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The Efficiency Gains of Long–Short Investing

Richard C. Grinold and Ronald N. Kahn

Long–short strategies have generated controversy and institutional interest for more than 10 years. We analyzed the efficiency gains of long–short investing, where we defined efficiency as the information ratio of the implemented strategy (the optimal portfolio) relative to the intrinsic information ratio of the alphas. The efficiency advantage of long–short investing arises from the loosening of the (surprisingly important) long-only constraint. Long–short and long-only managers need to understand the impact of this significant constraint. Long–short implementations offer the most improvement over long-only implementations when the universe of assets is large, asset volatility is low, and the strategy has high active risk. The long-only constraint induces biases (particularly toward small stocks), limits the manager's ability to act on upside information by not allowing short positions that could finance long positions, and reduces the efficiency of traditional (high-risk) long-only strategies relative to enhanced index (low-risk) long-only strategies.

Institutions in the United States have used long–short (or market-neutral) strategies for investing since at least the late 1980s. These strategies have generated controversy but, over time, have gained increasing acceptance as a worthwhile innovation. According to the *Pensions & Investments* May 18, 1998, issue, 30 investment management firms were then offering market-neutral strategies, up from the 21 firms listed one year earlier.[1] The popularity of long–short strategies arises from the distinct advantage they offer over long-only strategies: the potential for more efficient use of information, particularly (but not exclusively) downside information.

Long–short investing refers to a method for implementing active management ideas. Any strategy can be implemented as long–short or long only. Long–short investing is general. It does not refer to a particular source of information. Every long-only portfolio has an associated active portfolio with zero net investment and often zero beta. Therefore, every long-only portfolio has an associated long–short portfolio. But the long-only constraint has a significant impact on this associated long–short portfolio. Long–short strategies provide for more opportunities—particularly in the size of short positions in smaller stocks (assuming a capitalization-weighted benchmark is being used).

We present analysis of several important aspects of long–short strategies and, by implication, some important and poorly understood aspects of long-only strategies. The analysis is thus important to all managers—not solely those offering long–short strategies.

The long–short strategies we studied were defined specifically as equity market-neutral strategies. The strategies have betas of zero and equal long and short positions. Some databases group these strategies in the more general category of "hedge fund." The hedge fund category, however, can include almost any strategy that allows short positions. We focus much more specifically on risk-controlled equity strategies with zero beta and zero net investment.

Framework and Notation

Before we can review past research and controversy, we need a basic framework for analyzing active strategies.[2] We use the framework of Grinold and Kahn (2000). We define asset residual returns, θ_n, as

$$\theta_n = r_n - \beta_n r_B, \tag{1}$$

where

r_n = the asset's excess return (return above the risk-free rate)

β_n = the asset's beta with respect to the benchmark

r_B = the benchmark excess return

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The residual return is the part of the asset's return unexplained by the benchmark return. The asset's expected residual return is

$$\alpha_n = E(\theta_n). \tag{2}$$

Its residual risk is

$$\omega_n = \text{std}(\theta_n), \tag{3}$$

where $\text{std}(\theta_n)$ is the standard deviation of the asset's residual return.

Grinold (1994) showed that α_n has the form

$$\alpha_n = \omega_n IC z_n, \tag{4}$$

where IC is the information coefficient (the correlation of the forecasted α_n with the realization θ_n) and z_n is a dimensionless score with mean of 0 and standard deviation of 1 over time.

We build portfolios $\mathbf{h}_P$ to maximize utility:

$$\begin{aligned} U &= \alpha_P - \lambda_R \omega_P^2 \\ &= (\mathbf{h}_P - \mathbf{h}_B)^T \alpha - \lambda_R (\mathbf{h}_P - \mathbf{h}_B)^T \mathbf{VR} (\mathbf{h}_P - \mathbf{h}_B), \end{aligned} \tag{5}$$

where

λ_R = the investor's aversion to residual risk

$\mathbf{h}_B$ = the benchmark portfolio

$\mathbf{VR}$ = the covariance matrix of the residual returns

The optimal positions are

$$(\mathbf{h}_P - \mathbf{h}_B) = \left(\frac{1}{2\lambda_R}\right) \mathbf{VR}^{-1} \alpha. \tag{6}$$

A key statistic for measuring active strategies is the information ratio:

$$IR = \frac{\alpha_P}{\omega_P}. \tag{7}$$

This statistic is important because the maximum possible utility depends on the information ratio of the strategy:

$$U_{max} = \frac{(IR)^2}{4\lambda_R}. \tag{8}$$

Grinold (1989) showed that the information ratio depends on the strategy's information coefficient and its breadth:

$$IR = IC\sqrt{BR}, \tag{9}$$

where the breadth, BR, measures the number of independent bets per year. Basically, Equation 9 states that strategies earn high information ratios by applying their forecasting edge many times over.

Previous Research and Controversy

Proponents of long–short investing offer several arguments in its favor. What was probably the original argument claims that the complete dominance of long-only investing has preserved short-side inefficiencies; hence, the short side may offer higher alphas than the long side.

The second argument depends on diversification. A long–short implementation includes, effectively, a long portfolio and a short portfolio. If each of these portfolios separately has an information ratio of IR and the two portfolios are uncorrelated, then the combined strategy, simply through diversification, should exhibit an information ratio of $IR\sqrt{2}$. The problem with this argument is that it applies just as well to the active portfolio associated with any long-only portfolio. So, this argument cannot be the justification for long–short investing.

The third, and most important, argument for long–short investing is the enhanced efficiency that results from the loosening of the long-only constraint. The critical issue for long–short investing is not diversification but, rather, constraints.

These arguments in favor of long–short investing have generated considerable controversy. The first argument, short-side inefficiency, is difficult to prove and brings up the issue of the high implementation costs associated with shorting stocks. The second argument, based on diversification, is misleading, if not simply incorrect. Not surprisingly, it has attracted considerable attack. The third argument is the critical issue, and it has implications for both long–short and long-only investors.

The first criticism of long–short investing was by Michaud (1993). He criticized the diversification argument as overstated because the long and short portfolios are not uncorrelated. He pointed out that long-only investors also exploit short-side information. He questioned the cost of shorting. He also questioned whether risk-control technology was up to the task of building market-neutral portfolios.

From this opening, the debate moved to Arnott and Leinweber (1994), Michaud (1994), Jacobs and Levy (1995), and participants at a conference of the Institute for Quantitative Research in Finance (the Q-Group) on Long–Short Strategies in Equity and Fixed Income.[3] These articles and reports provide a point/counterpoint debate on several issues, especially the costs and risks of long–short strategies. Jacobs and Levy (1995) provided the first criticism of the diversification argument. Jacobs and Levy (1996), Freeman (1997), Jacobs, Levy, and Starer (1998, 1999), and Levin (1998) published further detailed analyses of aspects of long–short investing.

Other recent work has examined how long–short strategies fit into overall pension plans (Brush 1997) and the performance of long–short managers (Kahn and Rudd 1998).

Surprising Impact of the Long-Only Constraint

We set out to investigate the costs imposed by the most widespread institutional constraint—the restriction on short sales—or equivalently, the benefits of easing that constraint. How does the long-only constraint restrict the investor's opportunity set? To answer that question, we ignored transaction costs and all other constraints and focused on how this constraint affects the active frontier—the trade-off between exceptional return α and risk ω.

