The European Journal of Finance

ISSN: 1351-847X (Print) 1466-4364 (Online) Journal homepage: www.tandfonline.com/journals/rejf20

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The European Journal of Finance,
Vol. 15, Nos. 5–6, July–September 2009, 533–553

Diversification benefits for bond portfolios

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Finance research has focused primarily on the diversification of stock portfolios. Various metrics are used herein to assess the diversification benefits, and the optimal bond portfolio sizes (PSs) for investment opportunity (IO) sets differentiated by issuer type, credit ratings and term-to-maturity. While PSs of 25–40 bonds appear optimal for the marginal reduction of dispersion with increasing PS, larger (smaller) PSs are optimal if the investor is concerned about left tail weight (positive skewness or reward-to-downside risk). Although the marginal reduction of dispersion is less than 1% beyond these optimal PSs, much potential diversification benefits still remain unrealized for many of the IO sets studied herein.

Keywords: diversification benefits; portfolio size; derived and realized dispersion; skewness; Sortino ratio; tail shape

JEL Classification: G11; G23; C15; D81

1. Introduction

A cornerstone of the modern portfolio theory is the study by Markowitz (1952), which illustrates the benefits of forming portfolios with less than perfectly positively correlated assets. The subsequent literature, which has focused on the benefits and drawbacks of using this approach primarily for equity portfolios, reaches different conclusions concerning the minimum portfolio size (henceforth, PS) needed to achieve a ‘well-diversified’ equity portfolio. Early studies find a required PS for US equities that varies from eight to ten (Evans and Archer 1968; Latane and Young 1969; Elton and Gruber 1977) to 15 (Jennings 1971; Kryzanowski et al. 1985) to 30 (Statman 1987) to at least 100 (Fama 1965). More recent studies find that the required number of stocks has increased to 50 (Malkiel and Xu 2006) or as high as 300 (Statman 2004; Statman and Scheid 2005) due to increases in idiosyncratic volatility, increases in the correlations between stocks, and a change in the size and structure of industries (Bennett and Sias 2006). Other empirical studies find that stock diversification benefits diminish with large negative movements in stock returns (Silvapulle and Granger 2001) due to higher firm-level return dispersions when market returns are largely negative (Demirer and Lien 2004), and that the increasing importance of correlations during market downturns is related to the market’s tail distribution (Sancetta and Satchell 2007). Van Nieuwerburgh and Veldkamp (2005) argue that informational advantages explain the observation that investors tend to hold fewer assets than suggested by the literature on diversification benefits.

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ISSN 1351-847X print/ISSN 1466-4364 online

DOI: 10.1080/13518470902890758
http://www.informaworld.com

Numerous authors document improved return-risk combinations from international diversification in stocks or in stocks and bonds (Solnik et al. 1996; Chollerton et al. 1986; Jorion 1987, 1989; Kaplanis and Schaefer 1991; Thomas 1989; De Santis and Sarno 2008). Cappiello et al. (2006) and Hunter and Simon (2004, 2005) find that the average correlation in the international bond market has increased over time but not to the same extent as observed in the equity market by Goetzman et al. (2005), among others. Hunter and Simon (2004) find that US investors who hold a well-diversified portfolio of domestic fixed-income and equity investments can obtain incremental diversification benefits from investing in international government bonds if currency risk is hedged. Varotto (2007) finds that interest rate factors followed by maturity diversification are important and credit rating and seniority diversification effects are unimportant causes of diversification of total portfolio returns in international corporate bond portfolios. He also finds that industry diversification, unlike interest rate and maturity diversifications, have little impact on the volatility of corporate bond portfolios.

The study by McEnally and Boardman (1979) appears to the only study that investigates how many bonds are necessary to obtain a target level of diversification benefits in terms of volatility risk reduction for IO sets differentiated by bond ratings. These authors conclude that eight to sixteen bonds significantly reduce volatility risk in bond portfolios. They also find that diversified portfolios of high-yield bonds have lower systematic risk than portfolios of investment grade bonds, which could be attributed to an industry effect since the lowest risk bonds are in the utility sector, whereas the high risk bonds are industrial bonds.

However, the implications of the results of McEnally and Boardman may not be applicable to more recent periods for a number of reasons. First, they examine a randomly chosen sample of 515 corporate *straight* bonds for the period 1972–1976. Second, their findings are likely to be outdated and limited because the bond opportunity set has not only expanded substantially in terms of credit quality, industry, country and bond maturity, but the operational efficiency of the market has improved. Third, as McEnally and Boardman (1979) note, this time period is characterized by extreme instability in the corporate bond market in terms of interest rate volatility and default premia. As a result, Moody's re-rated approximately one-fourth of the bonds in their sample during the studied period. Fourth, the only metrics used to assess diversification benefits in their study is unconditional variance. More recent tests of the benefits of equity diversification use a much broader set of metrics that reflect higher-order moments, alternate definitions of risk, and reward-to-risk measures.

Thus, the primary purpose of this article is to re-examine the diversification benefits associated with different-sized portfolios of bonds using various metrics. These metrics investigate the diversification benefits in terms of dispersion of returns, reward to risk, downside risk and the probability of underperforming a target rate of return. In addition, the IO sets are categorized by industry sector and credit ratings. Also, the impact on the minimum PSs of an investor's preference for long- versus short-term investments is assessed by dividing the IO sets by maturities of more than and less than 10 years.

This article makes five contributions to the literature. The first contribution deals with the benefits of diversification of bond portfolios for IO sets that are differentiated not only by credit ratings but also by industry sectors, domesticity and/or maturities. The second contribution is the investigation of not only the straight bonds previously investigated in the literature but also bonds with additional characteristics such as callability, puttability and convertibility. The third contribution is an examination of the diversification benefits using various metrics, including some that were only recently introduced into the literature on stock diversification benefits. Fourth, we show that there is no minimum PS. The choice of the minimum PS depends on the objectives of investors in terms of risk, return and bond maturity, and on issuer and bond characteristics, such as industry and rating. Finally, we show that while the marginal reduction of dispersion with increasing PS is achieved with PSs of 25–40 bonds, much potential diversification benefits still remain unrealized for many of the IO sets studied herein. Together with the investment cost of obtaining such portfolios on own account, this may explain while individuals purchase bond mutual funds although studies find that bond mutual funds exhibit neutral and under performance when evaluated using gross and net-of-fees returns, respectively (Kahn and Rudd 1995).

The remainder of the article is organized as follows. The sample, data and IO sets are discussed in the next section. In Section 3, we report our results for the various performance metrics and discuss the minimum PS beyond which most of the marginal diversification benefits are exhausted. In Section 4, we conduct a sensitivity analysis to determine if our results change materially for a straight bond sample or different sample years. Section 5 concludes the article.

2. Samples and data

Our bond sample is extracted from the Lehman Brothers Fixed Income Database distributed by Warga (1998), which ceased to be updated in March 1998, and has been widely used in various bond studies (Elton et al. 2001; Liu et al. 2007). The database consists of 39,132 bonds and 1,289,010 monthly bond prices from January 1985 until December 1997. The database contains monthly quoted and matrix prices, and descriptive bond information, such as industry, rating, duration, convexity, monthly total dollar returns, coupons, maturities, and embedded option features. Since monthly dollar returns are reported in the database, the monthly rate of return at time $t+1$ is calculated using the formula:

$$r_{t+1} = \frac{C_{t+1} + P_{t+1} + A_{t+1} - P_t - A_t}{P_t + A_t}, \tag{1}$$

where P_t and P_{t+1} are the clean (bid) prices at time t and $t+1$, respectively; A_t and A_{t+1} are the accrued interest at time t and $t+1$, respectively; and C_{t+1} is the coupon payment at time $t+1$. The monthly rate of return is obtained by dividing the total dollar return (numerator) by the beginning of the period dirty price.

Our initial sample includes all bonds with quoted bid prices. This initial sample is divided into many IO sets depending on the deemed preferences of our hypothetical investor (Table 1).[1] There are 27,497 unique bonds and 939,267 bond prices when differentiating by issuer type, and 30,758 unique bonds with 927,295 bond prices when differentiating by credit ratings.

3. Diversification benefits measured using various metrics

In this section of the article, various metrics are used to measure the benefits of portfolio diversification and to identify the minimum PS needed to diversify a specific percentage of nonsystematic risk or to capture a specific percentage of the reward from bearing risk. This is implemented by selecting bonds randomly using a Monte Carlo approach in order to create 5000 portfolios for each IO set j and each portfolio size s. We test for a PS ranging from 2 to 100 and 'All', where the latter includes $N-1$ bonds and N is the number of bonds in the IO set j.

Since the form of the distribution changes as the IO set, metric and PS change, the values of the dispersion metrics used in the determination of the minimum PS also will change. Therefore, we examine various metrics for different PS and different IO sets to determine how the optimal PS changes when the return distribution is time varying.

