Home Bias in International Macroeconomics

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Summary

Home bias in international macroeconomics refers to the fact that investors around the world tend to allocate majority of their portfolios into domestic assets, despite the potential benefits to be had from international diversification. This phenomenon has been occurring across countries, over time, and across equity or bond portfolios. The bias towards domestic assets tends to be larger in developing countries relative to developed economies, with Europe characterized by the lowest equity home bias, while Central and South America—by the highest equity home bias. In addition, despite the secular decline in the level of equity home bias over time in all countries and regions, home bias still remains a robust feature of the data.

Whether home bias is a puzzle depends on the portfolio allocation that one uses as a theoretical benchmark. For instance, home bias in equity portfolio is a puzzle when assessed through the lens of a simple international capital asset pricing model (CAPM) with homogeneous investors. This model predicts that investors should hold world market portfolios, namely a portfolio with the share of domestic asset equal to the share of those assets in the world market portfolio. For instance, since the share of US equity in the world capitalization in 2016 was 56%, then US investors should allocate 56% of their equity portfolio into local assets, while investing the remaining 44% into foreign equities. Instead, foreign equity comprised just 23% of US equity portfolio in 2016, hence the equity home bias.

Alternative portfolio benchmark comes from the theories that emphasize costs for trading assets in international financial markets. These include transaction and information costs, differential tax treatments, and more broadly, differences in institutional environments. This research, however, has so far been unable to reach a consensus on the explanatory power of such costs.

Yet another theory argues that equity home bias can arise due to the hedging properties of local equity. In particular, local equity can provide insurance from real exchange rate risk and non-tradable income risk (such as labor income risk), and thus a preference towards home equities is not a puzzle, but rather an optimal response to such risks.

These theories, main advances and results in the macroeconomic literature on home bias are discussed in this article. It starts by presenting some empirical facts on the extent and dynamics of equity home bias in developed and developing countries. It is then shown how home bias can arise as an equilibrium outcome of the hedging demand in the model with real exchange rate and non-tradable labor income risk. Since solving models with portfolio choice is challenging, the recent advances in solving such models are also outlined in this article.

Integrating the portfolio dynamics into models that can generate realistic asset price and exchange rate dynamics remains a fruitful avenue for future research. A discussion of additional open questions in this research agenda and suggestions for further readings are also provided.

Keywords: equity, bonds, home bias, portfolios, international risk-sharing

Subjects: International Economics, Macroeconomics and Monetary Economics

Introduction

Investors around the world allocate the majority of their asset portfolios to domestic assets despite the potential diversification gains to be had from holding foreign assets. This lack of diversification in spite of the potential welfare gains from international diversification has been termed the "home bias puzzle." The existing literature has put forward two main explanations for equity home bias. The first strand argues that investors favor local equity to hedge risk, such as real exchange rate risk and non-tradable income risk. The second class of explanations relies on costs for trading assets in international financial markets. These include transaction costs, differential tax treatments, and more broadly, differences in institutional environments. This article reviews some of the channels and results in this literature, as well as discusses some of the methodological advances and presents home bias data facts. More precisely, the objectives of this article are threefold.

First, to document empirical properties of equity home bias in a large sample of developed and developing countries and to show how home bias have evolved over time. Dispersion in the equity home bias across countries is shown to be significant. On average, equity home bias is higher in developing countries than in developed economies. Among developed countries, Europe has the most diversified equity portfolio, while Japan and Australia have the highest equity home bias. Importantly, however, the degree of the bias has declined over time for almost all countries under study.

Second, to discuss several factors that can explain the equity home bias. For this purpose, a simple two-country, two-period model with international trading in goods, as well as equities and real bonds, is presented. The model features two key sources of risk that have been highlighted in the literature on portfolio choice and risk-sharing: the labor income risk, as emphasized by Baxter and Jermann (1997), and the relative price risk emphasized by Cole and Obstfeld (1991). To illustrate the effects of these risks on equilibrium portfolios, consider a simple two-country world where each country is endowed with stochastic output (Lucas tree) as in Lucas (1982). Further, suppose countries can trade claims to their respective endowment streams. If the preferences of the countries are symmetrical, holding half the shares in each tree will ensure perfect international risk-sharing, where the representative household in each country consumes half of the world endowment. This is a benchmark result of full diversification.

Now, suppose that households in each country work and receive a constant share of their country's endowment as their labor income. Since local workers are not allowed to work in the foreign country, this labor income in non-diversifiable. The remaining share of the output of each tree is paid out as dividends to the shareholders. In this world, labor income is perfectly correlated with the payouts on local equity. Therefore, households will choose portfolios that are 100% foreign-biased as this will allow them to again consume half of the world endowment. Thus, adding non-tradable risk worsens the equity home bias puzzle, which is the key message of Baxter and Jermann (1997).

Next, extend the simple example above to introduce relative price movements. In particular, assume that each country now is endowed with a different good and that the two goods can be freely traded across countries. Households in each country have preference for consumption of both goods. Now any changes in the relative endowments of the two goods will lead to relative price movement in the opposite direction. As emphasized by Cole and Obstfeld (1991), these relative price movements provide automatic insurance against country-specific shocks. In the special case of the unitary elasticity of intra-temporal substitution between local and foreign goods, the adjustments in the relative price will completely offset the changes in relative endowments, effectively delivering perfect insurance of the country-specific shocks, even in the absence of any financial instruments. As a result, any portfolio allocation of stocks, including fully home biased, will be compatible with the perfect risk-sharing equilibrium. When the elasticity of substitution is different from one, portfolio allocations will be sensitive to the model parameters and shocks.

The model presented in this article embeds the two risk factors discussed above and incorporates multiple asset classes—equities and bonds. The presence of a rich menu of assets allows for an interaction between equities and bonds in portfolio decisions—a feature that was recently emphasized in several papers (see Coeurdacier & Gourinchas, 2016; Coeurdacier, Kollmann, & Martin, 2010). In particular, these papers show that in the model payoffs on real bonds are strongly correlated with the real exchange rate (or terms of trade). Thus, households can structure their bond portfolios to hedge real exchange rate movements. Equities can then be used to hedge the impact of any additional source of risk (for instance, labor income risk) on investors' wealth. The equity home bias arises if non-financial income risk is negatively correlated with equity returns, after controlling for bond returns.[1] We illustrate these insights in detail in the model.

Solving models with portfolio choice is challenging for several reasons. One is that portfolio choices and real decisions are inherently intertwined, which requires a joint solution for portfolio and real allocations (unless markets are complete). Second is that standard linear approximation methods do not work for portfolio choice problems since portfolio allocations up to the first-order of approximation are indeterminate as all assets provide the same expected return. In this article I discuss the recent advances in solving portfolio choice models and use a version of one of the approaches to solve the illustrative model.

The third goal of the article is to discuss the challenges, open questions in this research agenda, and provide suggestions for further readings.

The article is structured as follows. Section "Introduction" presents data facts on equity home bias. Section "Empirical Evidence" discusses the methodological advances in the portfolio choice literature. The model is presented and solved for the optimal equity and bond portfolios in section "Solving Portfolio Choice Models." Section "Model" concludes. The appendix contains information about data sources, additional empirical results, and detailed model derivations.

Empirical Evidence

How home-biased are international portfolios? Home bias is defined here in the simplest possible way, using an international capital asset pricing model (CAPM) with homogeneous investors. This model predicts that investors should hold world market portfolios, namely a portfolio with the share of domestic asset equal to the share of those assets in the world market portfolio. For instance, since the share of Canada's equity in the world capitalization in 2016 was 4%, Canadian investors should allocate 4% of their equity portfolio into local assets, while investing the rest in foreign equities. Instead, local equity comprised 58% of Canadian equity portfolio in 2016, hence the equity home bias. A similar portfolio allocation characterizes other countries in the world. To show this formally, we construct a measure of equity home bias for country i in period t as follows:

$$HB_i = 1 - \frac{\text{Share of foreign equity in portfolio of country } i}{\text{Share of foreign equity in the world market portfolio}},$$

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where the numerator (Share of foreign equity in portfolio of country i) is computed as

$$\frac{\text{Portfolio Investment in Equity Assets}_i}{\text{Portfolio Investment in Equity Assets}_i + \text{Market Capitalization}_i - \text{Portfolio Investment in Equity Liabilities}_i},$$

$$\frac{\text{Portfolio Investment in Equity Assets}_i}{\text{Portfolio Investment in Equity Assets}_i + \text{Market Capitalization}_i - \text{Portfolio Investment in Equity Liabi}}$$

and denominator is computed as

$$1 - \frac{\text{Market capitalization}_i}{\Sigma_j \text{Market capitalization}_j}$$

$$1 - \frac{\text{Market capitalization}_i}{\Sigma_j \text{Market capitalization}_j}$$

Thus, $HB_i = 0$ (i.e., there is no home bias) when investors hold a world market portfolio; and $HB_i = 1$ (i.e., home bias is full) if investors allocate their entire portfolio to domestic equity.

Data on Portfolio Investment in Equity for both Assets and Liabilities is taken from the Coordinated Portfolio Investment Survey (CPIS) and International Financial Statistics database (IFS), both from the International Monetary Fund. Market capitalization comes from the World Bank Development Indicators. More details on the data and calculations can be found in the Appendix.[2]

The resulting equity home bias in a sample of developed countries and its evolution over time since 2001 are presented in Figure 1. In our sample of countries, Japan has the highest equity home bias—at close to 90% in 2001, declining to 68% in 2016. At the other extreme, Norway has consistently had the lowest equity home bias, at 56% in 2001, falling to just over 19% in 2016.

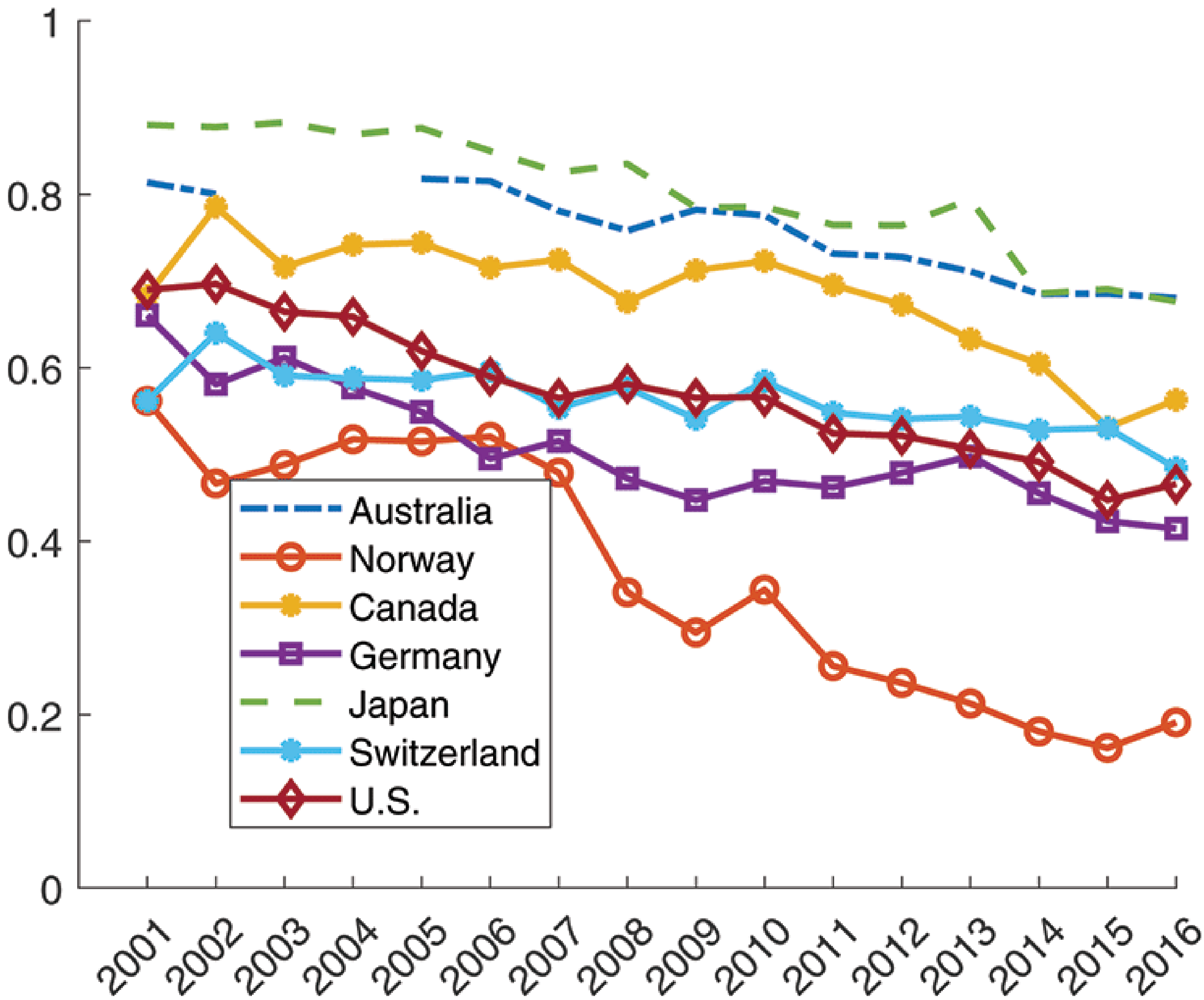

Figure 1. Equity home bias in developed countries.

