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Gains from International Diversification: 1968–85 Returns on Portfolios of Stocks and Bonds

ROBERT R. GRAUER and NILS H. HAKANSSON*

ABSTRACT

This paper applies the multi-period investment model to a universe of international securities on the basis of the simple probability assessment approach. Our principal findings are: 1) the gains from including non-U.S. asset categories in the universe were remarkably large (in some cases statistically significant), especially for the highly risk-averse strategies, 2) the gains from removing the no leverage constraint were more substantial than they were in the absence of non-U.S. securities, and 3) there is strong evidence of market segmentation in that the optimal levels of investment in U.S. securities were mostly zero in the presence of the non-U.S. asset categories.

I. Introduction

IN EARLIER PAPERS (Grauer and Hakansson [4, 5, 6]), we applied the multi-period portfolio model (see Mossin [17], Hakansson [8, 9], Leland [14], Ross [18], and Huberman and Ross [10]) to the construction and rebalancing of portfolios composed of U.S. stocks, corporate bonds, government bonds, and a risk-free asset. Borrowing was ruled out in the first article while margin purchases were permitted in the other two. The third article also included small stocks as a separate investment vehicle. The probability distributions used were naively estimated from *past* realized returns in the Ibbotson 1926–84 data base, and both annual and quarterly holding periods were employed from the mid-thirties forward. The results revealed that the gains from active diversification among the major asset categories were substantial, especially for the highly risk-averse strategies. In some cases, the realized returns from the "active" strategies were significantly higher than for fixed-weight policies of similar riskiness.

In the present study, the multi-period portfolio model, again using the simple probability assessment approach, is applied to a universe consisting of the four principal U.S. asset categories and up to 14 non-U.S. equity and bond categories. This is in contrast to previous studies of international diversification, which have principally been based on the mean-variance model of portfolio choice (see e.g. Grubel [7], Levy and Sarnat [15], Solnik [19], Solnik and Noetzlin [20], Logue

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[16], and Adler and Dumas [1]). Our principal findings are: 1) the gains from including non-U.S. asset categories in the universe were remarkably large, especially for the highly risk-averse strategies, 2) the gains from removing the no leverage constraint were more substantial than they were in the absence of non-U.S. securities, and 3) there is strong evidence of market segmentation in that the optimal levels of investment in U.S. securities were mostly zero in the presence of the non-U.S. asset categories. Based on the paired t-test, realized portfolio returns when non-U.S. asset classes were included in the universe exceeded those generated in the U.S. only universe at the 5% level of significance for many strategies.

II. The Model

The model used is the same as the one employed in Grauer and Hakansson [6] and the reader is therefore referred to that paper (specifically pp. 288–293) for details. It is based on the pure reinvestment version of multiperiod investment theory.[1] In particular, if $U_n(w_n)$ is the *induced* utility of wealth with n periods to go (to the horizon) and r is the single-period return, the important result (see e.g. Hakansson [9]),

$$U_n(w_n) \rightarrow \frac{1}{\gamma} w^{\gamma} \quad \text{for some} \quad \gamma < 1,$$

which holds for a very broad class of terminal utility functions $U_0(w_0)$ when returns are independent from period to period, implies that the stationary, myopic decision rule

$$\text{Max } E\left[\frac{1}{\gamma}(1 + r)^{\gamma}\right] \quad \text{in each period} \tag{1}$$

encompasses a broad variety of different goal formulations for investors with intermediate- to long-term investment horizons.[2] Since the relative risk aversion function for (1) is $1 - \gamma$, the family (1) incorporates the full range of risk attitudes from zero to infinity.

The inputs to the model are based on the "simple probability assessment" approach. Suppose quarterly revision is used. Then, at the beginning of quarter t, the portfolio problem for that quarter uses the following inputs: the (observable) risk-free return for quarter t, the (observable) call money rate + 1% at the beginning of quarter t, and the (observable) realized returns for common stocks, government bonds, corporate bonds, etc., for the previous n quarters. Each joint realization in quarters $t - n$ through $t - 1$ is given probability $1/n$ of occurring in quarter t. Thus, estimates are obtained on a moving basis and used in raw

[1] The simple reinvestment formulation does ignore consumption of course.

[2] A plot of the functions $\frac{1}{\gamma}(1 + r)^{\gamma}$ for several values of γ was given in Grauer and Hakansson [4, p. 42].

form without adjustment of any kind. Since the whole joint distribution is specified and used, there is no information loss; all moments—marginal and conditional for example—and every bit of correlation are implicitly taken into account.[3] As a starting point, we have consequently resisted all temptations not only to parameterize the input distributions but also to reduce estimation risk via, for example, the Stein estimator (see Jorion [13]).

With these inputs in place, the portfolio weights for the various asset categories and the proportion of assets borrowed are calculated by solving system (1) plus applicable constraints via nonlinear programming methods.[4] At the end of quarter t, the realized returns on the various assets are observed, along with the *realized* borrowing rate r^r_{Bt} (which may differ from the decision borrowing rate r^d_{Bt}).[5] Then, using the weights selected at the beginning of the quarter, the realized return on the portfolio chosen for quarter t is recorded. The cycle is then repeated in all subsequent quarters.[6] All reported returns are gross of transaction costs and taxes and assume that the investor in question had no influence on prices.

III. Data

The data used to estimate the probabilities of next period's returns on risky assets, and to calculate each period's realized returns, came from several sources. The (monthly and annual) returns series for the U.S. asset categories, and for U.S. inflation, were obtained from Ibbotson Associates [12]. The quarterly data base on non-U.S. equity returns for 1960–1985 covering seven countries (Canada, France, Germany, Japan, Netherlands, Switzerland, and the United Kingdom) used in our quarterly portfolio revision runs was supplied by First Chicago Investment Advisors. Finally, we obtained annual returns for 14 non-U.S. equity and government bonds from Ibbotson, Carr and Robinson [11] for the period 1960–80 and from Ibbotson Associates for the 1981–85 period for stocks and for 1981–84 for government bonds. The 1985 non-U.S. bond data were provided by Salomon Brothers. All returns are expressed in U.S. dollars and represent total returns since both dividends (net of foreign taxes withheld) and capital appreciation or depreciation are taken into account.

