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The home bias is here to stay

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ABSTRACT

Over the last 15 years, dramatically decreasing foreign investment costs have not reduced the home bias. We show that the home bias induced by a given cost is proportional to the factor $\rho/(1-\rho)$, where ρ is the average correlation between markets. This factor is very sensitive to the correlation, especially when the correlation is high. Empirically, correlations have been steadily increasing from 0.4 in the 90's to about 0.9 today. Thus, the decreasing extra costs are increasingly magnified, explaining the persistence of the home bias, and predicting its continuation.

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1. Introduction

It is well documented that investors exhibit a tendency to grossly overweight the proportion they invest in their domestic market. This phenomenon, known as the home bias, has been shown to hold for both individual and institutional investors, for stocks and for bonds, and for virtually all countries examined (see, for example, French and Poterba (1991), Lewis (1999), Chan et al. (2005), and Vanpée and De Moor (2012)). The home bias seems to imply investment inefficiency, as investors forgo the benefits of international diversification. Several explanations have been suggested for this phenomenon. The main ones are based on various types of extra costs, direct and indirect, associated with foreign investments. These include transaction costs, regulatory constraints, exchange rate risk, and information asymmetries due to barriers to information flow, different accounting standards and corporate culture, and even language barriers.

While there is an ongoing discussion about whether the home bias can be fully rationalized by the above extra costs for foreign investments⁴, we would like to focus attention on the following puzzling phenomenon. Over the last 15 years the foreign investments costs have decreased dramatically. Direct transaction costs have declined significantly⁵, and the internet has revolutionized both the volume and speed of information flow. In addition, there is a continuing trend of unification of accounting standards⁶. Surprisingly, this dramatic reduction in costs has not reduced the home bias. In fact, the home bias has remained remarkably steady, and it is as large today as it was 15 years ago. Fig. 1 documents the magnitude of the U.S. home bias over time. Since 1998 the home bias has leveled-off to about 40%. This persistence of the home bias despite of the dramatic decline in costs seems enigmatic, and it is the focus of the present study.⁷

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² The potential gain from international diversification was first discussed by Grubel (1968) and Levy and Sarnat (1970).

³ See, for example, Black (1974), Solnik (1974), Stulz (1981), Bohn and Tesar (1996), Tesar and Werner (1995), Brennan and Cao (1997), Coval and Moskowitz (2001), Huberman (2001), Chan et al. (2005), Ivkovic and Weisbenner (2005), Kalev et al. (2008), Van Nieuwerburgh and Veldkamp (2009), Beugelsdijk and Frijns (2010), Philips et al. (2012), Cooper et al. (2013), and Fedenia et al. (2013).

⁴ For two excellent reviews and analysis, see Lewis (1999), and Karolyi and Stulz

⁵ For example, Cooper and Kaplanis (1994) estimate the extra fees for foreign investments in 1989 as 0.68% per year. Today, the extra foreign investment fees are about one third of this value, at 0.22%. See section 5A for more detail.

⁶ As of August 2008, 85 countries around the world require International Financial Reporting Standards (IFRS) reporting (see "SEC Proposes Roadmap Toward Global Accounting Standards to Help Investors Compare Financial Information More Easily", U.S. Securities and Exchange Commission press release, 28 August 2008.). There is also convergence between IFRS and US GAAP standards. See Ball (2006) for a discussion of the convergence of accounting standards and their implications.

⁷ Another explanation that has been suggested for the home bias is based on keeping-up with the Joneses preference. If investors care not only about their wealth, but also about their relative standing with respect to the domestic market, it *seems* that this may induce them to increase their exposure to the local market (Lauterbach and Riesman, 2004). While this logic may hold in the case that the correlation between

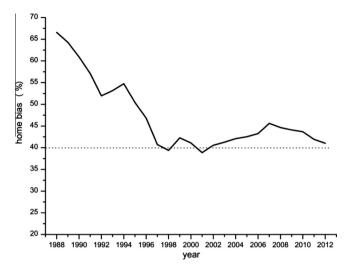


Fig. 1. The home bias over time. The figure shows the development of the U.S. home hias over time. The home hias is calculated following the standard practice of using a country's weight in the world market portfolio as the benchmark (see, for example, Chan et al. (2005), Fidora et al. (2007), and Chen and Yuan (2011)). The home bias is taken as $w_i - w_i^*$, where w_i is the average investment proportion of investors from country i in country i, and w; is the weight of country i in the world market portfolio. The proportion of the U.S. market in the world market is obtained from the World Development Indicators of the World Bank (http://data.worldbank.org/indicator). The average domestic investment proportion in the U.S. is taken from the Board of Governors of the Federal Reserve System (http:// www.federalreserve.gov/datadownload). It is calculated as the holdings of U.S. investors in the U.S. market (total U.S. market capitalization minus foreign investments in the U.S.) divided by the total investments of U.S. investors (including investments outside the U.S.). While the home bias has decreased from about 65% in the 80's to about 40% in 1997, it has since persisted at this high level, despite of the dramatic reduction in foreign investment costs. A similar pattern of a persisting home bias is obtained if one employs alternative measures of the home bias, such as w_i/w_i^* or $\log(w_i/w_i^*)$.

We show that the effect of foreign investments extra costs (or reciprocally, the home-advantage of domestic investments) on the optimal portfolio allocation depends crucially on the average correlation between markets. The additional foreign investment costs relative to the domestic investment cost can be manifested as lower net expected returns to investments in foreign markets (due to transaction costs or information asymmetries, for example), as a higher variance of these returns (due to lower information quality and exchange-rate risk, for example), or as a

the domestic and foreign markets is low, it no longer generally holds when the correlation between the markets is high, and the foreign market is more volatile than the domestic market (which is the realistic case, because of the effects of exchangerate risk, see Section 5). To see this, consider Galí's (1994) classic keeping-up preference $u(c,C)=\frac{1}{1-\alpha}c^{1-\alpha}C^{\alpha\gamma}$ where c is the person's own consumption, C is the consumption of the reference group ("the Joneses"), and $0 < \alpha$, $\gamma < 1$. For the case $\gamma = 0$ this reduces to the standard univariate CRRA function, and the keeping-up with the Joneses motive disappears. In the context of international investments, the reference group is taken as the total return in the investor's domestic market, \widetilde{R}_D , and the expected utility can be written as: $EU = \frac{1}{1-\alpha} E[\tilde{R}_P^{1-\alpha} \cdot \tilde{R}_D^{\alpha\gamma}]$, where \tilde{R}_P is the total return on the investor's portfolio. If the investor invests a proportion x in the domestic market and a proportion 1-x in the foreign market, then $\widetilde{R}_P=x\widetilde{R}_D+(1-x)\widetilde{R}_F$. The following numerical example illustrates that keeping-up with the Joneses preferences can actually decrease the home bias, rather than increase it. Suppose that in one state of the world the return in the domestic market is -10%, and the return in the foreign market is -50%. In the second state of the world the return in the domestic market is 20%, and the return in the foreign market is 80%. The two states are equally likely. Assume that $\alpha = 0.8$. If $\gamma = 0$, the investor has no keeping-up motive, and his optimal proportion in the domestic market is numerically found to be 77%. However, if the investor does have a keeping-up motive and γ = 0.8, the optimal proportion in the domestic market is only 53%. Thus, keeping-up preferences can actually decrease the proportion invested domestically. While in the above simplified example the correlation between the two markets is 1, the same result holds in the case of ρ < 1 and with more than two markets.

combination of both. We show that the effect of both of these types of extra costs on the home bias is proportional to the factor $\rho/(1-\rho)$, where ρ is the correlation between markets. We therefore call this term the "Home Bias Magnification" (HBM) factor:

$$HBM \equiv \rho/(1-\rho)$$
.

This result implies that a given cost will have a home bias effect proportional to 0.5/(1-0.5) = 1 if the correlation is $\rho = 0.5$, but an effect that is 9 times larger if the correlation is $\rho = 0.9$ (0.9) (1-0.9) = 9), and 19(!) times larger if the correlation is $\rho = 0.95$ (0.95/(1-0.95) = 19). This extreme correlation dependence is practically very relevant to the home bias persistence puzzle, as the average correlation between international markets has risen from about 0.4 in 1990 to about 0.9 today. This increase is systematic, and is primarily due to the liberalization of foreign investment regulation and capital account openness (Quinn and Voth, 2008), Fig. 2 depicts the growth of the average correlation over time (panel A), and the corresponding growth of the HBM factor, $\rho/(1-\rho)$, in panel B. We argue that this increased magnification offsets the decreasing foreign investment extra costs, resulting in the persistence of the home bias. We will show that the persisting 40% U.S. home bias can be rationalized by the increasing correlations, even with the prevailing low extra foreign investment costs.

The findings in this paper complement and extend the results of Quinn and Voth (2008) and Levy (2013), who show that the increasing correlations imply a significant reduction in the benefits of international diversification. Quinn and Voth who study the reasons for the increasing correlations note:

"If markets fluctuate in parallel, the advantages of moving money into overseas markets will be much smaller than previously thought – the "home bias" may be smaller than advertised." (p. 535)

Levy (2013) quantifies this statement and shows that a bounded-rational investor who does not diversify internationally does not lose much when correlations are as high as they are today. However, one could argue that a rational investor should take advantage of the international diversification benefits, even if they are not very large. Here we show that even perfectly rational investors should optimally invest much more domestically, because of the small home-advantages (due to even minor extra foreign investment costs) that are magnified by the high correlations.