Source: Author's calculations.

The degree of equity home bias is generally larger in developing countries (see Figure 2). For instance, in our sample of countries, the degree of equity home bias is at its highest in Turkey, at close to 1 in 2016, and at its lowest in Chile at 63% in 2016. In 2001 equity home bias was at its lowest in South Africa, but interestingly, it has slightly increased since, reaching 85% by the end of 2016.

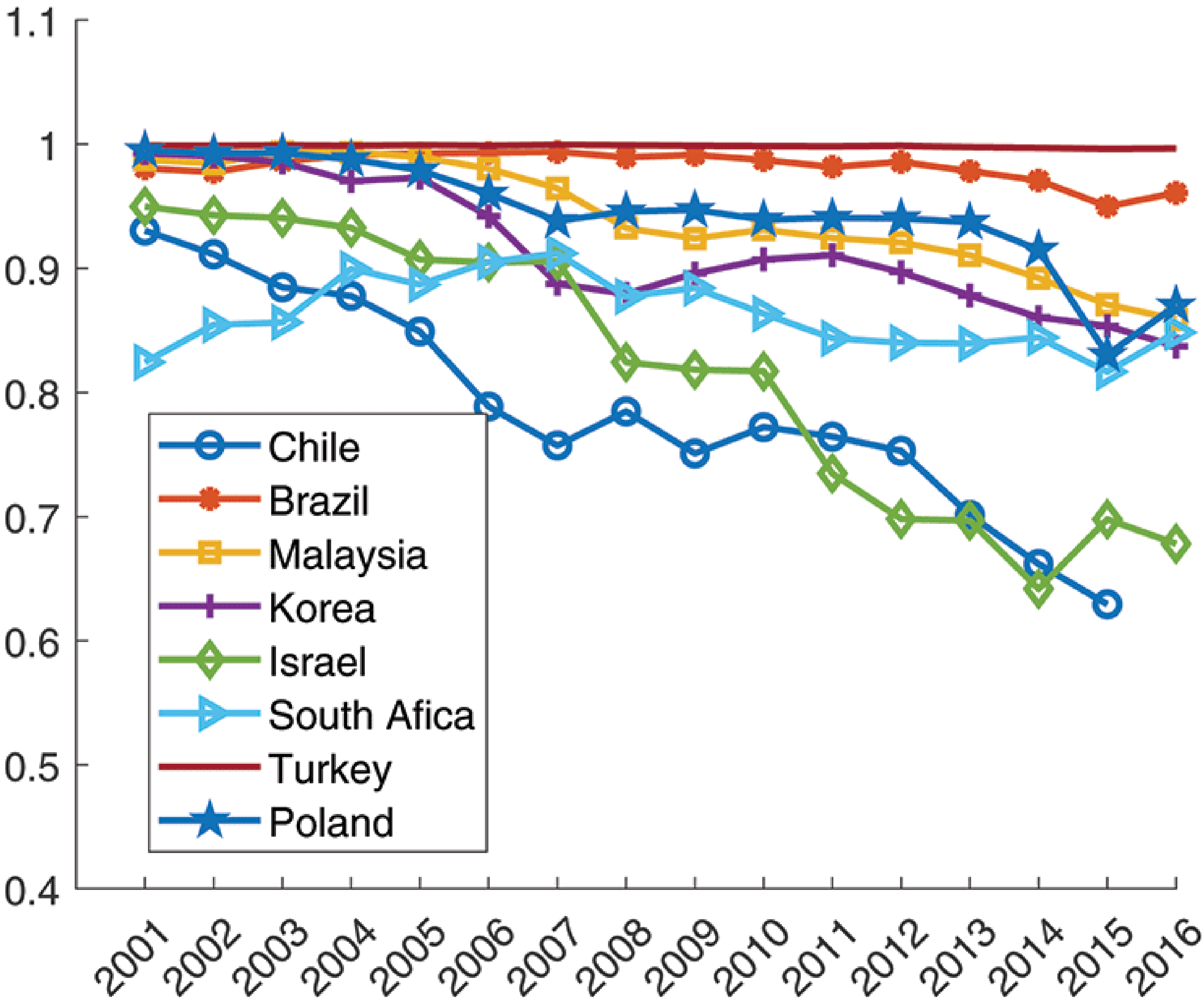

Figure 2. Equity home bias in developing countries.

Source: Author's calculations.

Regional trends in equity home bias are presented in Figure 3.[3] Europe is characterized by the lowest equity home bias, consistently since the introduction of the euro in 1999, while Central and South America feature the highest equity home bias.The important characteristic of all regions though is the secular decline in the level of equity home bias over time.

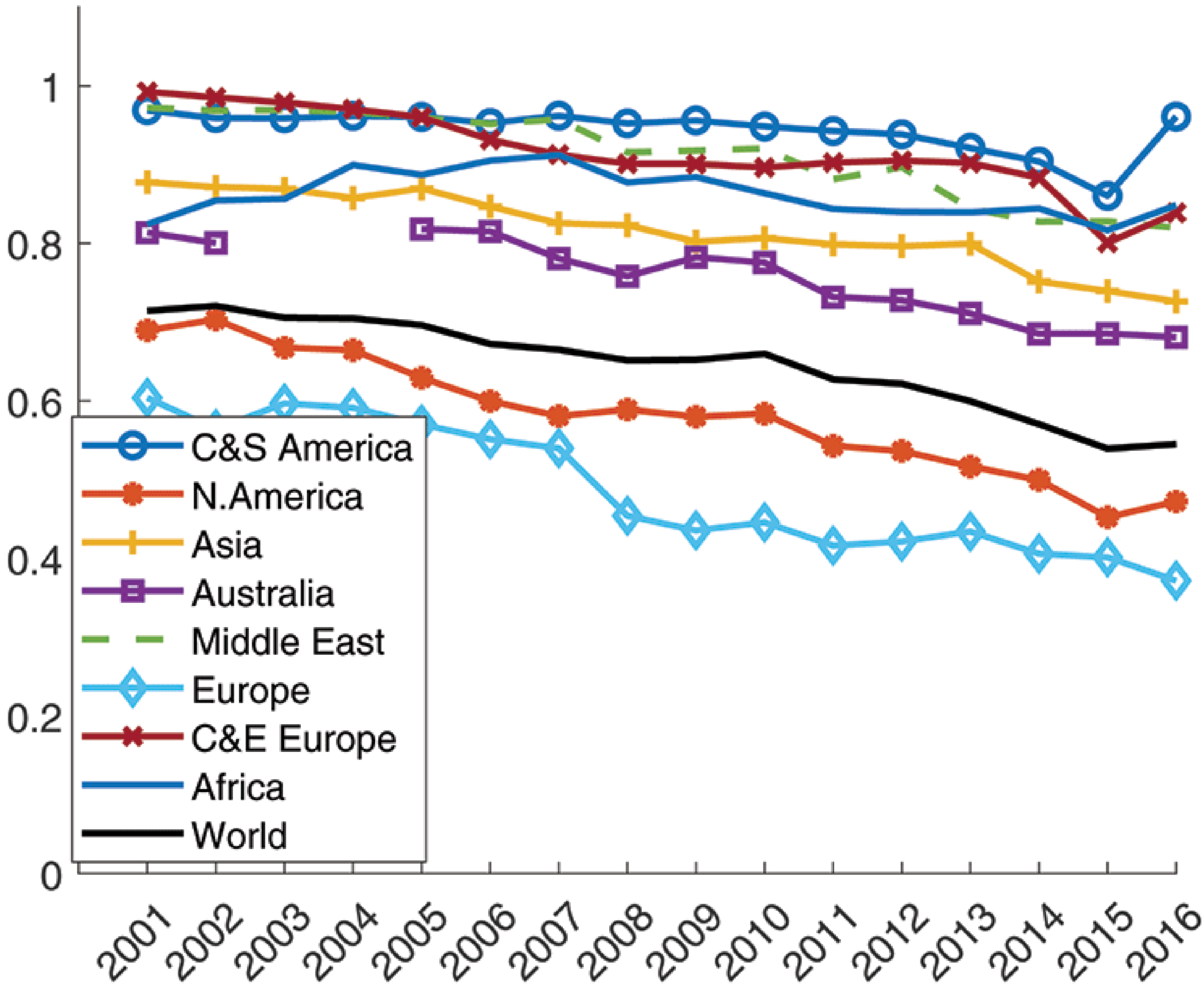

Figure 3. Equity home bias across regions.

Source: Author's calculations.

Appendix contains plots that reproduce Figures 1, 2, and 3 as far back as the data permits (only IFS dataset contains historical data before 2001) and confirms the clear downward trend in the equity home bias.

To summarize, the analysis above makes it clear that international equity portfolios around the globe have become more diversified over time, but a significant degree of home bias still remains, especially in developing countries.

Solving Portfolio Choice Models

To understand the empirical regularities presented in section "Empirical Evidence," the macroeconomic models were extended to incorporate explicit portfolio choices for investors. Prior research mainly focused on models that had a rather simplistic asset market structure, where a single, non-contingent bond or a complete set of Arrow-Debreu securities were available. More recent models have allowed for richer asset market structure, for instance, by allowing for international trades in equities and bonds. Such extensions, however, come with significant computational challenges. This is especially so when markets are incomplete, which is when the spanning condition is not satisfied. Roughly speaking, this occurs when the number of assets is smaller than the number of shocks in the model. In this case, the following problems must be taken into account.

First, in solving such models, one must address the complex interactions between the real and financial sides of the economy. For instance, portfolio decisions affect the degree of risk-sharing between agents, which in turn affects equilibrium real allocations. At the same time, real allocations influence asset returns, which in turn affect portfolio choices. In contrast, in complete markets models portfolio can be found independently of the real variables. In particular, one could first use the risk-sharing conditions to find the real allocations as the solution to a social planning problem. Then, one could solve for the equilibrium prices and portfolio choices that support these allocations in a decentralized market setting.

Second, with a large number of assets, the state space needed to characterize the equilibrium dynamics of the economy expands, thus increasing the difficulty of solving for the model equilibrium. A large number of state variables is not a concern when the model can be solved using methods that rely on the first-order approximations of the equilibrium equations around the steady state (i.e., Blanchard & Kahn (1980) and its extension in Klein (2000), or Christiano (2002), and others). Unfortunately, these methods are inapplicable to the models with portfolio choice, since up to the first-order of approximation portfolio allocations are indeterminate. In particular, up to the first order, expected asset returns are equalized and any combination of portfolios is consistent with the asset Euler equations.

Several methodological approaches have been proposed to deal with these difficulties. Devereux and Sutherland (2011) adapt the standard linearization solution techniques to models with portfolio choice by showing that one requires at least the first-order approximations of non-portfolio equations, and second-order approximations to portfolio equations (Euler equations) to solve for the steady state (or zero-order) portfolios. These portfolios are invariant since they are functions of the variance-covariance matrix of expected returns (or second-order moments), which are invariant in the second-order approximations.

Analogously, Devereux and Sutherland (2010) show that to solve for the first-order dynamics of portfolios, the non-portfolio equations of the model have to be approximated to the second order, while portfolio equations—to the third order. This makes the variance-covariance matrix of expected returns (second moments) vary with the first-order terms in the approximations, thus producing portfolio changes around the steady state.

In a closely related work, Tille and van Wincoop (2010) provide analogous insights into portfolio determination. The main distinction is that Tille and van Wincoop (2010) present an iterative solution method to solve for portfolio choices that applies to any order of approximation.

Another related methodology is proposed by Evans and Hnatkovska (2012) and utilized in Evans and Hnatkovska (2014), Hnatkovska (2010). Their method combines a perturbation technique commonly used in solving macro models (see Jin & Judd, 2002) with continuous-time approximations common in solving finance models of portfolio choice (see Campbell, Chan, & Viceira, 2003). This method differs from Devereux and Sutherland (2010) and Tille and van Wincoop (2010) in that it allows to characterize optimal portfolio holdings to second order from second-order approximations of the equilibrium conditions. In particular, the method consists of approximating both *first* and *second* moments of the state vector to the second order, thus allowing to characterize the portfolio optimality conditions to the fourth order. This yields the second-order accurate dynamics of portfolio holdings.

