

Risk and Return of Equity Index Collar Strategies

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Loss aversion leads many investors to seek tail-protection strategies, but put options that provide the most direct insurance are expensive (both in terms of premium paid and lower expected returns). One popular solution for mitigating this cost is to finance the protective put option by selling call options. This strategy is referred to as a *collar*, and this article investigates the collar's risk and return characteristics.

Index collars are typically described as providing protection at little to no cost for those who are willing to trade upside potential for reduced downside risk. For instance, the Chicago Board Options Exchange (CBOE) answers the question of who should use equity collars (CBOE [2012a, 2012b]):

- an investor who is looking to limit the downside risk of a stock position *at little to no cost* [emphasis added]
- an investor who is willing to forgo upside potential in return for obtaining this downside protection

Do collar strategies successfully protect a position at little to no cost? The answer depends on how one chooses to define "cost." Although there may be no upfront outlay if the put option and call option prices are the same, this alone reveals absolutely nothing about the investment attractiveness of the trade and its impact on returns.

In fact, the collar strategy buys one expensive instrument (a put option) and sells another expensive instrument (a call option). Even if the dollar "costs" offset, this is a bad deal relative to an alternative that *does not* systematically buy overpriced instruments, particularly when the purchased instrument is more overpriced than the one being sold.

We will show that the collar strategy's expected return can be decomposed into its equity risk premium and volatility risk premium components. A collar has lower equity beta than the equity index, indicating that it collects less equity risk premium. In the case of the volatility risk premium, there is a netting of volatility exposures from the put and call options. The purchased put options provide negative alpha to the equity index because of the volatility risk premium paid to put sellers, whereas the sold call options provide some positive alpha. In the special case of a zero-cost collar, the put option's negative alpha should dominate because of the implied volatility smile, which has been attributed to demand pressure by Bollen and Whaley [2004]; Gârleanu, Pedersen, and Poteshman [2009]; and Constantinides and Lian [2015].

Lower expected return due to reduced equity risk premium is incontrovertible: A collar mechanically has lower equity exposure than its underlying index. Buying a put option reduces the portfolio's equity

exposure, as does selling a call option. This is true whether the collar provides a credit, provides a debit, or is self-financing. This is also true whether the options are fairly, cheaply, or expensively priced.

Negative alpha due to the put option's volatility risk premium is a well-documented, economically rational empirical observation.¹ The collar's upfront net cost may be negligible, but its negative alpha can have a material impact on investors' returns. Investing in a typical collar implementation has provided lower returns and a lower Sharpe ratio than investing directly in the S&P 500 Index.² In other words, investors would be better off simply reducing their equity exposure.

CONSTRUCTING A COLLAR

Exhibit 1 plots an example payoff diagram for the simple collar strategy in which the purchased put option and written call option expire on the same date. The exhibit illustrates how the collar strategy trades off capped upside for limited downside.

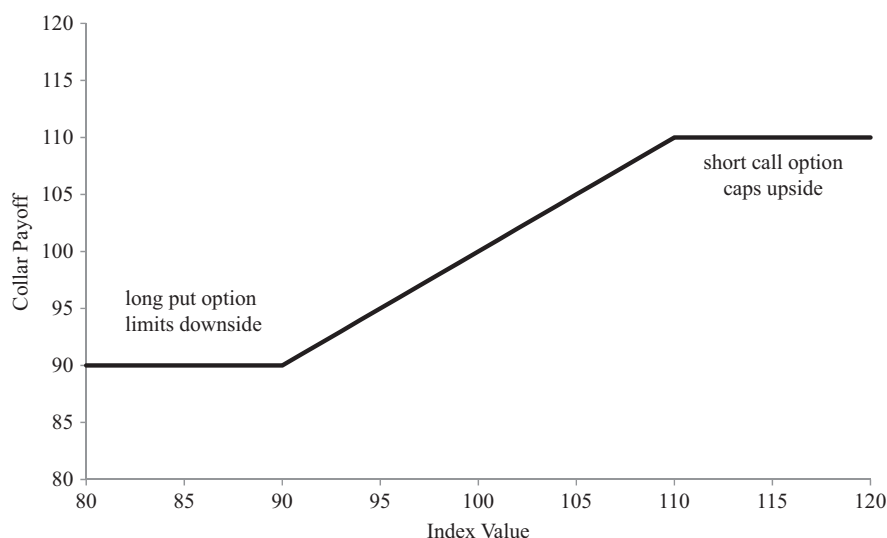
Many degrees of freedom exist when constructing a collar strategy. Most important are the strikes and maturities of the purchased put option and written call option. Collar strategies are often intended to be mostly or fully self-financed (in that the call option premium

received mostly or fully offsets the put option premium paid). This objective typically drives collar portfolio construction. For instance, a desired protection level might be selected, such as three-month protection at 95% of the current index value. The call option's strike then might be selected such that its premium most closely matches that of the purchased 5% out-of-the-money put option. This approach provides the *zero-cost* collar. Alternatively, the call option's strike and maturity can be selected such that the strategy is zero-cost on average.

The recently launched CBOE S&P 500 95-110 Collar Index (CLL) is an example of a collar implementation that buys three-month put options that are about 5% out-of-the-money at the time of purchase and sells one-month call options that are about 10% out-of-the-money at the time of sale. This portfolio construction is actually far from zero-cost. In fact, on average this strategy spends roughly \$7.50 on put options for every dollar it collects from selling calls. As far as we are aware, however, CLL is the only publicly available index that tracks an S&P 500 Index collar strategy. Given that it was intended to serve as a benchmark for these strategies, we feel that it is reasonable to treat CLL's performance as representative (or at least illustrative) of collars in general.³

EXHIBIT 1

Illustrative Collar Payoff Diagram



Note: Illustration is long the equity index, long a put option with \$90 strike price, and short a call option with \$110 strike price. The prices of the two options are equal, and they expire at the same time.

