the original smaller matrix to derive, from the two-equation system (2), the outputs of agricultural (X_1) and manufactured (X_2) goods required to satisfy any given combination of final demands Y_1 and Y_2. Second, determine the corresponding "output" of pollutants, i.e., $\bar{X}_3$, by entering the values of X_1 and X_2 thus obtained in the last equation of set (5).

Let $Y_1 = 55$ and $Y_2 = 30$; these are the levels of the final demand for agricultural and manufactured products as shown on the input-output table 1. Inserting these numbers on the right-hand side of (5), we find — using the general solution (2) of the first two equations — that $X_1 = 100$ and $X_2 = 50$. As should have been expected they are identical with the corresponding total output figures in table 1. Using the third equation in (5) we find, $X_3 = 60$. This is the total amount of the pollutant generated by both industries.

By performing a similar computation for $Y_1 = 55$ and $Y_2 = 0$ and then for $Y_1 = 0$ and $Y_2 = 30$, we could find out that 42.62 of these

60 grams of pollution are associated with agricultural and manufactured activities contributing directly and indirectly to the delivery to Households of 55 bushels of wheat, while the remaining 17.38 grams can be imputed to productive activities contributing directly and indirectly to final delivery of the 30 yards of cloth.

Had the final demand for cloth fallen from 30 yards to 15, the amount of pollution traceable in it would be reduced from 17.38 to 8.69 grams.

V

Before proceeding with further analytical exploration, it seems to be appropriate to introduce the pollution-flows explicitly in the original table 1:

TABLE 3.—INPUT-OUTPUT TABLE OF THE NATIONAL ECONOMY WITH POLLUTANTS INCLUDED (IN PHYSICAL UNITS)

From \ Into	Sector 1 Agriculture	Sector 2 Manufacture	Households	Total Output
Sector 1 Agriculture	25	20	55	100 bushels of wheat
Sector 2 Manufacture	14	6	30	50 yards of cloth
Sector 3 Air pollution	50	10		60 grams of pollutant

The entry at the bottom of final column in table 3 indicates that Agriculture produced 50 grams of pollutant and 0.50 grams per bushel of wheat. Multiplying the pollutant-output-coefficient of the manufacturing sector with its total output we find that it has contributed 10 to the grand total of 60 grams of pollution.

Conventional economic statistics concern themselves with production and consumption of goods and services that are supposed to have in our competitive private enterprise economy some positive market value. This explains why the production and consumption of DDT is, for example, entered in conventional input-output tables while the production and the consumption of carbon-monoxide generated by internal combustion engines is not. Since private and public bookkeeping, that constitutes the ultimate source of the most conventional economic statistics, does not concern itself with such "non-market" transactions, their magnitude has to be estimated indirectly through detailed analysis of the underlying technical relationships.

Problems of costing and of pricing are bound, however, to arise as soon as we go beyond explaining and measuring pollution toward doing something about it.

VI

A conventional national or regional input-output table contains a "value-added" row. It shows, in dollar figures, the wages, depreciation charges, profits, taxes and other costs incurred by each producing sector in addition to payments for inputs purchased from other producing sectors. Most of that "value-added" represents the cost of labor, capital, and other so-called primary factors of production, and depends on the physical amounts of such inputs and their prices. The wage bill of an industry equals, for example, the total number of man-years times the wage rate per man-year.

In table 4 the original national input-output table is extended to include labor input or total employment row.

TABLE 4.—INPUT-OUTPUT TABLE WITH LABOR INPUTS INCLUDED (IN PHYSICAL AND IN MONEY UNITS)

From \ Into	Sector 1 Agriculture	Sector 2 Manufacture	Households	Total Output
Sector 1 Agriculture	25	20	55	100 bushels of wheat
Sector 2 Manufacture	14	6	30	50 yards of cloth
Labor inputs (value-added)	80 ($80)	180 ($180)		260 man-years ($260)

The "cooking recipes" as shown on table 2 can be accordingly extended to include the labor input coefficients of both industries expressed in man-hours as well as in money units.

