On the competitiveness of memoryless strategies for the k-Canadian Traveller Problem

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Abstract

The k-Canadian Traveller Problem, defined and proven PSPACE-complete by Papadimitriou and Yannakakis, is a generalization of the Shortest Path Problem which admits blocked edges. Its objective is to determine the strategy that makes the traveller traverse graph G between two given nodes s and t with the minimal distance, knowing that at most k edges are blocked. The traveller discovers that an edge is blocked when arriving at one of its endpoint.

We study the competitiveness of randomized memoryless strategies to solve the k-CTP. Memoryless strategies are attractive in practice as a decision made by the strategy for a traveller in node v of G does not depend on his anterior moves. We establish that the competitive ratio of any randomized memoryless strategy cannot be better than $2k + O(1)$. This means that randomized memoryless strategies are asymptotically as competitive as deterministic strategies which achieve a ratio $2k + 1$ at best.

1. Introduction

The *Canadian Traveller Problem* (CTP), a generalization of the *Shortest Path Problem*, was introduced in [6]. Given an undirected weighted graph $G = (V, E, \omega)$ and two nodes $s, t \in V$, the objective is to design a strategy to make a traveller walk from s to t through G on the shortest path possible. Its particularity is that edges of G from set E_*, $E_* \subset E$, are blocked. The traveller does not know, however, which edges are blocked. He discovers blocked edges, also called blockages, when arriving to its endpoints. This implies that we solve the CTP with online algorithms, called strategies. The *k-Canadian Traveller Problem* (k-CTP) is the parameterized variant of CTP, where an upper bound k for the number of blocked edges is given. Both CTP and k-CTP are PSPACE-complete [2, 6].

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State-of-the-art. Strategies for the k-CTP are studied through the competitive analysis, which evaluates their quality [4]. The competitive ratio of a strategy is the maximum, over all satisfiable instances, of the ratio of the distance traversed by the traveller following the strategy and the *optimal offline cost*, which is the distance he would traverse if he knew blocked edges from the beginning.

There are two classes of strategies: deterministic and randomized. Westphal [7] proved that there is no deterministic strategy that achieves a competitive ratio better than $2k + 1$. This ratio is reached by REPOSITION and COMPARISON strategies [7, 8]. The REPOSITION strategy repeats an attempt to reach t through the shortest (s, t)-path going back to s after the discovery of an obstacle. As in practical cases, such as urban networks, it does not seem realistic, Xu *et al.* [8] introduced the GREEDY algorithm. For grids, it achieves ratio $\mathcal{O}(1)$, regardless of k. However, for any graph, this ratio is $\mathcal{O}\left(2^k\right)$.

We evaluate the competitiveness of the randomized strategies by calculating the maximal ratio of the mean distance traversed by the traveller following the strategy and the optimal offline cost. Westphal [7] proved that no randomized algorithm can attain a ratio smaller than $k + 1$. However, unlike the deterministic case, no $(\alpha k + 1)$-competitive randomized strategy, $\alpha < 2$, was identified, excepted for two very particular cases for which randomized strategies have been proposed. Demaine *et al.* [5] designed a strategy with a ratio $\left(1 + \frac{\sqrt{2}}{2}\right) k + 1$, executed in time of $\mathcal{O}\left(k\mu^2 |E|^2\right)$, where parameter μ may be exponential. It is dedicated to graphs that can be transformed into apex trees. Bender *et al.* studied in [3] a restriction of k-CTP for graphs composed of node-disjoint (s, t)-paths and proposed a polynomial-time strategy with ratio $(k + 1)$.

Contributions and paper plan. We study the competitiveness of memoryless strategies [1, 4]. The choice the strategy makes for the traveller at node v (to decide where he should go next) is independent of the path traversed before reaching this node. Memoryless strategies are worth of interest because they are easy to be implemented as they do not memorize the edges already visited by the traveller to make a decision. The only information they use is the graph $G \backslash E'_*$, which is the graph G deprived of the blocked edges discovered $E'_* \subseteq E_*$. Given that deterministic strategies cannot achieve a ratio better than $2k+1$, our goal is to prove that randomized memoryless strategies are not more competitive asymptotically. To do this, we identify a lower bound $c_k = 2k + O(1)$ of the competitiveness of randomized memoryless strategies for the k-CTP.