EXHIBIT 2

CBOE Collar Performance Summary (1986–2014)

	S&P 500 Index	CBOE Collar Index
Excess Return	7.3%	3.2%
Volatility	15.7%	10.7%
Sharpe Ratio	0.47	0.30
Beta	1.00	0.62
Upside Beta	1.00	0.68
Downside Beta	1.00	0.58

Notes: The table shows summary statistics for the S&P 500 Index (SPX) and the CBOE S&P 500 95-110 Collar Index (CLL). The date range is July 1, 1986, to December 31, 2014. Returns are excess of cash. "Excess Return" is an arithmetic average annualized return. Volatility, beta, and upside/downside beta were computed using 21-day overlapping returns. We define downside beta as $\frac{\sum_{t|x_t < 0} (y_t - \bar{y}) * (x_t - \bar{x})}{\sum_{t|x_t < 0} (x_t - \bar{x})^2}$ where y_t is the collar's trailing 21-day return on day t , x_t is the S&P 500's trailing 21-day return on day t , and \bar{y} and \bar{x} are their full-sample average 21-day returns. The upside beta, similarly, is $\frac{\sum_{t|x_t > 0} (y_t - \bar{y}) * (x_t - \bar{x})}{\sum_{t|x_t > 0} (x_t - \bar{x})^2}$.

COLLAR PERFORMANCE

Exhibit 2 reports performance characteristics for CLL and the S&P 500 Index (SPX) over the period July 1, 1986, through December 31, 2014. Over this period, the collar earned significantly less return in excess of cash than had the S&P 500 Index: 3.2% per year for CLL versus 7.3% per year for the S&P 500 Index. It has also had significantly lower volatility than the S&P 500 Index: 10.7% for CLL versus 15.7% for the S&P 500 Index. The collar has realized a Sharpe ratio approximately 35% lower than the S&P 500 Index.

The collar is expected to have lower average returns than the S&P 500 Index because its limited loss and capped gain reduce its equity exposure. This floor and cap are visible in Exhibit 3, which scatters the collar returns against the S&P 500 over the period, using 21-day overlapping returns. CLL's 0.62 equity beta is also visible as the slope of the regression line in the scatter. This number indicates that, as a starting point, the collar is expected to earn 38% less equity risk premium than the underlying S&P 500 Index.

However, there is more to the story than just earning less equity risk premium. Had lost equity risk premium been the only issue, the collar would have had 4.5% annualized excess return. Its 3.2% annualized excess return indicates the collar strategy has provided –1.3% of alpha per year on average. Somehow, the options have significantly detracted from performance

even after accounting for their reduced equity exposure. This article explores the reasons why.

The collar might be constructed such that it is self-financing, but the prices of the options traded are not really what matter. What really matters is the options' prices versus their fundamental or actuarially fair values. Equity index options tend to be expensive because of investor preference for loss avoidance, and out-of-the-money put options tend to be more expensive than out-of-the-money call options. Hence, buying protection is expected to hurt performance on a risk-adjusted basis because put options are expensive. Selling upside helps risk-adjusted performance for the same reason, but not to a great enough extent, which is why the collar strategy has realized 1.3% per year of negative alpha.

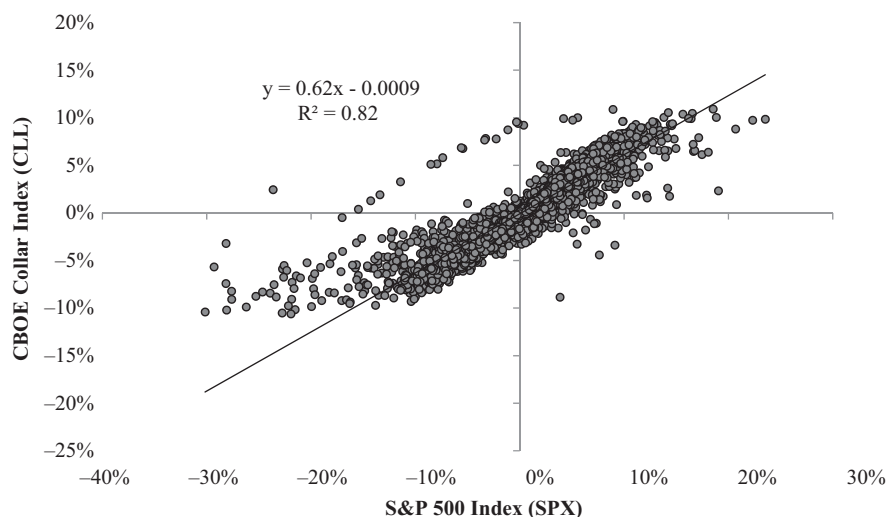
To visualize, Exhibit 4 plots the payoff diagram to an illustrative self-financing collar in which the sold call is cheap relative to the purchased put option. Comparing the mispriced collar to a fairly priced one, the call option strike in the former will be below the call option strike in the latter. Therefore, it can be observed that the mispriced collar's payoff is either identical to or worse than the fairly priced collar's payoff in all cases.

COLLAR PERFORMANCE ATTRIBUTION

When attributing the performance of option-related portfolios, we find it more instructive to focus on risk exposures rather than payoff diagrams.⁴

EXHIBIT 3

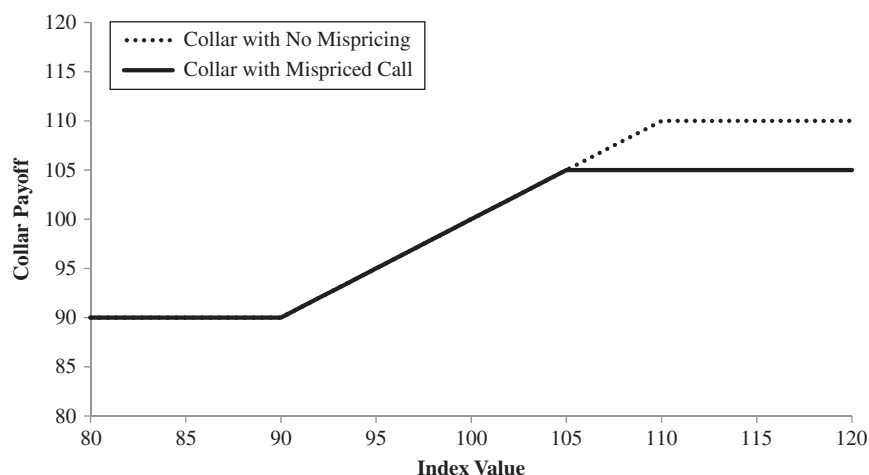
CBOE Collar Index vs. S&P 500 Index 21-Day Returns (1986–2014)



Notes: The chart plots overlapping 21-day returns for the SPX and the CLL. Data are present for the period from July 1, 1986, to December 31, 2014.

