

---

# Strategic currency hedging

Received: 22nd March, 2001

## Andrew Dales\*

is a currency researcher and strategist at Barclays Global Investors in London. Andrew has an MA in Mathematics and a Diploma in Statistics from Cambridge University.

## Richard Meese

oversees research on currencies and real estate investment trusts (REITS) in the Advanced Strategies & Research Group at Barclays Global Investors. Prior to joining BGI, Dick was a professor at the Haas School of Business at UC Berkeley from 1982, and he served as Associate Dean for Academic Affairs from 1996.

\*Barclays Global Investors, Murray House, 1 Royal Mint Court, London EC3N 4HH, UK.

Tel: +44 (0)20 7688 8957; Fax: +44 (0)20 7668 8910; e-mail: andrew.dales@barclaysglobal.com

**Abstract** This paper examines the rationale for strategic hedging policy, whereby some fixed proportion of the currency exposure associated with international assets is hedged. We begin with a review of both the theoretical and empirical literature on hedging policy. This literature provides a strong case for hedging some portion of the currency exposure associated with international investing. Differences in opinion remain, however, as to the appropriate methodology to use when constructing hedge ratios in practice. We advocate the use of portfolio optimisation methods and provide examples, along with caveats and guidelines, for the calculation of hedge ratios from a number of different currency perspectives.

**Keywords:** *currency hedging; currency risk; hedge ratio; portfolio optimisation*

## International investing and currency exposure

Most investors have two key goals: to maximise the expected return of their investments and to minimise the risk that something will go wrong. These two goals are neatly summarised by William Sharpe's famous ratio, where portfolio risk is measured by the standard deviation of asset return. These two goals are also the motivation behind the decision to invest internationally. Investors either believe that they can expect to receive extra returns from investing in non-domestic assets, or that by doing so they can reduce the risk of their overall portfolio.

There is, however, an interesting divergence in investment practice between investing in domestic assets and in investing internationally. In domestic investing, the ideas underlying the capital asset pricing model (CAPM) have had an enormous impact on investment practice. Many investors choose to apply CAPM to its final conclusion and invest in index funds. But even active funds follow many of the lessons of CAPM. Typical institutional active equity portfolios will hold a large number of stocks, and ensure that their exposures to various risk themes, such as market capitalisation, sectors and growth versus

**Table 1** Examples of typical strategic asset allocations for international investors

	US %	Japan %	UK %	Index %
European Equity	10.1	5.3	11.4	14.0
Japanese Equity	4.5	31.0	3.9	6.2
Pacific Rim Equity	1.2	0.6	2.8	1.7
UK Equity	4.2	2.2	56.3	5.8
US Equity	45.0	12.1	3.5	30.8
European Bonds	0.0	4.7	3.3	15.4
Japanese Bonds	0.0	37.7	2.3	10.6
UK Bonds	0.0	0.6	8.3	2.1
US Bonds	30.0	3.5	2.7	10.5
Cash	5.0	2.3	5.5	0.0
Other	0.0	0.0	0.0	2.9

Source: MSCI, Salomon Brothers and Barclays Global Investors

value will tend to be broadly similar to that of a representative cap-weighted index.

This is less true of international investing. Typical fund allocations from around the world tend to show a heavy bias towards investing in the domestic market. Unlike the domestic investing case, it is rare to find investors whose allocations to international assets are close to an international cap-weighted index, as suggested for example by some variants of the international capital asset pricing model. Table 1 shows strategic asset allocation weights for typical pension funds in the US, the UK and Japan; and also from an international cap-weighted index.<sup>1</sup> The funds all have an asset allocation that is much less diversified than the cap-weighted index, with significant weight being given to domestic equities and bonds.

Why should this home country bias be so prevalent? Many explanations, for example in Solnik (1989), have been offered: investors feel more comfortable about investing in assets with which they are more familiar; there are costs and administrative difficulties in investing internationally, although these have greatly diminished in recent years; and to the extent that investors seek to hedge

liabilities denominated in their home currency, domestic assets may provide more liability hedging credit than international assets.<sup>2</sup>

Another important explanation for investors' relatively small allocations to international investments is currency risk. In particular, most investors consider international assets on an unhedged basis. Doing so can impart a bias against international investing; see Grinold and Meese (2000). These authors argue that the primary reason for investing internationally is the diversification benefit. If investors only consider international assets on an unhedged basis, then they will fail to realise the full diversification benefits of international investing and will thus engage in less of it.

In the remainder of this paper, we consider a number of methodologies for making hedging policy decisions. Our preferred strategy is to use optimisation methods, although we readily acknowledge that all empirical solutions to hedging policy decisions are subject to estimation error. The remainder of the paper is organised as follows. In the next section, we review the literature on the hedging policy, including mean-variance optimisation. In the following section,

we discuss operational issues associated with the construction of optimal hedge ratios from asset return data. The penultimate section contains empirical examples, and is followed by the conclusion.

