



# Performance evaluation of portfolio insurance strategies using stochastic dominance criteria

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## ABSTRACT

This paper evaluates the performance of the stop-loss, synthetic put and constant proportion portfolio insurance techniques based on a block-bootstrap simulation. We consider not only traditional performance measures, but also some recently developed measures that capture the non-normality of the return distribution (value-at-risk, expected shortfall, and the Omega measure). We compare them to the more comprehensive stochastic dominance criteria. The impact of changing the rebalancing frequency and level of capital protection is examined. We find that, even though a buy-and-hold strategy generates higher average excess returns, it does not stochastically dominate the portfolio insurance strategies, nor vice versa. Our results indicate that a 100% floor value should be preferred to lower floor values and that daily-rebalanced synthetic put and CPPI strategies dominate their counterparts with less frequent rebalancing.

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## 1. Introduction

In the early eighties Leland and Rubinstein (1988) developed the portfolio insurance (PI) technique based on the option pricing formula of Black and Scholes (1973). The underlying idea was that a strategy which provides protection against market losses, while preserving the upward potential, should have considerable appeal to a wide range of investors. This payoff pattern can be achieved by synthetically creating a put option on a portfolio. Benninga and Blume (1985) show theoretically that the optimality of PI strategies depends on the investor's utility function. Assuming that the continuous returns of the risky asset are independently and normally distributed, Brooks and Levy (1993) also show that (very) risk averse investors may prefer PI to a buy-and-hold strategy. Of course, it is well-documented that financial returns are not normally distributed. More specifically, their variance clusters in time and both their conditional and unconditional distributions have fatter tails than the normal distribution. To the best of our knowledge, no analytical results about the relative merits of PI strategies have been derived for more realistic distributions. Empirical results do exist, but are mixed. Garcia and Gould (1987) argue that PI cannot outperform static mix portfolios in the long run. Conversely, some recent papers (e.g. Cesari and Cremonini, 2003) provide evi-

dence on the benefits of PI in bear markets, but they do not perform formal statistical tests. Hence, the continuing use of PI as well as the mixed research evidence suggests that so far no consensus has been reached about its effectiveness.

In this paper we provide an in-depth performance evaluation of several popular PI strategies. We contribute to the literature in several ways. Firstly, we introduce the stochastic dominance (SD) framework to evaluate PI strategies. Traditionally, investment performance is measured from a mean-variance point of view. However, since PI is developed to provide an upward potential combined with a downward protection, its evaluation should take into account the entire return distribution, which is precisely what SD does. Despite its obvious advantages, SD is not widely used in an empirical context because it is marred by sampling error considerations (see e.g. Kroll and Levy, 1980). Recent theoretical work on sampling error in SD tests with iid data (e.g. Davidson and Duclos, 2000 and Barrett and Donald, 2003), has triggered a new wave of SD literature (e.g. Post, 2003). Moreover, the subsampling method of Politis and Romano (1994) and insights of Linton et al. (2005) and Kläver (2005) allow to handle sampling error in SD tests with non iid data. We compare the SD results both with traditional mean-variance performance measures and with more recently developed measures that focus either on the left tail of the return distribution (value-at-risk and expected shortfall) or the entire distribution (the Omega measure). Secondly, as PI strategies have relatively long horizons – 12 months or even longer are no exception

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for many capital guaranteed funds – empirical testing is hampered by relatively few observations. Like Benninga (1990) and Bird et al. (1990) amongst other, we set up a simulation experiment, but we do not impose a (log)normal distribution. Instead, we simulate from the empirical distribution in order to retain the salient features of the data, such as the fat tails and volatility clustering. We use a block bootstrapping procedure in which a one-year block is repeatedly randomly selected from data on different stock markets. In this way, the performance of a PI strategy can be examined for different market scenarios. The block bootstrapping technique is used to preserve both the limited autocorrelation and the substantial heteroscedasticity of real world data in the sample. Thirdly, most PI research is focused on the synthetic put strategy (e.g. Bird et al., 1990), while little attention has been devoted to a profound comparison of this strategy with a stop-loss and CPPI strategy. We compare these three PI techniques not only to each other, but also to the passive buy-and-hold (B&H) strategy. Moreover, despite its practical importance, few studies have examined different choices of the floor value, rebalancing time, and CPPI multiple (e.g. Do and Faff, 2004). Our final simulation results are devoted to the impact of these choices.

In the performance evaluation a three-stage procedure is followed. First, we examine whether PI strategies outperform a B&H strategy in terms of downside protection and risk/return trade-off. Second, we demonstrate how choosing a different floor value and CPPI multiple affects this performance. Thirdly, the impact of a departure from the continuous rebalancing assumption is analyzed by studying lower rebalancing frequencies.

Our results indicate that PI strategies are valuable alternatives to a B&H strategy for at least some investors. More specifically, all three strategies provide a significantly better downside protection than a B&H strategy, albeit at the cost of a reduction in return. SD tests reject any dominance relation between the B&H and PI strategies. The SD results point out that both for the stop-loss and synthetic put strategy a 100% floor value should be preferred to lower floor values. Furthermore, we find, unsurprisingly, that a higher CPPI multiple enhances the upward potential of the CPPI strategies, but harms the protection level. Relaxing the rebalancing discipline substantially harms the strategies' performance. Both for the synthetic put and CPPI strategy daily rebalancing should be preferred.

This paper is organized as follows. In the next section, we discuss the PI alternatives stop-loss, synthetic put and CPPI. In Section 3 the performance measures are discussed. Section 4 presents the empirical setup and Section 5 contains the simulation results. Section 6 concludes.

## 2. Portfolio insurance strategies

Using *stop-loss portfolio insurance* (SL) (e.g. Rubinstein, 1985) the portfolio is fully invested in the risky asset ('equity') as long as its value is above the discounted value of the floor, which is the minimum target level the portfolio has to reach at the end of the investment horizon  $T$ . Once the portfolio drops below the discounted floor, the portfolio switches entirely to the risk-free asset, thereby ensuring that the target is reached at  $T$ . Hence, transaction costs are incurred only once, but can be substantial, as they are computed on the entire portfolio.

