Algorithms for Wireless Communications II

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Lecture 4

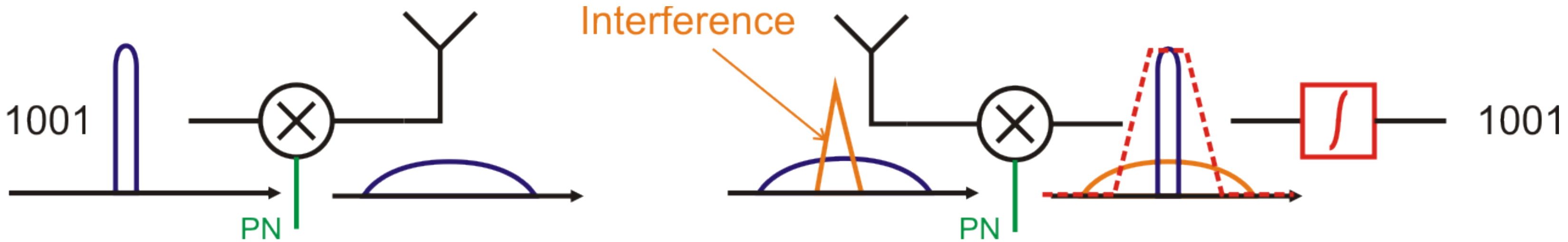
Direct Sequence Spread Spectrum Modulation (DSSS)

Spread Spectrum Modulation

- Assume we want to communicate with a symbol rate of T_s
- Minimum bandwidth requirement $B_s = 1/T_s$

Spread spectrum: Distribute the signal across a bandwidth that is larger than required

- Time domain view: transmit power and symbol duration remain unchanged
- Frequency domain view: power spectral density is reduced (same signal power is distributed over wider bandwidth)



Advantages

- Hide signal below the noise floor
- Avoid interference
- Provide easy multiple-access
- Exploit diversity from multipath propagation
- Improve accuracy of localization (GPS)

Flavors of Spread Spectrum Modulation

- Direct Sequence (DS)
  A carrier is modulated by a digital code sequence in which bit rate is much higher than the information signal bandwidth.

- Frequency Hopping (FH)
  A carrier frequency is shifted in discrete increments in a pattern dictated by a code sequence.

- Time Hopping (TH)
  Bursts of the carrier signal are initiated at times dictated by a code sequence.

- Hybrid Systems
  Use of combination of the above.

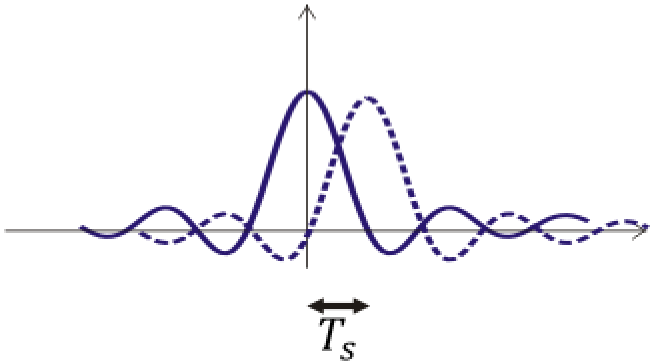
We consider only Direct Sequence Spread Spectrum (DSSS) modulation

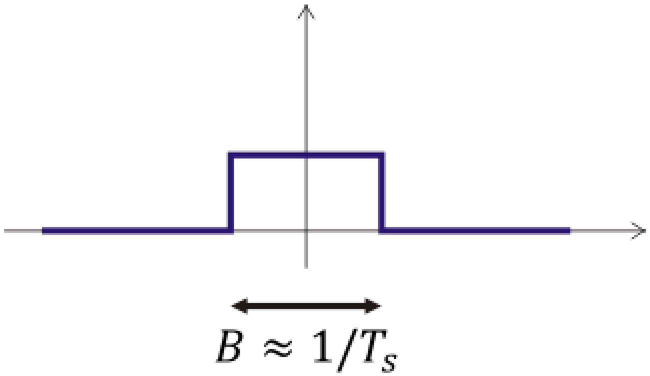
Signal Representation

Remember: Narrow-band modulation with symbol duration T_s

$$x(t) = \sum_{j=-\infty}^{+\infty} b_j g(t - jT_s)$$

- Each symbol is represented by the waveform of a (narrow band) pulse shaping filter
- Bandwidth of the pulse shape: $B \approx 1/T_s$



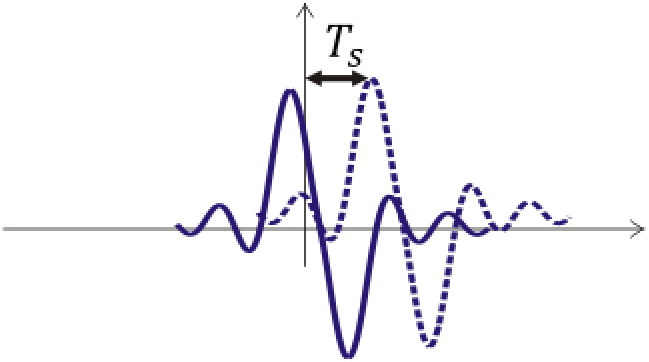


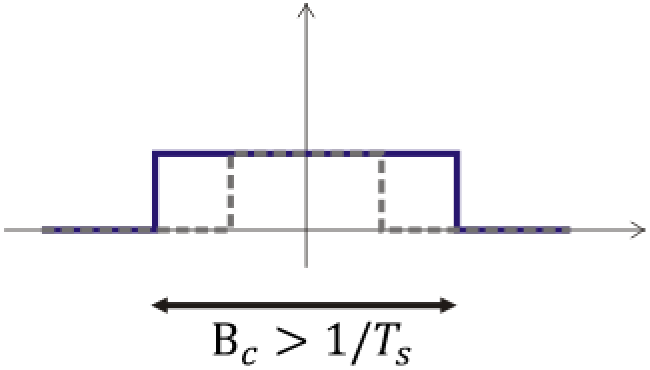
Spread spectrum modulation

- Replace the narrow band pulse shaping filter $g(t)$ with a signal waveform $c(t)$ with bandwidth B_c

$$x(t) = \sum_{j=-\infty}^{+\infty} b_j c(t - jT_s)$$

- For $\mathrm{B}_c > 1/T_s$ we get a bandwidth expansion
- Bandwidth expansion factor (spreading factor): $G = \mathrm{B}_c/B$





Implications of Increasing Bandwidth

What is the difference between the "narrow band" pulse shape and the "wideband" symbol waveform?

Consider the signal-space representation of $g(t)$ and $c(t)$

- Write as sum of orthogonal basis functions $\varphi_j(t)$

$$\mathrm{g}(t) = \sum_{j=1}^{N} g_j \varphi_j(t) \qquad\qquad c(t) = \sum_{j=1}^{N_c} c_j \varphi_j(t)$$

- **Signal space dimension**: number of orthogonal basis functions required to represent a signal
- Bandwidth and symbol duration define the signal space dimension

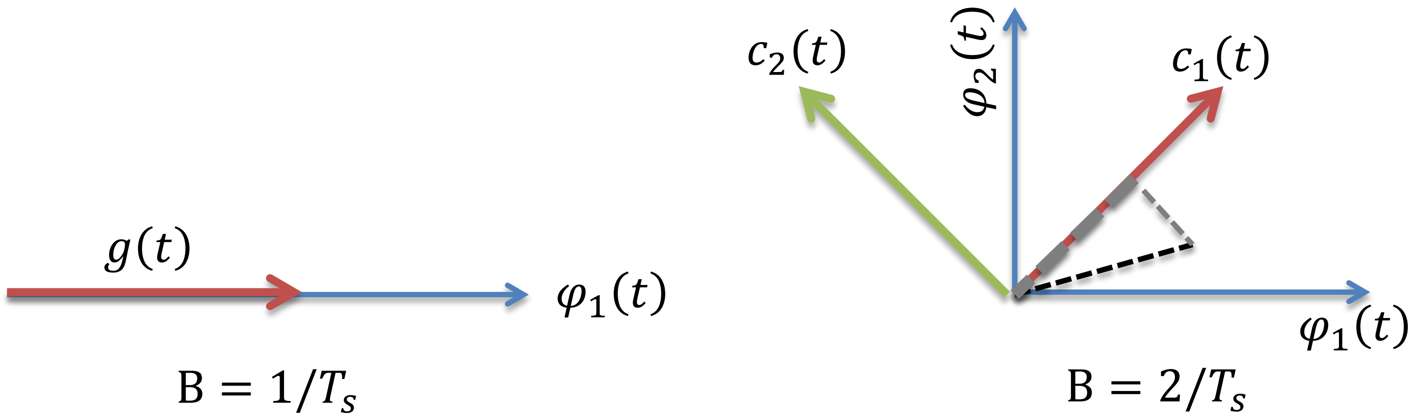
$$N \approx T_s B \qquad\qquad N_c \approx T_s B_c = GN$$

- Narrowband signal (pulse shape): $B \approx 1/T_s$ => $N = 1$
- Wideband signal (spreading waveform): $B_c \approx G/T_s$ => $N_c = G$

Spread spectrum modulation increases the dimension of the signal space

Signal Space Representation

Geometric interpretation



- Signal space dimensions can be for example: subsequent pulses, orthogonal carriers, …

- Wideband signals
  - Symbols can span more than one independent dimensions
  - Multiple spreading sequences are possible
  - Up to N_c spreading sequences are completely orthogonal

Direct Sequence Spread Spectrum (DSSS)

Basis functions for wideband signal waveform (spreading sequence)

