# Is Complete Hedging Optimal for International Bond Portfolios?

**Victor S. Filatov** and **Peter Rappoport**

*International investments offer diversification benefits for investors, but they also expose portfolios to highly volatile currency movements. Some have argued that hedging currency exposure completely offers investors a free lunch—lower risk with no sacrifice of expected return. Complete hedging did turn out to be a sound strategic position for U.S.-dollar-based investors in the 1980s.*

*Over the 1980s, however, non-U.S. investors could have lowered their international portfolios' risks by taking on some currency exposure. For these investors, the benefits of diversification across currencies more than offset the cost of the currencies' volatilities. Evidently the free-lunch argument misses something, and investors should be concerned that the missing link might change across countries or over time.*

*The conditions under which currency hedging does or does not contribute to diversification turn on the correlations between asset and currency returns—standard parameters in optimal portfolio selection. Investors need to take care in estimating these parameters, given their importance to the hedging decision. On the other hand, investors need have no strong views about or special expertise in forecasting exchange rate movements in order to avail themselves of the diversification benefits of currency exposure.*

A number of recent studies of internationally diversified portfolios have concluded that investors can lower their risks by *completely* hedging their exposure to exchange rate movements.[1] Because long-run exchange rate movements are difficult to predict, this reduction in risk is accompanied by little change in expected returns. Currency hedging has thus come to be regarded as presenting investors with a "free lunch." Deliberate *exposure* to foreign currency movements has, correspondingly, come to be regarded as a "tactical" decision, an attempt to exploit one's views on the direction of future currency movements, rather than a "strategic" attempt to lower long-run risks through diversification. Some investors believe that complete hedging permits one to separate the role of the fund manager from that of the currency specialist: The former need only decide on a position in hedged assets, while the latter need only be concerned with the extent of currency exposure.

This article shows that the apparent superiority of **complete hedging** is based on an inappropriate analysis of international diversification. That something is amiss with the free-lunch argument can readily be seen from Table I, which shows the risks associated with several bond portfolios viewed from the differing perspective of investors in the four countries that collectively dominate world bond issuance.[2] The first three columns compare the risks of domestic bonds and hedged and unhedged foreign bonds. They confirm observations made in the currency hedging literature: Returns on unhedged foreign assets are significantly more volatile than both hedged foreign returns and returns on domestic assets.

The last two columns show the risk from portfolios comprised of 90% domestic and 10% foreign bonds. The figures for the U.S. are consistent with the currency hedging literature: Hedged international diversification would have produced the lowest risk for dollar-based investors (2.83% per month, as opposed to 2.91% per month). But it is apparently not true in general that internationally diversified portfolios produce lower risks when currency risks are fully hedged. Investors based in the other three currencies would have incurred higher risks if they were fully hedged rather than unhedged. Moreover, on the assumption that systematic predictions of exchange rate movements cannot be any better than the forecasts implicit in forward exchange rates, the hedged and unhedged portfolios have equal expected returns. Remaining unhedged thus unambiguously dominates hedging for non-U.S. investors. Far from enjoying a free lunch, these investors would have been "stuck with the tab" had they hedged their currency exposures.

Table I International Investment Risks (percentage points per month)[a]

| Domestic Country | Domestic Bond | Foreign Bonds[b] | | Internationally Diversified Portfolios[c] | |
|---|---|---|---|---|---|
| | | Hedged | Unhedged | Hedged | Unhedged |
| United States | 3.05 | 1.52 | 3.92 | 2.83 | 2.91 |
| Germany | 1.47 | 2.17 | 3.09 | 1.46 | 1.38 |
| Japan | 1.59 | 2.57 | 3.78 | 1.57 | 1.48 |
| United Kingdom | 2.88 | 2.13 | 3.23 | 2.70 | 2.62 |

a. The figures shown are the standard deviations of monthly returns over the period June 1980–April 1989.
b. Market-capitalization-weighted indexes of foreign securities.
c. 90% domestic assets, 10% foreign.

This simple exercise reveals the important difference between the risks of foreign assets treated in isolation and their contribution to the risks of internationally diversified portfolios. It shows that the hedging decision should not be driven by the observation that hedged foreign returns are less volatile than unhedged returns. During the 1980s, investors based in the major international currencies would probably have agreed with this observation, but all except U.S. investors would have lowered their risks by *not* hedging. It also shows that the ability to forecast exchange rate movements is not a prerequisite for taking on currency exposure; such exposure can provide "strategic" benefits in the absence of forecasting ability.

Evidently, the literature has not analyzed adequately the conditions that make (not) hedging the optimal strategy for (non-) U.S. investors. This creates cause for concern. These unidentified conditions may change over time, motivating different strategic hedging positions. But without an understanding of what these conditions are, and how they have influenced the performance of hedging strategies in the past, recommendations about how to hedge future foreign currency exposures rest on shaky ground.

To pinpoint the factors that are important in determining optimal hedging strategy, this article develops an explicit model of the allocation problem faced by an investor wishing to minimize risks by diversifying internationally. The central result is that the optimal degree of currency hedging is determined by the **covariances** between exchange rate movements on the one hand and domestic and foreign asset returns on the other. If these covariances are zero or positive, it will be optimal to be fully or even over-hedged. If they are negative, it will be optimal to remain partially unhedged. In other words, **currency exposure** contributes to diversification if depreciation of the domestic currency tends to be accompanied by lower-than-average foreign and domestic asset returns, denominated in their local currencies. In that case, losses due to low asset returns will tend to be offset by gains from foreign currency exposures.

This explains the results in Table I. Hedging lowered risks for U.S. investors because, for the 1980s as a whole, movements in the dollar-yen, dollar-Deutschemark and dollar-pound exchange rates were *positively* correlated with U.S. and foreign asset returns, denominated in their local currencies. But from the viewpoint of non-U.S. investors (looking at ¥/$, DM/$ and £/$ exchange rates), the same data give inverse results—*negative* correlations between asset returns and currency movements, implying that hedging would not have reduced risks for non-U.S. investors. The same underlying phenomena that made it optimal for investors

***Glossary***

▶***Complete Hedging:***
A portfolio is *completely hedged* (with regard to exposure to currency movements) if all anticipated foreign currency cash flows have been sold into the investor's domestic currency at a known exchange rate (typically the forward rate for the appropriate time horizon).

▶***Covariance:***
A statistical measure of the degree of association between two random variables—for example, returns on stocks and bonds. The covariance will be positive if stock and bond returns are typically high at the same time and will be negative if one is high when the other is low.

▶***Currency Exposure:***
A portfolio has *currency exposure* if it contains assets whose returns will be received in foreign currency and these proceeds have not been completely hedged.