A very simple model will provide some insight before we tackle the more realistic case. This model consists of N assets and an equal-weighted benchmark. In addition, all assets have identical residual risk ω and uncorrelated residual returns. This model opens a small window on the workings of the long-only constraint.

With these assumptions, Equation 5 dictates that the active position for asset n is

$$h_P(n) - h_B(n) = \frac{\alpha_n}{2(\lambda_R)\omega^2}. \tag{10}$$

The overall residual (and active) risk, ω_P, is

$$\omega_P^2 = \frac{1}{4(\lambda_R^2)\omega^2} \sum_{n=1}^{N} \alpha_n^2. \tag{11}$$

From Equations 4 and 9, the active positions and portfolio active risk become

$$h_P(n) - h_B(n) = \frac{IRz_n}{2(\lambda_R)\omega\sqrt{N}} \tag{12}$$

and

$$\begin{aligned} \omega_P &= \frac{IR}{2\lambda_R}\sqrt{\left(\frac{1}{N}\right)\sum_{n=1}^{N} z_n^2} \\ &\approx \frac{IR}{2\lambda_R}, \end{aligned} \tag{13}$$

where the number of stocks, N, is used for the strategy breadth because this illustration assumes uncorrelated residual returns (and annual portfolio construction).

Equations 12 and 13 can be used to link the active position with the desired level of active risk, ω_P, the stock's residual risk, ω, and the square root of the number of assets to produce

$$h_P(n) - h_B(n) = \frac{\omega_P z_n}{\sqrt{N}\omega}. \tag{14}$$

The limitation on short sales becomes binding when the active position plus the benchmark hold-surprisingly, Figure 2 shows that it also handles the ing is negative. For an equal-weighted benchmark, this moment occurs when

$$z_n \leq -\frac{\omega}{\sqrt{N}\omega_P}. \tag{15}$$

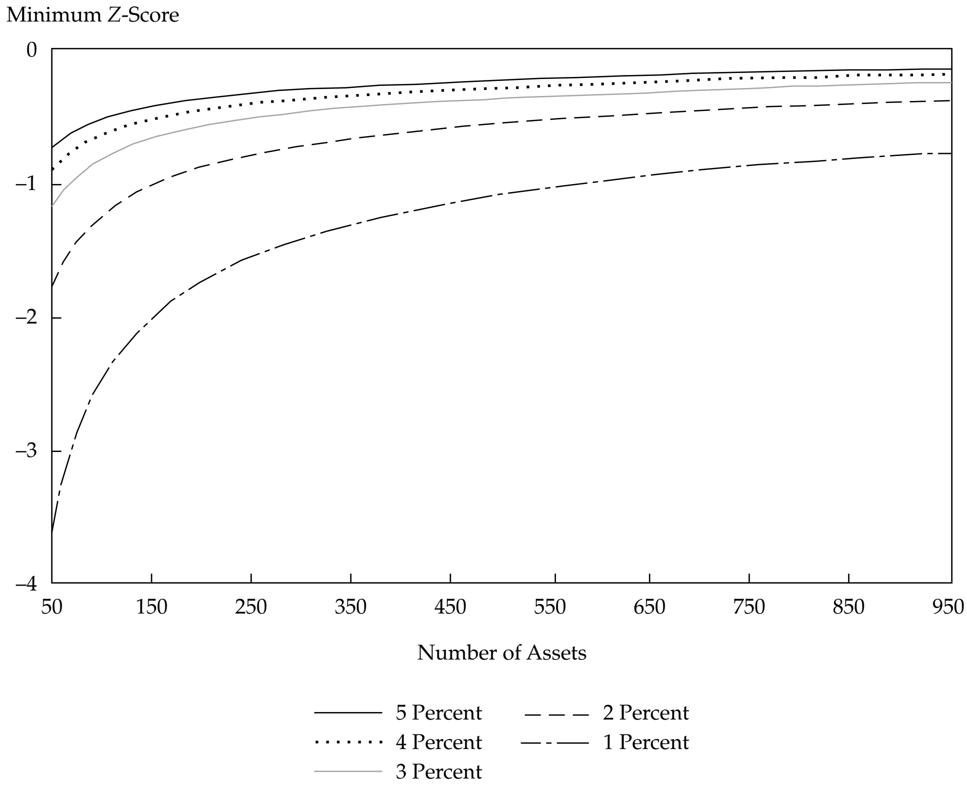
Figure 1 shows this information boundary as a function of the number of stocks for various levels of active risk. Information is wasted if the Z-score falls below the minimum level. The higher the minimum level, the more information an investor is likely to leave on the table. For example, suppose we have a strategy with 500 stocks, active risk of 5 percent, and typical residual risk of 25 percent. Whenever the Z-score falls below –0.22, we will waste information. Assuming normally distributed scores, this lost opportunity will occur 41 percent of the time.

This rough analysis indicates that an aggressive strategy involving a large number of low-volatility assets should reap the largest benefits from easing the restriction on short sales. The more aggressive the strategy, the more likely it is to hit bounds. The lower the asset volatility, the larger the active positions the investor would desire. The more assets in the benchmark, the lower the average benchmark holding and the more likely that the investor will hit the boundary.

In a long-only optimization, the restriction against short selling has both a direct and an indirect effect. The direct effect precludes exploiting the most negative alphas. The indirect effect grows out of the desire to stay fully invested. In this case, the investor must finance positive active positions with negative active positions. Hence, a scarcity of negative active positions can affect the long side: Overweights require underweights. Put another way, without the long-only constraint, an investor could take larger underweights relative to the benchmark. But because underweights and overweights balance, without the long-only constraint, the investor will take larger overweights as well.

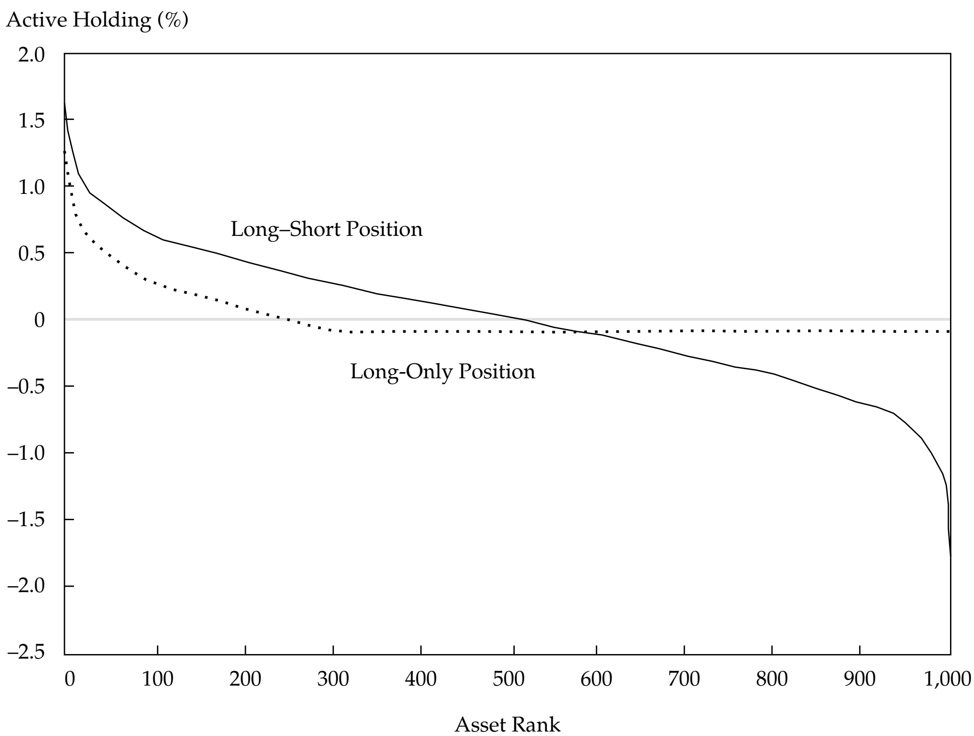
A simple case will illustrate this "knock-on" effect. Suppose we start with an equal-weighted benchmark and generate random alphas for each of the 1,000 assets in it. Then, we construct optimal portfolios in the long-only and long–short cases. **Figure 2**displays the active positions in the long–short and long-only cases with assets ordered by their alphas from highest to lowest. In the long–short case, a rough symmetry exists between the positive and negative active positions. The long-only case essentially assigns all assets after the first 300 the same negative alpha. We expected that the long-only portfolio would handle negative alphas less efficiently than the long–short portfolio, but largest positive alphas less efficiently.