## Currency risk in international investing

The usual process for investing in an international asset involves two transactions:

1. cash in the domestic currency must be translated into the foreign currency at the current exchange rate;
2. the foreign currency is then used to buy the asset.

When the foreign asset is sold, the two steps must be reversed. Selling the asset generates foreign cash, which must then be converted back into domestic cash. Returns that investors receive in their own currency from time  $t$  to time  $s$  ( $s > t$ ) are:

$$R_{\text{int,dom}}(t,s) = (1 + R_{\text{int,local}}(t,s)) \frac{s_{\text{int/dom}}(t)}{s_{\text{int/dom}}(s)} - 1 \quad (1)$$

for

$R_{\text{int,local}}$ : Return from an international asset in the currency of that asset.

$R_{\text{int,dom}}$ : Return from an international asset in the currency of the investor.

$s_{\text{int/dom}}$ : Exchange rate (unit of foreign currency per unit of domestic currency).

Using log returns this can be expressed more neatly as

$$ru_{i,d} = rl_i + e_{i,d} \quad (2)$$

where

$$\begin{aligned} ru_{i,d} &= \log(R_{\text{int,dom}} + 1) \\ rl_i &= \log(R_{\text{int,local}} + 1) \\ e_{i,d} &= \log\left(\frac{s_{\text{int/dom}}(t)}{s_{\text{int/dom}}(s)}\right) \end{aligned}$$

## Hedged returns

The above return calculation gives the *unhedged* return to investing in an international asset. It is unhedged because it leaves an exposure to movements in the exchange rate. An alternative way to invest in international assets involves borrowing and depositing cash. Instead of converting domestic cash into foreign cash to buy the foreign asset, the domestic cash is deposited in a domestic bank account. The overseas asset is bought by borrowing foreign cash at the prevailing overseas interest rate. In continuous (log) return space, the return from this strategy is

$$rh_{i,d} = rl_i - i_i + i_d \quad (3)$$

where  $rh_{i,d}$  is the *hedged* return to an international investment;  $i_i$  and  $i_d$  are the interest rates in the international and domestic markets.

The same hedged result can be achieved with lower transaction costs by investing unhedged, as in Equation (2), and also investing in currency forward contracts. This is because the return from a contract that promises to exchange a certain value of currency from foreign to domestic is

$$-f_{i,d} = f_{d,i} = i_d - i_i - e_{i,d} \quad (4)$$

## The hedge ratio

If a proportion  $h$  of the value of the international asset is hedged back into the domestic currency, then the return to

that investment is

$$r_{i,d}(h) = r_i + e_{i,d} - hf_{i,d} \quad (5)$$

It is useful to write this in an alternative form:

$$\begin{aligned} r_{i,d}(h) &= r_i + e_{i,d} - hf_{i,d} \\ &= i_d + [r_i - i_i] + (1 - h)f_{i,d} \end{aligned} \quad (6)$$

In words, the return to investing in international assets with a hedge ratio  $h$  is domestic interest rate plus the foreign excess return plus  $(1 - h)$  times the forward currency return. Excess returns are defined as the asset local return minus the local interest rate. If  $h = 1$ , then the exposure to the forward exchange rate is fully hedged, and Equation (6) will match Equation (3).

### Currency exposure in an international portfolio

Equation (6) can be used to formulate the returns for an international portfolio. Let  $w_i$  be the portfolio weight in assets 1 to  $n$ . Let assets be domestic assets such that

$$\sum_{i=1}^d w_i = D$$

If a proportion  $h_i$  of each international asset is hedged, then the portfolio return is

$$\begin{aligned} r_{\text{port}}(\mathbf{h}) &= i_d + \mathbf{w}' \mathbf{r}_{\text{excess}} \\ &\quad + \sum_{i=d+1}^n (1 - h_i) w_i f_{i,d} \end{aligned} \quad (7)$$

In this equation,  $\mathbf{w}$  is the vector of portfolio holdings,  $\mathbf{h}$  the vector of hedge ratios, and  $\mathbf{r}_{\text{excess}}$  is the vector of excess returns. An active currency strategy can attempt to add value by adjusting the hedge ratios  $h_i$  based on predictions for future currency movements.

Rather than predicting long-term currency movements, many investors will seek to set a strategic hedge ratio  $h$ , which they will apply to all their foreign currency exposure. In this case, Equation (7) can be rewritten

$$\begin{aligned} r_{\text{port}}(h) &= i_d + D r_{\text{ex},d} + (1 - D) r_{\text{ex},i} \\ &\quad + (1 - h)(1 - D) f_{i,d} \end{aligned} \quad (8)$$

Considerable debate has focused on the appropriate strategic choice for  $h$ , a topic we turn to next.

### Literature review<sup>3</sup>

Black (1989, 1990) is one of the few authors to rely on a general equilibrium framework to derive optimal hedging policy. He first notes that the gains from hedging are like the gains from international diversification. Hedging reduces risk for both sides, even as one side gains from the hedge while the other loses. When performance is measured in local currency, domestic investors gain on their hedging when the currency return on their international portfolio does badly and vice versa for non-domestic investors. Why not hedge 100 per cent? Because of differences in consumption baskets, investors in different countries increase their expected returns by taking on *some* currency risk.