The risk-free asset used for quarterly revision was assumed to be 90-day U.S. Treasury bills maturing at the end of the quarter; we used the *Survey of Current Business* and *The Wall Street Journal* as sources. In the annual portfolio revision case, the risk-free return was obtained from the yield, as of the beginning of the year, on that U.S. government obligation (note, bond, or bill) that matured on the date closest to the end of the year in question; we obtained the 1968–76 data privately from Roger Ibbotson and the remainder from *The Wall Street Journal.*

[3] For a comprehensive overview of these issues and problems associated with the estimation of return distributions, see Bawa, Brown, and Klein [2].

[4] The nonlinear programming algorithm employed is described in Best [3].

[5] The realized borrowing rate r^r_{Bt} was calculated as a monthly average.

[6] Note that if $n = 32$ under quarterly revision, then the first quarter for which a portfolio can be selected is the first quarter of 1968, since the period 1960–67 is required to develop the estimated return distributions used for that quarter's portfolio choice.

Table I

Asset Category and Fixed-Weight Portfolio Symbols

RL	Risk-free lending (quarterly or 1-year U.S. Treasury bills or notes)	JA	Japanese equities
GB	Long-term U.S. government bonds	NE	Dutch equities
CB	Long-term U.S. corporate bonds	SI	Swiss equities
CS	U.S. common stocks (S & P 500)	SW	Swedish equities
B	Borrowing	UK	British equities
UKB	British government bonds	E10	Equal-weighted portfolio of risky assets
GEB	German government bonds	E2	20% in E10, 80% in RL
JAB	Japanese government bonds	E4	40% in E10, 60% in RL
		E6	60% in E10, 40% in RL
		E8	80% in E10, 20% in RL
AU	Australian equities	E12	120% in E10, 20% in B
AS	Austrian equities	E14	140% in E10, 40% in B
CA	Canadian equities	E16	160% in E10, 60% in B
DK	Danish equities	E18	180% in E10, 80% in B
FR	French equities	E20	200% in E10, 100% in B
GE	German equities	IN	U.S. inflation

Margin requirements for stocks were obtained from the *Federal Reserve Bulletin*. These requirements were assumed to apply to non-U.S. equities as well. Initial margins were set at 10% for U.S. government bonds, at 20% for non-U.S. government bonds, and at 35% for corporate bonds. These levels are on the conservative side and designed to compensate for the absence of maintenance requirements.[7]

The borrowing rate was assumed to be the call money rate + 1%; for *decision* purposes (but not for rate of return calculations), the applicable beginning of period rate, r_{Bt}^{d}, was viewed as persisting throughout the period and thus as risk-free. For 1968–76, the call money rates were obtained from the *Survey of Current Business*; for later periods, *The Wall Street Journal* was the source.

IV. Results

Because of space limitations, only a portion of the results can be reported here. However, Tables II through VI and Figures A through D provide a fairly representative sample of our findings.

For comparison, we have calculated and included the returns for five unlevered and five levered equal-weighted fixed-weight portfolios. The compositions of these portfolios are shown in Table I along with an enumeration of the asset categories included in the study.

Quarterly Revision—No Leverage Case

Quarterly revision strategies were run (i) incorporating all of the 11 asset categories in the quarterly data base (risk-free lending, U.S. governments, U.S.

[7] There was no practical way to take maintenance margins into account in our programs. In any case, it is evident from the results that they would come into play only for the more risk-tolerant strategies, and even for them only occasionally, and that the net effect would be relatively neutral.

corporates, and U.S., French, German, Dutch, Swiss, British, Japanese, and Canadian equities) and (ii) based on the four U.S. asset classes only. Table II shows, and Figure A plots, the geometric means and standard deviations[8] of the realized returns for each of the 11 asset components (see squares), for the equal-weighted portfolios (see triangles), as well as for 16 strategies corresponding to γ's in (1) ranging from -75 (extremely risk-averse) to 1 (risk-neutral) in the no leverage case for the 16-year period 1970–85 both with and without the non-U.S. asset classes (see round dots vs. diamonds). The estimating period is 40 quarters. Among the asset categories, British equities had the highest volatility (34.36%) and Japanese equities the highest geometric mean (18.68%), with Dutch equities a distant second.

Among the active strategies in the "global" case, the most risk-averse (powers -75 to -5) did very similarly to the equal-weighted portfolios. Power .25 had the highest geometric mean (18.78%) and the risk-neutral strategy the highest volatility (28.72%). In the U.S.-only case, the risk-neutral strategy attained highest geometric mean (9.80%) as well as the largest standard deviation (11.62%).

Table III shows the portfolio compositions and the quarter-by-quarter returns for the power .25 strategy in the no leverage case both when non-U.S. equities are included in the universe and when they are not. In the U.S.-only case, holdings tended to be concentrated in either the risk-free asset or in common stocks, with an occasional allocation to corporate bonds. When non-U.S. equity markets were included, however, we see a very different pattern. Between the beginning of 1972 and through the end of the first quarter in 1982, the power .25 strategy kept 100% of assets in Japanese equities with only one exception (the second quarter of 1980). At other times, allocations were made to Dutch, British, Canadian, and Swiss equities. U.S., French, and German stocks, however, were ignored, as were U.S. government and corporate bonds. Note that of the U.S. asset categories, only the risk-free asset was ever touched and only in three quarters out of 64 at that.

Quarterly Revision—Leverage Case

When the borrowing constraint is removed in the previous analysis, Figure A is replaced by Figure B and the left side of Table II is replaced by the middle of Table II. In the U.S.-only case, only powers -1 through 1 ever engage in borrowing and do so with relative infrequency.

In the global case, leverage, even though available, was never used by powers -75 through -7, as Table II reflects. While the use of leverage uniformly increased geometric means for the other strategies, these increases were on the modest side. The increases in standard deviation, however, were rather large, especially for powers -1 and up. While primarily placed in Japanese equities, borrowed funds were also employed in the Dutch, British, and Australian equity markets.

[8] The standard deviation of r is very similar to the standard deviation of $\ln(1 + r)$, especially for levels less than 25%.