Figure 1. Sensitivity of Minimum *Z*-Score to Active Risk



Note: Curve labels indicate level of active risk.

Figure 2. Long-Only and Long–Short Active Positions with Assets Ordered by Highest to Lowest Alphas



Importance of the Benchmark Distribution

The impact of the long-only constraint depends on the weighting of the benchmark and can be more dramatic than shown in the previous section if the benchmark is not equally weighted. To calculate the impact in realistic environments, we need a model of the capitalization distribution. And this model requires a short detour.

We use Lorenz curves to measure distributions of capitalization. By definition, to construct the curves, we

- calculate benchmark weight as a fraction of total capitalization,
- order the assets from highest to lowest weight, and
- calculate the cumulative weight of the first n assets as n moved from largest to smallest.

Such a Lorenz curve plots the series of cumulative weights. It starts at 0 and increases in a concave fashion until it reaches 1. If all assets have the same capitalization, it will be a straight line. **Figure 3** shows Lorenz curves for the Russell 1000 Index, for a model portfolio designed (as described later) to resemble the Russell 1000, and for an equal-weighted portfolio.

One summary statistic for the Lorenz curve is the "Gini coefficient," which is twice the area under the curve less the area under the equal-weighted curve. Gini coefficients must range between 0 (for equal-weighted benchmarks) and 1 (for single-asset benchmarks). So, Lorenz curves can be drawn for benchmarks with any arbitrary distribution of capitalization and any distribution can be summarized with a Gini coefficient.

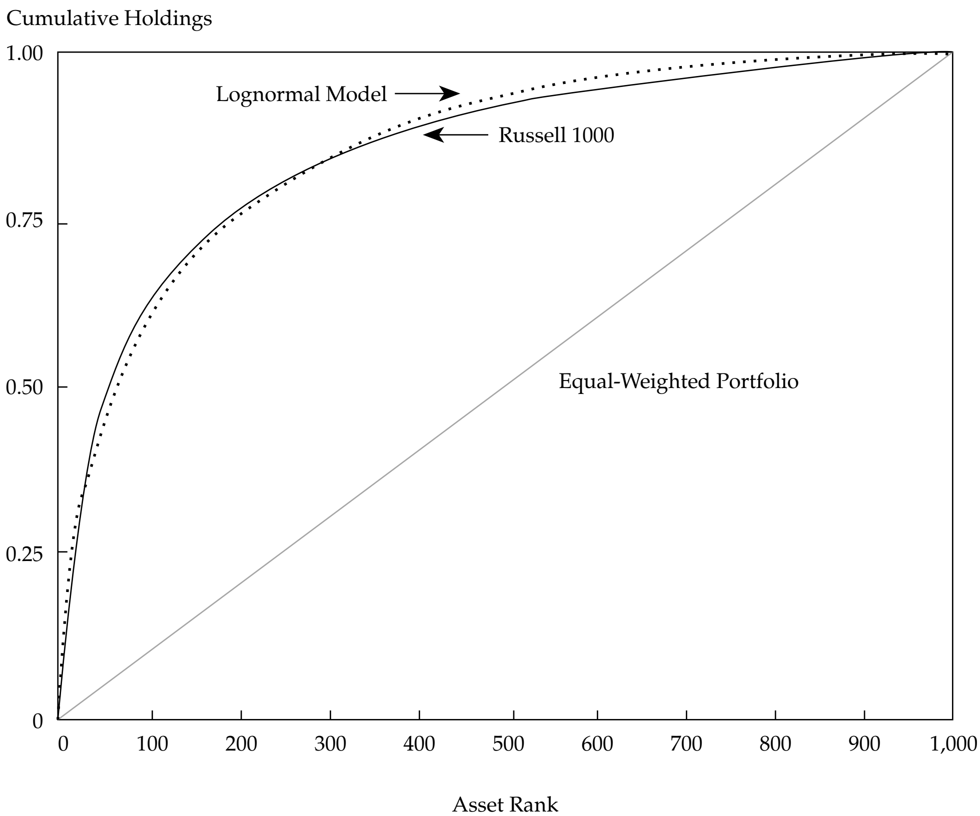
Further progress requires a specific form for the distribution of capitalization.

Capitalization Model. Assume that the distribution of capitalization is lognormal. A one-parameter model that will produce such a distribution is as follows: First, order the N assets by capitalization from largest ($n = 1$) to smallest ($n = N$). Define

$$p_n \equiv 1 - \left(\frac{1}{2N} + \frac{n-1}{N}\right). \tag{16}$$

These values resemble probabilities: They start close to 1 and move toward 0 as capitalization decreases.

Figure 3. Lorenz Curves



Next, calculate a normally distributed quantity y_n such that the probability of observing y_n is p_n:

$$p_n = \Phi(y_n), \tag{17}$$

where $\Phi(\bullet)$ is the cumulative normal distribution.

So far, linear ranks have been converted to normally distributed quantities y_n. The next step is to generate capitalizations:

$$CAP_n = \exp(cy_n). \tag{18}$$

The constant c can be generated to match the desired Gini coefficient or to match the Lorenz curve of the market.[4]

We used this model to match the Russell 1000 in Figure 3. **Table 1** contains similar results for several markets covered by Morgan Stanley Capital International (MSCI) as of September 1998. It also includes equal-weighted and cap-weighted examples from the hypothetical land of "Freedonia,"[5] whose market consists of 1 stock comprising 99 percent of total capitalization and 100 other stocks each with 0.01 percent of capitalization. To analyze the loss in efficiency as a result of the long-only constraint, we used the value 1.55 for the constant c because c ranges from 1.30 to 1.60 in a large number of countries, but the MSCI indexes necessarily trim out a great many of the smaller stocks in a market.