While the methods outlined above have a number of advantages, such as their ease of implementation, proximity to the existing linear approximation methods, and applicability to a broad range of environments (e.g., complete and incomplete markets, large menu of assets, variety of shocks, and high-dimensional problems more generally), they do suffer from a number of limitations. These limitations are mainly driven by the fact that in their essence these are local methods, i.e., they rely on approximations around a deterministic steady state. As a result, their accuracy tends to suffer in the presence of country asymmetries, large shocks, non-stationarity, or occasionally binding constraints. Assessing accuracy of these methods is not easy as it generally requires either comparison with a solution obtained using a global method, or an analytical solution obtained under some simplifying assumptions. Rabitsch, Stepanchuk, and Tsyrennikov (2015) give an example of the first approach. The authors use global method to solve two models. The first model is typical of the "macroeconomics" literature, while the second model contains features that capture key "financial" aspects, such as a realistic equity premium. Both models are studied in the symmetrical and asymmetrical setup, where countries experience shocks of different size. They find that Devereux and Sutherland's (2010) method performs very well in the symmetrical setting, and in the asymmetrical setting when short simulated paths of various variables are considered. However, the method's accuracy deteriorates at long horizons in asymmetrical settings. Examples of the second approach can be found in Evans and Hnatkovska (2014), Pavlova and Rigobon (2010b), and Pavlova and Rigobon (2010a).

Model

The macro literature has proposed two main explanations for equity home bias. The first argues that investors favor local equity to hedge risk, such as real exchange rate risk and non-tradable income risk. The second class of explanations relies on trade costs for assets in international financial markets. These include transaction and information costs, differential tax treatments, and more broadly, differences in institutional environments. The literature studying the effects of asset trade costs on equity home bias has been inconclusive. Some studies have found that only small costs are needed to induce a substantial bias for home equities or even to deter households from participating in a foreign market altogether (see Michaelides, 2003; Bhamra, Coeurdacier, & Guibaud, 2014; and others). Others have shown that costs need to be high (see, for instance, French & Poterba, 1991) to rationalize the observed equity home bias. A more telling observation on the limited role of asset trade costs for explaining equity home bias comes from the fact that despite the low share of foreign assets in investors' portfolio, cross-border equity flows by domestic residents are large in magnitude and volatile. This suggests that they do try to take advantage of diversification opportunities available abroad. In fact, the size and volatility of portfolio flows across borders exceed the size and volatility of portfolio flows in and out of domestic assets (see Tesar & Werner, 1995; Hnatkovska, 2010). In this article, therefore, the focus is on the hedging motives for equity home bias. Coeurdacier and Rey (2013) provide an excellent overview of both strands of the home bias literature.

To illustrate the workings of the hedging motive, a simple two-country model is presented. The model is essentially a simplified version of the model presented in Coeurdacier, Kollmann, and Martin (2009) and Coeurdacier and Rey (2013). To streamline the derivations, assume that there are only two periods, t and $t+1$. Two countries, Home (H) and Foreign (F), are fully symmetrical, and are each populated by a representative household and a firm. Two versions of the model will be considered: (i) with international trades in equities only, and (ii) with international trades in equities and bonds. The latter version allows to illustrate the effects of interactions in bond and equity holdings for portfolio home bias. For portfolios to be determinate, one requires the number of available assets to be no larger than the number of exogenous sources of uncertainty (the spanning condition). Thus, in the first version of the model, only country-specific productivity shocks will be considered, while in the extended version of the model, country-specific shocks to factor income shares (redistributive shocks) will also be introduced. Let us begin by outlining a simple model with equities only.

Households

Household preferences in country H are given by:

$$E_t\left[\frac{C_{H,t}^{1-\sigma}}{1-\sigma}+\beta\frac{C_{H,t+1}^{1-\sigma}}{1-\sigma}\right],$$

$$E_t\left[\frac{C_{H,t}^{1-\sigma}}{1-\sigma}+\beta\frac{C_{H,t+1}^{1-\sigma}}{1-\sigma}\right],$$

where E_t denotes expectations conditional on date t information, $\beta \in (0,1)$ is the subjective discount factor, σ is the coefficient of relative risk aversion. $C_{H,t}$ is the composite consumption by country H in period t given by the CES aggregator:

$$C_{H,t} = \left[\lambda^{1/\theta} \left(c_{H,t}^{H} \right)^{(\theta-1)/\theta} + (1-\lambda)^{1/\theta} \left(c_{F,t}^{H} \right)^{(\theta-1)/\theta} \right]^{\theta/(\theta-1)}.$$

$$C_{H,t} = \left[\lambda^{1/\theta} \left(c_{H,t}^{H} \right)^{(\theta-1)/\theta} + (1-\lambda)^{1/\theta} \left(c_{F,t}^{H} \right)^{(\theta-1)/\theta} \right]^{\theta/(\theta-1)}.$$

Here $c_{H,t}^{H}$ and $c_{F,t}^{H}$ denote H consumption of the good produced by H and F country at date t, respectively. θ is the elasticity of substitution between the two goods, while λ and $(1-\lambda)$ are the weights the household assigns to the consumption of H- and F-produced goods , respectively. Assume that $\lambda > 1/2$ so that households in each country have a preference for their local good. This is known as "consumption home bias," and is assumed to be exogenous in the model. Consumption aggregator in period $t+1$ is defined analogously.

The consumer price indices that correspond to these preferences are given by:

$$P_{H,t} = \left[\lambda p_{H,t}^{1-\theta} + (1-\lambda) p_{F,t}^{1-\theta} \right]^{1/(1-\theta)},$$

$$P_{H,t} = \left[\lambda p_{H,t}^{1-\theta} + (1-\lambda) p_{F,t}^{1-\theta} \right]^{1/(1-\theta)},$$

where $p_{H,t}$ is the price of H good, while $p_{F,t}$ is the price of foreign good. Preferences of households in country F are similarly defined in terms of foreign consumption of goods produced in countries H and F.

Households finance their consumption expenditures from wage income and by holding equities issued by H and F firms. Equities are claims to firms' dividend stream $\left\{ d_{i,t}^{A} \right\}, i = H, F$. A_{H}^{H} and A_{F}^{H} will be used to denote H holdings of equity issued by H and F firms, respectively. The flow budget constraint for H households in period t can be expressed as:

$$P_{H,t}C_{H,t} + p_{H,t}^{A}A_{H,t+1}^{H} + p_{F,t}^{A}A_{F,t+1}^{H} = w_{H,t}l_{H,t} + A_{H,t}^{H}(p_{H,t}^{A} + d_{H,t}^{A}) + A_{F,t}^{H}(p_{F,t}^{A} + d_{F,t}^{A}),$$

$$P_{H,t}C_{H,t} + p_{H,t}^{A}A_{H,t+1}^{H} + p_{F,t}^{A}A_{F,t+1}^{H} = w_{H,t}l_{H,t} + A_{H,t}^{H}(p_{H,t}^{A} + d_{H,t}^{A}) + A_{F,t}^{H}(p_{F,t}^{A} + d_{F,t}^{A}), \tag{4.1}$$

while period $t+1$ budget constraint is given by:

$$P_{H,t+1}C_{H,t+1} = w_{H,t+1}l_{H,t+1} + A^{H}_{H,t+1}d^{A}_{H,t+1} + A^{H}_{F,t+1}d^{A}_{F,t+1}.$$

$$P_{H,t+1}C_{H,t+1} = w_{H,t+1}l_{H,t+1} + A^{H}_{H,t+1}d^{A}_{H,t+1} + A^{H}_{F,t+1}d^{A}_{F,t+1}.$$

(4.2)

Here p^A_H and p^A_F are prices of equities issued by H and F firms, respectively.

The problem facing the household in the country F is symmetrical.

Firms

There is a continuum of perfectly competitive firms in each country. A representative H firm produces output $y_{H,t}$ using capital $k_{H,t}$ and labor $l_{H,t}$ according to a Cobb-Douglas production function:

$$y_{H,t} = z_{H,t}\left(k_{H,t}\right)^{\alpha}\left(l_{H,t}\right)^{1-\alpha},$$

$$y_{H,t} = z_{H,t}(k_{H,t})^{\alpha}(l_{H,t})^{1-\alpha},$$

where $\alpha \in (0,1)$ and $z_{H,t}$ is a stochastic total factor productivity (TFP). For simplicity, let us assume that capital stock $k_{H,t}$ is given and fixed. Since leisure does not enter the utility function, households spend their entire time endowment working. With Cobb-Douglas technology factor payments are easy to obtain. Thus, the wage income accrued to workers is:

$$w_{H,t}l_{H,t} = (1-\alpha)p_{H,t}y_{H,t}.$$

$$w_{H,t}l_{H,t} = (1-\alpha)p_{H,t}y_{H,t}.$$

(4.3)

Capital income $\alpha p_{H,t}y_{H,t}$ is paid out as dividends to the firm shareholders:

$$d^{A}_{H,t} = \alpha p_{H,t}y_{H,t}.$$

$$d^{A}_{H,t} = \alpha p_{H,t}y_{H,t}.$$

(4.4)

Note that at the moment, factor payment shares α and $1 - \alpha$ are assumed to be fixed. In the extended version of the model with redistributive shocks, these factor shares are assumed to be stochastic.

Market Clearing

The market clearing requirements of the model are most easily stated if we normalize the national populations as well as the population of firms to unity. The goods market clearing conditions are then given by:

$$c_{H,t}^{H} + c_{H,t}^{F} = y_{H,t} \text{ and } c_{F,t}^{H} + c_{F,t}^{F} = y_{F,t}$$

$$c_{H,t}^{H} + c_{H,t}^{F} = y_{H,t} \text{ and } c_{F,t}^{H} + c_{F,t}^{F} = y_{F,t}$$

(4.5)

Normalize the number of outstanding shares issued by firms in each country to unity. Market clearing in financial markets then requires:

$$A_{H,t}^{H} + A_{H,t}^{F} = 1 \text{and} A_{F,t}^{H} + A_{F,t}^{F} = 1.$$

$$A_{H,t}^{H} + A_{H,t}^{F} = 1 \text{and} A_{F,t}^{H} + A_{F,t}^{F} = 1.$$

(4.6)

Equilibrium

An equilibrium in the model comprises a set of asset prices and relative goods prices that clear markets given the state of productivity, and the optimal consumption, savings and portfolios decisions of households. The first-order conditions from the decision problem of the household in country H are:

$$c_{H,t}^{H} = \lambda \left(\frac{p_{H,t}}{P_{H,t}} \right)^{-\theta} C_{H,t} \text{and} c_{F,t}^{H} = (1-\lambda) \left(\frac{p_{F,t}}{P_{H,t}} \right)^{-\theta} C_{H,t}$$

$$c_{H,t}^{H} = \lambda \left(\frac{p_{H,t}}{P_{H,t}} \right)^{-\theta} C_{H,t} \text{and} c_{F,t}^{H} = (1-\lambda) \left(\frac{p_{F,t}}{P_{H,t}} \right)^{-\theta} C_{H,t}$$

(4.7)

$$1 = E_t \left[\beta \left(\frac{C_{H,t+1}}{C_{H,t}} \right)^{-\sigma} \frac{P_{H,t}}{P_{H,t+1}} \frac{d_{H,t+1}^{A}}{p_{H,t}^{A}} \right] \text{and}$$

$$1 = E_t \left[\beta \left(\frac{C_{H,t+1}}{C_{H,t}} \right)^{-\sigma} \frac{P_{H,t}}{P_{H,t+1}} \frac{d_{H,t+1}^{A}}{p_{H,t}^{A}} \right] \text{and}$$

(4.8)

$$1 = E_t \left[\beta \left(\frac{C_{H,t+1}}{C_{H,t}} \right)^{-\sigma} \frac{P_{H,t}}{P_{H,t+1}} \frac{d_{F,t+1}^{A}}{p_{F,t}^{A}} \right].$$

$$1 = E_t \left[\beta \left(\frac{C_{H,t+1}}{C_{H,t}} \right)^{-\sigma} \frac{P_{H,t}}{P_{H,t+1}} \frac{d_{F,t+1}^{A}}{p_{F,t}^{A}} \right].$$

(4.9)

Equations (4.7) give the optimal consumption allocations across H- and F-produced goods. Equations (4.8) and (4.9) represent the Euler equations with respect to H- and F-issued equities.

An analogous set of conditions characterizes the behavior of F households.

To complete the description of the economy, define terms of trade as the relative price of F goods, $q_t = p_{H,t}/p_{F,t}$ so the terms of trade improves for the H when q rises. The real exchange rate, Q_t, is defined as the relative price of H to F consumption, $Q_t = P_{H,t}/P_{F,t}$. With this definition, an appreciation of the real exchange rate is captured by a rise in Q.

