Algorithms for Wireless Communications II

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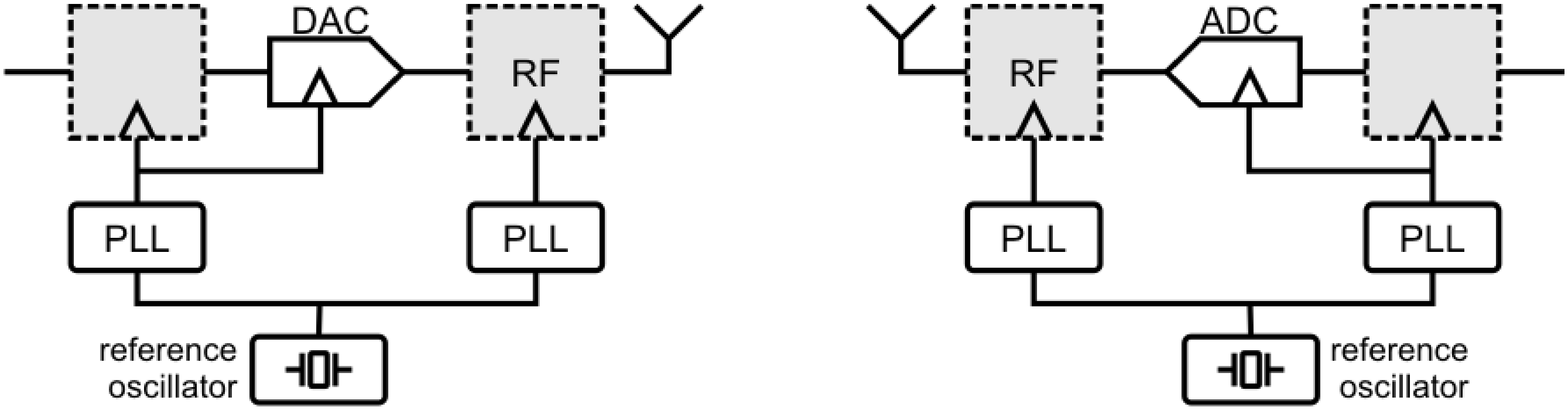
OFDM Synchronization

The Need for Synchronization



- A reference oscillator provides a reference frequency
  - Typical references: 1 MHz, 10 MHz, or 20 MHz (good quartz oscillators available)
- All system frequencies (samling frequency and carrier frequency) are derived from this reference using Phase Locked Loops

Each device has ist own reference

- References are never 100% accurate
- Signal propagation causes an unknown delay (phase) between devices

The Need for Synchronization

- Frequency reference oscillators are specified only by ist nominal value

- Individual parts can and do deviate from this nominal value

- Oscillator accuracy is espressed in *Parts per Million [ppm]*
  - Frequency independent metric
  - PPM is typically specified on both directions

- Impact of oscillator accuracy on frequency offset/error

Reference: $\Delta f_{\mathrm{ref}} = \varphi \frac{f_{\mathrm{ref}}}{10^6}$

After PLL: $\Delta f_0 = \varphi \frac{f_{\mathrm{ref}}}{10^6} \frac{N}{K} = \varphi \frac{f_0}{10^6}$

- Typical reference accuracy: 8 – 20ppm

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Synchronization Errors

- Initial Timing Offset
- Carrier Frequency Offset
- Signal to Noise Ratio
- Carrier Phase Noise
- Sampling Frequency Offset

Example: IEEE 802.11n WLAN

- Carrier frequency: 2.4 GHz
- Bandwidth (sampling frequency): 40 Mhz (128 tones: 312.5 KHz carrier spacing)
- Reference oscillator: ± 10 ppm

- **Carrier frequency**

$$-24 \text{ kHz} < \Delta f_0^{Tx} < +24 \text{ kHz} \qquad -24 \text{ kHz} < \Delta f_0^{Rx} < +24 \text{ kHz}$$

$$-48 \text{ kHz} < \left| f_0^{Tx} - f_0^{Rx} \right| < +48 \text{ kHz}$$

  - More than 10% of the carrier spacing
  - 70deg per OFDM symbol

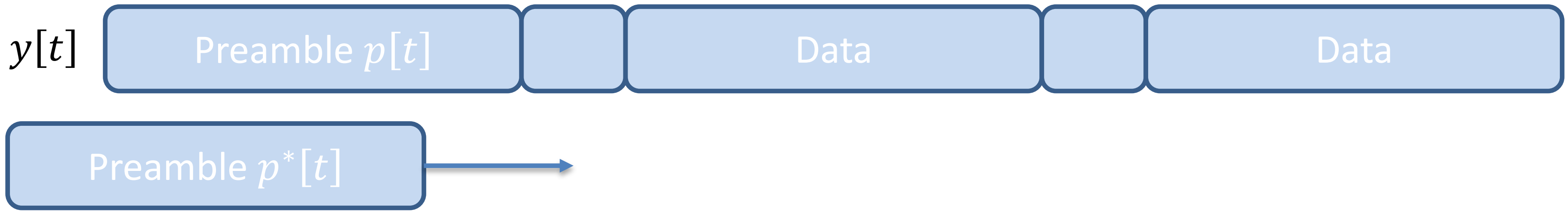
- **Sampling frequency**

$$-400 \text{ Hz} < \Delta f_s^{Tx} < +400 \text{ Hz} \qquad -400 \text{ Hz} < \Delta f_s^{Rx} < +400 \text{ Hz}$$

$$-800 \text{ Hz} < | f_s^{Tx} - f_s^{Rx} | < +800 \text{ Hz}$$

  - 1 sample per 300 OFDM symbols

Synchronization (Tx Matched Filter)



Received signal with frequency offset (no noise) – Dirac channel

$$y[t] = e^{\frac{2\pi \Delta f t}{N}} p[t - t_0]$$

Δf: Frequency offset, normalized to the symbol duration. One period in $\frac{1}{NT_s} \Rightarrow \Delta f = 1$