In section III it was shown how the general solution of the original input-output system (2) can be used to determine the total outputs of agricultural and manufactured products (X_1 and X_2) required to satisfy any given combination of deliveries of these goods (Y_1 and Y_2) to final Households. The corresponding

TABLE 5. — INPUT REQUIREMENTS PER UNIT OF OUTPUT (INCLUDING LABOR OR VALUE-ADDED)

From \ Into	Sector 1 Agriculture	Sector 2 Manufacture
Sector 1 Agriculture	0.25	0.40
Sector 2 Manufacture	0.14	0.12
Primary input-labor in man-hours (at $1 per hour)	0.80 ($0.80)	3.60 ($3.60)

total labor inputs can be derived by multiplying the appropriate labor coefficients (k_1 and k_2) with each sector's total output. The sum of both products yields the labor input L of the economy as a whole.

$$L = k_1X_1 + k_2X_2. \tag{6}$$

Assuming a wage rate of $1 per hour we find (see table 5) the payment for primary inputs per unit of the total output to be $0.80 in Agriculture and $3.60 in Manufacture. That implies that the prices of one bushel of wheat (p_1) and of a yard of cloth (p_2) must be just high enough to permit Agriculture to yield a "value-added" of v_1 (= 0.80) and Manufacture v_2 (= 3.60) per unit of their respective outputs after having paid for all the other inputs specified by their respective "cooking recipes."

$$p_1 - 0.25p_1 - 0.14p_2 = v_1$$
$$p_2 - 0.12p_2 - 0.40p_1 = v_2$$

or in a rearranged form,

$$\begin{aligned} 0.75p_1 - 0.14p_2 &= v_1 \\ -0.40p_1 + 0.88p_2 &= v_2 \end{aligned} \tag{7}$$

The "general solution" of these two equations permitting to compute p_1 and p_2 from any given combination of values-added, v_1 and v_2 is,

$$\begin{aligned} p_1 &= 1.457v_1 + 0.232v_2 \\ p_2 &= 0.662v_1 + 1.242v_2 \end{aligned} \tag{8}$$

with v_1 = $0.80 and v_2 = $3.60 we have, p_1 = $2.00 and p_2 = $5.00. Multiplying the physical quantities of wheat and cloth entered in the first and second rows of table 4 with appropriate prices, we can transform it into a familiar input-output table in which all transactions are shown in dollars.

VII

Within the framework of the open input-output system described above any reduction or increase in the output level of pollutants can be traced either to changes in the final demand for specific goods and services, changes in the technical structure of one or more sectors of the economy, or to some combination of the two.

The economist cannot devise new technology, but, as has been demonstrated above, he can explain or even anticipate the effect of any given technological change on the output of pollutants (as well as of all the other goods and services). He can determine the effects of such a change on sectoral, and, consequently, also the total demand for the "primary factor of production." With given "values-added" coefficients he can, moreover, estimate the effect of such a change on prices of various goods and services.

After the explanations given above, a single example should suffice to show how any of these questions can be formulated and answered in input-output terms.

Consider the simple two-sector economy whose original state and structure were described in tables 3, 4, 5 and 6. Assume that a process has been introduced permitting elimination (or prevention) of pollution and that the input requirements of that process amount to two man-years of labor (or $2.00 of value-added) and 0.20 yards of cloth per gram of pollutant prevented from being discharged — either by Agriculture or Manufacture — into the air.

TABLE 6. — STRUCTURAL MATRIX OF A NATIONAL ECONOMY WITH POLLUTION OUTPUT AND ANTI-POLLUTION INPUT COEFFICIENTS INCLUDED

Inputs and Pollutants' Output \ Output Sectors	Sector 1 Agriculture	Sector 2 Manufacture	Elimination of Pollutant
Sector 1 Agriculture	0.25	0.40	0
Sector 2 Manufacture	0.14	0.12	0.20
Pollutant (output)	0.50	0.20	
Labor (value-added)	0.80 ($0.80)	3.60 ($3.60)	2.00 ($2.00)

Combined with the previously introduced sets of technical coefficients this additional information yields the following complex structural matrix of the national economy.