A comment about terminology: the observation that investors tilt their portfolios toward their domestic market is typically termed a "home bias". Studies suggesting rational explanations for this phenomenon, including the present paper, imply that the increased domestic investment is actually not a bias at all - it is the result of rational optimization by investors who face various economic advantages to investing domestically. Yet, it is common in the literature to use the term "home bias" to describe the increased domestic investment, whether this is rational or not. To avoid confusion, we will use the standard term "home bias" for the empirically observed phenomenon, but will use Increased Domestic Investment (IDI) for the theoretically rationalized domestic tilt. The main message of this paper is that the IDI has remained high, even though the extra costs associated with foreign investments have dramatically reduced, and this is because of the increase in the correlations. Thus, one should rationally tilt investment toward his domestic market.

The structure of the paper is as follows. In the next section we derive analytical results for the magnification factor in the simplified case of two markets. The two market case is important as there is only one correlation and there is no need to make any assumption about the correlation matrix. In Section 3 we extend the results to the more general case of *N* markets. To convey the main idea of this study and to simplify the mathematical analysis, in this section we make the assumption of equal correlations

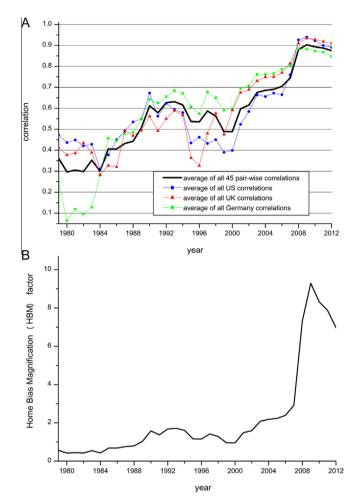


Fig. 2. Correlations and the home bias magnification factor over time. Panel A shows the growth of correlations over time. The bold line depicts the average of all 45 pair-wise correlations between the 10 markets investigated: U.S., U.K., Germany, France, Canada, Japan, Norway, Sweden, The Netherlands, and Australia. The correlations are calculated for annual returns, in terms of \$ US, over a 10-year moving window. Returns are taken from the MSCI database. The average correlation has increased from about 0.3 in the early 1980's to about 0.9 today. The three additional lines show the average correlations for three specific countries, the U.S., the U.K., and Germany, each averaged over the nine other countries. Panel B shows the corresponding growth of the HBM factor, $\rho/(1-\rho)$, calculated with the average correlation.

between any two markets. In Section 4 we relax this assumption and employ the empirical correlation matrix to study the case of heterogeneous correlations. We find that the results with heterogeneous correlations are very similar to those obtained analytically for the case of homogeneous correlations. In Section 5 we calibrate the magnitude of the induced IDI with the empirical estimates of the extra costs involved with foreign investments. We find that at the present level of correlations the relatively small homeadvantages implied by lower domestic direct transaction costs and the lack of exchange-rate risk for domestic investment rationalize the observed home bias. Thus, the "home bias" actually does not seem to constitute a bias. Section 6 concludes with a discussion of the implications for the expected future trends in the home bias. Most mathematical derivations are relegated to the appendix.

2. The case of two markets

Our analysis is conducted in the Mean-Variance (MV) framework, which is optimal in the expected utility paradigm with risk

aversion and normal distributions (more precisely the elliptic family of distributions, see Chamberlain (1983)). Even if the return distributions are not normal, the MV framework provides an excellent approximation for expected utility maximization (see Markowitz and Blay (2014), Chapter 2, and Levy and Markowitz (1979)).

Consider a domestic market with an expected excess return μ_D and standard deviation of returns σ_D , and a foreign market with an expected excess return μ_F and standard deviation σ_F (expected returns are in excess of the risk-free rate). Denote the correlation between the returns in these two markets by ρ . In the case of equal means and variances the investor obviously diversifies equally between the two markets. However, we assume some economic home-advantage, hence the investor tilts the portfolio toward more domestic investment. The economic home-advantage of the domestic market can be manifested by a higher expected return. due to lower transaction costs and less information asymmetries. or by a lower standard deviation, due to more precise information and the absence of exchange-rate risk.8 We denote the first type of home-advantage by Δ , $\Delta \equiv \frac{\mu_D}{\mu} - 1$, where $\bar{\mu} = (\mu_D + \mu_F)/2$, and where the home-advantage implies that $\Delta > 0$ We denote the second type of home-advantage by ε , where $\varepsilon\equiv 1-\frac{\sigma_D}{\sigma_E}$ and the homeadvantage implies that $\varepsilon > 0.9$ In what follows we analyze the effects of these two types of home-advantages on the Increased Domestic Investment (IDI), and show that they both depend on the correlation $\boldsymbol{\rho}$ as a linear function the newly introduced "Home Bias Magnification" (HBM) defined by $\frac{\rho}{1-\rho}$.

We employ the standard procedure for finding the optimal Mean–Variance investment weights (see, for example, Merton (1972) and Roll (1977)). The two international markets' covariance matrix is given by $C = \begin{bmatrix} \sigma_D^2 & \rho \sigma_D \sigma_F \\ \rho \sigma_D \sigma_F & \sigma_F^2 \end{bmatrix}$, and its inverse is given by: $C^{-1} = \frac{1}{1-\rho^2} \begin{bmatrix} \frac{1}{\sigma_D^2} & \frac{-\rho}{\sigma_D \sigma_F} \\ \frac{-\rho}{\sigma_D \sigma_F} & \frac{1}{\sigma_F^2} \end{bmatrix}$.

Denoting the vector of expected excess returns corresponding to the two countries under consideration by μ , standard portfolio optimization implies that $C^{-1} \cdot \mu$ yields the vector of *unscaled* optimal portfolio weights, which we denote by ω :

$$\omega_D = \frac{1}{1 - \rho^2} \left(\frac{\mu_D}{\sigma_D^2} - \frac{\mu_F \rho}{\sigma_D \sigma_F} \right), \quad \text{and}
\omega_F = \frac{1}{1 - \rho^2} \left(\frac{\mu_F}{\sigma_E^2} - \frac{\mu_D \rho}{\sigma_D \sigma_F} \right). \tag{1}$$

These weights are unscaled, i.e. they generally do not add up to 1. The weights in the optimal international tangency portfolio, which we denote by x, are the normalized ω 's given by:

$$\begin{aligned}
\mathbf{x}_{D} &= \frac{\omega_{D}}{\omega_{D} + \omega_{F}} = \left(\frac{\mu_{D}}{\sigma_{D}^{2}} - \frac{\mu_{F}\rho}{\sigma_{D}\sigma_{F}}\right) / \left(\frac{\mu_{D}}{\sigma_{D}^{2}} + \frac{\mu_{F}}{\sigma_{F}^{2}} - \rho \frac{\mu_{D} + \mu_{F}}{\sigma_{D}\sigma_{F}}\right), \\
\mathbf{x}_{F} &= \frac{\omega_{F}}{\omega_{D} + \omega_{F}} = \left(\frac{\mu_{F}}{\sigma_{F}^{2}} - \frac{\mu_{D}\rho}{\sigma_{D}\sigma_{F}}\right) / \left(\frac{\mu_{D}}{\sigma_{D}^{2}} + \frac{\mu_{F}}{\sigma_{F}^{2}} - \rho \frac{\mu_{D} + \mu_{F}}{\sigma_{D}\sigma_{F}}\right),
\end{aligned} (2)$$

In what follows, we use the above formulas for the optimal weights to separately analyze the home-advantage manifested by a higher expected return in the domestic market ($\Delta > 0$), and by a lower volatility in the domestic market ($\varepsilon > 0$).

⁸ Some of the exchange-rate risk may be hedged, but as the future foreign market return is unknown ex-ante, the hedge cannot be perfect, and residual exchange-rate risk remains. We expand on this point in Section 5.

⁹ These two definitions of Δ and ε are not perfectly symmetric, as Δ is expressed in terms of $\bar{\mu}$ and ε is expressed in terms of σ_F , however, they are analytically convenient, as will become evident below.