Table II

Comparison of Geometric Means and Standard Deviations of Annual Portfolio Returns With and Without Seven Non-U.S. Equity Markets, 1970–1985 (Quarterly portfolio revision, 40-quarter estimating period)

	U.S. Plus Seven Countries				U.S. Only	
	Without Leverage		With Leverage		Without Leverage	
Portfolio	Geometric Mean	Standard Deviation*	Geometric Mean	Standard Deviation*	Geometric Mean	Standard Deviation*
CS	10.08	17.27	10.08	17.27	10.08	17.27
GB	8.23	10.98	8.23	10.98	8.23	10.98
CB	9.07	11.68	9.07	11.68	9.07	11.68
FR	10.39	27.43	10.39	27.43		
GE	13.44	25.05	13.44	25.05		
NE	14.16	18.42	14.16	18.42		
SI	12.20	22.80	12.20	22.80		
UK	11.63	34.36	11.63	34.36		
JA	18.68	26.78	18.68	26.78		
CA	9.91	17.75	9.91	17.75		
RL	8.09	2.86	8.09	2.86	8.09	2.86
IN	6.88	3.22	6.88	3.22	6.88	3.22
Power 1	16.83	28.72	17.43	52.01	9.80	11.62
Power .75	17.32	27.60	19.00	50.39	8.96	10.96
Power .50	18.16	26.52	20.61	49.20	8.65	10.74
Power .25	18.78	25.96	21.65	46.91	8.54	10.48
Power 0	18.40	25.85	22.24	43.20	8.60	9.92
Power −1	16.69	24.43	19.34	33.05	9.07	8.53
Power −2	15.59	22.44	17.03	26.37	9.25	7.40
Power −3	14.75	19.96	15.33	21.58	9.25	6.32
Power −5	13.42	15.75	13.43	15.76	8.89	4.65
Power −7	12.42	12.30	12.42	12.30	8.70	3.91
Power −10	11.30	8.88	11.30	8.88	8.54	3.39
Power −15	10.33	6.10	10.33	6.10	8.40	3.07
Power −20	9.81	4.73	9.81	4.73	8.33	2.94
Power −30	9.27	3.49	9.27	3.49	8.25	2.86
Power −50	8.81	2.79	8.81	2.79	8.19	2.83
Power −75	8.57	2.63	8.57	2.63	8.15	2.83
E2	9.25	3.34	9.25	3.34		
E4	10.31	6.02	10.31	6.02		
E6	11.27	9.13	11.27	9.13		
E8	12.12	12.34	12.12	12.34		
E10	12.88	15.60	12.88	15.60		
E12			12.88	19.02		
E14			12.76	22.49		
E16			12.52	26.01		
E18			12.16	29.60		
E20			11.68	33.25		

* Standard deviation is for the variable $\ln(1 + r_t)$.

Annual Revision—No Leverage Case

In the annual holding period case, the following asset categories were included in addition to risk-free lending: U.S. corporate bonds, U.S., German, Japanese, and U.K. government bonds, and Austrian, Australian, British, Canadian, Dan-

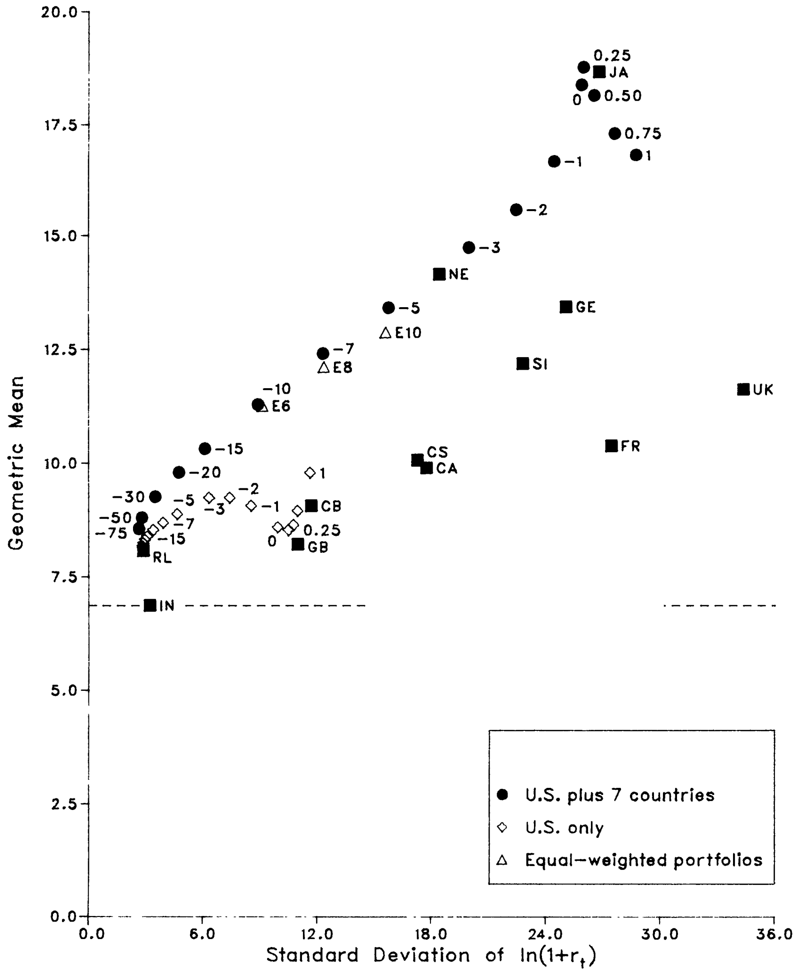

Figure A. Geometric Means and Standard Deviations of Annual Portfolio Returns for 16 Strategies With and Without Seven Non-U.S. Equity Markets, 1970–85 (Quarterly revision, without leverage, 40-quarter estimating period).

ish, Dutch, French, German, Japanese, Swedish, Swiss, and U.S. equities. Figure C plots the geometric means and standard deviations for each of the 18 asset components (see squares), for the equal-weighted portfolios (see triangles), as well as for the 16 strategies considered earlier for the 18-year period 1968–85, both with and without the non-U.S. asset classes (see round dots vs. diamonds). Most of the underlying information can be found in Table IV. Note that 100% investments in Australian equities did not keep pace with inflation; the risk-free asset and U.S. and British bonds barely stayed ahead of the inflation rate. British equities had the highest volatility and Japanese equities the highest geometric mean, with Swedish equities a distant second.