The *synthetic put strategy* (SP) is a dynamic portfolio strategy that replicates the payoff of a protective put (i.e. long positions in a put option and in the underlying risky asset). The strike price  $F$  is set equal to the desired floor at  $T$ . The portfolio invests changing proportions in the risky and risk-free assets. The proportions follow from an option pricing model (see e.g. Rubinstein and Leland, 1981). When the traditional Black and Scholes valuation is

used Benninga (1990) shows that the proportion invested in the risky asset is given by

$$S \cdot N(d) / \left( S \cdot N(d) + F \cdot e^{-rt} \cdot N(\sigma\sqrt{T} - d) \right), \quad (1)$$

where  $S$  is the initial stock price,  $r$  is the risk-free rate,  $N(\cdot)$  is the cumulative standard normal distribution function, and  $d = (\ln(S/F) + (r + \frac{1}{2}\sigma^2)T) / \sigma\sqrt{T}$ . The standard deviation ('volatility') of stock returns is given by  $\sigma$ . Taking into account transaction costs, we follow the Leland (1985) approach and adjust the option volatility as follows:

$$\sigma_{\text{adjusted}}^2 = \sigma^2 \left[ 1 + \left( \sqrt{2/\pi} \right) c / \sigma\sqrt{t} \right], \quad (2)$$

where  $c$  is the relative transaction cost and  $t$  is the rebalancing interval.

Portfolio protection can also be obtained by implementing a *constant proportion portfolio insurance* (CPPI) strategy, which was introduced by Black and Jones (1987).<sup>1</sup> This strategy also implies portfolio positions in both a risky and a risk-free asset. CPPI requires investing an amount equal to the product of the multiple  $m$  and the cushion in the risky asset, while investing the remainder in the risk-free asset. The cushion is the difference between the portfolio value and the floor  $F$ , whereas  $m$  represents the desired sensitivity to market changes. We impose short sale and credit constraints in order to follow the common practice in commercial applications. The risky proportion at time  $t$  is then given by:  $\max\{\min[m(W_t - F_t), W_t], 0\}$ , where  $W_t$  represents the portfolio value at time  $t$ . Similar to the SL strategy, a time subscript is added to the floor  $F$  to indicate the discounted value of the minimum target.

## 3. Performance measurement

Portfolio theory assumes that investors select optimal portfolios by maximizing expected utility of wealth. As such, choosing between portfolios amounts to choosing between return distributions. Of course, in the absence of knowing the exact return distributions, such choices are difficult to perform. In practice, investors therefore focus on specific moments or other statistics of these distributions, such as average returns in excess of a risk-free rate ('excess returns'), return volatility, or the Sharpe ratio. Under some conditions, it can be shown that portfolio selection based on comparisons of these statistics is consistent with expected utility (see e.g. Elton et al., 2003). Unfortunately, they are not sufficient for adequate selection of PI techniques, where special attention is paid to the left tail of the return distribution. High volatility can be due to positive return outliers, which would attract rather than shy away investors. Informal performance measures such as the occurrence of negative excess returns are therefore also contemplated. Recently, value-at-risk (VaR), an asymmetric risk measure, has been put forward as an alternative to the symmetric volatility measure (see e.g. Jorion, 2001; Ibragimov and Walden, 2007). VaR denotes the maximum loss at a certain confidence level. A major drawback of VaR is that no indication is given of the loss magnitude when the VaR is exceeded. The expected shortfall (ES) addresses this disadvantage by measuring the average loss below the VaR (see e.g. Acerbi and Tasche, 2002). Of course, higher confidence levels will entail more negative VaR and ES values. Hence, choosing a higher confidence level indicates a higher degree of risk aversion, but it is difficult to link both concepts formally for non-normal return distributions. Next, a larger skewness makes a protection strategy more appealing (see e.g. Harvey and Siddique, 2000; Post et al., 2008). Shadwick and Keating (2002) introduced

<sup>1</sup> See also Black and Perold (1992).

the Omega performance measure to compare non-normal return distributions.<sup>2</sup> Rather than focusing on a particular moment of the return distribution like volatility or skewness, Omega considers the whole distribution: it evaluates the gains with respect to a predefined return threshold and the losses with respect to this threshold. Mathematically, the Omega measure for a return threshold  $r$  is defined as  $\Omega(r) = \int_r^b (1 - F(x))dx / \int_a^r F(x)dx$ , where  $F(x)$  is the cumulative return distribution and  $(a, b)$  is the possible range of returns. From this definition it is clear that Omega is a function  $\Omega(r)$  of the return threshold  $r$ . Generally, a high Omega makes a strategy more appealing.

The problem with all these measures is that they are difficult to associate with expected utility and often contain parameters like the confidence level or the return threshold which are to some extent arbitrary. Comparing different measures across strategies may also lead to contradictory results. SD rules provide a framework that is explicitly based on expected utility. Moreover, like Omega they are based on the entire return distribution. Because of the asymmetry in PI distributions the SD framework seems particularly attractive. Yet, up to now it has not been popular because the theoretical distributions are unknown and all inference is plagued by severe sampling error. Short of a statistical framework to deal with this sampling error, researchers have relied on more informal measures like those introduced above.<sup>3</sup> Fortunately, recent work referred to below, has developed tests that allow testing for SD even when portfolio returns are correlated and exhibit time dependency. In the remainder of this section, we first discuss the intuition behind SD rules and then summarize the testing procedure.

### 3.1. Stochastic dominance rules

In general, selecting an expected utility maximizing portfolio is complex and often intractable. To avoid these complexities, investors often rely on decision rules. The mean–variance criterion provides one of these decision rules, which is consistent with expected utility for investors with quadratic utility functions and arbitrary return distributions. SD rules offer decision criteria that apply to a much broader set of utility functions by taking the whole return distribution into account. They are thus more appropriate in situations where fat-tailed or skewed return distributions are evaluated. SD rules partition the investment opportunity set into an efficient and an inefficient set.<sup>4</sup> The latter contains all investment prospects no investor satisfying certain characteristics would ever hold. The efficient set contains all investments from which investors will select their optimal portfolio. Depending on the restrictions placed on the utility function, different SD orders can be formulated. All versions assume investors have von Neumann–Morgenstern utility functions and that they maximize expected utility. In addition, first-order SD (FSD) requires that investors prefer higher returns to lower returns, which implies a utility function with a non-negative first derivative. Second-order SD (SSD) also presumes risk aversion. It posits diminishing marginal utility, which is sufficient for risk aversion. Whitmore (1970) introduced third-order SD (TSD) by adding the condition that utility functions have a non-negative third derivative. This implies the empirically attractive feature of decreasing absolute risk aversion. It is clear that higher order efficient sets are subsets of the lower order efficient sets.

Testing for SD can be based on comparing (functions of) the cumulative return distributions  $F_k(x)$  of the different investment prospects  $k$ . Prospect  $X_1$  stochastically dominates prospect  $X_2$  at or

der  $s$  if  $D_1^{(s)}(x) \leq D_2^{(s)}(x)$  for all  $x$  and with strict inequality for some  $x$ , where  $D_k^{(s)}(x) = \int_{-\infty}^x D_k^{(s-1)}(t)dt$  for  $k = 1, 2, s > 1$ , and  $D_k^{(1)}(x) = F_k(x)$ .