Table 1. Modeling Capitalization Distributions, September 1998

Index	Assets	Gini	Constant c
United States			
Russell 1000	1,000	0.71	1.55
MSCI	381	0.66	1.38
MSCI United Kingdom	135	0.63	1.30
MSCI Japan	308	0.65	1.35
MSCI Netherlands	23	0.64	1.38
Freedonia			
Equal weighting	101	0.00	0.00
Cap weighting	101	0.98	11.15

Armed with this one-parameter model of the distribution of capitalization, we could derive our rough estimates of the potential benefits of long–short investing.

Estimate of Long–Short Benefits. We could not derive any analytical expression for the loss in efficiency resulting from the long-only constraint because the problem contains an inequality constraint. But we could use a computer simulation to obtain a rough estimate of the magnitude of the impact.

As our previous simple analysis showed, the important variables in the simulation are the number of assets and the desired level of active risk. We considered 50, 100, 250, 500, and 1,000 assets, with desired risk levels from 1 percent to 8 percent by 1 percent increments and from those increments to 20 percent by 2 percent increments.[6]

For each of the five levels of assets and the 14 desired risk levels, we solved 900 randomly generated long-only optimizations. For each case, we assumed uncorrelated residual returns, identical residual risks of 25 percent, a full investment constraint, and an information ratio of 1.5. We ignored transaction costs and all other constraints. We generated alphas using

$$\alpha_n = \omega\left(\frac{IR}{\sqrt{N}}\right)z_n. \tag{19}$$

Figure 4 shows the active efficient frontier—the alpha as a function of active risk. Each observation point in Figure 4 displays the mean active return and mean active risk from a sample of 900 simulations.

The efficient frontiers in Figure 4 can be roughly estimated as[7]

$$\alpha(\omega, N) = 100IR\left\{\frac{\left[1 + \frac{\omega}{100}\right]^{1-\gamma(N)} - 1}{1-\gamma(N)}\right\}, \tag{20}$$

where

$$\gamma(N) = (53 + N)^{0.57} \tag{21}$$

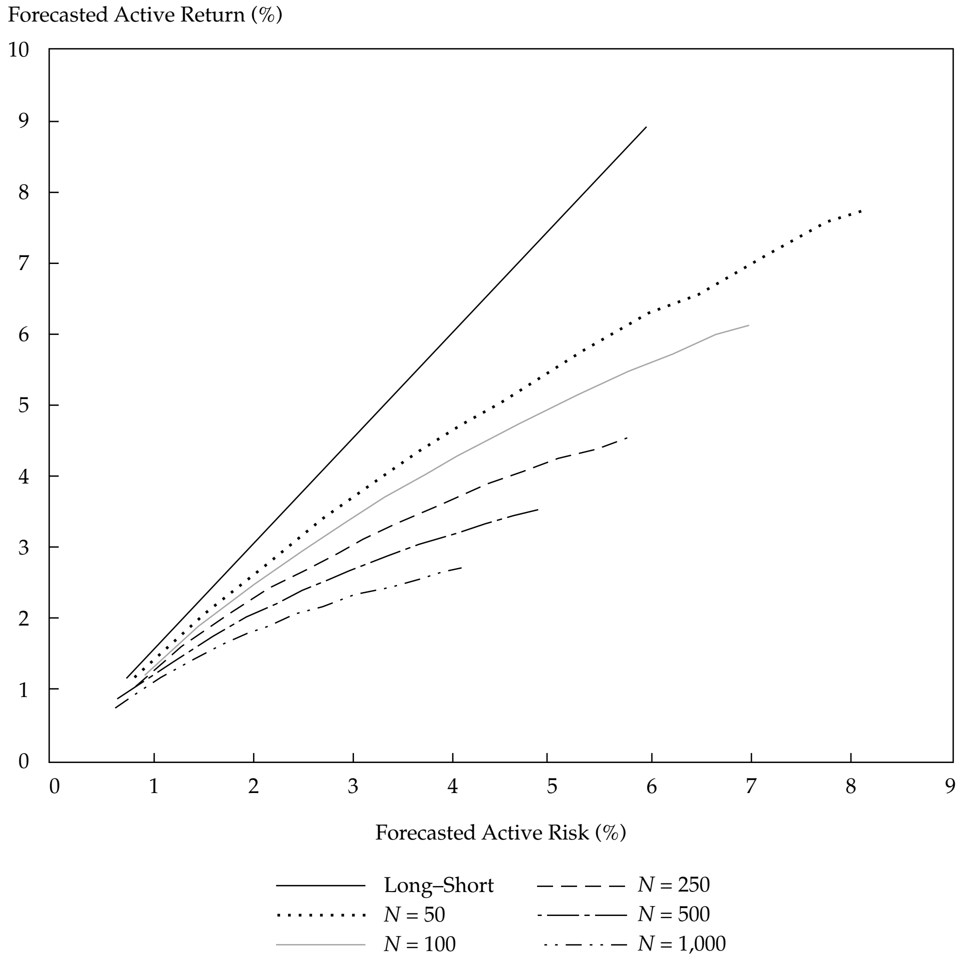
and α and ω are measured in percentages.

As anticipated, with the information ratio held constant, long-only implementations become less and less effective as the number of assets increases. Also clear is that higher desired active risk lowers efficiency. In fact, efficiency can be *defined* as the shrinkage in the information ratio (and information coefficient):

$$\begin{aligned}\text{Efficiency} &= \left\{\frac{\left[\frac{\alpha(\omega, N)}{\omega}\right]}{IR}\right\} \\ &= \left(\frac{100}{\omega}\right)\left\{\frac{\left[1 + \frac{\omega}{100}\right]^{1-\gamma(N)} - 1}{1-\gamma(N)}\right\}.\end{aligned} \tag{22}$$

Figure 5 illustrates the dependency of efficiency on risk and number of assets. For typical U.S. equity strategies—500 assets and 4.5 percent risk—the efficiency is 49 percent according to Equation 22, which agrees with Figure 5. The long-

Figure 4. The Active Efficient Frontier



only constraint has enormous impact: It cuts information ratios for typical strategies in half![8]

Equation 22 also allows quantification of the appeal of enhanced index (i.e., low-active-risk) strategies. The efficiency is 71 percent for a long-only strategy involving 500 assets with only 2 percent active risk. At this low level of risk, an investor loses only 29 percent of the original information ratio.

At high levels of active risk, long–short implementations can have a significant advantage over long-only implementations. At low levels of active risk, this advantage disappears. And given the higher implementation costs of long–short strategies (e.g., the uptick rule, costs of borrowing), at low levels of active risk, long-only implementations may offer an advantage.