Table 1. Sample sizes for the IO sets differentiated by issuer type and credit rating.

	IOs differentiated by issuer type						IOs differentiated by credit rating					
Characteristic	All	Tr./Ag.	Industrial	Utility	Financial	Foreign	All	Aaa	Aa	A	Baa	Spec.
Panel A: Sample sizes for IO sets differentiated by issuer type and credit rating												
Unique bonds(#)	27,497	9113	7511	4453	4991	1429	30,758	10,206	3714	7442	4722	4674
Bond prices (#)	939,267	291,229	260,869	159,892	163,108	64,169	927,295	340,761	112,965	238,113	120,663	114,793
Average coupon (%)	7.69	5.43	7.57	8.54	8.42	8.49	8.30	5.55	8.39	8.35	8.80	10.42
Average maturity (years)	12.04	11.59	11.61	16.56	7.92	12.50	11.89	11.47	14.22	12.13	12.66	8.99
Panel B: Statistics for IO set differentiated by issuer type and credit rating and with maturities $<$ 10 years												
Unique bonds (#)	19,194	5841	5741	2477	4145	990	21,355	7195	2233	5016	3083	3828
Bond prices (#)	546,547	162,281	165,061	57,911	126,021	35,273	549,376	201,504	54,677	139,772	69,209	84,214
Average coupon (%)	7.75	5.99	7.90	7.90	8.48	8.51	8.27	6.28	8.08	8.15	8.59	10.24
Average maturity (years)	5.44	4.12	5.31	5.98	5.00	5.87	5.43	4.20	5.20	5.43	5.77	6.57
Panel C: Statistics for IO set differentiated by issuer type and credit rating and with maturities $>$ 10 years												
Unique bonds (#)	11,497	3,899	2960	2554	1421	663	12395	3741	1812	3176	2067	1599
Bond prices (#)	392,702	128,939	95,806	101,979	37,085	28,893	377,902	139,247	58,285	98,340	51,451	30,579
Average coupon (%)	7.48	4.73	7.09	8.90	8.23	8.47	8.36	4.51	8.68	8.64	9.09	10.90
Average maturity (years)	20.53	20.99	20.68	22.57	17.83	20.59	20.78	22.00	22.69	21.64	21.93	15.64

Note: This table summarizes the sample sizes in terms of the total number (#) of unique bonds, the total number (#) of bond prices, the average coupon (%), and average maturity in years (yrs.) in the IO sets investigated in this article. These IO sets are differentiated by issuer type and credit rating (panel A) and maturities (short maturities in panel B and long maturities in panel C). 'Tr./Ag.' refers to Treasury/Agency and 'Spec.' refers to Speculative. Note that the sum of sample sizes for short maturities and long maturities does not add up to the size of the IO sets not differentiated by maturity because a unique bond could be listed both in short and long maturities if its time to maturity moves from more than 10 years to less than 10 years.

The most common method used to estimate the overall benefits of diversification as PS increases is to estimate the ratio in percentage terms of the potential benefits that are achieved, on average, for the specific PS versus the potential benefits achievable from holding all the assets in the IO set. The most common method for estimating the marginal benefits of diversification as the PS increases is to estimate the speed at which the value of the diversification metric changes (Campbell et al. 2001). Since the average correlation among security returns limits the power of diversification to reduce risk, a PS level should be reached at which an increase in PS produces only a small change in the metric measuring the marginal benefits of diversification. Due to the costs associated with further diversification, rational investors will be adverse to increasing the PS when the diversification benefits from incrementing the PS to the next larger PS are ‘small’, which is taken herein to be a marginal change in the value of the diversification metric of 1% or less. However, this criterion for the determination of a ‘minimum’ PS, which is based on a small marginal benefit (SMB), may leave a substantial proportion of the overall potential benefits from further diversification unrealized, as is shown below.

3.1 *Correlations of bond returns*

The first metric used in this section is the correlation of bond returns. This metric enables us to identify which IO sets have low or negative correlations, on average, and consequently may produce the highest diversification benefits. For each month, for each IO set j (un)differentiated by issuer type, rating category and maturity, the cross-sectional mean of the correlations between every unique pair of bonds contained therein is calculated using only the bonds with at least 27 returns over the 36-month moving window ending during that month.[2]

Summary statistics for various time-series distributions of the cross-sectional mean correlations for the (un)differentiated IO sets are reported in Table 2. The industrial and financial sectors have the lowest means and medians for the time-series of cross-sectional mean correlations over the studied period. For a fixed PS, portfolios composed of bonds issued by industrial or financial firms can be expected to eliminate idiosyncratic risk faster than portfolios consisting of bonds issued by firms of the other issuer types. As is the case for short- versus long-term maturity bonds (i.e., less than versus greater than 10 years), speculative grade bonds have a lower mean for the time-series of cross-sectional mean correlations over the studied period. All else held equal, this implies that investors may achieve diversification benefits faster, on average, for any PS by holding bonds with shorter maturities or lower quality ratings.

Summary statistics for this metric for various pairs of the differentiated IO sets are reported in Table 3. The potentially superior diversification properties of speculative grade bonds persist. The maximum and minimum time-series correlations for speculative grade bonds are 0.19 and 0.03 for A- and Aaa-rated bonds, respectively. Furthermore, the time-series correlations between speculative grade bonds with maturities less than 10 years and the other IO sets even become negative. Similarly, the categories of utilities and foreign bonds show relatively low levels of time-series average correlations with the other differentiated IO sets. For instance, utilities and foreign bonds are negatively correlated with the treasury/agency category for bonds with maturities longer than 10 years.[3]

3.2 *Dispersion of bond return metrics*

The first metric examined in this sub-section of the article is the excess standard or mean derived deviation (MDD) for a randomly selected portfolio, which is defined as the difference between

Table 2. Summary statistics for the time-series of the cross-sectional mean correlations of all individual bond-return pairing within each IO set differentiated by issuer type and credit rating.

Statistic	IOs differentiated by issuer type						IOs differentiated by credit rating					
	All	Tr./Ag.	Industrial	Utility	Financial	Foreign	All	Aaa	Aa	A	Baa	Spec.
Panel A: Statistics for IO sets differentiated by issuer type or credit rating												
Mean	0.347	0.526	0.224	0.419	0.27	0.426	0.324	0.458	0.414	0.365	0.329	0.108
Median	0.311	0.533	0.218	0.444	0.243	0.448	0.316	0.48	0.407	0.339	0.314	0.108
Standard deviation	0.080	0.101	0.073	0.067	0.097	0.063	0.044	0.063	0.061	0.083	0.092	0.039
Minimum	0.244	0.353	0.106	0.276	0.151	0.324	0.264	0.357	0.301	0.256	0.202	0.046
Maximum	0.482	0.687	0.342	0.524	0.455	0.514	0.435	0.557	0.528	0.521	0.493	0.176
Panel B: Statistics for IO set differentiated by issuer type or credit rating and with maturities $<$ 10 years												
Mean	0.268	0.409	0.173	0.291	0.249	0.351	0.245	0.344	0.321	0.303	0.278	0.098
Median	0.239	0.387	0.169	0.28	0.229	0.356	0.223	0.335	0.293	0.281	0.247	0.099
Standard deviation	0.078	0.096	0.056	0.096	0.091	0.069	0.046	0.055	0.083	0.085	0.096	0.035
Minimum	0.175	0.282	0.103	0.149	0.131	0.241	0.197	0.268	0.213	0.179	0.141	0.044
Maximum	0.399	0.574	0.273	0.459	0.421	0.467	0.362	0.482	0.478	0.462	0.449	0.162
Panel C: Statistics for IO set differentiated by issuer type or credit rating and with maturities $>$ 10 years												
Mean	0.513	0.817	0.363	0.484	0.385	0.53	0.531	0.728	0.506	0.476	0.416	0.163
Median	0.479	0.841	0.323	0.515	0.345	0.543	0.500	0.752	0.528	0.453	0.409	0.149
Standard deviation	0.109	0.10	0.130	0.075	0.130	0.075	0.106	0.098	0.075	0.092	0.094	0.079
Minimum	0.348	0.415	0.139	0.321	0.229	0.384	0.383	0.518	0.353	0.333	0.266	0.055
Maximum	0.688	0.928	0.546	0.587	0.635	0.648	0.714	0.884	0.642	0.63	0.573	0.310

Note: Summary statistics for the time-series of cross-sectional mean correlations of all individual bond return pairings within IO sets differentiated by issuer type and credit rating but undifferentiated by maturity are reported in panel A of this table. Summary statistics for these IO sets, when further differentiated by maturities of less than and more than 10 years, are reported in panels B and C, respectively. For each month, the mean cross-sectional correlation for each differentiated IO set j is calculated from the correlations between every unique pair of bonds in each IO set for bonds that have at least 27 monthly returns over a 36-month moving window. ‘Tr./Ag.’ refers to Treasury/Agency.