Portfolio Choices

To solve for the optimal portfolio allocations, the first-order conditions are log-approximated around the deterministic steady state. The approach follows Coeurdacier et al.'s (2009). The model described above features "locally-complete" markets since the spanning condition is satisfied (i.e., simply put, the number of assets equals the number of exogenous sources of uncertainty). Therefore, in the first-order approximation of the model, efficient consumption risk-sharing is achieved. Because of this, equilibrium portfolios can be obtained by taking just the first-order Taylor series expansions of the equilibrium conditions in the neighborhood of the deterministic and symmetrical steady state. Higher-order terms are not necessary to obtain the steady state portfolio allocations. If markets were incomplete, second-order approximations of the portfolio equilibrium conditions would be at least necessary to solve for the steady state portfolios, as shown in Devereux and Sutherland (2011) and Tille and van Wincoop (2010).

Some notation pertinent to log-linearization of the first-order conditions needs to be introduced. In particular, $x_t \equiv \frac{x_{H,t}}{x_{F,t}}$ is used to denote the ratio of H to F variables, and $\widehat{x}_t = \ln x_t - \ln x$ to denote the log deviation of variable x_t from its steady state value $x.\Delta z_{t+1} = z_{t+1} - z_t$ also denotes the first-difference of variable z_{t+1}.

Euler equations and risk-sharing condition: Let us begin by linearizing the Euler equations for H-issued equity held by H and F households:

$$1 = E_t\left[\beta\left(\frac{C_{H,t+1}}{C_{H,t}}\right)^{-\sigma}\frac{P_{H,t}}{P_{H,t+1}}\frac{d^A_{H,t+1}}{p^A_{H,t}}\right]\text{and}$$

$$1 = E_t\left[\beta\left(\frac{C_{F,t+1}}{C_{F,t}}\right)^{-\sigma}\frac{P_{F,t}}{P_{F,t+1}}\frac{d^A_{H,t+1}}{p^A_{H,t}}\right].$$

$$1 = E_t\left[\beta\left(\frac{C_{H,t+1}}{C_{H,t}}\right)^{-\sigma}\frac{P_{H,t}}{P_{H,t+1}}\frac{d^A_{H,t+1}}{p^A_{H,t}}\right]\text{ and}$$
$$1 = E_t\left[\beta\left(\frac{C_{F,t+1}}{C_{F,t}}\right)^{-\sigma}\frac{P_{F,t}}{P_{F,t+1}}\frac{d^A_{H,t+1}}{p^A_{H,t}}\right].$$

Let $M_{i,t+1} = \beta\left(\frac{C_{i,t+1}}{C_{i,t}}\right)^{-\sigma}\frac{P_{i,t}}{P_{i,t+1}}$ denote the stochastic discount factor of households in country $i = H, F$. Also let stock returns be defined as $R^A_{H,t+1} = d^A_{H,t+1}/p^A_{H,t}$ and $R^A_{F,t+1} = d^A_{F,t+1}/p^A_{F,t}$.

The Euler equations above can then be written as:

$$1 = E_t\left[M_{H,t+1}R^A_{H,t+1}\right] \text{and } 1 = E_t\left[M_{F,t+1}R^A_{H,t+1}\right]$$

$$1 = E_t\left[M_{H,t+1}R^A_{H,t+1}\right] \text{ and } 1 = E_t\left[M_{F,t+1}R^A_{H,t+1}\right]$$

Their log-linearized version is:

$$0 = E_t\hat{M}_{H,t+1} + E_t\hat{R}^A_{H,t+1} \text{ and } 0 = E_t\hat{M}_{F,t+1} + E_t\hat{R}^A_{H,t+1},$$

$$0 = E_t\widehat{M}_{H,t+1} + E_t\widehat{R}^A_{H,t+1} \text{ and } 0 = E_t\widehat{M}_{F,t+1} + E_t\widehat{R}^A_{H,t+1},$$

which implies:

$$E_t\hat{M}_{H,t+1} = E_t\hat{M}_{F,t+1}.$$

$$E_t\widehat{M}_{H,t+1} = E_t\widehat{M}_{F,t+1}. \tag{4.10}$$

Thus, up to the first order, the expected stochastic discount factors of H and F households are equalized.

The goal is to solve for portfolios that reproduce the efficient allocation in the first-order approximation of the model, that is, by looking for portfolios that are consistent with the condition $\widehat{M}_{H,t+1} = \widehat{M}_{F,t+1}$ i.e. perfect risk-sharing. Such portfolios obviously satisfy the approximate condition (4.10). Using the definition of $\widehat{M}_{i,t+1}$, perfect risk-sharing requires:

$$-\sigma\left(\Delta\hat{C}_{H,t+1} - \Delta\hat{C}_{F,t+1}\right) = \Delta\hat{P}_{H,t+1} - \Delta\hat{P}_{F,t+1}$$

$$-\sigma\left(\Delta\widehat{C}_{H,t+1} - \Delta\widehat{C}_{F,t+1}\right) = \Delta\widehat{P}_{H,t+1} - \Delta\widehat{P}_{F,t+1}$$

or

$$-\sigma \Delta \hat{C}_{t+1} = \Delta \hat{P}_{t+1} = \Delta \hat{Q}_{t+1}.$$

$$-\sigma \Delta \widehat{C}_{t+1} = \Delta \widehat{P}_{t+1} = \Delta \widehat{Q}_{t+1}. \tag{4.11}$$

This equation equates growth rates in marginal utilities of consumption across countries, adjusted for differences in price levels. It states that, in equilibrium, consumption must be allocated between households in countries H and F such that the marginal rates of substitution (converted into the same units using the real exchange rate) are equalized across countries (see Backus & Smith, 1993; and Kollmann, 1995, for more detail).

Real exchange rate: Linearizing the real exchange rate gives:

$$\hat{Q}_t = \hat{P}_{H,t} - \hat{P}_{F,t} = (2\lambda - 1)\hat{q}_t.$$

$$\widehat{Q}_t = \widehat{P}_{H,t} - \widehat{P}_{F,t} = (2\lambda - 1)\hat{q}_t. \tag{4.12}$$

See the Appendix for derivations.

Market clearing: Let us now turn our attention to the market clearing conditions in Equations (4.5). From there one can express relative production and demand as:

$$y_t \equiv \frac{y_{H,t}}{y_{F,t}} = \frac{c_{H,t}^H + c_{H,t}^F}{c_{F,t}^H + c_{F,t}^F}.$$

$$y_t \equiv \frac{y_{H,t}}{y_{F,t}} = \frac{c_{H,t}^H + c_{H,t}^F}{c_{F,t}^H + c_{F,t}^F}.$$

Substituting in the equilibrium consumption choices from Equations (4.7), we obtain:

$$y_t = \left(\frac{p_{H,t}}{p_{F,t}}\right)^{-\theta}\left[\frac{\lambda C_{H,t}P^{\theta}_{H,t} + (1-\lambda)C_{Ft}P^{\theta}_{F,t}}{(1-\lambda)C_{H,t}P^{\theta}_{H,t} + \lambda C_{Ft}P^{\theta}_{F,t}}\right].$$

$$y_t = \left(\frac{p_{H,t}}{p_{F,t}}\right)^{-\theta}\left[\frac{\lambda C_{H,t}P^{\theta}_{H,t} + (1-\lambda)\,C_{Ft}P^{\theta}_{F,t}}{(1-\lambda)\,C_{H,t}P^{\theta}_{H,t} + \lambda C_{Ft}P^{\theta}_{F,t}}\right].$$

Log-linearizing this expression and taking first-difference give:

$$\begin{aligned}\Delta\hat{y}_{t+1} &= -\left[\theta\left(1-(2\lambda-1)^2\right)+\frac{1}{\sigma}(2\lambda-1)^2\right]\Delta\hat{q}_{t+1}\\ &= -\phi\Delta\hat{q}_{t+1},\end{aligned}$$

$$\begin{aligned}\Delta\hat{y}_{t+1} &= -\left[\theta\left(1-(2\lambda-1)^2\right)+\tfrac{1}{\sigma}(2\lambda-1)^2\right]\Delta\hat{q}_{t+1}\\ &= -\phi\Delta\hat{q}_{t+1},\end{aligned}$$

(4.13)

where $\phi = \theta + \left(\frac{1}{\sigma} - \theta\right)(2\lambda - 1)^2$. See the Appendix for derivations. This condition postulates that higher relative production of H goods must coincide with a fall (deterioration) of the terms of trade.

Equity market clearing conditions together with the assumption of a symmetric steady state imply that equilibrium portfolios satisfy:

$$A^H_H = 1 - A^F_H = A^F_F = 1 - A^H_F \equiv A.$$

$$A^H_H = 1 - A^F_H = A^F_F = 1 - A^H_F \equiv A.$$

(4.14)

Budget constraint: Lastly, period $t+1$ budget constraint (4.2) is under study. The optimal portfolio must satisfy the linearized version of the budget constraint given by:

$$\hat{P}_{t+1} + \hat{C}_{t+1} = (1-\alpha)\hat{w}_{t+1} + (2A-1)\alpha\hat{d}_{t+1}^{A}.$$

$$\widehat{P}_{t+1} + \widehat{C}_{t+1} = (1-\alpha)\widehat{w}_{t+1} + (2A-1)\,\alpha\hat{d}_{t+1}^{A}.$$

Recall that $\widehat{P}_{t+1} = \widehat{P}_{t+1}^{H} - \widehat{P}_{t+1}^{F}$ and $\widehat{C}_{t+1} = \widehat{C}_{t+1}^{H} - \widehat{C}_{t+1}^{F}$. Taking the first-difference of the left hand-side of the expression above, and combining it with Equations (4.11) and (4.12), enables it to be rewritten as:

$$\Delta\hat{P}_{t+1} + \Delta\hat{C}_{t+1} = \left(1-\frac{1}{\sigma}\right)\Delta\hat{Q}_{t+1}.$$

$$\Delta\widehat{P}_{t+1} + \Delta\widehat{C}_{t+1} = \left(1-\frac{1}{\sigma}\right)\Delta\widehat{Q}_{t+1}. \tag{4.15}$$

Substituting it back into the linearized budget constraint yields:

$$\left(1-\frac{1}{\sigma}\right)\Delta\hat{Q}_{t+1} = (1-\alpha)\Delta\hat{w}_{t+1} + (2A-1)\alpha\Delta\hat{d}_{t+1}^{A},$$

$$\left(1-\frac{1}{\sigma}\right)\Delta\widehat{Q}_{t+1} = (1-\alpha)\Delta\widehat{w}_{t+1} + (2A-1)\,\alpha\Delta\hat{d}_{t+1}^{A}, \tag{4.16}$$

which shows that changes in the relative consumption expenditures in country H (relative to country F) have to coincide with changes in country H relative income, where the latter consists of relative wage income and dividend income.

Portfolios: One can solve for the optimal portfolio from budget constraint Equation (4.16) by expressing it in terms of covariances with the relative equity payouts $\hat{d}_{t+1}^{A}$:

$$\left(1-\frac{1}{\sigma}\right)cov(\hat{Q}_{t+1}, \hat{d}_{t+1}^{A}) = (1-\alpha)cov(\hat{w}_{t+1}, \hat{d}_{t+1}^{A}) + (2A-1)\alpha var(\hat{d}_{t+1}^{A}),$$

$$\left(1-\frac{1}{\sigma}\right) cov(\widehat{Q}_{t+1}, \hat{d}^{A}_{t+1}) = (1-\alpha)cov(\widehat{w}_{t+1}, \hat{d}^{A}_{t+1}) + (2A-1)\,\alpha var(\hat{d}^{A}_{t+1}),$$

where cov and var denote the covariance and variance, respectively, conditional on period t information.

The expression above can be solved for A as:

$$A = \frac{1}{2} - \frac{1}{2}\frac{1-\alpha}{\alpha}\beta_{w,d} + \frac{1}{2}\frac{1-\frac{1}{\sigma}}{\alpha}\beta_{Q,d}.$$

$$A = \frac{1}{2} - \frac{1}{2}\frac{1-\alpha}{\alpha}\beta_{w,d} + \frac{1}{2}\frac{1-\frac{1}{\sigma}}{\alpha}\beta_{Q,d}.$$

(4.17)

Here $\beta_{w,d} = cov(\widehat{w}_{t+1}, \hat{d}^{A}_{t+1})/var(\hat{d}^{A}_{t+1})$ and $\beta_{Q,d} = cov(\widehat{Q}_{t+1}, \hat{d}^{A}_{t+1})/var(\hat{d}^{A}_{t+1})$ are the *hedge ratios*. (See Heathcote & Perri, 2013; and Coeurdacier & Gourinchas, 2016, for a discussion of portfolios in terms of hedge ratios. Note that these hedge ratios are equilibrium objects in the model and thus are functions of model parameters. They will be derived below. The empirical counterparts of these objects, however, can be easily obtained from simple regressions of real exchange rate Q and wage income w on financial income, and are independent of the model details.