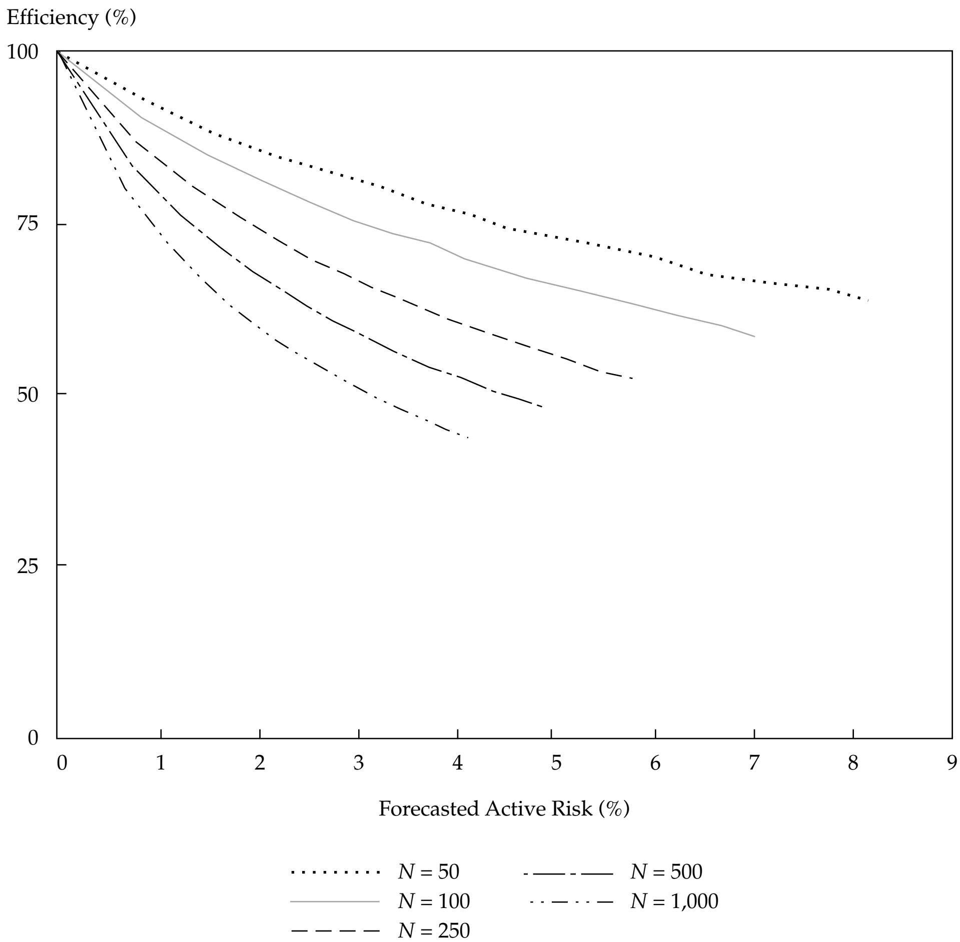
With a large number of assets and the long-only constraint, achieving high levels of active risk is difficult. From Equation 20, an empirical analog of Equation 13 can be derived (see Appendix A for details):

$$\lambda_R = \frac{IR}{2\omega\left(1+\frac{\omega}{100}\right)^{\gamma}}. \tag{23}$$

To corroborate the validity of our results on efficiency, we analyzed the sensitivities of the empirical results to the assumptions used for the efficient frontiers in Figures 4 and 5: an inherent information ratio of 1.5; a lognormal size distribution constant, c, of 1.55; and identical and uncorrelated residual risks of 25 percent. Changing the inherent information ratio did not affect our conclusions at all. As Equation 20 implies, the efficient frontier simply scales with the information ratio.

Changing the lognormal size distribution constant through the range from 1.2 to 1.6, a wider range than we observed in examining several markets, had a minor impact. Lower coefficients were found to be closer to equal weighting, so the long-only constraint was less restrictive in those cases. At 4.5 percent active risk and 500 assets, however,

Figure 5. Efficiency as a Function of Risk and Number of Assets



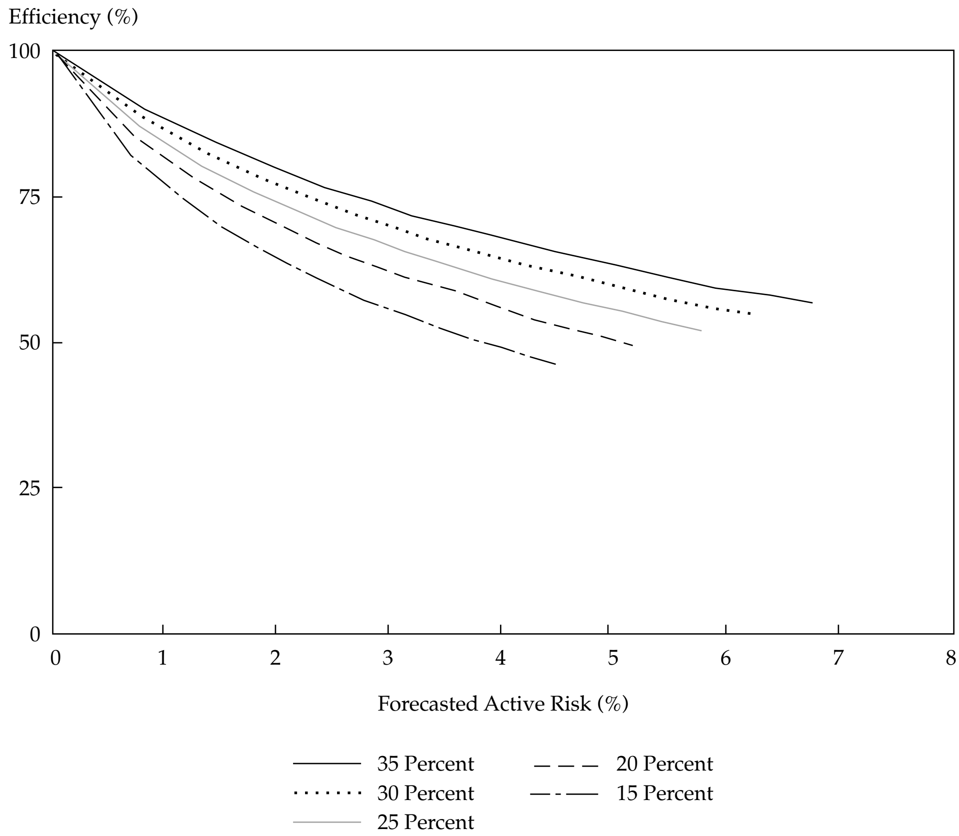
as we varied this coefficient, the efficiency ranged only from 0.49 to 0.51.

Figure 6 shows how our results changed with changing asset residual risk. Our base-case assumption of 25 percent asset residual risk is very close to the median U.S. equity residual risk, but an investor may be investing in a particular universe of assets with higher or lower average residual risk. As asset residual risk increases, the investor can achieve more risk with smaller active positions, thus making the long-only constraint less binding. At the extremely low level of 15 percent, the long-only constraint has a considerable impact. In the (more reasonable) range of 20–35 percent, the efficiency at 4.5 percent active risk and 250 assets ranges from 65 percent to 54 percent.

We also analyzed the assumption that every asset has equal residual risk. Given an average residual risk of 25 percent and assuming 500 assets, we analyzed possible correlations between size (as measured by the log of capitalization) and the log of residual risk. We expected a negative correlation because larger stocks tend to exhibit lower residual risks. Examination of large U.S. equities (the BARRA HICAP universe of roughly the largest 1,200 stocks) shows that the correlation between cap size and residual risk has varied from roughly –0.51 to –0.57 in the past 25 years. This negative correlation improves efficiency in general because it implies that smaller-cap stocks (for which the long-only constraint is most binding) are riskier than larger-cap stocks, which leads to smaller desired active positions in small-cap stocks). **Figure 7** shows the frontier as we varied that correlation from 0 to –0.6. With a correlation of 0, we found an efficiency of 49 percent at 4.5 percent active risk. With a correlation of –0.6, the situation improved to an efficiency of 0.63.