Table 3. Correlations between the time-series of cross-sectional mean correlations of monthly returns for various IO sets differentiated by issuer type and credit rating.

IO	All [All]	Tr./Ag. [Aaa]	Industrial [Aa]	Utility [A]	Financial [Baa]	Foreign [Spec.]
Panel A: Correlations for IO sets differentiated by issuer type and by credit rating						
All [All]	1.00 [1.00]					
Tr./Ag. [Aaa]	0.89 [0.89]	1.00 [1.00]				
Industrial [Aa]	0.93 [0.22]	0.78 [0.32]	1.00 [1.00]			
Utility [A]	0.45 [0.58]	0.12 [0.38]	0.28 [0.94]	1.00 [1.00]		
Financial [Baa]	0.94 [0.62]	0.74 [0.52]	0.82 [0.92]	0.65 [0.98]	1.00 [1.00]	
Foreign [Spec.]	0.63 [0.08]	0.47 [0.03]	0.42 [0.06]	0.75 [0.19]	0.67 [0.12]	1.00 [1.00]
Panel B: Correlations for IO sets differentiated by issuer type, credit rating and maturity						
All [All]	1.00 [1.00]	0.84 [0.91]	0.97 [0.33]	0.23 [0.84]	0.90 [0.74]	0.34 [0.76]
Tr./Ag. [Aaa]	0.92 [0.83]	1.00 [1.00]	0.87 [0.07]	−0.13 [0.57]	0.62 [0.44]	−0.07 [0.82]
Industrial [Aa]	0.94 [0.86]	0.81 [0.49]	1.00 [1.00]	0.08 [0.71]	0.83 [0.81]	0.17 [0.15]
Utility[A]	0.92 [0.74]	0.73 [0.42]	0.84 [0.98]	1.00 [1.00]	0.55 [0.97]	0.92 [0.48]
Financial [Baa]	0.94 [0.90]	0.79 [0.67]	0.84 [0.97]	0.98 [0.97]	1.00 [1.00]	0.64 [0.39]
Foreign [Spec.]	0.83 [−0.21]	0.85 [−0.21]	0.72 [0.21]	0.74 [0.10]	0.76 [0.11]	1.00 [1.00]

Note: This table reports the correlations between the time-series of cross-sectional mean correlations of monthly returns for the IO sets differentiated by issuer type and credit rating in panel A, where the latter are reported in the brackets. For each month, the mean cross-sectional correlation for each differentiated IO set j is calculated from the correlations between every unique pair of bonds in the IO set for bonds that have at least 27 monthly returns over a 36-month moving window. The correlations, which are further differentiated by bond maturity, are reported in panel B, where the values not in and in the brackets are based on maturities less than and greater than 10 years, respectively. 'Tr./Ag.' refers to Treasury/Agency and 'Spec.' refers to Speculative.

the time-series standard deviations of the random portfolio and the whole IO set to which that portfolio belongs. This metric, which is calculated for 5000 randomly selected portfolios for each (un)differentiated IO set, is given by[4]

$$\mathrm{MDD}_{j,s} = \bar{\sigma}_{j,s} - \sigma_J, \tag{2}$$

where $\bar{\sigma}_{j,s}$ is the mean of the standard deviations for the 5000 randomly selected portfolios with a PS or PS of s for (un)differentiated IO set j, and σ_J is the average standard deviation of all the bonds in (un)differentiated IO set j.[5]

As expected, the MDD decreases with increasing PS (see Figures 1 and 2). The minimum PS that satisfies the SMB criterion ranges from 35 to 45 bonds for IO sets differentiated by issuer type. The overall diversification benefit at this minimum PS is substantial (MDD reductions range from 75% to 96%). For issuer-type-differentiated IO sets for bond maturities less than 10 years, we observe not only lower SMB-determined PSs in the range of 30 to 35 bonds but also similar

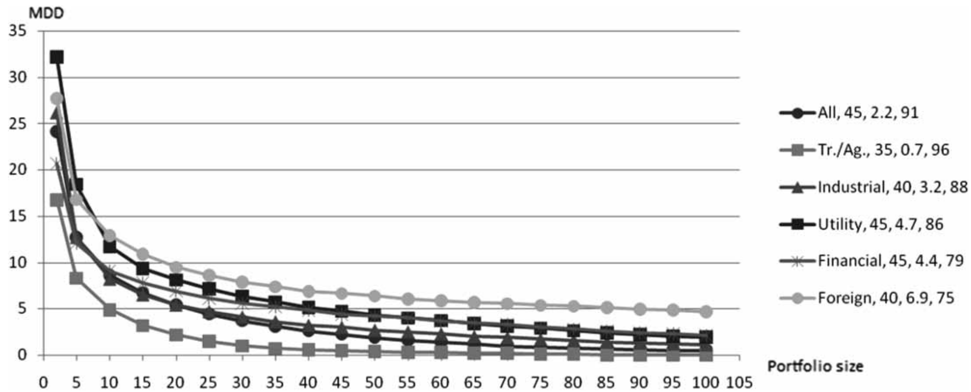

Figure 1. Excess Standard Deviations (MDDs) for IO Sets Differentiated by issuer type and maturity.
This figure depicts the excess standard deviations (MDDs) multiplied by 100 of quoted returns (i.e., differences between the standard deviations of the 5000 random portfolios and an equally weighted index of all bonds in that IO set j for the whole period) differentiated by PS and issuer type for all maturities for various PSs. The optimal PS is reached when the reduction in the MDD is not more than 1% from incrementing the PS to the next larger PS provided that the difference in the mean MDDs for PS s of two and All are significantly different at the 0.05 level. 'Tr./Ag.' refers to Treasury/Agency. The series name, optimal PS, the MDD value, and the percentage reduction in the MDD from a benchmark PS of two when the optimal PS is reached are reported in that order for each series in the legend to the figure.

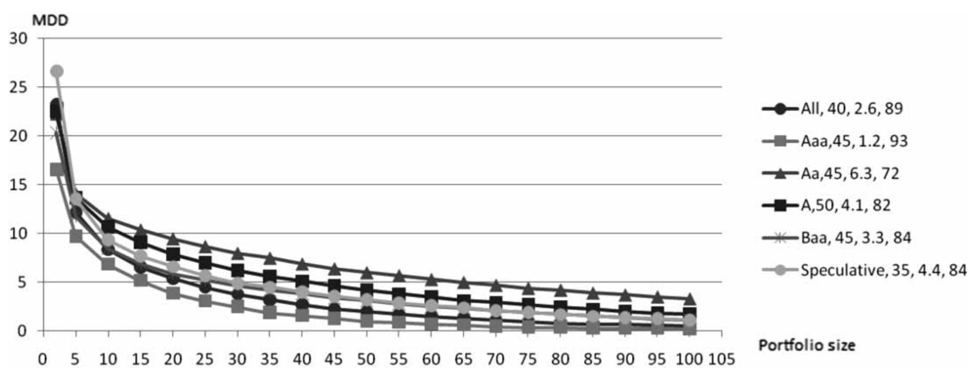

Figure 2. Excess Standard Deviations (MDDs) for IO Sets Differentiated by credit rating.
This figure depicts the excess standard deviations (MDDs) multiplied by 100 of quoted returns (i.e., differences between the standard deviations of the 5000 random portfolios and an equally weighted index of all bonds in the IO set j) differentiated by PS and rating category for all maturities for various PSs. The optimal PS is reached when the reduction in the MDD is not more than 1% from incrementing the PS to the next larger PS provided that the difference in the mean MDDs for PS s of two and All are significantly different at the 0.05 level. 'Tr./Ag.' refers to Treasury/Agency. The series name, optimal PS, the MDD value, and the percentage reduction in the MDD from a benchmark PS of two when the optimal PS is reached are reported in that order for each series in the legend to the figure.

reductions in the MDD of 75 to 95% (except for the 62% reduction in the MDD for the foreign IO set). A comparison of the MDD for a specific PS for shorter versus longer maturities clearly shows that the former is never smaller with a wide range of PS of 35–55 bonds but with a similar range of overall reductions in MDDs of 72–97%. This is due, most probably, to the higher sensitivity of long-term bonds to changes in economic factors.

The IO sets differentiated by rating category have a wider range for SMB-determined PS than the issuer-type-differentiated IO sets. For the rating category IO sets, the SMB-determined PS range from 35 to 50 bonds with an overall reduction in MDD ranging between 72 and 93%. When differentiated by maturities, the short maturities IO sets also show in general lower SMB-determined PS (40–55 bonds) than the longer maturities (35–60 bonds), except for the Baa and Speculative IO sets where the shorter maturities have a higher SMB-determined PS. The overall reductions in MDD are considerable for both long and short maturities (80–96%), except for the Aa short maturity IO set that exhibits a slightly lower reduction in MDD of 65%.

