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Investment principles for individual retirement accounts

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Abstract

The phenomenal growth of individual retirement accounts in the US, and globally, challenges both individuals and their advisors to rationally manage these investments. The two essential differences between an individual retirement account and an institutional portfolio are the length of the investment horizon and the regularity of monthly contributions. The purpose of this paper is to contrast principles of institutional investing with the management of individual retirement accounts. Using monthly historical data from 1926 to 2005 we evaluate the suitability for managing individual retirement portfolios of seven principles employed in institutional investing. We discover that some of these guidelines can be beneficially applied to the investment management of individual retirement accounts while others need to be reconsidered.

JEL classification: G10; G11

Keywords: Individual retirement accounts; Risk and return; Investment management; Risk management of individual retirement accounts

1. Introduction

Extensive academic research on risks and returns of various investment classes has been incorporated in investment primers, technical and statistical books and popular monographs. For example, the investment textbooks of Bodie et al. (2007), Reilly and Brown (2006) and Viceira and Campbell (2002), the classic financial econometrics book of Campbell et al. (1997) and the highly successful investment book of Siegel (2002), all collect, document and elucidate numerous stylized facts about asset returns, risks and long-term performance of stocks, bonds, cash and other classes of investments. From these findings certain investment guidelines have been proposed, statistically documented, and debated using very long time series of, mostly annual, returns of various aggregate indexes for capital markets. In this paper we select a few such standard stylized principles or guidelines for long-run institutional investing and explore their relevance in managing individual retirement accounts.

2. Individual retirement accounts

The 2005 Publication 590 of the US Internal Revenue Service describes in detail the numerous individual retirement arrangements that are legally recognized. Such arrangements include the traditional Individual Retirement Account (IRA), the Roth IRA, the Savings Incentive Match Plan for Employees (SIMPLE) IRA, the Self-Employed IRA, the Self-Directed IRA, 401(K) Account, the Roth 401(K) and others. For our analysis we do not distinguish between these accounts because we are not addressing tax and withdrawal issues. Rather, we use the concept of an individual retirement account to refer to a plan that provides some tax advantages to an individual who saves regularly for retirement. The two main characteristics of an individual retirement account we wish to focus on are the relatively short investment horizon of such investment vehicles and the regularity of contributions. For example, a typical individual retirement account has an

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investment horizon of 30–40 years while the investment horizon of an institutional endowment fund could be very long or hypothetically infinite.

3. Importance of individual retirement accounts

Individual retirement accounts have recently experienced a phenomenal growth as corporations have moved away from defined benefit retirement plans to defined contribution retirement plans. Furthermore, the present concerns of population aging and the financing of its retirement needs have resulted in discussions about the resources needed to be put aside over a lifetime of work to finance the consumption of the retirees. Greenspan (2005) has emphasized that retirement is a relatively new phenomenon in human history. Hence, the financial issues associated with saving and managing retirement investments are essentially new.

About a century ago, the average American life expectancy was only 47 years and very few individuals had the opportunity to live long past their retirement. In contrast, today the average life expectancy for both sexes and for all races, according to the US Department of Health and Human Services (2006), is about 77.8 years and a significant percent of workers will live in retirement for about 20 years. Moreover, the average life expectancy for all races and both sexes for those having reached the age of 65 is another 18.7 years, or a total of 83.7 years.

Social Security in the US will face challenging financial issues because by design it is not a fully funded program but rather a pay-as-you go system. Such a system depends on an appropriate ratio of workers to retirees to keep the system financially sound. Garrett and Rhine (2005) report that while in 1950 there were about 16 workers for each retiree, by 2004 there were only 3.1 workers for each retiree and the projection for 2030 is for 2.17 workers for each retiree. Current and projected ratios are substantially low and imply future financial shortfalls. Although Kotlikoff (2006) appears to be pessimistic about long-term US public finances, it is reasonable to argue that the financing of future retirement expenditures is rapidly becoming a major concern for the labor force. The three developments of (a) the future uncertainty of Social Security benefits, (b) the dramatic decline in defined benefit retirement programs and (c) the demographic reality of large numbers of retiring baby boomers, are all contributing to the recent concerns about retirement financing as reported in Dwyer (2005).

Efforts by the Bush Administration to partially privatize Social Security by introducing personal retirement accounts have been met with enough opposition to temporarily abandon their implementation. Accordingly, today, individuals realize they need to both save more and invest wisely on their own while policy measures to address this national problem are debated. Individuals also recognize that risks associated with retirement portfolios once assumed by firms or the government will be, in the future, substantially borne by them. This phenomenon, often described as the democratization of risk, induces individuals to manage these risks by seeking portfolio management advice from professionals.

4. Investment principles

We consider seven guidelines or principles for the financial management of a representative individual retirement account. Some of these principles are the same as the ones used in lump-sum investing, some are modified and some are new. We obtain these principles by asking certain questions or by proposing certain hypotheses. Then we do the analysis to confirm or reject these hypotheses. The principles proposed are not mathematical theorems because we do not prove them analytically. We simply give empirical evidence using a specific data set from Ibbotson Associates (2006) to support or reject the hypotheses.

Suppose that an individual saves \$1 per period, say every month, for a period of 20–40 years. This monthly contribution could be invested in an equity index such as the S&P 500 Index or in bonds or in cash or in various proportions among them. Both the individual and his/her advisor face several decisions that we formulate as hypotheses or questions.

First, using average returns (monthly or annually) from long series of data it is simple to calculate the growth of \$1 invested in a certain asset class. The first hypothesis translates this result to individual retirement accounts by stating that it is equally straightforward to calculate the accumulations of individual retirement accounts.

When \$1 is invested over a long horizon, its growth becomes phenomenal because of the power of compounding. The second hypothesis claims that individual retirement plans achieve accumulations that are similarly substantial, also because of the power of compounding.

The third hypothesis considers the investment risks associated with the length of the investment horizon. Siegel (2002, p. 11)) calculates the standard deviation of annual stock market returns and finds a risk of 17.5% over the very long period of 1802–2001, that is for almost 200 years. This risk is very similar to 17.2% which is the risk during the much shorter period of 1966–1981. We likewise hypothesize that an individual investing in equities for a 40 year period faces similar risks to individuals with a shorter horizon of 30 years.