Finally, **Figure 8** displays the size bias that we anticipated for various correlations between size and residual risk. The correlation did not significantly change the result. We measured size as log of capitalization, standardized to a mean of 0 and

Figure 6. Sensitivity of Efficiency to Asset Residual Risk
(250 assets)



standard deviation of 1. So, an active size exposure of –0.3 means that the active portfolio had an average size exposure 0.3 standard deviations below the benchmark.

These size biases are significant. Figure 8 implies that a typical manager following 500 stocks and targeting 4.5 percent risk will have a size exposure of –0.65.[9] In the United States, from October 1997 through September 1998, the size factor in the BARRA U.S. equity model exhibited a return of 1.5 percent: Large stocks outperformed small stocks. This set of circumstances would have generated a 98 basis point (bp) loss simply as a result of this incidental size bet. From September 1988 through September 1998, the same size factor experienced a cumulative gain of 361 bps, generating a loss of 235 bps over that 10-year period.

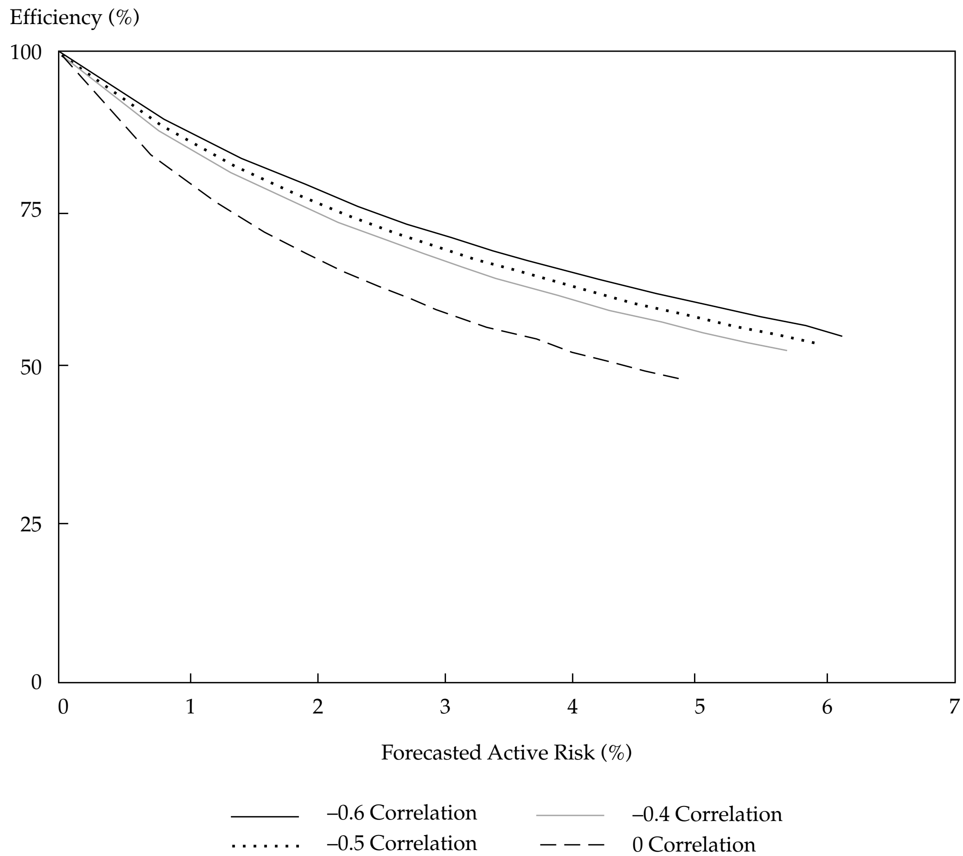
The Appeal of Long–Short Investing

Who should offer long–short strategies? Who should invest in them? Clearly, long–short strategies are a "pure" active management bet. The consensus expected return to long–short strategies is zero because the strategies have betas of zero. Put another way, the consensus investor does not invest in long–short strategies. Therefore, the most skillful active managers should offer long–short strategies: Such strategies allow them the freedom to implement their superior information most efficiently.

Long–short strategies offer no way to hide behind a benchmark. A long-only manager delivering 15 percent while the benchmark is delivering 20 percent is arguably in a better position than a long–short manager losing 5 percent. Although not an intrinsic benefit of long-only strategies, this aspect can be a practical benefit for investment managers.

Long–short strategies do offer investment managers the freedom to trade *only* on their superior information. They can build a long–short market-neutral portfolio by using only utility stocks if this is how they can add value. They have no need to buy stocks just because the stocks are members

Figure 7. Sensitivity of Efficiency to Correlations between Size and Residual Risk
(500 assets; 25 percent average residual risk)



Note: Correlations measured for log of capitalization and log of residual risk.

of the benchmark. Both the long and the short sides of the portfolio may have large active risk relative to the S&P 500 Index, just not to each other.

Long–short strategies offer the most benefit to those investors who are best able to identify skillful managers. Long–short strategies are quite appealing because of the (engineered) low correlations such strategies have with equity market benchmarks. Long–short strategies can in this way successfully compete against bonds.

Long–short investing also offers the appeal of easy alpha portability. Futures contracts can move active return from one benchmark to another. If an investor starts with a strategy managed relative to the S&P 500 and sells S&P 500 futures and buys Financial Times Stock Exchange 100 futures, the investor will transfer the alpha to the FTSE 100. In a conventional long-only strategy, this transfer requires an extra effort. It is not the natural thing to do. With a long–short strategy, the investor starts with the pure active return and must choose a benchmark. The potential for transfer is thrust upon the investor. So, long–short strategies place the notion of a portable alpha in center stage.

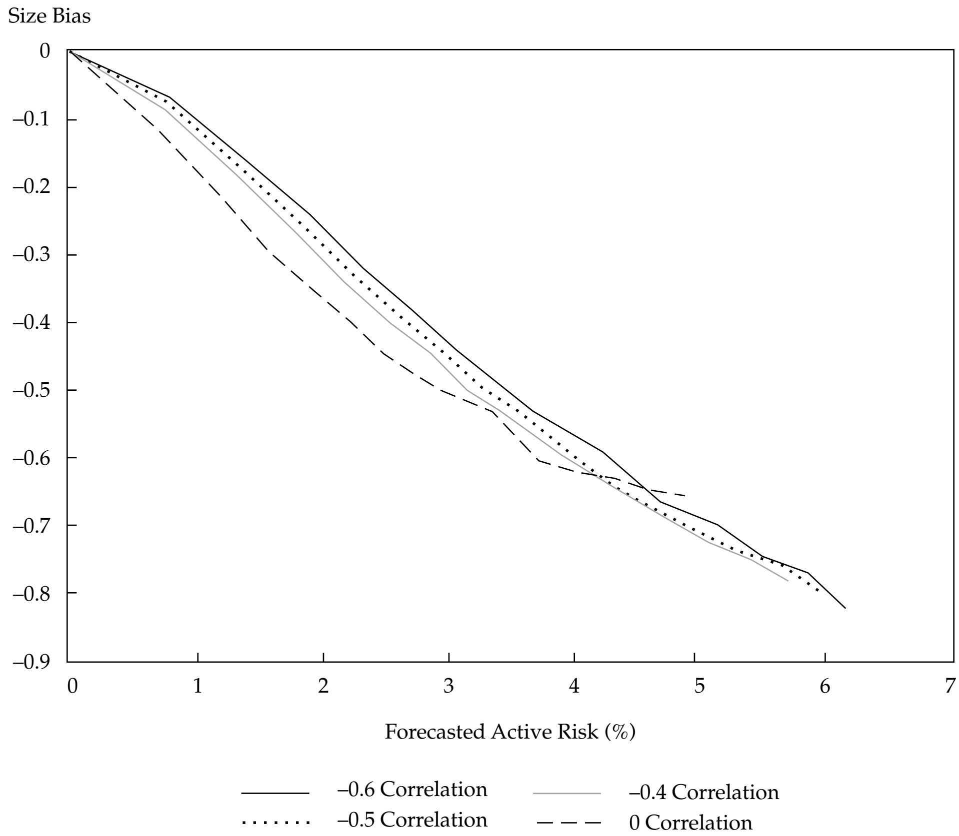
Finally, long–short investing offers the possibility of more-targeted active management fees than does traditional investing. Long-only portfolios largely contain the benchmark stocks. Long-only investors pay active fees for that passive core.[10] Long–short investors pay explicitly for the active holdings.