The second metric examined in this sub-section is the average cross-sectional standard deviation (de Silva et al. 2001; Ankrim and Ding 2002), which sometimes is referred to as the mean realized dispersion (MRD). When cross-sectional variations in returns are high, a fund manager is operating in a high-risk environment where the probabilities of market over- and under-performance are high. Consequently, risk-averse managers seek to reduce their exposure to higher MRDs, which for a fixed PS s and IO set j is given by

$$\mathrm{MRD}_{j,s} = \frac{1}{N}\sum_{\tau=1}^{N}\sigma_{j,s,\tau}, \tag{3}$$

where $\sigma_{j,s,\tau}$ is the cross-sectional standard deviation for the 5000 randomly selected portfolios for IO set j with a PS of s for month τ; and N is the number of months in the sample (i.e., 156 months from January 1985 until December 1997). The diversification benefits, which are shown in Figures 3 and 4, exhibit similar patterns across all (un)differentiated IO sets. The overall MRD is reduced, on average, by 76–80% for a SMB-determined PS of 35–40 bonds.

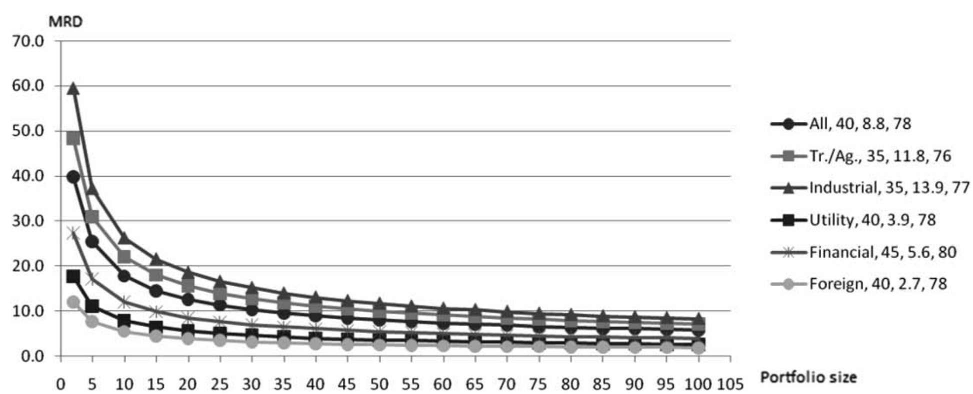

Figure 3. MRDs Differentiated by issuer type.
This figure depicts the MRDs multiplied by 100 of quoted returns (i.e., the mean of the cross-sectional standard deviations of IO set j for the whole period) differentiated by (PSs) and issuer type for all maturities. The optimal PS is reached when the reduction in the MRD is not more than 1% from incrementing the PS to the next larger PS provided that the difference in the mean MRDs for PS s of two and All are significantly different at the 0.05 level. ‘Tr./Ag.’ refers to Treasury/Agency. The series name, optimal PS, the MRD value, and the percentage reduction in the MRD from a benchmark PS of two when the optimal PS is reached are reported in that order for each series in the legend to the figure.

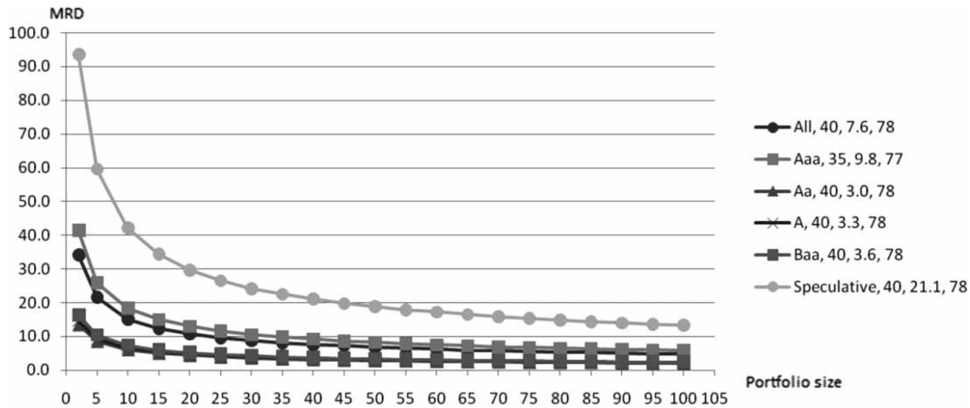

Figure 4. MRDs differentiated by PS and credit rating.
This figure depicts the MRDs multiplied by 100 of quoted returns (i.e., the mean of the cross-sectional standard deviations of IO set j for the whole period) differentiated by PS and rating category for all maturities. The optimal PS is reached when the reduction in the MRD is not more than 1% from incrementing the PS to the next larger PS provided that the difference in the mean MRDs for PS s of two and All are significantly different at the 0.05 level. 'Tr./Ag.' refers to Treasury/Agency. The series name, optimal PS, the MRD value, and the percentage reduction in the MRD from a benchmark PS of two when the optimal PS is reached are reported in that order for each series in the legend to the figure.

3.3 *Composite return and risk metrics*

Investors are interested in holding portfolios that provide the best return-risk tradeoffs. Consequently, a diversification strategy, such as increasing PS, which diminishes risk also, needs to result in a higher return-to-risk tradeoff. Accordingly, we now examine how different return-to-risk metrics react to a changing PS for the various IO sets.

Metrics commonly used for this purpose normalize the excess return over the risk-free rate of the portfolio by the risk of that portfolio. One such metric is the Sortino ratio, which is defined as

$$Sor_{j,s} = (\bar{r}_{j,s} - \bar{r}_f)/\bar{\sigma}_{j,s}, \tag{4}$$

where $\bar{r}_{j,s}$ is the mean return on the portfolios of size s for IO set j; $\bar{\sigma}_{j,s}$, the average semi standard deviation of returns for portfolios of size s for IO set j; and $\bar{r}_f$, the mean risk-free rate.

The results for the Sortino metrics are shown in Figures 5 and 6.[6] Based on the results for IO sets differentiated by issuer type, the SMB-determined minimum PSs range between 20 and 45 bonds for all IO sets, and the associated overall increases in their Sor are in the range of 72–94%. Based on a further differentiation by maturity, the SMB-determined minimum PSs are in the range of 20–65 bonds for short-term maturity IO sets (with associated increases in their overall Sor of 74–98%). They are in the range of 10 (foreign) to 30 (Tr./Ag.) bonds for the long-term maturity IO sets (with associated increases in their overall Sor of 74–95%).

3.4 *Metrics based on higher-order moments*

Although the metrics used so far have the advantage of being simple, robust and independent of any reference index, they do not capture higher dimensions of risk that may differ across PSs for the same IO sets. For example, the Sortino ratio ignores the existence of third and fourth moments (i.e., skewness and kurtosis), which may be unfavorable to the investor. Similarly, lower second

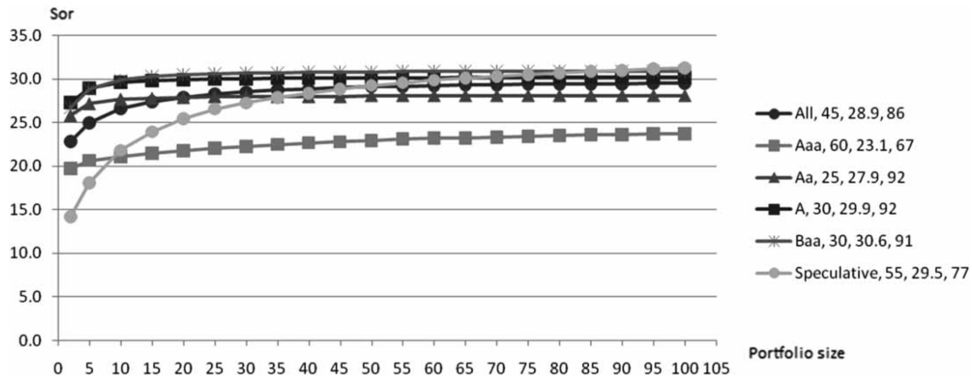

Figure 5. Sortino ratios differentiated by PS and issuer type.
This figure depicts the Sortino ratios (Sor) multiplied by 100 of quoted returns differentiated by PS and issuer type for all maturities. The optimal PS is reached when the reduction in the Sor is not more than 1% from incrementing the PS to the next larger PS provided that the difference in the mean Sor for PS s of two and All are significantly different at the 0.05 level. 'Tr./Ag.' refers to Treasury/Agency. The series name, optimal PS, the Sor value, and the percentage reduction in the Sor from a benchmark PS of two when the optimal PS is reached are reported in that order for each series in the legend to the figure.