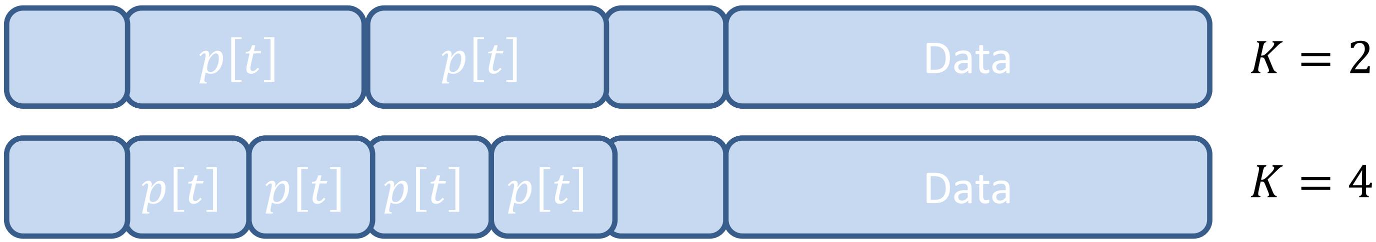
Correlation with transmitted sequence

$$c_{\Delta t} = \sum_{t=0}^{N} y[t]\, p^*[t - \Delta t] = \sum_{t=0}^{N} e^{\frac{2\pi \Delta f t}{N}} p[t - t_0]\, p^*[t - \Delta t]$$

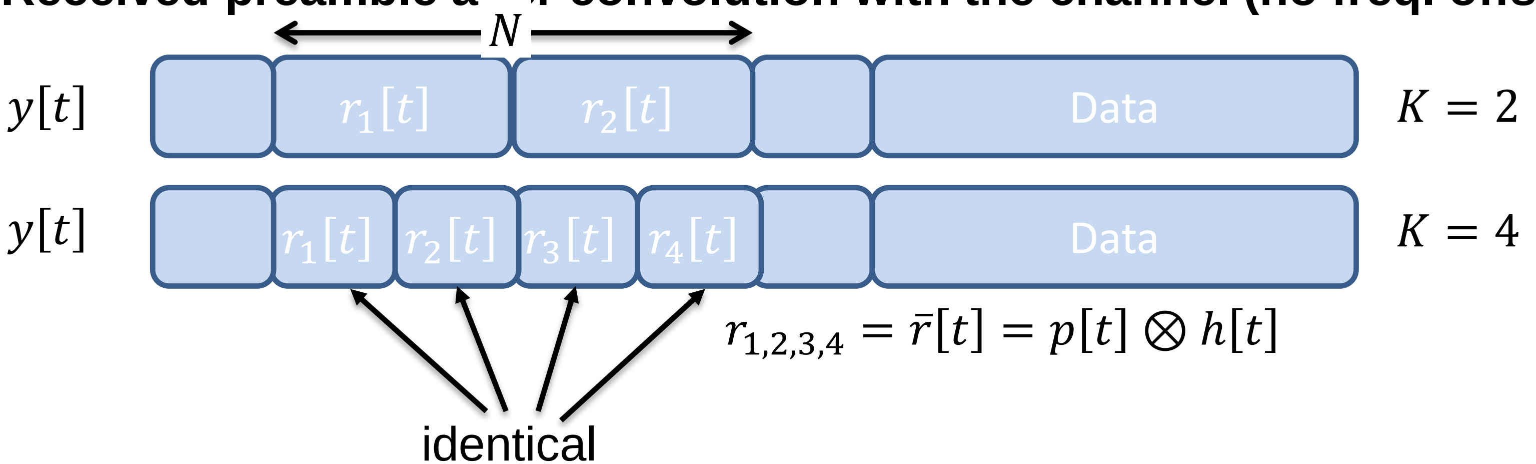
Assume $\Delta t = t_0$: what happens when the normalized frequency offset $\Delta f = 1$?

Synchronization with a Periodic Preamble

New preamble design: OFDM symbol is periodic in itself



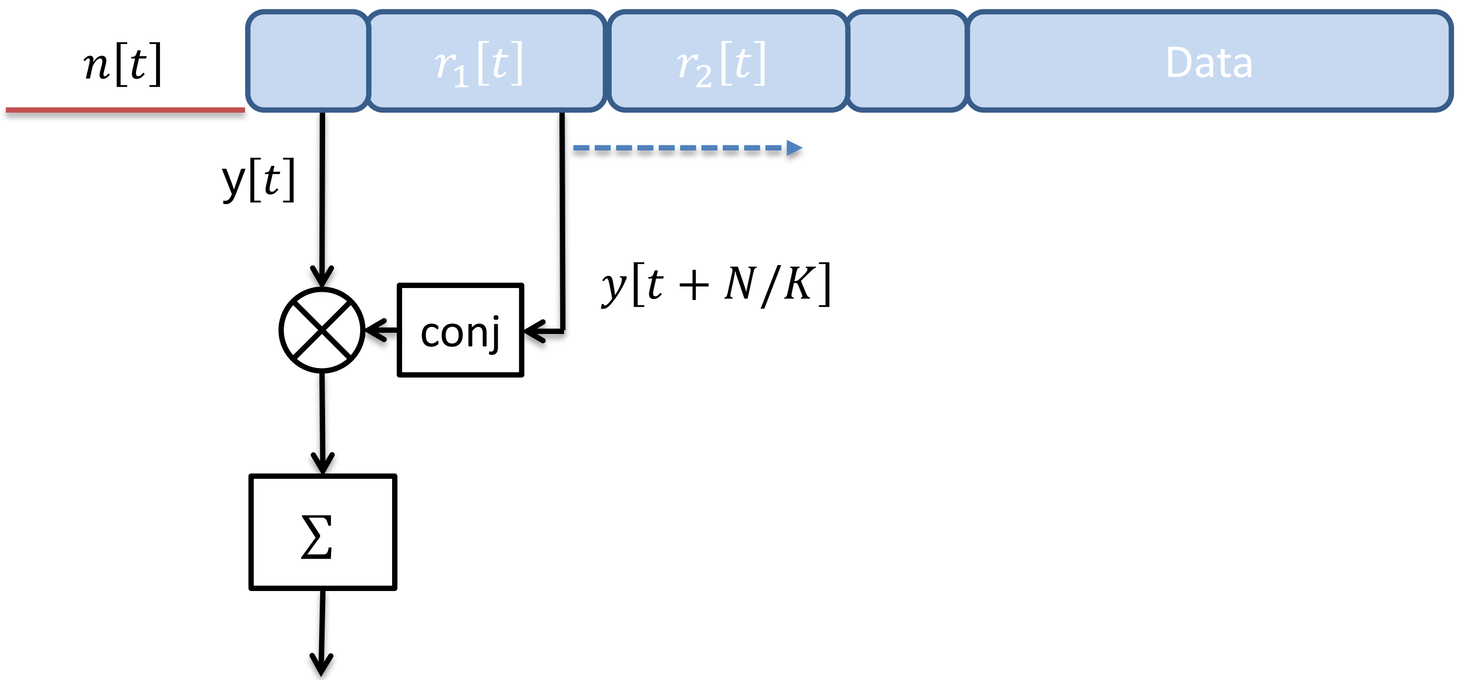
Received preamble after convolution with the channel (no freq. offset)