Empirical Observations

We have only preliminary observations about long–short strategies. The strategies do not have a sufficiently long track record to allow us to definitively compare their performance with the performance of long-only strategies. But understanding can begin with long–short strategies' risk profile and initial performance record.

For this purpose, we studied the performance of 14 U.S. long–short strategies with histories of

Figure 8. Sensitivity of Size Bias to Correlations between Size and Residual Risk (500 stocks)



Note: Correlations measured for log of capitalization and log of residual risk.

varying lengths (but all in the 1990s) ending in March 1998.[11] These 14 strategies are those of large, sophisticated quantitative managers. Most of these managers are BARRA clients. **Table 2** shows the relevant observations. (Keep in mind the small and potentially nonrepresentative sample behind these data.)

First, note that the risk levels shown for these strategies do not differ substantially from the typical active risk levels of about 4.5 percent.[12] So, at least based on these 14 sophisticated implementations, long–short strategies do not exhibit substantially higher levels of risk than long-only strategies.

Second, according to Table 2, these strategies achieved market neutrality. Their realized betas and market correlations are close to zero. In fact, the highest observed correlations with the S&P 500 correspond to managers with the shortest track records. No statistical evidence indicates that any of these strategies had true betas different from zero, and the realized numbers are all quite small. This

Table 2. Performance of 14 Long–Short Strategies, 1990s

Percentile	History (months)	Volatility	Beta	S&P 500 Correlation	Information Ratio
90	96	10.90%	0.10	0.23	1.45
75	86	6.22	0.06	0.15	1.23
50	72	5.50	0.02	0.04	1.00
25	50	4.12	–0.03	–0.07	0.69
10	28	3.62	–0.16	–0.20	0.44

(admittedly limited) sample thus refutes the argument that achieving market neutrality is difficult.

Third, at least in this historical period, these long–short strategies *as a group* provided remarkable performance. Although the performance results of 14 strategies over a particular market period do not prove that long–short implementations boost information ratios,[13] the results do help explain the increasing popularity of these strategies.

Summary

Long–short investing is an increasingly popular approach to implementing active strategies. Long–short strategies offer the potential to implement superior information more efficiently than long-only strategies. Because the long-only constraint is an inequality constraint and because its impact depends on the distribution of benchmark holdings, we could not derive many detailed analytical results on exact differences in efficiency. But both simple models and detailed simulations showed that the benefits of long–short investing can be significant, particularly when the universe of assets is large, asset volatility is low, and the strategy has high active risk.

From the opposite perspective, long-only managers should understand the surprising and significant impact of the long-only constraint on their portfolios. Among the surprises: This constraint induces a significant negative size bias; it affects active long as well as short positions; and enhanced index (low-risk) long-only strategies are more efficient than traditional (high-risk) long-only strategies.

Empirical observations on long–short investing are preliminary but should certainly inspire further interest and investigation.

We thank Naozer Dadachanji, Uzi Levin, Bruce Jacobs, and Bill Jacques for helpful comments and suggestions. Andrew Rudd contributed to the section on the appeal of long–short investing.

Appendix A. Further Explanations

We present here details about four items from the "Framework and Notation" section, especially underlying validation for Equations 4, 5, 8, and 9, and details of the derivation of Equation 23, the risk aversion required to achieve a given level of risk.[14]

General Form of an Alpha (Equation 5). We define a stock's alpha as its expected residual return, $E(\theta)$, conditional on information, which we represent as a signal, g, so that

$$\alpha = E(\theta|g). \tag{A1}$$

The best linear unbiased estimate of θ conditional on g is

$$E(\theta|g) = E(\theta) + \mathrm{cov}(\theta, g)\mathrm{var}^{-1}(g)[g - E(g)]. \tag{A2}$$

Assuming the unconditional expected residual return, $E(\theta)$, is zero, we can write Equation A1 as the desired result:

$$\alpha = \omega(IC)z, \tag{A3}$$

where the information coefficient is

$$IC \equiv \mathrm{corr}(\theta, g), \tag{A4}$$

the residual risk is

$$\omega \equiv \mathrm{std}(\theta), \tag{A5}$$

and the Z-score is

$$Z \equiv \frac{g - E(g)}{\mathrm{std}(g)}. \tag{A6}$$

Utility Function (Equation 5). Start with a general mean–variance utility function of the form

$$U = f_P - \lambda\sigma_P^2, \tag{A7}$$

where f_P is the expected excess (above the risk-free) return and σ_P is the standard deviation of that return.

This form of utility makes no reference to the benchmark. But we can express the excess return as a component driven by the benchmark and an independent component. When we do so, and then substitute these expressions into Equation A7 and delete terms that should be zero or are irrelevant to utility maximization (e.g., are constants), the utility simplifies to

$$U = \alpha_P - \lambda\omega_P^2 + (\beta_{PA}\Delta f_B - \lambda\beta_{PA}^2\sigma_B^2). \tag{A8}$$

where

- β_{PA} = the portfolio's active beta ($\beta_P - 1$)
- λ = the investor's risk aversion (trade-off between expected return and risk)
- Δf_B = the exceptional expected return to the benchmark

The benchmark has zero alpha by definition. For a portfolio that optimizes this utility, the first two terms on the right-hand side of Equation A8 determine stock selection and the last two terms determine only the overall portfolio beta in response to any expected exceptional benchmark return. For purposes of our study, we assumed no benchmark timing. Therefore, we set Δf_B and β_{PA} to zero and were left with the utility function of Equation 5.

When we optimize this resulting utility function, we still face a budget constraint—for example, the full investment constraint, $\mathbf{h}_P\mathbf{e} = 1$, where $\mathbf{h}_P$ is the portfolio and $\mathbf{e}$ is a vector of 1's. Adding this constraint and assuming a fully invested benchmark leads to optimal holdings:

$$(\mathbf{h}_P - \mathbf{h}_B) = \left(\frac{1}{2\lambda}\right)\left[\mathbf{VR}^{-1}\alpha - \left(\frac{\alpha^T\mathbf{VR}^{-1}\mathbf{e}}{\mathbf{e}^T\mathbf{VR}^{-1}\mathbf{e}}\right)\mathbf{VR}^{-1}\mathbf{e}\right], \quad \text{(A9)}$$

where $\mathbf{h}_B$ is the benchmark portfolio and **VR** is the covariance matrix of the residual returns.