### 3.2. Testing for stochastic dominance

Of course, the true cumulative distributions are not known in practice and SD tests have to rely on the empirical distribution function (EDF), which is subject to sampling error. Linton et al. (2005) show how consistent critical values for testing SD can be estimated from the data with only a limited number of assumptions. Most importantly, their test allows for dependency in the data. Consider  $K$  prospects  $X_1, \dots, X_K$ . Let  $N$  denote the full sample size of return observations of  $X_k$  for  $k = 1, \dots, K$ , i.e.  $\{X_{ki}; i = 1, \dots, N\}$ . Under the null hypothesis that a particular prospect  $k$  stochastically dominates all other outcomes at order  $s$ , a three-step procedure is suggested. Firstly, the test statistic  $T_N^{(s)}(k)$  for the full sample is computed:

$$T_N^{(s)}(k) = \max_{l: k \neq l} \sup_{x \in \chi} \sqrt{N} [\hat{D}_k^{(s)}(x) - \hat{D}_l^{(s)}(x)] \quad \text{for } s \geq 1, \quad (3)$$

where  $\hat{D}_k^{(1)}(x) = \hat{F}_{kN}(x) = \frac{1}{N} \sum_{i=1}^N 1(X_{ki} \leq x)$ , and  $\hat{D}_k^{(s)}(x) = \int_{-\infty}^x \hat{D}_k^{(s-1)}(t)dt$  for  $s \geq 2$ .

Here,  $\chi$  denotes the union of supports of all distributions  $\hat{F}_{jN}$  for  $j = 1, \dots, K$  and  $1(\cdot)$  denotes the indicator function. To find out whether prospect  $k$   $s$ -th order stochastically dominates all other prospects, Linton et al. (2005) suggest to take the maximum over all  $l$  with  $k \neq l$  in (3), see also Cho et al. (2007).

Secondly, following the circular block method of Kläver (2005), subsamples of size  $b$  are used to recompute test statistic (3) for each of the  $N - b + 1$  different subsamples  $\{W_i, \dots, W_{i+b-1}\}$ , where  $W_i = \{X_{ki}; k = 1, \dots, K\}$  and  $i = 1, \dots, N - b + 1$ , and additionally for the subsamples  $\{W_i, \dots, W_N, W_1, \dots, W_{i+b-N-1}\}$ , where  $i = N - b + 2, \dots, N$ . We denote the recomputed statistic by  $t_{N,b,i}^{(s)}(k)$ . Note that this test statistic is multiplied by the square root of the subsample size  $b$ . The underlying rationale is that since each of these subsamples is also a sample of the true sampling distribution, the distribution of the subsample test statistics yields an approximation of the sampling distribution of the full sample test statistic when  $b/N \rightarrow 0$  and  $b \rightarrow \infty$  as  $N \rightarrow \infty$ . Following Kläver (2005), we consider a subsample size  $b(N) = 10\sqrt{N}$ . Finally, the  $p$ -value of the test,  $\hat{p}_N^{(s)}(k)$ , is computed to decide whether or not to reject the null hypothesis of dominance:  $\hat{p}_N^{(s)}(k) = \frac{1}{N} \sum_{i=1}^N 1(t_{N,b,i}^{(s)}(k) > T_N^{(s)}(k))$  for  $s = 1, 2, 3$ .

In Section 5 we use the SD tests for several sets of strategies. We proceed in the following way. For each strategy  $k$ , we test whether it dominates all other strategies in the set by FSD, SSD, or TSD. This gives us three  $p$ -values per strategy. We next determine for each strategy the highest SD order for which the null hypothesis of SD cannot be rejected at the 5% level. We call a strategy FSD when we cannot reject the null that it first-order dominates all other outcomes. Note that first-order dominance also implies second- and third-order dominance. Likewise, a strategy is called SSD when the null that the strategy dominates all other outcomes by FSD is rejected, but not by SSD. Finally, if a strategy is TSD we can reject that the strategy dominates all other prospects by FSD and SSD, but not by TSD.

## 4. Simulation setup

To examine the performance of the strategies, we conduct a bootstrap simulation on an elaborate data set containing equity return data from the US, the UK, Japan, Australia, and Canada for the past 30 years. Although these markets are not independent, we considerably increase the number of possible return scenarios on which to test the PI strategies. By doing so it is hoped to obtain a better approximation of the population distribution. Daily simple returns in local currency are retrieved from Datastream for the

<sup>2</sup> We would like to thank an anonymous referee for suggesting this performance measure.

<sup>3</sup> In the context of PI, only Trennepohl et al. (1988) use the SD approach to compare their strategies, albeit without accounting for sampling error.

<sup>4</sup> This section is based on Levy (2006).

**Table 1**  
Summary statistics

Country	Average return (%)	Standard deviation ( $\sigma$ ) (%)	Skewness	Kurtosis	p-Value autocorrelation (Ljung–Box test)	p-Value autocorrelation (Ljung–Box test squared returns)	p-Value heteroscedasticity (Engle's ARCH test)
UK	19.69	16.28	−0.01***	11.28***	0.00	0.00	0.00
US	15.03	15.64	−0.94***	24.45***	0.00	0.00	0.00
Japan	7.25	17.49	−0.14***	13.04***	0.00	0.00	0.00
Australia	12.07	12.74	−0.31***	9.74***	0.85	0.00	0.00
Canada	13.31	13.05	−0.64***	16.89***	0.00	0.00	0.00

Summary statistics for the five markets in our data set. The variable UK contains 7714 observations, US contains 7655 observations, Japan contains 6647 observations, Australia contains 2050 observations, and Canada contains 7674 observations. The daily average return is reported on an annual basis, using  $\bar{r}_{\text{annual}} = (1 + \bar{r}_{\text{daily}})^{252} - 1$ . Daily standard deviations are transformed into annual standard deviations, using  $\sigma_{\text{annual}} = \sigma_{\text{daily}} \cdot \sqrt{252}$ . \*, \*\*, and \*\*\* denote significance at the 10%, 5% and 1% level, respectively.

time period 1 January 1973–31 March 2005, thereby deleting all non-trading days. The respective interest rates from the euro-interbank market are also downloaded. We only use stock return data for which we also have risk-free rates (see Table 1). In the bootstrap simulation it is assumed that a year contains 252 days. Summary statistics are presented in Table 1. The results confirm the empirical regularity that financial return series exhibit fat tails and negative skewness. The Ljung–Box test detects significant serial correlation in almost all countries. We also find significant heteroscedasticity in all series.

Bootstrapping offers a way of generating simulated return series without making any assumption regarding the return distribution. It retains both the skewness and the fat tails from the original data sets. Even so, a simple bootstrapping procedure would destroy the dependency effects (i.e. autocorrelation and heteroscedasticity). Block bootstrapping counters this problem (see e.g. Sanfilippo, 2003). First, we randomly draw a market (UK, US, Japan, Australia, or Canada) with replacement. Next, a random date (i.e. a starting date) is drawn with replacement. Starting from this date we analyze the one-year performance of each PI strategy for the drawn market, that is, the 252 days following the starting date are used to evaluate the different PI strategies. This procedure is repeated for 10,000 ‘years’.