With frequency offset: $r_k[t] = e^{\frac{2\pi \Delta f t}{N} + \frac{2\pi \Delta f (k-1)}{K}} \bar{r}[t]$

Synchronization with a Periodic Preamble

Idea: delayed autocorrelation finds periodic sequence



$$c_t = \sum_{\tau=0}^{N/K-1} y[t+\tau]y^*[t+\tau+N/K]$$

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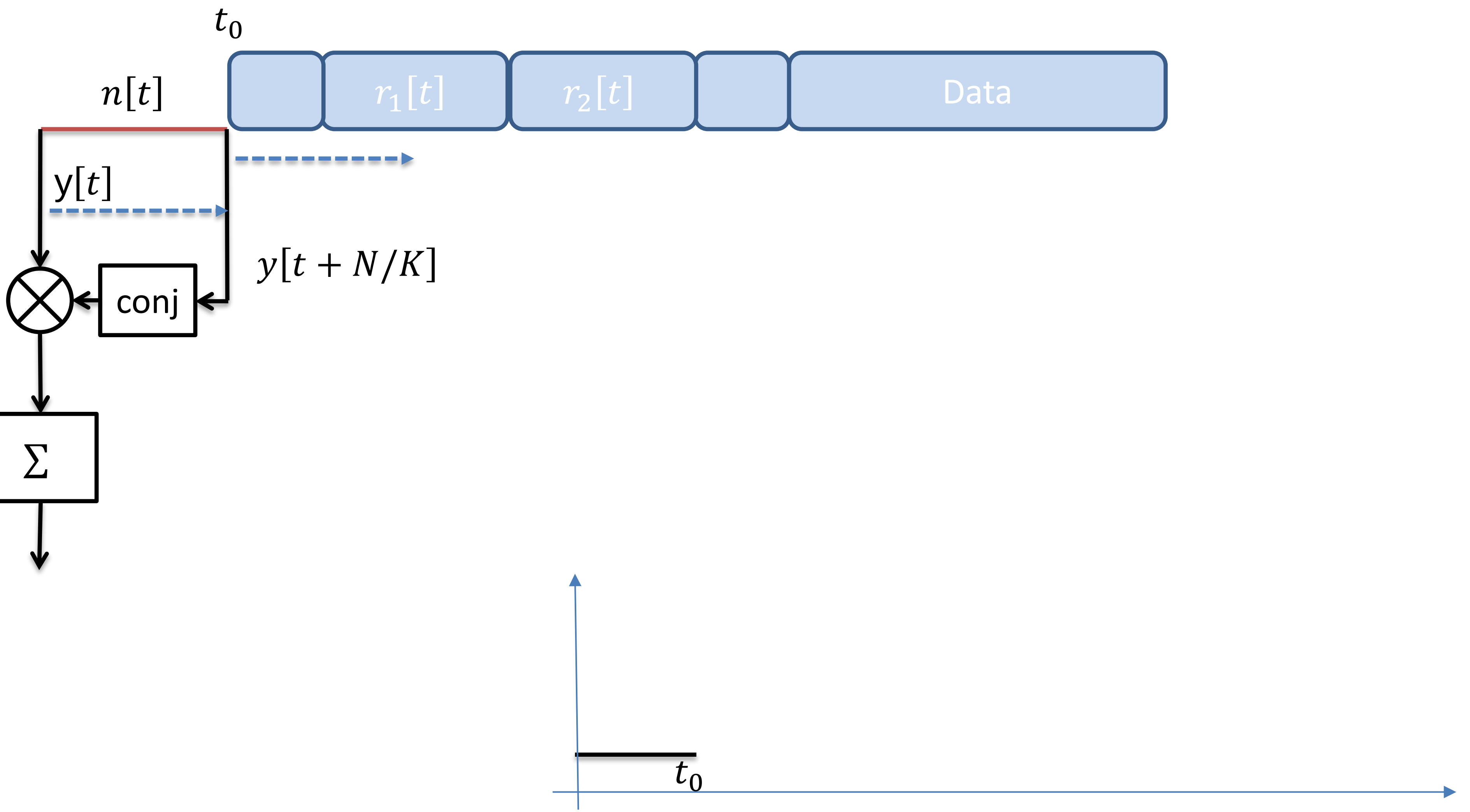

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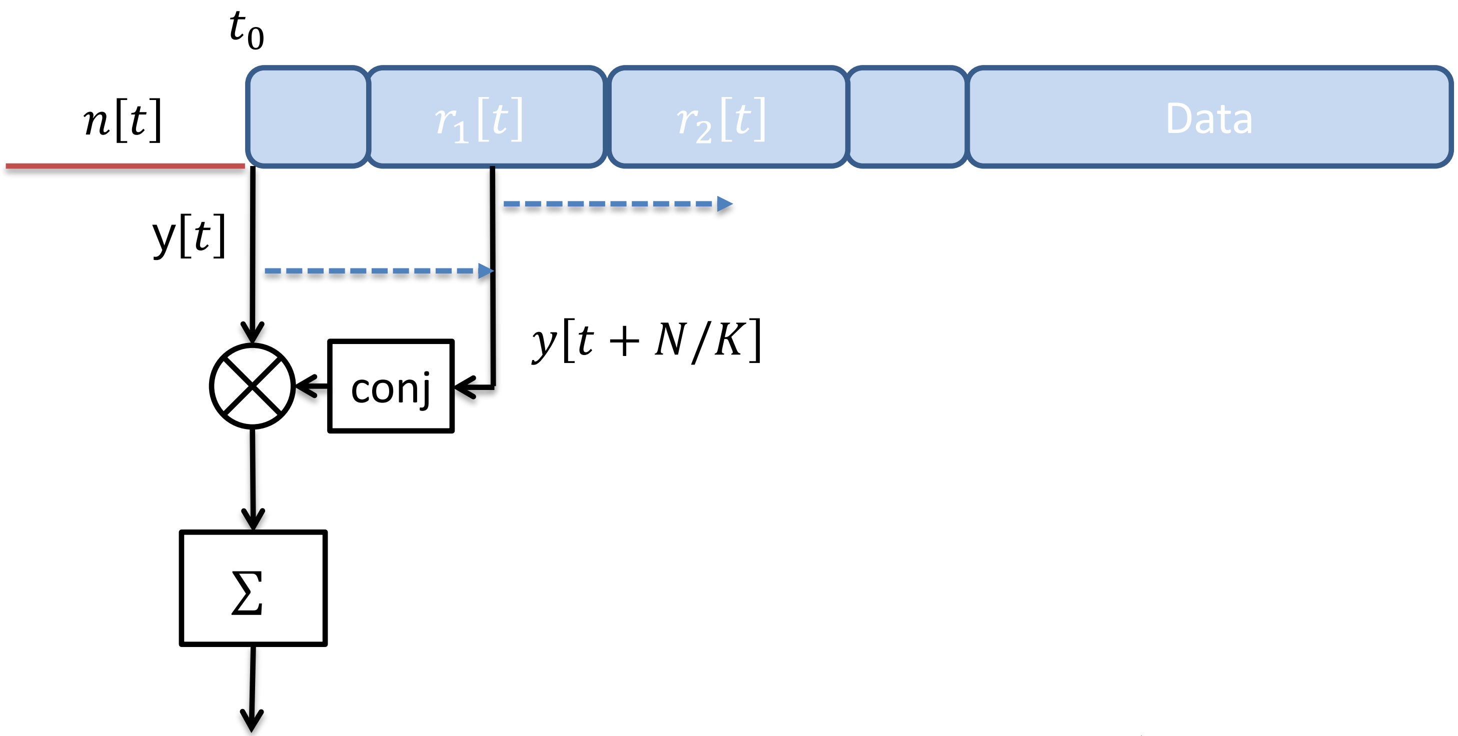