2.1. Higher expected return in the domestic market

Consider the case where $\sigma_D = \sigma_F$ and the home-advantage due to lower domestic investment cost is expressed as $\mu_D > \mu_F$ (i.e. $\Delta > 0$, $\varepsilon = 0$). In this case, the optimal portfolio weight in the domestic market, given by Eq. (2), simplifies to:

$$x_D = \frac{\mu_D - \rho \mu_F}{(\mu_D + \mu_E)(1 - \rho)}.$$

As $\frac{\mu_D}{\mu_D + \mu_E} = \frac{1}{2} \frac{\mu_D}{\bar{\mu}}$, and $\frac{\mu_F}{\mu_D + \mu_E} = 1 - \frac{1}{2} \frac{\mu_D}{\bar{\mu}}$, where $\bar{\mu} \equiv \frac{\mu_D + \mu_F}{2}$, we have:

$$\begin{split} x_D &= \frac{1}{1-\rho} \left[\frac{1}{2} \frac{\mu_D}{\bar{\mu}} - \left(1 - \frac{1}{2} \frac{\mu_D}{\bar{\mu}} \right) \rho \right] = \frac{1}{2} \frac{\mu_D}{\bar{\mu}} \left(\frac{1+\rho}{1-\rho} \right) - \frac{\rho}{1-\rho} \\ &= \frac{1}{2} \frac{\mu_D}{\bar{\mu}} + \frac{\rho}{1-\rho} \left(\frac{\mu_D}{\bar{\mu}} - 1 \right). \end{split}$$

Recalling the definition $\Delta \equiv \frac{\mu_D}{\bar{\mu}} - 1$, we have:

$$x_D = \frac{1}{2}(1+\Delta) + \frac{\rho}{1-\rho}\Delta.$$

If there is no home-advantage we have $\mu_D = \mu_F = \bar{\mu}$, and therefore Δ = 0, and as a result $x_D = \frac{1}{2}$, as expected. We define the deviation from this symmetric benchmark induced by the home-advantage Δ , as the expected-return driven Increased Domestic Investment, IDI_{EP} :

$$IDI_{ER} \equiv x_D - \frac{1}{2} = \Delta \cdot \left[\frac{1}{2} + \frac{\rho}{1 - \rho} \right] = \Delta \cdot \left[\frac{1}{2} + HBM \right].$$
 (3)

Recall that the IDI is the rationalized increased domestic investment. Eq. (3) reveals first, that if there is no extra cost in foreign investment, Δ = 0, there is no IDI. For the case Δ > 0, the homeadvantage Δ has two effects on the optimal diversification. The first term, $\Delta/2$, is unrelated to the correlation between the two markets, while the second term is proportional to the HBM factor, $\rho/(1-\rho)$. For ρ = 1/3 the HBM term is ½, hence these two terms are equal. However, for ρ = 0.9 the HBM is 9, i.e. it is 18 times larger than the first term. Thus, the effect of the expected return homeadvantage is very sensitive to the correlation, especially when the correlation is large, as is the case in recent years (see Fig. 2).

2.2. Lower volatility in the domestic market

When the home-advantage is manifested as a lower domestic volatility, we have $\mu_D = \mu_F$ and $\sigma_D < \sigma_F$ (i.e. $\Delta = 0$, $\varepsilon > 0$). The optimal proportion in the domestic market, given by Eq. (1), simplifies to:

$$\chi_D = \frac{\frac{1}{\sigma_D^2} - \frac{\rho}{\sigma_D \sigma_F}}{\frac{1}{\sigma_D^2} + \frac{1}{\sigma_E^2} - \frac{2\rho}{\sigma_D \sigma_F}}.$$

Multiplying both the numerator and denominator by σ_D^2 , and recalling that $\frac{\sigma_D}{\sigma_c} \equiv 1 - \varepsilon$ we obtain:

$$x_D = \frac{1 - (1 - \varepsilon)\rho}{1 + (1 - \varepsilon)^2 - 2\rho(1 - \varepsilon)}.$$
 (4)

Assuming that the home-advantage is relatively small, we can neglect second-order terms in ε and employ the Taylor series approximation $(1-\varepsilon)^2\approx 1-2\varepsilon$ to obtain:

$$\begin{split} x_{D} &\approx \frac{1 - \rho + \varepsilon \rho}{1 + (1 - 2\varepsilon) - 2\rho + 2\rho\varepsilon} = \frac{1 - \rho + \varepsilon \rho}{2(1 - \rho)(1 - \varepsilon)} \\ &= \frac{1}{2} \frac{1}{1 - \varepsilon} + \frac{1}{2} \frac{\varepsilon}{1 - \varepsilon} \frac{\rho}{1 - \rho}. \end{split} \tag{5}$$

For ε = 0 we have no home-advantage and, as expected, x_D = 1/2. We denote the deviation from this benchmark case as the volatility-driven Increased Domestic Investment, IDI_{VOL} :

$$IDI_{VOL} \equiv x_D - \frac{1}{2} \approx \frac{1}{2} \cdot \frac{\varepsilon}{1 - \varepsilon} \left[1 + \frac{\rho}{1 - \rho} \right] = \frac{1}{2} \cdot \frac{\varepsilon}{1 - \varepsilon} [1 + HBM].$$
 (6)

Again, the effect on the optimal portfolio weight is composed of two parts, one which is invariant to the correlation, and the other is the HBM factor given by $\rho/(1-\rho)$. For small values of ρ the two terms are comparable, but as ρ becomes larger the HBM term dominates, and the investment proportion becomes very sensitive to ρ .

2.3. C. The role of the Sharpe ratio

In the preceding two sub-sections we separately analyze the home-advantage induced by a higher domestic expected return, and by a lower domestic volatility. In the general case, where both the expected returns and the volatilities differ across markets, the sensitivity of the IDI to the correlation is primarily drive by the difference in the Sharpe ratios across markets¹⁰. To see this, let us denote the domestic home-advantage in terms of Sharpe ratio as:

$$\psi \equiv \frac{SR_D}{SR_F} - 1 = \frac{(\mu_D/\sigma_D)}{(\mu_E/\sigma_F)} - 1,\tag{7}$$

where SR denotes the Sharpe ratio (recall that μ is the expected return in excess of the risk-free rate). Thus, $\psi > 0$ implies a domestic advantage. Employing the domestic investment formula in Eq. (2) we prove in Appendix A that the rationalized increased domestic investment induced by the combined effect is given by:

$$IDI_{combined} \equiv x_D - \frac{1}{2} \approx \frac{\varepsilon}{4} + \psi \frac{1 - \varepsilon}{4} \left(\frac{1 + \rho}{1 - \rho + \psi} \right), \tag{8}$$

where, as before, ε denotes the volatility home-advantage. The interesting insight from Eq. (8) is that if the two markets have the same Sharpe ratios (ψ = 0), the IDI does not depend on the correlation. This represents the unrealistic case of no home-advantage. Realistically, the domestic expected return is typically higher, and the domestic volatility is lower than the foreign counterparts, and the domestic Sharpe ratio is thus higher (ψ > 0). As the Sharpe difference increases, so does the sensitivity of the IDI to the correlation. This is illustrated graphically in Fig. A1 in the Appendix. This result about the role of the Sharpe ratio generalizes to the case of many markets and heterogeneous correlations, as shown numerically in Section 4.

3. The case of N markets

In order to analyze the case with several markets we first make the assumption that all pair-wise correlations are identical, i.e. $\rho_{i,j} = \rho$ for all $i \neq j$, as in Elton and Gruber (1973). In the next section we numerically investigate the case where the correlations are heterogeneous. As we shall see below, the homogenous correlation case greatly simplified the mathematical analysis, and yields results which are not very different result than those obtained in the case of heterogeneous correlations.

In the equal correlations case with *N* markets the unscaled portfolio weight in the *i*th market is given by:

$$\omega_{i} = C^{-1} \cdot \mu = (a - b) \frac{\mu_{i}}{\sigma_{i}^{2}} + Nb \frac{1}{\sigma_{i}} \overline{\left(\frac{\mu}{\sigma}\right)}$$

$$= \frac{1}{\sigma_{i}} \left[\left(\frac{1 + (N - 1)\rho}{1 + (N - 2)\rho - (N - 1)\rho^{2}} \right) \frac{\mu_{i}}{\sigma_{i}} - \left(\frac{\rho N}{1 + (N - 2)\rho - (N - 1)\rho^{2}} \right) \overline{\left(\frac{\mu}{\sigma}\right)} \right] \tag{9}$$

where $\overline{\left(\frac{\mu}{\sigma}\right)} \equiv \frac{1}{N} \sum_{i=1}^{N} \frac{\mu_{i}}{\sigma_{i}}$, and a and b are given by:

$$a = \frac{1 + (N-2)\rho}{1 + (N-2)\rho - (N-1)\rho^2}, \ b = \frac{-\rho}{1 + (N-2)\rho - (N-1)\rho^2}$$

¹⁰ We thank the referee for suggesting this analysis.

The derivation of Eq. (9) is given in Appendix B. (Note that Eq. (1) for two markets is obtained as a special case of Eq. (9) with N = 2). As in the two-market case, we will separately analyze the IDI induced by a higher domestic expected return ($\Delta > 0$), and the one manifested as a lower domestic volatility ($\varepsilon > 0$). In Section 4 we will also numerically analyze these two effects jointly.

In general, the foreign markets may differ in their expected returns and volatilities. In what follows, we analyze the benchmark homogenous case where these *ex-ante* parameters do not differ across the different foreign markets. This homogeneous case facilitates analytical tractability, and it also seems as a reasonable assumption about the *ex-ante* parameters, which are unknown and can only be estimated with estimation errors that are typically large. In Section 4 we also numerically analyze the home bias with the heterogeneous empirical sample parameters, and find that the results are similar to those of the homogeneous case.