In the global case, the most risk-averse active strategies, (powers −75 to −7) clearly did very well. Powers −75 to −20, for example, had geometric means above 10.9% and smaller standard deviations (less than 10.7%) than any asset

Table III

Portfolio Compositions and Realized Returns for Power .25 With and Without Seven Non-U.S. Equity Markets, 1970–85 (Quarterly revision, without leverage, 40-quarter estimating period)

		U.S. Plus Seven Countries											U.S. Only					
		Investment Fractions													Inv. Fractions			
Period	r_t	RL	CS	GB	CB	FR	GE	NE	SI	UK	JA	CA	Period	r_t	RL	CS	GB	CB
1970Q 1	3.04								.12		.88		1970Q 1	.41	.58	.42		
1970Q 2	−12.39								.33		.67		1970Q 2	−18.03		1.00		
1970Q 3	2.61									.01	.80	.19	1970Q 3	4.79	.79	.21		
1970Q 4	6.19											1.00	1970Q 4	10.43		1.00		
1971Q 1	7.87											1.00	1971Q 1	9.69		1.00		
1971Q 2	8.21										.65	.35	1971Q 2	.17		1.00		
1971Q 3	−2.08									.13	.87		1971Q 3	−0.59		1.00		
1971Q 4	14.68									.19	.81		1971Q 4	4.66		1.00		
1972Q 1	23.51										1.00		1972Q 1	5.74		1.00		
1972Q 2	19.13										1.00		1972Q 2	.67		1.00		
1972Q 3	16.36										1.00		1972Q 3	3.91		1.00		
1972Q 4	31.94										1.00		1972Q 4	7.56		1.00		
1973Q 1	10.64										1.00		1973Q 1	−4.89		1.00		
1973Q 2	−7.21										1.00		1973Q 2	−5.77		1.00		
1973Q 3	−4.05										1.00		1973Q 3	2.36	.87	.13		
1973Q 4	−18.76										1.00		1973Q 4	−7.72	.13	.87		
1974Q 1	4.98										1.00		1974Q 1	1.94	1.00			
1974Q 2	1.88										1.00		1974Q 2	2.06	1.00			
1974Q 3	−19.24										1.00		1974Q 3	1.94	1.00			
1974Q 4	−2.38										1.00		1974Q 4	1.81	1.00			
1975Q 1	23.02										1.00		1975Q 1	1.62	1.00			
1975Q 2	1.56										1.00		1975Q 2	1.42	1.00			
1975Q 3	−14.98										1.00		1975Q 3	−0.59	.83	.17		
1975Q 4	12.90										1.00		1975Q 4	1.52	1.00			
1976Q 1	6.96										1.00		1976Q 1	2.68	.90	.10		
1976Q 2	6.19										1.00		1976Q 2	2.09	.31	.69		
1976Q 3	2.41										1.00		1976Q 3	1.79	.22	.78		
1976Q 4	8.17										1.00		1976Q 4	3.14		1.00		

1977Q 1	1.13				1.00	1977Q 1	−7.44		1.00	
1977Q 2	4.84				1.00	1977Q 2	2.68	.28	.63	.10
1977Q 3	5.25				1.00	1977Q 3	−0.10		.30	.70
1977Q 4	3.89				1.00	1977Q 4	1.47	1.00		
1978Q 1	22.46				1.00	1978Q 1	1.24	.81		.19
1978Q 2	9.60				1.00	1978Q 2	1.61	1.00		
1978Q 3	11.36				1.00	1978Q 3	1.75	1.00		
1978Q 4	2.52				1.00	1978Q 4	1.99	1.00		
1979Q 1	−7.22				1.00	1979Q 1	2.43	1.00		
1979Q 2	−0.76				1.00	1979Q 2	2.37	1.00		
1979Q 3	1.82				1.00	1979Q 3	2.15	1.00		
1979Q 4	−5.97				1.00	1979Q 4	2.52	1.00		
1980Q 1	−4.48				1.00	1980Q 1	2.96	1.00		
1980Q 2	16.71	.17			.83	1980Q 2	3.68	1.00		
1980Q 3	6.99				1.00	1980Q 3	10.88	.04	.96	
1980Q 4	7.15				1.00	1980Q 4	2.85	1.00		
1981Q 1	6.43				1.00	1981Q 1	3.73	1.00		
1981Q 2	8.27				1.00	1981Q 2	3.24	1.00		
1981Q 3	−9.63				1.00	1981Q 3	3.73	1.00		
1981Q 4	10.91				1.00	1981Q 4	3.77	1.00		
1982Q 1	−17.74				1.00	1982Q 1	2.86	1.00		
1982Q 2	0.91	.53			.47	1982Q 2	3.46	1.00		
1982Q 3	2.93	.96			.04	1982Q 3	3.33	1.00		
1982Q 4	17.13		.07	.46	.47	1982Q 4	1.95	1.00		
1983Q 1	21.27		1.00			1983Q 1	3.24	.85	.15	
1983Q 2	11.12		.93	.07		1983Q 2	6.21	.55	.45	
1983Q 3	0.36		.84	.16		1983Q 3	.21	.14	.86	
1983Q 4	1.71		.78	.22		1983Q 4	.85	.27	.73	
1984Q 1	16.43		.53	.23	.24	1984Q 1	−2.27		1.00	
1984Q 2	−11.04		.20	.51	.29	1984Q 2	−1.23	.26	.74	
1984Q 3	4.60		.35	.65		1984Q 3	7.16	.35	.65	
1984Q 4	1.47		.17	.83		1984Q 4	1.76		1.00	
1985Q 1	13.58			1.00		1985Q 1	9.35		1.00	
1985Q 2	3.54			.44	.56	1985Q 2	7.49		1.00	
1985Q 3	14.25			.38	.62	1985Q 3	−4.05		1.00	
1985Q 4	12.60		.14		.86	1985Q 4	17.18		1.00	