EXHIBIT 4

Illustrative Mispriced Collar Payoff Diagram



Notes: The “Collar with Mispriced Call” is long the equity index, long a put option with a \$90 strike price, and short a call option with a \$105 strike price. The “Collar with No Mispricing” is long the equity index, long a put option with a \$90 strike price, and short a call option with a \$110 strike price. In both scenarios, it is assumed that the prices of the two options are equal and that they expire at the same time.

Exhibit 5 shows the risk exposure arising from each component of the strategy. The collar begins with a long equity position, which provides positive equity exposure and no volatility exposure. A put option is purchased, which reduces equity exposure because a put option has negative delta and introduces long volatility exposure.

Both of the put option exposures reduce the strategy’s expected return. Lower equity exposure means less equity risk premium is collected, and long volatility exposure means that volatility risk premium is paid to the put option seller. A call option is then sold, which further reduces equity exposure because it has positive delta and introduces

EXHIBIT 5

Contributions to Strategy Returns

	Equity Risk Premium	Volatility Risk Premium	Dynamic Equity Timing
Long Equity	Earns	—	—
Long Call Option	Earns	Pays	Momentum
Long Put Option	Pays	Pays	Momentum
Short Call Option	Pays	Earns	Reversal
Short Put Option	Earns	Earns	Reversal

Note: This table shows the risk exposure arising from each component of a collar strategy.

short volatility exposure. Whereas the long put option and short call option have offsetting volatility exposures, both option positions reduce the portfolio's equity exposure.

Israelov and Nielsen [2015a] proposed a performance attribution methodology for options-related portfolios. Their attribution identifies and measures a portfolio's equity timing arising from option convexity. Both option positions within the collar have dynamic equity exposure, but they partially offset each other: Long options bet on momentum and short options bet on reversal. These timing bets do not perfectly offset because the call and put options may have different strikes and maturities. From a risk management perspective, the put option's dynamic equity exposure may be desired because it reduces the strategy's equity exposure as losses accumulate in order to create a floor—it is a form of drawdown control. The short call option's dynamic exposure also reduces the collar's equity exposure as gains accumulate; however, it is unclear why this would be desired unless an investor seeks to express a very specific view on equity market reversals.

To better understand the drivers of the CBOE Collar Index's performance, we construct a portfolio that mimics the collar index and decompose its return in excess of cash, as suggested by Israelov and Nielsen [2015a], into (1) passive equity exposure, (2) dynamic equity exposure, and (3) volatility exposure. The decomposition is reported in Exhibit 6. Results are reported over the period March 25, 1996, through December 31, 2014, a smaller window than considered in Exhibit 2 because of data availability.

The strategy characteristics over the shorter 18-year sample are similar to those reported earlier over the

original 28-year window. The collar realized 3.6% annualized excess return, 3.2% lower than the S&P 500 Index. We define the passive equity exposure as the strategy's average equity exposure. *Passive equity* exposure provides 0.75 beta and 5.1% per year in equity risk premium. Equity exposure arising from option path dependence is attributed to *dynamic equity* (−0.07 beta), and equity exposure arising from correlation between equity returns and changes in implied volatility is attributed to *volatility* (−0.03 beta).

The option positions have reduced the collar's returns in three ways. First, 1.7% per year of equity risk premium is lost because of a 25% reduction in passive equity exposure. Second, the collar's time-varying equity exposure has further detracted performance by 0.5% per year. However, there is no compelling ex ante expectation that the net dynamic exposure should have non-zero returns, and the 0.5% loss is not statistically significant. Finally, the strategy's net volatility exposure has reduced returns by 1.0% per year. The volatility risk premium is therefore responsible for roughly one-third of the CBOE Collar Index's underperformance relative to the S&P 500 Index.

Exhibit 7 further decomposes the strategy's net volatility exposure into the put and call options' respective contributions. As the −0.29 correlation indicates, these two components partially offset each other by providing volatility exposures in opposite directions. Buying protection via put options significantly detracts from the strategy's performance because options tend to be richly priced, costing 1.6% per year on average in volatility risk premium paid to put option sellers. In this case, selling call options recovered less than 40% of

EXHIBIT 6

Collar Return Decomposition (1996–2014)

	S&P 500 Index	Collar Strategy	Passive Equity	Dynamic Equity	Volatility
Excess Return	6.8%	3.6%	5.1%	−0.5%	−1.0%
Volatility	16.5%	11.4%	12.3%	4.0%	2.1%
Sharpe Ratio	0.41	0.32	0.41	−0.13	−0.45
Risk Contribution	100%	100%	109%	−10%	0%
Beta	1.00	0.64	0.75	−0.07	−0.03
Upside Beta	1.00	0.69	0.74	−0.05	0.00
Downside Beta	1.00	0.61	0.75	−0.09	−0.05

Notes: The table shows summary statistics for the S&P 500 Index, an S&P 500 collar strategy mimicking the methodology of the CLL, and the collar strategy's decomposition. The collar backtest is long the S&P 500 Index, long 5% out-of-the-money front-quarter S&P 500 put options, and short 10% out-of-the-money front-month S&P 500 call options, all held to expiry. Following Israelov and Nielsen [2015a], the collar returns are decomposed into (1) passive equity exposure, (2) dynamic equity exposure, and (3) volatility exposure.