3.1. Higher domestic expected return

In this case we assume that $\sigma_F = \sigma_D$ for all foreign markets ($\varepsilon = 0$) and that $\mu_D > \bar{\mu}$ (i.e. $\Delta > 0$), where $\bar{\mu}$ is the expected return averaged across all markets, i.e. $\bar{\mu} \equiv (1/N) \sum_{i=1}^N \mu_i$. The unscaled weights in Eq. (9) simplify to:

$$\begin{split} \omega_{i} &= A \cdot [(1 + (N - 1)\rho)\mu_{i} - \rho N\bar{\mu}] \\ &= A \cdot [(1 - \rho)\mu_{i} + \rho N(\mu_{i} - \bar{\mu})], \end{split} \tag{10}$$

where *A* is a constant identical for all markets,¹¹ which thus disappears when normalizing the unscaled weights to obtain the scaled tangency portfolio weights. The scaled weight of the domestic market in the tangency portfolio is:

$$\begin{split} x_D &= \frac{(1-\rho)\mu_D + \rho N(\mu_D - \bar{\mu})}{(1-\rho)N\bar{\mu}} \\ &= \frac{1}{N} \frac{\mu_D}{\bar{\mu}} + \frac{\rho}{1-\rho} \left(\frac{\mu_D}{\bar{\mu}} - 1\right). \end{split} \tag{11}$$

Recalling that $\frac{\mu_D}{n} - 1 \equiv \Delta$, we have:

$$x_D = \frac{1}{N}(1+\Delta) + \frac{\rho}{1-\rho}\Delta. \tag{12}$$

For the benchmark case with no home-advantage we have $\Delta = 0$, and, as expected, $x_D = 1/N$. The IDI induced by the expected return domestic advantage is the deviation from this benchmark, hence the IDI is given by:

$$IDI_{ER} = x_D - \frac{1}{N} = \Delta \cdot \left[\frac{1}{N} + \frac{\rho}{1 - \rho} \right] = \Delta \cdot \left[\frac{1}{N} + HBM \right].$$
 (13)

Notice that Eq. (3) emerges as a special case of Eq. (13) with N = 2. We again find that the effect of the home-advantage on the domestic investment proportion is linear in the HBM factor $\rho/(1-\rho)$. When N is large, the term that is independent of the correlation, Δ/N , is small. In contrast, the correlation-dependent HBM term, $\Delta \cdot \rho/(1-\rho) = \Delta \cdot HBM$, is independent of N, and is thus generally much larger than the first term, especially for large correlation values. ¹²

3.2. Lower domestic volatility

In this case we assume the same expected return for all markets $(\Delta = 0)$ but a lower volatility in the domestic market $(\varepsilon > 0)$. To analyze this case we need the following two additional assumptions:

(i) The number of markets, N, is relatively large, and (ii) the home-advantage is not very large (which conforms to the empirical data, see Section 5), such that second-order terms of ε can be neglected. In the next section we numerically show that the analytical results obtained under the assumption of large N yield a good approximation even when the number of markets, N, is only 10.

The unscaled weights in Eq. (9) become in this case:

$$\omega_i = B \cdot \left(\frac{(1 - \rho)}{\sigma_i^2} + \frac{\rho N}{\sigma_i} \left(\frac{1}{\sigma_i} - \overline{\left(\frac{1}{\sigma}\right)} \right) \right),$$

and the scaled proportion is given by:

$$\chi_D \approx \frac{1+2\varepsilon}{N} + \varepsilon \frac{\rho}{1-\rho}$$
(14)

(see Appendix C). For ε = 0 we have the case of no home-advantage, and thus, x_D = 1/N, as expected. The deviation from this benchmark is the volatility-induced IDI:

$$IDI_{VOL} = x_D - \frac{1}{N} = \varepsilon \left[\frac{2}{N} + \frac{\rho}{1 - \rho} \right] = \varepsilon \left[\frac{2}{N} + HBM \right].$$
 (15)

As in the case of the home-advantage expressed as a higher expected return, when the home-advantage is expressed by a lower volatility, the HBM term, $\rho/(1-\rho)$, dominates the term 2/N, especially for large N and large ρ . ¹³ Thus, when the correlation is high, the investment proportion in the domestic market is very sensitive to the home-advantage.

4. Heterogeneous correlations

The previous analysis shows that in the case of two markets, as well as in the case of N markets with homogeneous pair-wise correlations ρ , for a given home-advantage, the induced IDI is a linear function of the HBM factor, $\rho/(1-\rho)$. One might suspect that this result is special to the case of equal correlations. In order to examine the robustness of this result, in this section we numerically analyze the case of heterogeneous correlations. We employ the empirical correlation matrix and change the *average* correlation, while maintaining the empirical heterogeneity of the correlations. This procedure allows us to analyze the average correlation effect on the home bias in the heterogeneous correlation case.

As a basis for the analysis we employ the empirical sample correlation matrix for 10 major international markets, as shown in Table 1. To vary the average correlation between markets we "inflate" or "deflate" the empirical correlation matrix ρ by a factor α :

$$\boldsymbol{\rho}^* = \alpha \boldsymbol{\rho} + (1 - \alpha) \boldsymbol{I},\tag{16}$$

where I is the identity matrix, ρ^* is the modified ("inflated" or "deflated") correlation matrix, and α is a positive constant. This transformation multiplies all the pair-wise correlations by a factor α , while keeping the diagonal of the correlation matrix to be ones. For example, for $\alpha=1$, the modified correlation matrix is simply the empirical matrix. For $\alpha=0$ it is the identity matrix, and for $\alpha=0.5$, we obtain a modified correlation matrix where each pairwise correlation is half of its original sample value.

As in the previous sections, we first separately analyze the effect of a home-advantage manifested as a higher domestic expected return ($\Delta > 0$, $\varepsilon = 0$), and as a lower domestic variance ($\Delta = 0$, $\varepsilon > 0$). We then also examine the joint effect of both factors simultaneously ($\Delta > 0$, $\varepsilon > 0$). We employ two frameworks for setting the expected returns and standard deviations. In the first

¹¹ $A = \frac{1}{\sigma^2} \frac{1}{1 + (N-2)\rho - (N-1)\rho^2}$

¹² Note that the derivation of Eq. (13) does not require the assumption that the examte expected returns are the same across the foreign markets. This assumption is only required for the derivation of the volatility induced IDI.

¹³ Note that Eq. (15) is similar to Eq. (6), that is derived for the case of two markets, but plugging N = 2 in Eq. (15) yields a slightly different result. This is because the derivation of (15) assumes that N is large, thus, (15) does not hold exactly for low values of N.

Table 1The empirical correlation matrix. Correlations are calculated from annual returns during the relatively recent 2003–2012 10-year period. Returns are calculated from the perspective of a U.S. investor, in dollars. We do not employ a longer sample period, because the correlations lave been increasing systematically over time (see Fig. 2).

| | US | Canada | Germany | France | The Netherlands | Norway | Sweden | UK | Australia | Japan |
|-----------------|------|--------|---------|--------|-----------------|--------|--------|------|-----------|-------|
| US | 1 | 0.90 | 0.86 | 0.89 | 0.91 | 0.88 | 0.93 | 0.96 | 0.91 | 0.77 |
| Canada | 0.90 | 1 | 0.86 | 0.87 | 0.91 | 0.92 | 0.89 | 0.90 | 0.96 | 0.80 |
| Germany | 0.86 | 0.86 | 1 | 0.94 | 0.89 | 0.79 | 0.83 | 0.85 | 0.85 | 0.76 |
| France | 0.89 | 0.87 | 0.94 | 1 | 0.97 | 0.91 | 0.90 | 0.94 | 0.92 | 0.74 |
| The Netherlands | 0.91 | 0.91 | 0.89 | 0.97 | 1 | 0.95 | 0.88 | 0.96 | 0.96 | 0.68 |
| Norway | 0.88 | 0.92 | 0.79 | 0.91 | 0.95 | 1 | 0.90 | 0.96 | 0.98 | 0.66 |
| Sweden | 0.93 | 0.89 | 0.83 | 0.90 | 0.88 | 0.90 | 1 | 0.94 | 0.92 | 0.79 |
| UK | 0.96 | 0.90 | 0.85 | 0.94 | 0.96 | 0.96 | 0.94 | 1 | 0.96 | 0.71 |
| Australia | 0.91 | 0.96 | 0.85 | 0.92 | 0.96 | 0.98 | 0.92 | 0.96 | 1 | 0.68 |
| Japan | 0.77 | 0.80 | 0.76 | 0.74 | 0.68 | 0.66 | 0.79 | 0.71 | 0.68 | 1 |

Source: MSCI country index database, http://www.msci.com/products/indexes/performance.html.

framework we assume that these parameters are the same across all foreign markets. This seems as a realistically reasonable assumption, as the ex-ante parameters are unknown, and their estimation typically involves large estimation errors. This framework is also consistent with the analytical setting of the preceding section. In the second framework we take the expected return and standard deviation for each market as its empirical sample value (see Table A1 in the appendix for these parameter values). In both frameworks we take the domestic parameters with one of the two types of home-advantages (or both advantages simultaneously). We measure the induced IDI as the difference between the optimal investment proportions with the home-advantage minus the optimal investment proportion without the home-advantage. The optimal proportions are obtained numerically by the standard meanvariance optimization method with a no-short constraint. We assign the home-advantage to one country at a time, and measure the induced IDI for this country. We report the IDI averaged across all countries.