▶***Risk Minimization:***
The objective of the approach to portfolio optimization taken in this article is to select the portfolio with the lowest possible risk among all portfolios that are admissible, where risk is defined as the standard deviation of the return on the portfolio.

▶***Constrained Optimal Hedge Ratio:***
The solution to a portfolio optimization problem that is constrained; we rule out the possibility of short sales of currencies, hence the optimal hedge ratio for each currency is constrained to lie between 0 and 1.

based in U.S. dollars to hedge foreign exposures made it optimal for investors based in foreign currencies *not* to do so. Complete hedging cannot be appropriate for *all* investors at *all* times. (Note, once again, that expected currency returns do not enter this argument.)

This article considers explicitly the role played by the share of the domestic asset in the portfolio. It turns out that the volatility of portfolio returns is typically more sensitive to changes in the distribution of the portfolio across foreign and domestic assets than it is to changes in the hedge ratio, particularly when the share of domestic assets exceeds 75%. This suggests that hedging is not necessarily the most effective way of controlling currency risks in global portfolios.

The theoretical framework of the article provides a perspective on international investment that differs in significant respects from the existing literature. Specifically, it indicates that significant benefits from hedging can typically be reaped only when the hedge ratio and the division of the portfolio between domestic and foreign assets are decided simultaneously. Consequently, the bond portfolio manager should be kept informed of, and take into account, decisions concerning currency exposure. It is not necessarily optimal to make bond allocation and currency exposure decisions independently. Judging the performance of international bond managers relative to a hedged benchmark may therefore result in investments that are detrimental to overall portfolio risk.

This article indicates that the hedging decision requires knowledge of several covariance parameters. The standard practice is to estimate these from long historical samples, typically of monthly data, and the allocation exercises carried out in the article use this approach. However, there is no reason to believe that these covariances are constant over time. They may change, for example, as the degree of integration and focus of attention of international asset markets changes. We present some evidence that suggests that the covariance between U.S. bond returns and dollar-foreign currency exchange rates has moved from positive to negative in recent years. Should this trend continue, U.S. investors may minimize risks in the future by less-than-complete hedging. Correct implementation of strategies to control risk requires timely estimates of the changing inputs to the portfolio problem.

## A Simple Portfolio Model

The correlations between returns on the assets of different countries have historically been low, offering investors the opportunity to reduce risks by diversifying internationally. However, because total return performance is measured in an investor's domestic currency, holding foreign assets exposes the portfolio to exchange rate risks, which have, historically, been large. Consequently, investors have been advised to sell foreign currency proceeds forward; this immunizes them against currency risks, at a cost, in terms of expected return, equal to the forward premium.[3]

The problem with this justification for complete hedging is that, while the declared objective is to minimize portfolio risks without sacrificing return, the superiority of complete hedging is inferred solely from the lower volatility of hedged assets relative to unhedged assets. The correct approach would view the hedging decision as part of an overall strategy of **risk minimization**. One can think of the problem in the following way. The investor chooses the proportions in which he or she will hold three assets—domestic, hedged foreign and unhedged foreign. Returns on unhedged foreign assets are more volatile than those on hedged foreign assets. Does this mean that the unhedged asset should be assigned a weight of zero? Not necessarily. Its returns may covary with the other returns in such a way that holding it is advantageous for overall portfolio risk.

A simple model, which casts both currency hedging and international asset allocation in the same framework, captures the issues involved. The model is framed in terms of the most elementary case, involving a choice between one domestic and one foreign asset. The foreign asset can be thought of as an aggregate of assets of individual foreign countries, with shares equal to their respective world market capitalizations. Changes in the "local currency" of this composite foreign asset will then be the market-capitalization-weighted average of the respective individual currency movements. This simplification permits us to characterize the solution to the optimization problem explicitly, and to identify the factors that determine the hedge ratio.

We use the following notation:

- $D$ = the percentage return on the domestic asset over the horizon of the investment;
- $L$ = the percentage return on the foreign asset over the horizon, when returns are valued in the foreign currency;
- $C$ = the percentage change in the domestic-currency/foreign-currency exchange rate over the horizon;
- $F$ = the current foreign exchange forward premium for the horizon date, expressed as a percentage of the spot exchange rate at the time the assets are purchased;
- $R$ = the percentage return on the portfolio;
- $w_i$ = the portfolio's share of asset i;

$\sigma_i^2$ = the variance of returns on asset i; and
$\sigma_{ij}$ = the covariance of returns on assets i and j.

The return on the foreign asset valued in the domestic currency is L + C. Hedging currency risk essentially involves swapping the unknown currency movement C for the known forward premium F. Hence the return on the composite hedged foreign asset is L + F. The return on the overall portfolio is then:

**Eq. 1**

$$R = w_D D + w_{L+C}(L + C) + w_{L+F}(L + F),$$

where the weights of the three assets must sum to one. The hedge ratio, h, is defined as the proportion of hedged foreign assets to total foreign assets:[4]

**Eq. 2**

$$h = \frac{w_{L+F}}{w_{L+F} + w_{L+C}}.$$

Consider an investor who wishes to minimize risk by selecting appropriate values of $w_D$, $w_{L+C}$ and $w_{L+F}$. The optimal shares of the three assets will be determined by the variances and covariances of their returns, while the optimal hedge ratio will be 1 only if $w_{L+C}$, the share of the unhedged foreign asset, is zero. Using Equation (2), the portfolio return can be reexpressed as:

**Eq. 3**

$$R = w_D D + (1 - w_D)[L + hF + (1 - h)C],$$

which breaks returns down into the individual contributions of the basic sources of uncertainty—D, C and L.[5] The variance of returns is:

**Eq. 4**

$$\mathrm{Var}(R) = w_D^2\sigma_D^2 + (1 - w_D)^2\sigma_L^2 + (1 - w_D)^2(1 - h)^2\sigma_C^2 + 2[w_D(1 - w_D)\sigma_{DL} + w_D(1 - w_D)(1 - h)\sigma_{DC} + (1 - w_D)^2(1 - h)\sigma_{LC}].$$

The advantage of hedging completely (h = 1) is that it removes $\sigma_C^2$—the variance of currency returns—from the variance of the portfolio. However, this also simultaneously removes the interactions of currency returns with asset returns—that is, the covariances $\sigma_{DC}$ and $\sigma_{LC}$. If these covariances are sufficiently negative, the benefit of immunizing against currency volatility will be outweighed by the loss of the diversification benefits of currency exposure.

