

Canadians should use memory

Pierre Bergé^{*,1}, Julien Hemery², Arpad Rimmel², and Joanna Tomasik²

¹LRI, Université Paris-Sud, Université Paris-Saclay, 91405 Orsay Cedex, France

²LRI, CentraleSupélec, Université Paris-Saclay, 91405 Orsay Cedex, France

Abstract

We establish that the competitive ratio of any randomized memoryless strategy is not less than $2k + 1$. Only randomized strategies using memory potentially overpass this ratio now.

1 Definitions

We start by introducing the notation. For any graph $G = (V, E, \omega)$, let $G \setminus E'$ denotes its subgraph $(V, E \setminus E', \omega)$. If P is a path, we note its cost as $\omega(P) = \sum_{e \in P} \omega(e)$.

1.1 Memoryless Strategies for the k -CTP

We remind the definition of CTP. Let $G = (V, E, \omega)$ be an undirected graph with positive weights. The objective is to make a traveller traverse the graph from a source node s to a target one t , with $s, t \in V$. There is a set $E_* \subsetneq E$ of blocked edges. The traveller does not know a priori which edges are blocked. He discovers a blocked edge only when arriving to one of its endpoints. For example, if (v, w) is a blockage he will discover it when arriving to v (or w). The goal is to design the strategy A with the minimal competitive ratio.

We focus on *memoryless strategies* (MS). Concretely, we suppose that the traveller remembers the blocked edges he has discovered but forgets the nodes which he has already visited. In other words, a decision of an MS is independent of the nodes already visited. In the literature, the term *memoryless* was used in the context of online algorithms (e.g. PAGING PROBLEM [4], LIST UPDATE PROBLEM [1]) which take decisions according to the current state, ignoring past events. An MS can be either deterministic or randomized.

Definition 1 (Memoryless Strategies for the k -CTP) *A deterministic strategy A is an MS if and only if (iff) the next node w the traveller visits depends on graph G deprived of blocked edges already discovered E'_* and the current traveller position v : $w = A(G \setminus E'_*, v)$. Similarly, a randomized strategy A is an MS iff node w is the realization of a discrete random variable $X = A(G \setminus E'_*, v)$.*

MSes are easy to be implemented because they do not use past moves to take a decision. For the same reason, they do not need any extra memory either.

For example, the GREEDY strategy [8] is an MS. It consists in choosing at each step the first edge of the shortest path between the current node v and the target t . This strategy, illustrated

*Corresponding author: Pierre.Berge@lri.fr

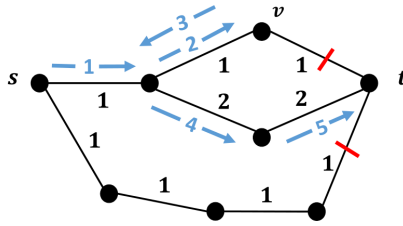


Figure 1: Illustration of the GREEDY strategy on a graph with 2 blocked edges

in Figure 1, does not refer to anterior moves. In contrast, the REPOSITION strategy [7] is not an MS as any decision refers to the past moves of the traveller.

We propose the following process to identify whether a strategy A is a deterministic MS. Let us suppose that a traveller T_1 executes strategy A : he has already visited certain nodes of the graph, he is currently at node v but he has not reached target t yet. Let us imagine a second traveller T_2 who is airdropped on node v and starts applying strategy A . If the traveller T_2 always follows the same path as T_1 until reaching t , A is a deterministic MS. If T_1 and T_2 may follow different paths, then A is not an MS. Formally, proving that a strategy is an MS consists in finding the function which transforms the pair $(G \setminus E_*, v)$ into node $w = A(G \setminus E'_*, v)$.

1.2 Competitive ratio

Let (G, E_*) be a *road map*, i.e. a pair with graph $G = (V, E, \omega)$ and blocked edges $E_* \subsetneq E$, such that there is an (s, t) -path in graph $G \setminus E_*$ (nodes s and t always remain in the same connected component). We note $\omega_A(G, E_*)$ the distance traversed by the traveller reaching t with strategy A on graph G with blocked edges E_* and $\omega_{\min}(G, E_*)$ the cost of the shortest (s, t) -path in graph $G \setminus E_*$.

The ratio $\omega_A(G, E_*) / \omega_{\min}(G, E_*)$ is abbreviated as $c_A(G, E_*)$. A strategy A is c_A -competitive [4, 8] iff for any (G, E_*) , $\omega_A(G, E_*) \leq c_A \omega_{\min}(G, E_*)$. Otherwise stated, for any (G, E_*) , $c_A(G, E_*) \leq c_A$. If strategy A is randomized, $\omega_A(G, E_*)$ is replaced by $\mathbb{E}(\omega_A(G, E_*))$ which is the expected distance traversed by the traveller to reach t with strategy A .

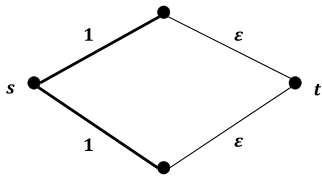
1.3 Graphs G_k

We define recursively a sequence of graphs G_i for $i \geq 1$ with weights from $\{1, \varepsilon\}$, $0 < \varepsilon \ll 1$. Graphs G_1 and G_{i+1} are represented in Figures 2a and 2b, respectively, edges with weight 1 are thicker than edges with weight ε . There is a vertical symmetrical on any graph G_i which is noted Δ_{vert} . Let $E_{i,\text{left}}$ denote the set of edges of G_i on the left to axis Δ_{vert} and $E_{i,\text{right}}$ the set of edges on the right of Δ_{vert} . We focus on the competitiveness of MSes on graphs in set \mathcal{G}_k , defined below.

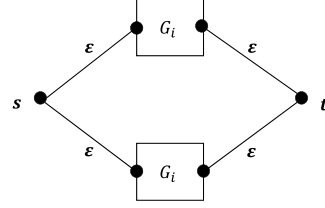
$$\mathcal{G}_k = \{G_i \setminus E' : E' \subsetneq E_{i,\text{right}}, |E'| \leq k\}$$

References

- [1] Susanne Albers. Online algorithms: a survey. *Math. Program.*, pages 3–26, 2003.
- [2] Amotz Bar-Noy and Baruch Schieber. The Canadian Traveller Problem. In *Proc. of ACM/SIAM SODA*, pages 261–270, 1991.



(a) Graph G_1



(b) Graph G_{i+1}

Figure 2: Recursive construction of graphs G_i

- [3] Marco Bender and Stephan Westphal. An optimal randomized online algorithm for the k-Canadian Traveller Problem on node-disjoint paths. *J. Comb. Optim.*, 30(1):87–96, 2015.
- [4] Allan Borodin and Ran El-Yaniv. *Online computation and competitive analysis*. Cambridge University Press, 1998.
- [5] Erik D. Demaine, Yamming Huang, Chung-Shou Liao, and Kunihiro Sadakane. Canadians Should Travel Randomly. *Proc. of ICALP*, pages 380–391, 2014.
- [6] Christos Papadimitriou and Mihalis Yannakakis. Shortest paths without a map. *Theor. Comput. Sci.*, 84(1):127–150, 1991.
- [7] Stephan Westphal. A note on the k-Canadian Traveller Problem. *Inform. Proces. Lett.*, 106(3):87–89, 2008.
- [8] Yinfeng Xu, Maolin Hu, Bing Su, Binhai Zhu, and Zhijun Zhu. The Canadian traveller problem and its competitive analysis. *J. Comb. Optim.*, 18(2):195–205, 2009.