To implement the SP strategies the volatility of the underlying asset and a risk-free rate are needed. Since we do not know the actual volatility in advance, we are confronted with a forecasting problem. We use the standard deviation of the 252 continuously compounded daily stock returns prior to the randomly drawn date as an estimate of future volatility. We adjust the estimate using Eq. (2) to take into account the effect of transaction costs. In addition, the continuously compounded one-year risk-free rate on the starting date is applied to compute (1). For the CPPI strategy, the floor grows daily using the one-year risk-free rate on the starting date. Similarly, we use this one-year rate for discounting the SL floor. During the investment horizon, we use the short-term (S/T) (i.e. 2 days notice) risk-free rate to gross up the risk-free part of the portfolios. Since the one-year risk-free rate is generally somewhat higher than the S/T risk-free rate, this procedure implies that sometimes the floor will be missed by the end of the investment horizon.

## 5. Performance measurement results

### 5.1. Portfolio insurance versus buy-and-hold

To answer the question whether PI is a useful alternative to a B&H investment for at least some investors, Table 2 compares the latter to three basic PI strategies. To address the effect of frequent rebalancing, a proportionate transaction cost of 0.1% is taken into account.<sup>5</sup> We study the following strategies: a SL with a 100%

**Table 2**  
Performance results: PI strategies versus buy-and-hold strategy

Portfolio insurance strategy	Stop-loss	Synthetic put	CPPI	Buy-and-hold
Rebalancing discipline	Daily	Daily	Daily	–
Floor value (%)	100	100	95	0
Multiple	–	–	14	–
Initial equity allocation (%)	100	70	70	100
Average excess return	3.52***	4.62***	3.87***	5.62
Standard deviation	15.12***	14.38***	13.14***	17.98
Sharpe	0.23	0.32**	0.29***	0.31
% < 0	57.55***	43.13***	58.69***	35.95
Average negative excess return	−7.14***	−8.18***	−4.81***	−12.86
VaR 5%	−13.16***	−13.04***	−6.33***	−24.44
ES 5%	−17.12***	−16.48***	−7.50***	−31.65
Skewness	1.20***	0.93***	1.73***	0.14
Omega measure	1.82	2.28	2.31	2.23
Stochastic dominance efficiency	No SD***	No SD***	No SD***	No SD***

Performance results of three basic portfolio insurance strategies and a buy-and-hold strategy obtained from a block bootstrap simulation. In the simulation we repeatedly draw a one-year block from the return index series (covering the UK, US, Japan, Australia, and Canada). This procedure is repeated for 10,000 years. Returns are in excess of the one-year rate corresponding to the drawn market and date. Transaction costs of 0.1% are taken into account. The VaR is obtained by first sorting the portfolio returns into ascending order and then looking at the return at the 5% level. ES is the average of VaR exceedences. The Omega measure is defined relative to a zero (excess) return threshold. \*, \*\*, and \*\*\* denote a significant difference between the portfolio insurance and buy-and-hold strategy at the 10, 5, and 1% level, respectively. For the stochastic dominance (SD) tests, we consider a set of strategies containing the present four strategies. Under the null, a particular strategy dominates all other strategies in the set. If the null is rejected, no dominance relation is present. The table displays the highest SD order for which the null hypothesis of dominance cannot be rejected. First-order SD (FSD) implies that we cannot reject the null that the strategy first-order dominates all other strategies. Note that FSD also implies second- and third-order dominance. Similarly, second-order SD (SSD) means that we can reject the null that the strategy dominates all other outcomes by FSD, but not that it dominates the other outcomes by SSD (and thus also TSD). Finally, if a strategy is TSD we can reject that the strategy dominates all other prospects by FSD and SSD, but not that it dominates the other strategies in the set by TSD. No SD means that we reject the null of SD by any order. \*, \*\*, and \*\*\* denote the significance level of the hypothesis test for SD identification, i.e. 10%, 5%, and 1% level.

floor value, a SP with a 100% floor value, and a CPPI strategy with a 95% floor value and multiple 14. For the latter we do not opt for a 100% floor value, since it would then only have a very small equity exposure. As an alternative, we consider a CPPI strategy that has an equal average initial equity allocation as the SP, which was found to be 70%. We consider a CPPI strategy with a 95% floor and multiple 14 as an alternative to the SL and SP strategies with a 100% floor value.<sup>6</sup> Fig. 1 plots the payoff functions of the three PI strategies. The

<sup>5</sup> As such dynamic strategies are generally implemented using futures transactions, a 0.1% transaction cost is reasonable. Note that we also analyzed 0% and 0.3% transaction costs with qualitatively similar SD results.

<sup>6</sup> Note that this initial equity allocation can be obtained by different combinations of CPPI floor values and multiples. We find that for the (95% floor, multiple 14) combination, the null hypothesis of third-order SD relative to the (arbitrary) combinations (80%, 3.5), (85%, 4.67), (90%, 7), and (97.5%, 28) cannot be rejected.



floor values are indicated by dotted horizontal lines and a full 45° line is drawn to facilitate comparisons. The payoff functions confirm that PI strategies provide payoffs similar to a protective put strategy. Note that PI is no free lunch: portfolio insurers pay for the downside protection in terms of a reduction in upside capture. As typical for 'buy high – sell low' strategies, the payoff function of the synthetic put and CPPI strategy (with multiple >1) is convex.