The theoretical framework that underpins Black's analysis is a highly stylised real international capital asset pricing model. In the Black framework, *all* investors (regardless of domicile) have the same hedge ratio, and every investor holds the same diversified portfolio of world equities. Since there must be a borrower for every lender and currency longs for every short, in equilibrium, asset returns (and their volatilities and correlations) will adjust until all available

equities are willingly held and some investor is willing to take each side of all currency contracts.

The universal hedging formula that comes out of Black's analysis is

$$\frac{\mu_m - \sigma_m^2}{\mu_m - \sigma_c^2/2} \quad (9)$$

where  $\mu_m$  is the average (across investors) of the expected excess return on the world market portfolio,  $\sigma_m^2$  is the variance of the world market portfolio, and  $\sigma_c^2$  is the average exchange rate variance across all currency pairs.

Suppose that the expected excess return on the world market portfolio is 6 per cent, its risk is 15 per cent, and average currency risk is 10 per cent. Then the universal hedge ratio is 70 per cent. For reasonable variations in the three inputs to Equation (9) the universal hedge ratio ranges from 40 to 90 per cent.<sup>4</sup>

The rest of the literature on hedging policy is not as well grounded in general equilibrium financial theory. That said, most of the assumptions underpinning Black's analysis are not true, as, for example, market participants cannot perfectly hedge the real purchasing power of their international investments with forward contracts. This has led researchers to pursue more pragmatic solutions to the hedging policy question, i.e. what is the optimal  $h$  for a long-term investor?

Early researchers, Perold and Schulman (1988), advocate a 100 per cent hedge ratio. These authors argue that, since currencies have zero expected return in the long run, hedging is a free lunch: i.e. hedging reduces risk for no loss of expected return.<sup>5</sup> The analysis is based on quarterly real (CPI inflation adjusted) returns over 1978–87, and shows that the risk reduction is still large when domestic purchasing power is taken into account. Their recommendation does not necessarily provide maximal risk

reduction, as there is no consideration of cross-hedging or optimal hedge ratio analysis; see below. Finally, the authors also note the lack of precision of hedge ratio estimates, which are essentially regression coefficients, a theme we return to below.

Many authors<sup>6</sup> have advocated the use of mean-variance optimisation to solve for the optimal hedge ratio. This procedure can be interpreted as a cost-benefit analysis for currency hedging. The cost of hedging includes the cost of forward contracts and the costs associated with the periodic portfolio rebalancing that is necessary to maintain the hedge. Indirect costs include the small return that one can expect from bearing currency risk (Black, 1990), which is lost when all currency exposure is hedged. Finally, the clear benefit of hedging is risk reduction.

Portfolio returns for a given hedge ratio are given by Equation (8). Mean-variance optimisation maximises the objective function

$$\max_h f(h) = \mathbf{E}[r_{\text{port}}(h)] - \lambda \text{Var}[r_{\text{port}}(h)] \quad (10)$$

The coefficient of risk aversion  $\lambda$  is used to determine the investor's appetite for taking additional risk to gain additional return. In order to solve (10), we need to make some assumptions about the expected return from hedging currency using forward contracts. If we have no long-term predictions for currency movements, we can assume that the expected return from holding currency forwards is simply the costs of implementing the hedge. We shall assume a fixed hedging cost of  $k$  times the size of the hedge. For an institutional investor  $k$  is approximately 15 basis points on an annual basis. Thus we assume that

$$\mathbf{E}[r_{\text{port}}(h)] = C - kh(1 - D) \quad (11)$$

where  $C$  does not depend on the hedge ratio,  $k$  is the cost of hedging,  $h$  is the hedge ratio, and  $D$  is the proportion of assets invested in domestic assets. The only impact on expected returns from currency hedging is assumed to be the cost of implementing the hedges.

The variance of portfolio returns for a hedge ratio  $h$  is

$$\sigma_p^2 = \sigma_{ex}^2 + (1-h)^2(1-D)^2\sigma_f^2 + 2(1-h)\{(1-D)D\sigma_{d,f} + (1-D)^2\sigma_{i,f}\} \quad (12)$$

Here,  $\sigma_{ex}$  is the volatility of excess returns (both domestic and international);  $\sigma_f$  is the volatility of currency forward returns;  $\sigma_{d,f}$  and  $\sigma_{i,f}$  are the covariances of forward returns with domestic and international excess returns.