The equilibrium equity portfolio in Equation (4.17) has three terms:

(i) a constant term equal to 1/2, which is a *fully diversified portfolio* with equal weights in local and foreign equities as in Lucas (1982). It arises if $\beta_{w,d} = 0$ and $\beta_{Q,d} = 0$;

(ii) a term that captures the propensity of equity to hedge *non-tradable income risk* (labor income risk here). It is given by the hedge ratio $\beta_{w,d}$. If $\beta_{w,d} < 0$, so that the relative equity payoffs covary negatively with relative labor income, then investors would favor local equity. Intuitively, since households cannot trade claims to their labor income risk, they have to rely on the available assets to hedge against this risk. If local equity pays relatively more when labor income is relatively low, it will be a good hedge against labor income risk.

(iii) a term that captures the propensity of equity to hedge *real exchange rate risk*. It is given by the hedge ratio $\beta_{Q,d}$. Investors will bias their portfolios toward local equity if $\sigma > 1$ and $\beta_{Q,d} > 0$, that is, if equity pays more when real exchange rate appreciates. As can be seen from Equation (4.15), with $\sigma > 1$ household consumption expenditure rises when the real exchange rate appreciates. Therefore, an asset that pays more in those states of the world will allow households to smooth their consumption and will be a good hedge against the real exchange rate risk.[4]

Portfolio solution in (4.17) provides a partial equilibrium version of optimal portfolio allocation since the hedge ratios that it uses are themselves endogenous variables determined within the general equilibrium model. While the model only provides a very stylized theory of these hedge ratios, it does allow to link the degree of home bias to the structural parameters.

Recall that labor income and dividend payouts as defined in Equations (4.3) and (4.4) are constant shares of output. Thus, relative labor income and relative dividends are just:

$$\begin{aligned} \frac{w_{H,t} l_{H,t}}{w_{F,t} l_{F,t}} &= \frac{p_{H,t} y_{H,t}}{p_{F,t} y_{F,t}} = q_t y_t \\ \frac{d_{H,t}^A}{d_{F,t}^A} &= \frac{p_{H,t} y_{H,t}}{p_{F,t} y_{F,t}} = q_t y_t . \end{aligned}$$

$$\begin{aligned} \frac{w_{H,t} l_{H,t}}{w_{F,t} l_{F,t}} &= \frac{p_{H,t} y_{H,t}}{p_{F,t} y_{F,t}} = q_t y_t \\ \frac{d_{H,t}^A}{d_{F,t}^A} &= \frac{p_{H,t} y_{H,t}}{p_{F,t} y_{F,t}} = q_t y_t . \end{aligned}$$

Linearizing these expressions and combining them with Equation (4.13) gives:

$$\hat{w}_t = (1-\phi)\hat{q}_t \text{ and } \hat{d}_t^A = (1-\phi)\hat{q}_t .$$

$$\widehat{w}_t = (1-\phi)\hat{q}_t \text{ and } \hat{d}_t^A = (1-\phi)\hat{q}_t .$$

This implies:

$$\begin{aligned} \beta_{w,d} &= 1 \\ \beta_{Q,d} &= \frac{(2\lambda - 1)}{(1-\phi)} . \end{aligned}$$

$$\begin{aligned} \beta_{w,d} &= 1 \\ \beta_{Q,d} &= \tfrac{(2\lambda-1)}{(1-\phi)}. \end{aligned}$$

The optimal equity portfolio in general equilibrium then becomes:

$$A = \frac{1}{2} - \frac{1}{2}\frac{1-\alpha}{\alpha} + \frac{1}{2}\frac{1-\frac{1}{\sigma}}{\alpha}\frac{(2\lambda-1)}{(1-\phi)}.$$

$$A = \frac{1}{2} - \frac{1}{2}\frac{1-\alpha}{\alpha} + \frac{1}{2}\frac{1-\frac{1}{\sigma}}{\alpha}\frac{(2\lambda-1)}{(1-\phi)}. \tag{4.18}$$

As before, the equilibrium equity portfolio consists of three terms:

(i) a full diversification term that gives optimal portfolio share at 1/2. This solution would obtain if there were no hedging motives for holding equity (i.e., when $\alpha \to 1$, so that there is no labor income risk; and when $\lambda \to 1/2$ so that there is no home bias in consumption, and therefore the real exchange rate is constant);

(ii) a term for hedging non-tradable income risk, given by $-\frac{1}{2}\frac{1-\alpha}{\alpha}$. Since in the model both labor income and equity payouts are proportional to total output, the correlation between them is positive (equal to 1, in fact). This implies that local equity is not a good hedge for labor income risk and investors would actually want to go short in their local equity. This is the well-known result presented in Baxter and Jermann (1997).

(iii) a term for hedging the real exchange rate risk, given by $\frac{1}{2}\frac{1-\frac{1}{\sigma}}{\alpha}\frac{(2\lambda-1)}{(1-\phi)}$. The model implies that this hedging term depends on several parameters: the degree of risk aversion, σ; the degree of consumption home bias, λ; the elasticity of intra-temporal substitution between H and F goods in household's preferences, θ (through the ϕ term in Equation (4.13)). The hedging term is positive when $\sigma > 1$,$\lambda > 1/2$, and $\phi < 1$. The size of the composite term ϕ will be affected by parameter θ. In particular, $\phi < 1$ when $\theta < 1$ and $\phi > 1$ when $\theta > 1$.

In the first case, the elasticity of intra-temporal substitution $\theta < 1$ implies that changes in relative output across countries are more than offset by the changes in the real exchange rate in the opposite direction. For instance, a decline in country H relative output will be accompanied by a more than proportional rise in the real exchange rate (appreciation). Since relative equity payouts are proportional to the real exchange rate, local stocks are paying more when consumption is more expensive. This makes local equity valuable and biases investor's portfolios toward these equities.

In the second case, when $\theta > 1$, the adjustments in the real exchange rate are more muted and respond less than one-for-one to relative output changes. In this case, a decline in country H relative output will be accompanied by a small rise in the real exchange rate and a fall in relative equity returns. This makes foreign equity valuable and biases investor's portfolios toward foreign equities.

Note that when $\phi = 1$, local equity share A is indeterminate. This is because any changes in relative output are exactly offset by the changes in the terms of trade. That is, terms of trade movements provide full insurance against any output shocks, even in the absence of financial instruments. As a result, any portfolio ensures perfect risk-sharing across countries. This is a counterpart of the result in Cole and Obstfeld (1991).

The discussion above makes it clear that equity home bias in the model can arise only due to equity's propensity to hedge real exchange rate risk, which requires $\theta < 1$. This sensitivity of equity portfolios to the intra-temporal elasticity of substitution has been highlighted in several existing studies (for instance, see Kollmann, 2006; Hnatkovska, 2010; Heathcote & Perri, 2013).

Extended Model with Bonds

The model extension is now under consideration with two new features: (i) each country issues a risk-free real bond that can be traded internationally; (ii) there are shocks to factor income shares (i.e., redistributive shocks).

Several authors have emphasized the importance of interactions between various assets in investor's portfolios for the optimal asset allocations. For instance, Coeurdacier and Gourinchas (2016) show how trading in real bonds across countries affects equity portfolios. Hnatkovska (2010) argues that access to equities issued by firms in non-tradable goods sectors matter for equity portfolios in tradable goods sectors. Other recent studies that considered models with multiple asset classes include Engel and Matsumoto (2009), Coeurdacier et al. (2010), Devereux and Sutherland (2009) among others.

The extended model with bonds features four international assets—H and F equities, and H and F bonds. If there are only two shocks in the model (i.e., H and F productivity shocks), the spanning condition is not satisfied, as the number of assets now exceeds the number of shocks, resulting in portfolio indeterminacy. To address this issue, shocks are added to factor income shares in the two countries. That is, α is now stochastic in each country. The role of such shocks for optimal portfolios has been explored in several existing studies (see, for instance, Coeurdacier et al., 2009; Coeurdacier & Gourinchas, 2016).

Next, the key features of this extended model that differ from the baseline model are outlined. Let B_H^H and B_F^H denote H holdings of bonds issued by H and F countries, while p_H^B and p_F^B denote their respective prices, and d_H^B and d_F^B denote their respective payouts. Bonds are real bonds, so that a unit of H(F) bond purchased in period t yields one unit of H(F) composite consumption good in period $t + 1$.

The flow budget constraints in the extended model are as follows. The constraint of H household in period t is:

$$P_{H,t}C_{H,t} + p^{A}_{H,t}A^{H}_{H,t+1} + p^{A}_{F,t}A^{H}_{F,t+1} + p^{B}_{H,t}B^{H}_{H,t+1} + p^{B}_{F,t}B^{H}_{F,t+1}$$
$$= w_{H,t}l_{H,t} + A^{H}_{H,t}(p^{A}_{H,t} + d^{A}_{H,t}) + A^{H}_{F,t}(p^{A}_{F,t} + d^{A}_{F,t}) + B^{H}_{H,t}d^{B}_{H,t} + B^{H}_{F,t}d^{B}_{F,t},$$

$$P_{H,t}C_{H,t} + p^{A}_{H,t}A^{H}_{H,t+1} + p^{A}_{F,t}A^{H}_{F,t+1} + p^{B}_{H,t}B^{H}_{H,t+1} + p^{B}_{F,t}B^{H}_{F,t+1}$$
$$= w_{H,t}l_{H,t} + A^{H}_{H,t}(p^{A}_{H,t} + d^{A}_{H,t}) + A^{H}_{F,t}(p^{A}_{F,t} + d^{A}_{F,t}) + B^{H}_{H,t}d^{B}_{H,t} + B^{H}_{F,t}d^{B}_{F,t},$$

(4.19)

while in period $t+1$ budget constraint is:

$$P_{H,t+1}C_{H,t+1} = w_{H,t+1}l_{H,t+1} + A^{H}_{H,t+1}d^{A}_{H,t+1} + A^{H}_{F,t+1}d^{A}_{F,t+1} + B^{H}_{H,t+1}d^{B}_{H,t+1} + B^{H}_{F,t+1}d^{B}_{F,t+1}.$$

$$P_{H,t+1}C_{H,t+1} = w_{H,t+1}l_{H,t+1} + A^{H}_{H,t+1}d^{A}_{H,t+1} + A^{H}_{F,t+1}d^{A}_{F,t+1} + B^{H}_{H,t+1}d^{B}_{H,t+1} + B^{H}_{F,t+1}d^{B}_{F,t+1}.$$

(4.20)

Assume that bonds are in zero net supply, so that the market clearing conditions for bonds are:

$$B^{H}_{H,t} + B^{F}_{H,t} = 0$$
$$B^{H}_{F,t} + B^{F}_{F,t} = 0.$$

$$B^{H}_{H,t} + B^{F}_{H,t} = 0$$
$$B^{H}_{F,t} + B^{F}_{F,t} = 0.$$

With the shocks to factor shares, the labor income becomes:

$$w_{H,t}l_{H,t} = (1-\alpha_{H,t})p_{H,t}y_{H,t},$$

$$w_{H,t}l_{H,t} = (1-\alpha_{H,t})p_{H,t}y_{H,t},$$

(4.21)

which capital income is given by:

$$d^A_{H,t} = \alpha_{H,t} p_{H,t} y_{H,t}.$$

$$d^A_{H,t} = \alpha_{H,t} p_{H,t} y_{H,t}.$$

(4.22)

As before, F country setup is symmetrical.

One can follow the same steps as above to derive the optimal equity and bond portfolios. Due to changes in the setup, a few linearized equations are now different. In particular, the linearized period $t+1$ budget constraint in first-differences becomes:

$$\left(1 - \frac{1}{\sigma}\right) \Delta \hat{Q}_{t+1} = (1-\alpha)\Delta \hat{w}_{t+1} + (2A-1)\alpha \Delta \hat{d}^A_{t+1} + 2B\Delta \hat{d}^B_{t+1}.$$

$$\left(1 - \frac{1}{\sigma}\right) \Delta \widehat{Q}_{t+1} = (1-\alpha)\Delta \widehat{w}_{t+1} + (2A-1)\,\alpha \Delta \hat{d}^A_{t+1} + 2B\Delta \hat{d}^B_{t+1}.$$

(4.23)

As before, one can solve for the optimal portfolio from the budget constraint Equation (4.23) by expressing it in terms of covariances with the asset payouts. With two assets in each country, one can solve for the equilibrium equity share by using covariances with relative equity payouts $\hat{d}^A_{t+1}$ *conditional* on the relative bond payout $\hat{d}^B_{t+1}$. The equilibrium bond share can be obtained by using covariances with the relative bond payout, $\hat{d}^B_{t+1}$, *conditional* on the relative equity payout $\hat{d}^A_{t+1}$. The resulting equity allocation is:

$$A = \frac{1}{2} - \frac{1}{2}\frac{1-\alpha}{\alpha}\beta_{(w,d^A)/d^B} + \frac{1}{2}\frac{1-\frac{1}{\sigma}}{\alpha}\beta_{(Q,d^A)/d^B},$$

$$A = \frac{1}{2} - \frac{1}{2}\frac{1-\alpha}{\alpha}\beta_{(w,d^A)/d^B} + \frac{1}{2}\frac{1-\frac{1}{\sigma}}{\alpha}\beta_{(Q,d^A)/d^B},$$

(4.24)

while bond holding is:

$$B=\frac{1}{2}\left(1-\frac{1}{\sigma}\right)\beta_{(Q,d^B)/d^A}-\frac{1-\alpha}{2}\beta_{(w,d^B)/d^A}$$

$$B=\frac{1}{2}\left(1-\frac{1}{\sigma}\right)\beta_{(Q,d^B)/d^A}-\frac{1-\alpha}{2}\beta_{(w,d^B)/d^A} \tag{4.25}$$

Here $\beta_{(w,d^i)/d^j}=cov_{\hat{d}^j_{t+1}}(\widehat{w}_{t+1},\hat{d}^i_{t+1})/var_{\hat{d}^j_{t+1}}(\hat{d}^i_{t+1})$ and $\beta_{(Q,d^i)/d^j}=cov_{\hat{d}^j_{t+1}}(\widehat{Q}_{t+1},\hat{d}^i_{t+1})/var_{\hat{d}^j_{t+1}}(\hat{d}^i_{t+1}),i,j=\{A,B\}$ are the *conditional hedge ratios*.