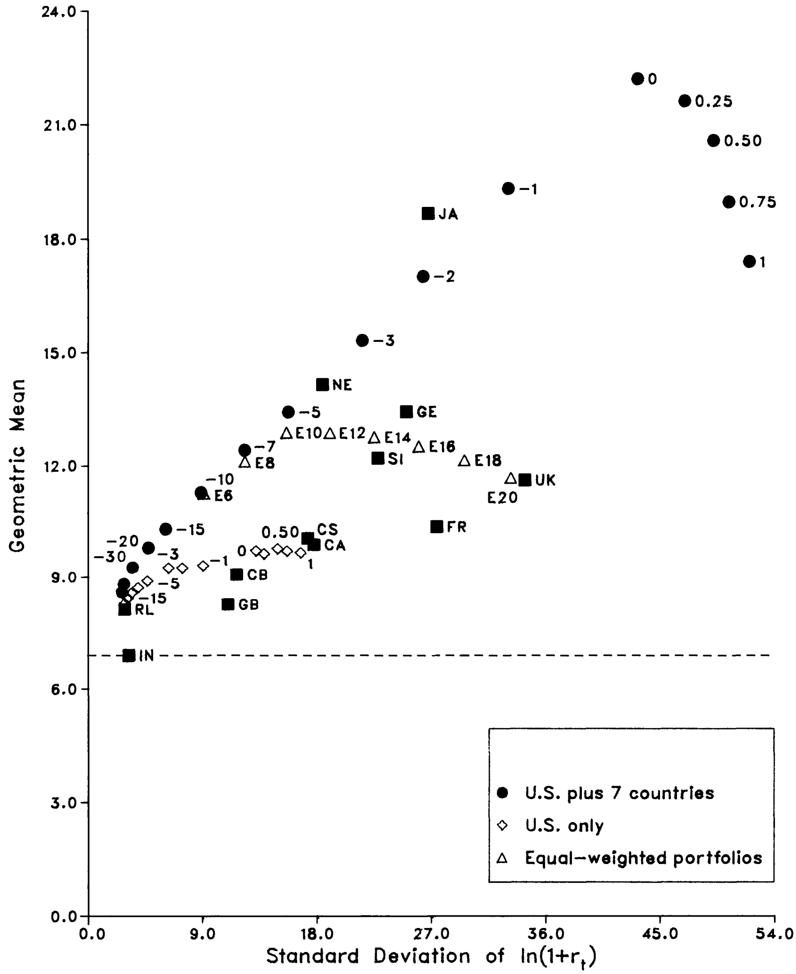

Figure B. Geometric Means and Standard Deviations of Annual Portfolio Returns for 16 Strategies With and Without Seven Non-U.S. Equity Markets, 1970–85 (Quarterly revision, with leverage, 40-quarter estimating period).

category except the risk-free asset (which had a geometric mean of 7.96% and a standard deviation of 2.59%). The more risk tolerant active strategies, however, did poorly—several were "dominated" by Japanese equities. In the domestic case, powers 0 through 1 had both the highest geometric mean (8.45%) and the highest standard deviation (9.98%).

Turning to the optimal global investment policies, only British government bonds were consistently ignored among the 18 asset categories. However, U.S. equities were chosen at most once. Japanese government bonds were heavily favored by the more risk-averse during the 1971–73 period while German government bonds tended to be important during the 1974–80 period. Japanese equities were held almost every year by all strategies. Remarkably, the risk-free asset was *ignored* in 12 years of 18 even by the extremely conservative power −75 strategy.

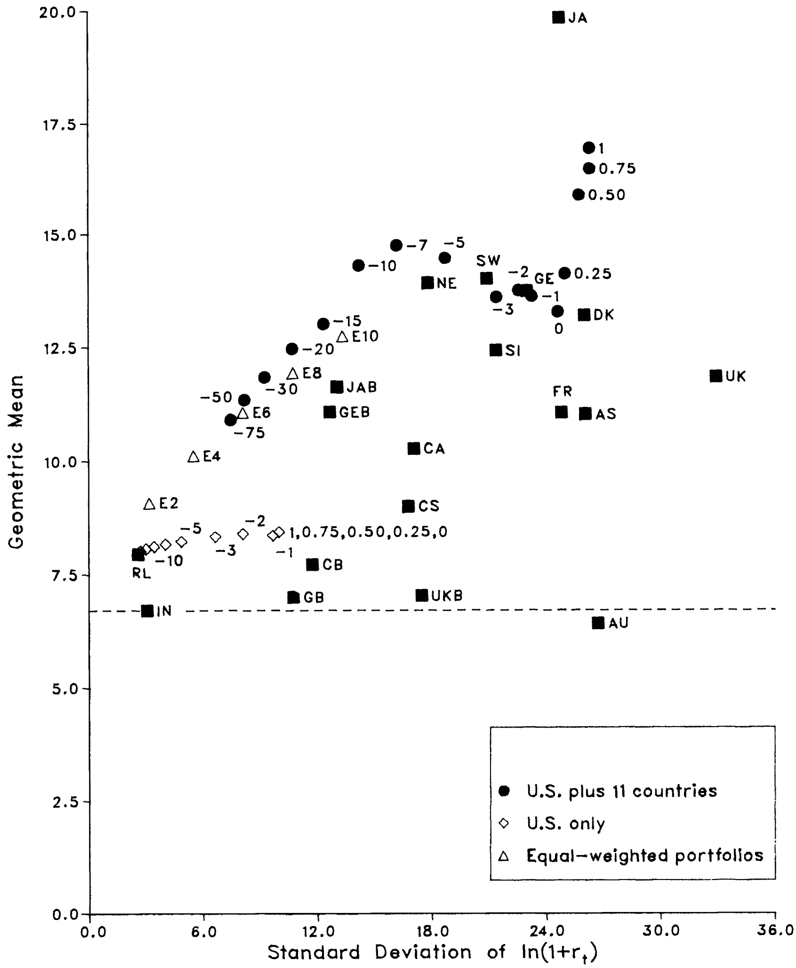

Figure C. Geometric Means and Standard Deviations of Annual Portfolio Returns for 16 Strategies With and Without Eleven Non-U.S. Equity Markets and Three Non-U.S. Bond Markets, 1968–85 (Annual revision, without leverage, 8-year estimating period).