Returns are excess of cash. The date range is March 25, 1996, to December 31, 2014. "Excess Return" is an arithmetic average annualized return. Risk contribution is defined as the covariance of the component with the full strategy, divided by the variance of the full strategy. Volatility, beta, and upside/downside beta were computed using 21-day overlapping returns. We define downside beta as $\frac{\sum_{t|x_t < 0} (y_t - \bar{y}) * (x_t - \bar{x})}{\sum_{t|x_t < 0} (x_t - \bar{x})^2}$ where y_t is the collar's trailing 21-day return on day t , x_t is the S&P 500's trailing 21-day return on day t , and \bar{y} and \bar{x} are their full-sample average 21-day returns. The upside beta, similarly, is $\frac{\sum_{t|x_t > 0} (y_t - \bar{y}) * (x_t - \bar{x})}{\sum_{t|x_t > 0} (x_t - \bar{x})^2}$.

EXHIBIT 7

Volatility Component Return Decomposition (1996–2014)

	Volatility	Long Volatility (Put Option)	Short Volatility (Call Option)
Excess Return	−1.0%	−1.6%	0.6%
Volatility	2.1%	2.2%	0.8%
Sharpe Ratio	−0.45	−0.70	0.75
Risk Contribution	0%	−2%	2%
Beta	−0.03	−0.04	0.02
Upside Beta	0.00	−0.02	0.02
Downside Beta	−0.05	−0.07	0.01
Correlation	—		−0.29

Notes: The table shows summary statistics for the volatility component of an S&P 500 collar strategy mimicking the methodology of the CLL, and it further decomposes this component into the put and call options' respective contributions. The collar backtest is long the S&P 500 Index, long 5% out-of-the-money front-quarter S&P 500 put options, and short 10% out-of-the-money front-month S&P 500 call options, all held to expiry. Following Israelov and Nielsen [2015a], the collar returns are decomposed into (1) passive equity exposure, (2) dynamic equity exposure, and (3) volatility exposure. For this table, the volatility component is then decomposed into the put and call options' contributions.

Returns are excess of cash. The date range is March 25, 1996, to December 31, 2014. "Excess Return" is an arithmetic average annualized return. Risk contribution is defined as the covariance of the component with the full strategy, divided by the variance of the full strategy. Volatility, beta, upside/downside beta, and correlation were computed using 21-day overlapping returns. We define downside beta as $\frac{\sum_{t|x_t < 0} (y_t - \bar{y}) * (x_t - \bar{x})}{\sum_{t|x_t < 0} (x_t - \bar{x})^2}$ where y_t is the collar's trailing 21-day return on day t , x_t is the S&P 500's trailing 21-day return on day t , and \bar{y} and \bar{x} are their full-sample average 21-day returns. The upside beta, similarly, is $\frac{\sum_{t|x_t > 0} (y_t - \bar{y}) * (x_t - \bar{x})}{\sum_{t|x_t > 0} (x_t - \bar{x})^2}$.

the volatility risk premium paid out by purchasing put options, about 0.6% per year.

Net long volatility exposure is not a necessity of collar construction. Alternative approaches to option selection in terms of maturities or strikes can potentially lead to a collar that has positive alpha because it is net short volatility. The benchmark collar example is nevertheless a good reminder that buying put options and selling call options is not necessarily neutral in terms of expected return.

The collar successfully provides asymmetric beta to the S&P 500, although the asymmetry is moderate. To estimate it, we can calculate the strategy's downside and upside betas. Specifically, we define downside beta as:

$$\frac{\sum_{t|x_t < 0} (\gamma_t - \bar{\gamma}) \star (x_t - \bar{x})}{\sum_{t|x_t < 0} (x_t - \bar{x})^2} \quad (1)$$

in which γ_t is the collar's trailing 21-day return on day t ; x_t is the S&P 500's trailing 21-day return on day t ; and $\bar{\gamma}$ and \bar{x} are their full-sample average 21-day returns. The upside beta, similarly, is:

$$\frac{\sum_{t|x_t > 0} (\gamma_t - \bar{\gamma}) \star (x_t - \bar{x})}{\sum_{t|x_t > 0} (x_t - \bar{x})^2} \quad (2)$$

Using these definitions, the collar's downside beta is 0.61 versus its overall 0.64 equity beta and its 0.69 upside beta. Interestingly, approximately half of the asymmetric

downside beta may be attributed to dynamic equity exposure, indicating that investors could have achieved much of the desired downside protection by dynamically trading the index and without trading options. Dynamic equity trading can protect against downtrends; long option positions can protect against both downtrends and gaps. Thus, on the margin, options provide gap protection.

PATH DEPENDENCE

Path dependence is one of the significant challenges posed by portfolios containing options. The portfolio's exposure to equity and volatility are meaningfully affected by its options' strikes and maturities as well as by changing market conditions. Unfortunately, the resulting exposures might not necessarily reflect the manager's views.

To illustrate the effect of path dependence, Exhibits 8 and 9 provide examples of the CBOE Collar Index's risk exposures on two option expiration (rebalance) dates representing different risk environments. December 19, 2008, represents a high-volatility environment with the VIX Index at 44.9, and September 19, 2014, represents a low-volatility environment with the VIX Index at 12.1.

The collar strategy is the same on both dates, but its risk exposures on these two dates markedly differ. On December 19, 2008, the collar had approximately 0.5 S&P 500 Index exposure. On September 19, 2014, its exposure to the S&P 500 was 50% higher. It is not

EXHIBIT 8

High-Volatility Environment (December 19, 2008)—S&P 500: 887.88 and VIX: 44.9

	Long 3 Month Put	Short 1 Month Call	Collar Greek Exposure (Combined)
Strike	855	995	—
Strike/Index Value	96%	112%	—
Price/Index Value	7.0%	0.5%	—
Implied Volatility	45.1%	34.4%	—
Delta Contribution	−0.39	−0.12	0.49
Gamma Contribution	1.7%	−2.1%	−0.43%
Vega Contribution	0.19%	−0.05%	0.14%

Note: This table provides an example of the risk exposures for the CLL on December 19, 2008 (a high-volatility environment).