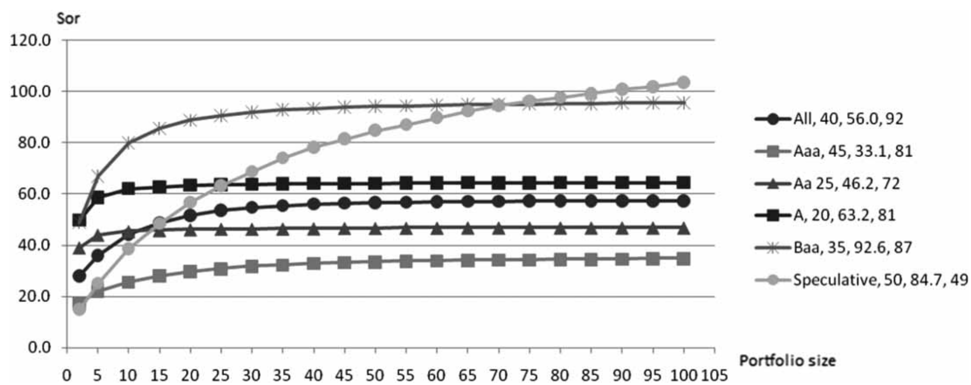

Figure 6. Sortino ratios differentiated by PS and credit rating.
This figure depicts the Sortino ratios (Sor) multiplied by 100 of quoted returns differentiated by PS and rating category for all maturities. The optimal PS is reached when the reduction in the Sor is not more than 1% from incrementing the PS to the next larger PS provided that the difference in the mean Sor for PS s of two and All are significantly different at the 0.05 level. 'Tr./Ag.' refers to Treasury/Agency. The series name, optimal PS, the Sor value, and the percentage reduction in the Sor from a benchmark PS of two when the optimal PS is reached are reported in that order for each series in the legend to the figure.

return moments may occur for PSs along with fatter tails. In addition, the Sortino ratio can be manipulated by transferring part of the risk from the first and second-order moments to the third and fourth-order moments (Lo 2001).[7]

The time-series mean of the cross-sectional Skew and Kurt for a fixed PS s and IO set j are given by

$$\mu_{\mathrm{Skew}_{j,s}} = \frac{1}{N}\sum_{\tau=1}^{N} \mathrm{Skew}_{j,s,\tau} \quad \text{and} \quad \mu_{\mathrm{Kurt}_{j,s}} = \frac{1}{N}\sum_{\tau=1}^{N} \mathrm{Kurt}_{j,s,\tau}, \tag{5}$$

where $\text{Skew}_{j,s,\tau}$ and $\text{Kurt}_{j,s,\tau}$ are the cross-sectional skewness and kurtosis, respectively, for the 5000 randomly selected portfolios for IO set j with a PS of s for month τ; and N is the number of cross-sections.

The literature documents that investors prefer to construct portfolios with positive skewness given that the mean of returns generally falls above the median (Harvey and Siddique 2000; Premaratne and Tay 2002). Consequently, an increase in PS that makes skewness more positive or less negative is considered valuable. Based on Figures 7 and 8, the mean of the time-series of cross-sectional mean skewnesses is highly positive at a PS of two, and decreases monotonically as the PS increases from 2 to all bonds for all IO sets. Thus, the SMB-determined minimum PS of two bonds is preferred for skewness for all IO sets. These results are consistent with those documented by Kryzanowski and Singh (2009) for Canadian equity IO sets where further diversification diminishes the positive skewness associated with not well-diversified portfolios. Interestingly, the returns of the 'All' portfolio are negatively skewed, which is consistent with the results of many studies that find negative skewness for an index (French et al. 1987; Brown et al. 1993; and Campbell and Hentschel 1992, where the latter authors claim that the price reaction tends to be greater for unfavorable compared with favorable events).

In contrast, the kurtosis metric, as shown in Figures 9 and 10, decreases monotically with an increase in PS from 2–100 for all IO sets. If risk-averse investors weigh potential downside returns more than potential upside returns, then these investors will prefer a distribution with low kurtosis, since the tails are more likely to fall closer to the mean. Thus, the risk of an extreme loss decreases as PS increases from 2–100 bonds.[8] PSs of 20–30 bonds capture most of the decrease in kurtosis as PS increases (84–88% except for the 46% for the foreign IO set). When differentiated by short maturities, the minimum PS range remains at 20–30 bonds with the corresponding average decreases in kurtosis in the range of 41–89%. The minimum PS range drops to 15–25 bonds for longer maturities, and the corresponding average reductions range from 25 (foreign IO set) to 86% ('All' IO set). When further differentiated by credit rating, the range of minimum PS is 20–30 bonds, and the corresponding average reductions in kurtosis range from 55 to 87%.

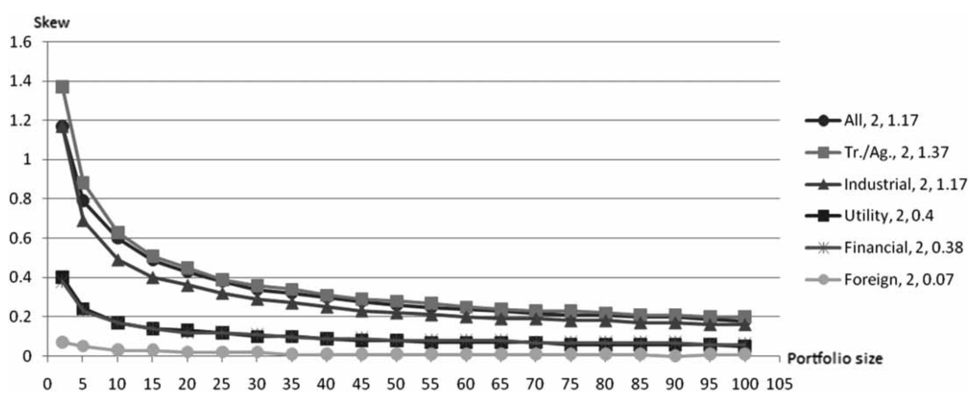

Figure 7. Skewness differentiated by PS and issuer type.
This figure depicts the skewnesses (Skews) of quoted returns differentiated by PS and issuer type for all maturities. The optimal PS is reached when the reduction in the Skew is not more than 1% from incrementing the PS to the next larger PS provided that the difference in the mean Skews for PS s of two and All are significantly different at the 0.05 level. 'Tr./Ag.' refers to Treasury/Agency. The series name, optimal PS, and the Skew value are reported in that order for each series in the legend to the figure.

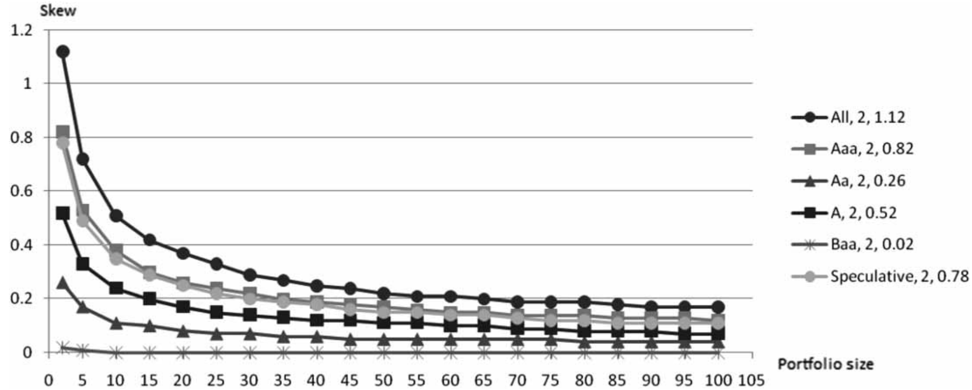

Figure 8. Skewness differentiated by PS and credit rating.
This figure depicts the skewnesses (Skews) of quoted returns differentiated by PS and rating category for all maturities. The optimal PS is reached when the reduction in the Skew is not more than 1% from incrementing the PS to the next larger PS provided that the difference in the mean Skews for PS s of two and All are significantly different at the 0.05 level. 'Tr./Ag.' refers to Treasury/Agency. The series name, optimal PS, and the Skew value are reported in that order for each series in the legend to the figure.