Fig. 3 describes the results of the heterogeneous correlation case obtained in the first framework, where the ex-ante expected returns are assumed to be equal across the foreign markets, and the standard deviations are also assumed to be equal across the foreign markets (but there is a home-advantage to the domestic market). The figure shows the induced IDI, averaged across all countries, as a function of the average correlation. Panel A shows the case of an expected return home-advantage with Δ = 0.05. This value of Δ means that if the foreign market expected return is, for example, 10%, then the domestic expected return is $10 \cdot 1.05 = 10.5$ %. I.e., this does not constitute a very large home-advantage. Panel B shows the case of a lower volatility home-advantage with ε = 0.05 (meaning that if the foreign standard deviation is, say, 20%, then the domestic standard deviation is $0.95 \cdot 20\% = 19\%$). The dashed lines in Panels A and B show the theoretically derived IDI for the case of equal correlations: $IDI_{ER} = \Delta \cdot [\frac{1}{N} + \frac{\rho}{1-\rho}]$ in Panel A (Eq. (13)), and $IDI_{VOL} = \varepsilon \left[\frac{2}{N} + \frac{\rho}{1-\rho} \right]$ in panel B (Eq. (15)).

The figure reveals that the induced IDI in the case of heterogeneous correlations exhibits the same sensitivity to the average correlation as theoretically found in the case of equal correlations, and that the analytical formulas derived under the assumptions of homogeneous correlations and large N yield results very close to those obtained numerically with heterogeneous correlations and N of only 10. Panel C shows the joint effect with both $\Delta = 0.05$ and $\varepsilon = 0.05$. It reveals that the joint effect of both of the homeadvantages simultaneously is very close to the sum of the two separate effects. Thus, the two effects are approximately independent of one another, and their joint influence on the home bias is approximately additive.

Fig. 4 shows the corresponding results in the second framework, where the expected returns and standard deviations are taken as their sample counterparts (and again, the home-advantages are

applied to the domestic market). The results are similar to those obtained under the assumption of homogeneous expected returns and standard deviations across all markets: the induced IDI increases sharply with the correlation when the correlation is high. In addition, the joint effect of both home-advantages is close to the sum of the two separate effects. ¹⁴ Thus, the results are not sensitive to the relaxation of the assumption that the *ex-ante* expected returns and standard deviations are homogeneous across the foreign markets.

The approximate additivity of the two home-advantage effects is closely related and consistent with the central role of the Sharpe ratio in determining the sensitivity of the IDI to the correlation, as derived analytically for the 2-market in Section 2. The additivity result implies that when both home-advantage effects are present, the correlation-dependent term in the IDI is approximately $(\Delta + \varepsilon) \frac{\rho}{1-\rho}$ (see Eqs. (13) and (15)). When the number of markets is large, $\Delta + \varepsilon \approx \psi$, 15 i.e. the sensitivity of the IDI to the correlation depends on ψ , which is the Sharpe ratio domestic home-advantage $(\psi \equiv \frac{SR_D}{SR_C} - 1)$, as also found in the two-market case.

5. Calibration of the induced home bias

How much of the empirically observed home bias can be rationalized with realistic values of the various home-advantages? It is difficult to quantify all of the home-advantages. We focus here on two types of foreign investment costs that can be quantified: transaction costs ($\Delta > 0$) and foreign exchange rate risk ($\varepsilon > 0$). Other types of costs (e.g. poorer information quality and more asymmetry of information in foreign markets), may likely also be important, but are much harder to quantify.

5.1. Transaction costs

Cooper and Kaplanis (1994) use the expense ratio of various types of mutual funds to estimate the additional cost involved with

¹⁴ Mean-variance optimization with the sample parameters typically yields a positive investment proportion in only a few assets, while for most assets the proportions are zero (see, for example, Black and Litterman (1992); If short selling is allowed, the optimization typically yields about half of the assets in short positions, some of which are very extreme). This implies rather unrealistic diversification, and is due to the large sampling errors and the sensitivity of the portfolio weights to the assumed expected returns. This effect tends to mitigate the average induced IDI, because in some cases the investment proportion in the domestic market is zero, and it remains zero also after the home-advantages are introduced (i.e. no IDI is induced). It may also explain the small "bump" in the IDI around an average correlation of about 0.25 revealed in Fig. 4 – below this value there is on average only 1 market with zero investment for our sample of 10 countries. At this correlation value the average number of markets with zero investment jumps to 5, and hence the average IDI drops. When the correlation continues to increase the HBM factor becomes large, and the induced IDI increases again.

¹⁵ To see this, note that for large $N \Delta \equiv \frac{\mu_D}{\mu} - 1 \approx \frac{\mu_D}{\mu_F} - 1$ (as there is only one domestic market, but many foreign markets). Recall that ε is defined as $\varepsilon \equiv 1 - \frac{\sigma_D}{\sigma_F}$, and that this number is not large. Thus: $\psi \equiv \left(\frac{\mu_D}{\sigma_D}\right) / \left(\frac{\mu_F}{\sigma_F}\right) - 1 = \frac{\mu_D}{\mu_E} \frac{\sigma_F}{\sigma_F} - 1 \approx \frac{1+\Delta}{1-\varepsilon} - 1 = \frac{\Delta+\varepsilon}{1-\varepsilon} \approx \Delta + \varepsilon$.

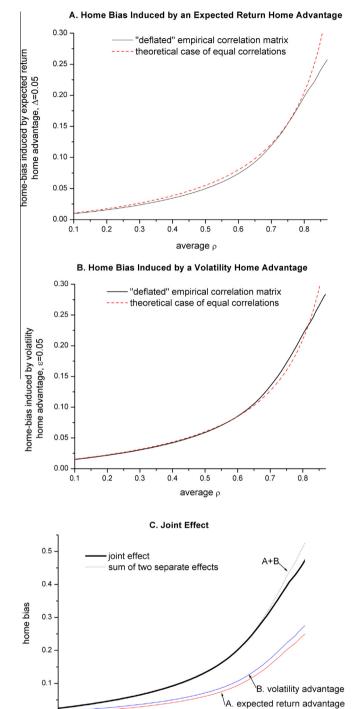


Fig. 3. The IDI as a function of the average correlation, for a given home-advantage. In panel A the home-advantage is expressed as a higher domestic expected return of Δ = 0.05. In panel B it is expressed as a lower domestic volatility of ε = 0.05. The portfolio optimization is conducted with a no-short constraint. The dashed lines show the theoretical predictions derived for the case of equal correlations (Eqs. (13) and (15)). The average correlation is varied by "inflating" or "deflating" the empirical sample correlation matrix according to Eq. (16). Panel C shows that the joint effect of both advantages (bold) is similar to the sum of the two separate effects (dashed).

0.5

average ρ

0.6

0.7

0.3

0.4

investing in foreign markets. They find that the expense ratio of US diversified mutual funds in 1989 was 1.20% while it was 1.88% for diversified funds investing overseas. Thus, they estimate the additional annual cost of foreign investments to be 0.68% (see Cooper

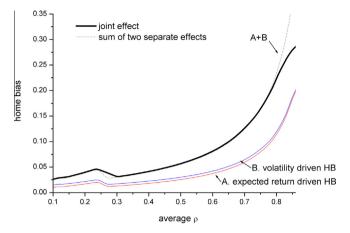


Fig. 4. The IDI as a function of the average correlation, for a given home-advantage with the empirical parameters, namely, the case of the heterogeneous sample expected returns and standard deviations (see Table A1 in the appendix for these values). The induced IDI increases sharply with the correlation when the correlation becomes large, just as in the case of homogenous expected returns and homogeneous standard deviations (compare with Fig. 3). The joint effect of both higher expected return and lower volatility home-advantages (bold line) is similar to the summation of the two separate effects (dotted line), thus, the effects are approximately additive. See footnote 13 for a possible explanation of the "bump" around the average correlation value of 0.25.

and Kaplanis (1994) p. 56). Taking a ballpark estimate of an expected annual return of μ_D = 10% in the domestic market, we have μ_F = 10% – 0.68% = 9.32% in the foreign market, and a Δ value of Δ = (μ_D/μ_F) – 1 = (10/9.32) – 1 = 0.073. Table 2 provides the IDI induced by the expected return home-advantage $IDI_{ER} = \Delta \cdot \left[\frac{1}{N} + \frac{\rho}{1-\rho}\right]$, as derived in Section 3 (see Eq. (11)), for various levels of the home-advantage and the correlation. For Δ = 0.07 the table implies an IDI of 5.4% when a correlation of ρ = 0.4, typical of the late 80's and early 90's, is employed. At the current correlation level of ρ = 0.9, the same home-advantage implies a much-higher IDI of 63.7% (see shaded cells in Table 2).

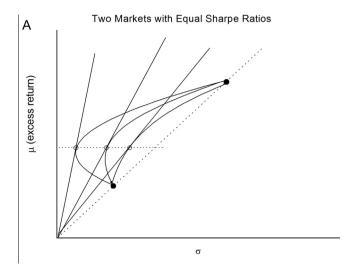
Nowadays, the extra transaction costs for foreign investments are much lower. ETFs have reduced not only the absolute fees, but also the difference between foreign and domestic investments. For instance, the average annual fee for the top 10 U.S. large cap ETFs in the 2013 U.S. News Report is 0.087%. The average fee for the top 10 international ETFs in the same report is 0.311%. Thus, following the Cooper and Kaplanis (1994) methodology, the extra cost to foreign investments can be estimated as the difference between these fees, which is 0.224%. With the ballpark estimate of μ_D = 10% this translates into a Δ value of $\Delta = (\mu_D/\mu_F) - 1 = (10/9.776) - 1 = 0.023$. At the current correlation of ρ = 0.9 this Δ value implies an IDI in excess of 18% (see Table 2). Thus, even though the foreign investment extra transaction costs are only a third of what they were in the late 80's, because of the increased correlations they induce an IDI which is more than three times its value with the 80's parameters (namely, with ρ = 0.4 and Δ = 0.07, which prevailed in the 80's).