EXHIBIT 9

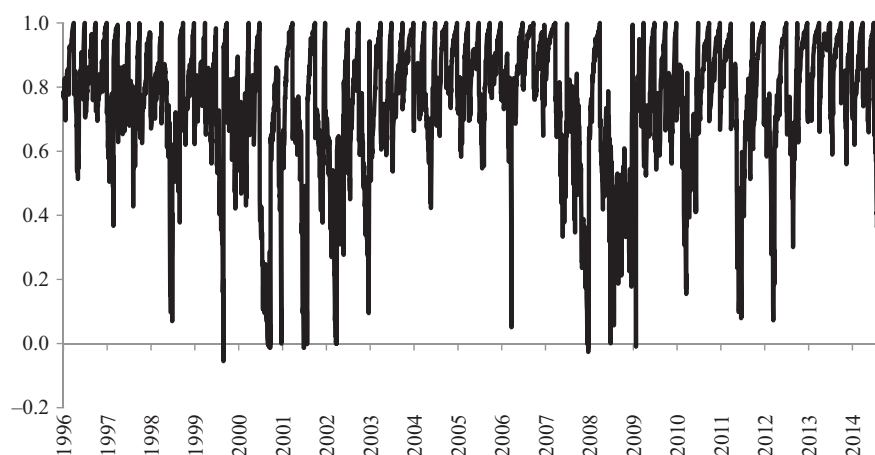
Low-Volatility Environment (September 19, 2014)—S&P 500: 2010.40 and VIX: 12.1

	Long 3 Month Put	Short 1 Month Call	Collar Greek Exposure (Combined)
Strike	1910	2300	—
Strike/Index Value	95%	114%	—
Price/Index Value	1.1%	0.0%	—
Implied Volatility	14.5%	16.1%	—
Delta Contribution	−0.24	−0.00	0.76
Gamma Contribution	4.4%	−0.1%	4.28%
Vega Contribution	0.16%	0.00%	0.15%

Note: This table provides an example of the risk exposures for the CLL on September 19, 2014 (a low-volatility environment).

EXHIBIT 10

CBOE Collar Index's Equity Exposure (1996–2014)



Notes: The chart plots the delta exposure for an S&P 500 collar strategy mimicking the methodology of the CLL. The backtest is long the S&P 500 Index, long 5% out-of-the-money front-quarter S&P 500 put options, and short 10% out-of-the-money front-month S&P 500 call options, all held to expiry. The date range is from March 25, 1996, to December 31, 2014.

clear why a collar investor should want to have 50% higher equity exposure on September 19, 2014, than on December 19, 2008. Exhibit 10 plots the collar index's equity exposure over the period beginning in 1996 and ending in 2014 and shows how the collar index's equity exposure has considerable variation over time. Its 95% confidence interval is a low of 0.16 and a high of 0.99.

In fact, on a few occasions the collar index remarkably has close to zero or slightly *negative* equity exposure.

For example, on March 14, 2008, the strategy would have been long a deep in-the-money put option with a delta of −1.00 and short a deep out-of-the-money call option providing a delta exposure of −0.03, resulting in a portfolio delta exposure of −0.03. This unusual situation arose because the S&P 500 had fallen over 13% since the put option had originally been purchased, so instead of straddling the spot index value, the call and put strikes were now both above it. Further exacerbating the issue,

the strikes were at this point unusually close together since the call strike was selected to be 10% above the February 2008 spot value whereas the put had been selected to be 5% below the much higher December 2007 spot value.

The collar strategy also exhibits significant variation in its exposure to volatility as defined by both its gamma, which measures an option's convexity to the underlying security price, and by its vega, which measures an option's sensitivity to implied volatility changes. On December 19, 2008, the collar was short gamma, whereas on September 19, 2014, the collar was long gamma. Not only was the sign different, but so, too, was the magnitude of the exposure. Gamma was nearly 10 times larger on September 19, 2014, than on December 19, 2008. It is not clear (to us) why a collar investor should want to be long significant gamma in September 2014 and short moderate gamma in December 2008.

Exhibit 11 plots the collar index's gamma exposure over the period beginning in 1996 and ending in 2014 and shows how the collar index's volatility exposure also has considerable variation over time. Consistent with the collar paying out volatility risk premium because of its net long volatility exposure, it is long gamma 84% of the time, and its average gamma is 0.02. Likewise, the collar's net vega exposure also changed signs multiple

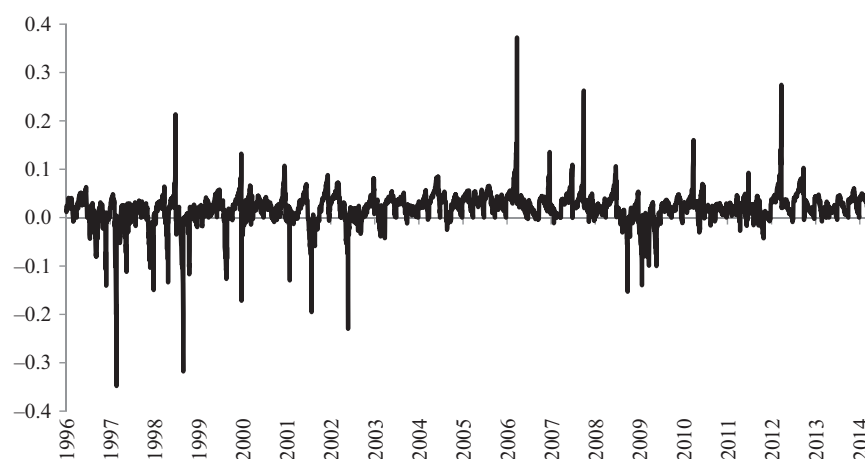
times over the period, meaning that the portfolio was sometimes positively exposed to changes in implied volatility, whereas at other times it was negatively exposed.⁵ Its net vega exposure was 0.077% on average, and it was positive 94% of the time.