- Time orthogonal Nyquist pulses with bandwidth $B_c = \mathrm{G}/T_s$

$$\varphi_i(t) = g_c(t - iT_c)$$

- Individual pulses are called **chips**
- Chip duration (chip rate) $T_c = \frac{T_s}{G} = 1/B_c$

Signal waveform is fully described by its G dimensional signal space representation at sampling rate G/T_s

$$\mathbf{c} = [c_1 \quad \cdots \quad c_G]$$

$$c(t) = \sum_{j=1}^{G} c_j g_c(t - jT_c)$$

Direct Sequence Spread Spectrum (DSSS)

Power normalization

- Before spreading, signal power was located in one signal dimension $g(t)$ and

$$\int_{-\infty}^{+\infty} g(t)\, g^*(t) dt = 1$$

- After spreading power is distributed over G orthogonal dimensions of the signal space $g_c(t - jT_c)$

$$\int_{-\infty}^{+\infty} c(t)\, c^*(t) dt = \sum_{j=1}^{G} \left|c_j\right|^2 = 1$$

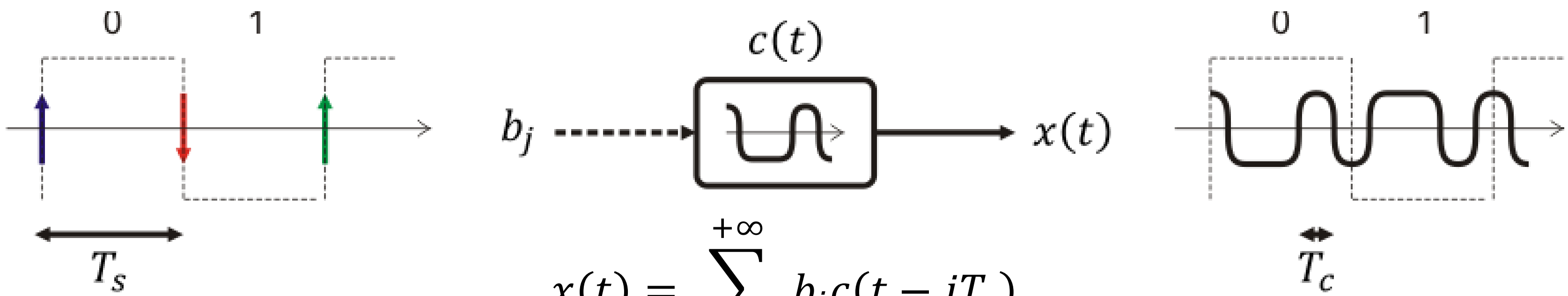
- To ensure that the spreading does not alter the signal power we set

$$\left|c_j\right|^2 = \frac{1}{G}$$

Often we choose $c_1 \in \left\{\pm \frac{1}{\sqrt{G}}\right\}$

DSSS Modulation (1)

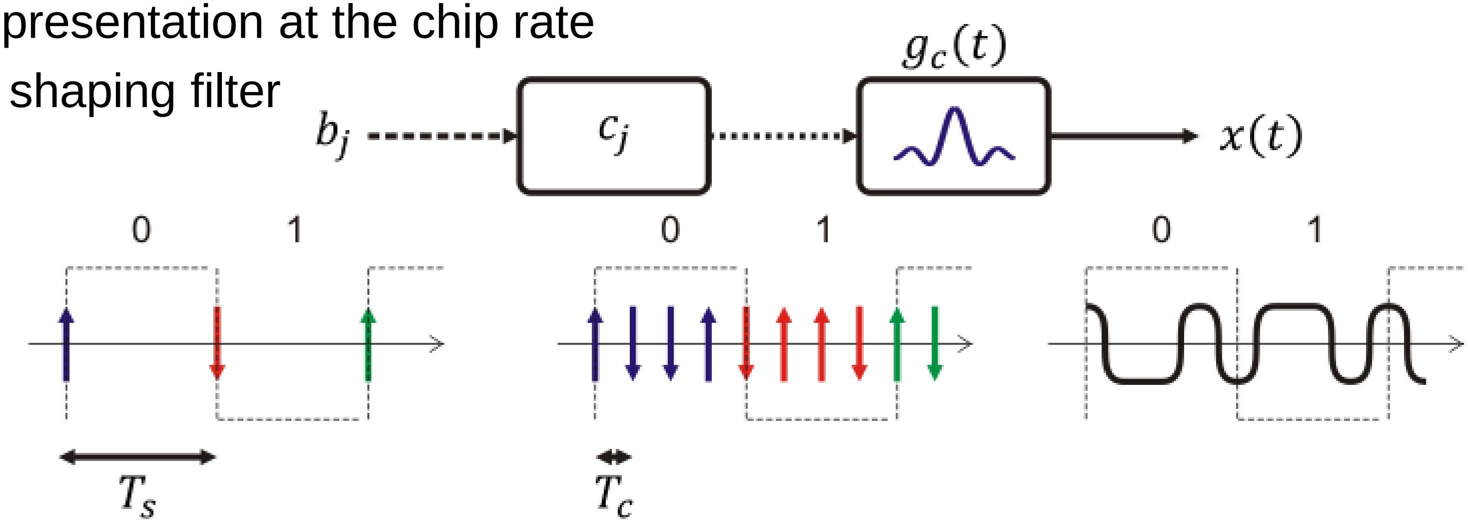
- Discrete-time signal b_j at symbol rate T_s used to excite a *modulation filter* with the impulse response of the chosen (cont. time) wideband signal waveform



$$x(t) = \sum_{j=-\infty}^{+\infty} b_j c(t - jT_s)$$

Modulation in two steps

- Signal space representation at the chip rate
- Chip-rate pulse shaping filter

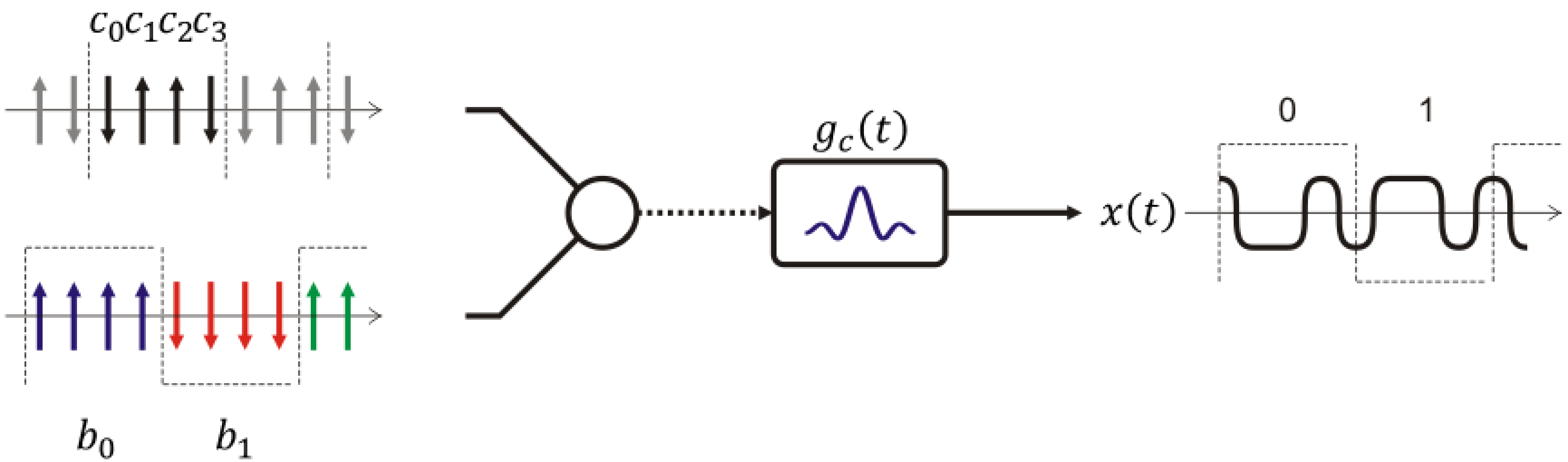


DSSS Modulation with Spreading

- Spreading = multiplication of a narrow band signal with another signal with a wider bandwidth

$$x(t) = \frac{1}{\sqrt{G}} \sum_{i=-\infty}^{+\infty} \sum_{j=1}^{G} b_i c_j g_c(t - jT_c - iT_s)$$

- Discrete time data signal is upsampled (repeated) to the chip-rate
- Multiplication with the discrete time chip sequence
- Pulse shaping with $g_c(t)$: controls the shape of the wideband spectrum



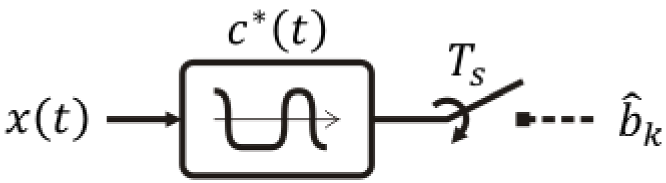
DSSS Receiver Structure

Optimum AWGN receiver for the k-th symbol

- **Matched Filter** with signal waveform

- Sampling of the filter output at $t = kT_s$ (assume perfect synchronization)

$$\hat{b}_k = \int_{-\infty}^{+\infty} y(t - kT_s)\, c^*(t) dt$$



DSSS Receiver Structure

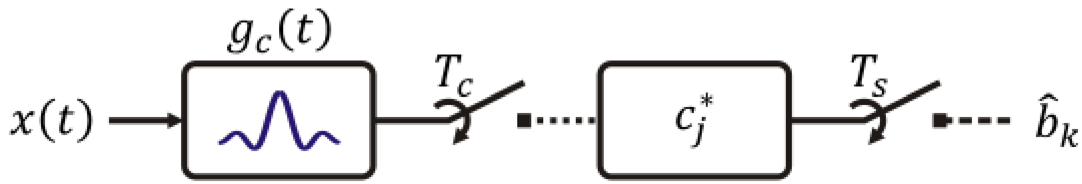
- Substituting $c^*(t)$ yields

$$\hat{b}_k = \int_{-\infty}^{+\infty} y(t-kT_s) \sum_{i=1}^{G} \frac{c_i^*}{\sqrt{G}} g_c^*(t-iT_c)\,dt = \sum_{i=1}^{G} \frac{c_i^*}{\sqrt{G}} \boxed{\int_{-\infty}^{+\infty} y(t-kT_s)\, g_c^*(t-iT_c)dt}$$