We remind, in Section 2, the definitions of k-CTP, memoryless strategies and the competitive ratio. In Section 3, we present sets $\mathcal{R}_k$ of *road maps*, *i.e.* pairs (G, E_*), which are a means to study the performance of memoryless strategies. We prove in Section 4 that randomized memoryless strategies cannot drop below a ratio $c_k = 2k + O(1)$ on road maps in $\mathcal{R}_k$, where expression $O(1)$ is clarified. Eventually, we draw conclusions and highlight the future work in Section 5.

2. Definitions

We start by introducing the notation. For any graph $G = (V, E, \omega)$, let $G \backslash E'$ denotes its subgraph $(V, E \backslash E', \omega)$.

2.1. Memoryless Strategies for the k-CTP

We remind the definition of CTP. Let $G = (V, E, \omega)$ be an undirected graph with positive weights. The objective is to make a traveller traverse the graph from a source node s to a target one t, with $s, t \in V$ and a set $E_* \subsetneq E$ of blocked edges. The traveller does not know a priori which edges are blocked. He discovers a blocked edge only when arriving to one of its endpoints. For example, if (v, w) is a blockage he will discover it when arriving to v (or w). The goal is to design the strategy A with the minimum competitive ratio.

We focus on *memoryless strategies* (MS). Concretely, we suppose that the traveller remembers the blocked edges he has discovered but forgets the nodes which he has already visited. In other words, a decision of an MS is independent of the nodes already visited. In the literature, the term *memoryless* was used in the context of online algorithms (e.g. PAGING PROBLEM [4], LIST UPDATE PROBLEM [1]) which take decisions according to the current state, ignoring past events. An MS can be either deterministic or randomized.

Definition 1 (Memoryless Strategies for the k-CTP). *A deterministic strategy A is an MS if and only if (iff) the next node w the traveller visits depends on graph G deprived of blocked edges already discovered E'_* and the current traveller position v: $w = A\left(G \backslash E'_*, v\right)$. Similarly, a randomized strategy A is an MS iff node w is the realization of a discrete random variable $X = A\left(G \backslash E'_*, v\right)$.*

For example, the GREEDY strategy [8] is a deterministic MS. It consists in choosing at each step the first edge of the shortest path between the current node v and the target t. In contrast, the REPOSITION strategy [7] is not an MS as its decision refers to the past moves of the traveller. The polynomial-time strategies proposed in the literature do not use much memory information in the decision-making process. Either they are memoryless or they use a small amount of memory. For example, REPOSITION (deterministic [7], randomized [3]) can be implemented with a one bit memory given that the only information to retain is whether the traveller tries to reach t or to return to s.

The following process allows us to identify whether a deterministic strategy A is an MS. Let us suppose that a traveller T_1 follows strategy A: he has already visited certain nodes of the graph, he is currently at node v but he has not reached target t yet. Let us imagine a second traveller T_2 who is airdropped on node v and is guided by strategy A. If the traveller T_2 always follows the same path as T_1 until reaching t, A is a deterministic MS. If T_1 and T_2 may follow different paths, then A is not an MS. Formally, prooving that a strategy is a MS consists in finding the function which transforms the pair $(G \backslash E_*, v)$ into node $w = A\left(G \backslash E'_*, v\right)$.

2.2. Competitive ratio

Let (G, E_*) be a *road map*, *i.e.* a pair with graph $G = (V, E, \omega)$ and blocked edges $E_* \subsetneq E$, such that there is an (s,t)-path in graph $G \backslash E_*$ (nodes s and t remain in the same connected component when all blocked edges are discovered). We note $\omega_A(G, E_*)$ the distance traversed by the traveller reaching t with strategy A on graph G with blocked edges E_* and $\omega_{\min}(G, E_*)$ the cost of the shortest (s,t)-path in graph $G \backslash E_*$.