The solution for equity share is very similar to the one obtained in the simple model, with the key exception being that the hedge ratios are now conditional on the bond payouts. Bond portfolio is also a function of hedge ratios, conditional on equity payouts.

The solution for bond holdings has two terms:

(i) a terms that captures bonds' propensity to hedge the real exchange rate risk. Investors will go long in local bonds if $\sigma>1$ and $\beta_{(Q,d^B)/d^A}>0$, which requires that real exchange rate covaries positively with the bond payout, after controlling for equity payoffs. Intuitively, as in the simple model, the real exchange rate appreciation leads to an increase in the required relative consumption expenditure. Therefore, if bond payments rise when real exchange rate appreciates, then bonds will be a good hedge against real exchange rate risk.

(ii) a terms that reflects the capacity of bonds to hedge the non-tradable income risk. Bonds will be a good hedge if $\beta_{(w,d^B)/d^A}<0$, that is, if bond payoffs are negatively correlated with labor income risk. In this case, investors will take long positions in local bonds.

Next, the general equilibrium solutions for bond and equity portfolio is obtained by linking the conditional hedge ratios to the model's structural parameters. First, one needs to find linearized expressions for labor income, equity payouts, and bond payouts. Linearizing Equations (4.21) and (4.22) and computing the relative labor income and equity payout give:

$$\begin{aligned}\hat{w}_{t+1} &= -\frac{\alpha}{1-\alpha}\hat{\alpha}_{t+1}+\hat{q}_{t+1}+\hat{y}_{t+1}\\ \hat{d}^A_{t+1} &= \hat{\alpha}_{t+1}+\hat{q}_{t+1}+\hat{y}_{t+1}.\end{aligned}$$

$$\begin{aligned}\widehat{w}_{t+1} &= -\frac{\alpha}{1-\alpha}\widehat{\alpha}_{t+1}+\hat{q}_{t+1}+\hat{y}_{t+1}\\ \hat{d}^A_{t+1} &= \widehat{\alpha}_{t+1}+\hat{q}_{t+1}+\hat{y}_{t+1}.\end{aligned}$$

(4.26)

Since a unit of H(F) bond purchased in period t yields one unit of H(F) composite consumption good in period $t+1$, the following is obtained:

$$d^{B}_{i,t+1} = P_{i,t+1},\ i = \left\{H, F\right\}.$$

$$d^{B}_{i,t+1} = P_{i,t+1},\ i = \left\{H, F\right\}.$$

Therefore, the relative payout on bonds is equal to the real exchange rate, $\widehat{Q}_{t+1}$.

Conditional hedge ratios become:

$$\begin{aligned} &\beta_{(w,d^A)/d^B} = -\frac{\alpha}{1-\alpha}, \beta_{(Q,d^A)/d^B} = 0 \\ &\beta_{(Q,d^B)/d^A} = 1, \beta_{(w,d^B)/d^A} = \frac{1}{1-\alpha}\frac{1-\phi}{2\lambda-1}. \end{aligned}$$

$$\begin{aligned} &\beta_{(w,d^A)/d^B} = -\frac{\alpha}{1-\alpha}, \beta_{(Q,d^A)/d^B} = 0 \\ &\beta_{(Q,d^B)/d^A} = 1, \beta_{(w,d^B)/d^A} = \frac{1}{1-\alpha}\frac{1-\phi}{2\lambda-1}. \end{aligned}$$

Substituting these into the equilibrium equity and bond portfolios yields:

$$\begin{aligned} &A = 1 \\ &B = \frac{1}{2}\left[\left(1-\frac{1}{\sigma}\right) - \frac{1-\phi}{2\lambda-1}\right]. \end{aligned}$$

$$\begin{aligned} &A = 1 \\ &B = \frac{1}{2}\left[\left(1-\frac{1}{\sigma}\right) - \frac{1-\phi}{2\lambda-1}\right]. \end{aligned}$$

These conditions imply that there is full equity home bias, with investors in each country allocating their entire equity portfolios into local stocks.

The solution for bond holdings has two terms that depend on several parameters: the degree of risk aversion, σ; the degree of consumption home bias, λ; the elasticity of intra-temporal substitution between H and F goods in household's preferences, θ, through the ϕ term in Equation (4.13):

(i) The first term in the solution for bond reflects demand for bond to hedge the real exchange rate risk, $1 - 1/\sigma$, and it is positive with $\sigma > 1$. This is not surprising since bonds provide a perfect hedge against real exchange rate fluctuations ($\beta_{(Q,d^B)/d^A} = 1$).

(ii) The second term reflects hedging for non-tradable labor income risk, $-\frac{1-\phi}{2\lambda-1}$, conditional on the relative dividend payoffs on equity. With home bias in preferences ($\lambda > 1/2$), the sign of this term depends on the sign of ($\phi - 1$). When $\phi > 1$, which occurs roughly when $\theta > 1$ the hedging terms is positive and investors will choose a long position in local bonds. Intuitively, when θ is large, the responses of real exchange rate to changes in relative output are weak. Therefore, a decline in relative output will be accompanied by a fall in relative wages, even though real exchange rate appreciates. Thus, relative wages are negatively correlated with the real exchange rate, which makes bonds a good hedge for labor income risk. In contrast, when θ is small, the correlation between relative wages and the real exchange rate turns positive, making them a poor hedge for labor income risk.

The discussion above makes it clear that equilibrium bond positions in the model are sensitive to the intra-temporal elasticity of substitution. The equity position, however, is independent of this elasticity, in contrast to the results in equity-only model. Clearly, access of investors to international bonds has first-order repercussions for equilibrium equity portfolios. Allowing bond trading in the model is essential because bonds facilitate hedging of the real exchange rate risk. Equities can then be used to insure the fluctuations of non-financial labor income risk. This is the key result in Coeurdacier and Gourinchas (2016).

The extended model implies that investors should hold only local equities in their portfolios. While this is a stark prediction, it can be relaxed if capital accumulation was added to the model (see Coeurdacier et al., 2010; Heathcote & Perri, 2013). In this case, equity portfolio will exhibit home bias but it will be less than full for a choice of standard parameter values.

Conclusion

The macroeconomic literature has made significant progress in understanding international portfolio allocations, factors affecting them, and even their dynamics. A number of questions, nevertheless, still remain open. First, one of the key challenges is to develop and solve models with international portfolio choice that also have realistic asset pricing implications. One such attempt is found in Evans and Hnatkovska (2014) who analyze a model with international equities and a non-state contingent bond under three different regimes of financial integration. They show that the model does a good job of matching properties of capital flows in the data, but fails to reproduce the volatility of returns found in the data. However, the model correctly predicts the fall in the volatility of equity, bond and foreign exchange returns following an increase in integration. Other contributions to this area include Rabitsch et al. (2015) who analyze a model with portfolio choice and differential risk attitudes across countries, which allows them to generate significant equity premium; and Benigno and Nisticò (2012) who introduce ambiguity aversion into a model with portfolio choice and show that it can generate significant equity home bias and is also helpful in accounting for the equity premium puzzle.

Second, real exchange rate plays a key role in portfolio choice models where it constitutes an important source of risk. The existing models, however, have difficult time matching the properties of the real exchange rates in the data. In particular, the volatility and persistence of the real exchange rate tends to be significantly higher in the data relative to the models. Moreover, the vast majority of international business cycle models (with and without portfolio choice) would predict that real exchange rate is positively correlated with the relative consumption growth between countries (see, for instance, Equation (4.11) in this chapter). This is in sharp contrast with the data, where this correlation is negative (see Backus & Smith, 1993, for the original discussion). Several explanations have been proposed in the literature to solve the "Backus-Smith puzzle." They all rely on breaking the perfect risk-sharing assumption. Corsetti, Dedola, and Leduc (2008) argue that when the elasticity of intra-temporal substitution between domestic and imported goods in the production function is very low, the Backus-Smith puzzle can be resolved. Alternatively, Corsetti et al. (2008) also show that when the elasticity of substitution is instead very high, a negative correlation between the real exchange rate and relative consumption can be obtained, if productivity shocks are very persistent. In both cases, however, the model does not produce enough volatility in the real exchange rate. Moreover, the implications of these fixes for international portfolios have not been explored.

Third, the empirical regularities presented at the beginning of this article beg the question of what factors are behind the differences in portfolio home bias across countries, and what are the drivers of the changes in the degree of home bias over time. Explanations for the heterogeneity of equity home bias across countries have emphasized several factors. Heathcote and Perri (2013) linked the degree of equity home bias to countries trade openness. Coeurdacier and Gourinchas (2016) computed the hedge ratios of equity returns with non-financial risk and real exchange rate risk in the data for G-7 countries and showed that they can explain a significant share of equity home bias in these countries. They also found that the hedge ratios for bond returns, however, lack predictive power. Explanations for the decline in equity home bias have emphasized the moderation in the exchange rate risk, specifically for the Euro area countries since the introduction of the Euro in 1999 (see, for instance, De Santis & Gérard, 2009). However, given the widespread fall in equity home bias, understanding the factors behind the decline remains an open area of research.

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References

Backus, D. K., & Smith, G. W. (1993). Consumption and real exchange rates in dynamic economies with non-traded goods. *Journal of International Economics*, *35*(3–4), 297–316.

Baxter, M., & Jermann, U. J. (1997). The international diversification puzzle is worse than you think. *American Economic Review*, *87*(1), 170–80.

Benigno, P., & Nisticò, S. (2012). International portfolio allocation under model uncertainty. *American Economic Journal: Macroeconomics*, *4*(1), 144–189.

Bhamra, H. S., Coeurdacier, N., & Guibaud, S. (2014). A dynamic equilibrium model of imperfectly integrated financial markets. *Journal of Economic Theory*, *154*(C), 490–542.

Blanchard, O. J., & Kahn, C. M. (1980). The solution of linear difference models under rational expectations. *Econometrica*, *48*(5), 1305–1311.

Campbell, J. Y., Chan, Y. L., & Viceira, L. M. (2003). A multivariate model of strategic asset allocation. *Journal of Financial Economics*, *67*(1), 41–80.

Christiano, L. J. (2002). Solving dynamic equilibrium models by a method of undetermined coefficients. *Computational Economics*, *20*(1–2), 21–55.

Coeurdacier, N., & Gourinchas, P.-O. (2016). When bonds matter: Home bias in goods and assets. *Journal of Monetary Economics*, *82*(C), 119–137.

Coeurdacier, N., Kollmann, R., & Martin, P. (2009). International portfolios with supply, demand and redistributive shocks. In R. H. Clarida & F. Giavazzi (Eds.), *NBER International Seminar on Macroeconomics 2007* (pp. 231–263). Chicago, IL: University of Chicago Press.

Coeurdacier, N., Kollmann, R., & Martin, P. (2010). International portfolios, capital accumulation and foreign assets dynamics. *Journal of International Economics*, *80*(1), 100–112.

Coeurdacier, N., & Rey, H. (2013). Home bias in open economy financial macroeconomics. *Journal of Economic Literature*, *51*(1), 63–115.

Cole, H. L., & Obstfeld, M. (1991). Commodity trade and international risk sharing: How much do financial markets matter? *Journal of Monetary Economics*, *28*(1), 3–24.

Corsetti, G., Dedola, L., & Leduc, S. (2008). International risk sharing and the transmission of productivity shocks. *Review of Economic Studies*, *75*(2), 443–473.

