Universal Nuclear Scaling: A Core Compression Law For Isotopes

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Abstract: We report the discovery of a mathematical relationship that describes the nuclear charge Q of all known isotopes as a function of mass number A with high precision. Our "Core Compression Law," $Q = c_1A^{(2/3)} + c_2A$, fits 3,355 isotopes—including both stable and unstable species—with $R^2 = 0.977$. This challenges the prevailing view that only stable isotopes follow predictable patterns, revealing instead that all nuclear matter obeys universal scaling laws governed by electron charge as the fundamental organizing principle.

Introduction : Nuclear stability has traditionally been understood through the valley of stability, where approximately 250-300 stable isotopes follow predictable neutron-to-proton ratios [1,2]. The >3,000 known unstable isotopes are generally considered to exhibit complex, unpredictable behavior dominated by quantum shell effects and exotic decay modes [3,4]. Here we demonstrate that this conventional wisdom is incomplete: all known isotopes, stable and unstable alike, conform to a simple two-parameter scaling law governed by electron charge. The form of the equation $Q = c_1A^{2/3} + c_2A$ is similar to the result obtained when one finds the line of beta stability by differentiating the Bethe-Weizsäcker SEMF with respect to Z. The SEMF includes a surface term ($\propto A^{2/3}$) and a Coulomb term ($\propto Z^2/A^{1/3}$) which, when optimized for Z(A), produces a similar functional form. Several key distinctions differentiate this work:

Scope: While SEMF-derived stability conditions apply primarily to the ~250 stable isotopes, our empirical relationship fits all 3,355 known isotopes with an average deviation of approximately 4 charge units (RMSE = 4.22).

Target quantity: SEMF models binding energy and mass; we directly fit the charge-mass relationship across the entire nuclear chart.

Predictive power: SEMF cannot account for the behavior of exotic unstable isotopes, yet our relationship shows these follow the same scaling law as stable isotopes ($R^2 = 0.976$).

Discovery method: Rather than deriving this form from nuclear theory, we discovered it through comprehensive data analysis, interpreted through electron charge dynamics.

The similarity suggests deep underlying physics, but the empirical precision and universal applicability across all isotope types represents a fundamentally new finding.

Methods: We analyzed 3,355 isotopes from the comprehensive nuclear database [5], comprising 226 stable isotopes (6.7%) and 3,129 unstable isotopes (93.3%). For each isotope, we recorded the nuclear charge Q (number of protons), mass number A, and stability classification.

We hypothesized that nuclear charge scales according to competing surface and volume effects under electron charge influence:

$Q = c_1A^{(2/3)} + c_2A$ (1)

where the $A^{(2/3)}$ term represents surface effects and the A term represents core compression effects driven by electron charge dynamics. We fitted this model using least squares regression and evaluated goodness-of-fit using R^2 statistics.

Results: When the universal fit derived from all isotopes was applied to the subset of 3,129 unstable isotopes, it still accounted for 97.6% of the variance ($R^2 = 0.9756$). The Core Compression Law provides an extraordinary fit to all isotopes:

All isotopes (n = 3,355):

- Coefficients: $c_1 = 0.481147$, $c_2 = 0.321893$
- $R^2 = 0.9772$
- RMSE = 4.22

Stable isotopes (n = 226):

- Coefficients: $c_1 = 0.673917$, $c_2 = 0.284380$
- $R^2 = 0.9988$
- RMSE = 0.91

Unstable isotopes (n = 3,129):

- $R^2 = 0.9756$ (using all-isotope fit)
- RMSE = 4.36

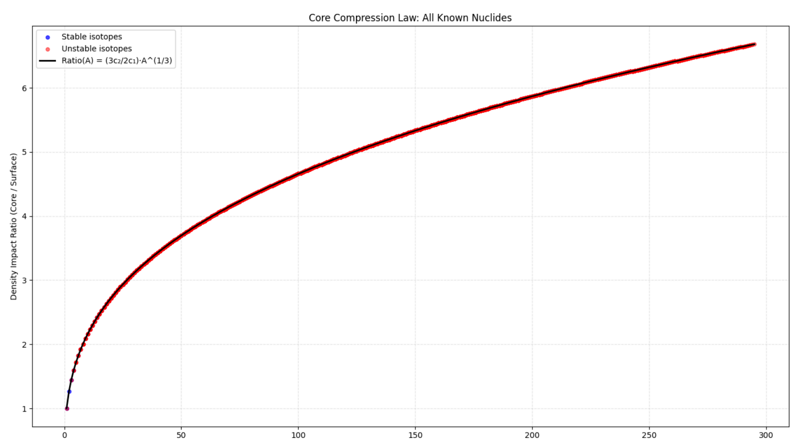

Figure 1 shows agreement between predicted and observed values across the nuclear chart, with stable and unstable isotopes following the same fundamental relationship, which we attribute to an underlying charge dynamics principle under development.