The ratio $\omega_A(G, E_*) / \omega_{\min}(G, E_*)$ is abbreviated as $c_A(G, E_*)$. A strategy A is c_A-competitive [4, 8] iff for any $(G, E_*), \omega_A(G, E_*) \leq c_A \omega_{\min}(G, E_*)$. Otherwise stated, for any (G, E_*), $c_A(G, E_*) \leq c_A$. If strategy A is randomized, $\omega_A(G, E_*)$ is replaced by $\mathbb{E}(\omega_A(G, E_*))$ which is the expected distance traversed by the traveller to reach t with strategy A. The competitive ratio can also be evaluated on a family $\mathcal{R}$ of road maps, put formally:

$$c_{A,\mathcal{R}} = \max_{(G,E_*) \in \mathcal{R}} c_A(G, E_*) \,. \tag{1}$$

This "local" competitive ratio fulfils $c_{A,\mathcal{R}} \leq c_A$. The definition of the competitive ratio can also be extended to families of strategies. We note c_{MS} the competitive ratio of MSes, which is the minimum over competitive ratios of any MSes: $c_{\mathrm{MS}} = \min_{A \text{ MS}} c_A$.

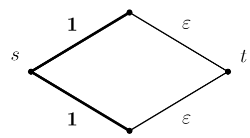
3. Road atlas used to study randomized MSes

Before specifying *road atlases* $\mathcal{R}_k$, *i.e.* families of road maps we construct to evaluate the competitiveness of randomized MSes, we need to introduce the concepts used in their definition.

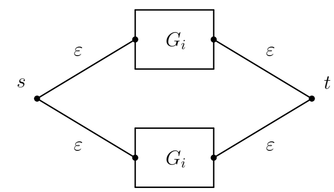
We define recursively a sequence of graphs G_i for $i \geq 1$ with weights from $\{1, \varepsilon\}$, $0 < \varepsilon \ll 1$. Graphs G_1 and G_{i+1} are represented in Figures 1a and 1b, graphs G_2 and G_3 are shown in Figures 1c and 1d. Edges with weight 1 are thicker than edges with weight ε. For any graph G_i, axis Δ_{vert} is its vertical axis of symmetry (Figures 1c and 1d).

We focus on a traveller traversing a road map (G_i, E_*) composed of graph G_i and at most i blocked edges, guided by an MS. All blocked edges from E_* are on the right side of axis Δ_{vert}. Indeed, blocking edges on the left of Δ_{vert} affects negligibly the total distance traversed by a traveller. Let us suppose that he traverses graph G_i and has already discovered some blocked edges $E' \subseteq E_*$. He now tries to reach t on graph $G_i \backslash E'$, ignoring that the undiscovered blocked edges are $E'_* = E_* \backslash E'$. The graph considered at this step is $G_i \backslash E'$ as past moves are not taken into account.

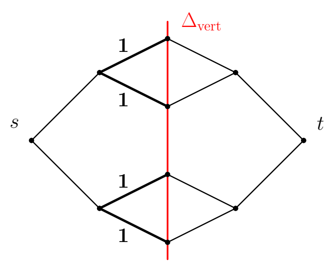
We note $\mathcal{G}$ the set of all the subgraphs of G_i, *i.e.* graphs $G_i \backslash E'$ with at most i edges E' on the right of Δ_{vert}, for any $i \geq 1$. We call them *diamond graphs* because of their appearance, diamonds joined together. Formally, we set $\mathcal{G} = \bigcup_{i=1}^{+\infty} \{G_i \backslash E' : |E'| \leq i\}$. For any graph $G \in \mathcal{G}$, we partition its edges, noted E_G, into two sets $E_{G,\mathrm{left}}$ (on the left side of axis Δ_{vert}) and $E_{G,\mathrm{right}}$ (on the right of axis Δ_{vert}).