For all intents and purposes, on September 19, 2014, the collar was a protective put strategy. The call option's premium, delta, gamma, and vega were all nearly zero; only the equity position and put option contributed to the collar's risk exposure. This is potentially troublesome because the collar approach is typically recommended as an alternative to buying expensive put options for protection. For an investor who finds a protective put to be unattractive, it is confusing that the collar would be deemed an attractive alternative on dates such as this.

In fact, over the period 1996 through 2014, we estimate that the collar strategy's risk exposures were effectively equivalent⁶ to those of a protective put strategy on 19% of days. These dates were determined solely by the path dependence of the portfolio and not by any active decision on the part of an investor. Because protective put strategies are long volatility, however, this implies that the collar was (at least in part) unintentionally acting as a volatility timing strategy. Given a belief in reasonably efficient markets, it seems extremely dubious that

EXHIBIT 11

CBOE Collar Index's Gamma Exposure (1996–2014)



Notes: The chart plots the gamma exposure for an S&P 500 collar strategy mimicking the methodology of the CLL. The backtest is long the S&P 500 Index, long 5% out-of-the-money front-quarter S&P 500 put options, and short 10% out-of-the-money front-month S&P 500 call options, all held to expiry. The date range is from March 25, 1996, to December 31, 2014.

such a timing strategy (which is easily computable *ex ante*) should work. Therefore, collar investors who view protective puts unfavorably should be concerned that a collar may resemble one so frequently.

POTENTIAL COLLAR BENEFITS

Although the drawbacks discussed in the previous two sections (significant path dependence, dynamic equity timing exposure, and partially offsetting volatility exposures) apply to all collars to some degree, the precise magnitudes of their impacts will vary depending on the particular option-selection strategy used. These drawbacks seem to be particularly acute for common collar constructions, but a case could potentially be made for collar strategies or collar-like strategies that attempt to minimize the flaws while keeping intact most of the downside protection benefits.

In particular, one potentially promising collar construction could sell at-the-money, short-dated call options while buying deep-out-of-the-money, longer-dated puts. Such a construction would have the advantage that, on net, it should be short volatility and thus *collect* the volatility risk premium rather than pay it out, as in more typical constructions. However, unlike with a pure equity or covered call construction, the purchased put options would still provide a floor to protect against large gaps.

A similar case could be made for a collar-like strategy that sells delta-hedged at-the-money call options while buying delta-hedged deep-out-of-the-money puts. This portfolio can be thought of as a protected short volatility strategy, selling at-the-money options to earn volatility risk premium and buying deep out-of-the-money to protect against gaps (i.e., large spikes in volatility). Careful selection of option strike and maturity should provide a strategy that maintains a net short volatility exposure.

As these examples show, we do not intend to make the claim that all possible index collar strategies should be avoided in all possible investment environments. We are simply pointing out that much of the traditional framework for thinking about these strategies is muddled, and a clearer picture of both the positives and negatives emerges from a closer, more careful analysis. Rather than indicting collar strategies *in general*, we believe that, *as typically implemented*, a collar would likely be a poor choice for most investors.

COLLAR ALTERNATIVES

An investor who demands equity gap protection has limited alternatives to expensive protection-buying strategies. Gap protection requires long volatility exposure, and long volatility exposure has negative expected returns because someone has to underwrite this financial insurance and will not do so for free.

Alternatives to collar strategies can help investors reduce their downside risk if they are willing to forgo gap protection. We will analyze four candidates. As a starting point, the most direct method of reducing downside risk is simply to reduce overall risk itself by selling some equity. Similar to the collar strategy, this approach has lower expected return because it collects less equity risk premium, but it does not purchase negative-alpha options and thus should have a superior risk-adjusted return. A second alternative is a protective put strategy that, like the collar, reduces its equity exposure by buying put options. Since it does not attempt to offset the cost through call option sales, however, this strategy has negative-alpha long volatility exposure. The third alternative we consider is the covered call strategy, which also has lower equity exposure than the underlying index but adds alpha to the portfolio in the form of volatility risk premium collected by selling call options. As a final alternative, we look at the risk-managed covered call strategy proposed by Israelov and Nielsen [2015a], which dynamically trades the equity index to maintain a constant equity exposure, hedging the traditional covered call's time-varying equity exposure. Since the various strategies differ significantly (by construction) in their average return, volatility, and betas, for ease of comparison we lever each strategy to provide the same *ex post* compounded return as the equity index collar and then compare their risk characteristics.

Exhibit 12 reports summary characteristics for these portfolios. Since 1996, CLL has provided 2.3% annual compounded excess return.⁷ A portfolio that invests 36% of its net asset value (NAV) in the S&P 500 Index and the remaining 64% in cash would have earned the same 2.3% compound excess return. Investing 49% of NAV in the CBOE S&P 500 BuyWrite Index (BXM) and 51% in cash or 45% of NAV in the risk-managed BuyWrite strategy and 55% in cash also matches the collar's annual compounded return.

EXHIBIT 12

Collar Alternatives Performance Summary (1996–2014)

	S&P 500 Index	CBOE Collar Index	S&P 500 Index (36%)	CBOE PPUT Index (101%)	CBOE BXM Index (49%)	Hedged BXM Index (45%)
Excess Return (Geom.)	5.2%	2.3%	2.3%	2.3%	2.3%	2.3%
Volatility	16.5%	11.4%	5.9%	13.2%	5.6%	4.1%
Sharpe Ratio	0.32	0.20	0.39	0.17	0.41	0.55
Beta	1.00	0.63	0.36	0.73	0.30	0.24
Upside Beta	1.00	0.69	0.36	0.82	0.25	0.23
Downside Beta	1.00	0.59	0.36	0.68	0.34	0.25

Notes: The table shows summary statistics for the S&P 500 Index, the CLL, as well as four strategies levered to have the same return as CLL: the S&P 500 Index, the CBOE S&P 500 5% Put Protection Index (PPUT), the CBOE S&P 500 BuyWrite Index (BXM), and the risk-managed covered call strategy proposed by Israelov and Nielsen [2015a] (a strategy that dynamically trades the equity index to maintain a constant equity exposure, hedging the traditional covered call's time-varying equity exposure). These four strategies were levered 36%, 101%, 49%, and 45%, respectively.