Fourth, recall that, for \$1 invested over 40 years, the final accumulation is the same when using the actual monthly returns or the sample average over 40 years. Do we get the same answer for an individual retirement account over 40 years by computing accumulations either by averages or by the actual term structure of returns? The fourth hypothesis says that the results are the same.

Fifth, we hypothesize that as in institutional investing with very long horizons, returns are stable over shorter periods that are typical for individual retirement accounts.

Sixth, for individual investment horizons, of 20–40 years that are much shorter than investment horizons of institutions we hypothesize that equity still outperforms bonds and bonds outperform cash.

Seventh, we perform numerous calculations for accumulations of individual retirement accounts with 20–40 year horizons investing in equities, bonds or cash and hypothesize that these accumulations are normally distributed. Confirming normality of accumulations simplifies the calculation of probabilities associated with extreme accumulations of plus or minus two standard deviations from a given mean. If normality is rejected the calculation of such extreme accumulations becomes more complicated.

Finally, institutional investing is not concerned with the size of the periodic contributions or with targeting certain accumulations. In contrast, managing individual retirement accounts is uniquely concerned with how much must be saved consistently over specified periods, to provide for sufficient funds to finance consumption expenditures during retirement.

Many more stylized facts have been discovered concerning large, medium and small company stocks as well as value versus growth stocks. Furthermore, the importance of global investing has been emphasized as a tool for enhancing diversification. These important issues are not discussed in this paper.

5. Data and methodology

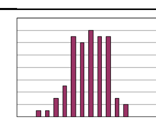
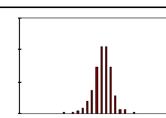
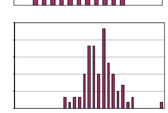
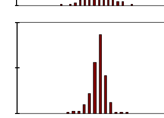
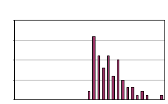
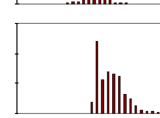
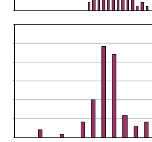
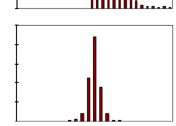
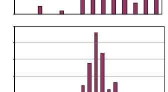
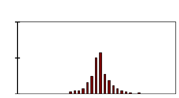
We consider a representative individual who invests $1 per period over a number of periods in stocks or bonds. The period of contributions we consider is one month and the investment horizon is 20, 30 or 40 years.

The data in this paper are those described and reported in Ibbotson Associates (2006). In particular we use data reporting the growth of $1 invested in large company stocks as measured by the S&P 500 Index, small company stocks, long-term government bonds and Treasury Bills from December 31, 1925 to December 31, 2005. Ibbotson Associates (2006) offer a detailed description of these investment classes as well as data about inflation measured by the Consumer Price Index for All Urban Consumers, not seasonally adjusted, and anchored on December 31, 1925. So $1 on December 31, 1925 had the same purchasing power as $10.64 on December 31, 2005.

Ibbotson Associates explain that returns of large company stocks are calculated by the total return that include both capital appreciation and reinvestment of dividends of the S&P 500 index with several modifications that they describe in detail. With such modifications explained, Ibbotson Associates provide monthly (total) returns for the various wealth indices. In all our calculations, Tables and Figures, a monthly return is the monthly return between the value of the index at the last trading day of a given month and its value the last trading day of the previous month. For example, the January 2005 return is that between December 31, 2004 and January 31, 2005. If there was no trading on either date, it is understood that the return is calculated from the last day of the prior month to the last trading day of the current month.

In our sample we have a total of 960 monthly returns (80 years times 12 monthly returns from 1926 to 2005 inclusive). The monthly and annual statistics for these returns and their corresponding monthly and annual distributions are presented in Table 1.

Table 1
Summary statistics for monthly and annual nominal total returns 1926–2005

Series	Annual			Monthly			Annual distribution (%)	Monthly distribution (%)
	Geometric mean (%)	Arithmetic mean (%)	Standard deviation (%)	Geometric mean (%)	Arithmetic mean (%)	Standard deviation (%)		
Large company stocks: S&P 500 Index	10.40	12.30	20.20	0.83	1.03%	5.83%		
Small company stocks	12.60	17.40	32.90	0.99	1.45	9.50		
US treasury bills	3.70	3.80	3.10	0.30	0.32	0.89		
Inflation	3.00	3.1	4.3	0.25	0.26	1.24		
Long-term government bonds	5.5	5.8	9.2	0.45	0.48	2.66		

6. Investment horizons

Most individual investors begin around the age of 25 investing a small percentage of their monthly income for retirement purposes, often matched by their employers. They continue to do so monthly over their working lifetime of about 40 years. The initial contributions have a very long period to compound but subsequent contributions have shorter periods of compounding. To complicate matters, the initial monthly investments may be small but as wages and salaries increase because of productivity gains and adjustments for inflation, contributions towards the last third or fourth of the investment horizon may be larger than earlier ones but are not invested for a sufficiently long period to benefit from the long-term acceleration of compounding.

To enrich our findings we also calculate accumulations for 20 and 30 year horizons. Anecdotal evidence suggests that individuals may devote the period they are between 25 and 35 years old to form a family and save for a house down payment. They actually may dissave during the time they are between 35 and 45 years old as they accumulate mortgage and other debts and may only have a 20 year investment horizon between 45 and 65 or 50 and 70. This simple story does not account for major college expenses, leaving even a shorter period for retirement investing. Whatever scenario one wishes to develop to explain the length of investment horizons for individual retirement accounts, our sample calculations for 20–40 years can accommodate it.

7. Calculation of accumulations

We begin our analysis with the first two hypotheses. Recall that the first hypothesis claims that for an individual retirement account with specific monthly contributions and random monthly returns there is a closed form formula to compute final accumulations. The second hypothesis claims that such final accumulations are equivalent in size to accumulations of \$1 invested over long periods of time.