5.2. Exchange rate risk

Consider an American investor investing in a foreign market, say in Germany. The fluctuation in the foreign exchange rate may decrease or increase the return variance in dollar terms relative to the return variance in Euros, depending on the correlation between the exchange rate fluctuations and the fluctuations in the rates of return in the German equity market.

To examine whether exchange rate fluctuations increase or decrease the volatility from the perspective of a foreign investor,

¹⁶ http://money.usnews.com/funds/etfs.



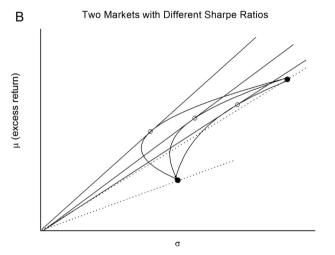


Fig. A1. The Sharpe ratio and correlation sensitivity. When the two markets have the same Sharpe ratios (ψ = 0) the investment proportions, and hence the IDI, do not depend of the correlation, as shown in Panel A (recall that μ denotes the expected return in excess of the risk-free rate). Panel B shows the case of markets with different Sharpe ratios. In this case, the investment proportions and IDI are sensitive to the correlation between the two markets, as implied by Eq. (8).

and by how much, we calculated the annual rates of return on the 10 international markets for the 25-year period 1988-2012. The rates of return are calculated in local currencies as well as in US dollars, in Euro and in British pound. Then we calculated for each country the variance in its domestic currency as well as in the three major currencies mentioned above. Table 3 provides the average variances in the local currencies as well as the average variance in the three major currencies across all 10 countries. As we can see from the table, the risk is larger for the foreign investor relative to the risk for the domestic investor, as measured by local currencies. For example, the average risk for the American investor who invests in all 10 countries, when returns are measured in dollar terms is 8.8% higher than the average risk to domestic investors as measured in local currencies. Thus, the exchange-rate risk is a substantial factor, which may induce a home bias. In terms of our notations, σ_F/σ_D = 1.088 implies that $\varepsilon=1-\frac{\sigma_D}{\sigma_F}=0.08$. For a UK investor the exchange rate risk implies a similar value of ε = 0.07, while for an investor with the EURO as the domestic currency the value is eve larger at ε = 0.23 (see Table 3).

One may suspect that by employing currency hedging one can eliminate the exchange-rate risk. However, this is not the case, for at least two sources of basis risk. First, as we are dealing with stochastic investments, the amount to hedge is not known in advance (i.e. the American investor does not know how many Euros she will have at the end of the period to exchange into dollars). One could attempt partial hedging, but this introduces a new kind of risk – pure exchange rate risk. For example, if the American investor entered a forward contract to exchange 100 Euros for \$120, but at the end of the period her foreign equity is worth only 80 Euros, she ends up taking an extra foreign exchange gamble on the missing 20 Euros. Second, currency derivatives have a pre-specified maturity date. Thus, they require the foreign investor to either liquidate her foreign equity holdings at that date, or alternatively, to keep them but to be exposed to the risk of the naked derivative. In sum, the foreign exchange-rate risk cannot be eliminated by hedging, and a large part of this risk is inherent to international diversification.

Table 4 provides the IDI induced by the higher volatility cost $IDI_{VOL} = \varepsilon \cdot \left[\frac{2}{N} + \frac{\rho}{1-\rho}\right]$ as a function of the volatility home-advantage, ε , and the correlation, ρ . The empirically observed ε is 0.08 for the U.S. investor. With an average correlation of 0.4, the exchange rate risk can implies an IDI of only 6.9%. However, at the current average correlation of 0.9, the same exchange rate risk induces an IDI of over 73% (see shaded cells in Table 4). Even if most of the exchange rate risk can be hedged away, and the volatility home-advantage is only ε = 0.03 (which is a very optimistic assumption), the induced IDI is 27.6%. As the exchange rate risk seems to be an unavoidable element of international diversification, the high international correlations imply that a large IDI seems to be also unavoidable.

To summarize, realistic values of the extra transaction costs and the exchange rate risk involved with foreign investments induce a large IDI and can rationalize the empirically observed persisting home bias.

5.3. C. The role of correlations

Table 1 reveals that Japan is systematically less correlated with the other 9 markets than they are amongst themselves: Japan's average correlation is 0.73, while the average correlation among the other 9 markets excluding Japan is 0.91. Intuitively, this may suggest a lower home bias in Japan, because Japanese investors have more to gain by diversifying internationally. Of course, this is under the assumption that all else is equal, and even then, this is only a rough intuition, because it is well-known that the portfolio weights are determined as a complex function of *all* correlations (and means and standard deviations).¹⁷ Still, it is interesting to note that this intuition about Japan conforms with the empirical findings of Chan et al. (2005) who examine the international holdings of more than 20,000 mutual funds from 26 countries. They find that the domestic home bias in Japan is 1.86, much lower than the average domestic home bias of 3.53 in all 26 countries (see Table 3, p. 1510 in Chan et al.)¹⁸. Japan's low correlations also imply that non-Japanese investors have more to gain in terms of diversification by investing in Japan. Indeed, Chan et al. find that the foreign bias is lower than average in Japan. 19

 $^{^{17}}$ For example, even if all the assets have exactly the same mean returns and the same variances, and asset A has a lower average correlation than asset B, this does not necessarily imply that the weight of asset A in the optimal mean-variance portfolio will be higher than the weight of asset B – it could be lower because of the overall correlation structure among all assets.

¹⁸ Chan et al. (2005) define the domestic home bias of country j as $\log(w_{jj}/w_j^*)$, where w_j^* is the weight of country j in the world market portfolio, and w_{jj} is the average investment proportion of mutual funds from country j in country j.

¹⁹ The foreign bias is defined as $\log(w_{ij}/w_j^*)$ where w_{ij} is the average investment proportion of mutual funds from country i in country j. Chan et al. (2005) report the average foreign bias of country j, averaged across all countries $i \neq j$. The average foreign bias across all 26 countries is -1.42, while it is only -0.97 in Japan (see Table 3 in Chan et al.). This means that non-Japanese investors under-invest in Japan (because they over-invest domestically), but to a *lesser* extent than their average under-investment in other foreign countries with higher correlations.

 Table 2

 Calibration of the Increased Domestic Investment (IDI) induced by extra foreign investment transaction costs.

| Δ ϱ | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
|--------------------|-----|-----|-----|-----|------|------|------|------|------|-------|
| 0.01 | 0.1 | 0.2 | 0.4 | 0.5 | 0.8 | 1.1 | 1.6 | 2.4 | 4.1 | 9.1 |
| 0.02 | 0.2 | 0.4 | 0.7 | 1.1 | 1.5 | 2.2 | 3.2 | 4.9 | 8.2 | 18.2 |
| 0.03 | 0.3 | 0.6 | 1.1 | 1.6 | 2.3 | 3.3 | 4.8 | 7.3 | 12.3 | 27.3 |
| 0.04 | 0.4 | 0.8 | 1.4 | 2.1 | 3.1 | 4.4 | 6.4 | 9.7 | 16.4 | 36.4 |
| 0.05 | 0.5 | 1.1 | 1.8 | 2.6 | 3.8 | 5.5 | 8.0 | 12.2 | 20.5 | 45.5 |
| 0.06 | 0.6 | 1.3 | 2.1 | 3.2 | 4.6 | 6.6 | 9.6 | 14.6 | 24.6 | 54.6 |
| 0.07 | 0.7 | 1.5 | 2.5 | 3.7 | 5.4 | 7.7 | 11.2 | 17.0 | 28.7 | 63.7 |
| 0.08 | 0.8 | 1.7 | 2.8 | 4.2 | 6.1 | 8.8 | 12.8 | 19.5 | 32.8 | 72.8 |
| 0.09 | 0.9 | 1.9 | 3.2 | 4.8 | 6.9 | 9.9 | 14.4 | 21.9 | 36.9 | 81.9 |
| 0.10 | 1.0 | 2.1 | 3.5 | 5.3 | 7.7 | 11.0 | 16.0 | 24.3 | 41.0 | 91.0 |
| 0.11 | 1.1 | 2.3 | 3.9 | 5.8 | 8.4 | 12.1 | 17.6 | 26.8 | 45.1 | 100.1 |
| 0.12 | 1.2 | 2.5 | 4.2 | 6.3 | 9.2 | 13.2 | 19.2 | 29.2 | 49.2 | 109.2 |
| 0.13 | 1.3 | 2.7 | 4.6 | 6.9 | 10.0 | 14.3 | 20.8 | 31.6 | 53.3 | 118.3 |
| 0.14 | 1.4 | 3.0 | 4.9 | 7.4 | 10.7 | 15.4 | 22.4 | 34.1 | 57.4 | 127.4 |
| 0.15 | 1.5 | 3.2 | 5.3 | 7.9 | 11.5 | 16.5 | 24.0 | 36.5 | 61.5 | 136.5 |

The table shows the IDI induced by direct extra transaction costs as a function of the cost and the correlation between markets. The IDI is calculated by Eq. (13), for the case of N=10 markets. The foreign investment extra transaction cost is estimated by Cooper and Kaplanis (1994) as 0.68% annually. Taking a benchmark expected return of 10% for the domestic market, this cost implies an expected return of 9.32% for the foreign market, and therefore a Δ of (10/9.32) -1=0.073. The shaded row in the table shows the induced IDI for a slightly lower value of $\Delta=0.07$. For a correlation value of 0.4, which was typical of the 1980's, the transaction cost induces an IDI of only 5.4%. However, with the current correlation values of 0.9, the same cost induces an IDI of 63.7%. Even if the extra cost has been reduced to 0.22% (a Δ of approximately 0.023), with current correlation values of 0.9 the IDI induced is more than 18%.