In our performance analysis, we focus on returns in excess of the one-year rate, since we want to assess the excess performance of the strategies relative to a risk-free investment. Tests are performed to check whether there is a significant difference between the performance statistics of the PI strategies and the B&H strategy.<sup>7</sup> A *t*-test reveals that the B&H strategy outperforms all strategies in terms of average excess return. This foregone excess return illustrates the implicit cost inherent in PI investments. On the other hand, a Levene test<sup>8</sup> points out that PI strategies deliver a significantly lower risk than a B&H strategy. We apply the Jobson and Korkie (1981) test to test for differences in Sharpe ratios. We find that the Sharpe ratio of the SL and CPPI strategy is significantly lower than the Sharpe ratio of the B&H strategy. Conversely, the SP reports a (statistically) significantly higher Sharpe ratio than the B&H strategy although the ratios are numerically close.<sup>9</sup> Furthermore, a *t*-test on frequencies points out that all PI strategies have a significantly higher frequency of negative excess returns than the B&H strategy. However, this result should be interpreted with caution, since *t*-test results also indicate significantly better average negative excess returns for PI strategies. The lower risk of the PI strategies is corroborated by the VaR and ES measures. To test for differences in VaR, we conduct an unconditional coverage test (Christoffersen, 2003; Patton, 2005). Under the null, the B&H VaR equals the VaR of a particular PI strategy. A 95% confidence level is used, which implies that under the null, we expect 5% of the observations of a PI strategy to exceed the B&H VaR.<sup>10</sup> This is formally tested by calculating the fraction of B&H VaR violations (i.e. 'the hit ratio') for each PI strategy and performing the following *t*-test:  $(\frac{1}{N} \sum_{n=1}^N \text{Hit}_n - \alpha) / \sqrt{\alpha(1-\alpha)/N} \rightarrow N(0, 1)$  as  $N \rightarrow \infty$ , where  $\text{Hit}_n$  equals one if the PI return is lower than the B&H VaR and zero else,  $N$  is the number of strategy returns and  $\alpha$  equals 5%. For all strategies, the null is rejected at the 1% significance level, implying that the PI strategies have statistically significantly better VaR levels. For the ES, we test whether the PI strategies and the B&H strategy have a significantly different ES by using a bootstrap approach (again we use a 95% confidence level). First, we randomly draw 5000 returns pairwise from the full sample of 10,000 empirical PI returns and 10,000 B&H returns. From these bootstrapped return observations, we calculate the ES for both strategies. This procedure is repeated 10,000 times to obtain two ES distributions. We use a paired-samples *t*-test to check whether the two ES distributions are statistically different. We find that all *t*-statistics are extremely significant. Again, this points to less risky PI strategies. This result is robust to alternative sizes of the sampled return observations. Note that the CPPI strategy shows considerably better VaR and ES levels than any other strategy in the set.

<sup>7</sup> Significance tests are not performed for the Omega measure, since we are not aware of a statistical framework to test for significant differences in Omega measures.

<sup>8</sup> Levene's test (Levene, 1960) is used to test for equality of variances between the PI strategies and the B&H strategy. Levene's test evaluates whether  $k$  samples have equal variances by performing a one-way ANOVA on the absolute value of observations centered by subtracting their respective sample means. It is robust to non-normality.

<sup>9</sup> Note that in a PI context the Sharpe ratio is not necessarily an adequate performance measure, since portfolio insurers do not only care about the mean and variance of returns. As emphasized in Leland (1999), PI strategies will typically be undervalued by CAPM performance measures.

<sup>10</sup> The results for a 90 and 99% confidence level are available upon request.

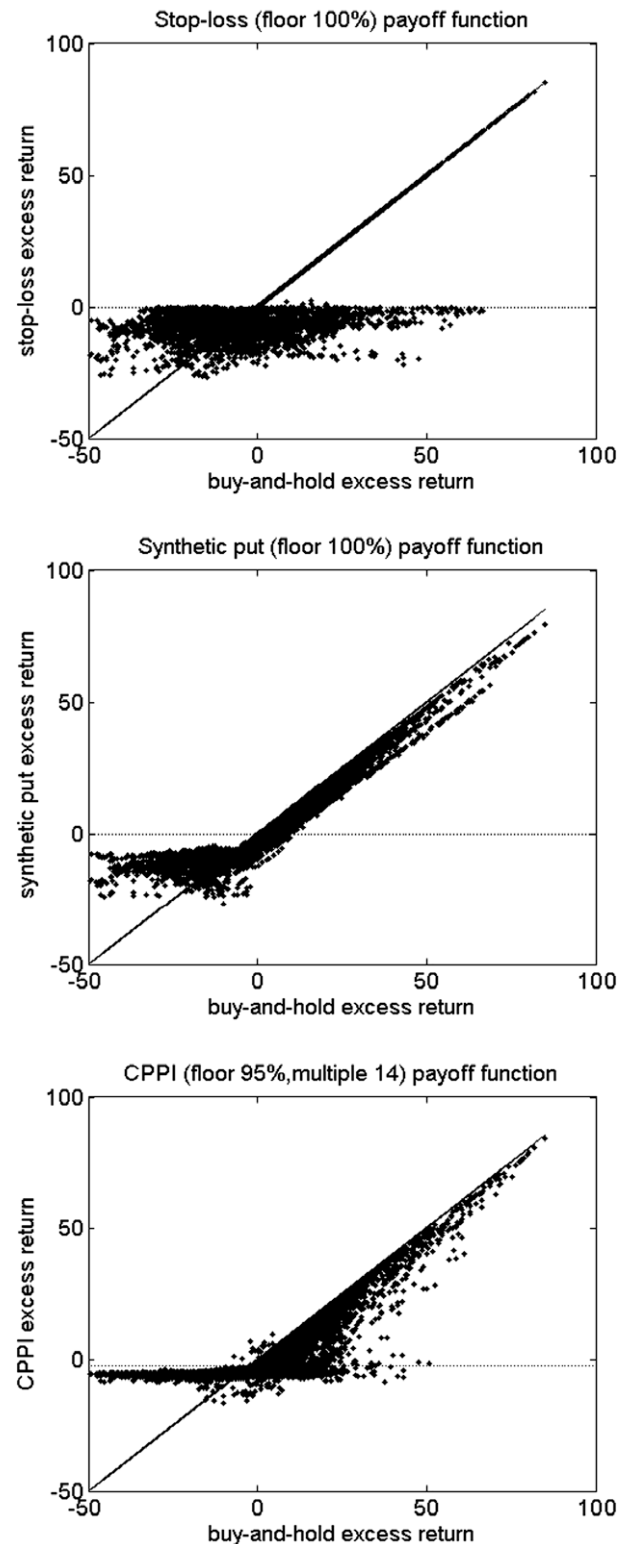


Fig. 1. Payoff functions of the SL, SP, and CPPI portfolio insurance strategies (daily rebalancing and 0.1% transaction costs).