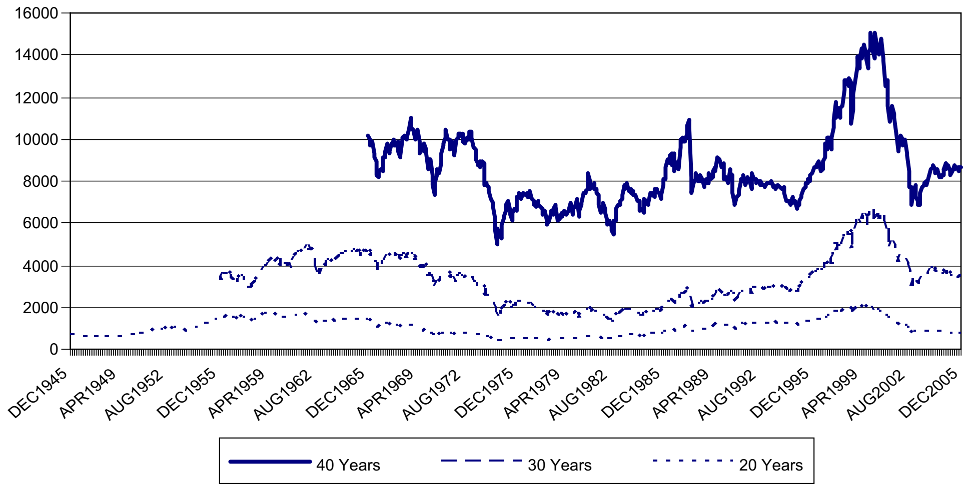
Consider a long lived institution that invested \$1 on December 31, 1925 in the S&P 500 Index. This investment would have grown to $1^{*}(1.104\hat{)}80 = \$\ 2{,}738.58$ by December 31, 2005. All investment books perform similar calculations to illustrate the simplicity of calculating the final accumulation of an initial investment and the power of compounding. Some textbooks that use significantly longer periods, such as Siegel (2002, p.22)) who considers the period 1802–2001, report even more spectacular accumulations. This immediately raises the question: how relevant are such impressive accumulations for individual retirement accounts? Put differently, how applicable is the principle of straightforward calculation and how relevant is the significance of compounding to individual retirement account management?

Two modifications need to be considered. First, individuals have limited investment horizons ranging from 20 to 40 years. What is the accumulation of \$1 invested over 40 years in large stocks? The answer is $1^{*}(1.104\hat{)}40 = \$52.33$. Although considerably lower than \$2,738.58, this is still an impressive accumulation because an initial \$1 grows to about \$52, that is, the ratio of investment to accumulation is 1:52. The second modification is more drastic because contributions to a retirement account occur monthly over the investment horizon of 40 years rather than as a lump sum at the beginning. So compounding is effective for early investments while contributions made later do not have sufficient time to compound substantially.

How can we calculate the accumulations of individual retirement accounts? In contrast to the simple compounding formula of institutional investing, there is no explicit formula for calculating accumulations for individual retirement accounts, except for very simple cases of constant contributions growing at a constant rate. The intuitive reason a simple formula does not exist is that for every month a random contribution is added to the past accumulation and this sum grows at a random monthly return during one period. Thus the final accumulation is the sum and product of two long random sequences. One sequence is the random monthly returns and the other is the random monthly contributions added to partial accumulations. One may argue that monthly contributions need not be random. However, realistically, over a horizon of 40 years one would expect both productivity and inflation adjustments to monthly contributions. Also, as individuals age they may accelerate their monthly contributions.

Malkiel (2003) has argued that periodic investments of equal dollar amounts to an investment account can reduce the risks associated with equity investment by insuring that the entire portfolio will not be purchased at possibly inflated prices. Monthly contributions over many years translate to buying at a whole range of prices, both high and low. Constantinides (1979) investigates this notion of optimality of dollar-cost averaging and highlights the difficulties associated with this complex problem.

Since we cannot use a formula to compute the accumulation of monthly investments, we perform the actual calculations using a monthly contribution of \$1 over 20–40 year horizons. The monthly total returns are the actual data from (Ibbotson Associates, 2006). Note we do not use average monthly returns but actual returns to capture the impact of the specific term structure of returns. Since the entire sample of monthly returns consists of 960 observations, we get accumulations for large numbers of generations. We define a generation as follows: Suppose that the investment horizon is 20 years. The first generation of investors will contribute \$240 in current value invested monthly during the period December 31, 1925 to December 31, 1945. The second generation will begin a month later and also invest a total of \$240 at monthly return rates starting January 31, 1926 and ending on January 31, 1946. When the investment horizon is 20 years we compute accumulations for a total of 721 generations. In a similar fashion, if the investment horizon is 30 years we get



Graph 1. 20–40 Year accumulations per generation with $1 contributed monthly to the S&P 500.

accumulations for 601 generations and finally, if the investment horizon is 40 years we get accumulations for 481 generations. Graph 1 describes the time series of total accumulations at the end of the appropriate period when contributions are invested in the S&P500 Index for the three investment horizons of 20–40 years.

Graph 1 illustrates that total accumulations are dependent on both the term structure of returns and also the length of the investment horizon. For individuals who invested longer their accumulations are, of course, larger. In our samples of 721, 601 and 481 generations corresponding to investment horizons of 20–40 year we get accumulations averaging $1,027.48, $3,302.69 and $8,604.32 with standard deviations of $416, $1,199 and $1,912. The means indicate that on average a $240 contribution over 20 years grows to $1,027.48, which is 1: 4.3. For the 30 year investment horizon the ratio of amount contributed to total accumulation is 1:13.8 and for the 40 year horizon it is 1:35.8. All ratios are lower compared to 1:52 computed previously. This illustrates that on average accumulations of individual retirement accounts grow to smaller multiples of the amount invested compared to investments with only an initial contribution. The explanation, already suggested earlier, is simple. Only the early contributions in a retirement account enjoy the full benefits of compounding while later contributions are invested for much shorter periods.