The input-output balance of the entire economy can be described by the following set of four equations:

$$\begin{aligned}
&0.75X_1 - 0.40X_2 = Y_1 && \text{(wheat)}\\
&-0.14X_1 + 0.88X_2 - 0.20X_3 = Y_2 && \text{(cotton cloth)}\\
&0.50X_1 + 0.20X_2 - X_3 = Y_3 && \text{(pollutant)}\\
&-0.80X_1 - 3.60X_2 - 2.00X_3 + L = Y_4 && \text{(labor)}
\end{aligned} \tag{9}$$

Variables:

X_1 : total output of agricultural products
X_2 : total output of manufactured products
X_3 : total amount of eliminated pollutant
L : employment
Y_1 : final demand for agricultural products
Y_2 : final demand for manufactured products
Y_3 : total uneliminated amount of pollutant
Y_4 : total amount of labor employed by Household and other "final demand" sectors.[1]

Instead of describing complete elimination of all pollution, the third equation contains on its right-hand side Y_3, the amount of uneliminated pollutant. Unlike all other elements of the given vector of final deliveries it is not "demanded" but, rather, tolerated.[2]

The general solution of that system, for the unknown X's in terms of any given set of Y's is written out in full below

$$\begin{aligned}
X_1 &= 1.573Y_1 + 0.749Y_2 - 0.149Y_3 + 0.000Y_4 && \text{Agriculture}\\
X_2 &= 0.449Y_1 + 1.404Y_2 - 0.280Y_3 + 0.000Y_4 && \text{Manufacture}\\
X_3 &= 0.876Y_1 + 9.655Y_2 - 1.131Y_3 + 0.000Y_4 && \text{Pollutant}\\
L &= 4.628Y_1 + 6.965Y_2 - 3.393Y_3 + 0.000Y_4 && \text{Labor}
\end{aligned} \tag{10}$$

The square set of coefficients (each multiplied with the appropriate Y) on the right-hand side of (10) is the inverse of the matrix of constants appearing on the left-hand side of (9). The inversion was, of course, performed on a computer.

The first equation shows that each additional bushel of agricultural product delivered to final consumers (i.e., Households) would require (directly and indirectly) an increase of the total output of agricultural sector (X_1) by 1.573 bushels, while the final delivery of an additional yard of cloth would imply a rise of total agricultural outputs by 0.749 bushels.

The next term in the same equation measures the (direct and indirect) relationship between the total output of agricultural products (X_1) and the "delivery" to final users of Y_3 grams of uneliminated pollutants.

The constant -0.149 associated with it in this final equation indicates that a reduction in the total amount of pollutant delivered to final consumers by one gram would require an increase of agricultural output by 0.149 bushels.

Tracing down the column of coefficients associated with Y_3 in the second, third and fourth equations we can see what effect a reduction in the amount of pollutant delivered to the final users would have on the total output levels of all other industries. Manufacture would have to produce additional yards of cloth. Sector 3, the anti-pollution industry itself, would be required to eliminate 1.131 grams of pollutant to make possible the reduction of its final delivery by 1 gram, the reason for this being that economic activities required (directly and indirectly) for elimination of pollution do, in fact, generate some of it themselves.

The coefficients of the first two terms on the right-hand side of the third equation show how the level of operation of the anti-pollution industry (X_3) would have to vary with changes in the amounts of agricultural and manufactured goods purchased by final consumers, if the amount of uneliminated pollutant (Y_3) were kept constant. The last equation shows that the total, i.e., direct and indirect, labor input required to reduce Y_3 by 1 gram amounts to 3.393 man-years. This can be compared with 4.628 man-years required for delivery to the final users of an additional bushel of wheat and 6.965 man-years needed to let them have one more yard of cloth.

Starting with the assumption that Households, i.e., the final users, consume 55 bushels

[1] In all numerical examples presented in this paper Y_4 is assumed to be equal zero.

[2] In (6) that describes a system that generates pollution, but does not contain any activity combating it, the variable X_3 stands for the total amount of uneliminated pollution that is in system (8) represented by Y_3.

of wheat and 30 yards of cloth and also are ready to tolerate 30 grams of uneliminated pollution, the general solution (10) was used to determine the physical magnitudes of the intersectoral input-output flows shown in table 7.