In order to find the optimal hedge ratio  $h^*$ , we can look at the first-order condition for (10), namely

$$\frac{\partial f}{\partial h} = -k(1-D) + 2\lambda\{(1-h)(1-D)^2\sigma_f^2 + (1-D)D\sigma_{d,f} + (1-D)^2\sigma_{i,f}\} \quad (13)$$

Setting the differential to zero gives

$$h^* = 1 - \frac{k}{2\lambda(1-D)\sigma_f^2} + \frac{1}{(1-D)\sigma_f} \{D\sigma_{d,f} + (1-D)\sigma_{i,f}\} \quad (14)$$

The solution for the optimal hedge ratio has a number of interesting features. First, if we set  $k$  to zero, then (14) becomes the equation for the risk-minimising hedge ratio. The risk-minimising hedge ratio is therefore 100 per cent plus or minus a component that is determined by the covariances of the forward currency returns with excess returns from the domestic and international parts of the portfolio. Secondly, increasing the cost of hedging,

or decreasing risk aversion will always make the optimal hedge ratios lower.

Thirdly, looking at the correlation terms ( $\rho_{d,f}$  and  $\rho_{i,f}$ ), the optimal hedge ratio will increase if there is a positive correlation between domestic excess returns and forward currency returns or between international excess returns and forward currency returns. Since the forward returns are for a long international currency and short domestic currency exposure, however, the correlation between currency and assets have the opposite effect on the optimal hedge ratio for domestic and international asset exposure. That is, if international currency strength is positively correlated with international asset strength, then the optimal hedge ratio will increase, while a positive correlation between domestic currency strength and domestic asset strength will decrease the optimal hedge ratio. A common argument for using a low or zero hedge ratio for an equity portfolio has been that the strength of an equity market is likely to be *negatively* correlated with the strength of the currency. This rationale, however, only leads to a low optimal hedge ratio in the case where you assume that the fund is mostly invested in international equity assets. If the fund is predominately invested in domestic equity assets, then this correlation argument actually leads to a higher hedge ratio. That is, if a portfolio holds a high proportion of domestic equity, and these assets perform poorly in times when the domestic currency is strong, then it is particularly dangerous to be exposed to risk from exposure to foreign currency, since this exposure will also yield negative returns when the domestic currency is strong.

A final feature of the optimal hedge ratio equation is that correlation terms will be more important in determining the ratio if the investment in

international assets is small, or if currency risk  $\sigma_f$  is small compared with the risk of excess domestic or international returns ( $\sigma_d$  and  $\sigma_i$ ).

Gardner and Stone (1995) argue, on the basis of stochastic simulations, that the estimation of portfolio shares from mean-variance optimisation problems such as (10) have such wide confidence intervals that they are suspect for policy analysis (i.e. confidence bands for the hedge ratio for each currency can often be above 100 per cent or below 0 per cent). Sampling variability issues are most acute for estimates of expected asset returns, a topic we shall return to below. In a specialised example, where only minimum variance hedging is considered, the authors' estimated hedge ratios are precise enough for hedging policy purposes, in our view.

In a related paper, Gardner and Wuilloud (1995) argue that, if the investment horizon is short, (say 1–2 years), and the investor has moderate to low risk aversion, there is a substantial probability that the optimal hedge ratio will underperform another portfolio that makes use of a simple alternative hedging strategy. In other words, the international investor will frequently experience regret, and the regret can be large in magnitude.

Grant and Wuilloud define regret as

$$\text{Regret} = \max [U(A) - U(\text{OHR}), 0] \quad (15)$$

where  $U(A)$  is the realised utility in the case where an alternative hedge ratio is used, and  $U(\text{OHR})$  is the realised utility in the case where the optimal hedge ratio is used. The regret can be sizeable, since in the authors' set up the optimal hedge ratio involves expected currency return (i.e. the term in Equation (11) summarised by  $k$ ). In the short run, this term can be substantial, and we have assumed its long-run value is zero in the

analysis above. Thus it is not surprising that regret can be quite large. The authors then recommend a 50 per cent hedge ratio, as it minimises the maximal expected regret relative to both a completely hedged portfolio and a completely unhedged portfolio.

Finally, Froot (1993) argues for a zero hedge ratio over long investment horizons. His analysis is based on estimates of asset return second moments over a 200 year sample. For international equity and exchange rate data that span the modern floating exchange rate period (1973–present), there are only five to six non-overlapping periods of 5-year returns. This makes any analysis based on the estimation of means and variances of long-dated asset returns over the last thirty years of post Bretton Woods data suspect in our view. We focus on estimation issues in the next section.

To sum up, virtually all of the existing theoretical and empirical work on hedging policy suggests that a policy of considering international assets only on an unhedged basis ( $h = 0$ ) is sub-optimal. Our reading of the literature is that hedge ratios in the range of 20–80 per cent are easily defensible, using a variety of theoretical and empirical approaches.

### How useful are optimal hedge ratios based on estimated parameters?

Given the results of Gardner and Douglas (1995) noted above, it is useful to review the conditions under which optimal hedge ratio analysis leads to reliable hedge ratio recommendations. We begin by appealing to the literature on stationary time series and the empirical distribution of asset returns.

More specifically, if the process generating returns is stationary,<sup>7</sup> researchers should use the longest time series available to estimate expected

returns and second moments (variances and correlations). The only way to reduce the sampling variability of the estimate of the mean is to increase the length of the sample. In contrast, estimates of second moments can be made more precise (for a data sample of given calendar length) by estimating variances and correlations using the highest frequency data available (e.g. Merton, 1980).