De Santis, R. A., & Gérard, B. (2009). International portfolio reallocation: Diversification benefits and European monetary union. *European Economic Review*, *53*(8), 1010–1027.

Devereux, M. B., & Sutherland, A. (2009). A portfolio model of capital flows to emerging markets. *Journal of Development Economics*, *89*(2), 181–193.

Devereux, M. B., & Sutherland, A. (2010). Country portfolio dynamics. *Journal of Economic Dynamics and Control*, *34*(7), 1325–1342.

Devereux, M. B., & Sutherland, A. (2011). Country portfolios in open economy macro-models. *Journal of the European Economic Association*, *9*(2), 337–369.

Engel, C., & Matsumoto, A. (2009). The international diversification puzzle when goods prices are sticky: It's really about exchange-rate hedging, not equity portfolios. *American Economic Journal: Macroeconomics*, *1*(2), 155–188.

Evans, M., & Hnatkovska, V. V. (2012). A method for solving general equilibrium models with incomplete markets and many financial assets. *Journal of Economic Dynamics and Control*, *36*(12), 1909–1930.

Evans, M., & Hnatkovska, V. V. (2014). International capital flows, returns and world financial integration. *Journal of International Economics*, *92*(1), 14–33.

French, K. R., & Poterba, J. M. (1991). Investor diversification and international equity markets. *American Economic Review*, *81*(2), 222–226.

Heathcote, J., & Perri, F. (2013). The international diversification puzzle is not as bad as you think. *Journal of Political Economy*, *121*(6), 1108–1159.

Hnatkovska, V. (2010). Home bias and high turnover: Dynamic portfolio choice with incomplete markets. *Journal of International Economics*, *80*(1), 113–128.

Jin, H.-H., & Judd, K. L. (2002). *Perturbation methods for general dynamic stochastic models* (working papers). Oxford, U.K.: Stanford University.

Klein, P. (2000). Using the generalized Schur form to solve a multivariate linear rational expectations model. *Journal of Economic Dynamics and Control*, *24*(10), 1405–1423.

Kollmann, R. (1995). Consumption, real exchange rates and the structure of international asset markets, *Journal of International Money and Finance*, *14*(2), 191–211.

Kollmann, R. (2006). *International portfolio equilibrium and the current account: A dynamic general equilibrium perspective*. CEPR Discussion Paper 5512.

Lucas, R. J. (1982). Interest rates and currency prices in a two-country world. *Journal of Monetary Economics*, *10*(3), 335–359.

Michaelides, A. (2003). International portfolio choice, liquidity constraints and the home equity bias puzzle. *Journal of Economic Dynamics and Control*, *28*(3), 555–594.

Pavlova, A., & Rigobo, R. (2010a). An asset-pricing view of external adjustment. *Journal of International Economics*, *80*(1), 144–156.

Pavlova, A., & Rigobo, R. (2010b). *Equilibrium portfolios and external adjustment under incomplete markets* (working paper series). London, U.K.: London Business School.

Rabitsch, K., Stepanchuk, S., & Tsyrennikov, V. (2015). International portfolios: A comparison of solution methods. *Journal of International Economics*, *97*(2), 404–422.

Tesar, L. L., & Werner, I. M. (1995). Home bias and high turnover. *Journal of International Money and Finance*, *14*(4), 467–492.

Tille, C., & van Wincoop, E. (2010). International capital flows. *Journal of International Economics*, *80*(2), 157–175.

Appendix

This appendix contains data description and additional results, as well as some model derivations.

Data

The data used in section "Empirical Evidence" come from the Coordinated Portfolio Investment Survey (CPIS) and International Financial Statistics database (IFS). CPIS does not report total Liabilities for all countries, so total Liabilities are computed by aggregating bilateral information available in the dataset. The years available in CPIS are 1997 and then 2001–2016. IFS contains data from 1980–2016 for a few countries. To construct the measures of equity home bias the Liabilities reported in the CPIS were used, whenever available, otherwise the aggregated Liabilities were used. Market capitalization data is from the World Bank Development Indicators database for the relevant sample period.

The following table contains the list of countries comprising each region in the dataset.

Table A1. List of Countries in Each Region

ID	Region	Countries in Each Region
1	Central and South America	Brazil, Chile
2	North America	United States and Canada
3	Asia	Hong Kong, Japan, Kazakhstan, Korea, Malaysia, Philippines, Singapore, and Thailand
4	Oceania	Australia
5	Middle East	Israel and Turkey
6	Europe	Belgium, Germany, Greece, Italy, the Netherlands, Norway, and Switzerland
7	Central and Eastern Europe	Hungary, Poland, Romania, and Slovak Republic
8	Africa	South Africa

The figures below present the equity home bias in developed (figure A1) and developing (A2) countries, as well as in different regions (figure A3) using IFS data going back to 1980.

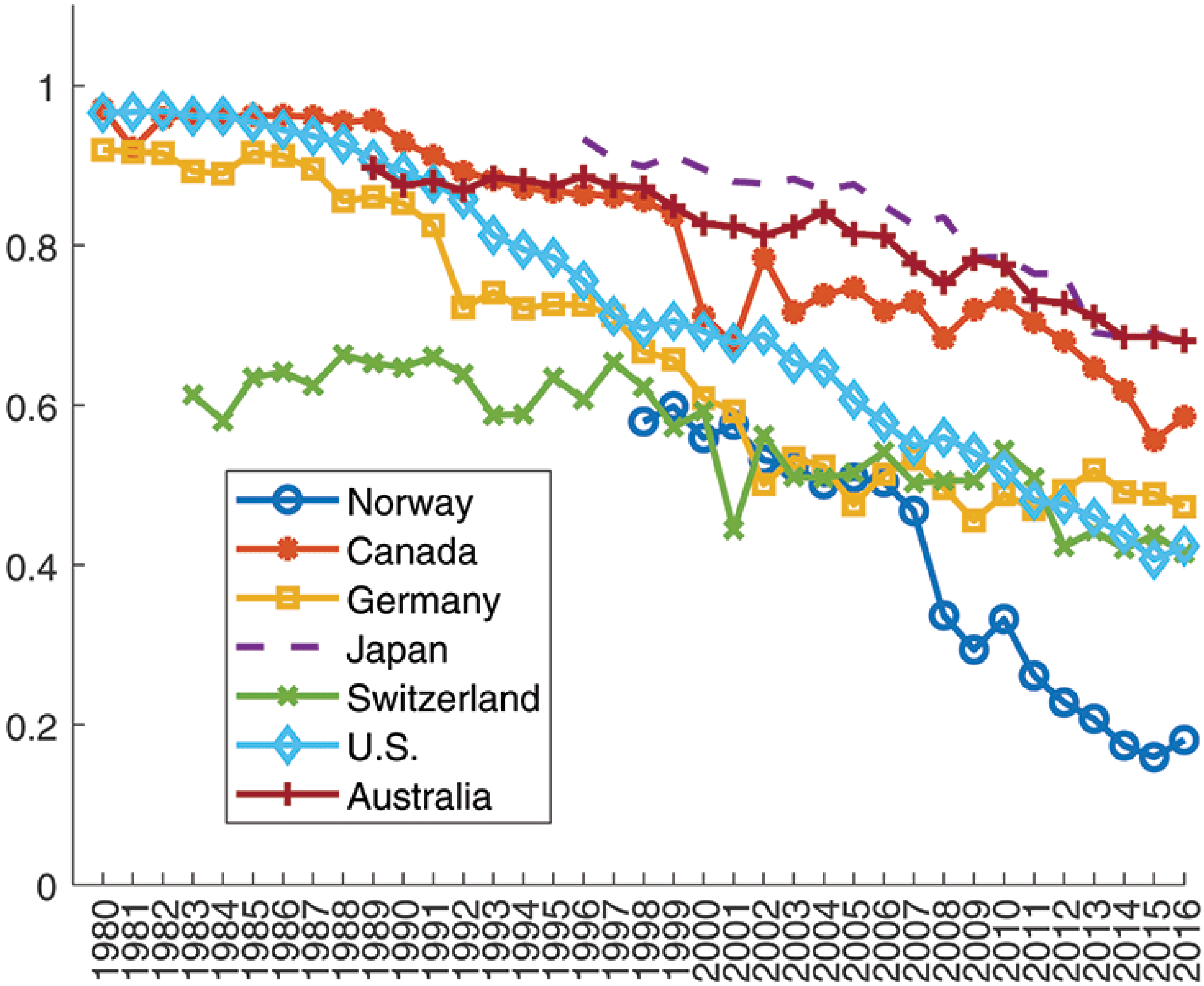

Figure A1. Equity home bias in developed countries, 1980–2016.

Source: Author's calculations.

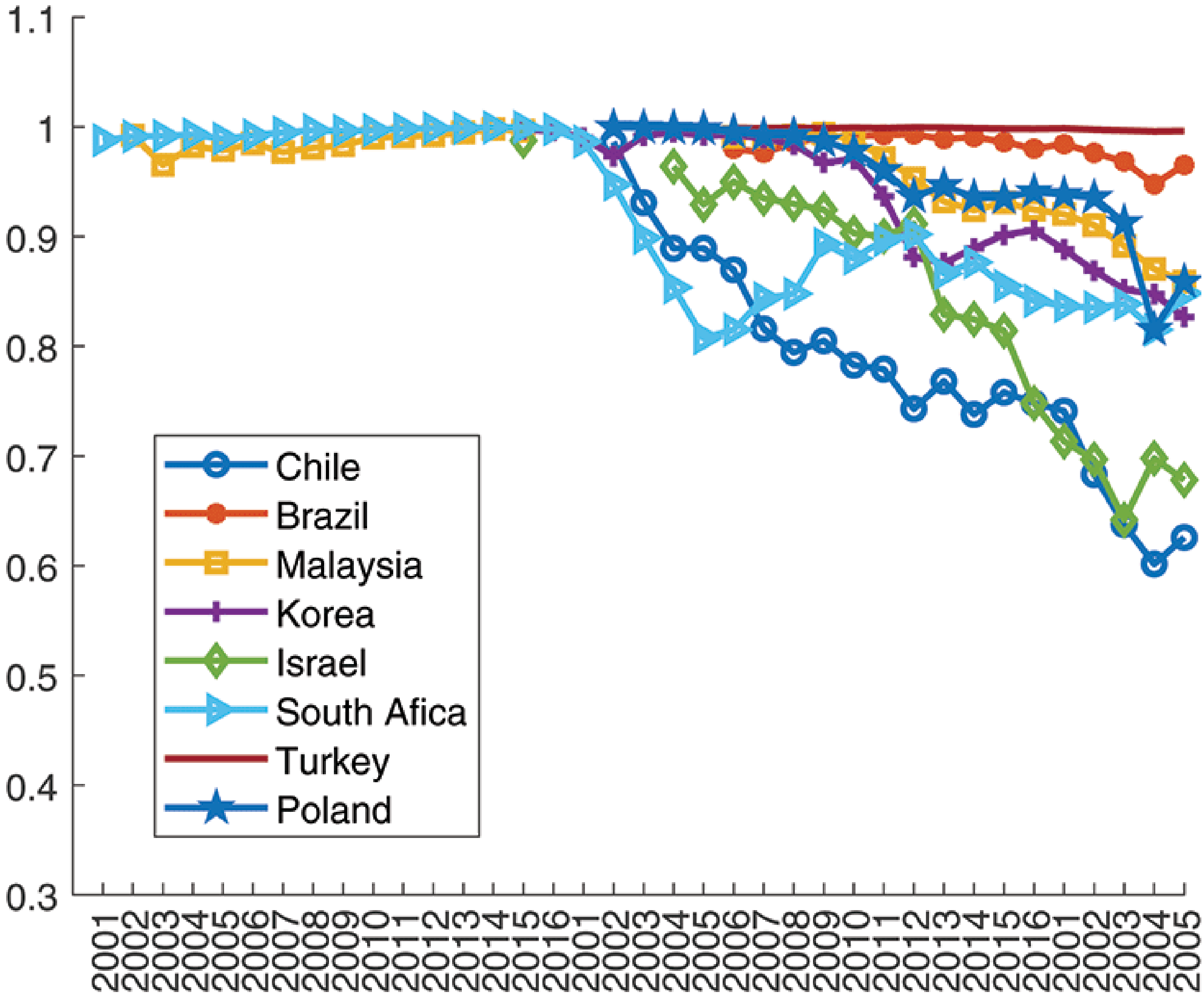

Figure A2. Equity home bias in developing countries, 1980–2016.

Source: Author’s calculations.