The entries in the third row show that the agricultural and manufactured sectors generate 63.93 (= 52.25 + 11.68) grams of pollution of which 33.93 are eliminated by anti-industry pollution and the remaining 30 are delivered to Households.

VIII

The dollar figures entered in parentheses are based on prices the derivation of which is explained below.

The original equation, system (7), describing the price-cost relationships within the agricultural and manufacturing sectors has now to be expanded through inclusion of a third equation stating that the price of "eliminating one gram of pollution" (i.e., p_3) should be just high enough to cover — after payment for inputs purchased from other industries has been met — the value-added, v_3, i.e., the payments to labor and other primary factors employed directly by the anti-pollution industry.

$$\begin{aligned} p_1 - 0.25p_1 - 0.14p_2 &= v_1 \\ p_2 - 0.12p_2 - 0.40p_1 &= v_2 \\ p_3 \qquad\qquad - 0.20p_2 &= v_3 \end{aligned}$$

or in rearranged form,

$$\begin{aligned} 0.75p_1 - 0.14p_2 \qquad &= v_1 \\ -0.40p_1 + 0.88p_2 \qquad &= v_2 \\ -0.20p_2 + p_3 &= v_3. \end{aligned} \tag{11}$$

The general solution of these equations — analogous to (8) is

$$\begin{aligned} p_1 &= 1.457v_1 + 0.232v_2 \\ p_2 &= 0.662v_1 + 1.242v_2 \\ p_3 &= 0.132v_1 + 0.248v_2 + v_3. \end{aligned} \tag{12}$$

Assuming as before, $v_1 = 0.80$, $v_2 = 3.60$ and $v_3 = 2.00$, we find,

$$\begin{aligned} p_1 &= \$2.00 \\ p_2 &= \$5.00 \\ p_3 &= \$3.00 \end{aligned}$$

The price (= cost per unit) of eliminating pollution turns out to be \$3.00 per gram. The prices of agricultural and manufactured products remain the same as they were before.

TABLE 7. — INPUT-OUTPUT TABLE OF THE NATIONAL ECONOMY (SURPLUS POLLUTION IS ELIMINATED BY THE ANTI-POLLUTION INDUSTRY)

Output Sectors / Inputs and Pollutants' Output	Sector 1 Agriculture	Sector 2 Manufacture	Anti-Pollution	Final Deliveries to Households	Totals
Sector I Agriculture (bushels)	26.12 (\$52.24)	23.37 (\$46.74)	0	55 (\$110.00)	104.50 (\$208.99)
Sector 2 Manufacture (yards)	14.63 (\$73.15)	7.01 (\$35.05)	6.79 (\$33.94)	30 (\$150.00)	58.43 (\$292.13)
Pollutant (grams)	52.25	11.68	−33.93	30 (\$101.80 paid for elimination of 33.93 grams of pollutant)	
Labor (man-years)	83.60 (\$83.60)	210.34 (\$210.34)	67.86 (\$67.86)	0	361.80 (\$361.80)
Column Totals	\$208.99	\$292.13	\$101.80	\$361.80	

$p_1 = \$2.00$, $p_2 = \$5.00$, $p_3 = \$3.00$, $p_k = \$1.00$ (wage rate).

Putting corresponding dollar values on all the physical transactions shown on the input-output table 7 we find that the labor employed by the three sectors add up to \$361.80. The wheat and cloth delivered to final consumers cost \$260.00. The remaining \$101.80 of the value-added earned by the Households will just suffice to pay the price, i.e., to defray the costs of eliminating 33.93 of the total of 63.93 grams of pollution generated by the system. These payments could be made directly or they might be collected in form of taxes imposed on the Households and used by the Government to cover the costs of the privately or publicly operated anti-pollution industry.

The price system would be different, if through voluntary action or to obey a special law, each industry undertook to eliminate, at its own expense, all or at least some specified fraction of the pollution generated by it. The added costs would, of course, be included in the price of its marketable product.