Graph 1 also illustrates the fluctuations and corresponding risks in the accumulations. For example, the 721 generations that invested for 20 years experienced accumulations that were as low as $342 (for the generation retiring on September 1974) and as high as $2,054 (for the generation retiring on June 1999). Similarly, for the 601 generations that invested for 30 years one experienced an accumulation as low as $1,318 (July 1982) and one as high as $6,734 (March 2000). Finally, the 481 generations that invested for 40 years had the lowest accumulation of $4,954 (September 1974) and the highest of $15,046 (March 2000).

Booms and busts in the stock market impact individual retirement final sums quite dramatically because a few months of above average returns, or of below average returns, towards the end of the retirement horizon are applied to substantial sums of already accumulated wealth over many years. Shiller (2005) discusses this in great detail. The risks associated with the 20–40 year accumulations are described by the standard deviations of $416, $1,199 and $1,912 reported earlier.

These standard deviations as measures of risk support the notion that longer investment horizons are riskier. Graph 1 also suggests a similar inference. However, one cannot conclude from the absolute magnitude of these standard deviations the relative riskiness of investment horizons of individual retirement accumulations because a 40 year accumulation is on average much larger than one of 20 years.

When the data are standardized, initialized or the volatility of sample accumulations is computed the hypothesis that longer investment horizons are always riskier cannot at all times be confirmed. Individual retirement horizons with random periodic contributions and random periodic returns do not allow for conclusive results concerning the associated risks. For our data, computing the volatility of accumulations we get 14.3%, 14.4% and 15% for horizons of 20–40 years respectively. So, for our particular calculations, the risk of a 40 year investment horizon is a little higher than the risk of a shorter horizon but this just holds in this particular case.

To sum up our results, the first hypothesis that claims that accumulations of individual retirement accounts can be computed simply is not true because the randomness of both the periodic contributions and returns does not yield a closed form solution. The second hypothesis that proposes that final accumulations are large multiples of the amount contributed needs to be modified to say that final accumulations are often a few times more than the amount contributed.

Translating these first two hypotheses into investment principles for individual retirement accounts we offer two guidelines. First, calculating individual retirement accumulations is complex and needs to be done with mathematical care. Second, the power of compounding is not in full force for individual retirement accounts because the early contributions that will experience the impact of full compounding are often only a fraction of later contributions. Although later contributions are much larger they will only be invested for a fraction of the investment horizon and will not benefit from the power of compounding.

Third, fluctuations in individual retirement accounts based upon our calculations of 721 generations investing for 20 years, 601 generations investing for 30 years and 481 generations investing for 40 years are quite substantial with several booms and busts. As already stated, the March 2000 generation had a total accumulation of $15,046 compared to the September 1974 generation that had an accumulation of only $4, 954. The third hypothesis indicates that risks associated with individual retirement accounts are substantial when contributions are fully invested in equities for long horizons of 40 years, but it is not conclusive that risks increase monotonically with the length of investment horizons.

Unlike institutional investing where very long investment horizons allow for a recovery of lost wealth during stock market busts, certain generations of individuals reaching retirement immediately after a stock market crash may not have sufficient time to recover lost wealth. The presence of few stock market crashes over a period of 80 years may not be adequately reflected in the standard deviations of accumulated wealth for a large number of generations but the few generations that experienced significant wealth declines due to these crashes faced substantial risks.

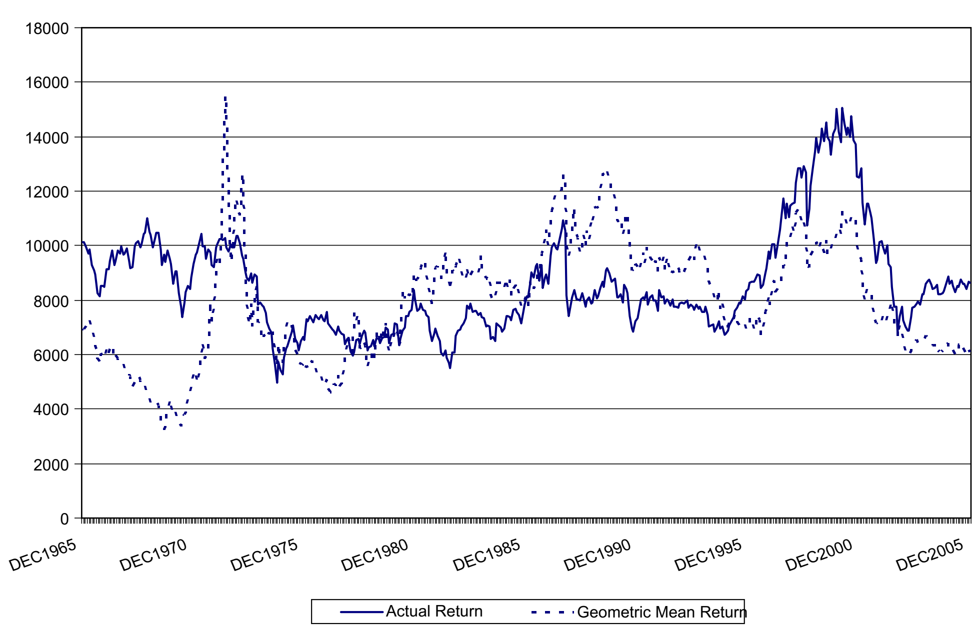
8. Term structure of returns vs. averages

When investing $1 for a number of years, using actual monthly or annual returns or their corresponding (geometric) averages yields the same result. Does the same result hold in individual retirement accounts with monthly contributions? In other words, suppose we calculate a $1 monthly contribution over a 40 year horizon first, by using the actual 40 year monthly return sequence and second by using its mean. Are both accumulations equal? The answer is no.

The fourth hypothesis claims that for calculating final accumulations for individual retirement accounts the term structure of returns cannot be replaced by its (geometric) mean, a practice regularly followed in institutional investing. Graph 2 gives one illustration of the different amounts of accumulations using the two methods. The fourth principle emphasizes that hypothetical illustrations of potential accumulations computed on calculated or assumed means may differ considerably from true accumulations.