Chip matched filter with subsequent sampling at iT_c

Matched filter in two steps

1. Chip matched filter followed by sampling at the chip rate T_c (integral)
2. Signal-space matched filter with the discrete-time chip sequence



$$z_{ki} = \int_{-\infty}^{+\infty} y(t-kT_s)\, g_c^*(t-iT_c)dt$$

$$\hat{b}_k = \sum_{i=1}^{G} c_i^* z_{ki}$$

DSSS Receiver Structure

- For simplicity, consider only one symbol ($k = 1$) and drop the index k

$$z_i = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{G}} \sum_{j=1}^{G} bc_j g_c(t - jT_c)\, g_c^*(t - iT_c) dt$$

- Exchange sum and integral

$$z_i = \frac{1}{\sqrt{G}} \sum_{j=1}^{G} bc_j \int_{-\infty}^{+\infty} g_c(t - jT_c) g_c^*(t - iT_c)\, dt$$

- Introduce the pulse shape autocorrelation function

$$R_{gg}(\tau) = \int_{-\infty}^{+\infty} g_c(t) g_c^*(t - \tau)\, dt$$

- Note that $R_{gg}(0) = 1$ and $R_{gg}(kT_c) = 0$ for $k \neq 0$

After chip matched filtering we retrieve the transmitted discrete time signal

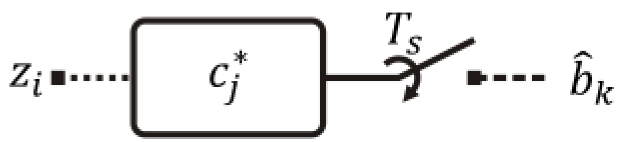
$$z_{\boldsymbol{i}} = \frac{1}{\sqrt{G}} bc_{\boldsymbol{i}}$$

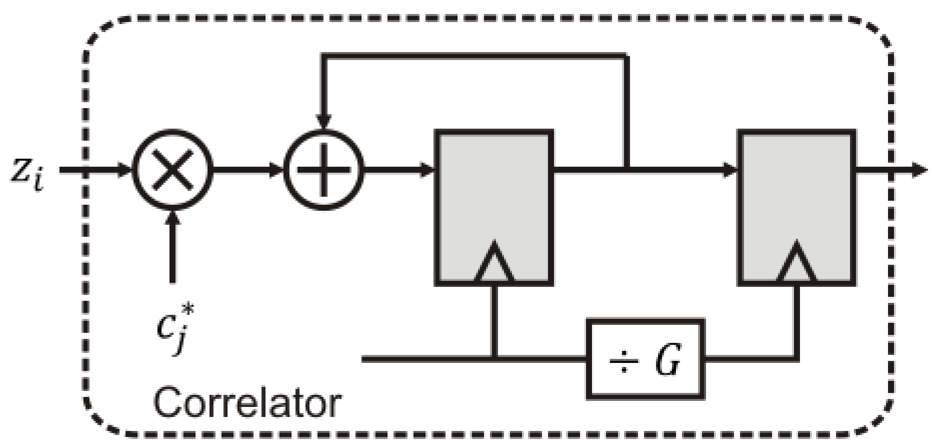
DSSS Receiver Structure (Chip Matched Filter)

Matched filtering (with the signal space representation) followed by down-sampling by a factor of G (T_c to T_s)

- Entails many unnecessary computations
- Corresponds to computing the correlation

Correlation can be implemented with low complexity using a multiply-accumulate operation (1 Multiplier, 1 adder, a register)





Performance of DSSS

We use the discrete time signal model (chip spaced sampling)

- Consider only the transmission of a single symbol b

$$y_i = bc_i + n_i + I_i$$

- c_i : Signal space representation of the spreading waveform $E_s = \sum_{i=1}^{G} |c_i|^2$
- n_i : Additive white Gaussian noise with PSD N_0, Hence, $n_i : \mathcal{CN}(0, N_0)$
- I_i : Interferer with constant power $E_I = \sum_{i=1}^{G} |I_i|^2$

Despreading (Signal-space matched filter)

$$\hat{b} = \sum_{i=1}^{G} c_i^* y_i = b \sum_{i=1}^{G} c_i^* c_i + \sum_{i=1}^{G} c_i^* n_i + \sum_{i=1}^{G} c_i^* I_i$$

- Signal, noise and interference power after despreading:

$$\overline{E_s} = E_s \qquad \mathcal{CN}(0, N_0) \qquad \overline{E}_I = E_I / G$$

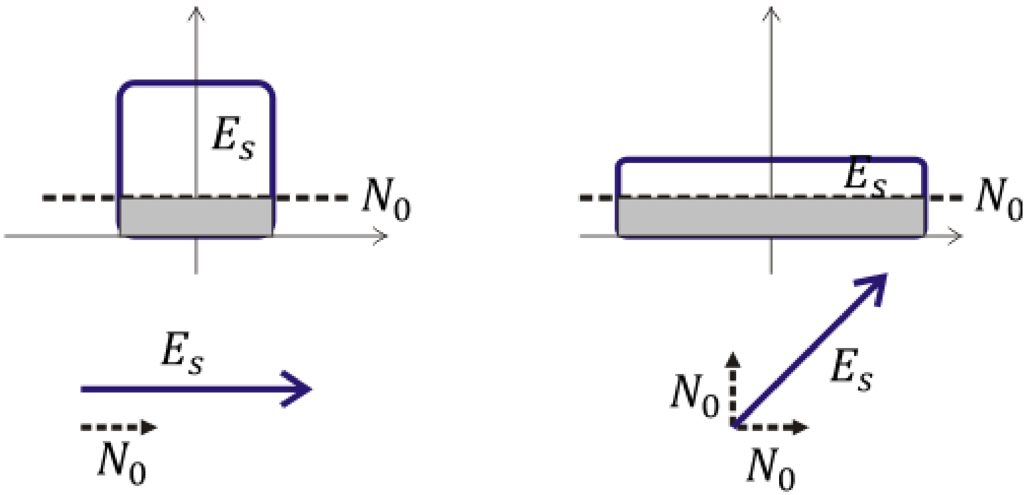
Performance of DSSS

Impact of spreading on SNR (ratio of signal energy to noise PSD)

- AWGN has constant power across all frequencies and signal dimensions
- AWGN can not be avoided by escaping into other dimensions of the signal space

- The total noise power increases with increasing bandwidth (dimensions)
- However, at least the amount of noise in direction of the signal remains the same

$$SNR = E_s/N_0 \qquad => \qquad \overline{SNR} = \frac{\overline{E_s}}{N_0} = E_s/N_0$$



Spreading factor does not influence SNR (Es/N0)

Performance of DSSS

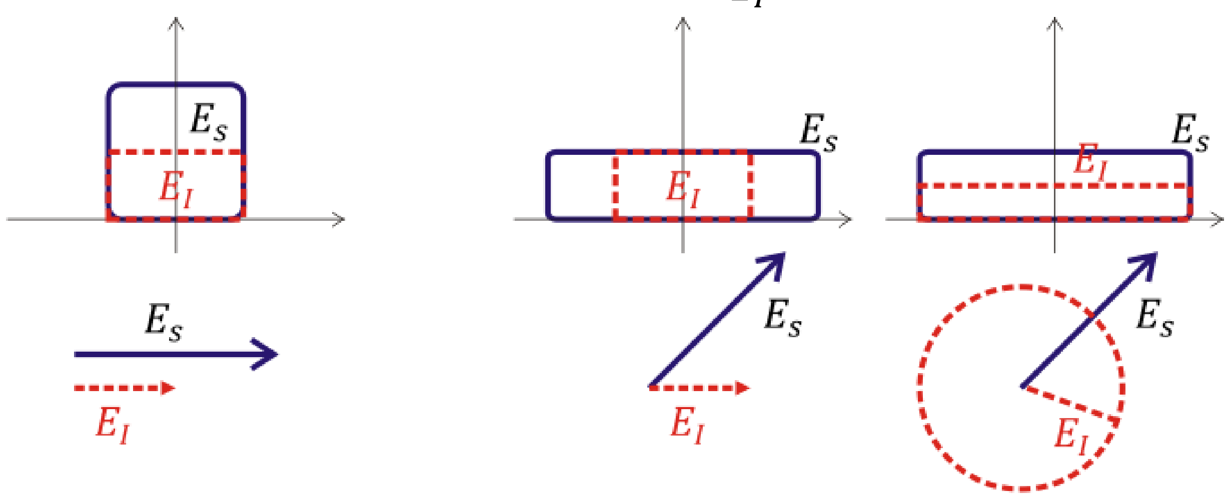
Impact of spreading on SIR (ratio of signal energy to interferer energy)

- Interferer has given bandwidth and energy E_I
- When increasing bandwidth (adding dimension) of the signal, interference lies only in a particular direction (subspace) of the signal space

- Spreading the signal across all dimensions (frequencies) reduces the component of the interference in the direction of the signal (better SIR)