Let, for example, the agricultural and manufacturing sectors bear the costs of eliminating, say, 50 per cent of the pollution that, under prevailing technical conditions, would be generated by each one of them. They may either engage in anti-pollution operations on their own account or pay an appropriately prorated tax.

In either case the first two equations in (11) have to be modified by inclusion of additional terms: the outlay for eliminating 0.25 grams and 0.10 grams of pollutant per unit of agricultural and industrial output respectively.

$$\begin{aligned} 0.75p_1 - 0.14p_2 - 0.25p_3 &= v_3 \\ -0.40p_1 + 0.88p_2 - 0.10p_3 &= v_2 \\ - 0.20p_2 + \quad p_3 &= v_3. \end{aligned} \tag{13}$$

The "inversion" of the modified matrix of structural coefficients appearing on the left-hand side yields the following general solution of the price system:

$$\begin{aligned} p_1 &= 1.511v_1 + 0.334v_2 + 0.411v_3 \\ p_2 &= 0.703v_1 + 1.318v_2 + 0.308v_3 \\ p_3 &= 0.141v_2 + 0.264v_2 + 1.062v_3. \end{aligned} \tag{14}$$

With "values-added" in all the three sectors remaining the same as they were before (i.e., v_1 = \$.80, v_2 = \$3.60, v_3 = \$2.60) these new sets of prices are as follows:

$$\begin{aligned} p_1 &= \$3.234 \\ p_2 &= \$5.923 \\ p_3 &= \$3.185 \end{aligned}$$

While purchasing a bushel of wheat or a yard of cloth the purchaser now pays for elimination of some of the pollution generated in production of that good. The prices are now higher than they were before. From the point of view of Households, i.e., of the final consumers, the relationship between real costs and real benefits remain, nevertheless, the same; having paid for some anti-pollution activities indirectly he will have to spend less on them directly.

IX

The final table 8 shows the flows of goods and services between all the sectors of the national economy analyzed above. The structural characteristics of the system — presented in the form of a complete set of technical input-output coefficients — were assumed to be given; so was the vector of final demand, i.e., quantities of products of each industry delivered to Households (and other final users) as well as the uneliminated amount of pollutant that, for one reason or another, they are prepared to "tolerate." Each industry is assumed to be responsible for elimination of 50 per cent of pollution that would have been generated in the absence of such counter measures. The Households defray — directly or through tax contributions — the cost of reducing the net output of pollution still further to the amount that they do, in fact, accept.

On the basis of this structural information we can compute the outputs and the inputs of all sectors of the economy, including the anti-pollution industries, corresponding to any given "bill of final demand." With information on "value-added," i.e., the income paid out by each sector per unit of its total output, we can, furthermore, determine the prices of all outputs, the total income received by the final consumer and the breakdown of their total expenditures by types of goods consumed.

The 30 grams of pollutant entered in the "bill of final demand" are delivered free of charge. The \$6.26 entered in the same box represent the costs of that part of anti-pollution activities that were covered by Households directly, rather than through payment of higher prices for agricultural and manufactured goods.

The input requirements of anti-pollution activities paid for by the agricultural and manufacturing sectors and all the other input requirements are shown separately and then combined in the total input columns. The figures entered in the pollution row show ac-

Table 8. — Input-Output Table of a National Economy with Pollution-Related Activities Presented Separately

	Agriculture			Manufacture				Final Deliveries to Households	National Totals
	Wheat	Anti-pollution	Total	Cloth	Anti-pollution	Total	Anti-pollution		
Agriculture	26.12 ($84.47)	0	26.12 ($84.47)	23.37 ($75.58)	0	23.37 ($75.58)	0	55 ($177.87)	105.50 ($337.96)
Manufacture	14.63 ($86.65)	5.23 ($30.98)	19.86 ($117.63)	7.01 ($41.52)	1.17 ($6.93)	8.18 ($48.45)	.39 ($2.33)	30 ($117.69)	58.43 ($346.07)
Pollutant	52.25	−26.13	26.12	11.69	−5.85	5.84	−1.97	30 ($6.26 paid for elimination of 1.97 grams of pollutant)	
Labor (value-added)	83.60 ($83.60)	52.26 ($52.26)	135.86 ($135.86)	210.34 ($210.34)	($11.70) ($11.70)	($222.04) ($222.04)	($3.93) ($3.93)		361.8 ($361.80)
Totals Costs	($254.72)	($83.24)	($337.96)	($327.44)	($18.63)	($346.07)	($6.26)	($361.80)	