As long as the portfolio weight assigned to the domestic asset, $w_D$, does not equal 1, overall portfolio risk will be minimized by hedging according to the following rule:[6]

**Eq. 5**

$$\tilde{h} = 1 + \frac{w_D\sigma_{DC} + (1 - w_D)\sigma_{LC}}{(1 - w_D)\sigma_C^2}.$$

Equation (5) allows us to identify the role of the covariances between asset returns and exchange rate movements and their interaction with the volatility of exchange rate movements.

First, it will be optimal to maintain some currency exposure (h < 1) only if at least one of $\sigma_{DC}$ and $\sigma_{LC}$ is sufficiently negative. If $\sigma_{DC}$ and $\sigma_{LC}$ are both positive, it will never be optimal to remain unhedged. But if these covariances are both positive from the perspective of, say, a dollar-based investor in sterling assets, they will both be *negative* from the perspective of a sterling-based investor holding dollar assets, for whom the currency changes are reversed.[7] Thus, while the U.S. investor should hedge over 100% in this case, Equation (5) shows that the U.K. investor should not fully hedge. Consequently, to advocate that all investors hedge currency exposures equally, irrespective of their base currencies, cannot be sound advice. It should be stressed, moreover, that this result has nothing to do with expectations of appreciation or depreciation of either currency. It follows solely from the part played by the asset return–currency covariances in the solution to the problem of minimizing risk.

Second, currency volatility, $\sigma_C^2$, scales the effect on the hedge ratio of the asset return–currency covariances: The larger $\sigma_C^2$ is, the smaller their effect. This follows the logic of the general asset allocation considerations raised above. High volatility by itself says nothing conclusive about whether one should be exposed to the asset's risks, although, all other things being equal, the larger the volatility of an asset, the smaller the weight assigned to it will be.

The literature on currency hedging has typically examined the risks of completely hedged and unhedged positions (h = 1 and h = 0, respectively).[8] For purposes of comparison, we will constrain the optimal hedge ratio to lie between 0 and 1, which effectively rules out short sales. This **constrained optimal hedge ratio**, h*, is then described by:

**Eq. 6**

$$h^* = \begin{cases} 0 \text{ if } \tilde{h} \le 0 \\ \tilde{h} \text{ if } 0 \le \tilde{h} \le 1 \\ 1 \text{ if } \tilde{h} \ge 1 \end{cases}$$

Other authors have considered the problem of minimizing the risk implied by the position in

Table II Portfolio Returns of Alternative Strategies, June 1980–April 1989[a]

| Domestic Country | | (a) Optimal | (b) Fully Hedged | (c) $w = 0.9$ $h = 1$ | (d) $w = 0.9$ | (e) $w = 0.9$ $h = 0$ | (f) Unhedged | Covariance/Correlation Matrix and Mean Returns[d] | D | L | C |
|---|---|---|---|---|---|---|---|---|---|---|---|
| United States | w | 0.00 | 0.00 | 0.90 | 0.90 | 0.90 | 0.70 | D | 3.05 | 1.56 | 1.30 |
| | h | 1.00 | 1.00 | 1.00 | 1.00 | 0.00 | 0.00 | L | 0.52 | 1.52 | 1.34 |
| | Risk[b] | 1.52 | 1.52 | 2.83 | 2.83 | 2.91 | 2.78 | C | 0.17 | 0.37 | 3.22 |
| | Mean Return[c] | 0.82 | 0.82 | 0.89 | 0.89 | 0.89 | 0.85 | Mean | 0.90 | 0.82 | 0.23 |
| Germany | w | 0.83 | 0.88 | 0.90 | 0.90 | 0.90 | 0.83 | D | 1.47 | 1.33 | −1.23 |
| | h | 0.00 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | L | 0.55 | 2.17 | −1.01 |
| | Risk[b] | 1.36 | 1.46 | 1.46 | 1.38 | 1.38 | 1.36 | C | −0.39 | −0.18 | 2.62 |
| | Mean Return[c] | 0.71 | 0.70 | 0.70 | 0.70 | 0.70 | 0.71 | Mean | 0.68 | 0.88 | 0.17 |
| Japan | w | 0.85 | 0.89 | 0.90 | 0.90 | 0.90 | 0.85 | D | 1.59 | 1.39 | −1.40 |
| | h | 0.00 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | L | 0.47 | 2.57 | −0.96 |
| | Risk[b] | 1.46 | 1.57 | 1.57 | 1.48 | 1.48 | 1.46 | C | −0.40 | −0.11 | 3.10 |
| | Mean Return[c] | 0.77 | 0.77 | 0.77 | 0.77 | 0.77 | 0.77 | Mean | 0.75 | 0.91 | −0.54 |
| United Kingdom | w | 0.27 | 0.22 | 0.90 | 0.90 | 0.90 | 0.56 | D | 2.88 | 1.75 | −1.63 |
| | h | 0.76 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | L | 0.50 | 2.13 | −0.97 |
| | Risk[b] | 1.99 | 2.05 | 2.70 | 2.62 | 2.62 | 2.20 | C | −0.33 | −0.16 | 2.80 |
| | Mean Return[c] | 0.90 | 0.89 | 1.07 | 1.07 | 1.07 | 0.98 | Mean | 1.10 | 0.83 | 0.46 |

a. Market-capitalization-weighted indexes of foreign securities are used for the international portfolio.
b. Standard deviation of monthly returns in percentage points.
c. Mean returns calculated on the assumption that expected currency returns and forward premiums are zero.
d. The signed square roots of the absolute values of covariances are presented in the table to permit comparison with the standard deviations on the main diagonal; correlation coefficients are displayed below the main diagonal.

foreign assets alone [L + hF + (1 − h)C]. This is equivalent to the special case of the problem examined here, in which $w_D = 0$. Under these circumstances, Equation (5) reduces to:[9]

**Eq. 7**

$$\tilde{h} = 1 + \frac{\sigma_{LC}}{\sigma_C^2}$$

Even if this approach is used, the U.K. investor discussed above should not hedge completely (because $\sigma_{LC}$ is negative). However, this formulation of the portfolio return whose risk is to be minimized (that is, the international asset return) is simply wrong. The main attraction of international assets to investors is as a means of achieving diversification benefits beyond those offered by closely correlated domestic assets.[10] Indeed, efficient frontiers and risk results offered in support of hedging always include U.S. assets in the portfolio.[11] Thus Equation (1) or (3) provides the appropriate description of returns, while Equation (5) is the correct formula for the optimal hedge ratio.