Table 3
The standard deviation of returns in local currencies, versus the standard deviation of a foreign investor who is also exposed to exchange rate risk. Annual returns for the 25-year period 1988–2012 are employed. For each country the standard deviation of its market's annual returns is calculated in local currency. It is then also calculated in US Dollar, Euro, and British Pound. The table shows the average results across all 10 countries. For a U.S. investor, the exchange rate risk increases the standard deviation by 8.8% on average, and implies a domestic volatility advantage of ε = 0.08.

| | Average standard deviation of returns (%) | Percentage increase of the standard deviation in foreign currency $\frac{\sigma_E}{\sigma_D}-1$ | $\varepsilon=1-rac{\sigma_{D}}{\sigma_{F}}$ |
|-------------------|---|---|--|
| In local currency | 22.8 | 0 | 0 |
| In US dollar | 24.8 | 0.088 (8.8%) | 0.08 |
| In Euro | 29.7 | 0.30 (30%) | 0.23 |
| In British pound | 24.5 | 0.075 (7.5%) | 0.07 |

Source: MSCI country index database, http://www.msci.com/products/indexes/performance.html.

 Table 4

 Calibration of the Increased Domestic Investment (IDI) Induced by Exchange-Rate Risk.

| ε ρ | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
|----------------------|-----|-----|-----|-----|------|------|------|------|------|-------|
| 0.01 | 0.2 | 0.3 | 0.5 | 0.6 | 0.9 | 1.2 | 1.7 | 2.5 | 4.2 | 9.2 |
| 0.02 | 0.4 | 0.6 | 0.9 | 1.3 | 1.7 | 2.4 | 3.4 | 5.1 | 8.4 | 18.4 |
| 0.03 | 0.6 | 0.9 | 1.4 | 1.9 | 2.6 | 3.6 | 5.1 | 7.6 | 12.6 | 27.6 |
| 0.04 | 0.8 | 1.2 | 1.8 | 2.5 | 3.5 | 4.8 | 6.8 | 10.1 | 16.8 | 36.8 |
| 0.05 | 1.0 | 1.6 | 2.3 | 3.1 | 4.3 | 6.0 | 8.5 | 12.7 | 21.0 | 46.0 |
| 0.06 | 1.2 | 1.9 | 2.7 | 3.8 | 5.2 | 7.2 | 10.2 | 15.2 | 25.2 | 55.2 |
| 0.07 | 1.4 | 2.2 | 3.2 | 4.4 | 6.1 | 8.4 | 11.9 | 17.7 | 29.4 | 64.4 |
| 0.08 | 1.6 | 2.5 | 3.6 | 5.0 | 6.9 | 9.6 | 13.6 | 20.3 | 33.6 | 73.6 |
| 0.09 | 1.8 | 2.8 | 4.1 | 5.7 | 7.8 | 10.8 | 15.3 | 22.8 | 37.8 | 82.8 |
| 0.10 | 2.0 | 3.1 | 4.5 | 6.3 | 8.7 | 12.0 | 17.0 | 25.3 | 42.0 | 92.0 |
| 0.11 | 2.2 | 3.4 | 5.0 | 6.9 | 9.5 | 13.2 | 18.7 | 27.9 | 46.2 | 101.2 |
| 0.12 | 2.4 | 3.7 | 5.4 | 7.5 | 10.4 | 14.4 | 20.4 | 30.4 | 50.4 | 110.4 |
| 0.13 | 2.6 | 4.0 | 5.9 | 8.2 | 11.3 | 15.6 | 22.1 | 32.9 | 54.6 | 119.6 |
| 0.14 | 2.8 | 4.4 | 6.3 | 8.8 | 12.1 | 16.8 | 23.8 | 35.5 | 58.8 | 128.8 |
| 0.15 | 3.0 | 4.7 | 6.8 | 9.4 | 13.0 | 18.0 | 25.5 | 38.0 | 63.0 | 138.0 |

The table shows the IDI induced by the exchange rate risk for different levels of the exchange rate risk and the assumed correlation. The IDI is calculated by Eq. (15), for the case of N = 10 markets. For the rather conservative estimate that the exchange rate risk increases the standard deviation by about 9% (i.e. $\sigma_F/\sigma_D = 1.09$) we have $v = 1 - \sigma_D/\sigma_F \approx 0.08$. With this value and a correlation of 0.4, the exchange rate risk can rationalize a home bias of only 6.9%. However, with the current correlation of 0.9, the same exchange-rate risk induces an IDI of over 73%.

Table A1Empirical average return and standard deviations.

| Country | Mean (%) | S.D. (%) | |
|-----------------|----------|----------|--|
| US | 11.5 | 18.7 | |
| Canada | 10.2 | 17.5 | |
| Germany | 12.1 | 25.8 | |
| France | 11.6 | 24.6 | |
| The Netherlands | 12.3 | 23.0 | |
| Norway | 14.5 | 29.4 | |
| Sweden | 17.8 | 31.9 | |
| UK | 9.9 | 16.2 | |
| Australia | 11.2 | 17.6 | |
| Japan | 1.2 | 23.4 | |

Means and standard deviations are calculated from annual returns in local currencies, for the 25-year period 1988–2012. Returns are taken from the MSCI country index database.

Japan is rather special in its relatively low correlations. For countries with more typical correlations the complex interactions between all correlations in the optimization process confounds the intuitive relation between correlations and investment weights observed for Japan. Chan et al. (2005) find mixed evidence regarding the role of correlations on the bias (see Tables V, VI, and VII in their paper), however, Coeurdacier and Guibaud (2011) find that controlling for various sources of market frictions, investors do tilt their international portfolios toward countries that have lower correlations with their domestic market, as theoretically anticipated.

6. Conclusions

Tilting investments toward the investor's domestic market is commonly termed the "home bias" phenomenon. However, this tilt can be at least partially rationalized by various extra costs to foreign investments. Indeed, there are various home-advantages to investing domestically: lower transaction costs, less information asymmetries, the absence of exchange-rate risk, etc. These advantages induce a rational Increased Domestic Investment (IDI). In this paper we investigate whether the empirically observed domestic tilt can be rationalized by the economic home-advantages.

Obviously, the lower the home-advantage, one would expect a lower IDI. Over the last 15 years most of these home-advantages have dramatically declined, in large part due to the internet revolution and the convergence to standardized international accounting conventions. However, counter-intuitively, the home bias has not decreased over this period, and it is as large today as it was 15 years ago, persisting at the 40% level. This paper suggests an explanation for this home bias persistence puzzle.

We show that the effect of a given home-advantage on the IDI depends crucially on the average correlation between markets, ρ . Specifically, both home-advantages that are manifested as a higher domestic expected return, and home-advantages that are manifested as a lower domestic volatility induce an IDI that depends on the Home Bias Magnification factor, a concept introduced in this study, given by HBM = $\rho/(1-\rho)$. This implies that advantages that induce a negligible IDI when correlations are in the range 0.4-0.5, induce a dramatically larger IDI when correlations are in the range 0.85-0.95. This suggests an explanation for the persistence of the home bias in face of the declining home-advantages - during the last 15 years the average correlation between international markets has increased steadily and systematically from about 0.5 to about 0.9. Thus, the declining home-advantages are magnified by the increasing correlations, resulting in a U.S. home bias that persists at its high level of about 40%.

Why have correlations been steadily increasing over the last few decades? Bekaert and Harvey (2000) and Quinn and Voth

(2008) show that the correlation increase is driven primarily by easing the regulation on foreign investment, expressed mainly by liberalization and capital account openness. Thus, the increase in the correlations is in part due to the increase in international diversification by investors. However, the higher the correlations, the lower the benefit from diversification. This, in turn, decreases and the motivation to continue increasing foreign investments. Therefore, one would expect correlations to reach an equilibrium value. As the global market became more open, and foreign investments costs have declined, the equilibrium correlation has shifted to higher and higher values. How further can this trend continue? The analysis in this paper shows that the IDI grows according to the HBM factor, $\rho/(1-\rho)$, which implies that when the correlations are large, the IDI becomes very sensitive to any further increase in the correlation. Thus, with average correlations of about 0.9, even a very little difference in transaction costs or in perceived risk rationalize a very large home bias. Therefore, the present high level of correlation seems to suggest that we are close to the point where the trend can no longer continue.