$p_1 = \$3.23$, $p_2 = \$5.92$, $p_3 = \$3.19$.
$v_1 = \$0.80$, $v_2 = \$3.60$, $v_3 = \$2.00$.

cordingly the amount of pollution that would be generated by the principal production process, the amount eliminated (entered with a minus sign), and finally the amount actually released by the industry in question. The amount (1.97) eliminated by anti-pollution activities not controlled by other sectors is entered in a separate column that shows also the corresponding inputs.

From a purely formal point of view the only difference between table 8 and table 7 is that in the latter all input requirements of Agriculture and Manufacture and the amount of pollutant released by each of them are shown in a single column, while in the former the productive and anti-pollution activities are described also separately. If such subdivision proves to be impossible and if, furthermore, no separate anti-pollution industry can be identified, we have to rely on the still simpler analytical approach that led up to the construction of table 3.

X

Once appropriate sets of technical input and output coefficients have been compiled, generation and elimination of all the various kinds of pollutants can be analyzed as what they actually are — integral parts of the economic process.

Studies of regional and multi-regional systems, multi-sectoral projections of economic growth and, in particular, the effects of anticipated technological changes, as well as all other special types of input-output analysis can, thus, be extended so as to cover the production and elimination of pollution as well.

The compilation and organization of additional quantitative information required for such extension could be accelerated by systematic utilization of practical experience gained by public and private research organizations already actively engaged in compilation of various types of input-output tables.

MATHEMATICAL APPENDIX

Static-Open Input-Output System with Pollution-Related Activities Built In

Notation

Commodities and Services

$1, 2, 3, \ldots i \ldots j \ldots m, m+1, m+2, \ldots g \ldots k \ldots n$

useful goods — pollutants

Technical Coefficients

a_{ij} — input of good i per unit of output of good j (produced by sector j)

a_{ig} — input of good i per unit of eliminated pollutant g (eliminated by sector g)

a_{gi} — output of pollutant g per unit of output of good i (produced by sector i)

a_{gk} — output of pollutant g per unit of eliminated pollutant k (eliminated by sector k)

r_{gi}, r_{gk} — proportion of pollutant g generated by industry i or k eliminated at the expense of that industry.

Variables

x_i — total output of good i

x_g — total amount of pollutant g eliminated

y_i — final delivery of good i (to Households)

y_g — final delivery of pollutant g (to Households)

p_1 — price of good

p_g — the "price" of eliminating one unit of pollutant g

v_i — "value-added" in industry i per unit of good i produced by it

v_g — "value-added" in anti-pollution sector g per unit of pollutant g eliminated by it.

Vectors and Matrices

$$A_{11} = [a_{ij}] \quad i, j = 1, 2, 3, \ldots, m$$

$$\left.\begin{array}{l} A_{21} = [a_{gi}] \\ A_{12} = [a_{ig}] \end{array}\right\} \begin{array}{l} i = 1, 2, 3, \ldots m \\ g = m+1, m+2, m+3, \ldots, n \end{array}$$

$$A_{22} = [a_{gk}] \quad g, k = m+1, m+2, m+3, \ldots, n$$

$$Q_{21} = [q_{gi}] \quad \begin{array}{l} i = 1, 2, \ldots m \\ g = m+1, m+2, \ldots n \end{array}$$

$$Q_{22} = [q_{gk}] \quad g, k = m+1, m+2, \ldots, n$$

where $q_{gi} = r_{gi}a_{gi}$

$q_{gk} = r_{gk}a_{gk}$

$$X_1 = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{Bmatrix} \quad Y_1 = \begin{Bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{Bmatrix} \quad V_1 = \begin{Bmatrix} v_1 \\ v_2 \\ \vdots \\ v_m \end{Bmatrix}$$

$$X_2 = \begin{Bmatrix} x_{m+1} \\ x_{m+2} \\ \vdots \\ x_n \end{Bmatrix} \quad Y_2 = \begin{Bmatrix} y_{m+1} \\ y_{m+2} \\ \vdots \\ y_n \end{Bmatrix} \quad V_2 = \begin{Bmatrix} v_{m+1} \\ v_{m+2} \\ \vdots \\ v_n \end{Bmatrix}$$