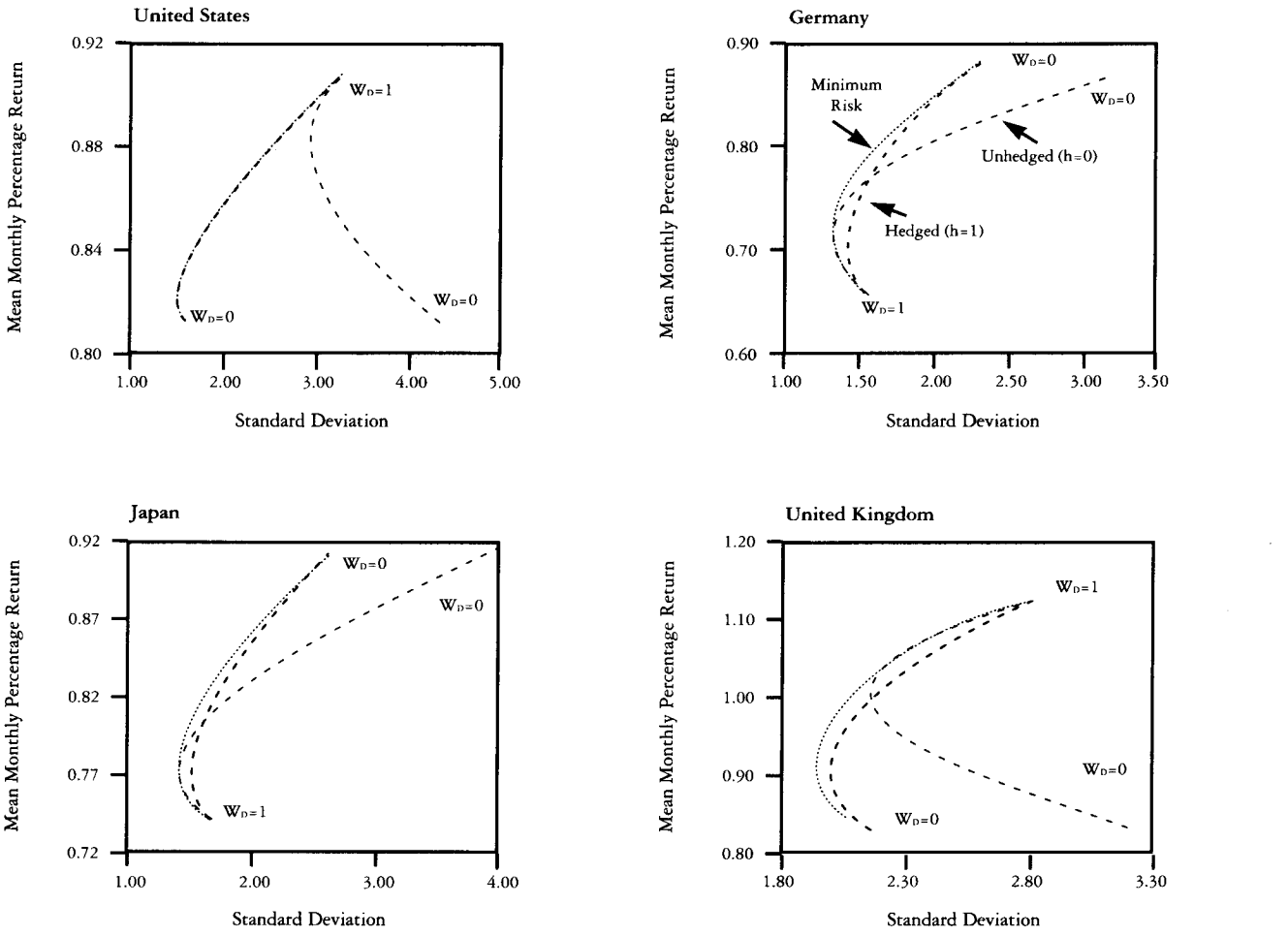
## Fixed-Weight Portfolios

The principles just developed can be illustrated by investigating further the example in Table I. Investors in each country are assumed to allocate their portfolios between domestic bonds and a market-capitalization-weighted composite of foreign bonds by choosing $w_D$, and to determine the degree of hedging by selecting h. Table II gives the outcomes of diversification and hedging strategies; it is again assumed that expected returns to currency exposure and forward contracts are zero.[12]

It was optimal for U.S. investors to hedge completely [column (a)], which is to be expected from the positive signs of $\sigma_{DC}$ (1.30) and $\sigma_{LC}$ (1.34) in the right-hand panel. In contrast, these covariances are all negative for investors based in the other three currencies. The optimal strategy for German and Japanese investors was to remain completely unhedged, while sterling-based investors would have minimized risks by hedging 76% of their currency exposures.

The free lunch from hedging can be assessed using Figure A, which shows, at each value of the average return on the four portfolios, the minimum risk attainable under three strategies. The bold dashed line indicates the frontier for completely hedged international diversification, while the dashed line shows the lowest risk attainable when the investor remains completely unhedged. Finally, the dotted line indicates the lowest risk attainable at each level of mean return, when both the hedge ratio and the weight of the domestic asset are chosen to minimize risk. By construction, this line will never lie to the right of the other two, although it could coincide with them at some points. Note that, for non-U.S. investors, the hedged strategy is strictly dominated by the optimal strategy and does not dominate the unhedged strategy. Contrast this with the argument that hedging offers a free lunch because unhedged foreign bonds are more volatile than hedged but have the same expected return.[13]