Processing gain

$$SIR = E_s/E_I \qquad => \qquad \overline{SIR} = \frac{\overline{E_s}}{\overline{E_I}} = E_s{\color{red}G}/E_I$$

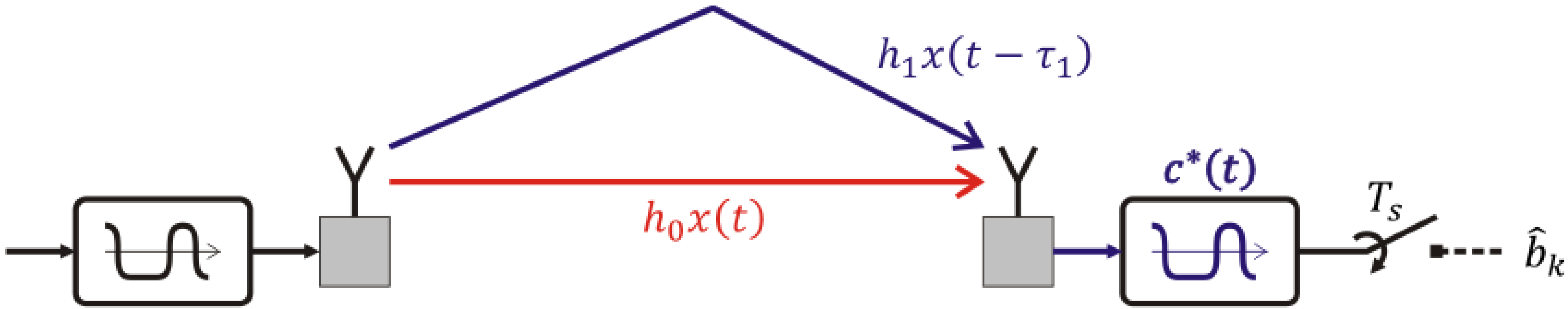


SIR improves with the increasing spreading factor

DSSS with Multipath Channels

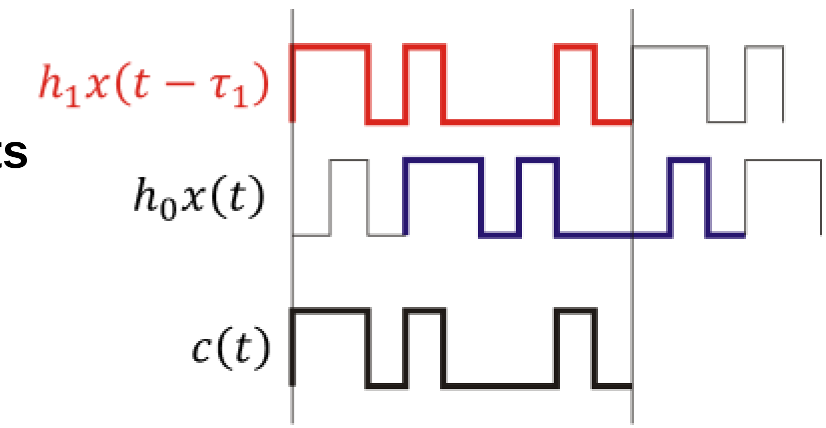
For simplicity consider a model with two paths

- AWGN receiver considers only the first arriving path and correlates with $c^*(t)$



Despreading with the spreading sequence as for AWGN yields three signal components

- Desired signal
- Intersymbol interference (ISI)
- Self interference



$$\widehat{b_k} = b_k h_0 \int_0^{T_s} c(t)\, c^*(t) dt + b_{k-1} \int_0^{\tau} h_1 c(T-\tau+t)\, c^*(t) dt + b_k \int_{\tau}^{T_s} h_1 c(t-\tau)\, c^*(t) dt$$

DSSS with Multipath Channels

Assumption: Delay spread is much smaller than the symbol duration ($T_s \gg \tau$), though usually greater than the chip duration ($T_c < \tau$)

$$\widehat{b_k} = b_k h_0 \int_0^{T_s} c(t)\, c^*(t) dt + \underbrace{\cancel{b_{k-1} \int_0^{\tau} h_1 c(T - \tau + t)\, c^*(t) dt}}_{\to 0} + b_k \int_{0}^{T_s} h_1 c(t - \tau)\, c^*(t) dt$$

- ISI becomes negligible
- Self interference can be approximated by the autocorrelation function

- With the autocorrelation function of the spreading sequence

$$R_{cc}(0) = 1$$

$$R_{cc}(\tau) = \int_{-\infty}^{+\infty} \boldsymbol{c}(t) \boldsymbol{c}^*(t - \tau)\, dt$$

we obtain

$$\widehat{b_k} = h_0 b_k + h_1 b_k R_{cc}(\tau)$$

Performance of DSSS in Multipath with AWGN

With AWGN (PSD: N_0) the signal after despreading (2-path model) is

$$\widehat{b_k} = h_0 b_k + h_1 b_k R_{\boldsymbol{cc}}(\tau) + n'$$

where n': $\mathcal{CN}(0, N_0)$

For a general L-path model we obtain

$$\widehat{b_k} = h_0 b_k + \sum_{i=1}^{L} h_i b_k R_{cc}(\tau_i) + n'$$

Performance is determined by the Signal to Interference plus Noise power

$$SINR = \frac{|h_0|^2}{\left|\sum_{i=1}^{L} h_i b_k R_{cc}(\tau_i)\right|^2 + N_0}$$

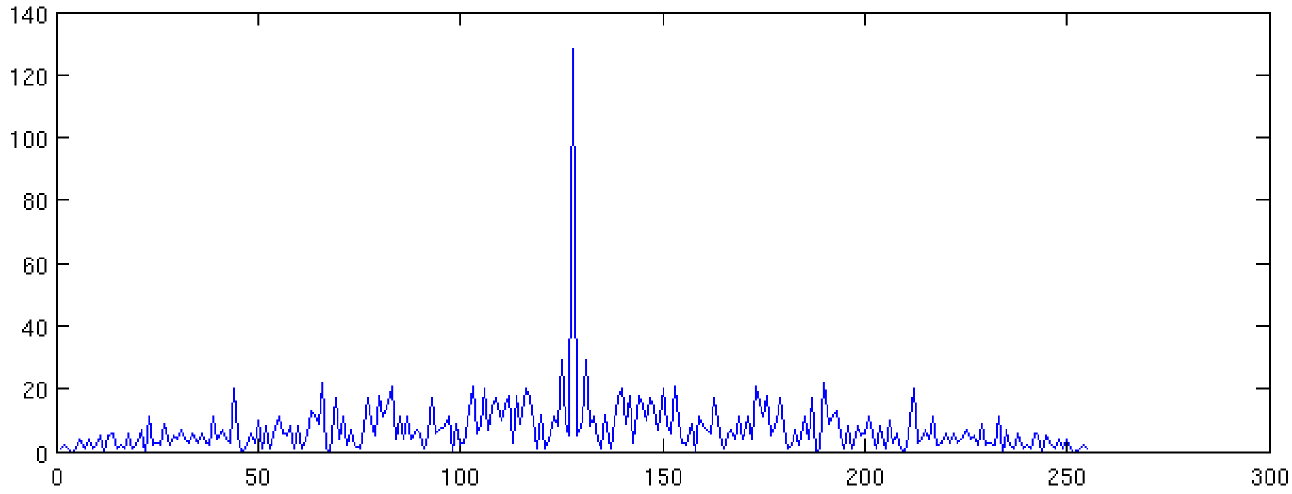
The autocorrelation properties of the spreading sequence determine the interference from the multipath component

DSSS with Multipath Channels

Autocorrelation functions of spreading sequences

$$c(t) = \sum_{j=1}^{G} c_j g_c(t - jT_c)$$

- Power normalization $R_{\boldsymbol{cc}}(0) = 1$ by design
- $R_{\boldsymbol{cc}}(\tau)$ is symmetric



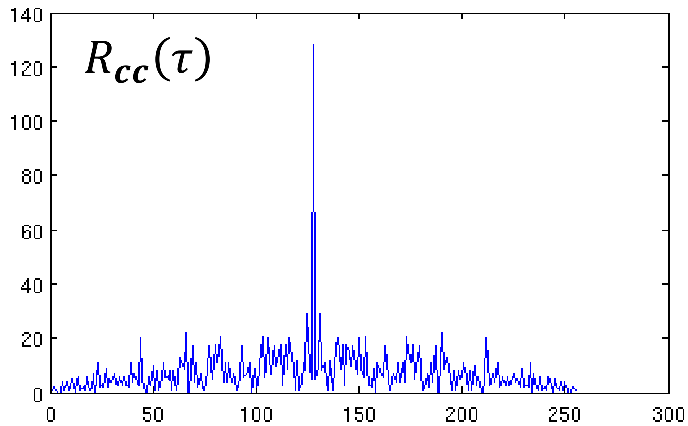
$R_{\boldsymbol{cc}}(\tau \neq 0)$ **depends on** c_j **and on the AKF of the pulse shape** $R_{gg}(\tau)$