It is well known that the outputs from mean-variance analyses are only as useful as the quality of the inputs, and input quality will depend on the precision with which we estimate expected returns and second moments. In order to reduce uncertainty in optimal hedge ratio estimates, we do not recommend the inclusion of estimated currency returns into optimal hedge ratio analysis. Instead, we prefer to impose our prior assumption that expected excess currency returns are zero over the long run (see note 5), at least for developed currency markets. If analysts have strong views about the direction of expected currency returns over the longer term, these views will dominate the other two terms in the expression for the optimal hedge ratio in Equation (14).

A somewhat paradoxical recommendation is that for *long*-term hedging policy calculations, one should rely on estimates of the second moments based on return data over *short* horizons (again, provided that the return series are stationary).<sup>8</sup> Correlations and variances of long-dated returns can then be inferred by careful aggregation of the estimates obtained from higher frequency data.

For some recent research that suggests that the distribution of asset returns are well characterised by fractionally integrated (also called long memory) processes, see Andersen *et al.* (2001a, b). These processes are stationary, but have auto-correlations that decay very slowly

over time (hence the 'long memory' label). Unfortunately, it is hard to distinguish return series with this long-memory property from return series that are subject to changes in the underlying regime that generates the data. Davies and Harte (1987) recognise that distinguishing a fractional Gaussian process from an AR(1) process is a difficult task.

An important practical implication of our inability to assess the stationarity of return series is that analysts need to keep track of known changes in the economic environment when estimating returns means and second moments. Examples include a change from a managed currency regime to a floating regime, or changes in the regulations governing securities markets that permit financial innovation. Thus optimal hedge ratio analyses will need to be updated whenever the assumptions underlying their construction change.

Two of the important parameters in the equation for the optimal hedge ratio (14) are the correlation of currency returns with international asset returns, and the correlation of currency returns with domestic asset returns. To the extent that currencies and assets are driven by the same set of fundamental economic factors, these correlations can be inferred by reference to appropriate valuation models. It is generally agreed that currencies share more fundamental drivers with bonds than with equities, as for example, expected inflation is unambiguously bad for both bonds and currencies. Thus there is also potential for analysts to impose priors on second moments (as well as on expected returns), based on their beliefs about the overlap in fundamental drivers of currency and asset returns.

Finally, optimal hedge ratio analysis can be expanded to consider separate hedge ratios for international stocks and bonds, or separate hedge ratios for



**Table 2** Empirical volatilities and correlations for typical pension fund portfolios using monthly returns data 1978–2001

Base currency	Currency risk ( $\sigma_i$ ) (%)	Domestic risk ( $\sigma_d$ ) (%)	International risk ( $\sigma_i$ ) (%)	Domestic correlation ( $\rho_{d,i}$ )	International correlation ( $\rho_{i,i}$ )
US	9.6	10.0	13.5	0.05	−0.12
Japan	10.8	8.9	10.3	−0.16	−0.07
UK	8.3	13.8	11.0	0.05	−0.01

Source: MSCI, Salomon Brothers and Barclays Global Investors

**Table 3** Hedge ratio calculations for typical pension funds (%)

Base currency	Cost term	Domestic covariance term	International covariance term	Overall hedge ratio	Standard error of hedge ratio
US	−14	22	−17	91	33
Japan	−7	−32	−6	54	18
UK	−12	21	−2	100	32

different currencies. Care must be taken when disaggregate analysis of this type is desired. In general, aggregate hedge ratios will be more precisely estimated, as averaging (across currencies or assets) will tend to reduce the variance of the estimates of the mean and second moments.

### Empirical examples of optimal hedge ratios

Formula (14) was used to calculate hedge ratios based on returns to asset allocation strategies taken from typical US, UK and Japanese pension funds. The asset allocation weights used were those shown in Table 1. For each strategy, we calculated monthly excess domestic returns, hedged international returns and forward currency returns from 1978 to 2001. The volatilities and correlations for the return components are shown in Table 2.

Most studies (e.g. Solnik, 1989) find a small negative correlation between stock market strength and currency strength. We also find some evidence for this: the