Returns are excess of cash. The date range is March 25, 1996, to December 31, 2014. "Excess Return (Geom.)" is a geometric average annualized return. Volatility, beta, and upside/downside beta were computed using 21-day overlapping returns. We define downside beta as $\frac{\sum_{t|x_t < 0} (y_t - \bar{y}) * (x_t - \bar{x})}{\sum_{t|x_t < 0} (x_t - \bar{x})^2}$ where y_t is the collar's trailing 21-day return on day t , x_t is the S&P 500's trailing 21-day return on day t , and \bar{y} and \bar{x} are their full-sample average 21-day returns. The upside beta, similarly, is $\frac{\sum_{t|x_t > 0} (y_t - \bar{y}) * (x_t - \bar{x})}{\sum_{t|x_t > 0} (x_t - \bar{x})^2}$.

A very small amount of leverage would be required to match the collar's compounded return by investing in the CBOE S&P 500 5% Put Protection Index (PPUT), with 101% of NAV needed.

Over the time period, the collar's performance has been superior to the protective put strategy's. While PPUT realized 13.2% annualized volatility in achieving its return, the collar required only 11.4%. This outperformance should be expected based on the two strategies' portfolio constructions because the collar's call option sales provide a positive alpha boost relative to the protective put by having a smaller long-volatility exposure.

The comparison to the other alternatives, however, is much less favorable for the collar. In all three instances, the collar has been a significantly more volatile approach to achieving the same return: 11.4% annualized volatility versus 5.9%, 5.6%, and 4.1% for the S&P 500, BuyWrite, and risk-managed BuyWrite portfolios, respectively. The partially invested S&P 500 portfolio matches the collar's return with lower volatility because the S&P 500 has no negative alpha. Investing in the BuyWrite strategy allows for even lower volatility because selling call options earns the volatility risk premium, and the risk-managed BuyWrite further reduces risk by hedging the BuyWrite's uncompensated dynamic equity exposure.

All three of these alternatives have achieved the same return as the CBOE Collar Index, but with lower risk, higher Sharpe, and lower beta. In fact, compared to each the collar has even had higher downside beta per unit of average return. Unlike these alternatives, however, the collar *does* provide downside gap protection. But is it worth the cost?

BLACK SWANS

Based on the historical evidence, one should expect selling delta-hedged options to be more profitable than buying them, since the volatility risk premium is positive on average. As we have shown, this implies that the collar strategy should perform poorly relative to alternate ways of reducing downside risk. One possible objection, however, is that forming an expectation by just looking at the historical record may implicitly underweight the probability of rare, extremely negative events—*black swans*. A skeptic can argue that option sellers have merely been "lucky" in recent years, so the collar may start looking better again if one adjusts one's assumption about the expected frequency of black swans.

Although this is a reasonable objection, it turns out that black swans would need to be unreasonably frequent

to make the collar to start looking attractive. Mimicking an argument made by Israelov and Nielsen [2015b], we considered hypothetical market crashes of a magnitude similar to the October 1987 crash. Specifically, in each of these events we assumed that the S&P 500 falls 20% in one day and implied volatility spikes to 150%.⁸

With these assumptions, we can estimate the black swan return earned by a collar portfolio (based on CLL construction) consisting of a long position in the S&P 500 Index, a long position in a 5% out-of-the-money put option with three months to expiration, and a short position in a 10% out-of-the-money call option with one month to expiration. Although the index would be down 20% on the day and the short call option position would also be down 5.3%, these losses would be offset by a 32.2% gain in the long put position, resulting in the overall collar portfolio being up 6.9%.

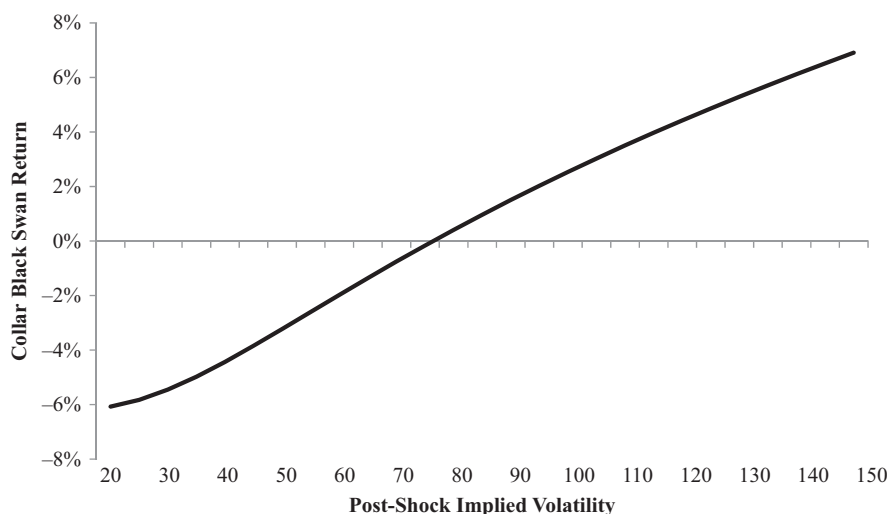
The fact that the collar portfolio *made* money in our simulated black swan event may seem surprising. Although a collar should provide some downside protection, its long-term beta to the underlying index is still positive, so at first glance one would expect it to

lose money in an equity crash. The explanation is that this collar construction has a significantly net long vega exposure (0.15%), so all else equal, it should make money as implied volatility increases if the increase is similar across the term structure. As shown in Exhibit 13, the 150% post-shock implied volatility in our simulation is extreme enough that the gains from the portfolio's positive vega exposure outweigh the losses one would expect from its beta alone.