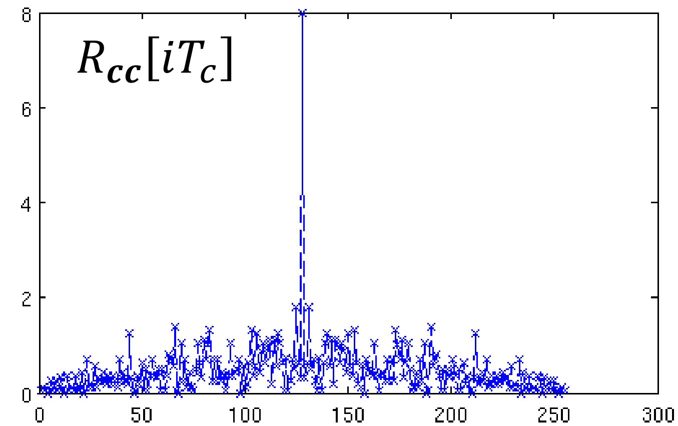
DSSS with Multipath Channels

- The magnitude (energy) in the sidelobes decreases as G increases ($\propto 1/G$)
- For $\tau < T_c$, $R_{\boldsymbol{cc}}(\tau \neq 0)$ follows closely the AKF of the chip matched filter $R_{gg}(\tau)$
- At the sampling points, i.e.,$\tau = iT_c$, $R_{\boldsymbol{cc}}(iT_c)$ is defined by the AKF of the chip sequence

$$R_{\boldsymbol{cc}}(iT_c) = R_{\boldsymbol{cc}}[iT_c] = \sum_{j=1}^{G} c_j c_{j-i}^*$$

- Away from $\tau = 0$, resembles closely the AKF of the discrete chip sequence





Improving the AKF properties of the signal space spreading waveform improves the AKF of the continuous time AKF and thus reduces interference

Matched Filter for Multipath Channels

Consider an arbitrary multipath channel

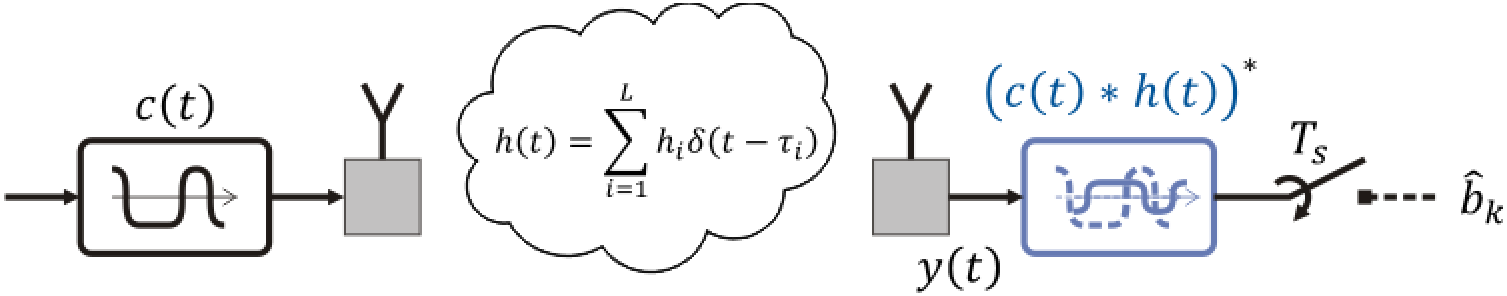
$$h(t) = \sum_{i=1}^{L} h_i \delta(t - \tau_i)$$

with the received signal

$$y(t) = x(t) * h(t) + n(t)$$

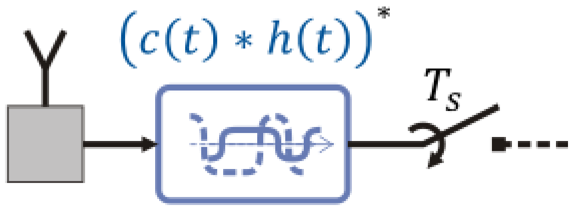
Matched filter (optimum in a sense of maximizing SNR)

$$y(t) * \left(c(t) * h(t)\right)^* = y(t) * \int c^*(t') \sum_{i=1}^{L} h_i^* \delta(t - t' - \tau_i) dt' = y(t) * \sum_{i=1}^{L} h_i^* c^*(t - \tau_i)$$



The RAKE Receiver

Rearranging the equations of the multipath matched filter for implementation



$$y(t) * \left(c(t) * h(t)\right)^* = y(t) * \int c^*(t') \sum_{i=1}^{L} h_i^* \delta(t - t' - \tau_i) dt'$$

- After exchanging summation and the integral we get

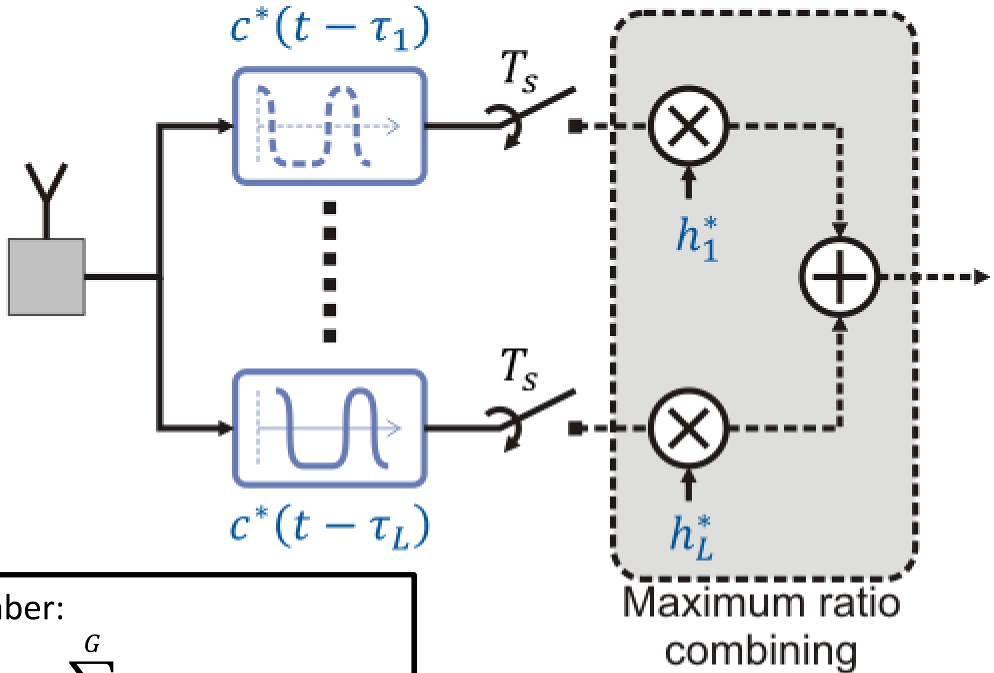
$$y(t) * \sum_{i=1}^{L} h_i^* c^*(t - \tau_i) = \sum_{i=1}^{L} h_i^* y(t) * c^*(t - \tau_i)$$

Instead of convolving the received signal with the superpositions of the delayed and weighted spreading sequences we can

Extracting a suitable architecture => RAKE

1. Filter with delayed versions of the reference sequence
2. Combine results after weighting with the channel gains **(Maximum Ratio Combining)**

$$\sum_{i=1}^{L} h_i^* y(t) * c^*(t-\tau_i)$$



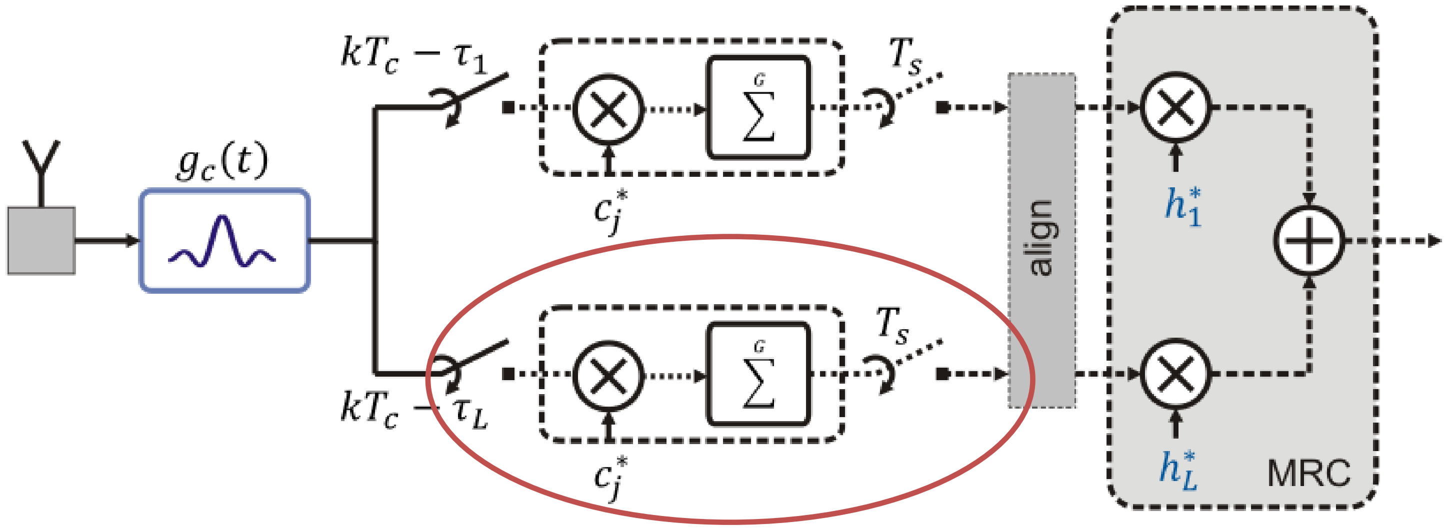
Remember:

$$c(t) = \sum_{j=1}^{G} c_j g_c(t - jT_c)$$

Further simplifications

- Replace the matched filter with a chip matched filter followed by a correlator
- Use only a single matched filter whose output is sampled at appropriate times

The RAKE Receiver



Rake finger

- Each RAKE finger needs to sample the signal at the corresponding delay τ_i
- If all delays are chip-spaced ($\tau_i = k_i T_c$), sampling at different delays is easy, if delays are not sample spaced (usually this is not the case), interpolation must be performed for each RAKE finger
- Often, for complexity reduction integer delays are assumed
- Correlator outputs must be time-aligned before MRC