When we calculate the accumulation of $1 invested over a 20, 30 or 40 year investment horizon growing at a given term structure of monthly returns or growing by the average of the returns over the same horizon, we obtain the same final amount. For example, the growth of $1 to $2,738.58 is the result of letting the initial investment grow or decline at the actual annual rate of growth of the S&P 500 Index over a period of 80 years. The average rate of growth over the same period is calculated by computing the average annual rate x that satisfies the equation $\$1(1+x)^{80} = \$2{,}738.58$. The solution is $x = 10.4\%$. Thus, by construction the initial investment of $1 grows to the same accumulation either growing at the actual term structure of returns or by its calculated average mean over the same period.

The situation is different when contributions are made on a monthly basis for two reasons. First, for any given generation with an investment horizon of 40 years, there is a very small probability that the actual term structure



Graph 2. 40 Year accumulations per generation using actual returns versus generation mean returns.

of returns will have the same mean as the population mean of 80 years. By term structure of monthly returns we define the actual sequence of monthly returns of the S&P 500 Index during a period of 480 months, corresponding to an investment horizon of 40 years.

Second, even if the average returns over all investment horizons are almost the same this will not imply that the term structure of returns is exactly the same. There is no unique correspondence between a term structure of monthly returns and its (geometric) mean. Consider 480 monthly returns and their (geometric) mean. This mean is unique, yet the 480 monthly returns can form 480! permutations of sequences of monthly returns, each generating a different final accumulation because the order of returns affects the final sum.

Put differently, the term structure of returns for an initial institutional investment with no further contributions can be substituted mathematically by its (geometric) average. However, if the investor makes monthly contributions over an investment horizon of 40 years, it does not necessarily follow that the accumulated wealth of these contributions growing by the actual term structure of monthly returns and the accumulated wealth of monthly returns growing at the mean of these monthly returns are the same. The actual term structure of returns plays an important role in the determination of the total accumulation at the end of a period of 40 years.

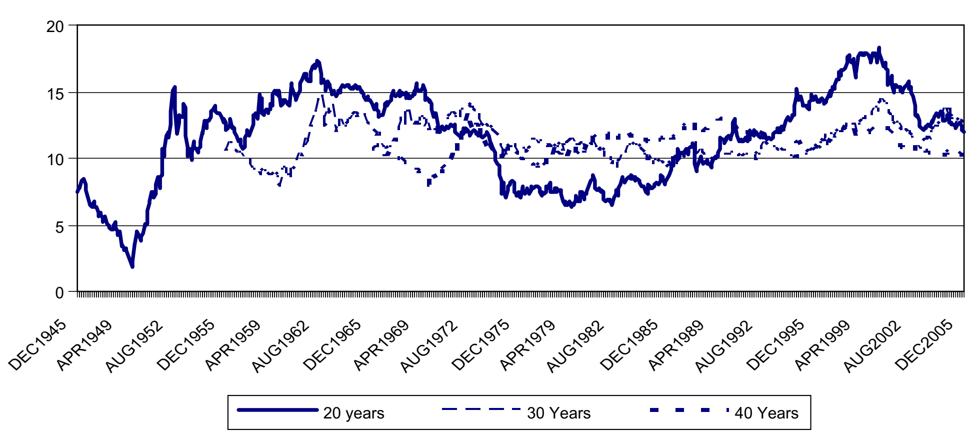
In Graph 2 the horizontal axis denotes 481 successive generations of investors each contributing $1 monthly to the S&P500 Index for a 40-year period. The first generation begins investing on December 31, 1925 and stops on December 31, 1965. The second generation starts and ends a month later and the very last generation in our sample begins investing on December 31, 1965 and ends on December 31, 2005. Graph 2 illustrates the time series of accumulated wealth of 481 overlapping generations computed in two different ways. First, by using the 40 year mean return that is specific for each generation and second, by using the actual term structure of returns during the 40 year period (also specific to each generation). The two accumulations coincide only in few years while for the majority of generations they diverge.

Note that accumulations calculated by the actual term structure of returns are more stable than those calculated by the (geometric) mean return. For the data in Graph 2, the standard deviations of accumulations are $1912 (reported earlier) and $2160 respectively. This is easy to explain. At any point in time, the younger generation begins and ends a month later than the one immediately before it. Computing the accumulation by using the term structure of returns is much more stable because the two nearby generations overlap over 478 out of 480 returns. However, when the (geometric) mean is computed, it may be the case that two nearby generations have slightly different means, again because of the long overlap of 478 identical monthly returns, but applying two slightly different means to a long sequence of monthly contributions causes greater variability to the final accumulation.

Thus, our fourth hypothesis that is motivated from institutional investing where replacing returns by their sample means is both common and reasonable, is illustrated here to be incorrect for individual retirement management. The term structure of monthly returns plays a definitive role in individual retirement accumulations and cannot be substituted by their means. Also the risks associated with these two methods are different. In principle, when managing individual retirement accounts one needs to be aware that substituting means for the term structure of returns yields incorrect results.

9. Stability of returns

Nominal and real total returns from a well-diversified portfolio of stocks are quite stable over major sub-periods. For example, Siegel (2002, p. 13) reports annual total nominal and total real stock market returns for 1802–2001 and also for major sub-periods such as 1802–1870, 1871–1925 and 1926–2001 to illustrate that the average sample returns (and risks discussed earlier) of these three sub-periods do not vary much from the mean of the population.



Graph 3. Time series of generational annualized means for 20–40 year investment horizons.

The stability, however, of these results depends on a very large sample of over 50 years. Our fifth hypothesis postulates that this stability found in US capital markets also holds for individual retirement accounts with much briefer (relatively) investment horizons. This section discusses the stability of the S&P 500 returns in terms of 20–40 year investment horizons.

In Graph 3 we present the time series of (geometric) means for 20–40 year investment horizons. These means are obtained by forming a rolling window of generations of investors beginning with the first generation from December 31, 1925 to December 31, 1945 for the 20 year horizon, to December 31, 1955 for the 30 year horizon and December 31, 1965 for the 40 year horizon. Then we drop one month and add a new month until we end with December 31, 2005. We have as before a total of 721, 601 and 481 generations with horizons of 20–40 years respectively. The reader will recall that in the previous section we suggested caution in using means to compute accumulations. Here we use means only as a tool for checking stability.