Annual Revision—Leverage Case

When the opportunity to use leverage is introduced, we obtain, in the global case, a notable shift upward and to the right in the geometric means and standard deviations of the realized returns (see the left and middle of Table IV and the round dots vs. the diamonds in Figure D). For the most risk-averse strategies, the increases in geometric means are rather dramatic, while the increases in volatility are only modest, while a deterioration occurs for powers 0 through 1. Incredibly, the ultra-conservative power −75 strategy had a higher geometric mean (17.10%) than any asset category except Japanese equities. It should be noted that we were extremely leery about using leverage with only annual revision—unfortunately, no quarterly bond data were available.

Table IV

Comparison of Geometric Means and Standard Deviations of Annual Portfolio Returns With and Without Eleven Non-U.S. Equity Markets and Three Non-U.S. Bond Markets, 1968–1985
(Annual portfolio revision, 8-year estimating period)

	U.S. Plus Eleven Countries				U.S. Only	
	Without Leverage		With Leverage		With Leverage	
Portfolio	Geometric Mean	Standard Deviation*	Geometric Mean	Standard Deviation*	Geometric Mean	Standard Deviation*
CS	9.01	16.80	9.01	16.80	9.01	16.80
GB	7.01	10.74	7.01	10.74	7.01	10.74
CB	7.73	11.73	7.73	11.73	7.73	11.73
AS	11.03	26.09	11.03	26.09		
DK	13.20	26.06	13.20	26.06		
FR	11.07	24.84	11.07	24.84		
GE	13.73	23.02	13.73	23.02		
NE	13.91	17.82	13.91	17.82		
SW	14.01	20.93	14.01	20.93		
SI	12.43	21.39	12.43	21.39		
UK	11.85	32.96	11.85	32.96		
JA	19.83	24.75	19.83	24.75		
AU	6.43	26.70	6.43	26.70		
CA	10.28	17.09	10.28	17.09		
GEB	11.09	12.66	11.09	12.66		
JAB	11.64	13.05	11.64	13.05		
UKB	7.04	17.45	7.04	17.45		
RL	7.96	2.59	7.96	2.59	7.96	2.59
IN	6.72	3.06	6.72	3.06	6.72	3.06
Power 1	16.93	26.33	15.89	39.02	2.16	30.29
Power .75	16.47	26.33	15.56	36.99	7.33	17.72
Power .50	15.87	25.76	12.17	38.66	7.98	17.02
Power .25	14.11	25.02	12.04	38.78	8.42	13.84
Power 0	13.28	24.66	12.46	39.42	8.63	11.31
Power −1	13.62	23.26	16.44	35.64	8.29	9.80
Power −2	13.75	22.58	19.32	31.84	8.41	8.08
Power −3	13.60	21.41	20.23	29.98	8.35	6.63
Power −5	14.47	18.74	18.79	27.10	8.24	4.85
Power −7	14.74	16.18	17.57	24.83	8.18	4.02
Power −10	14.30	14.20	18.18	21.37	8.13	3.42
Power −15	13.02	12.34	18.35	19.30	8.08	3.01
Power −20	12.47	10.69	18.22	18.18	8.06	2.84
Power −30	11.85	9.24	17.76	17.38	8.03	2.71
Power −50	11.36	8.18	17.48	17.12	8.00	2.64
Power −75	10.92	7.46	17.10	16.90	7.99	2.61
E2	9.09	3.19	9.09	3.19		
E4	10.13	5.51	10.13	5.51		
E6	11.08	8.11	11.08	8.11		
E8	11.95	10.74	11.95	10.74		
E10	12.75	13.34	12.75	13.34		
E12			12.95	16.12		
E14			13.07	18.90		
E16			13.10	21.70		
E18			13.04	24.54		
E20			12.89	27.41		

* Standard deviation is for the variables $\ln(1 + r_t)$.