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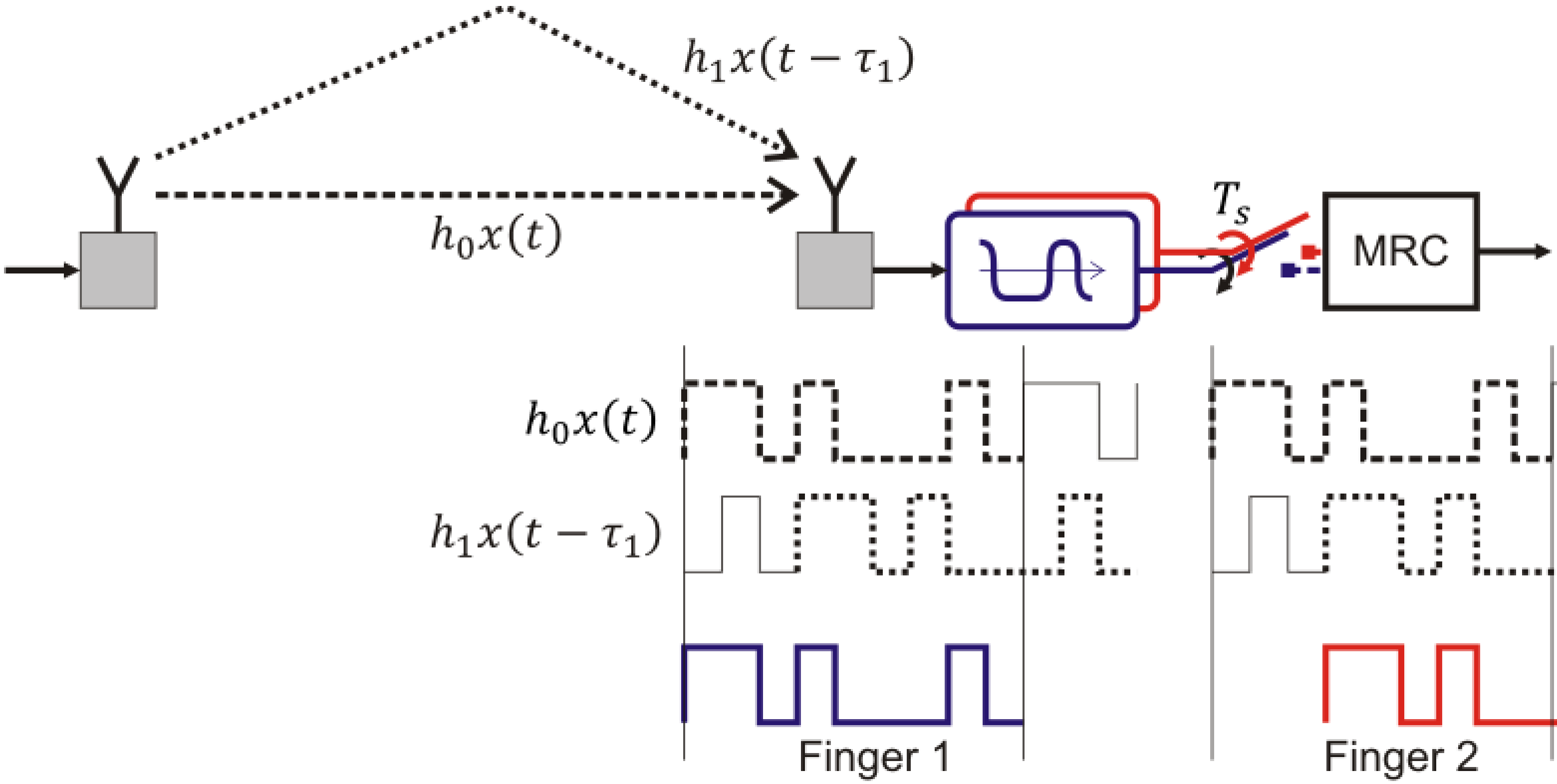
Performance of the RAKE Receiver

Consider again a system with a 2-path channel *and* AWGN



$$\widehat{b_k} = h_0^* y(t) * c^*(t)\Big|_{t=kT_s} + h_1^* y(t) * c^*(t-\tau_1)\Big|_{t=kT_s+\tau_1}$$

Performance of the RAKE Receiver

- Simplifying assumption: neglect ISI for analysis

$$\widehat{b_k} = |h_0|^2 b_k + h_0^* h_1 b_k R_{cc}(\tau_1) + h_0^* n_0' + |h_1|^2 b_k + h_1^* h_0 b_k R_{cc}(-\tau_1) + h_1^* n_1'$$

- Useful signal: $|h_0|^2 b_k + |h_1|^2 b_k = \underbrace{\left(|\boldsymbol{h_0}|^2 + |\boldsymbol{h_1}|^2\right)}_{\textbf{Effective channel}} b_k$

- Interference from multipath: $h_0^* h_1 b_k R_{cc}(\tau_1) + h_1^* h_0 b_k R_{cc}(-\tau_1)$
- Noise: $h_0^* n_0' + h_1^* n_1'$

For a general L-path model and a RAKE with L fingers we obtain

$$\widehat{b_k} = \underbrace{b_k \sum_{i=1}^{L} |h_i|^2}_{\text{Signal}} + \underbrace{b_k \sum_{i=1}^{L} \sum_{j \neq i}^{L} h_i^* h_j R_{cc}(\tau_j - \tau_i)}_{\text{Interference}} + \underbrace{\sum_{i=1}^{L} h_{\boldsymbol{i}}^* n_i'}_{\text{Noise}}$$

Performance of the RAKE Receiver

What is the impact of using a RAKE on the SINR?

- Replace the interference term with $I = b_k \sum_{i=1}^{L} \sum_{j \neq i}^{L} h_i^* h_j R_{cc}(\tau_j - \tau_i)$

$$\widehat{b_k} = b_k \sum_{i=1}^{L} |h_i|^2 + I + \sum_{i=1}^{L} h_{\boldsymbol{i}}^* n_i'$$

Compute the signal to interference plus noise ratio

$$SINR = \frac{\left(\sum_{i=1}^{L} |h_i|^2\right)^2}{|I|^2 + N_0 \sum_{i=1}^{L} \left|h_{\boldsymbol{i}}^*\right|^2}$$

- Consider only the SNR (neglect the interference term)
  - Valid on the low-SNR regime, where AWGN dominates or for large spreading factors

Observation

- Signal power grows with the squared sum of the channel gains
- Noise power only grows with the sum of the squared channel gains

Performance of the RAKE Receiver

Example: All channel gains are equal ($h_1 = h_2 = \cdots = h_L = h$)

- Signal power: $\left(\sum_{i=1}^{L} |h_i|^2\right)^2 = (L|h|^2)^2 = L^2|h|^4$
- Noise power: $N_0 \sum_{i=1}^{L} |h_{\boldsymbol{i}}^*|^2 = N_0 L|h|^2$

$$SNR = \frac{L^2|h|^4}{N_0 L|h|^2} = \frac{L|h|^2}{N_0}$$

Signal to noise ratio improves with increasing number of fingers
(of course only for fingers associated with a new tap with non-zero gain)

Impact of the RAKE receiver in a fading multipath channel

- Effective channel is given by $\sum_{i=1}^{L} |h_i|^2$ (combination of all paths)
- Intuition: if one path is in a fade, the effective channel is still good, since another path (picked up by another finger) is likely to be still good

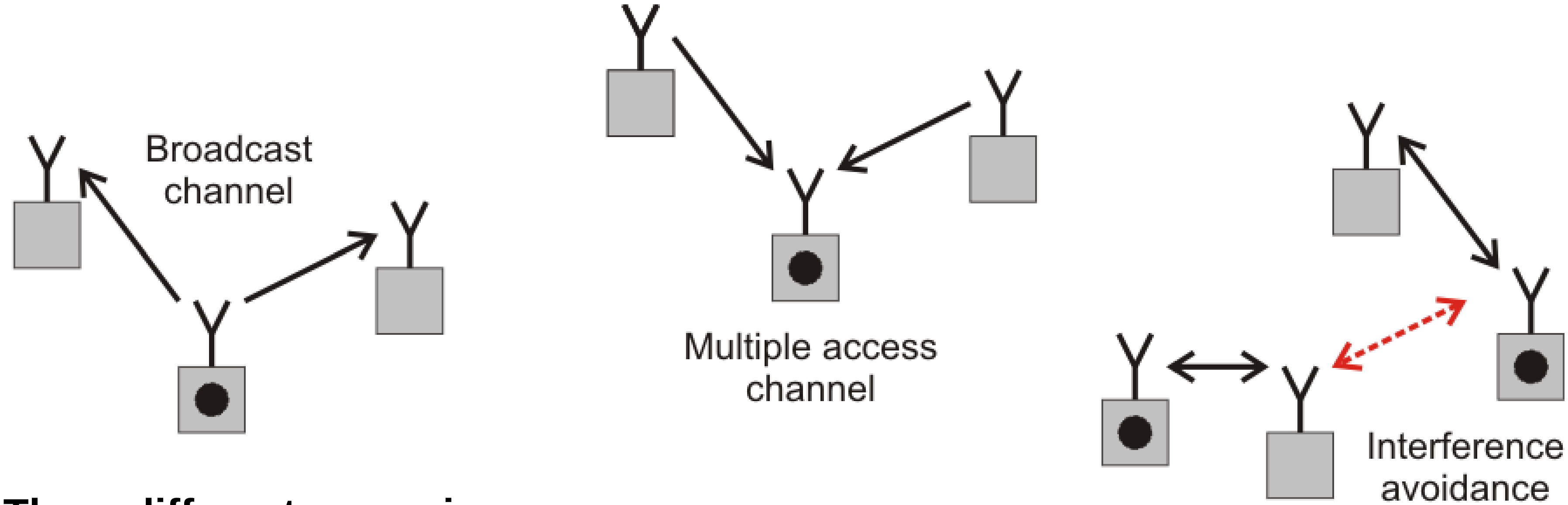
- Mathematically: if h_i is Gaussian, $\sum_{i=1}^{L} |h_i|^2$ has Chi-square distribution of degree L