**Figure A** Efficient Frontiers, June 1980–April 1989*



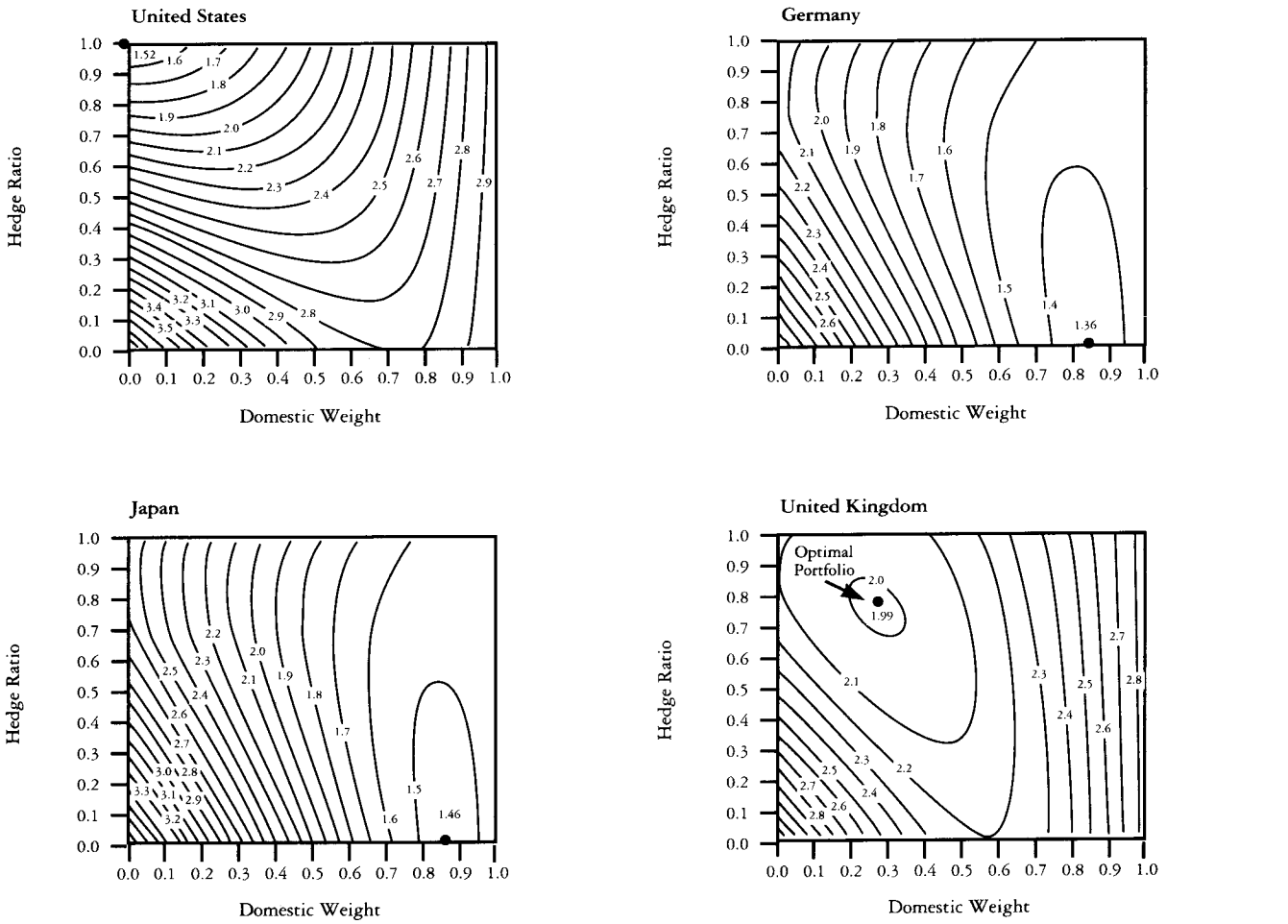

* Expected currency changes and forward premiums set equal to zero.

This view ignores the potential diversification benefits of unhedged assets and turns out to have been valid historically only for U.S. investors.[14]

Where an investor would actually choose to end up on these diagrams will depend on her preferences. Any regular utility function would, in general, place the investor on the upward-sloping portion of the efficient frontier, but specific information is required to say where. An alternative, and more conservative, approach is to imagine that one of the investor's objectives is to keep the value of the portfolio above some target level. The investor will thus be concerned with the probability that, during some time interval, the value of the portfolio declines below this given level. The investor uses this minimum-risk point as a benchmark and is not prepared to incur greater risk unless compensated by an increase in expected return sufficiently large to ensure that the probability of underperformance is not increased. If portfolio returns are normally distributed, this probability will remain constant if the standard deviation of returns and the mean return increase by the same amount as the investor moves up the frontier. Using this criterion, the minimum-risk point will be optimal for all investors.

Because currency returns are so volatile, hedging has been regarded as an important means of controlling the risks of international investments. However, the data analyzed here suggest that, at least for the extent of international investment practiced by most funds, hedging is less significant than varying the proportion of the portfolio in foreign assets. This outcome, at least for U.S. and U.K. investors, may have been expected from the data already presented in Table II. For investors

**Figure B** International Risk Contours (monthly standard deviations of domestic minus international portfolio returns)*




* Market-capitalization-weighted indexes of foreign securities are used for the international portfolio.

based in those two countries, the optimal portfolio has an unrealistically large concentration of foreign assets. Columns (c), (d) and (e) show the effects of constraining foreign assets to 10% of the portfolio; they show that risk varies little across alternative hedge ratios.

Figure B, showing the contours of the standard deviations of portfolio returns, illustrates clearly the roles played by the hedge ratio and the domestic weight.[15] A vertical contour indicates that, at the given value of $w_D$, variations in the hedge ratio have no effect on overall portfolio risk. In view of this, U.S. and U.K. investors who devote only 10% or so of their portfolios to foreign assets really need not be concerned with the hedging decision. Indeed, when the weight of the domestic asset exceeds 75%, risk changes very little as the hedge ratio changes; that is, the contours are close to vertical. Only when the domestic weight is below about 75% does varying the hedge ratio have a significant impact on risk. For Japanese and German investors holding less than 25% of their portfolios in foreign assets, changing the hedge ratio from 0 to 1 while keeping the domestic weight fixed raises risk by less than 15 basis points.

Apparently, when the concentration of the portfolio in the domestic asset is high, the hedge ratio has only a "second-order" effect on risk. From the diagrams, it appears that investors who wish only to place a small fraction of their portfolios in foreign bonds do not really need to devote their energies to deciding whether being hedged or unhedged is the right neutral currency exposure. Either position is appropriate.[16] Of course, this in no way diminishes the role of active currency management.

Most analyses of currency hedging have been conducted using after-the-fact calculations similar to those presented above. Performance is calculated on the assumption that the investor knew