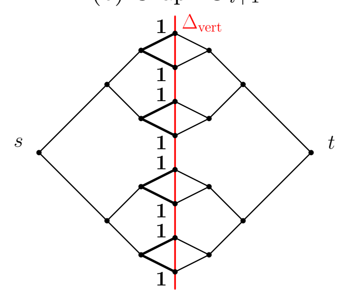
(a) Graph G_1



(b) Graph G_{i+1}



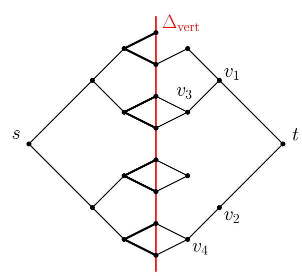
(c) Graph G_2 and axis Δ_{vert}



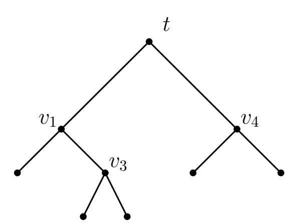
(d) Graph G_3 and axis Δ_{vert}

Figure 1: Recursive construction of graphs G_i (weights ε are omitted in Figures 1c and 1d).

To any diamond graph G of $\mathcal{G}$, we associate a *diamond binary tree* (DBT) noted T_G. Tree T_G is obtained from the right half of graph G rooted in t (on the right side of axis Δ_{vert}) by successive contractions of edges: any node with a single son is merged with its father (in Figure 2a: edge (t, v_2) is contracted, v_2 merges with t). We note $T_\varnothing$ the empty tree. Any nonempty tree is a triplet (v, T_a, T_b) with a root $v \in V$ and two trees T_a and T_b. Figures 2a and 2b illustrate the construction of the DBT.



(a) Graph $G \in \mathcal{G}$, subgraph of G_3.



(b) The DBT T_G

Figure 2: An example of graph $G \in \mathcal{G}$ and its DBT T_G.

To put the definition of DBTs, let $L(v)$ denote the set of sons of node v. The function L is only defined for the nodes on the right side of Δ_{vert} and for all $v \in \Delta_{\text{vert}}$, we have $L(v) = \emptyset$. For graph G of Figure 2a, $L(t) = \{v_1, v_2\}$, $L(v_2) = \{v_4\}$ for example.

Function BIN-TREE gives the construction of tree T_G, which is BIN-TREE (t).

$$\text{BIN-TREE}\,(v) = \begin{cases} T_\varnothing \text{ if } L(v) = \emptyset, \\ \text{BIN-TREE}\,(v_{\text{next}}) \text{ if } L(v) = \{v_{\text{next}}\}, \\ (v, \text{BIN-TREE}(v_{\text{up}}), \text{BIN-TREE}(v_{\text{down}})) \text{ if } L(v) = \{v_{\text{up}}, v_{\text{down}}\}. \end{cases}$$

We say that the depth of a node v in a DBT T, noted $d(v)$, is equal to the number of edges separating it from the root. We note $d_{\min}(T)$ the minimum depth of all T leaves. For example, for DBT T_G of Figure 2b, $d_{\min}(T_G) = 2$.

Definition 2 (Sets $\mathcal{D}_k$). *Set $\mathcal{D}_k$ contains graphs of $\mathcal{G}$ such that their DBT T_G fulfils $d_{min}(T_G) \geq k$: $\mathcal{D}_k = \{G \in \mathcal{G} : d_{min}(T_G) \geq k\}$.*

Finally, we define road atlases $\mathcal{R}_k$:

Definition 3 (Road atlas $\mathcal{R}_k$). *Road atlas $\mathcal{R}_k$ is composed of road maps (G, E_*) where $|E_*| = k$ and $G \in \mathcal{D}_k$: $\mathcal{R}_k = \{(G, E_*) : G \in \mathcal{D}_k, |E_*| = k\}$.*

4. Competitiveness of randomized MSes

We study the competitiveness of a randomized MS for road atlases $\mathcal{R}_k$. The MS performance is determined by properties of corresponding DBTs T_G. These properties result from relations which exist between DBT edges.