The arithmetic average of the time series of 721, 601 and 481 mean returns for generations of investors having horizons of 20–40 years respectively are: 11.56%, 11.36% and 11.12%. This confirms stability of averages across investment horizons. Corresponding standard deviations of these 721, 601 and 481 generations of returns are 9.9%, 4.75% and 3.55% suggesting that longer investment horizons are relatively more stable than shorter ones.

10. Relative performance

Ibbotson Associates (2006) and several other investment books document that over the long-run small company stocks outperform large company stocks and these outperform bonds and bonds outperform Treasury Bills. Data presented in Table 1 give precise returns and risks in US capital markets. This evidence motivates our sixth hypothesis that states that similar results hold for individual retirement accounts. To evaluate this hypothesis we use means for investment horizons of 20–40 years and within each horizon we compute the mean return of various investment classes. Evaluating the validity of the relative performance of various asset classes for individual retirement accounts relies heavily on two modifications when compared to long term institutional investing. First, the length of investment horizons is much shorter than the very large horizons of Ibbotson's 80 years. Second, using means is appropriate for institutional investing but for individual retirement management their use is limited by the observations we made earlier.

Our computations summarized in Graph 4 confirm that for the 20 year investment horizon small company stocks outperform large stocks for 85% of the 721 generations and large company stocks outperform bonds and Treasury Bills. Note that long-term government bonds do not consistently outperform Treasury Bills. For the 30 year investment horizon, small company stocks outperform large company stocks for 91% of the 601 generations while large company stocks outperform bonds and Treasury Bills and again bonds outperform Treasury Bills for the majority of generations. For the 40 year investment horizon, the superiority of small stocks to large stocks and large stocks to bonds and to Treasury Bills is for all 481 generations. However, still bonds do not outperform consistently Treasury Bills.

This graph suggests that findings about the relative performance of various classes of investments derived from very long investment horizons also hold for relatively shorter horizons of 20–40 years that are typical for individual retirement accounts. The relatively shorter horizons of individual retirement accounts do not allow bonds to clearly outperform Treasury bills. The high returns of small stocks, of course were not known to investors for the majority of the 1925–2005 period. In a similar way it is too soon to conclude that the recent outperformance of various global indexes relative to the S&P 500 Index will continue over the next 50 years.

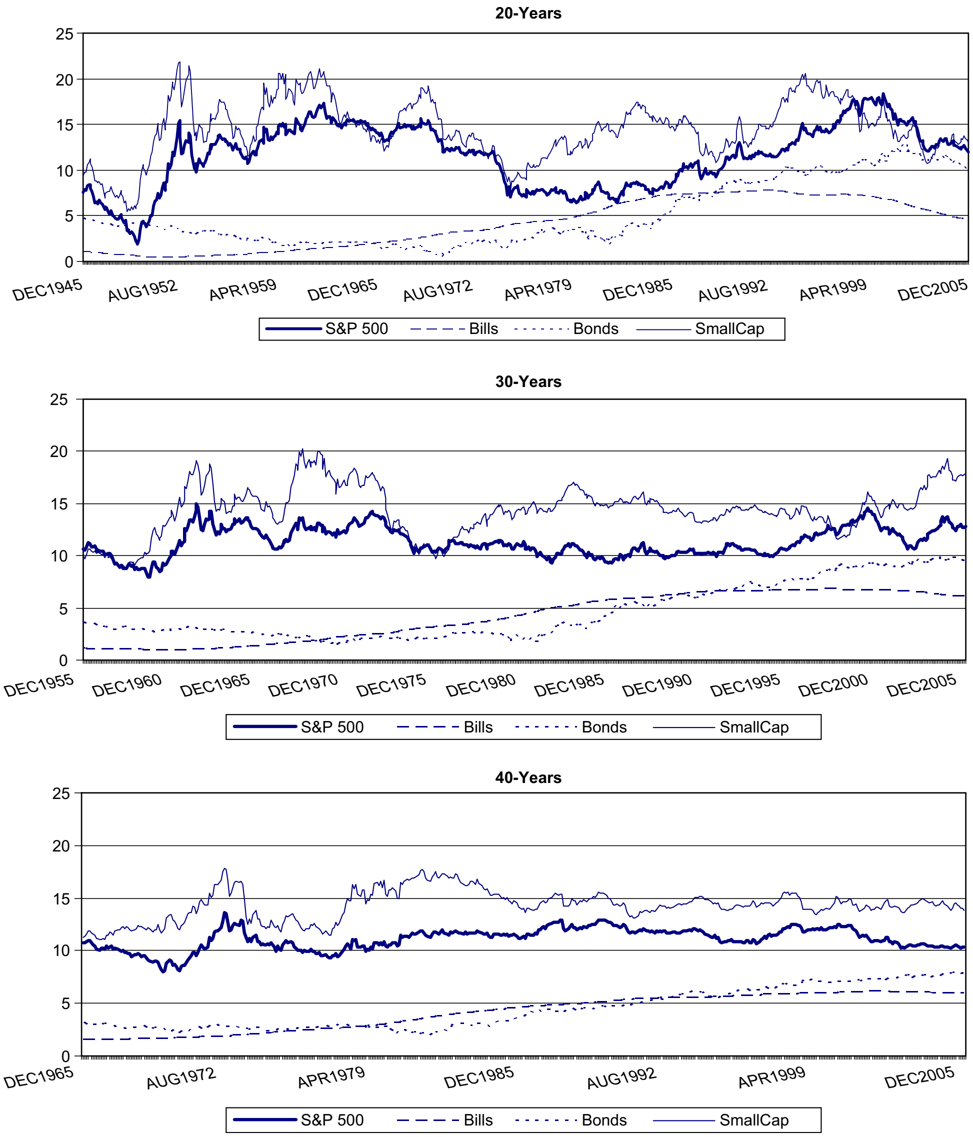
11. Distributions of accumulations

For capital markets, annual stock returns (nominal, real, total) appear to be normally distributed as the sample size increases. Exact normality cannot be established statistically because returns of a stock index such as the S&P 500 exhibit fat tails. This near normality of returns over long period samples and the log-normality behavior of asset prices are presented in Campbell et al. (1997).