Universal Nuclear Scaling Law. Nuclear charge Q versus mass number A for all 3,355 known isotopes. Blue points represent stable isotopes (n = 226), red points represent unstable isotopes (n = 3,129). The black line shows the Core Compression Law fit: $Q = 0.481A^{(2/3)} + 0.322A$ ($R^2 = 0.977$). The precision of this relationship across the entire nuclear chart reveals universal order in nuclear structure governed by electron charge dynamics.

The density impact ratio, $(3c_2/2c_1)A^{(1/3)}$, reveals how core compression scales relative to surface effects as nuclei grow larger under electron charge influence. This ratio increases monotonically from ~1 for light nuclei to ~7 for superheavy elements, indicating the growing dominance of volume effects over surface effects.

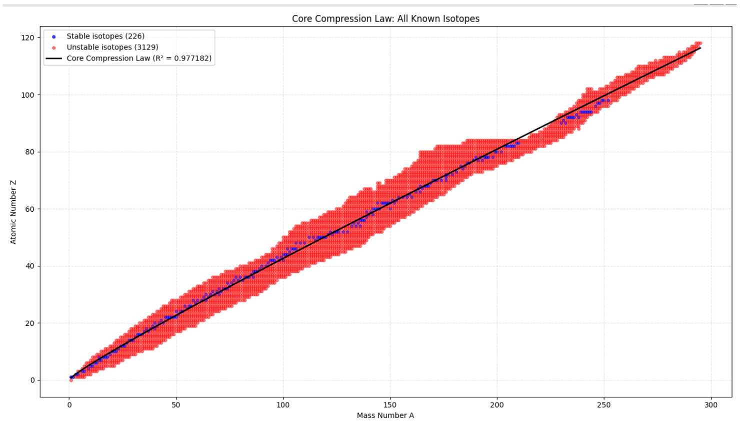

Figure 2 shows the traditional Nuclide Distribution.

Residual analysis reveals systematic deviations at magic numbers (Q = 2, 8, 20, 28, 50, 82), confirming that shell effects represent secondary perturbations to the underlying universal scaling law driven by electron charge.

Discussion: These findings fundamentally challenge current understanding of nuclear structure. The traditional paradigm holds that stable and unstable isotopes represent qualitatively different regimes: stable nuclei following predictable patterns while unstable nuclei exhibit chaotic, quantum-dominated behavior [6,7]. Interestingly, the scaling coefficients for the subset of stable isotopes differ from the universal fit, suggesting that the precise locus of maximum stability is governed by a subtly different balance of core and surface effects than the bulk of nuclear matter.

Our Core Compression Law demonstrates instead that:

1. **Universal scaling governs all nuclear matter**, with electron charge as the fundamental organizing principle
2. **Unstable isotopes are as predictable as stable ones** when viewed through the electron charge framework
3. **Nuclear structure exhibits order** across the nuclides driven by charge dynamics
4. **Shell effects are perturbations** to fundamental charge-based scaling laws rather than primary organizing principles

The physical interpretation suggests that nuclear charge Q is determined by the competition between surface tension effects ($\propto A^{(2/3)}$) and core compression effects ($\propto A$), both mediated by electron charge interactions. As nuclei grow, core compression dominates under electron charge influence, driving the deviation from the $Q \approx A/2$ relationship observed in light nuclei.

The precision of this relationship ($R^2 > 0.97$ for all isotopes) implies that nuclear structure is governed by more fundamental charge-based scaling laws than previously recognized. This may have profound implications for understanding nucleosynthesis, nuclear stability, and the synthesis of superheavy elements through electron charge dynamics.

Conclusion: The Core Compression Law reveals unexpected universality in nuclear structure, unifying stable and unstable isotopes under a simple mathematical framework. This discovery suggests that nuclear physics may be more predictable and orderly than previously thought, potentially revolutionizing our approach to nuclear theory and applications.

Data and Code Availability: All data and analysis code are freely available at: https://github.com/tracyphasespace/Quantum-Field-Dynamics/tree/main/NuclidePredictionCurve

- `nuclides.csv`: Complete (standard) dataset of 3,355 isotopes
- `core_compression_law.py`: Full analysis pipeline reproducing all results
- Complete documentation for reproducing all figures and statistical analyses

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