The depth of an edge (u, v), $D(u, v)$, is defined as $D(u, v) = \min\{d(u), d(v)\}$. Edge e' is the mother of edge e if $e \cap e' \neq \emptyset$ and $D(e') = D(e) - 1$, putting it shortly $e' = P(e)$. Edge e^* is the aunt of edge e if $e^* \cap P(e) \neq \emptyset$ and $D(P(e)) = D(e^*)$. We indicate this fact as $e^* = U(e)$. Observe that the aunt of e and its mother share the same ancestor which may be written as $U(e) = \{e' \in T : P(e') = P(P(e)) = P^2(e)\}$.

The following Theorem 1 states that cutting one edge from $G \in \mathcal{D}_k$ produces a graph $G \setminus \{e\} \in \mathcal{D}_{k-1}$.

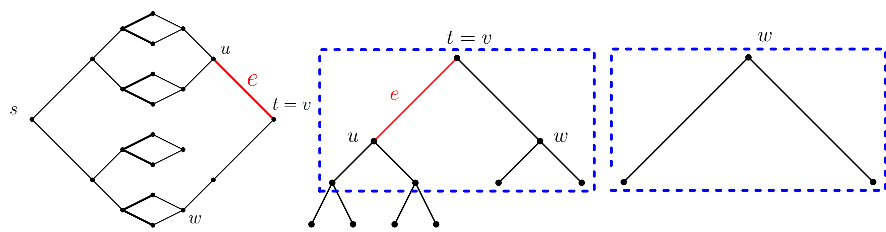
Theorem 1. *For any $G \in \mathcal{D}_k$ and edge $e \in E_{G,\text{right}}$, graph $G \setminus \{e\} \in \mathcal{D}_{k-1}$.*

Proof. Let $G \in \mathcal{D}_k$ and e be an edge in $E_{G,\text{right}}$. There is an edge e^* in T_G for which $T_{G\setminus\{e\}}$ is obtained by removing e^* and its descendants from T_G and next applying the edge contraction, if necessary. Let v be the "shallower" endpoint of edge $e^* = \{u, v\}$, *i.e.* $d(v) = \min\{d(u), d(v)\}$. Edge e^* and its mother have this node in common, $v \in P(e^*)$. We distinguish two cases :

- **The depth of node u is greater or equal to $d_{\min}(T_G)$:** If u is the unique leaf of depth $d_{\min}(T_G)$, the depth of leaves of the DBT $T_{G\setminus\{e\}}$ is $d_{\min}(T_G) - 1 \geq k - 1$. Otherwise, in DBT $T_{G\setminus\{e\}}$, the depth of leaves is still equal to $d_{\min}(T_G) \geq k$. In both cases, $G \setminus \{e\} \in \mathcal{D}_{k-1}$.
- **The depth of node u is strictly inferior to $d_{\min}(T_G)$:** Let T_v be the subtree of T_G with root v. We note w the brother of node u, *i.e.* the other son of node v (Figures 3a and 3b). When edge e is removed from G, edge e^* and

its descendants are withdrawn in the DBT (in the DBT of Figure 3b, edge e has not been contracted so $e = e^*$). So, after the contraction, T_v becomes T_w, the subtree rooted in w. All the leaves of T_w have initially a depth greater than k so by removing e from G, all the leaves of T_w have a depth greater than $k-1$. All leaves outside T_v, preserve their depth which is greater than k. Therefore, the depth of all leaves of $T_{G\setminus\{e\}}$ is greater than $k-1$.

After examining these two cases, we conclude that $G\setminus\{e\}$ belongs to $\mathcal{D}_{k-1}$. ■



(a) Example of graph $G \in \mathcal{D}_2$. Edge e (red) is about to be removed from this graph.

(b) The DBT T_G. Blue frame covers nodes of depth 0 to 2. Two leaves are at depth 2.

(c) The DBT $T_{G\setminus\{e\}}$. Nodes of depth 0 to 1 are framed. Graph $G\setminus\{e\}$ belongs to $\mathcal{D}_1$.