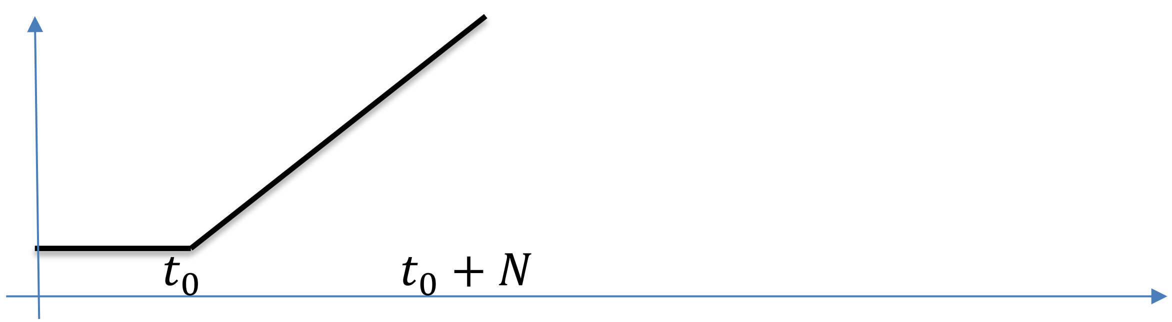

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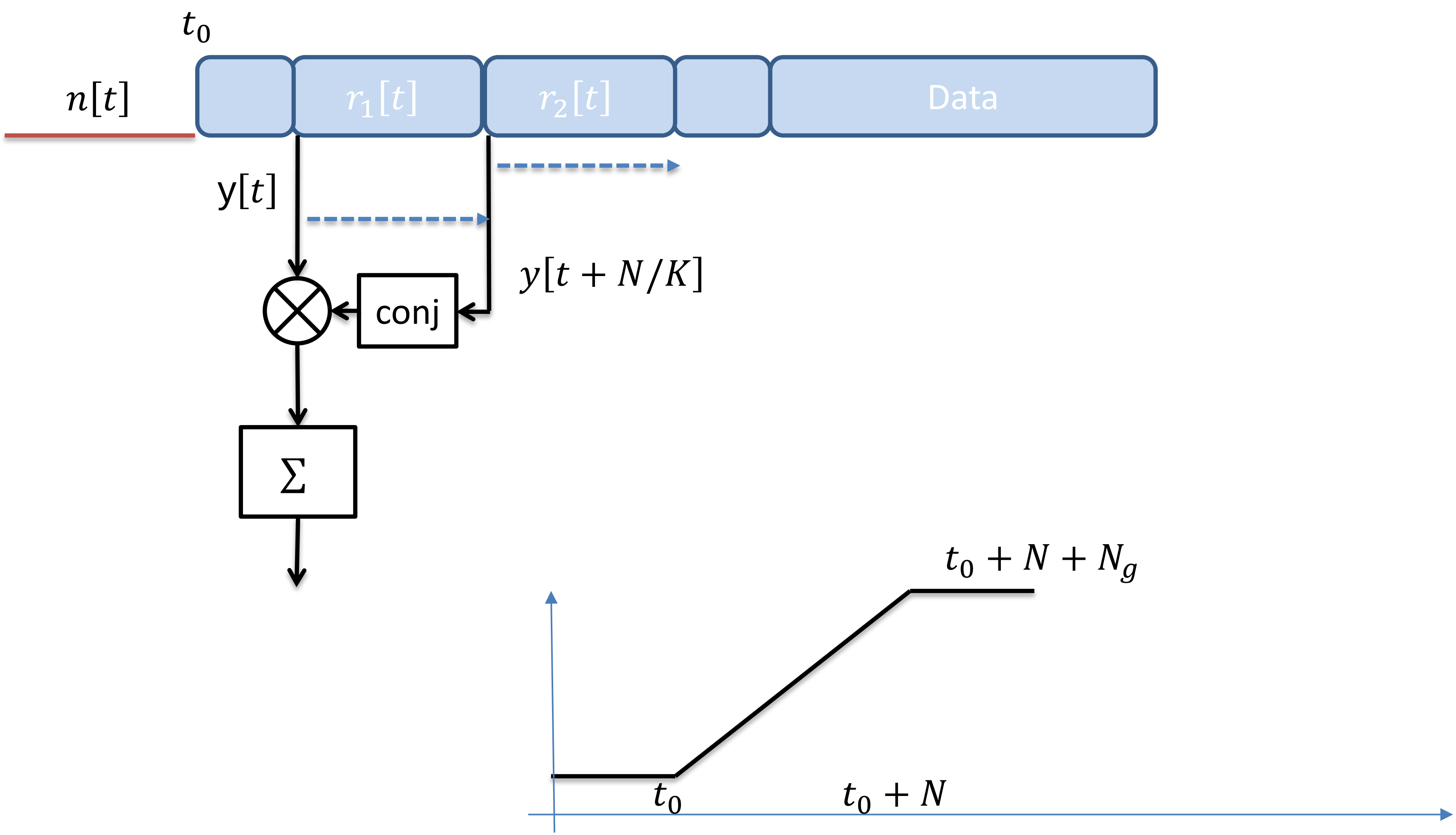