Will the home bias eventually disappear if costs to international investments continue to decline? We believe that a substantial home bias will remain in the future, for two reasons. First, one source of foreign investment disadvantage, the exchange rate risk, is not expected to disappear. The additional risk implied by exchange rates cannot be fully eliminated by hedging, and at the current high level of correlations can easily induce a substantial IDI in the range of 20–30%. Second, as some costs decrease, and international diversification increases, this induces a further increase in the equilibrium correlations, as discussed above. Thus, the remaining costs have an even larger effect on the home bias. So it seems that the home bias is here to stay...

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Appendix A.

A. Deivation of Eq. (8)

Eq. (2) can be expressed in terms of the Sharpe ratios:

$$x_D = \frac{\frac{1}{\sigma_D}(SR_D - \rho SR_F)}{\frac{1}{\sigma_D}(SR_D - \rho SR_F) + \frac{1}{\sigma_F}(SR_F - \rho SR_D)}.$$

Employing the definition of the Sharpe ratio home-advantage, $\psi \equiv \frac{SR_D}{SR_F} - 1$, we have $SR_D = (1 + \psi) SR_F$ and therefore:

$$\textit{x}_D = \frac{\frac{1}{\sigma_D} \textit{SR}_F(1+\psi-\rho)}{\frac{1}{\sigma_D} \textit{SR}_F(1+\psi-\rho) + \frac{1}{\sigma_F} \textit{SR}_F(1-\rho(1+\psi))}, or:$$

$$\chi_D = \frac{1+\psi-\rho}{1+\psi-\rho+\frac{\sigma_D}{\sigma_F}(1-\rho-\rho\psi)}$$

Recalling that $\frac{\sigma_D}{\sigma_E} \equiv 1 - \varepsilon$, and dividing by $1 + \psi - \rho$ we obtain:

$$x_D = \frac{1}{1 + (1 - \varepsilon)(\frac{1 - \rho - \rho \psi}{1 - \rho + \psi})} = \frac{1}{2 - \varepsilon - \frac{\psi(1 + \rho)(1 - \varepsilon)}{1 - \rho + \psi}}.$$

Let us define $a \equiv \varepsilon + \frac{\psi(1+\rho)(1-\varepsilon)}{1-\rho+\psi}$. By the assumption that the homeadvantages are small ($\varepsilon < < 1$, $\psi < < 1$), a is also small for as long as the correlation is not perfect. Employing the Taylor series expansion $\frac{1}{2-a} \approx \frac{1}{2} + \frac{a}{4}$ for small a, we have:

$$x_D \approx \ \frac{1}{2} + \frac{\varepsilon}{4} + \psi \frac{1 - \varepsilon}{4} \left(\frac{1 + \rho}{1 - \rho + \psi} \right).$$

Finally,

$$IDI_{combined} \equiv x_D - \frac{1}{2} \approx \frac{\varepsilon}{4} + \psi \frac{1 - \varepsilon}{4} \left(\frac{1 + \rho}{1 - \rho + \psi} \right)$$

B. Derivation of Eq. (9)

In the equal correlations case with N markets the covariance matrix can be written as:

with its inverse given by:

$$C^{-1} = \begin{bmatrix} 1/\sigma_1 & 0 & \cdots & 0 \\ \vdots & 1/\sigma_2 & & & \\ & & \ddots & \vdots \\ & & & 0 \\ 0 & & \cdots & 0 & 1/\sigma_N \end{bmatrix} \begin{bmatrix} a & b & \cdots & b \\ b & a & & \\ \vdots & \ddots & \vdots \\ & & b \\ b & \cdots & b & a \end{bmatrix} \begin{bmatrix} 1/\sigma_1 & 0 & \cdots & 0 \\ \vdots & 1/\sigma_2 & & & \\ & & \ddots & \vdots \\ & & & 0 \\ 0 & & \cdots & 0 & 1/\sigma_N \end{bmatrix}$$

where a and b are given by: $a = \frac{1 + (N-2)\rho}{1 + (N-2)\rho - (N-1)\rho^2}$, $b = \frac{-\rho}{1 + (N-2)\rho - (N-1)\rho^2}$ (see Elton and Gruber 1973, and Levy and Ritov 2011 p. 1462). The unscaled portfolio weights are given by:

$$\begin{split} \omega_i &= C^{-1} \cdot \mu = (a-b) \frac{\mu_i}{\sigma_i^2} + Nb \frac{1}{\sigma_i} \overline{\left(\frac{\mu}{\sigma}\right)} \\ &= \frac{1}{\sigma_i} \left[\left(\frac{1 + (N-1)\rho}{1 + (N-2)\rho - (N-1)\rho^2} \right) \frac{\mu_i}{\sigma_i} - \left(\frac{\rho N}{1 + (N-2)\rho - (N-1)\rho^2} \right) \overline{\left(\frac{\mu}{\sigma}\right)} \right] \end{split}$$

where $\overline{\binom{\mu}{\sigma}} \equiv \frac{1}{N} \sum_{j=1}^{N} \frac{\mu_j}{\sigma_j}$ (note that Eq. (1) for two markets is obtained as a special case of the above equation with N=2).

C. Derivation of Eq. (14)

When the expected returns are equal across all markets, and the home-advantage is expressed as a lower domestic volatility, the unscaled weights in Eq. (9) become:

$$\omega_i = B \cdot \left(\frac{(1 - \rho)}{\sigma_i^2} + \frac{\rho N}{\sigma_i} \left(\frac{1}{\sigma_i} - \overline{\left(\frac{1}{\sigma}\right)} \right) \right).$$

where $\overline{\left(\frac{1}{\sigma}\right)} \equiv \frac{1}{N} \sum_{j=1}^{N} \frac{1}{\sigma_{i}}$, and *B* is a constant, which is identical for all markets $\left(B = \frac{\mu}{1 + (N-2)\rho - (N-1)\rho^2}\right)$. The scaled proportions are obtained by dividing each of the ω_i 's by their sum. This sum, $\sum_{i=1}^N \omega_i$, is given

$$\begin{split} \sum_{i=1}^{N} \omega_{i} &= B \cdot \left[(1 - \rho + \rho N) \left(\sum_{i=1}^{N} \frac{1}{\sigma_{i}^{2}} \right) - \rho N \overline{\left(\frac{1}{\sigma}\right)} \left(\sum_{i=1}^{N} \frac{1}{\sigma_{i}} \right) \right] \\ &= B \cdot N \overline{\left[(1 - \rho + \rho N) \overline{\left(\frac{1}{\sigma^{2}}\right)} - \rho N \left(\frac{1}{\sigma}\right)^{2} \right]} \end{split}$$

As N is assumed to be large, we have $\overline{(\frac{1}{\sigma^2})} \approx \overline{(\frac{1}{\sigma})^2} \approx \overline{(\frac{1}{\sigma})^2} \approx \frac{1}{\sigma_F^2}, ^{20}$ and the sum can thus be approximated by: $\sum_{i=1}^N \omega_i \approx B \cdot (1-\rho) N \frac{1}{\sigma_F^2}$. The scaled investment proportion in the domestic market is thus:

$$x_D = \frac{\omega_D}{\sum_{i=1}^N \omega_i} \approx \left(\frac{1 - \rho + \rho N}{\sigma_D^2} - \frac{\rho N}{\sigma_D \sigma_F}\right) / \left((1 - \rho)N \frac{1}{\sigma_F^2}\right).$$

Multiplying both the numerator and the denominator by σ_D^2 , and recalling that $\frac{\sigma_D}{\sigma_E} \equiv 1 - \varepsilon$, we have:

$$x_{\rm D} \approx \frac{1 - \rho + \rho N - \rho N (1 - \varepsilon)}{(1 - \rho)N(1 - \varepsilon)^2}.$$

By assumption ii) second-order terms of ε can be neglected. Employing the Taylor expansion $1/(1-\varepsilon)^2 \approx 1 + 2\varepsilon$, we have:

$$x_D \approx \frac{(1 - \rho + \rho N \varepsilon)(1 + 2\varepsilon)}{(1 - \rho)N} = \frac{1 + 2\varepsilon}{N} + \varepsilon \frac{\rho}{1 - \rho}$$

For $\varepsilon = 0$ we have the case on no home-advantage, and thus, $x_D = 1/N$, as expected. The deviation from this benchmark is the volatility-induced home bias:

$$HB_{VOL} = x_D - \frac{1}{N} = \varepsilon \left[\frac{2}{N} + \frac{\rho}{1 - \rho} \right] = \varepsilon \left[\frac{2}{N} + HBM \right].$$

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²⁰ For example, suppose that $\sigma_F = 0.20$, $\sigma_D = 0.18$, and N = 10. In this case: $\overline{(\frac{1}{\sigma^2})} = \frac{9}{10} (\frac{1}{0.2})^2 + \frac{1}{10} (\frac{1}{0.18})^2 = 25.59; \ \overline{(\frac{1}{\sigma})}^2 = \left[\frac{9}{10} \frac{1}{0.2} + \frac{1}{10} \frac{1}{0.18}\right]^2 = 25.56; \ \frac{1}{\sigma_x^2} = \frac{1}{0.2^2} = 25. \text{ Thus,}$ the difference between these terms is about 2%. For N = 30 the difference is less than

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