Translating these findings for individual retirement management we propose the seventh hypothesis that postulates that the means of 721 generations investing for 20 years, 601 generations investing for 30 years and 481 generations investing for 40 years are distributed normally. This hypothesis is novel for individual retirement accounts. We have computed distributions of means for all 4 classes of assets: small cap, S&P 500, bonds and bills, each for three investment horizons of 20–40 years. The normality hypothesis is rejected for all cases.

To avoid ambiguity, accumulations are computed at the exact term structure of returns with $1 invested per month. Changing the amount of monthly contribution affects the final accumulation and this accumulation may further be impacted if contributions accelerate as time passes. To illustrate the influence of changing contributions we repeat all our calculations using annual increases in the monthly contributions that incorporate a 2% productivity increase and a 3% inflation adjustment.

All results clearly reject the normality hypothesis of accumulations. Such non-normality of portfolio accumulations over 20–40 years for different generations of retirees gives no statistical justification to perform probabilistic calculations assuming normality. Evidence of non-normality necessitates a careful study of portfolio accumulation dis-



Graph 4. Relative performance: means of small cap, S&P 500, bonds, and T-bills per generation for 20–40 year investment horizons.

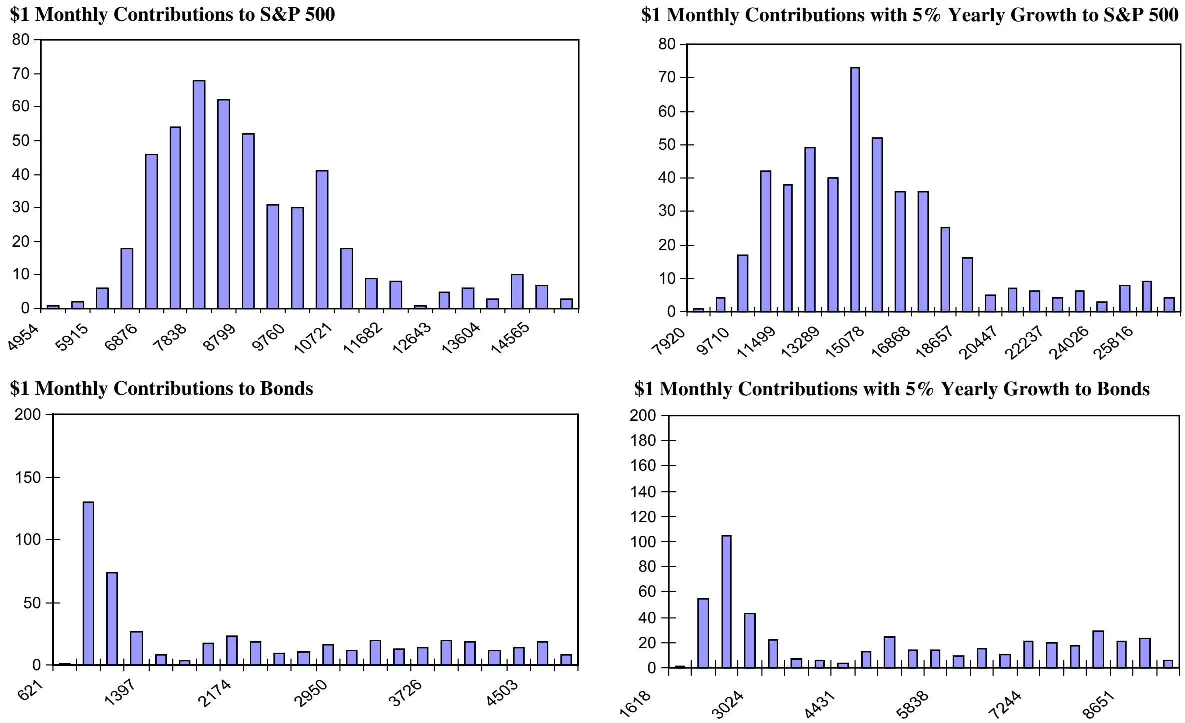
tributions and its associated probabilities for a random retiree achieving a certain accumulation.

Graph 5 selectively illustrates the 40 year accumulation when contributions are invested either in large stocks or long-term government bonds. These distributions appear in the upper half of the panel and are clearly skewed to the right. When monthly contributions grow over time to account for productivity increases and compensation for inflation, the distribution of accumulation becomes even more skewed to the right. The lower panel shows the distribution of accumulations when the initial monthly \$1 contribution grows annually by 5% to account for a 2% productivity increase and 3% for inflation.

12. Adequacy of accumulations

Next, we wish to evaluate the effectiveness of contributions made to an individual retirement account. Suppose that the representative investor sets as a goal during a 40 year retirement horizon to secure as income an amount equal of 60% of his/her pre-retirement annual income. Our calculations can be revised for a higher or lower percent and also for longer or shorter investment horizons as well as shorter or longer retirement horizons.

Logue and Rader (1998) describe that a representative defined benefit retirement plan usually offers the retiree 1.5% of final pay per year worked. Thus if an employee has worked for 40 years then $40 \times 1.5 = 60\%$ of final pay is expected to be received, possibly adjusted for inflation over an average retirement horizon of 20 years. The question of a representative employee who contributes a given amount per month over 40 years invested in a certain class of investments is the following: what is the necessary annual contribution as a percent of the employee's annual income that needs to be invested to contribute to the achievement of the 60% rule? Malliaris (2007) discusses in



Graph 5. Distribution of 40 year S&P 500 and bond accumulations for 481 generations.