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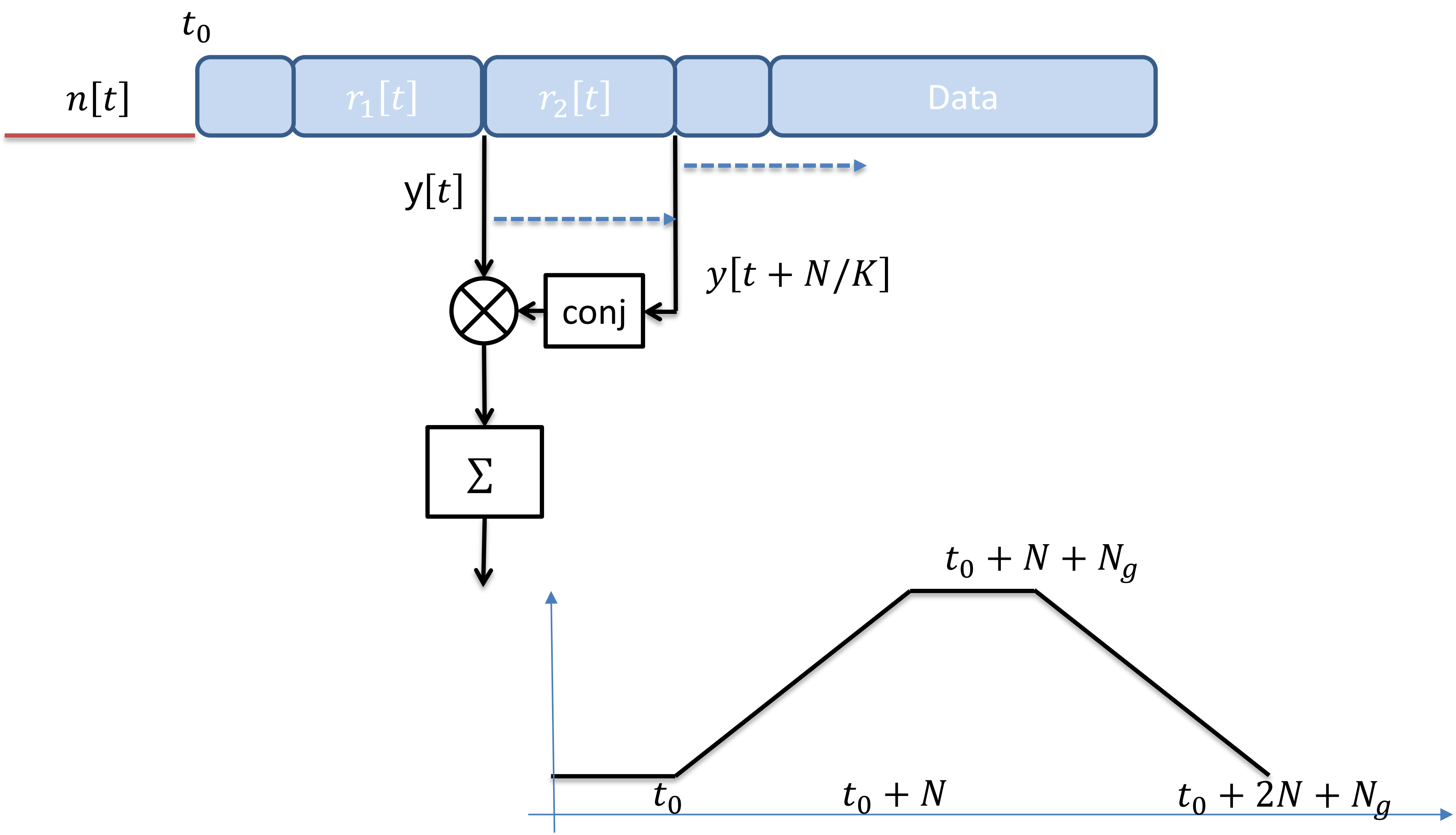

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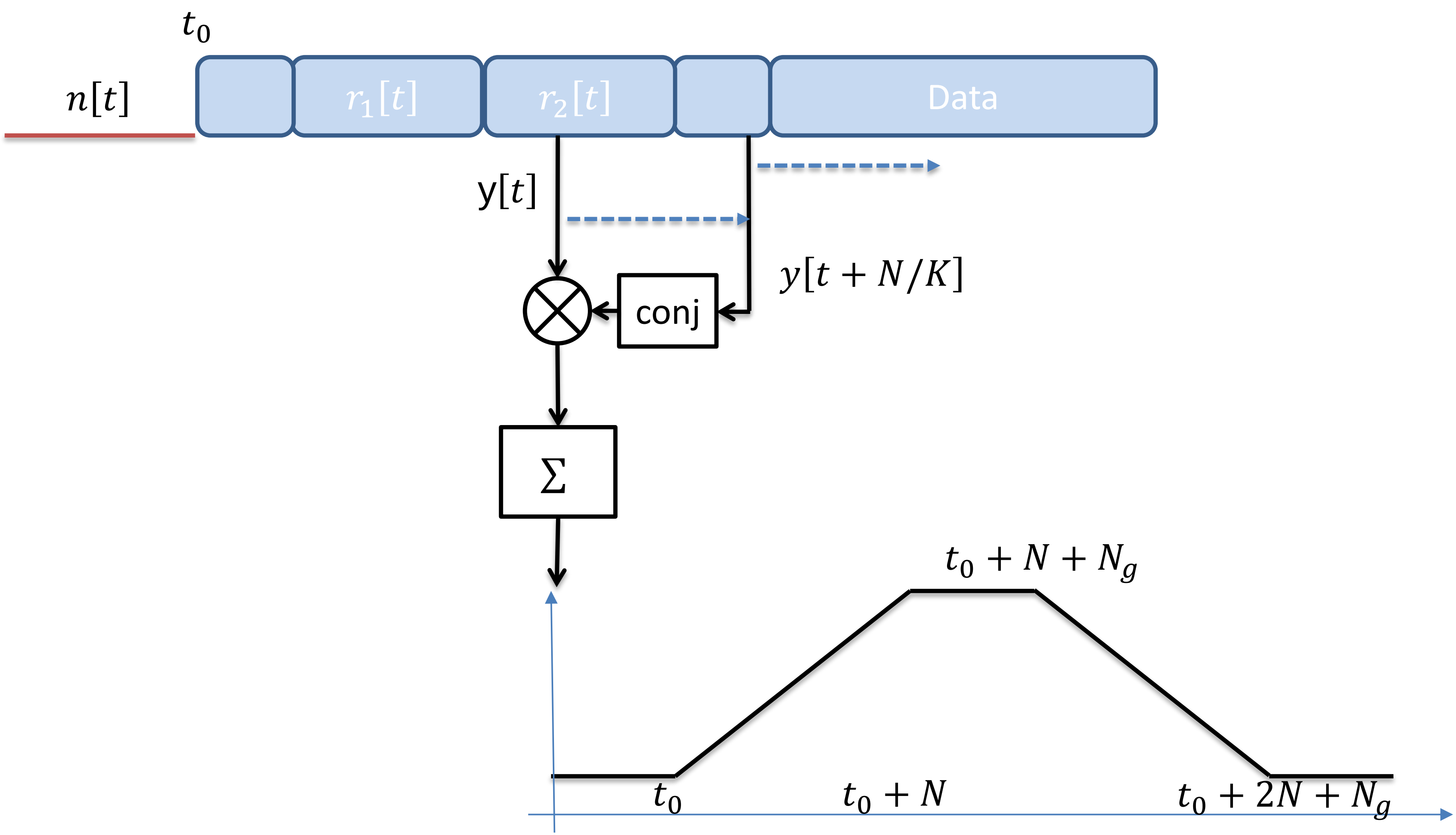