To test for differences in skewness, we also resort to bootstrapping. Given the null that the skewness of the B&H strategy exceeds or equals the skewness of a PI strategy, we first construct symmetric return distributions, by adding for each of the 10,000 original returns an observation that lies at the same distance at the other side of the mean, i.e. using the transformation  $-(r_i - \bar{r}) + \bar{r}$ , where

**Table 3**

Performance under different market conditions

Volatility subgroup Rebalancing discipline	Low volatility – 3333 obs				Medium volatility – 3334 obs				High volatility – 3333 obs			
	Daily				Daily				Daily			
Portfolio insurance strategy	SL	SP	CPPI	B&H	SL	SP	CPPI	B&H	SL	SP	CPPI	B&H
Floor value (%)	100	100	95	0	100	100	95	0	100	100	95	0
Multiple	–	–	14	–	–	–	14	–	–	–	14	–
Initial equity allocation (%)	100	70	70	100	100	70	70	100	100	70	70	100
Average excess return	7.09***	7.66***	6.84***	9.24	3.21***	4.00***	3.25***	5.37	0.27***	2.18	1.51***	2.27
Standard deviation	12.91***	11.13***	10.76***	12.23	14.42	12.93***	11.62***	15.77	16.96***	17.72***	15.89	23.41
Sharpe	0.55***	0.69***	0.64***	0.76	0.22***	0.31***	0.28***	0.34	0.02***	0.12***	0.10	0.10
% < 0	38.49***	24.93***	38.61***	22.23	55.28***	46.34***	60.41***	36.59	78.88***	58.12***	77.05***	49.03
Average negative excess return	–6.65***	–6.67***	–3.56***	–8.14	–7.84***	–7.11***	–4.70***	–10.43	–6.88***	–9.69***	–5.52***	–16.80
VaR 5%	–10.30***	–9.21***	–5.34***	–11.61	–14.51***	–12.68***	–5.86***	–19.60	–15.49***	–15.08***	–6.99***	–30.23
ES 5%	–12.26***	–11.16***	–5.74***	–15.54	–17.66***	–16.36***	–6.31***	–29.34	–19.37***	–18.96***	–9.09***	–36.12
Skewness	0.27***	0.23***	0.68***	–0.16	0.77***	0.66***	1.32***	–0.16	2.20***	1.44***	2.46***	0.59
Omega measure	3.78	5.59	5.93	6.44	1.72	2.20	2.11	2.52	1.02	1.36	1.32	1.26
Stochastic dominance efficiency	No SD***	No SD***	No SD***	SSD	No SD***	No SD***	TSD	TSD	SSD	TSD	SSD	TSD

Performance results of the strategies obtained from a block bootstrap simulation. Results are shown for a daily rebalancing frequency and a fixed floor value. In the simulation we repeatedly draw a one-year block from the return index series (covering the UK, US, Japan, Australia, and Canada). This procedure is repeated for 10,000 years. Returns are in excess of the one-year rate corresponding to the drawn market and date. Based on the ex post volatility of the drawn market, three volatility groups are constructed. The average volatility is 9.19%, 13.11%, and 20.09% for the low, medium, and high volatility scenario, respectively. Transaction costs of 0.1% are assumed. \*, \*\*, and \*\*\* denote a significant difference between the portfolio insurance and buy-and-hold strategy at the 10%, 5%, and 1% level, respectively. For the SD tests, we consider the set of strategies within each volatility scenario. Under the null, a particular strategy dominates all other strategies in the set. If the null is rejected, no dominance relation is present. The table displays the highest SD order for which the null hypothesis of dominance cannot be rejected.

$r_i$  is the  $i$ th original return of a particular strategy for  $i = 1, \dots, 10,000$  and  $\bar{r}$  is the average return. In this way, the mean and all even central sample moments are preserved, whereas all odd central sample moments (except the mean) are set to zero, which is a necessary condition for symmetry. This ensures that both series have identical skewness coefficients, i.e. zero. Next, we simulate the distribution of the difference in skewness by randomly drawing 10,000 returns pairwise from the 20,000 returns of a PI strategy and the B&H portfolio and computing the sample skewness difference. Repeating this 10,000 times, we obtain an approximation of its distribution, which is then used to compute the  $p$ -value of obtaining an even more extreme skewness difference than the empirical skewness difference. We find that all PI strategies have statistically significantly higher skewness coefficients than the B&H strategy.

Summarizing, we find that the three PI strategies entail less risk than the B&H strategy, regardless of how we measure risk – the only exception being the proportion of negative returns. They also have higher skewness. Both attractive features seem to come at the price of lower average returns. Therefore, it is not clear whether risk averse investors would prefer the PI strategies to the B&H strategy. As the Sharpe ratio is not necessarily an adequate statistic to compare asymmetric distributions, we next examine the ranking of the strategies based on the Omega measure (with a zero excess return threshold) and find that both the CPPI and SP strategy outperform the B&H strategy. However, in the absence of a statistical framework to test for differences in Omega measures, these ranking results should be interpreted with caution, certainly given that their numerical values are similar. As an alternative, we investigate whether any SD relations exist among the four strategies considered in Table 2.<sup>11</sup> We find that the null hypothesis of SD is rejected for all strategies and all SD orders. That is, none of the strategies in Table 2 is able to dominate the other strategies in the set.<sup>12</sup> In eco-

nomics terms, this implies that some investors will prefer one of the PI strategies, whereas others will favor the B&H strategy. This finding of a lack of dominance between insured and uninsured portfolios corroborates the theoretical results of Benninga and Blume (1985) and Brooks and Levy (1993). In sum, we find that even though PI strategies yield lower returns than a B&H strategy, the corresponding lower risk (both in terms of symmetric and downside risk) compensates this reduced return, making these strategies potentially valuable alternatives to B&H investments.

The analysis above relied on the assumption that the market phase is unknown at the start of the investment horizon, since it is not ex ante known to the investor whether or not the market phase will be bullish or bearish, turbulent or calm. Nevertheless, it is interesting to backtest the results and examine whether different performance results are obtained under different market conditions.<sup>13</sup> Table 3 reports the results when the sample is divided into three volatility groups: low, medium, and high market volatility, based on the stock market volatility in the simulated year. Obviously, this is an ex-post exercise, since the subgroups are constructed using realized volatility.

Comparing all volatility scenarios, we find that the PI strategies have the best average excess return in the low volatility scenario. Nevertheless, we cannot reject the null that a B&H strategy (which has the best Sharpe ratio and Omega in this scenario) second-order dominates all other PI strategies in this scenario. Indeed, in low volatility markets, PI is less appealing. For the moderate volatility scenario, we cannot reject that the CPPI and B&H strategy third-order dominate the other strategies in the set. This outcome is in line with the observation that the CPPI strategy delivers the best skewness in the moderate volatility scenario. For the highest volatility subgroup, we cannot reject the null that the SL and CPPI strategies second-order dominate all other strategies. The skewness of these strategies is better than the skewness of the SP and B&H strategy. The underperformance of the SP strategy is not surprising, as the Black–Scholes model requires a continuous stock price process without sudden jumps to value the put option. Underestimating the volatility will then result in seeking less protection than required. The CPPI delivers the same Sharpe ratio as the B&H strategy

<sup>11</sup> Here, as well as in the remaining part of this paper we only report the SD test results for a subsample size of 1000 ( $=10N^{0.5}$ ). Other subsample sizes generate similar  $p$ -values.

<sup>12</sup> The same conclusion holds when rebalancing is based on a market move trigger. In this setting, rebalancing occurs following a 1%, 4%, or 10% change in the equity portfolio of the PI strategies (as in Bird et al., 1990). Detailed results are available upon request.