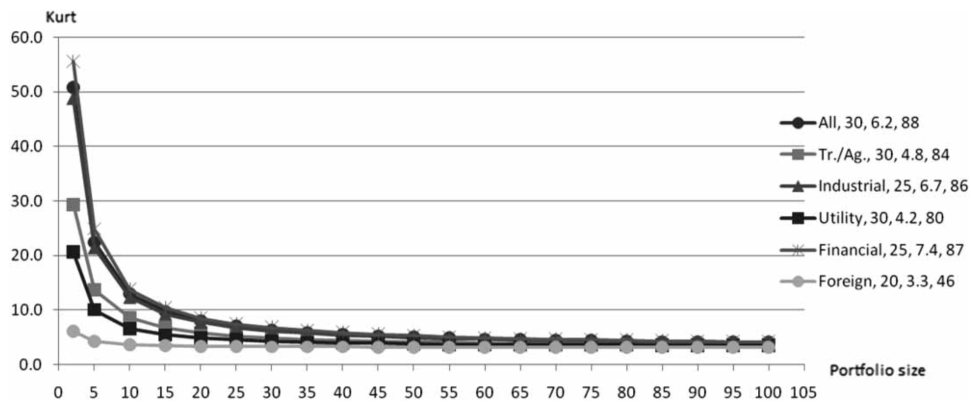

Figure 9. Kurtosises differentiated by PS and issuer type.
This figure depicts the kurtosises (Kurts) of quoted returns differentiated by PS and issuer type for all maturities. The optimal PS is reached when the reduction in the Kurt is not more than 1% from incrementing the PS to the next larger PS provided that the difference in the mean Kurts for PS s of two and All are significantly different at the 0.05 level. 'Tr./Ag.' refers to Treasury/Agency. The series name, optimal PS, the Kurt value, and the percentage reduction in the Kurt from a benchmark PS of two when the optimal PS is reached are reported in that order for each series in the legend to the figure.

Most interestingly, the relation between kurtosis and PS or s is convex; first decreasing as PS increases, and then increasing as PS increases further so that the kurtosis at a PS of All is considerably higher than its corresponding value at a PS of two for all (un)differentiated IO sets. This illustrates a potential difficulty when interpreting changes in kurtosis in isolation. Kurtosis not only measures the tail heaviness of a distribution relative to that of the normal distribution, but it also measures the peakedness of that distribution, and their relative impacts on the kurtosis measure can vary with changing PS.[9] Specifically, the convex relation between skewness and PS

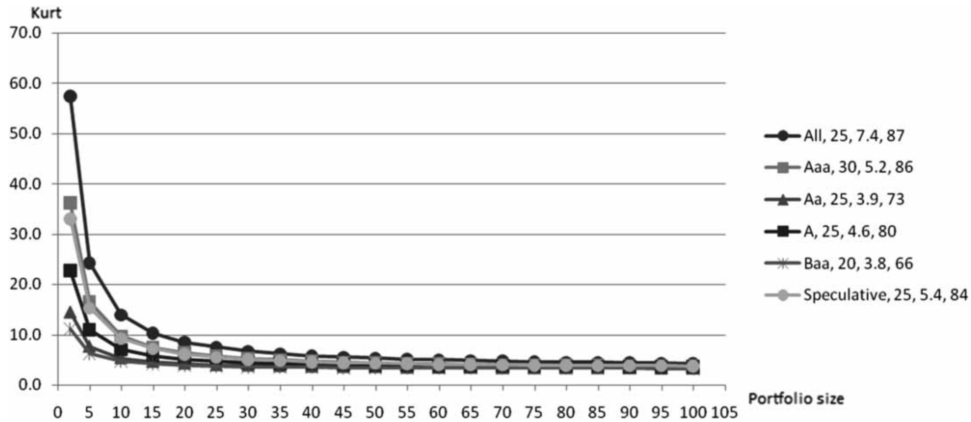

Figure 10. Kurtosises differentiated by PS and credit rating.
This figure depicts the kurtosises (Kurts) of quoted returns differentiated by PS and rating category for all maturities. The optimal PS is reached when the reduction in the Kurt is not more than 1% from incrementing the PS to the next larger PS provided that the difference in the mean Kurts for PSs of two and All are significantly different at the 0.05 level. 'Tr./Ag.' refers to Treasury/Agency. The series name, optimal PS, the Kurt value, and the percentage reduction in the Kurt from a benchmark PS of two when the optimal PS is reached are reported in that order for each series in the legend to the figure.

for a fixed IO set j for month τ occurs because the ratio, $(\sum_{i=1}^{5000}(r_{i,j,s,\tau} - \bar{r}_{j,s,\tau})^4)/\sigma_{j,s,\tau}^4$, first declines in value and then increases in value as s increases, where $(r_{i,j,s,\tau} - \bar{r}_{j,s,\tau})$ is the return deviation for the ith portfolio of size s for IO set j for month τ from its cross-sectional mean return for that month, and $\sigma_{j,s,\tau}$ is the cross-sectional standard deviation of returns for the portfolios of size s for IO set j for month τ. In turn, this means that $\sum_{i=1}^{5000}(r_{i,j,s,\tau} - \bar{r}_{j,s,\tau})^4$ initially declines at a faster rate than $\sigma_{j,s,\tau}^4$ as PS increases, and later declines at a slower rate than $\sigma_{j,s,\tau}^4$ as PS increases further. In contrast, the skewness, which is based on raising mean return deviations and the standard deviation of returns to the third and not fourth power declines monotonically with increasing PS.

To measure the left and right tails, we use the left (LQW) and right (RQW) quantile robust measures of tail weight as introduced by Brys et al. (2006).[10] These measures are not sensitive to the presence of outliers and provide robust measures of tail heaviness. Similar to Brys et al. (2006), we choose to measure the tail weight of the left and right 1/8 quantiles. The LQW(0.125) results are shown in Figures 11 and 12.[11] In all IO sets (without exception), the tail weight for the PS of 'All' is significantly higher than those of the PS of 100. This clearly contributes to the high Kurtosis measures of a PS of 'All'. More interestingly, however, is that the tail weight of a PS of 'All' is not always the highest reported for the IO sets. In fact, some PSs have a higher tail weight (e.g., the tail weight of the IO set 'All' for a PS of 10 is 0.380, whereas it is 0.360 for a PS of 'All'). Given the fact that the kurtosis measures for 'All' are the highest in the IO set even if some PS have a higher-tail weight leads us to conclude that the main factor contributing to the high kurtosis is the peakedness of the distribution.[12]

Unlike the other metrics examined above, the difference between the left tail weight (LTW) metric values at PSs of two and All are not helpful in measuring diversification benefits. This is due to the nonlinear relationship between the tail weights and the PSs, which results in some of the maximum and/or minimum LTW values being associated with a PS different than two or All. Consequently, the total potential diversification benefit for this metric is redefined as the

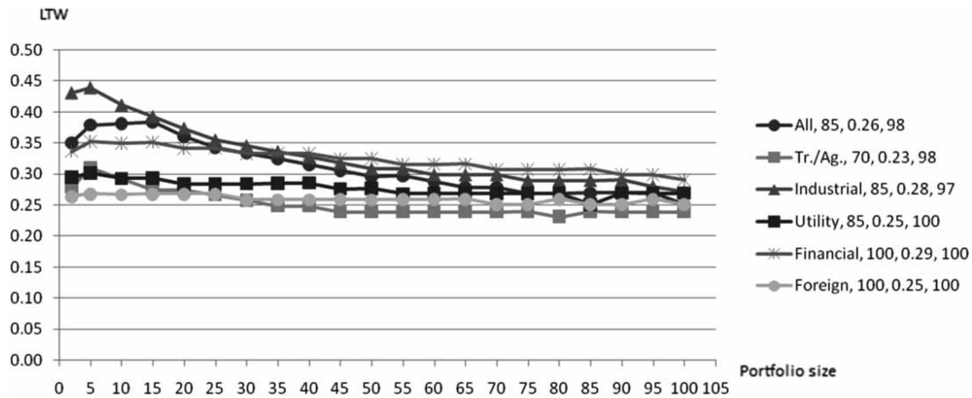

Figure 11. Left tail weights differentiated by PS and issuer type.
This figure depicts the left tail weights (LTWs) of quoted returns differentiated by PS and issuer type for all maturities. The optimal PS is reached when the reduction in the LTW based on the difference between the maximum and minimum LTW values for the IO set is not more than 1% from incrementing the PS to the next larger PS. 'Tr./Ag.' refers to Treasury/Agency. The series name, optimal PS, the LTW value, and the percentage reduction in the LTW from a benchmark PS of 2 when the optimal PS is reached are reported in that order for each series in the legend to the figure.