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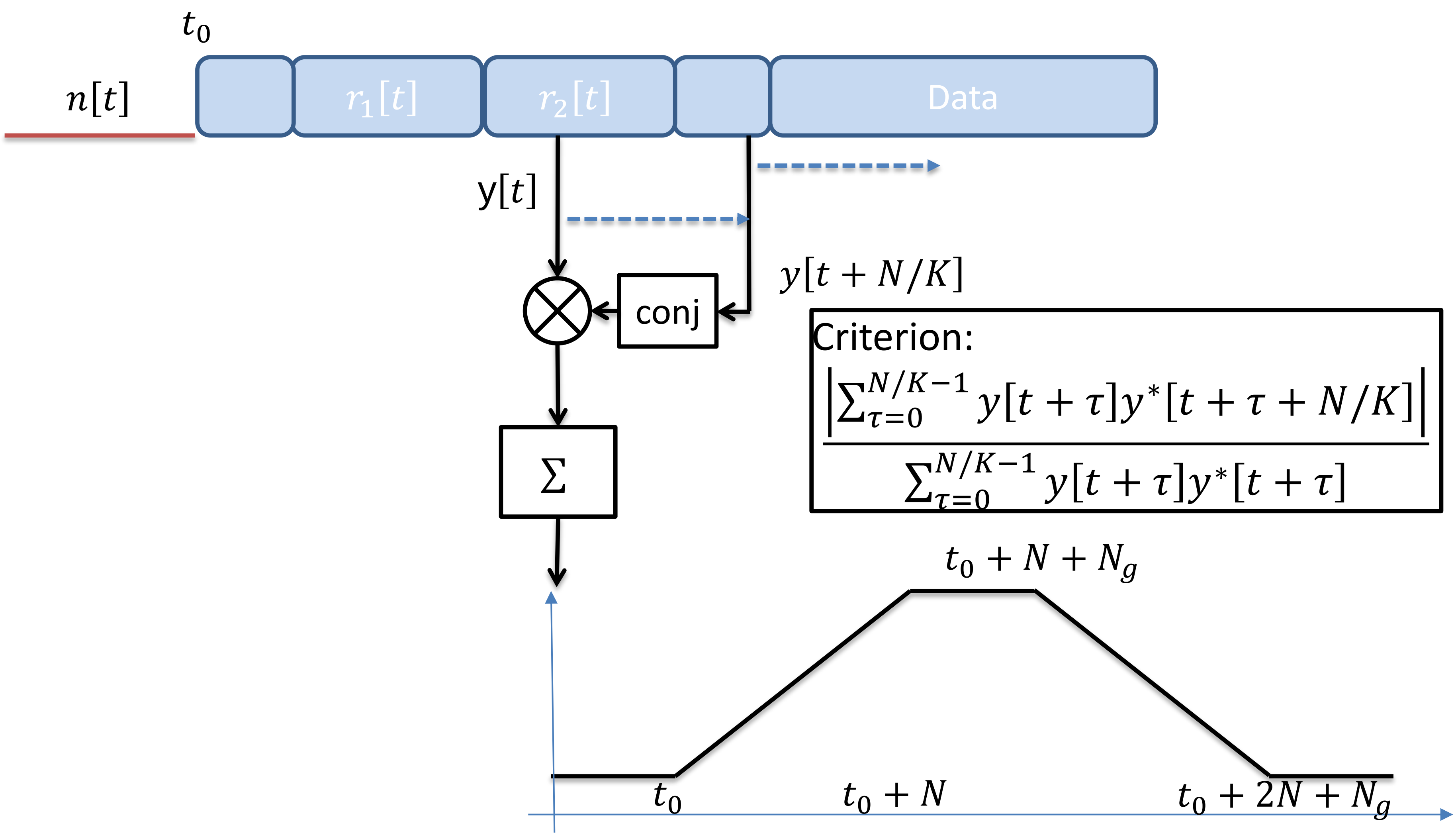
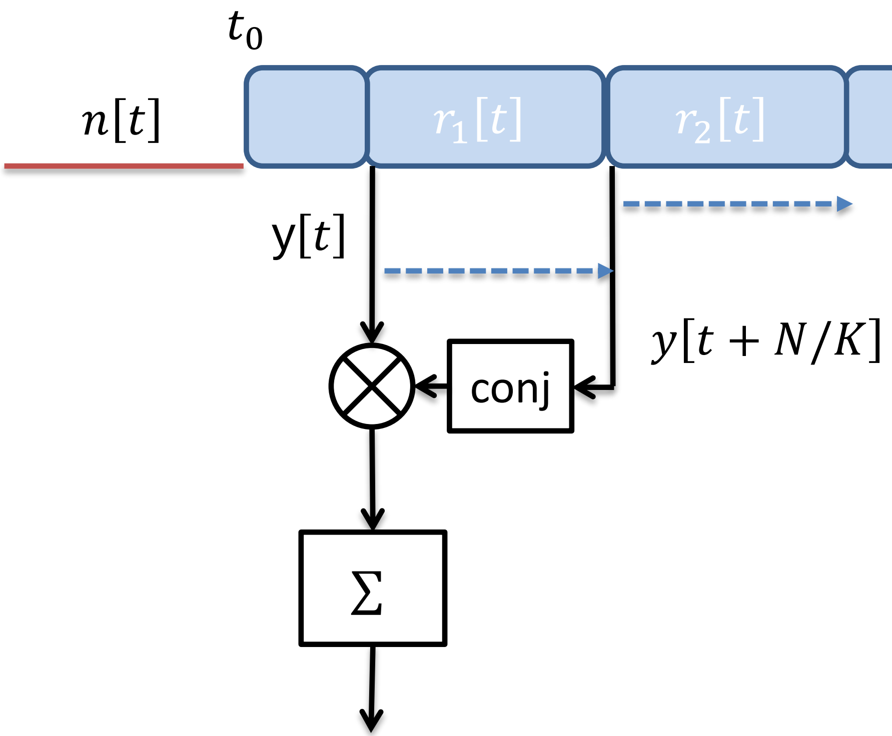