Code Division Multiple Access (CDMA)

Multiple Access Scenarios

- Electromagnetic spectrum is a shared resource

Multiplexing channel access and channel utilization across different devices



Three different scenarios:

1. Broadcast channel (downlink): a common source (e.g., base station) sends to multiple terminals at the same time (but different data)
2. Multiple-access channel (uplink): different devices send to the same destination
3. Interference channel: different device pairs communicate in the same channel and interfere with each other (insufficient spatial separation)

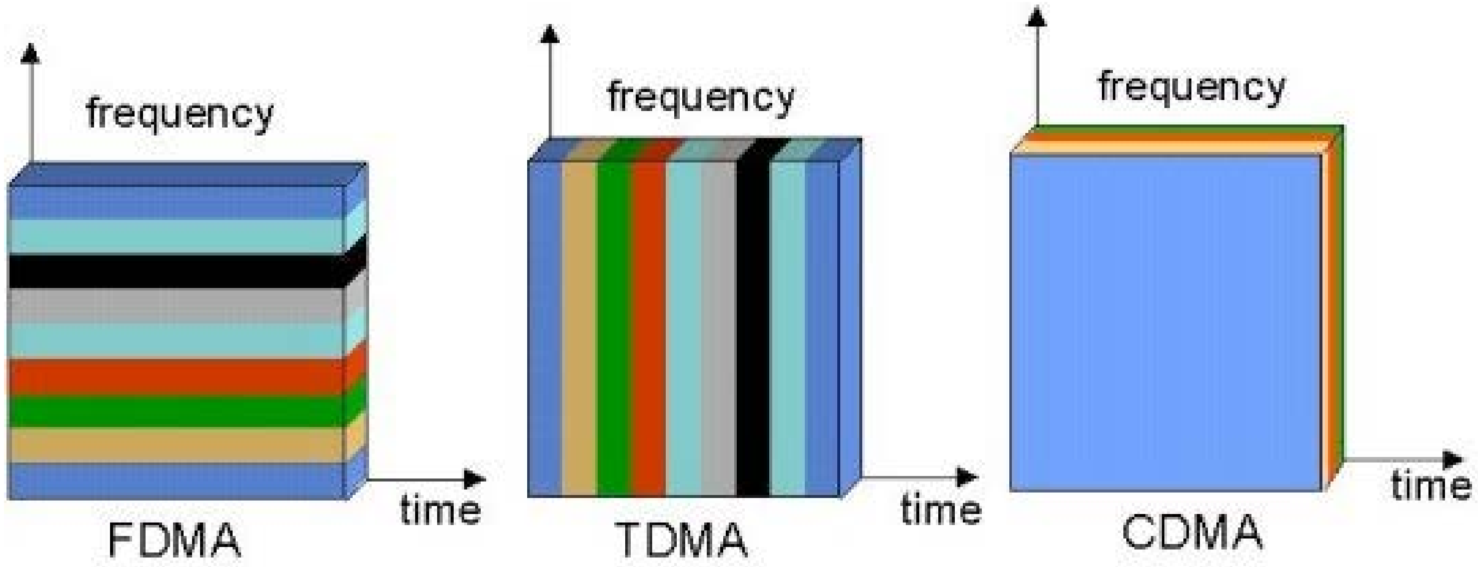
Multiplexing Strategies

How can we multiplex different data streams / communication links?

- Multiplexing (ideal) = putting signals into different (orthogonal) dimensions of the signal space

Options for multiplexing

- FDMA: orthogonalize in frequency
- TDMA: orthogonalize in time
- **CDMA**: orthogonalize in signal/code space (i.e., all users use all time and frequency slots, yet with orthogonal waveforms)

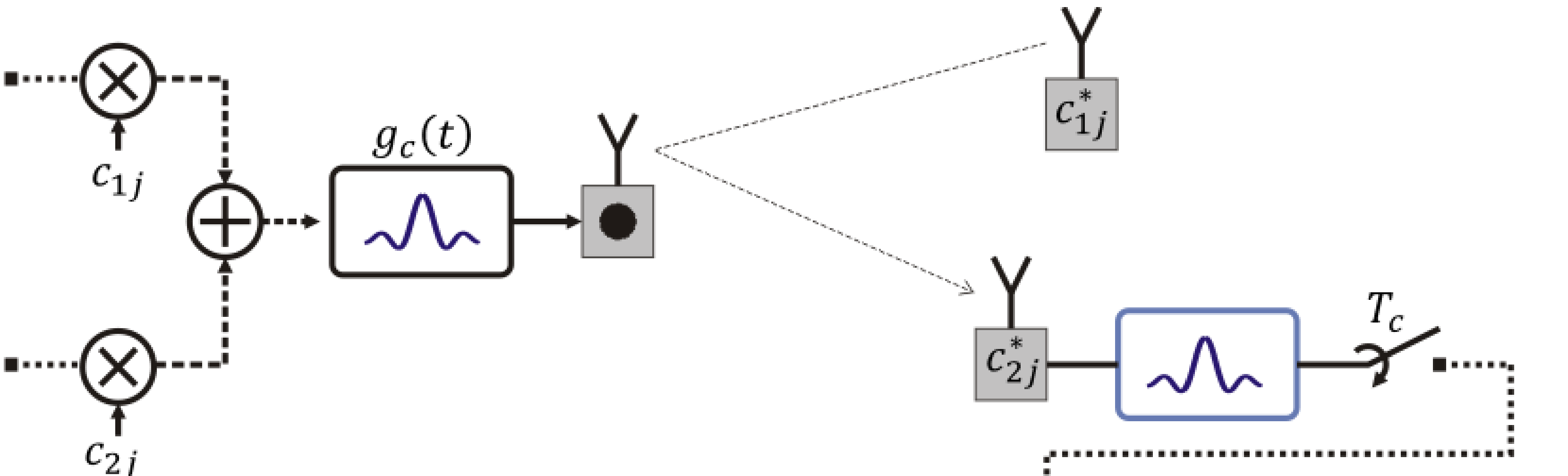


Code Division Multiple Access

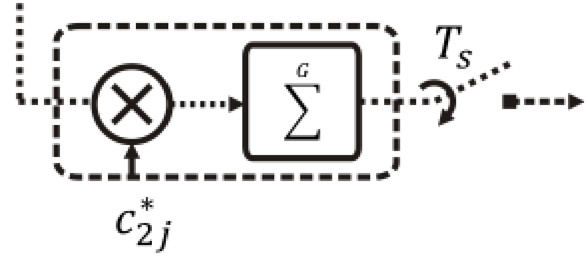
Consider a downlink scenario with multiple users

- Example UMTS with one base station transmitting to multiple users

- Each user k is assigned a **code** (spreading waveform) c_{kj}, where j is the time index
- **Spreading Factor (SF)** = bandwidth expansion of the data signal



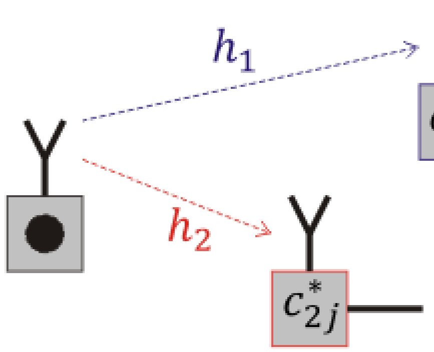
- **Receiver**: each user despreads the signal with the code (signal waveform) assigned to him



Code Division Multiple Access (Receiver)

For simplicity we make the following assumptions

- An AWGN channel with channel gains h_1 and h_2 to usets 1 and 2 respectively
- Perfect synchronization and a chip matched filter (sampled data after the MF is a sufficient statistics)



Consider only the following chip-spaced discrete time input-output model

- b_k is the symbol transmitted to user k (consider only a single symbol)
- t is the time index ($1 \le t \le SF$)
- $SF = T_s/T_c$ is the spreading factor (i.e., chips per symbol)

- Transmitted signal waveform

$$x_t = c_{1t}b_1 + c_{2t}b_2$$

- Received signal (sample=chip) at user k and time t

$$y_{kt} = h_k x_t = h_k c_{1t} b_1 + h_k c_{2t} b_2$$

Code Division Multiple Access (Receiver)

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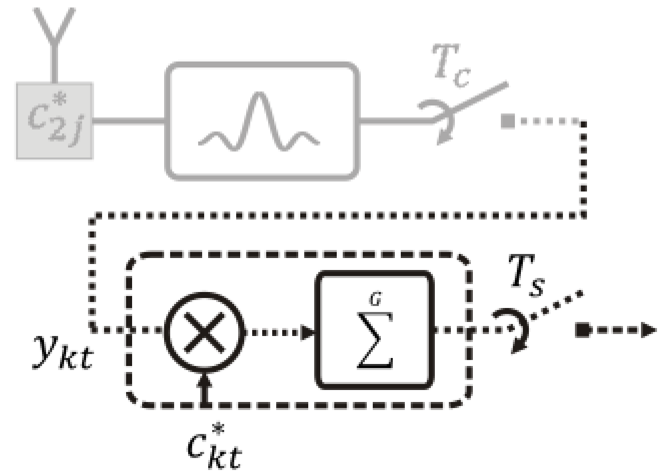


$$y_{kt} = h_k x_t = h_k c_{1t} b_1 + h_k c_{2t} b_2$$

Receiver of user 1

- Despreads (correlates) the received signal with its assigned code c_{1t}^*

$$\hat{b}_k = h_k^* \sum_{t=1}^{SF} c_{kt}^* y_{kt} = h_k^* h_k \left(b_1 \underbrace{\sum_{t=1}^{SF} c_{kt}^* c_{1t}}_{\psi_{11}^c[0]} + b_2 \underbrace{\sum_{t=1}^{SF} c_{kt}^* c_{2t}}_{\psi_{12}^c[0]} \right)$$