Figure 3: Illustration of the proof of Theorem 1 on a subgraph of G_3.

We note $c_{\mathrm{MS}}(G,k)$ the maximum over values $c_{\mathrm{MS}}(G,E_*)$, where $(G,E_*) \in \mathcal{R}_k$. This maximum represents the competitive ratio of MSes on road maps containing graph G from $\mathcal{D}_k$. Formally:

$$c_{\mathrm{MS}}(G,k) = \min_{A\in\ \mathrm{MS}} \max_{(G,E_*)\in\mathcal{R}_k} c_A(G,E_*)\,. \tag{2}$$

Observe that the competitive ratio $c_{\mathrm{MS}}(G,k)$ defined for a traveller starting from source s is still valid for a traveller starting his walk at any node on the left side of axis Δ_{vert} as weights ε are negligeable.

The next theorem states that value $c_{\mathrm{MS}}(G,k)$ is the same for all graphs of $\mathcal{D}_k$. In other words, graphs in $\mathcal{D}_k$ are equivalent regarding the competitiveness of MSes. Let c_k denote the competitive ratio of MSes over atlas $\mathcal{R}_k$. The proof of Theorem 2 provides a recursive formula of ratio c_k.

Theorem 2. *For any graphs $G, G' \in \mathcal{D}_k$, $c_{\mathrm{MS}}(G,k) = c_{\mathrm{MS}}(G',k) = c_k$.*

Proof. We prove it by induction. If $k = 0$, then for every graph of $\mathcal{D}_0$, $c_{\mathrm{MS}}(G,0) = c_0 = 1$ because any path the traveller takes is open.

We assume the theorem holds for graphs from $\mathcal{D}_{k-1}$. For any $G \in \mathcal{D}_k$, all leaves of T_G are at depth greater than or equal to k. So, T_G is complete up to depth k and has 2^k nodes of depth k. We suppose that the traveller, guided by the most competitive MS A over atlas $\mathcal{R}_k$, is standing at source s and starts his walk on a road map $(G,E_*) \in \mathcal{R}_k$. We determine an upper bound of $c_A(G,E_*)$.

As strategy A is the most competitive, the traveller using it either reaches t directly with distance 1 or meets a blocked edge and traverses a total distance

$2 + c_{k-1}$: distance 1 to reach the blockage, distance 1 to go back to a node on the left hand side of Δ_{vert} and distance c_{k-1} to reach t on graph G deprived of the blocked edges, which belongs to $\mathcal{D}_{k-1}$ (Theorem 1).

For any $1 \leq j \leq 2^k$, let $p^A_{k,j}$ denote the probability that the traveller visits the j^{th} node at depth k, noted $v_{k,j}$ (index j passes from left to right in the DBT representation). Let $P_{k,j}$ be the set of simple (s,t)-paths of length 1 which traverses node $v_{k,j}$. Given a set E_* of blocked edges, we note $\mathcal{N}(E_*)$ the set of nodes $v_{k,j}$ of depth k such that any path of $P_{k,j}$ is open. For any set of blocked edges, set $\mathcal{N}(E_*)$ is necessarily non-empty because the minimum number of edges needed to block at least one path in each set $P_{k,j}$ is $k+1$.

For any node $v_{k,j}$, we construct a set of blocked edges such that $\mathcal{N}(E_*) = \{v_{k,j}\}$ and all paths in any $P_{k,j'}$ with $j' \neq j$ are obstructed. Indeed, let e be the edge linking the father node of $v_{k,j}$ and its brother. By blocking the set of edges $E_{k,j}$ containing e, $U(e)$, $U(U(e)), \ldots, U^{k-1}(e)$, we block $1+2+4+\ldots+2^{k-1} = 2^k - 1$ nodes $v_{k,j'}$ at depth k, $j' \neq j$. It comes that:

$$c_A(G, E_{k,j}) = p^A_{k,j} + (2 + c_{k-1}) \sum_{j' \neq j} p^A_{k,j'} = p^A_{k,j} + (2 + c_{k-1})(1 - p^A_{k,j}).$$