<sup>13</sup> We would like to thank an anonymous referee for this suggestion.

**Table 4**  
Impact of changing the floor value

Portfolio insurance strategy	Stop-loss			Synthetic put			CPPI		
Floor value (%)	95	97.5	100	95	97.5	100	95	95	95
Multiple	–	–	–	–	–	–	16	15	14
Initial equity allocation (%)	100	100	100	80	75	70	80	75	70
Average excess return	4.26***	4.00***	3.52	4.95**	4.81	4.62	3.97	3.92	3.87
Standard deviation	16.98***	16.25***	15.12	15.68***	15.07***	14.38	13.51***	13.35***	13.14
Sharpe ratio	0.25***	0.25***	0.23	0.32***	0.32***	0.32	0.29	0.29*	0.29
% < 0	45.96***	50.49***	57.55	40.50***	41.73***	43.13	58.70	58.87	58.69
Average negative excess return	–10.79***	–9.18***	–7.14	–9.91***	–9.07***	–8.18	–5.09***	–4.95***	–4.81
VaR 5%	–16.67***	–15.04***	–13.16	–16.33***	–14.60***	–13.04	–6.49***	–6.43***	–6.33
ES 5%	–21.13***	–19.35***	–17.12	–19.94***	–18.17***	–16.48	–7.68***	–7.59***	–7.50
Skewness	0.75	0.94	1.20	0.68	0.80	0.93	1.67	1.70	1.73
Omega measure	1.83	1.83	1.82	2.20	2.24	2.28	2.27	2.29	2.31
SD efficiency	No SD <sup>ooo</sup>	No SD <sup>ooo</sup>	TSD	No SD <sup>ooo</sup>	No SD <sup>ooo</sup>	TSD	No SD <sup>ooo</sup>	No SD <sup>ooo</sup>	TSD

Performance results of the strategies obtained from a block bootstrap simulation. Results are shown for a daily rebalancing frequency and three different floor values. \*, \*\*, and \*\*\* denote a significant difference between the portfolio insurance strategy with a lower floor value and the benchmark portfolio insurance strategy (in the third column of each panel) at the 10%, 5%, and 1% level, respectively. For the SD tests, we consider the set of strategies within each panel. Under the null, a particular strategy dominates all other strategies in the panel. If the null is rejected, no dominance relation is present. The table displays the highest SD order for which the null hypothesis of dominance cannot be rejected. FSD implies that we cannot reject the null that the strategy first-order dominates all other strategies. Similarly, SSD means that we can reject the null that the strategy dominates all other outcomes by FSD, but not that it dominates the other outcomes by SSD. Finally, if a strategy is TSD we can reject that the strategy dominates all other prospects by FSD and SSD, but not that it dominates the other strategies in the set by TSD. No SD means that we reject the null of SD by any order. \*, \*\*, and \*\*\* denote the significance level of the hypothesis test for SD identification, i.e. 10%, 5%, and 1% level.

in this scenario, but with a lower downside risk and substantially better skewness. The SL strategy will often resemble a risk-free investment in this volatility scenario, as wealth will be transferred into a risk-free investment once the downward limit has been reached. This switch implies that investors cannot benefit from any subsequent upward market movements. Note that with high market volatility, the dynamic character of the PI strategies requires that financial markets are sufficiently liquid so that portfolios can be correctly rebalanced (cfr. the October 1987 crash where the financial markets were not sufficiently liquid to absorb the selling pressure of portfolio insurers). As expected, we find that, as market conditions worsen, the attractiveness of PI strategies increases.

### 5.2. Changing the floor value

In this section, we examine how sensitive performance is to lower target levels. A lower floor value implies that the SL strategy will be relatively longer invested in stocks before switching to a risk-free investment. The SP strategy on the other hand, will initially allocate more funds to the risky asset. Furthermore, the CPPI strategy will behave riskier because of the relatively larger equity exposure in case of lower floor values. We do not attempt to find an optimal floor value, which would be impossible without assuming a particular distribution for the risky asset returns (e.g. Brooks and Levy, 1993). Rather we wish to indicate the impact this particular choice may have on PI performance. The results for the different strategies are presented in Table 4, where a daily rebalancing frequency is maintained; results for weekly and monthly frequencies are qualitatively similar. The results in each panel refer to the same PI strategy, where only the floor value is different. The strategies with the lower floors are compared to the benchmark strategies from Table 2, whose results are duplicated to facilitate comparison.

In Table 4 floor values of 95% and 97.5% are considered for the SL strategy. The lower floor values yield better average excess returns and Sharpe ratios but with higher risk, both in terms of volatility and downside risk. Moreover, the return distributions are characterized by lower skewness. Based on SD, we reject the null hypothesis that these lower floor strategies stochastically dominate both other SL strategies at all orders. We cannot reject the

hypothesis that the 100% floor strategy third-order dominates the lower floor strategies.<sup>14</sup> In economic terms, this implies that investors who have a preference for positive skewness would prefer a SL strategy with a 100% floor value to a strategy with a lower floor value. The results for the SP strategy are similar to those for the SL strategy. The lower floor strategies lead to higher excess returns, but we cannot reject that the highest floor stochastically dominates the lower floors at the third order. The lower floor strategies on the other hand do not dominate. Again, the effect of the higher positive skewness seems to be an important characteristic. Note that the Omega measure corroborates the SD results.

For the CPPI strategy we change the floor-multiple combinations. As in the previous section, we choose combinations that result in the same initial equity allocation as the SP strategy. For each initial equity allocation we first compare several floor-multiple combinations and retain the combination for which we cannot reject the hypothesis that it stochastically dominates the other combinations. The CPPI combinations (95%, 15) and (95%, 16) are retained for the initial equity allocations of 75% and 80%, respectively.<sup>15</sup> The three CPPI strategies that are comparable to the SL and SP strategies with floor values 95%, 97.5% and 100%, are subsequently compared in last panel of Table 4. Again the 70% initial equity allocation strategy, i.e. combination (95%, 14), is preferred. Both other combinations do not stochastically dominate, whereas third-order dominance cannot be rejected for the (95%, 14) strategy. Again, the ranking based on the Omega measure confirms the SD results.

### 5.3. Changing the rebalancing frequency

The choice of the rebalancing frequency is a trade-off between seeking high protection (e.g. daily rebalancing) and reducing transaction costs (e.g. weekly or monthly rebalancing). In order to study this trade-off, the preferred strategies in Table 4 are replicated at both the weekly and monthly rebalancing frequency. In Table 5 their results are compared to those of the daily rebalancing variant. We find that both the SP and the CPPI strategy suffer from a

<sup>14</sup> It should be noted that the TSD *p*-value for the SL strategy with a 100% floor value is only 6.28%, while 5% would imply TSD rejection. Moreover, the Omega values are practically similar for all SL floor values, corroborating a no dominance conclusion.