- Desired signal component given by auto-correlation $\psi_{11}^c[0]$ of the code c_{1t}
- Interference from other user given by cross-correlation $\psi_{12}^c[0]$ of codes c_{1t} and c_{1t}

Code Division Multiple Access (Receiver)

Consider the case of N users

- Assume the desire user is user-1

$$y_{1t} = h_1 c_{1t} b_1 + h_1 \sum_{n=2}^{N} c_{nt} b_n$$

- After despreading

$$\hat{b}_k == |h|^2 \left(b_1 \psi_{11}^{c}[0] + b_2 \sum_{n=2}^{N} \psi_{1n}^{c}[0] \right)$$

Signal from desired user

Interference from all other users

Cross correlation properties between codes define the ability to separate users (interference between users)

Code Design for CDMA

Objective: minimize cross-correlation between (all) codes

- Perfect separation implies

$$\psi_{ln}^{c}[0] = \begin{cases} 1 & l = n \\ 0 & l \neq n \end{cases}$$

- This condition is met when codes are perfectly orthogonal (inner product of corresponding vectors is zero)

$$\mathbf{c}_l = \begin{bmatrix} c_{l,1} & \cdots & c_{l,SF} \end{bmatrix}$$

$$\mathbf{c}_l \boldsymbol{c}_n^{\mathrm{H}} = \begin{cases} 1 & l = n \\ 0 & l \neq n \end{cases}$$

- Number of orthogonal sequences (codes) is limited by the length of the code

Walsh-Hadamard Sequences

Recursive construction

- Codes are the columns of the orthogonal matrix $\mathbf{H}_n$ which is constructed as

$$\mathbf{H}_n = \begin{bmatrix} +\mathbf{H}_{n-1} & +\mathbf{H}_{n-1} \\ +\mathbf{H}_{n-1} & -\mathbf{H}_{n-1} \end{bmatrix} \qquad \mathbf{H}_0 = \begin{bmatrix} +1 & +1 \\ +1 & -1 \end{bmatrix}$$

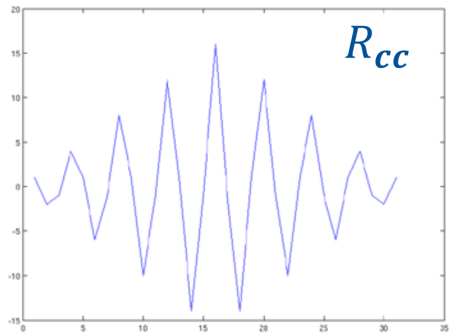
- Examples:

$$\mathbf{H}_0 = \begin{bmatrix} +1 & +1 \\ +1 & -1 \end{bmatrix}$$
$$SF = 2$$

$$\mathbf{H}_1 = \begin{bmatrix} +1 & +1 & +1 & +1 \\ +1 & -1 & +1 & -1 \\ +1 & +1 & -1 & -1 \\ +1 & -1 & -1 & +1 \end{bmatrix}$$
$$SF = 4$$

Drawbacks:

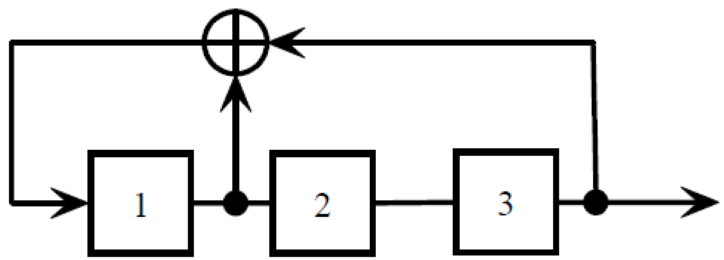
- Only available for spreading factors as powers of 2 ($SF = 2^K$)
- Number of orthogonal codes is always limited to SF
- Poor autocorrelation properties $R_{cc}[t > 0]$ large
  - Good when perfectly synchronized and channel has no multipath
  - Poor interference rejection from multipath
- Signal not well distributed across frequencies (NB interference)

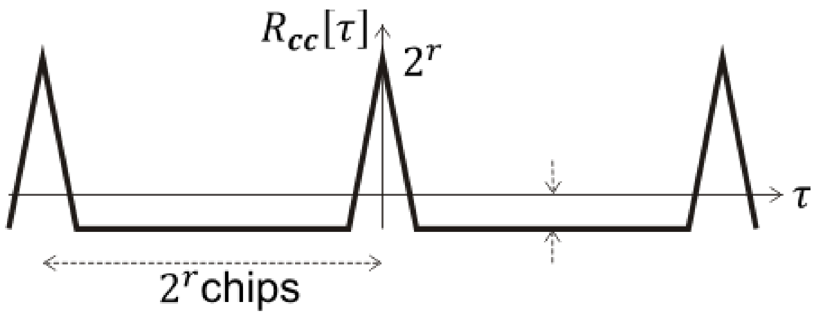


Maximum-Length (ML) and Gold Sequences

ML-Sequence: pseudo-random sequence

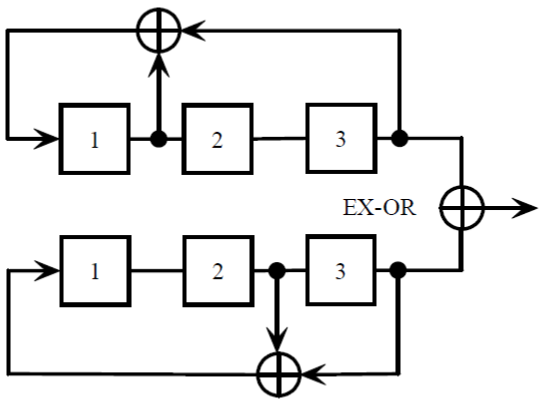
- Generated by an r-bit Linear Feedback Shift Register with period 2^r
- ML sequences have excellent autocorrelation properties and are almost zero-mean





Gold-sequence: constructed from two ML-sequences

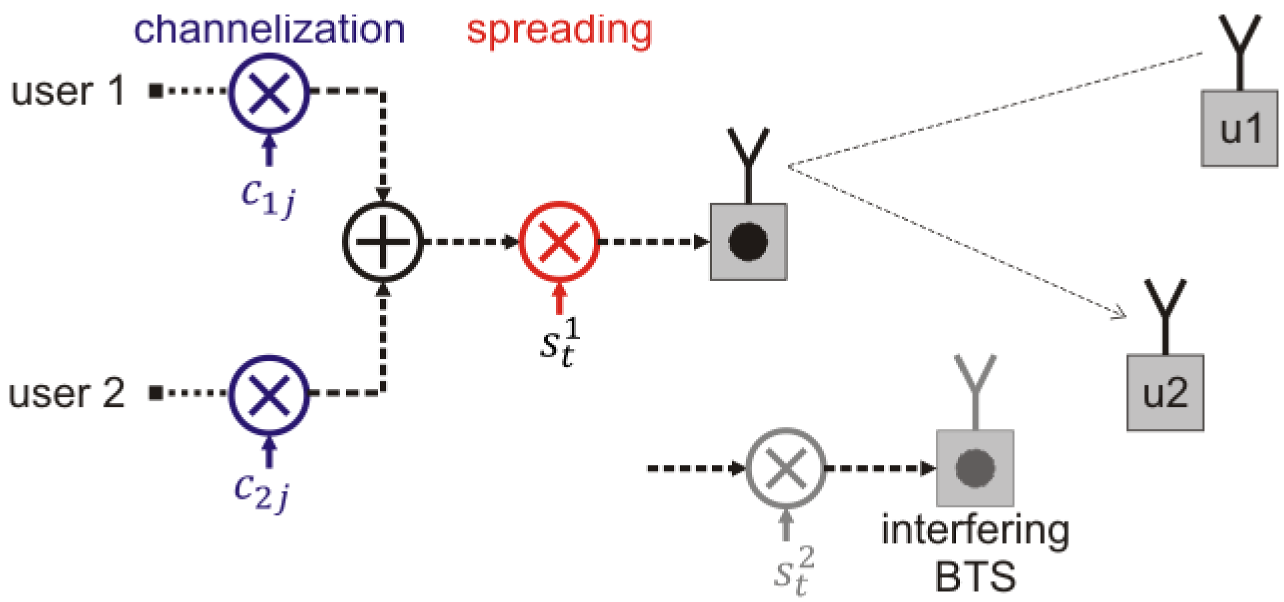
- Good autocorrelation properties
- Different initializations or time shifts lead to different codes with very good cross-correlation properties



Application in CDMA (Example: UMTS/HSDPA downlink)

Modulation in two steps

1. Use Walsh-Hadamard codes for channelization
2. Spread the signal with a base-station specific Gold Code



- Effective spreading sequence: product of the channelization and spreading code

Combines the advantages of orthogonal Hadamard sequences with the good auto- and cross-correlation properties of the Gold Sequences