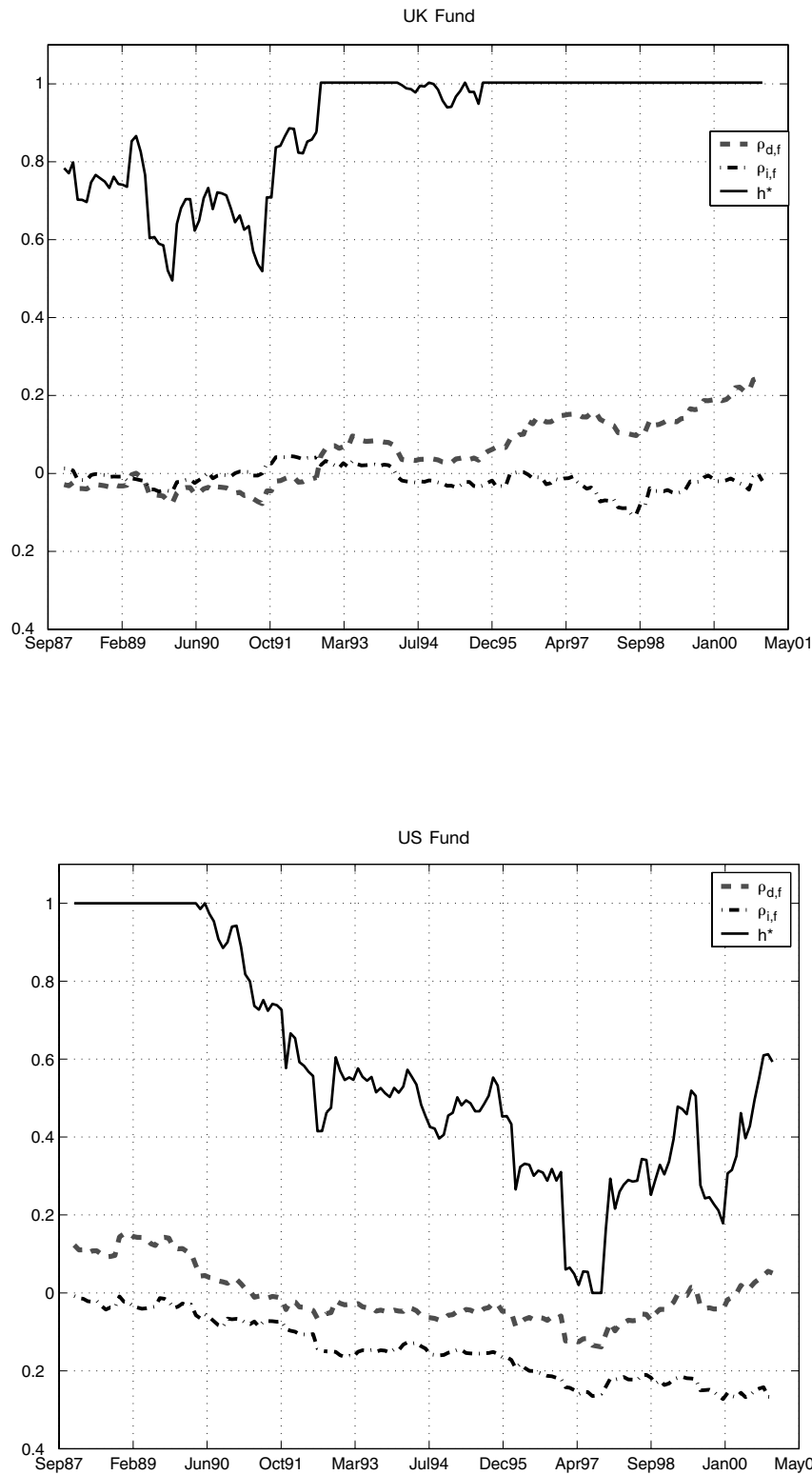
correlation between currency returns and international returns are all negative. Similarly, the correlations between domestic returns and currency returns are positive for the US and the UK, which again supports the theory that the US and UK equity returns are supported by weak domestic currency. The same is not true for Japan, where domestic stock and bond strength have tended to go hand in hand with domestic currency strength.

The impact that these variances and covariances have for the mean-variance optimal hedge ratio is shown in Table 3.

Based on these calculations, the UK and US investor would hedge most of their foreign currency exposure, while the Japanese investor would hedge around half of their currency exposure.

### Sensitivity of empirical hedge ratio estimates

As has been referred to in the literature review, the optimal hedge calculations can be sensitive to small changes in the correlations between market and currency returns. The asymptotic



**Figure 1** Correlation of currency returns with excess domestic and international returns, together with optimal hedge ratio for typical pension funds