<sup>15</sup> Detailed results are available upon request.

**Table 5**

Impact of changing the rebalancing frequency

Portfolio insurance strategy	Stop-loss			Synthetic put			CPPI		
Rebalancing discipline	Daily	Weekly	Monthly	Daily	Weekly	Monthly	Daily	Weekly	Monthly
Floor value (%)	100	100	100	100	100	100	95	95	95
Multiple	–	–	–	–	–	–	14	14	14
Initial equity allocation (%)	100	100	100	70	70	70	70	70	70
Average excess return	3.52	3.76	4.25***	4.62	4.55	4.49	3.87	3.42***	3.33***
Standard deviation	15.12	15.68***	16.43***	14.38	14.32***	14.29***	13.14	12.91***	13.47***
Sharpe	0.23	0.24***	0.26***	0.32	0.32***	0.31***	0.29	0.26***	0.25***
% < 0	57.55	54.58***	49.75***	43.13	42.87	42.51	58.69	60.03***	58.24
Average negative excess return	–7.14	–7.92***	–9.11***	–8.18	–8.25	–8.33*	–4.81	–4.85*	–5.67***
VaR 5%	–13.16	–14.40***	–16.22***	–13.04	–13.28**	–13.86***	–6.33	–6.66***	–8.77***
ES 5%	–17.12	–18.93***	–21.07***	–16.48	–16.68***	–17.81***	–7.50	–8.51***	–12.91***
Skewness	1.20	1.06	0.87	0.93	0.91	0.85	1.73	1.78***	1.54
Omega measure	1.82	1.84	1.92	2.28	2.26	2.25	2.31	2.13	1.99
Stochastic dominance efficiency	No SD <sup>ooo</sup>	No SD <sup>ooo</sup>	No SD <sup>ooo</sup>	TSD	No SD <sup>ooo</sup>	No SD <sup>ooo</sup>	SSD	No SD <sup>ooo</sup>	No SD <sup>ooo</sup>

Performance results of the strategies obtained from a block bootstrap simulation. Results are shown for a fixed floor value and a daily, weekly, or monthly rebalancing frequency. \*, \*\*, and \*\*\* denote a significant difference between the portfolio insurance strategy with a lower rebalancing frequency and the daily rebalancing variant at the 10%, 5%, and 1% level, respectively. For the SD tests, we consider the set of strategies within each panel. Under the null, a particular strategy dominates all other strategies in the panel. If the null is rejected, no dominance relation is present. The table displays the highest SD order for which the null hypothesis of dominance cannot be rejected. FSD implies that we cannot reject the null that the strategy first-order dominates all other strategies. Similarly, SSD means that we can reject the null that the strategy dominates all other outcomes by FSD, but not that it dominates the other outcomes by SSD. Finally, if a strategy is TSD we can reject that the strategy dominates all other prospects by FSD and SSD, but not that it dominates the other strategies in the set by TSD. No SD means that we reject the null of SD by any order. \*, \*\*, and \*\*\* denote the significance level of the hypothesis test for SD identification, i.e. 10%, 5%, and 1% level.

reduction in average excess return in case of weekly (five days) or monthly (21 days) rebalancing. In contrast, for the SL strategy we find that a lower rebalancing frequency yields a higher average excess return, albeit with a higher risk. The longer rebalancing interval may prevent too early a switch to a risk-free investment following a temporary stock price fall below the discounted floor, thereby explaining the higher average returns. Since the portfolios are managed less strictly, the average negative excess return as well as the VaR and ES becomes significantly worse for all strategies. Furthermore, the skewness decreases, indicating a less right-skewed return distribution.

The SD tests in Table 5 point out that for the SL strategy the null of dominance is rejected for each rebalancing frequency.<sup>16</sup> This absence of any dominance relation implies that the optimal rebalancing frequency will depend on the type of investor. Even though daily rebalancing delivers the best protection, a monthly rebalancing frequency seems to compensate a lower protection with a higher average excess return for the SL strategy. This is confirmed by the Omega measure, which increases for lower rebalancing frequencies. Even though the SP strategy suffers from frequent transaction costs with daily rebalancing, the SD results in Table 5 show that we cannot reject the null that the SP strategy with a daily rebalancing frequency third-order dominates the weekly and monthly rebalancing frequency variants. In other words, the benefits from daily rebalancing (i.e. better protection and average excess return) outweigh the advantage of reduced transaction costs provided by lower rebalancing frequencies. This implies that investors who have a preference for positive skewness would implement a daily rebalancing frequency. Similarly, we find that daily rebalancing is the preferred rebalancing frequency for the CPPI strategy. Based on SD tests we cannot reject the null that a daily rebalancing frequency dominates lower frequencies by second-order (and thus also third-order) degree. Again, the Omega measure corroborates these results, as it increases markedly when rebalancing is more frequent.

## 6. Concluding remarks

The present study provides additional insight into the continuing controversy over PI strategies. Our results are threefold. Firstly, we find that PI techniques outperform a B&H strategy in terms of downside protection, but provide lower excess returns in return. Based on SD, no dominance relations are identified between the PI and B&H strategies. This implies that even though PI strategies yield lower returns, the corresponding lower risk compensates sufficiently to make them valuable alternatives for at least some investors. Secondly, we show that the highest floor values deliver the best downside protection, i.e. a superior average negative excess return, VaR, ES, and skewness, albeit at the cost of a lower excess return. Based on SD tests, we cannot reject that a floor value of 100% dominates lower floor values by third-order SD. Thirdly, this paper reveals how sensitive performance is to the rebalancing frequency. We find that even though daily rebalancing will provide the best protection for the SL strategy, a monthly frequency seems to compensate a lower protection with a higher average excess return. However, for the SP and CPPI strategy, the benefits from daily rebalancing (i.e. better protection and average excess return) outweigh the advantage of lower transaction costs implied by lower rebalancing frequencies. Furthermore, our results illustrate that based on the more general SD analysis, a simple analysis of Sharpe ratios appears to be too short-sighted an approach for screening PI strategies. It should be noted that the performance of the SP strategy strongly depends on the accuracy of the volatility estimate in the Black and Scholes option pricing formula. We considered the volatility of the previous year to be an appropriate proxy for the ex post volatility. The performance of the SP strategy may be enhanced by using better volatility predictors. Moreover, lock-in thresholds can be incorporated in the strategies, so that intermediate gains are locked-in by dynamically adjusting the floor. We leave this for further research.

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<sup>16</sup> When using market move rebalancing triggers, the null of dominance is rejected for each rebalancing trigger and for each PI strategy. Similar to rebalancing based on a time interval, we find that the PI strategies offer less protection with higher market move triggers.



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