Criterion:

$$\frac{\left|\sum_{\tau=0}^{N/K-1} y[t+\tau]y^*[t+\tau+N/K]\right|}{\sum_{\tau=0}^{N/K-1} y[t+\tau]y^*[t+\tau]}$$

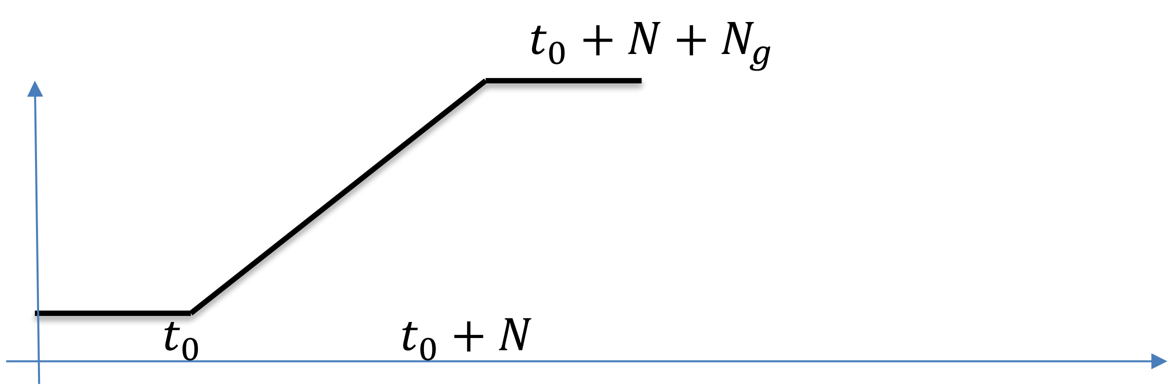
Synchronization with a Periodic Preamble

Robustness against frequency offset



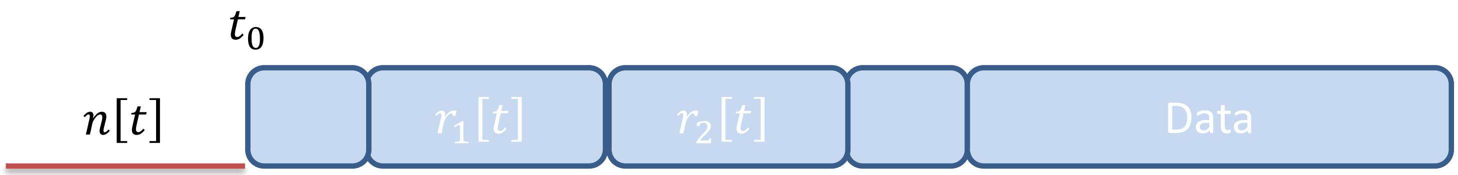
$$c_t = \sum_{\tau=0}^{N/K-1} e^{\frac{2\pi\Delta f t}{N}} \bar{r}[t] e^{-\frac{2\pi\Delta f t}{N} - \frac{2\pi\Delta f}{K}} \bar{r}^*[t] = \sum_{\tau=0}^{N/K-1} |\bar{r}[t]|^2 e^{-\frac{2\pi\Delta f}{K}}$$

Only a phase has no influence on magnitude



Freq. Offset Estimation with Periodic Preamble (OFDM)

Idea: use phase of the autocorrelation at $t_0 + N + N_g$ for freq. offset estimation



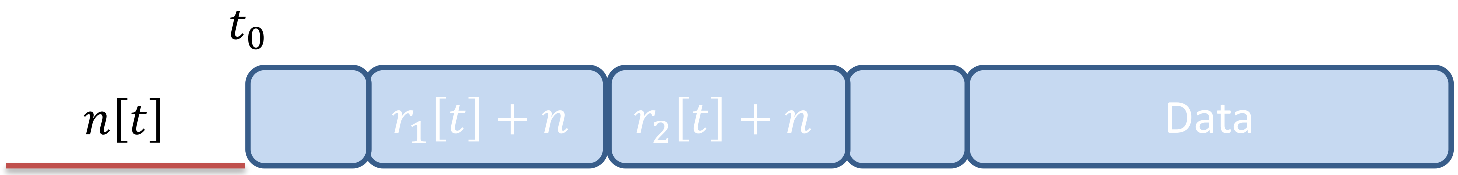
$$c_{t_0+N+N_g} = \sum_{\tau=0}^{N/K-1} |\bar{r}[t]|^2 e^{-\frac{2\pi\Delta f}{K}}$$

$$-2\pi\Delta f_0 K = \angle c_t$$

$$\Delta f_0 = -\frac{\angle c_t}{2\pi K}$$

SNR Estimation with Periodic Preamble (OFDM)

Introduce a preamble consisting of two identical OFDM training symbols



$$y[t + kN/K] = \bar{r}[t] + n[t + kN/K]$$

$$A = \frac{1}{N}\sum_{\tau=0}^{N-1} y[t]y^*[t + kN/K] = \frac{1}{N}\sum_{\tau=0}^{N-1} \bar{r}[t]\bar{r}^*[t] + n[t]n^*[t + kN/K] \approx$$
$$E\{|\bar{r}[t]|^2\} + \underbrace{E\{n[t]n^*[t + kN/K]\}}_{\to 0}$$

$$B = \frac{1}{N}\sum_{\tau=0}^{N-1} |y[t]|^2 \approx E\{|\bar{r}[t]|^2\} + E\{|n[t]|^2\}$$

$$SNR = \frac{E\{|\bar{r}[t]|^2\}}{E\{|n[t]|^2\}} = \frac{A}{B - A}$$

Residual carrier offset and phase noise

Signal after CFO compensation in time domain

- Assume phase $\Theta(t)$ changes only very slowly => constant within an OFDM symbol
- Does not cause inter carrier interference, but

$$y[t] = e^{j\Theta(t)} \sum_{k=0}^{\infty} s\left[t - k\left(N + N_g\right)\right] \approx \sum_{k=0}^{\infty} e^{j\Theta\left(k\left(N+N_g\right)\right)} s_k\left[t - k\left(N + N_g\right)\right]$$