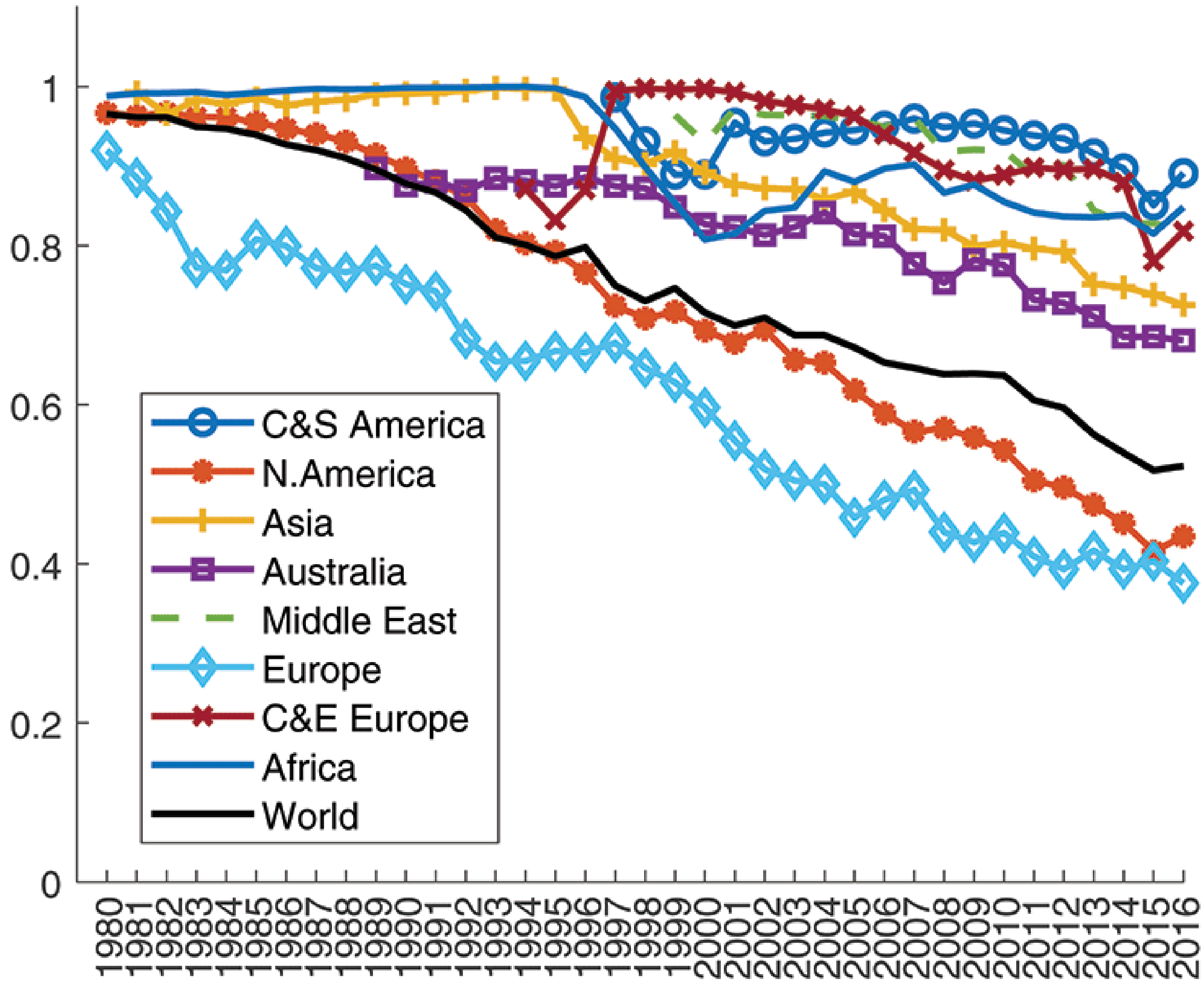

Figure A3. Equity home bias across regions, 1980–2016.

Source: Author's calculations.

Almost all countries exhibit secular decline in equity home bias over time, in line with the trends in the CPIS dataset presented in the main text.

Model

Real exchange rate: Real exchange rate is defined as:

$$Q_t = \frac{P_{H,t}}{P_{F,t}},$$

$$Q_t = \frac{P_{H,t}}{P_{F,t}},$$

which gives $\widehat{Q}_t = \widehat{P}_{H,t} - \widehat{P}_{F,t}$. To express the real exchange rate in terms of the terms of trade, recall that $P_{H,t} = \left[\lambda p_{H,t}^{1-\theta} + (1-\lambda) p_{F,t}^{1-\theta}\right]^{1/(1-\theta)}$ and $P_{F,t} = \left[\lambda p_{F,t}^{1-\theta} + (1-\lambda) p_{H,t}^{1-\theta}\right]^{1/(1-\theta)}$, which gives:

$$\begin{aligned} Q_t &= \left[\frac{\lambda p_{H,t}^{1-\theta} + (1-\lambda) p_{F,t}^{1-\theta}}{\lambda p_{F,t}^{1-\theta} + (1-\lambda) p_{H,t}^{1-\theta}}\right]^{1/(1-\theta)} \\ &= \left[\frac{\lambda \left(\frac{p_{H,t}}{p_{F,t}}\right)^{1-\theta} + (1-\lambda)}{\lambda + (1-\lambda)\left(\frac{p_{H,t}}{p_{F,t}}\right)^{1-\theta}}\right]^{\frac{1}{1-\theta}}. \end{aligned}$$

$$\begin{aligned} Q_t &= \left[\frac{\lambda p_{H,t}^{1-\theta} + (1-\lambda) p_{F,t}^{1-\theta}}{\lambda p_{F,t}^{1-\theta} + (1-\lambda) p_{H,t}^{1-\theta}}\right]^{1/(1-\theta)} \\ &= \left[\frac{\lambda \left(\frac{p_{H,t}}{p_{F,t}}\right)^{1-\theta} + (1-\lambda)}{\lambda + (1-\lambda)\left(\frac{p_{H,t}}{p_{F,t}}\right)^{1-\theta}}\right]^{\frac{1}{1-\theta}}. \end{aligned}$$

Taking logs on both sides of this expression yields:

$$\ln Q_t = \frac{1}{1-\theta}\left[\ln\left(\lambda q_t^{1-\theta} + 1 - \lambda\right) - \ln\left(\lambda + (1-\lambda) q_t^{1-\theta}\right)\right],$$

$$\ln Q_t = \frac{1}{1-\theta}\left[\ln\left(\lambda q_t^{1-\theta} + 1 - \lambda\right) - \ln\left(\lambda + (1-\lambda) q_t^{1-\theta}\right)\right],$$

where I used the definition of the terms of trade, $q_t = \frac{p_{H,t}}{p_{F,t}}$. The first-order Taylor series expansions of this expression around the deterministic symmetrical steady state give:

$$\begin{aligned}\hat{Q}_t &= \frac{1}{1-\theta}\left[\lambda(1-\theta)\hat{q}_t - (1-\lambda)(1-\theta)\hat{q}_t\right] \\ &= (2\lambda-1)\hat{q}_t.\end{aligned}$$

$$\begin{aligned}\widehat{Q}_t &= \tfrac{1}{1-\theta}[\lambda(1-\theta)\hat{q}_t - (1-\lambda)(1-\theta)\hat{q}_t] \\ &= (2\lambda-1)\hat{q}_t.\end{aligned}$$

This is Equation (4.12) in the main text.

Market clearing: Next, the log-approximation of the market clearing conditions is derived in Equations (4.5). In relative terms, the market clearing can be written as $y_t \equiv \frac{y_{H,t}}{y_{F,t}} = \frac{c^H_{H,t}+c^F_{H,t}}{c^H_{F,t}+c^F_{F,t}}$. Substituting in the equilibrium consumption choices from Equations (4.7), we obtain:

$$y_t = \left(\frac{p_{H,t}}{p_{F,t}}\right)^{-\theta}\left[\frac{\lambda C_{H,t}P^{\theta}_{H,t} + (1-\lambda)C_{Ft}P^{\theta}_{F,t}}{(1-\lambda)C_{H,t}P^{\theta}_{H,t} + \lambda C_{Ft}P^{\theta}_{F,t}}\right].$$

$$y_t = \left(\frac{p_{H,t}}{p_{F,t}}\right)^{-\theta}\left[\frac{\lambda C_{H,t}P^{\theta}_{H,t} + (1-\lambda)\, C_{Ft}P^{\theta}_{F,t}}{(1-\lambda)\, C_{H,t}P^{\theta}_{H,t} + \lambda C_{Ft}P^{\theta}_{F,t}}\right].$$

Using the definition of terms of trade, $q_t = \frac{p_{H,t}}{p_{F,t}}$, and factoring out $C_{H,t}P^{\theta}_{H,t}$ in the numerator and denominator yields:

$$y_t = q_t^{-\theta}\left[\frac{\lambda + (1-\lambda)\frac{C_{Ft}P^{\theta}_{F,t}}{C_{H,t}P^{\theta}_{H,t}}}{1-\lambda+\lambda\frac{C_{Ft}P^{\theta}_{F,t}}{C_{H,t}P^{\theta}_{H,t}}}\right].$$

$$y_t = q_t^{-\theta}\left[\frac{\lambda + (1-\lambda)\,\frac{C_{Ft}P^{\theta}_{F,t}}{C_{H,t}P^{\theta}_{H,t}}}{1-\lambda+\lambda\frac{C_{Ft}P^{\theta}_{F,t}}{C_{H,t}P^{\theta}_{H,t}}}\right].$$

Taking logs on both sides of this expression gives:

$$\ln y_t = -\theta \ln q_t + \ln\left(\lambda + (1-\lambda)\frac{C_{Ft}P^{\theta}_{F,t}}{C_{H,t}P^{\theta}_{H,t}}\right) - \ln\left(1-\lambda+\lambda\frac{C_{Ft}P^{\theta}_{F,t}}{C_{H,t}P^{\theta}_{H,t}}\right).$$

$$\ln y_t = -\theta \ln q_t + \ln\left(\lambda + (1-\lambda)\frac{C_{Ft}P^{\theta}_{F,t}}{C_{H,t}P^{\theta}_{H,t}}\right) - \ln\left(1-\lambda+\lambda\frac{C_{Ft}P^{\theta}_{F,t}}{C_{H,t}P^{\theta}_{H,t}}\right).$$

The first-order Taylor series expansions of this expression around the deterministic symmetrical steady state give:

$$\hat{y}_t = -\theta\hat{q}_t + (1-2\lambda)\left(\hat{C}_{F,t} - \hat{C}_{H,t}\right) + \theta(1-2\lambda)\left(\hat{P}_{F,t} - \hat{P}_{H,t}\right).$$

$$\hat{y}_t = -\theta\hat{q}_t + (1-2\lambda)\left(\widehat{C}_{F,t} - \widehat{C}_{H,t}\right) + \theta(1-2\lambda)\left(\widehat{P}_{F,t} - \widehat{P}_{H,t}\right).$$

We can now take the first-differences of this expression between $t+1$ and t and substitute in for $\Delta\widehat{C}_{F,t+1} - \Delta\widehat{C}_{H,t+1}$ from Equation (4.11) and for $\widehat{P}_{F,t+1} - \widehat{P}_{H,t+1}$ from Equation (4.12) to obtain:

$$\begin{aligned}\Delta\hat{y}_{t+1} &= -\theta\Delta\hat{q}_{t+1} - \frac{1}{\sigma}(2^{\lambda-1)2}\Delta\hat{q}_{t+1} + \theta(2^{\lambda-1)2}\Delta\hat{q}_{t+1} \\ &= -\phi\Delta\hat{q}_{t+1},\end{aligned}$$

$$\begin{aligned}\Delta\hat{y}_{t+1} &= -\theta\Delta\hat{q}_{t+1} - \tfrac{1}{\sigma}(2^{\lambda-1)2}\Delta\hat{q}_{t+1} + \theta(2^{\lambda-1)2}\Delta\hat{q}_{t+1} \\ &= -\phi\Delta\hat{q}_{t+1},\end{aligned}$$

where $\phi = \theta + \left(\frac{1}{\sigma} - \theta\right)(2\lambda-1)^2$. This is Equation (4.13) in the main text.

Notes

1. The conditioning on bond returns is important as conditional and unconditional covariances of equity returns with non-financial income risk tend to be different in the data.

2. The levels and trends of equity home bias are consistent in the CPIS and IFS datasets, so only CPIS results are reported here. The results from the IFS dataset for longer time horizon are presented in the Appendix.

3. See Table A1 in the Appendix for the list of countries in each region.

4. If $\sigma < 1$ then there will be local equity bias if $\beta_{Q,d} < 0$, that is equity payoffs are negatively correlated with the real exchange rate. This is because real exchange rate appreciation also implies that local goods are relatively more expensive, thus making it optimal for households to shift their consumption to the states where local goods are relatively less expensive. This effect dominates the smoothing effect described before when the inter-temporal elasticity of substitution is relatively high $\sigma < 1)$. The two effects exactly offset each other with $\sigma = 1$ and the real exchange rate hedging term in the portfolio solution disappears.