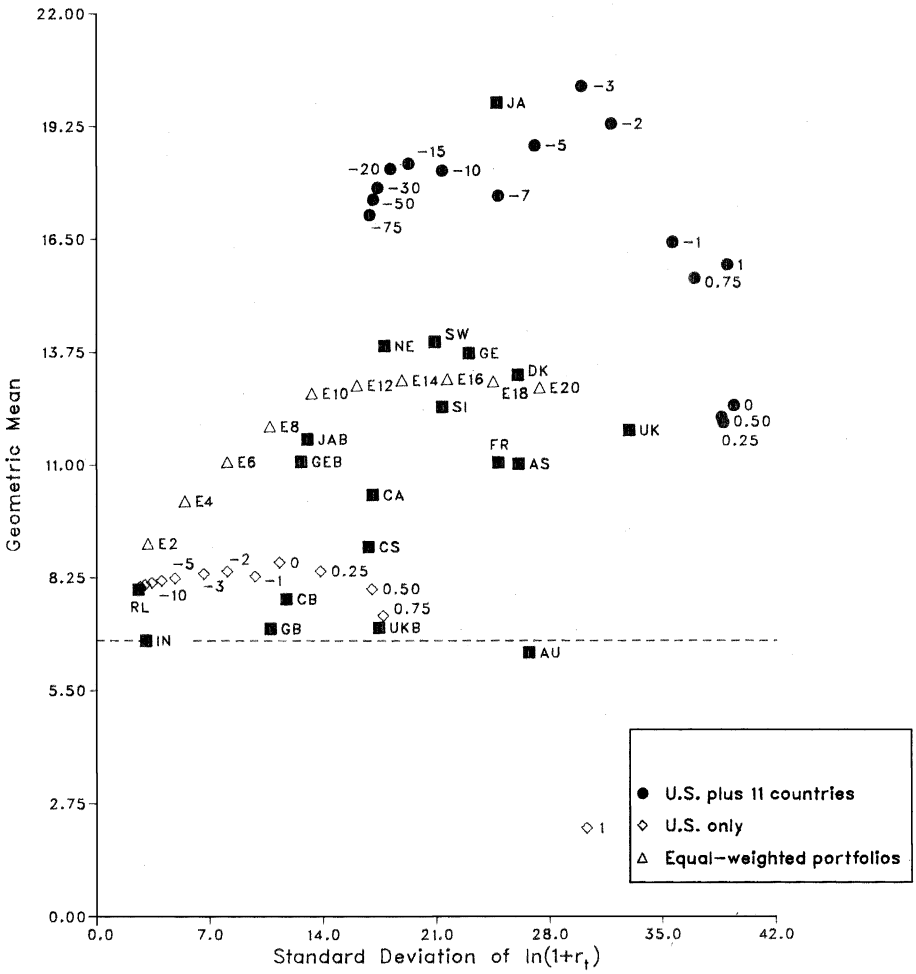

Figure D. Geometric Means and Standard Deviations of Annual Portfolio Returns for 16 Strategies With and Without Eleven Non-U.S. Equity Markets and Three Non-U.S. Bond Markets, 1968–85 (Annual revision, with leverage, 8-year estimating period).

Examining the strategies and returns from the power −15 strategy (Table V), we observe that this investor lent in only four years, borrowed (most heavily) in eight years, and did neither in the remaining six years. Highly levered positions were taken in Japanese government bonds during 1971–73 and in German government bonds during the 1976–79 period (with rather remarkable results). Among stocks, Japanese equities were the most important outlet, followed by Canadian, British, Australian and Austrian equities. Only two asset categories failed to attract this investor: British government bonds and U.S. equities.

The returns for case when the asset universe was composed of only U.S. securities, with leverage, are given on the right side of Table IV and are represented by the diamonds in Figure D. As the picture shows, the inclusion of the three non-U.S. bond categories and the 11 non-U.S. equities had a rather dramatic impact in the leverage case.

Table V

Portfolio Compositions and Realized Returns for Power −15 With and Without Eleven Non-U.S. Equity Markets and Three Non-U.S. Bond Markets, 1968–85 (Annual revision, with leverage, 8-year estimating period)

Year	r_t	Investment Proportions																		
		RL	B	CS	GB	CB	AS	DK	FR	GE	NE	SW	SI	UK	JA	AU	CA	GEB	JAB	UKB
U.S. Plus Eleven Countries																				
1968	25.18							.24					.03		.33	.32	.08			
1969	7.45							.44						.07	.03	.31	.01		.14	
1970	−10.32	.01													.67	.32				
1971	72.36		−3.14												.08	.10	.20	.37	3.38	
1972	41.93		−3.79												.01	.06	.05	.70	3.98	
1973	13.59		−3.71												.06	.03	.04	1.22	3.36	
1974	16.52														.12		.01	.86		
1975	7.56						.14						.06		.07		.02	.71		
1976	39.36		−1.83				.97							.16	.22		.11	1.38		
1977	68.94		−2.75			.32								.03	.24		.40	2.76		
1978	20.66		−2.27			.72	.63							.16				1.76		
1979	−5.70		−2.71					.29		.36				.18	.03			2.85		
1980	−5.79						.11		.06		.15				.02			.66		
1981	5.64	.68														.08		.23		
1982	12.67	.94												.02	.04					
1983	26.44											.33		.21	.17		.11	.19		
1984	−6.49		−0.15									.38		.22	.19		.13	.22		
1985	38.98	.07			.14							.12		.30	.30		.07			

U.S. Only						
1968	6.80	.85		.15		
1969	3.86	.82		.18		
1970	8.18	1.00				
1971	6.57	.83		.17		
1972	6.92	.81		.19		
1973	3.22	.88		.12		
1974	7.26	1.00				
1975	7.13	1.00				
1976	6.06	1.00				
1977	3.50	.74			.21	.06
1978	5.48	.80				.20
1979	10.47	1.00				
1980	11.33	1.00				
1981	13.78	1.00				
1982	13.25	1.00				
1983	11.34	.77		.23		
1984	9.47	.85		.15		
1985	11.71	.89		.11		

Table VI

Paired t-tests

Strategy	U.S. Plus Seven Non-U.S. Equity Markets vs. U.S. Only, 1970–85 (Quarterly revision, 40-quarter estimating period)						U.S. Plus Fourteen Non-U.S. Equity and Bond Markets vs. U.S. Only, 1968–85 (Annual revision, 8-year estimating period)					
	Without Leverage			With Leverage			Without Leverage			With Leverage		
	$\bar{d}$	$\sigma(d)$	t	$\bar{d}$	$\sigma(d)$	t	$\bar{d}$	$\sigma(d)$	t	$\bar{d}$	$\sigma(d)$	t
Power 1	.0155	.0982	1.27	.0171	.1866	.73	.0752	.2419	1.32	.1260	.5262	1.02
Power .75	.0185	.0962	1.54	.0203	.1817	.89	.0713	.2387	1.27	.0739	.3554	.88
Power .50	.0210	.0959	1.75*	.0235	.1790	1.05	0.662	.2319	1.21	.0381	.3615	.45
Power .25	.0225	.0958	1.88*	.0259	.1671	1.24	.0508	.2227	.97	.0328	.3575	.39
Power 0	.0216	.0951	1.82*	.0270	.1480	1.46	.0435	.2181	.85	.0346	.3691	.40
Power −1	.0169	.0882	1.53	.0219	.1095	1.60	.0473	.2015	1.00	.0725	.3327	.93
Power −2	.0141	.0791	1.43	.0172	.0857	1.61	.0481	.1996	1.02	.0959	.3045	1.34
Power −3	.0123	.0656	1.50	.0135	.0688	1.58	.0473	.1963	1.02	.1041	.2964	1.49
Power −5	.0102	.0495	1.65	.0102	.0495	1.65	.0560	.1811	1.31	.0930	.2736	1.44
Power −7	.0084	.0385	1.75*	.0084	.0385	1.75*	.0589	.1608	1.55	.0832	.2542	1.39
Power −10	.0063	.0281	1.79*	.0063	.0281	1.79*	.0555	.1447	1.63	.0889	.2237	1.69
Power −15	.0044	.0194	1.82*	.0044	.0194	1.82*	.0447	.1293	1.47	.0907	.2059	1.87*
Power −20	.0034	.0148	1.84*	.0034	.0148	1.84*	.0400	.1152	1.47	.0899	.1964	1.94*
Power −30	.0023	.0100	1.86*	.0023	.0100	1.86*	.0348	.1027	1.44	.0862	.1895	1.93*
Power −50	.0014	.0061	1.87*	.0014	.0061	1.87*	.0306	.0936	1.39	.0841	.1878	1.90*
Power −75	.0010	.0041	1.88*	.0010	.0041	1.88*	.0268	.0867	1.31	.0810	.1856	1.85*

* Significant at 5% level.