To reach Equation 6, we made the reasonable assumption that our alphas were cash neutral:

$$\alpha^T\mathbf{VR}^{-1}\mathbf{e} = 0. \quad \text{(A10)}$$

For the simple model discussed in the text (in which residual returns were independent and of equal volatility), Equation A10 requires that the alphas have a mean of zero.

Importance of the Information Ratio (Equation 8). Substituting the information ratio, *IR*, into our utility function produces

$$U = IR\omega_P - \lambda\omega_P^2. \quad \text{(A11)}$$

Optimizing Equation A11 as a function of risk leads to the first-order condition:

$$IR - 2\lambda\omega_P^* = 0, \quad \text{(A12)}$$

with

$$U(\omega_P^*) = \frac{(IR)^2}{4\lambda}, \quad \text{(A13)}$$

where ω^*p is the optimal level of risk. Equation A13 is Equation 8 in the text.

Information Ratios, Skill, and Breadth (Equation 9). The mathematical derivation of Equation 9 is too long to include here, but we can summarize the argument in spirit. It begins with the alpha forecasts and builds an optimal portfolio. It then analyzes the expected information ratio of that optimal portfolio, ultimately expressing it on the basis of the correlations between alpha forecasts and subsequent realized returns (the information coefficient) and on the number of independent forecasts (typically, the number of stocks in the portfolio). Intuitively, the information ratio measures expected return per unit of risk, the information coefficient is a component of the expected return, and the breadth is a measure of potential risk diversification.

Risk Aversion and Target Risk Level (Equation 23). Generalizing on Equation A11, we express utility in terms of risk as

$$U = \alpha(\omega) - \lambda_R\omega^2. \quad \text{(A14)}$$

Using Equation 20, Equation A14 becomes

$$U = 100IR\left\{\frac{\left[1 + \frac{\omega}{100}\right]^{1-\gamma(N)} - 1}{1 - \gamma(N)}\right\} - \lambda_R\omega^2. \quad \text{(A15)}$$

We solve for the optimal level of risk by taking the derivative of U with respect to ω and setting the result equal to zero:

$$IR\left[1 + \frac{\omega}{100}\right]^{-\gamma(N)} = 2\lambda_R\omega, \quad \text{(A16)}$$

which leads directly to Equation 23.

Notes

1. See various articles on market-neutral strategies in the May 12, 1997, and May 18, 1998, issues of *Pensions & Investments*.
2. Appendix A expands on several points in this section, in particular, the details behind Equations 4, 5, 8, and 9.
3. Refer to Dadachanji (1995) and Jacobs (1997).
4. As an alternative, set the constant c to the standard deviation of the log of the capitalization of all the stocks. The two criteria mentioned in the text place greater emphasis on fitting the larger-cap stocks.
5. Freedonia appeared in the 1933 Marx Brothers movie, "Duck Soup." During a 1994 Balkan eruption, when asked if the United States should intervene in Freedonia, several U.S. congressmen laughed, several stated that it would require further study, and several more were in favor of intervention if Freedonia continued its policy of ethnic cleansing.
6. We used Equation 13 to convert desired risk levels to risk aversions. We required extremely high levels of desired risk because the long-only constraint severely hampered our ability to take risk.
7. Equations 20 and 21 are estimates based on computer simulations. You may obtain slightly different results if you repeat the experiment yourself.
8. This is a best-case analysis assuming we have efficiently used our information. Poor portfolio construction will only reduce the efficiency.
9. Note that this analysis did not include size as a risk factor. Adding that would mitigate (but not eliminate) the bias.
10. See Freeman.
11. See Kahn and Rudd.
12. See Grinold and Kahn (2000) for empirical observations on long-only risk levels. The standard error of the mean risk level for these long–short strategies is 0.64 percent. So, although the medians displayed here exceed 4.5 percent, the difference is not significant at the 95 percent confidence level.
13. In fact, several of these managers struggled in 1999.
14. Even this technical appendix is insufficient to thoroughly cover the items in "Framework and Notation." Readers who would like the full treatment should refer to Chapters 4, 5, 6, and 10 of Grinold and Kahn (2000).

References

Arnott, Robert D., and David J. Leinweber. 1994. "Long–Short Strategies Reassessed." *Financial Analysts Journal*, vol. 50, no. 5 (September/October):76–78.

Brush, John S. 1997. "Comparisons and Combinations of Long and Long–Short Strategies." *Financial Analysts Journal*, vol. 53, no. 3 (May/June): 81–89.

Dadachanji, Naozer. 1995. "Market Neutral Long–Short Strategies: The Perception versus the Reality." Presentation at the October 1995 Q-Group Conference, La Quinta, CA.

Freeman, John D. 1997. "Investment Deadweight and the Advantages of Long–Short Portfolio Management." *VBA Journal* (September):11–14.

Grinold, Richard C. 1989. "The Fundamental Law of Active Management." *Journal of Portfolio Management*, vol. 15, no. 3 (Spring):30–37.

———. 1994. "Alpha Is Volatility Times IC Times Score, or Real Alphas Don't Get Eaten." *Journal of Portfolio Management*, vol. 20, no. 4 (Summer):9–16.

Grinold, Richard C., and Ronald N. Kahn. 2000. *Active Portfolio Management*. 2nd ed. New York: McGraw-Hill.

Jacobs, Bruce I. 1997. "The Long and Short on Long–Short." *Journal of Investing*, vol. 6, no. 1 (Spring):73–86. (Also presented at the October 1995 Q-Group Conference, La Quinta, CA.)

———. 1998. "Controlled Risk Strategies." In *Alternative Investing*. Charlottesville, VA: AIMR.

Jacobs, Bruce I., and Kenneth N. Levy. 1995. "More on Long–Short Strategies." *Financial Analysts Journal*, vol. 51, no. 2 (March/April):88–90.

———. 1996. "20 Myths about Long–Short." *Financial Analysts Journal*, vol. 52, no. 5 (September/October):81–85.

Jacobs, Bruce I., Kenneth N. Levy, and David Starer. 1998. "On the Optimality of Long–Short Strategies." *Financial Analysts Journal*, vol. 54, no. 2 (March/April):40–51.

———. 1999. "Long–Short Portfolio Management: An Integrated Approach." *Journal of Portfolio Management*, vol. 25, no. 2 (Winter):23–32.

Kahn, Ronald N., and Andrew Rudd. 1998. "What's the Market for Market Neutral?" *BARRA Preprint* (June).

Levin, Asriel. 1998. "Long–Short Investing—Who, Why, and How." In *Enhanced Index Strategies for the Multi-Manager Portfolio*. Edited by Brian Bruce. New York: Institutional Investor.

Michaud, Richard O. 1993. "Are Long–Short Equity Strategies Superior?" *Financial Analysts Journal*, vol. 49, no. 6 (November/December):44–49. (Also presented at the October 1995 Q-Group Conference, La Quinta, CA.)

———. 1994. "Reply to Arnott and Leinweber." *Financial Analysts Journal*, vol. 50, no. 5 (September/October):78–80.