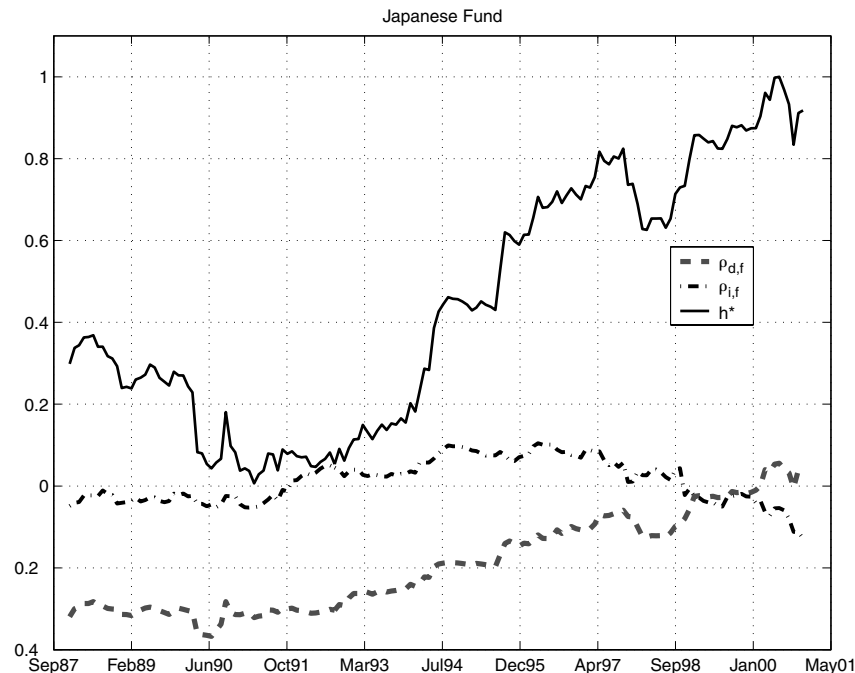


Figure 1 (continued)

standard error for the correlation between two time series is approximately  $\sqrt{1/n}$ , where  $n$  is the sample size (Wei, 1990). In the calculations above, we used 276 periods, so the standard error on the correlation calculations is in the order of 0.06. The size of the sample error can have a dramatic effect on the confidence limits of the hedge ratio calculations. For example, a one standard deviation change in the correlation between domestic returns and currency returns for the US investor would cause a 25 per cent change in the optimal hedge ratio.

As an alternative to an asymptotic standard error for the correlation coefficient, we can use the assumption of normality and regression theory to deduce that

$$(n-2)\rho^2/(1-\rho^2)$$

is distributed as an  $F(1, n-2)$  random variable.

The sensitivity of the optimal hedge ratio is heavily dependent on the proportion of international investment. This means that an investor with a higher proportion of assets invested internationally will be able to estimate an optimal hedge ratio with a higher degree of certainty than an investor with a lower proportion of international assets. Also, the gain from hedging, in terms of the potential for a low cost reduction of risk, will be greater. It is not necessarily the case that the optimal hedge ratio will increase if the investor puts a higher weight on their international investments.

As pointed out above, if we assumed that correlations were stationary over time, then more accurate estimates of correlation would be achievable by examining higher frequency data. There is a risk, however, that the structure of short-term correlations will not apply to longer investment periods.

### Time dependence of hedge ratio results

The standard error calculations for the hedge ratio also assume that the underlying correlations between the return series do not change over time.

Figure 1 shows the rolling 10-year correlations of currency returns with domestic and international excess returns for typical US, Japanese and UK pension funds. The funds are assumed to have the strategic asset allocations shown in Table 1. The hedge ratio based on the preceding 10 years of data is also shown. Figure 1 highlights the point that we made earlier, as the correlation of currency returns with domestic and international asset excess returns are reasonably stable over the sample period, but the variations in these correlations get magnified in the hedge ratio calculation.

### Concluding remarks

In this paper, we have examined the literature on currency hedging policy. Our recommendation is to use mean-variance optimisation methods to calculate optimal hedge ratios in practice. It is well known that optimisation methods are sensitive to the values of input parameters, and that input parameters can be difficult to estimate precisely. This is especially true of expected asset returns, so we do not recommend the inclusion of the estimated average currency return in optimal hedge ratio analysis. Rather, by appealing to long-run economic parity relations, such as uncovered interest parity, we suggest that one imposes the prior that longer-term expected currency returns are zero for developed market currencies. Second moments (volatility and correlation) can be estimated with greater confidence than expected returns given a sample of the same calendar length. By sampling more frequently, the

estimation error of the variance can be reduced, while sampling error of the mean cannot. This suggests that higher frequency returns are useful for estimating the second moment parameters required for optimal hedge ratio analysis.

Nonetheless, care must be taken to examine the stability of the economic environment over which second moment parameters are estimated. Here too, if the analyst suspects instability, then hedging policy recommendations can be based on parameters estimated over shorter sample periods, or by using the analyst's priors for the second moment parameters, again using economic valuation models for asset returns as a guide.

We provide estimated hedge ratios for representative investors domiciled in Japan, the US and UK respectively. The analysis indicates that both US- and UK-based investors should hedge a substantial fraction of their currency exposure, while a typical Japanese investor should hedge about half of their currency risk. These conclusions are not based on a model of expected currency return, but rather, are driven by asset return correlations and a reasonable assumption about the annual costs of currency hedging.

Finally, when assessing hedging policy for individual currencies or asset classes (e.g. stock or bonds separately) care must be exercised, as an estimate of the weighted average hedge ratio for all currencies and international assets will be estimated more precisely than individual currency hedge ratios or individual asset hedge ratios.