PHYSICAL INPUT-OUTPUT BALANCE

$$\left[\begin{array}{c:c} I - A_{11} & -A_{12} \\ \hdashline A_{21} & -I + A_{22} \end{array}\right] \left[\begin{array}{c} X_1 \\ \hdashline X_2 \end{array}\right] \left[\begin{array}{c} Y_1 \\ \hdashline Y_2 \end{array}\right] \tag{15}$$

$$\left[\begin{array}{c} X_1 \\ \hdashline X_2 \end{array}\right] = \left[\begin{array}{c:c} I - A_{11} & -A_{12} \\ \hdashline A_{21} & -I + A_{22} \end{array}\right]^{-1} \left[\begin{array}{c} Y_1 \\ \hdashline Y_2 \end{array}\right] \tag{16}$$

INPUT-OUTPUT BALANCE BETWEEN PRICES AND VALUES-ADDED

$$\left[\begin{array}{c:c} I - A'_{11} & -Q'_{21} \\ \hdashline -A'_{12} & I - Q'_{22} \end{array}\right] \left[\begin{array}{c} P_1 \\ \hdashline P_2 \end{array}\right] = \left[\begin{array}{c} V_1 \\ \hdashline V_2 \end{array}\right] \tag{17}$$

$$\left[\begin{array}{c} P_1 \\ \hdashline P_2 \end{array}\right] = \left[\begin{array}{c:c} I - A'_{11} & -Q'_{21} \\ \hdashline -A'_{12} & I - Q'_{22} \end{array}\right]^{-1} \left[\begin{array}{c} V_1 \\ \hdashline V_2 \end{array}\right] \tag{18}$$

Supplementary Notation and Equations Accounting for Pollution Generated Directly by Final Consumption

Notation

Technical Coefficients

$a_{gy,(i)}$ — output of pollutant generated by consumption of one unit of commodity i delivered to final demand.

Variables

$y_g{}^*$ — sum total of pollutant g "delivered" from all industries to and generated within the final demand factor,

$x_g{}^*$ — total gross output of pollutant g generated by all industries and in the final demand sector.

$$A_y = \begin{Bmatrix} a_{m+1,\,y(1)} & a_{m+1,\,y(1)} & \cdots & a_{m+1,\,y(m)} \\ a_{m+2,\,y(2)} & a_{m+2,\,y(2)} & \cdots & a_{m+2,\,y(m)} \\ \vdots & \vdots & & \\ a_n\ y_1 & a_n\ y_2 & \cdots & a_n\ y_m \end{Bmatrix}$$

$$Y_2{}^* = \begin{Bmatrix} y^*{}_{m+1} \\ y^*{}_{m+2} \\ \vdots \\ y_n{}^* \end{Bmatrix} \qquad x_g{}^* = \begin{Bmatrix} x^*{}_{m+1} \\ x^*{}_{m+2} \\ \vdots \\ x_n{}^* \end{Bmatrix}$$

In case some pollution is generated within the final demand sector itself, the vector Y_2 appearing on the right-hand side of (15) and (16) has to be replaced by vector $Y_2 - Y_2{}^*$, where

$$Y_2{}^* = A_yY_1. \tag{19}$$

The price-values added equations (17), (18) do not have to be modified.

Total gross output of pollutants generated by all industries and the final demand sector does not enter explicitly in any of the equations presented above; it can, however, be computed on the basis of the following equation,

$$X^* = [A_{21} \vdots A_{22}] \left[\begin{array}{c} X_1 \\ \cdots \\ X_2 \end{array}\right] + Y_2{}^*. \tag{20}$$