Other Results

We also constructed portfolios on the basis of 32-quarter and ten-year estimating periods, respectively. The differences in returns and portfolio compositions were fairly small, though substantially larger in the global case than in the U.S. only case.

V. Tests

In view of conventional wisdom, the small number of observations, and the large standard deviations commonly observed in equity markets, one would not expect that the differences in realized returns for the active strategies in the international and domestic cases to be statistically significant. However, given the size of the raw differences, it appeared desirable to conduct some tests.

Recall that terminal wealth w_0 in terms of beginning wealth w_n is given by

$$\begin{aligned} w_0 &= w_n(1 + r_n)(1 + r_{n-1}) \cdots (1 + r_1) \\ &= w_n \exp\{\textstyle\sum_{t=1}^{n} \ln(1 + r_t)\}. \end{aligned}$$

Since the returns themselves are not additive but compound multiplicatively, we employed the (one-tailed) paired t-test for dependent observations to the additive variables $\ln(1 + r_t)$ to test whether, under the simple active strategy approach used in this paper, the addition of non-U.S. investment outlets generated significantly different returns. Thus, for each active strategy (risk attitude) γ, we compare the return series $r_1^1, \cdots, r_n^1$ with the return series $r_1^2, \cdots, r_n^2$ for the two different universes. Specifically, we calculate the statistic

$$t = \frac{\bar{d}}{\sigma(d)/\sqrt{n}},$$

where $\bar{d} = \sum_{t=1}^{n} [\ln(1 + r_t^1) - \ln(1 + r_t^2)]/n$ and $\sigma(d)$ is the standard deviation of the differences between $\ln(1 + r_t^1)$ and $\ln(1 + r_t^2)$. In each case, the null hypothesis is that $E[\ln(1 + r_t^1)] = E[\ln(1 + r_t^2)]$ and the alternative hypothesis is that $E[\ln(1 + r_t^1)] > E[\ln(1 + r_t^2)]$.

The results are summarized in Table VI. The first six columns show that the returns when seven non-U.S. equity markets are added to the universe under quarterly reinvestment are sufficiently higher than the returns obtained without these markets for each of the more risk-averse strategies (power −7 and down) to be significant at the 5% level, both with leverage and without leverage. In the no leverage case, the 5% level of significance is also reached for powers 0, .25, and .50.

Under annual reinvestment, there are many fewer observations (18 only) and the results are therefore not nearly as strong. Thus, without leverage, the inclusion of eleven non-U.S. equity markets and three non-U.S. bond markets in the universe did not, despite their impressive appearance, improve returns significantly (with the exception of six strategies at the 10% level). With leverage, however, the 5% level of significance was reached for five (conservative) strategies, as the rightmost three columns show.

VI. Summary

Applying the multi-period investment model to a universe of international securities on the basis of the simple probability assessment approach led to some surprising findings. First, the returns on the more risk-tolerant strategies with non-U.S. equities in the opportunity set, and the returns on the most risk-averse strategies after also adding non-U.S. bonds to the universe, were rather impressive, reinforcing the earlier evidence based on U.S. data only that the past, joint empirical distribution contains exploitable timing information. Second, there is evidence of strong market segmentation since in the presence of the non-U.S. asset categories, the three risky U.S. investment outlets (government bonds, corporate bonds, and common stocks) were virtually ignored—this was true across the full spectrum of risk attitudes. The strength of this phenomenon is perhaps best measured by the following two findings: 1) even the extremely conservative power −75 strategy completely passed up risk-free U.S. government securities in favor of allocations ranging from 100% to 477% to portfolios of *risky non-U.S.* securities in 12 out of 18 years, and 2) for many strategies, the portfolio returns with non-U.S. asset classes included in the universe exceeded those generated from the domestic universe at the 5% level of significance.

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DISCUSSION

MICHEL CROUHY*: The paper by Robert Grauer and Nils Hakansson addresses a classical but still controversial empirical issue: Does diversification always pay? I like the paper and I am very sympathetic to the impressive empirical work which has been done, although more information is needed about your data set, especially the composition of the foreign portfolios and dividend adjustments in the various countries.

My comments will be directed towards two main questions:

— the underlying model,

— the risk of estimation,

and I will have a few minor additional remarks concerning the investment horizon.

First, the theoretical model. I agree with you that the mean-variance framework is clearly not suited to determine the optimal multiperiodic investment policy. Indeed, as shown by Merton (1973) and Landskroner (1977) the opportunity set is not stationary and investors cannot behave as if they were myopic. The only exception is, of course, for the case where the policy involved is the growth optimal policy associated with log-utility, as shown in Mossin (1968).

In general investors have to hedge against adverse shifts in the opportunity set. Hence, their optimal portfolio decisions do not only incorporate their estimation of current probability distributions, but also estimation of distributions to prevail in the future. This question relate to my second comment on the risk of estimation which I will address later on.

You rule out the continuous time portfolio model because it becomes intractable with transaction costs and taxes. But you do not introduce them either in your analysis, especially transaction costs. From my point of view, the continuous time model gives precious indications about transacting in a world where transaction costs do exist. Indeed, Constantinides (1986) has shown recently that large transaction costs are accommodated by investors in terms of expected utility, by drastically reducing the frequency and the value of trade. This last point is really important as the present paper discusses portfolio rebalancing on a quarterly basis. This type of revision might be sub-optimal due to transaction costs. The no tax assumption, on the other hand, can be accepted because of tax exempt investors.

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