**Table III Multicurrency Portfolios, June 1980–April 1989**

| Domestic Country Strategy | United States (US) | | | | Germany (WG) | | | | Japan (JAP) | | | | United Kingdom (UK) | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *Optimal* | *w = 0.75* | *w = 0.9* | *Unhedged* | *Optimal* | *w = 0.75* | *w = 0.9* | *Unhedged* | *Optimal* | *w = 0.75* | *w = 0.9* | *Unhedged* | *Optimal* | *w = 0.75* | *w = 0.9* | *Unhedged* | *Hedged* |
| $w_{US}$ | 0 | 0.75 | 0.9 | 0.70 | 0.09 | 0.07 | 0.05 | 0.08 | 0.07 | 0.08 | 0.07 | 0.10 | 0.07 | 0.01 | 0 | 0.10 | 0 |
| $h_{US}$[a] | — | — | — | — | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | — |
| $w_{WG}$ | 0.58 | 0.12 | 0.03 | 0.14 | 0.54 | 0.75 | 0.90 | 0.81 | 0.50 | 0.17 | 0.03 | 0.09 | 0.53 | 0.17 | 0.07 | 0.25 | 0.58 |
| $h_{WG}$ | 1 | 1 | 1 | 0 | — | — | — | — | 1 | 1 | 0 | 0 | 0.28 | 0 | — | — | 1 |
| $w_{JAP}$ | 0.42 | 0.13 | 0.07 | 0.16 | 0.37 | 0.18 | 0.05 | 0.10 | 0.42 | 0.75 | 0.90 | 0.81 | 0.40 | 0.07 | 0.03 | 0.14 | 0.42 |
| $h_{JAP}$ | 1 | 1 | 1 | 0 | 1 | 0.79 | 0 | 0 | — | — | — | — | 1 | 0 | 0 | 0 | 1 |
| $w_{UK}$ | 0.01 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.75 | 0.90 | 0.51 | 0.01 |
| $h_{UK}$ | 1 | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — | 1 |
| Hedge Ratio[b] | 1 | 1 | 1 | 0 | 0.71 | 0.56 | 0 | 0 | 0.87 | 0.68 | 0 | 0 | 0.93 | 0.19 | 0 | 0 | 1 |
| Mean Return | 0.71 | 0.85 | 0.88 | 0.85 | 0.73 | 0.71 | 0.69 | 0.70 | 0.72 | 0.75 | 0.76 | 0.76 | 0.72 | 1.00 | 1.06 | 0.93 | 0.71 |
| Risk[c] | 1.33 | 2.46 | 2.80 | 2.75 | 1.28 | 1.31 | 1.37 | 1.34 | 1.29 | 1.38 | 1.46 | 1.43 | 1.31 | 2.25 | 2.58 | 2.03 | 1.33 |

a. The hedge ratio for each country is the proportion of the weight assigned to that country that is invested in hedged assets. It is not defined when the weight assigned to the country is zero.
b. The overall hedge ratio is the ratio of the sum of the weights of hedged foreign assets to the sum of the weights of total foreign assets.
c. Standard deviation of monthly returns in percentage points.

the moments of returns necessary to reach the decisions described above. Using this approach, the case for automatic currency hedging has been shown to be limited. However, it may be argued that the strategies described here are more difficult to implement than those advocated in the literature, because they require estimation of more variance and covariance parameters—specifically, those involving currency movements.

If the covariance between domestic asset returns and currency movements varies significantly over time, for example, estimates based on prior data may not be appropriate for making decisions. Indeed, the losses from "estimation risk" may more than outweigh the gains from making optimal decisions based on the variance and covariance estimates that turned out to be correct *ex post*. On the other hand, controlling the size of samples in order to incorporate information about structural changes may produce more precise estimates than those used above and permit superior performance.

To address this objection, we repeated the optimization exercises of Table II on an out-of-sample basis. The investor was assumed to rebalance every six months, using 24 months of data to estimate the variances and covariances necessary to calculate the optimal rule under each strategy. These optimal values of $w_D$ and h constitute the investor's strategy for the next six months, after which the procedure is repeated. In general, the results were similar to those in Table II, both in the standard deviations of portfolio returns and in the orderings of different strategies.[17] Once again, constraining $w_D$ to be 0.9 for all time periods has a more detrimental effect on overall portfolio performance than does restricting the hedge ratio. These results demonstrate that it is *possible to use estimates based on past data* to formulate strategic hedging strategies that leave the investor exposed to currency fluctuations yet produce superior or at least competitive performances relative to strategies that require no estimation of parameters [(c) and (e) in Table II] or have no currency exposure [(b) and (c)].

## Flexible-Weight Portfolios

In general, further reductions in risk will be feasible if the investor is permitted to choose the weights assigned to individual foreign countries' securities.[18] It is not possible to predict, *a priori*, what the effect on hedge ratios will be, because many more covariance and variance terms are involved in the optimal solutions.

Table III presents the results for portfolios diversified over the assets of the four countries. The investor is assumed to minimize portfolio variance by choosing among the domestic asset and the hedged and unhedged versions of the three foreign assets, making seven assets in all.[19] The hedge ratio for each foreign asset is given by Equation (2), and an overall hedge ratio can be calculated as the proportion of hedged foreign assets in total foreign assets.

As long as forward premiums are regarded as nonstochastic, the optimal portfolio under complete hedging is the same for all countries in terms of its volatility; it is described in the last column of Table III. This portfolio is also the optimal one for U.S investors. It is thus optimal for U.S. investors to hedge in the multicurrency setting as well. However, as in Table II, this optimal portfolio has no investment in U.S. assets, hence is unlikely to be acceptable.

When the proportion of U.S. assets is raised to 0.9, the hedge ratio is still 1, but the risk reduction from diversification beyond the fixed-weight case of Table II is not great (2.80%, versus 2.83%). Moreover, the unhedged portfolio has lower risk than the 90% U.S. portfolio. This illustrates the point made above—that manipulation of the weight of the domestic asset can be more useful in terms of reducing volatility than selection of the optimal hedge ratio.

As far as currency hedging is concerned, the results for the other three countries repeat the overall message of Tables I and II. Investors in these countries would not have minimized risk by hedging completely. When devoting 75% or 90% of their wealth to their domestic asset, they should have remained largely unhedged. In all cases, significant gains over fixed-weight portfolios can be reaped only if the weight of the domestic asset is not constrained.

The results in Table III demonstrate that the use of a fixed-weight index for foreign assets (as in Table II) limits considerably the possibilities for reducing overall risk. When freed of these constraints, investors in all countries avoid dollar and sterling bonds in favor of Japanese and German bonds. For example, U.S. investors decrease their holdings of U.K. bonds from the 22% given by the market-capitalization portfolio [Table II, column (a)] to 1%. Similarly, in the fixed-weight case reported in Table II, German investors hold 82% of their optimal portfolio in domestic bonds, apparently because this is the only way to avoid the high concentrations of U.S. and U.K. bonds in the non-DM fixed-weight index. When free to choose among all four classes, they substitute Japanese for U.S. and U.K. bonds, and the very high concentrations of DM bonds becomes unnecessary. Once again, it appears that the decision to allocate assets across different countries is of considerable importance for overall risk.

## Organization of International Portfolio Management

Plan sponsors typically divide the tasks of managing their portfolios among a number of specialists with expertise in particular fields. These specialists operate independently of each other. A common practice is to judge each manager against a benchmark index or normal portfolio that he can replicate. While this approach makes it feasible to diversify risk across managers, it is worth noting that it has two particular drawbacks that are highlighted by the model and results presented here.

