

# Canadians should use memory

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## Abstract

We establish that the competitive ratio of any randomized memoryless strategy is not less than  $2k + 1$ . Only randomized strategies using memory potentially overpass this ratio now.

## 1 Definitions

We start by introducing the notation. For any graph  $G = (V, E, \omega)$ , let  $G \setminus E'$  denotes its subgraph  $(V, E \setminus E', \omega)$ . If  $P$  is a path, we note its cost as  $\omega(P) = \sum_{e \in P} \omega(e)$ .

### 1.1 Memoryless Strategies for the $k$ -CTP

We remind the definition of CTP. Let  $G = (V, E, \omega)$  be an undirected graph with positive weights. The objective is to make a traveller traverse the graph from a source node  $s$  to a target one  $t$ , with  $s, t \in V$ . There is a set  $E_* \subsetneq E$  of blocked edges. The traveller does not know a priori which edges are blocked. He discovers a blocked edge only when arriving to one of its endpoints. For example, if  $(v, w)$  is a blockage he will discover it when arriving to  $v$  (or  $w$ ). The goal is to design the strategy  $A$  with the minimal competitive ratio.

We focus on *memoryless strategies* (MS). Concretely, we suppose that the traveller remembers the blocked edges he has discovered but forgets the nodes which he has already visited. In other words, a decision of an MS is independent of the nodes already visited. In the literature, the term *memoryless* was used in the context of online algorithms (e.g. PAGING PROBLEM [4], LIST UPDATE PROBLEM [1]) which take decisions according to the current state, ignoring past events. An MS can be either deterministic or randomized.

**Definition 1 (Memoryless Strategies for the  $k$ -CTP)** *A deterministic strategy  $A$  is an MS if and only if (iff) the next node  $w$  the traveller visits depends on graph  $G$  deprived of blocked edges already discovered  $E'_*$  and the current traveller position  $v$ :  $w = A(G \setminus E'_*, v)$ . Similarly, a randomized strategy  $A$  is an MS iff node  $w$  is the realization of a discrete random variable  $X = A(G \setminus E'_*, v)$ .*

MSes are easy to be implemented because they do not use past moves to take a decision. For the same reason, they do not need any extra memory either.

For example, the GREEDY strategy [8] is an MS. It consists in choosing at each step the first edge of the shortest path between the current node  $v$  and the target  $t$ . This strategy, illustrated

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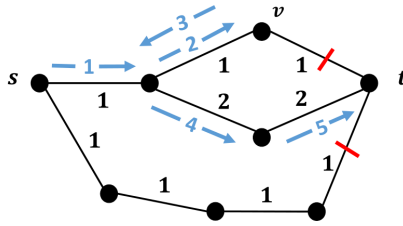


Figure 1: Illustration of the GREEDY strategy on a graph with 2 blocked edges

in Figure 1, does not refer to anterior moves. In contrast, the REPOSITION strategy [7] is not an MS as any decision refers to the past moves of the traveller.

We propose the following process to identify whether a strategy  $A$  is a deterministic MS. Let us suppose that a traveller  $T_1$  executes strategy  $A$ : he has already visited certain nodes of the graph, he is currently at node  $v$  but he has not reached target  $t$  yet. Let us imagine a second traveller  $T_2$  who is airdropped on node  $v$  and starts applying strategy  $A$ . If the traveller  $T_2$  always follows the same path as  $T_1$  until reaching  $t$ ,  $A$  is a deterministic MS. If  $T_1$  and  $T_2$  may follow different paths, then  $A$  is not an MS. Formally, proving that a strategy is an MS consists in finding the function which transforms the pair  $(G \setminus E_*, v)$  into node  $w = A(G \setminus E'_*, v)$ .

## 1.2 Competitive ratio

Let  $(G, E_*)$  be a *road map*, i.e. a pair with graph  $G = (V, E, \omega)$  and blocked edges  $E_* \subsetneq E$ , such that there is an  $(s, t)$ -path in graph  $G \setminus E_*$  (nodes  $s$  and  $t$  always remain in the same connected component). We note  $\omega_A(G, E_*)$  the distance traversed by the traveller reaching  $t$  with strategy  $A$  on graph  $G$  with blocked edges  $E_*$  and  $\omega_{\min}(G, E_*)$  the cost of the shortest  $(s, t)$ -path in graph  $G \setminus E_*$ .

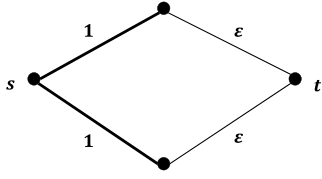
The ratio  $\omega_A(G, E_*) / \omega_{\min}(G, E_*)$  is abbreviated as  $c_A(G, E_*)$ . A strategy  $A$  is  $c_A$ -competitive [4, 8] iff for any  $(G, E_*)$ ,  $\omega_A(G, E_*) \leq c_A \omega_{\min}(G, E_*)$ . Otherwise stated, for any  $(G, E_*)$ ,  $c_A(G, E_*) \leq c_A$ . If strategy  $A$  is randomized,  $\omega_A(G, E_*)$  is replaced by  $\mathbb{E}(\omega_A(G, E_*))$  which is the expected distance traversed by the traveller to reach  $t$  with strategy  $A$ .

## 1.3 Graphs $G_k$

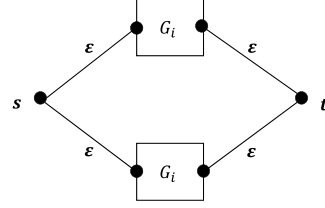
We define recursively a sequence of graphs  $G_i$  for  $i \geq 1$  with weights from  $\{1, \varepsilon\}$ ,  $0 < \varepsilon \ll 1$ . Graphs  $G_1$  and  $G_{i+1}$  are represented in Figures 2a and 2b, respectively, edges with weight 1 are thicker than edges with weight  $\varepsilon$ .

## References

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(a) Graph  $G_1$



(b) Graph  $G_{i+1}$

Figure 2: Recursive construction of graphs  $G_i$

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