### Notes

- 1 Index weights are calculated by combining 60 per cent MSCI World Equity Index with 40 per cent Salomon World Government Bond Index.
- 2 Sharpe and Tint (1990) define liability hedging credit (LHC). LHC measures the degree to which an asset provides utility for an investor due to the

- correlation of the asset's return with the growth rate of the investor's domestic liabilities.
- 3 Our discussion ignores the effect of economic currency exposure from investing in international equities. As an extreme example, consider a UK-listed company which operates solely in the US. All of the company's future cash flows are in US\$. Fluctuations of the dollar-pound exchange rate should have no effect on the value of the company measured in US\$, but a large effect when measured in its listing currency, UK£. In reality, it is unlikely that there would be such a clear-cut example of a company's economic exposure being so different from its listing exposure, not least because many companies choose to hedge their economic currency exposures internally. One could expect, however, that there are many cases where a large multinational, listed in a small currency region, would have substantial economic exposure outside the base currency of their listing. The net effect of this is that an international portfolio will have greater economic exposure to the dollar than that implied by the base currency of the listing locations of the portfolio components. In such cases, a US\$-based investor may need to hedge less of their international equity exposure than an international investor from a small currency block.
  - 4 The range of 40–90 per cent is obtained when the expected excess return on the world portfolio varies from 6 to 12 per cent, world portfolio risk varies from 15 to 20 per cent, and average currency risk varies from 10 to 15 per cent.
  - 5 The argument assumes a zero currency risk premium, or equivalently, that the forward rate is an unbiased predictor of the future spot rate. There is a considerable empirical and theoretical literature on the exchange rate risk premium. It is complicated by the fact that currency risk premiums can be positive or negative and change sign over time as asset supplies change. Studies examining returns to forward contracts over long time periods are consistent with very small, but highly variable risk premiums. See Meredith and Chinn (1998) for evidence that uncovered interest parity holds over long horizons.
  - 6 See for example Lee (1990), Glen and Jorion (1993), Jorion (1994), Kritzman (1993), or Nesbitt (1991), among others.
  - 7 A covariance stationary process  $x(t)$  is one whose mean and variance are independent of time  $t$ , and whose auto-covariances  $\text{cov}(x(t), x(t-s))$  depend only on  $s$ .
  - 8 Clearly, this recommendation can be taken to an extreme, as very high frequency data (e.g. intraday) is likely to contain distortions due to market microstructure frictions.

## References

- Andersen, T., Bollerslev, T., Diebold, F. and Labys, P. (2001a) 'The Distribution of Realized Exchange Rate Volatility', *Journal of the American Statistical Association*, forthcoming.
- Andersen, T., Bollerslev, T., Diebold, F. and Ebens, H. (2001b) 'The Distribution of Stock Return Volatility', *Journal of Financial Economics*, forthcoming.
- Black, F. (1989) 'Universal Hedging: Optimizing Currency Risk and Reward in International Equity Portfolios', *Financial Analysts Journal*, 45(4).
- Black, F. (1990) 'Equilibrium Exchange Rate Hedging', *Journal of Finance*, 45(3).
- Davies, R. and Harte, D. (1987) 'Tests for Hurst Effect', *Biometrika*, 74, 95–101.
- Froot, K. (1993) 'Currency Hedging over long horizons', NBER Working Paper, No. 4355.
- Gardner, G. and Wuilloud, T. (1995) 'Currency Risk in International Portfolios: How Satisfying is Optimal Hedging?', *Journal of Portfolio Management*, 21(3).
- Gardner, G. and Stone, D. (1995) 'Estimating Currency Hedge Ratios for International Portfolios', *Financial Analysts Journal*, 51(6).
- Glen, J. and Jorion, P. (1993) 'Currency Hedging for International Portfolios', *Journal of Finance*, 48(5).
- Grinold, R. and Meese, R. (2000) 'Strategic Asset Allocation and International Investing', *Journal of Portfolio Management*, 27(1).
- Jorion, P. (1994) 'Mean Variance Analysis of Currency Overlays', *Financial Analysts Journal*, 50(3).
- Kritzman, M. (1993) 'The Optimal Currency Hedging Policy with Biased Forward Rates', *Journal of Portfolio Management*, 19(4).
- Lee, A. (1990) 'Currency Hedging and Implementation Issues', in L. R. Thomas (ed.) *The Currency Hedging Debate*, IFR, London.
- Meredith, G. and Chinn, M. (1998) 'Long Horizon Uncovered Interest Rate Parity', NBER Working Paper No. 6797.
- Merton, R. (1980) 'On Estimating the Expected Return on the Market: An Exploratory Investigation', *Journal of Financial Economics*, 8(4).
- Nesbitt, S. (1991) 'Currency Hedging Rules for Plan Sponsors', *Financial Analysts Journal*, 47(2).
- Perold, A. and Schulman, E. (1988) 'The Free Lunch in Currency Hedging: Implications for Investment Policy and Performance Standards', *Financial Analysts Journal*, 44(3).
- Sharpe, W. and Tint, L. (1990) 'Liabilities — A New Approach', *Journal of Portfolio Management*, 17(2).
- Solnik, B. (1989) *International Investments*, Addison-Wesley, Reading, MA.
- Wei, W. (1990) *Time Series Analysis — Univariate and Multivariate Methods*, Addison-Wesley, Reading, MA, 349–350.