Canadians should use memory

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Abstract

We establish that the competitive ratio of any randomized memoryless strategy is not less than $2k+1$. Only randomized strategies using memory potentially overpass this ratio now.

1 Definitions

We start by introducing the notation. For any graph $G = (V, E, \omega)$, let $G \backslash E'$ denotes its subgraph $(V, E \backslash E', \omega)$. If P is a path, we note its cost as $\omega(P) = \sum_{e \in P} \omega(e)$.

1.1 Memoryless Strategies for the k-CTP

We remind the definition of CTP. Let $G = (V, E, \omega)$ be an undirected graph with positive weights. The objective is to make a traveller traverse the graph from a source node s to a target one t, with $s, t \in V$. There is a set $E_* \subsetneq E$ of blocked edges. The traveller does not know a priori which edges are blocked. He discovers a blocked edge only when arriving to one of its endpoints. For example, if (v, w) is a blockage he will discover it when arriving to v (or w). The goal is to design the strategy A with the minimal competitive ratio.

We focus on *memoryless strategies* (MS). Concretely, we suppose that the traveller remembers the blocked edges he has discovered but forgets the nodes which he has already visited. In other words, a decision of an MS is independent of the nodes already visited. In the literature, the term *memoryless* was used in the context of online algorithms (e.g. PAGING PROBLEM [**?**], LIST UPDATE PROBLEM [**?**]) which take decisions according to the current state, ignoring past events. An MS can be either deterministic or randomized.

Definition 1 (Memoryless Strategies for the k-CTP) *A deterministic strategy A is an MS if and only if (iff) the next node w the traveller visits depends on graph G deprived of blocked edges already discovered E'_* and the current traveller position v: $w = A(G \backslash E'_*, v)$. Similarly, a randomized strategy A is an MS iff node w is the realization of a discrete random variable $X = A(G \backslash E'_*, v)$.*

MSes are easy to be implemented because they do not use past moves to take a decision. For the same reason, they do not need any extra memory either.

For example, the GREEDY strategy [**?**] is an MS. It consists in choosing at each step the first edge of the shortest path between the current node v and the target t. This strategy, illustrated

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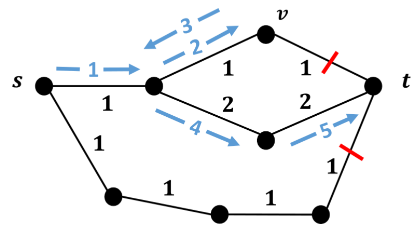


Figure 1: Illustration of the GREEDY strategy on a graph with 2 blocked edges

in Figure **??**, does not refer to anterior moves. In contrast, the REPOSITION strategy [**?**] is not an MS as any decision refers to the past moves of the traveller.

We propose the following process to identify whether a strategy A is a deterministic MS. Let us suppose that a traveller T_1 executes strategy A: he has already visited certain nodes of the graph, he is currently at node v but he has not reached target t yet. Let us imagine a second traveller T_2 who is airdropped on node v and starts applying strategy A. If the traveller T_2 always follows the same path as T_1 until reaching t, A is a deterministic MS. If T_1 and T_2 may follow different paths, then A is not an MS. Formally, prooving that a strategy is an MS consists in finding the function which transforms the pair $(G \backslash E_*, v)$ into node $w = A\left(G \backslash E'_*, v\right)$.

1.2 Competitive ratio

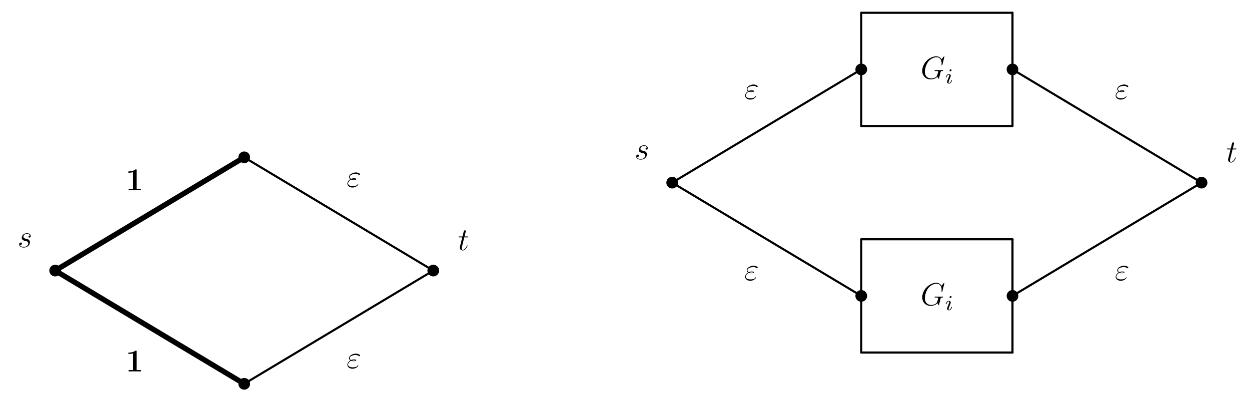
Let (G, E_*) be a *road map*, *i.e.* a pair with graph $G = (V, E, \omega)$ and blocked edges $E_* \subsetneq E$, such that there is an (s,t)-path in graph $G \backslash E_*$ (nodes s and t always remain in the same connected component). We note $\omega_A\left(G, E_*\right)$ the distance traversed by the traveller reaching t with strategy A on graph G with blocked edges E_* and $\omega_{\min}\left(G, E_*\right)$ the cost of the shortest (s,t)-path in graph $G \backslash E_*$.

The ratio $\omega_A\left(G, E_*\right)/\omega_{\min}\left(G, E_*\right)$ is abbreviated as $c_A\left(G, E_*\right)$. A strategy A is c_A-competitive [**?**, **?**] iff for any $\left(G, E_*\right), \omega_A\left(G, E_*\right) \leq c_A \omega_{\min}\left(G, E_*\right)$. Otherwise stated, for any (G, E_*), $c_A\left(G, E_*\right) \leq c_A$. If strategy A is randomized, $\omega_A\left(G, E_*\right)$ is replaced by $\mathbb{E}\left(\omega_A\left(G, E_*\right)\right)$ which is the expected distance traversed by the traveller to reach t with strategy A.

1.3 Graphs G_k

We define recursively a sequence of graphs G_i for $i \geq 1$ with weights from $\{1, \varepsilon\}$, $0 < \varepsilon \ll 1$. Graphs G_1 and G_{i+1} are represented in Figures **??** and **??**, respectively, edges with weight 1 are thicker than edges with weight ε. For every graph G_k, we will only consider the road maps where the blocked edges are at the right of the graph, the left edges affecting negligibly the total cost of a traveler. We note E_{right} the set of all the right edges.

When edges are blocked in G_k, a part of the graph without end. We assume that an optimal strategies won't go in these dead end parts. For every set E', we will note for $G \backslash E'$ the graph without these dead ends. We define $\mathcal{G}_k$ the set of all sub-graphs of G_k where edges have been removed. Formally, $\mathcal{G}_k = \{G_k \backslash E' : (G_k, E')\,\text{road map}, |E'| \leq k\}$. For each graph $G_k \backslash E'$, a road map $(G_k \backslash E', E_*)$ has the condition $|E'| + |E_*| = k$.



(a) Graph G_1

(b) Graph G_{i+1}

Figure 2: Recursive construction of graphs G_i