Table 2
Probability that a specified percentage of annual contributions will be sufficient to finance a certain percent of final pay, assuming a 40 year investment horizon and a 20 year retirement period

Percent of final pay	Percent of contribution									
	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
50.00	0.00	0.00	12.2	24.8	52.7	67.3	80.0	93.0	99.1	99.8
55.00	0.00	0.00	8.6	16.7	38.1	60.1	69.8	85.1	94.4	99.3
60.00	0.00	0.00	6.1	14.9	27.3	52.7	66.4	75.0	87.8	96.2
65.00	0.00	0.00	3.4	12.4	20.5	40.3	59.5	67.6	79.1	91.0
70.00	0.00	0.00	0.7	10.1	16.0	29.7	52.7	64.6	71.2	82.9
75.00	0.00	0.00	0.00	8.1	14.9	24.8	41.9	58.6	67.3	75.0
80.00	0.00	0.00	0.00	6.1	12.8	17.8	31.5	52.7	64.0	68.7
85.00	0.00	0.00	0.00	4.5	11.5	15.8	26.1	43.5	57.7	66.4
90.00	0.00	0.00	0.00	1.6	9.2	14.9	22.3	34.0	52.7	62.2
95.00	0.00	0.00	0.00	0.2	7.7	13.1	17.3	28.4	44.4	57.2
100.00	0.00	0.00	0.00	0.0	6.1	12.2	15.5	24.8	26.0	52.7

detail all the technical aspects of such an exercise and presents several tables. Here we report one representative table. Assuming a 40 year investment horizon and taking into account the actual monthly returns of the S&P 500 Index we compute multiple accumulations for monthly contributions that are a certain percent (1–10%) of the US Median Income. We then estimate the present value of a 20 year annuity that is required to finance a certain percent of final pay and calculate as a percent the number of generations in our sample that have achieved such a sufficient accumulation. Table 2 computes the probabilities for securing a certain percent of final pay during a 20 year retirement period at various levels of income contributed when all contributions are invested in large stocks. The main conclusion of this calculation is that in order for an individual to achieve with very high probability a retirement income that is 60% of his/her final pay requires a contribution of the individual's monthly income that is about 10%, provided the contributions are invested in large stocks for 40 years.

13. Summary and conclusions

The dramatic recent increase in the number of individual retirement accounts in the US and globally necessitates rational financial management of these accounts. In contrast to the exhaustive research about capital markets summarized in Ibbotson Associates (2006) and Constantinides

(2002), the academic literature is just beginning to address various aspects of individual retirement accounts. As an increasing number of individuals attempt to manage their own accounts, often with the help of advisors, it is informative to list certain useful guidelines. Furthermore it is helpful to relate these guidelines to ones that have been developed for institutional investing. Our paper proposes several such guidelines.

First, calculating individual retirement accumulations is not simple when compared to straightforward lump-sum investing, because both the monthly contributions and returns are unpredictable. The calculations of periodic contributions are not complex but not all investors are skilled to perform them. Using average return and average contributions may simplify the calculations but it also causes such an average accumulation to deviate, often substantially, from the actual one. Using also average returns as an approximation to the actual term structure of returns causes the standard deviations of accumulations to be larger.

Second, for investment horizons of 20, 30 and even 40 years of monthly contributions the accumulations, even when funds are invested 100% in stocks are not very large multiples of total contributions because the power of compounding works only partially. Early contributions have long periods to compound while later contributions have shorter time to compound, particularly if they are larger and cause contributions to increase.

Third, our calculations support the notion of stability of returns found in the literature. For investment horizons of 40, 30 and even 20 years, geometric means of monthly returns of the S&P 500 Index for numerous generations have similar means but different standard deviations. This stability of returns does not necessarily translate to stability of accumulations for two reasons. First, individuals with longer horizons of 40 years accumulate absolutely much larger sums than those who invest for only 30 or 20 years. The standard deviations of these accumulations are also different because longer horizons imply larger fluctuations. Second, for large numbers of generations the accumulations appear clustered but rapid booms and busts offset such clustering and common measures of risk do not discriminate between periods of stable accumulations and periods of rapid change. In other words, each generation, like vintage wine, is characterized by monthly returns that are path dependent, meaning that each nearby return is correlated with the previous one reflecting the state of the economy that does not act totally at random month to month. This means that individual investors live, invest and consume during their generation and cannot transport their investments at other times.

Typical examples are the 1987 October Crash or the bursting of the technology bubble in March 2000. The accumulations of those who retired before such episodes compared to the ones who retired few months later may be significantly different. This fact necessitates careful management of investment allocations for individuals approaching retirement age. In contrast if accumulations were very stable across generations, most generations contributing the same amount would accumulate approximately the same final amounts.

Fourth, investing in equities remains the favorite vehicle for highest returns. For horizons of 20–40 years, the overwhelming majority of generations accumulate the largest sums by investing in small company stocks vs large company stocks vs. bonds or bills. Bonds however do not consistently outperform Treasury Bills in our sample. This superior performance of stocks is also associated with relatively higher risk.

Fifth, calculations with fixed monthly contributions or contributions increasing annually by a 2% productivity growth and a 3% inflation rate, yield accumulations which are skewed to the right. This pattern of distribution necessitates more careful statistical inference about what percent of generations will achieve certain accumulations.

Finally, unlike institutional investing that often has a very long investment horizon, individual retirement accounts have both a terminal goal and horizon. Our analysis suggests that individuals contributing for a 40 year horizon can experience a very high probability of achieving their retirement goal of having sufficient funds to live comfortably for 20 years beyond the age of 65 if they invest in stocks at least 10% of their annual income. Employer contributions and social security payments may reduce the need to invest 10% percent of the investor's annual income.

In our analysis we have ignored transactions costs, taxes and the possibility that the investor lacks the discipline to adhere to his or her 40 year investment plan. These and other real world complications, such as sickness, loss of employment, family problems, all contribute to lowering accumulations and thus lowering probabilities of achieving one's retirement goals. On the positive side, many individuals after they pay off their mortgages and also pay fully or partially for their children's education, save for retirement substantial amounts during the last decade of their working lives.

As academic research and the practice of managing individual retirement accounts grow the findings discussed here will be further revised and several new guidelines will be proposed to help individuals manage their retirement portfolios more rationally.

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