Signal still rotates over time

Consider the freq. domain representation of the *k*th OFDM symbol

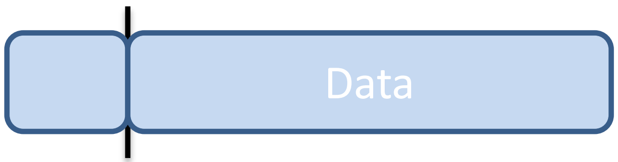
$$DFT\left\{e^{j\Theta\left(k\left(N+N_g\right)\right)} s_k[t]\right\} = e^{j\Theta\left(k\left(N+N_g\right)\right)} \mathcal{s}[f]\mathcal{h}[f]$$

- **All tones are affected equally by the frequency offset**
- **Solution: insert known Pilot tones at carrier f_p to estimate $\Theta\left(k\left(N + N_g\right)\right)$ as**

$$\Theta\left(k\left(N + N_g\right)\right) = \angle \mathcal{y}_{\boldsymbol{k}}[f_{\boldsymbol{p}}]\mathcal{p}^*[f]\mathcal{h}^*[f]$$

Timing offset in frequency domain

Sampling offset leads to a drift in the start of OFDM symbols over time



$$DFT\{y_1[t]\} = p[f]h[f]$$

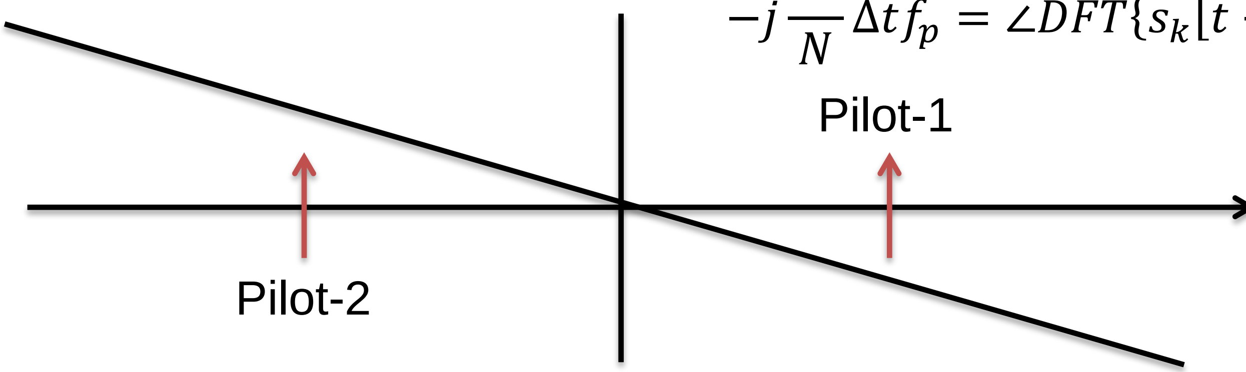
$$DFT\{y_k[t]\} = DFT\{s_k[t-\Delta t\,]\} = s[f]h[f]e^{-j\frac{2\pi}{N}\Delta t f}$$

Timing offset leads to a linear phase change over tones

Pilot tones @ f_p

$$y_k[f]h^*[f]p^*[f] = e^{-j\frac{2\pi}{N}\Delta t f_p}$$

$$-j\frac{2\pi}{N}\Delta t f_p = \angle DFT\{s_k[t-\Delta t\,]\}p^*[f_p]h^*[f_p]$$

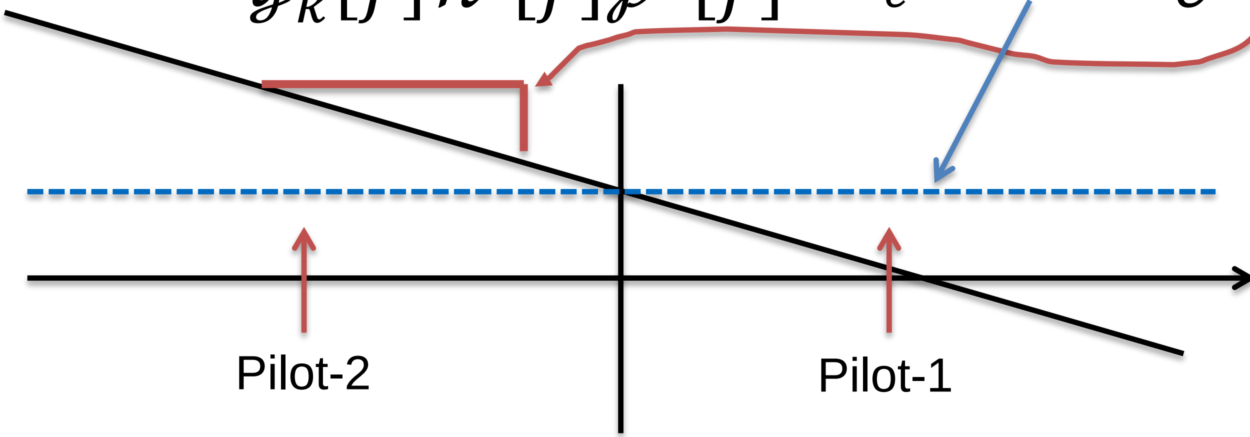


Timing offset and phase noise in frequency domain

Sampling offset leads to a drift in the start of OFDM symbols over time

Pilot tones @ f_p

$$\mathcal{y}_k[f]\mathcal{h}^*[f]\mathcal{p}^*[f] = e^{j\Theta\left(k\left(N+N_g\right)\right)}e^{-j\frac{2\pi}{N}\Delta t f_p}$$



$$-j\frac{2\pi}{N}\Delta t f_p + \Theta\left(k\left(N+N_g\right)\right) = \angle DFT\{s_k[t-{\scriptstyle \Delta t}]\}\mathcal{p}^*[f_p]\mathcal{h}^*[f_p]$$

- **Slope (across tones): timing offset; mean (across tones): phase noise**