For any set of blocked edges E_*, if $v_{k,j} \in \mathcal{N}(E_*)$, then the distance traversed when passing through it is 1. If $v_{k,j} \notin \mathcal{N}(E_*)$, it is at most $2 + c_{k-1}$:

$$c_A(G, E_*) \leq \sum_{v_{k,j} \in \mathcal{N}(E_*)} p^A_{k,j} + (2 + c_{k-1}) \sum_{v_{k,j} \notin \mathcal{N}(E_*)} p^A_{k,j}.$$

We note j^* the index of the node v_{k,j^*} in $\mathcal{N}(E_*)$, where p^A_{k,j^*} is the smallest. As $1 < 2 + c_{k-1}$, the greater $\sum_{v_{k,j} \in \mathcal{N}(E_*)} p^A_{k,j}$ is, the smaller $c_A(G, E_*)$ is:

$$c_A(G, E_*) \leq p^A_{k,j^*} + (2 + c_{k-1}) \sum_{j \neq j^*} p^A_{k,j} = c_A(G, E_{k,j^*}).$$

The ratio $c_A(G, E_*)$ attains this upper bound when blocked edges of road map is one of E_{k,j^*}. Consequently, we obtain the competitive ratio of strategy A over road atlas $\mathcal{R}_k$,

$$\begin{aligned} c_{A,\mathcal{R}_k} &= \max_{1 \leq j \leq 2^k} p^A_{k,j} + \left(1 - p^A_{k,j}\right)(2 + c_{k-1}) \\ &\geq \frac{1}{2^k} + \left(1 - \frac{1}{2^k}\right)(2 + c_{k-1}). \end{aligned}$$

The MS with the smallest value $c_{A,\mathcal{R}_k}$ fulfils $p^A_{k,j} = \frac{1}{2^k}$ for any $1 \leq j \leq 2^k$. This value depends on k and c_{k-1}, not on the graph itself. For $G, G' \in \mathcal{D}_k$, $c_{\text{MS}}(G,k) = c_{\text{MS}}(G',k) = c_k = \min_A c_{A,\mathcal{R}_k} = \frac{1}{2^k} + \left(1 - \frac{1}{2^k}\right)(2 + c_{k-1})$. ■

The Theorem 2 says that, for k blockages, the competitive ratio of MSes on road atlas $\mathcal{R}_k$ is given by sequence c_k. Consequently, $c_{\text{MS}} \leq c_k$. We observe that $c_k - c_{k-1} = 2 - \frac{1}{2^k} - \frac{c_{k-1}}{2^k}$ and we obtain the following iterative formula:

$$c_k = 2k + 1 - \sum_{j=0}^{k-1} \frac{c_j + 1}{2^{j+1}}.$$

As $\sum_{j=0}^{+\infty} \frac{c_j + 1}{2^{j+1}}$ is finite, $c_k = 2k + O(1)$. The numerical computations give $\sum_{j=0}^{10^4} \frac{c_j + 1}{2^{j+1}} = 3.213$, so the competitive ratio of MSes is larger than $2k - 2.22$. As c_k represents the competitive ratio of the best competitive MS over road atlas $\mathcal{R}_k$, no MS can go below $2k + O\,(1)$ in terms of competitiveness.

5. Conclusion and further work

We studied the competitiveness of the MSes for the k-CTP. An MS is a strategy which does not make decisions referring to the anterior moves of the traveller, in other words, the nodes the traveller visited until his current position.

Then, we constructed a series of k-CTP instances, called road atlases and noted $\mathcal{R}_k$. We foremost concluded that a randomized MS cannot reach a competitive ratio better than $2k + \mathcal{O}\,(1)$ on road atlas $\mathcal{R}_k$. That is to say that we identified an upper bound on the competitive ratio of randomized MSes which is significantly higher than the existing one $k + 1$. In future research, if we aim at designing a strategy with competitive ratio $\alpha k + O\,(1)$, $\alpha < 2$, we shall focus on strategies which are not only randomized but use memory as well.

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