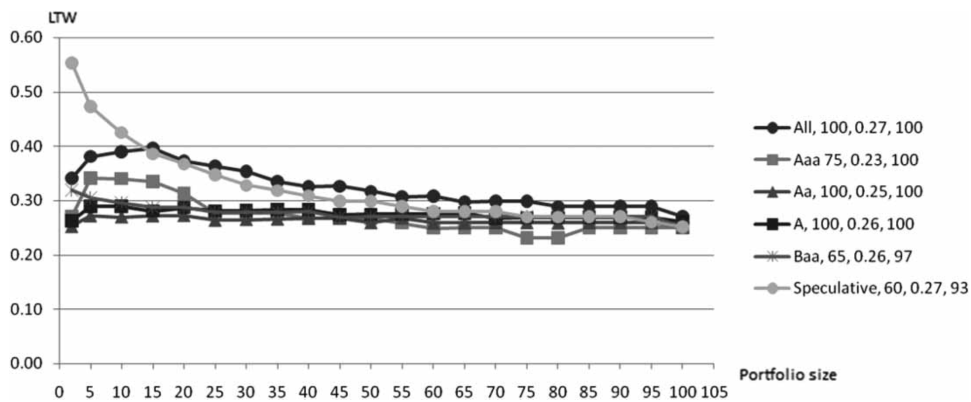

Figure 12. Left tail weights differentiated by PS and credit rating.
This figure depicts the left tail weights (LTWs) of quoted returns differentiated by PS and rating category for all maturities. The optimal PS is reached when the reduction in the LTW based on the difference between the maximum and minimum LTW values for the IO set is not more than 1% from incrementing the PS to the next larger PS. 'Tr./Ag.' refers to Treasury/Agency. The series name, optimal PS, the LTW value, and the percentage reduction in the LTW from a benchmark PS of two when the optimal PS is reached are reported in that order for each series in the legend to the figure.

difference between the maximum and minimum LTW values, and the optimal PS is redefined as the PS beyond which no other PS provides a marginal reduction of more than 1% in this measure of total potential diversification benefits. The optimal PSs are between 80 and 100 bonds for the IO sets differentiated by issuer type. Exceptions occur mainly in the long-term maturity IO sets where the optimal PSs for TR/Ag., Foreign and Industrial are 25, 45 and 65 bonds, respectively.

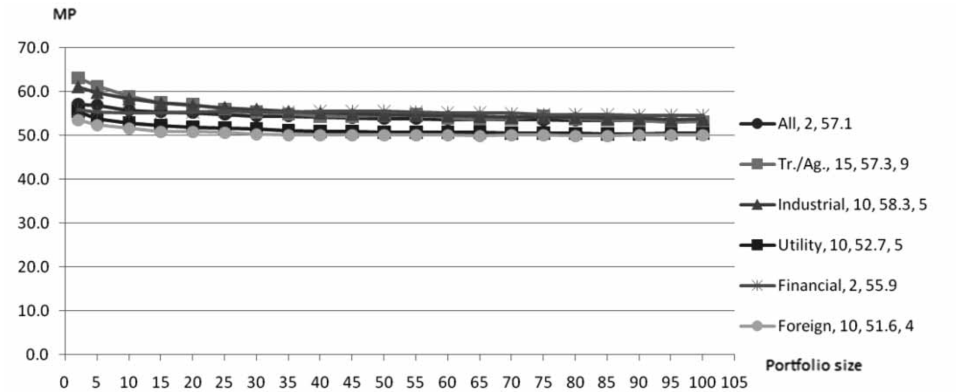

Figure 13. Probabilities of observing market underperformance differentiated by PS and issuer type.
This figure depicts the mean probabilities (MPs) multiplied by 100 that a portfolio of size s that is randomly drawn from IO set j differentiated by issuer type will, on average, underperform the market return over holding periods of 3 years. The optimal PS is reached when the reduction in the MP is not more than 1% from incrementing the PS to the next larger PS provided that the difference in the MPs for PS s of two and All are significantly different at the 0.05 level. 'Tr./Ag.' refers to Treasury/Agency. The series name, optimal PS, the MP value, and the percentage reduction in the MP from a benchmark PS of two when the optimal PS is reached are reported in that order for each series in the legend to the figure.

The optimal PSs are 60–100 bonds for the credit-rating IO sets when undifferentiated by maturity, and are wider at 45–100 and 35–100 bonds for short- and long-term maturities, respectively.

3.5 *Probability of underperforming a target rate of return*

The literature documents that investors are concerned about the probability of portfolio returns falling below a target rate of return (Mao 1970; Xu 2003; Byrne and Lee 2004). We now investigate the probability that the cumulative holding-period return of a portfolio of size s is lower than the cumulative return over the same holding period for an equal-weighted portfolio of all the bonds in the IO set.[13]

Based on the results summarized in Figures 13 and 14, the probability that a portfolio of size s underperforms the market varies somewhat across IO set and PS. Not unexpectedly, the probability of underperforming the market is almost zero, on average, when the PS is one less than all the available bonds in the IO set. The SMB minimum PS does not exceed 15 bonds for any IO set with corresponding potential benefits that do not exceed 9%. We find a higher probability of market underperformance for speculative compared with investment grade bonds for the various PSs. We caution that drawing an inference of relative underperformance of speculative bonds based solely on this metric would be incorrect. As we found earlier, the return-to-risk rewards as captured by the Sortino ratios (Figure 6) for portfolios of speculative bonds, on average, exceed the Sortino ratios for all other bond portfolios with the exception of Baa bonds at PSs above 25 bonds and exceed all other bond portfolios at PSs greater than 75 bonds.

4. Sensitivity analysis

In this section, we conduct various sensitivity analyses to investigate if our choice of bond sample affects the optimal PS. We begin with three samples of time periods of equal length (i.e.,

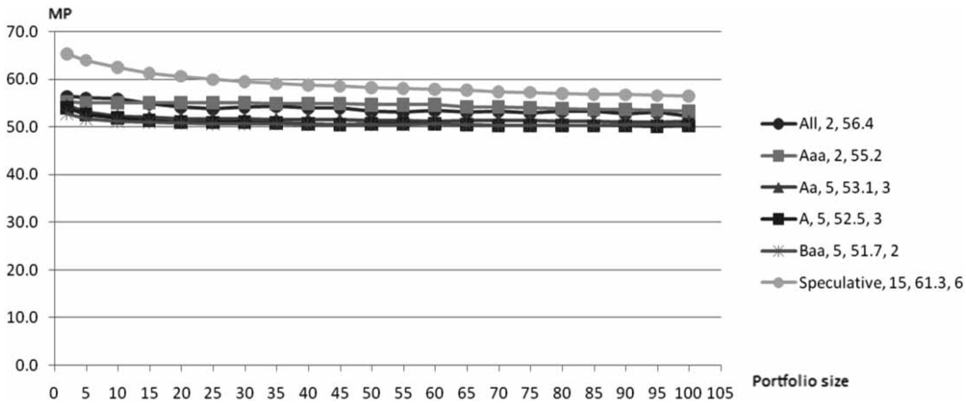

Figure 14. Probability of observing market underperformance differentiated by PS and credit rating.
This figure depicts the mean probabilities multiplied by 100 that a portfolio of size s that is randomly drawn from IO set j differentiated by rating category will, on average, underperform the market return over holding periods of three years. The optimal PS is reached when the reduction in the MP is not more than 1% from incrementing the PS to the next larger PS provided that the difference in the MPs for PS s of two and All are significantly different at the 0.05 level. 'Tr./Ag.' refers to Treasury/Agency. The series name, optimal PS, the MP value, and the percentage reduction in the MP from a benchmark PS of two when the optimal PS is reached are reported in that order for each series in the legend to the figure. Note that the Aa series is eclipsed by the A series.

1986–1989, 1990–1993 and 1994–1997), and investigate the SMB-determined minimum PSs using the MRD metric differentiated by issuer type for the different sample periods. Based on untabulated results, we find that there are no significant changes in the SMB-determined PSs across the samples even though the metric values, potential diversification benefits and the form of distribution differ across these three time periods. In general, the optimal PS is about 40 bonds with associated diversification benefits of about 80%.[14] Second, we restrict our bond choice to straight bonds by excluding bonds with embedded options. Based on untabulated results, we find that in general there is not much difference between the optimal PS for the straight bond IO sets and that for the IO sets that also include bonds with embedded options. The optimal PS for the IO sets of straight bonds exhibit an optimal PS of 30–40 bonds for the MRD metric compared with 35–40 bonds for the samples that include the bonds with embedded options. Similarly, the optimal PS using the skewness metric of two bonds and the kurtosis metric of 20–30 bonds are the same for the IO sets (with)out the bonds with embedded options.

5. Conclusion

In this article, the minimum PSs required to capture most of the diversification benefits from increasing PS for various measures of diversification benefits are examined for IO sets differentiated by issuer type or bond rating, and further differentiated by term to maturity. Most of the diversification benefits are defined herein as the PS for which the marginal benefits from further diversification are less than 1%.