The sign of the collar's return in our simulation is also consistent with the positive observed return of CLL on "Black Monday" (October 19, 1987), a date on which it gained 3.3% despite the S&P 500 falling 20.5%. As in our simulated case, much of the positive return can be explained as a result of the portfolio's positive vega exposure. However, the actual CLL (unlike our simulation) also benefited from some extra "luck" in the sense that it happened to have an unusually low exposure to equity moves on the eve of the crash. Specifically, the S&P 500 fell significantly between the September and October option expiration dates, which meant that on the Friday before the crash, the call option's strike

EXHIBIT 13

Collar Simulated Black Swan Return by Post-Shock Implied Volatility



Notes: The chart plots the simulated black swan return for a collar portfolio as a function of the post-shock implied volatility of the options in the portfolio. The collar portfolio (based on the CLL construction) consists of a long position in the S&P 500 Index, a long position in a 5% out-of-the-money put option with 3 months to expiration, and a short position in a 10% out-of-the-money call option with 1 month to expiration. Pre-shock, both the put and call options are assumed to have implied volatilities of 18%. To generate the black swan returns, we applied hypothetical market crashes of a similar magnitude to the October 1987 crash. Specifically, in each of these events we assumed that the S&P 500 falls 20% in one day, while the implied volatilities of both the put option and the call option increase to the specified post-shock implied volatility level.

would have been roughly 110% of the S&P 500 level, while the put option's strike would have been around 106% of the level. We can estimate that CLL therefore would have had only a 0.26 delta exposure on October 16, 1987, which is in the 4th percentile since 1996 and well below the median delta exposure of 0.78. Without this path dependence effect, CLL's return on "Black Monday" could have been considerably worse.

Returning to our simulation, we can use our calculated 6.9% return then to estimate the black swan frequency assumption required for the collar's expected return to equal the expected return of being long the S&P 500. Specifically, we divide the difference in the two strategies' black swan returns ($6.9\% - -20\% = 26.9\%$) by the difference in observed annualized returns ($7.3\% - 3.2\% = 4.1\%$) to arrive at a frequency assumption of one black swan event every 6.6 years.

If requiring the collar to match the return of the underlying index (collar descriptions give the impression that they do) seems too extreme, a collar investor may instead be content with just matching the *beta-adjusted* S&P 500 return. This investor would, in other words, be hoping that the long delta-hedged put option expected return and the short delta-hedged call option expected return offset each other. Using a calculation similar to the one described in the previous paragraph, the black swan frequency required for this to occur for the CLL construction is approximately once every 22 years.

Given the fact that there has only been one S&P 500 crash of a similar magnitude since at least 1950 (and one crash for the Dow Jones Industrial Average of a similar magnitude since 1897), we would argue that these frequencies seem unreasonably high. In that case, the results in the previous section hold.

CONCLUDING THOUGHTS

Reading a typical collar description can leave the impression that the collar should have approximately the same return as its underlying security: investors give up some upside so that they can reduce their downside, and the net cost is zero because the call option premium collected is roughly equal to that paid for the put option.

The truth is that a typical collar construction should be expected to have lower returns than its underlying security because (1) it has lower equity

exposure and thus earns less equity risk premium, and (2) it purchases put options that are more richly priced than the call options it sells and thus pays volatility risk premium. It is folly to believe that collaring one's equity exposure necessarily leaves one's expected return intact.

The equity index collar is a complex, low-beta strategy, often with negative alpha. It has time-varying equity and volatility exposures, and these dynamic exposures may be unrelated to desired risk allocations and forecasts of expected returns. Those who wish to reduce their equity risk have a simple, elegant, effective, and transparent solution in their toolkit: They can sell a portion of their equities. Buying and selling options each month is unnecessary.

However, reducing a portfolio's equity exposure clearly and mechanically reduces its expected return. If investors must maintain their portfolio's return target, then alternatives can replace the lost equity risk premium with alpha. Volatility risk premium is one potential candidate. An equity index covered call, which simultaneously reduces equity exposure and adds volatility risk premium, is one potential solution.

ENDNOTES

¹Bakshi and Kapadia [2003] analyzed delta-hedged index option returns and found evidence in favor of a volatility risk premium. Hill et al. [2006] showed that covered call returns are higher because of the spread between implied and realized volatility.

²In general, a typical collar strategy should have lower expected returns and a lower expected Sharpe ratio than its underlying, assuming a positive equity risk premium and volatility risk premium for that underlying in the long term. However, a collar strategy is likely to outperform during market downturns, owing to its lower equity exposure and often its net long volatility exposure as well. Szado and Schneeweis [2010], for example, found that a passive collar strategy on the PowerShares QQQ exchange-traded fund (ETF) outperformed its underlying from 1999 to 2009, a period that included both the technology bubble collapse and the financial crisis of 2007–2009.

³Although this particular collar implementation spends more on puts than it collects from selling calls, we arrived at very similar conclusions after simulating other portfolio constructions that were closer to zero-cost. These results are available upon request.

⁴The payoff diagram shown in Exhibit 1 is only appropriate when the call and put option expire on the same date. A collar strategy that buys a three-month put option at a specific strike and then sequentially sells three monthly call options at different strikes cannot be represented in a payoff diagram because of path dependence.

⁵It should be noted, however, that the put and call option implied volatilities did not necessarily move in sync, both because the options had different strikes and because they often had different expirations. Therefore, the interpretation of the net vega exposure is somewhat complicated.

⁶For this calculation, we considered a collar to be equivalent to a protective put on a given day if the call option has a delta exposure less than 0.02, a gamma exposure less than 0.2%, and a vega exposure less than 0.01%.

⁷The strategy's annual simple excess return is 2.9%. Volatility drag reduces its compounded return to 2.3%.

⁸For this scenario, we assumed that all options' implied volatilities spike to 150%, regardless of maturity. In an actual crash, however, the implied volatilities of longer-dated options would probably not increase as much as shorter-dated options' implied volatilities. Our simulation is therefore likely being generous to the collar's return in such an event, since applying a time-to-maturity adjustment to the collar's put option implied volatility shock size would decrease its simulated return.

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