First, our analysis cautions against a complete division of labor between foreign and domestic bond managers. We have shown that the efficacy of strategies such as hedging international currency risk depends on the proportion of the portfolio assigned to the domestic asset. The risk contours in Figure B suggest that, when the portfolio is concentrated in domestic assets, the overall risk reduction brought about by judicious choice of a hedge ratio may not be significant. Moreover, the optimal hedge ratio for the portfolio as a whole will change as the weight of the domestic asset changes. This is evident from a comparison of Equations (5) and (7). Equation (5) shows the optimal hedge ratio for the portfolio as a whole, while Equation (7) shows the optimal hedge ratio for foreign assets treated in isolation. As long as $\sigma_{DC}$ does not equal zero, these two hedge ratios will differ, and their (absolute) difference will be greater, the larger $w_D$, the share of the domestic asset.[20] Because the optimal hedge ratio for the portfolio as a whole depends on $w_D$, it follows that managers should take each other's decisions into account.

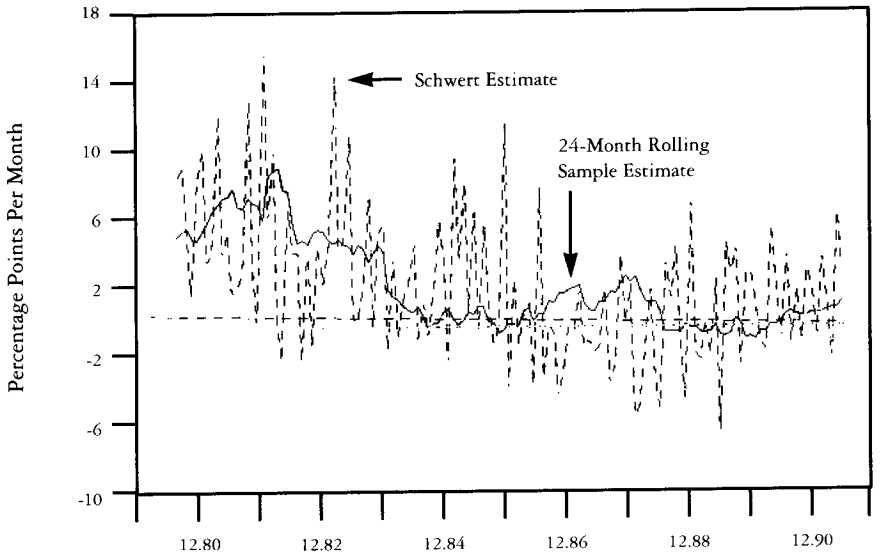
A second problem concerns the use of a hedged or unhedged international benchmark for judging the performance of the international manager. In practice, because their performance is assessed frequently and the costs of underperforming a benchmark are higher than the gains from outperforming, many bond managers tend to keep the expected risk of their portfolios reasonably close to the expected risk of their benchmark. They deviate significantly, in terms of residual risk and return, only when they think it is possible to produce sufficient excess returns to compensate for the added risk. If the manager is measured against a hedged benchmark, he will accept currency exposure only as *a tactical* move.[21] However, Equation (5) shows that currency exposure can have *strategic* value when the covariances between asset and currency returns are negative. Judging a manager against a fully hedged benchmark thus limits the diversification benefits available from strategic currency exposure.[22]

## Conclusions

We have demonstrated that currency hedging has a strategic role to play in international asset allocation. Investors should not be dissuaded from remaining unhedged by the high volatility of currency exposure; the costs of volatility can be more than offset by the benefits of diversification. The interaction of currency and asset returns determines whether currency has a strategic role to play.

The theoretical analysis and results presented so far confirm and explain the superiority of a strategy of complete hedging for U.S. investors—a strategy that has been widely discussed in the literature on currency hedging. But they also caution against putting this strategy on "automatic pilot" for the indefinite future. The configuration of bond return–currency covariances that obtained during the 1980s, making the strategy successful, may not persist. The simplest international financial models predict that ap-

**Figure C** U.S. Bonds and the Dollar: Covariance of Returns




preciation of the domestic currency will go hand-in-hand with a capital inflow and a concomitant rise in asset prices. As a consequence, $\sigma_{DC}$ is expected to be negative and $\sigma_{LC}$ positive. The fact that, during the 1980s, both covariances have been positive from the point of view of U.S. investors, but negative for Japanese, German and U.K. investors may reflect the United States' special place in world financial markets during this period. This may not continue as ease of access to the asset markets of other countries increases and as European integration becomes a reality.

This view is borne out by estimates of $\sigma_{DC}$ that attempt to capture its movements over time, two of which are shown in Figure C. The solid line shows the path of estimates of $\sigma_{DC}$ for 24-month rolling samples. The dashed lined shows the results of a "nonparametric" covariance estimator developed by William Schwert.[23] This estimate uses the fact that $D_tC_t$—the product of the currency and domestic bond returns in period t—is, on average, $\sigma_{DC}$ (after correcting for mean returns). Consequently, a regression of $D_tC_t$ on other variables (such as time trends and lags of $D_tC_t$) can capture systematic movements over time in $\sigma_{DC}$. The Schwert estimate of $\sigma_{DC}$ is much more volatile than the rolling sample estimate, but both reveal similar broad patterns: $\sigma_{DC}$ was much higher in the first four years of the 1980s than subsequently and frequently assumed negative values in the latter part of the decade.

It is evident that the covariance between U.S. bond returns and foreign currency returns has changed over time. Its average over the last decade is likely to provide a misleading guide to its value in the future. While this parameter has moved toward a value more in line with basic economic theory, there is no hard and fast reason why it or other parameters should not change in the future.

This points to an aspect of hedging (and portfolio allocation in general) that has not been addressed here—the sensitivity of optimal hedging decisions to the approach used to estimate variances and covariances. Using long historical samples of (say) monthly returns will result in reliable estimates if the underlying variances and covariances do not shift over time; the estimates will be biased if such shifts occur. In contrast, using shorter samples results in less reliable estimates, but less chance of bias. Developing estimates that strike a balance between bias and reliability is an area for future research that has important implications for practical implementation of asset allocation models.