Based on the results summarized in Table 4, we find that the minimum PSs vary not only by issuer type, term-to-maturity and bond rating but also by the metric used to measure the marginal benefits of further diversification. Further, while the marginal benefits of further diversification are generally achieved with PSs of 25–40 bonds, the untapped benefits of full diversification (i.e., holding all bonds in the IO set) at these PSs are still sizeable compared with IO sets of

Table 4. Summary of the minimum portfolio sizes beyond which the marginal benefits of increasing PS are less than 1%.

IO sets differentiated by issuer type							IO sets differentiated by credit rating						
IO set	MDD	MRD	Skew	LTW	Sor	Prob3yr.	IO set	MDD	MRD	Skew	LTW	Sor	Prob3yr.
Panel A: Minimum PS for IO sets differentiated by issuer type for all maturites													
All	45	40	2	85	35	2	All	40	40	2	100	35	2
Tr./Ag.	35	35	2	70	50	15	Aaa	45	35	2	75	45	2
Industrial	40	35	2	85	45	10	Aa	45	40	2	100	25	5
Utility	45	40	2	85	25	10	A	50	40	2	100	30	5
Financial	45	45	2	100	30	2	Baa	45	40	2	65	30	5
Foreign	40	40	2	100	30	10	Spec.	35	40	2	60	30	15
Panel B: Minimum PS for IO sets differentiated by issuer type for maturities $<$ 10 years													
All	35	35	2	95	30	2	All	40	40	2	100	35	2
Tr./Ag.	35	40	2	85	25	2	Aaa	45	40	2	90	25	2
Industrial	30	40	2	85	35	5	Aa	45	40	2	90	30	5
Utility	35	35	2	85	30	15	A	55	40	2	95	25	2
Financial	35	35	2	100	35	2	Baa	55	35	2	45	25	2
Foreign	35	35	2	80	20	2	Spec.	45	35	2	70	35	10
Panel C: Minimum PS for IO sets differentiated by issuer type for maturities $>$ 10 years													
All	50	40	2	95	35	2	All	50	40	2	95	35	2
Tr./Ag.	35	40	2	25	5	2	Aaa	60	40	2	100	45	2
Industrial	45	40	2	65	35	10	Aa	60	40	2	95	30	10
Utility	55	40	2	85	30	5	A	60	40	2	95	35	10
Financial	50	45	2	85	40	10	Baa	45	40	2	45	30	5
Foreign	35	45	2	45	25	10	Spec.	35	40	2	35	35	10

Note: This table reports the minimum PSs beyond which the marginal benefits of increasing the PS to the next PS are less than 1% for various diversification metrics. These include: MDD (mean excess standard deviation), MRD (mean cross-sectional dispersion), skewness (skew), left tail weight (LTW), Sor (Sortino ratio), and probabilities of earning less than the market return over a 3-year holding period (Prob3yr.). 'Tr./Ag.' refers to Treasury/Agency; and 'Spec.' refers to Speculative.

equities. This may explain the empirical findings that unlike equity funds, bond funds generally are value-neutral for unit holders based on gross returns and value-destroying based on net returns (Kahn and Rudd 1995). This is caused by the difficulty and cost for retail investors of forming their own bond portfolios that capture a high percentage of the potential benefits of full diversification and to earn value-neutral benchmark- and risk-adjusted returns through self management.

Acknowledgements

Financial support from the Concordia University Research Chair in Finance, IFM2, SSHRC and SSQRC-CIRPÉE are gratefully acknowledged. The usual disclaimer applies.

Notes

1. Since the foreign bonds are issued in US dollars by non-US companies from all sectors, all of our returns are based on US dollars. Clearly, the number of foreign bonds is much smaller than the number of US bonds, which means that our study has an investor home bias. Nevertheless, it accurately reflects the home-bias selection followed by investors that are referred to as 'the home bias puzzle' in the literature, since international diversification not subject to his bias could achieve higher diversification benefits.

2. The decision to require a minimum of 27 monthly returns to compute the correlations is based on a number of considerations. Requiring fewer monthly returns leads to a bias in the correlations due to the increase in missing observations, whereas requiring no missing values would result in the elimination of too many bonds from our sample. Having 75% of the observations as nonmissing, in our opinion, is an appropriate balance between the negative consequences of requiring too many or too few returns in order to include more bonds (i.e., a more representative sample of bonds) in our correlation computations.
3. When the number of bonds for a specific month is less than 101, the PS of 'All' represented by a portfolio of $N-1$ bonds is lower in size than a PS of 100. Consequently, we eliminate months from our metric calculations where the number of bonds available for selection is less than 101. This results in the elimination of four months for the foreign (short maturities) IO set, one month for financial (long maturities) IO set, one month for foreign (long maturities) IO set, and 32 months for speculative grade (long maturities) IO set.
4. We also examine the normalized portfolio variance metric of Goetzmann and Kumar (2008) that is equal to the ratio of the two variances in the MDD metric, and not to the difference of their standard deviations. Based on untabulated results, a SMB-determined minimum PS of around 20–30 bonds captures a high percentage of potential diversification benefits of 91–98% (except for a capture of only 83% for the foreign IO set for both short and long maturities). Similar results are found for most IO sets differentiated by credit rating. Their SMB-determined minimum PSs are 20–40 bonds that correspond to the capture of 87–98% of the potential overall benefits of diversification.
5. Equal weights are used in forming the portfolios given the findings reported in the literature for equities that no sample-based mean–variance portfolio formation strategy is consistently better in terms of out-of-sample performance than using equal weights. To illustrate, DeMiguel et al. (2007) find that none of the 14 models that they evaluate across seven empirical data sets is consistently better than the $1/N$ rule in terms of Sharpe ratio, certainty-equivalent return, or turnover. They conclude that this indicates that the out-of-sample gain from optimal diversification is more than offset by estimation error.
6. The SMB-determined minimum PSs for the Sharpe ratio range from 30 (Utility) to 50 (industrial) bonds, and the relative increase in the Sharpe ratios range from 80 (Tr./Ag.) to 95% (foreign) for IO sets differentiated by issuer type. The SMB-determined minimum PSs range from 25 (Aa) to 60 (Aaa) bonds with associated relative overall increases of 67% (Aaa) to 92% (Aa-A) for IO sets differentiated by credit rating.
7. By selling out-of-the-money put options on the S&P 500, Lo (2001) obtains a Sharpe ratio of 1.94 for the period from January 1992 to December 1999. This is higher than the corresponding Sharpe ratio of 0.98 for the S&P 500. In Lo's example, the maximum loss for his fund is −18.3% compared with −8.9% for the S&P 500.
8. Unlike the other metrics, diversification benefits are captured by the decrease in kurtosis between a PS of two, and the PS under investigation since measuring the potential diversification benefits as the difference in the kurtosises between a PS of two and PS of 'All' bonds is not applicable due to the very high kurtosis for a PS of 'All' bonds.
9. According to Ruppert (1987), kurtosis measures both peakedness and tail weight, because if probability mass is moved from the flanks to the center of a distribution, then mass has to be moved from the flanks to the tail to keep the scale fixed. As a result, Brys et al. (2006) conclude that, since no agreement exists on what kurtosis really estimates, its use is often restricted to symmetric distributions. They also note that the kurtosis coefficient is very sensitive to outliers in the data.
10. For a continuous univariate distribution F, $\mathrm{LQW}_F(p) = [Q((1-p)/2) + Q(P/2) - 2Q(0.25)]/[Q((1-p)/2) - Q(P/2)]$ and $\mathrm{RQW}_F(q) = [Q((1+q)/2) + Q(1-(q/2)) - 2Q(0.75)]/[Q((1+q)/2) - Q(1-(q/2))]$ in which $0 < p < 1/2$ and $1/2 < q < 1$and where $Q(p) = Q_F(p) = F^{-1}(P)$ is the quantile function. The results of the right tail weight are not reported to conserve valuable journal space.
11. We also test the left medcouple and right medcouple robust measures of tail weight, and the results emit similar implications as those discussed herein (for further details about these tests refer to Brys et al. 2006). We also test the tail behavior for up to 20,000 randomly selected portfolios and again the results have the same patterns. The right tail weight tables are not reported to conserve valuable journal space.
12. We also examine semi-variance with the risk-free rate as the target return given the evidence that bond return distributions are not symmetric and investors dislike negative returns. Based on untabulated results, the SMB minimum PS range is 20–25 bonds, and the overall reductions in the semi-variances range between 93 and 96% for IO sets differentiated by issuer type. Similar results are observed for further differentiation by maturities and for the various IO sets differentiated by credit ratings (un)differentiated by maturities.
13. For each month, the cumulative holding-period return is first calculated. Then, the probability is calculated based on the number of times that the holding-period returns for the specific PS underperforms the holding-period return on the market (the target return).
14. The results reported in this section tend to have the same pattern for the IO sets differentiated by rating category.

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