## Footnotes

1. *See, for example, L. R. Thomas, "The Performance of Currency Hedged Foreign Bonds" (Goldman Sachs, February 1989) and A. Perold and E. Schulman, "The Free Lunch in Currency Hedging: Implications for Investment Policy,"* Financial Analysts Journal, *May/June 1988. Articles that bear similarities to certain points made below are P. Jorion, "International Asset Allocation,"* Investment Management Review, *1987, pp. 44–49; M. Rosenberg, "How Strong is the Case for Currency-Hedged Foreign Bond Funds?" (Merrill Lynch, August 28, 1988); and J. D. Glen, "Exchange Rate Uncertainty, Forward Contracts and the Performance of Global Equity Portfolios" (Working paper #37-89 Rodney L. White Center for Financial Research); M. Adler, M. R. Granito and A. Lee, "Should International Portfolios Be Permanently Hedged?" in L. R. Thomas, III, ed.,* The Currency-Hedging Debate *(London: IFR Publishing, 1990) and A. Lee, "Hedging Decisions Within the Overall Asset Allocation Strategy," in* Managing Currency Risk *(Charlottesville, VA: The Institute of Chartered Financial Analysts, 1989).*
2. *The examples in Table I and in the rest of the article are confined to bond portfolios. The same quantitative results emerge when equities are included. However, as the diversity of assets increases, it becomes more difficult to trace the effects of the variables that drive the optimal hedging decision. In the interests of being able to link clearly our theoretical analysis and the empirical examples we present, we limit the portfolios considered to domestic and foreign-currency government bonds.*

3. *See, for example, Perold and Schulman, "The Free Lunch,"* op. cit.
4. *Equation (2) shows that a negative hedge ratio can arise if, for example, the investor is short hedged foreign assets ($w_{L+F} < 0$) but long foreign assets ($w_{L+C} + w_{L+F} > 0$). A hedge ratio in excess of one arises, for example, from a longer position in hedged foreign assets than in foreign assets overall ($w_{L+F} > 0$, $w_{L+C} < 0$, and $w_{L+C} + w_{L+F} > 0$).*
5. *To conform with the literature on currency hedging, we do not treat the forward premium as a random variable. This is the correct thing to do when considering a one-period hedge with the same termination date as the forward contract. However, for "rolling hedge" strategies it is not appropriate, because forward contracts after the first will be struck at values of the forward premium that are uncertain at the time the assets are purchased. In practice, the contribution to volatility of the forward premium is insignificant in comparison to that of exchange rate and asset returns.*
6. *A full description of the optimization problem and its solution is given in our original working paper, "Currency-Hedged International Fixed-Income Investment: A Reexamination" (J. P. Morgan Securities Inc., April 1990).*
7. *To be precise, $\sigma_{DC}^{US} = -\sigma_{LC}^{UK}$ and $\sigma_{LC}^{US} = -\sigma_{DC}^{UK}$, where the superscripts indicate the currency in which returns are valued.*
8. *See Footnote 1 for references.*
9. *This formula is to be found in Perold and Schulman, "The Free Lunch,"* op. cit.
10. *See B. Solnik, "Why Not Diversify Internationally Rather Than Domestically?,"* Financial Analysts Journal, *July/August 1979.*
11. *For example, Perold and Schulman, "The Free Lunch,"* op. cit., *offer results for portfolios containing 90% and 75% U.S. assets.*
12. *The results are based on monthly returns and foreign exchange data spanning the period June 1980 through April 1989. Data prior to 1986 are drawn from the Salomon Brothers World Bond Index, while J. P. Morgan Government Bond Index data are used for the later period. The longest interval over which internally consistent data were available was June 1978 through April 1989. The sample period for Table II starts in June 1980 to retain comparability with the "rolling sample" results of Figure C, where 24 months of initial observations are required to produce the first forecasts of variances and covariances. The data used were total return indexes for the United States, Germany, Japan and the United Kingdom. Results are presented for the full sample of data because average returns have fluctuated considerably during subperiods, and we want to minimize the possibility of the results being infected by the idiosyncracies of subsamples. Results for subperiods of the 1978–89 sample lead to the same qualitative conclusions. The weights used are the averages of the world government bond capitalization proportions of the United States, Germany, Japan and the United Kingdom over the period August 1987 through June 1989. Together, these countries accounted for 86% of world government bond market capitalization. As proportions of the total amount of securities outstanding of the four countries, their shares were 52.5%, 7.5%, 29.5%, and 10.5%, respectively. So, for example, U.K. securities would make up 14.9% of the composite foreign security available to Japanese investors [0.149 = 0.105/(1 − 0.295)].*
13. *See Perold and Schulman, "The Free Lunch,"* op. cit.
14. *Glen ("Exchange Rate Uncertainty,"* op. cit.*) demonstrates empirically that for both U.S. and U.K. investors, hedging an equity index with forward contracts would have incurred a significant risk premium. Therefore, hedging exchange rate risk is a type of insurance that is priced by the market in a manner similar to other types of risks. Consequently, hedged returns do not have the same expected return as unhedged returns; in the long run, the free lunch is consumed by the risk premium. In fact, Glen finds that, for U.K. investors, the risk premium for hedging was not justified* ex post *by the amount of risk reduction obtained.*
15. *Along a contour, the standard deviation of returns is constant.*
16. *This argument does not consider the extra transactions and administrative costs involved in hedging, which will make the expected returns of the hedging strategies less attractive.*
17. *For complete results of these simulations, the reader is referred to our original working paper (see Footnote 6).*
18. *In the studies referred to in Footnote 1, the weights in the foreign asset basket are kept fixed, although some studies also include stocks as well as bonds.*
19. *As in the two-country case, returns can be expressed equivalently in terms of currency and hedged assets, or "currency surprise" (C − F) and unhedged assets, i.e.:*

$$R = w_D D + \Sigma w_i L_i + \Sigma w_i (1 - h_i) C_i + \Sigma w_i h_i F_i$$

*or*

$$R = w_D D + \Sigma w_i (L_i + C_i) - \Sigma w_i h_i (C_i - F_i),$$

*where the i subscript indexes the foreign countries.*
20. *Further differences occur in the more realistic case where the international manager allocates across a group of foreign assets. If the international manager is isolated from the domestic manager, he or she will also not take into account the covariances between domestic and foreign asset returns in choosing the mix of foreign assets and currency exposure.*
21. *Similarly, an unhedged benchmark will cause hedging to become a purely tactical move.*
22. *The same analysis is valid if currencies are managed by an overlay manager, who is judged according to a benchmark.*
23. *W. Schwert, "Why Does Stock Market Volatility Change Over Time?"* Journal of Finance, *December 1989. Schwert examines the variance of stock returns, but his approach extends readily to covariances between returns. The results produced are similar to those of an alternative approach to time-varying measures of volatility and correlation, the "auto-regressive conditionally heteroscedastic" (ARCH) models introduced by R. F. Engle, "Auto-Regressive Conditional Heteroscedasticity and Estimates of the Variance of U.K. Inflation,"* Econometrica, *June 1982.*