

STEP :

$$\begin{array}{c} \models \varphi \rightarrow \bigvee_{\varphi_l \Rightarrow^{\exists} \varphi_r \in \mathcal{S}} \exists \text{FreeVars}(\varphi_l). \varphi_l \\ \models ((\varphi \wedge \varphi_l) \neq \perp_{\text{Cfg}}) \wedge \varphi_r \rightarrow \varphi' \quad \text{for each } \varphi_l \Rightarrow^{\exists} \varphi_r \in \mathcal{S} \\ \hline \mathcal{S}, \mathcal{A} \vdash_C \varphi \Rightarrow^{\forall} \varphi' \end{array}$$

AXIOM :

$$\begin{array}{c} \varphi \Rightarrow^Q \varphi' \in \mathcal{S} \cup \mathcal{A} \quad \psi \text{ is FOL formula (logical frame)} \\ \hline \mathcal{S}, \mathcal{A} \vdash_C \varphi \wedge \psi \Rightarrow^Q \varphi' \wedge \psi \end{array}$$

REFLEXIVITY :

$$\frac{}{\mathcal{S}, \mathcal{A} \vdash \varphi \Rightarrow^Q \varphi}$$

TRANSITIVITY :

$$\begin{array}{c} \mathcal{S}, \mathcal{A} \vdash_C \varphi_1 \Rightarrow^Q \varphi_2 \quad \mathcal{S}, \mathcal{A} \cup C \vdash \varphi_2 \Rightarrow^Q \varphi_3 \\ \hline \mathcal{S}, \mathcal{A} \vdash_C \varphi_1 \Rightarrow^Q \varphi_3 \end{array}$$

CONSEQUENCE :

$$\begin{array}{c} \models \varphi_1 \rightarrow \varphi'_1 \quad \mathcal{S}, \mathcal{A} \vdash_C \varphi'_1 \Rightarrow^Q \varphi'_2 \quad \models \varphi'_2 \rightarrow \varphi_2 \\ \hline \mathcal{S}, \mathcal{A} \vdash_C \varphi_1 \Rightarrow^Q \varphi_2 \end{array}$$

CASE ANALYSIS :

$$\begin{array}{c} \mathcal{S}, \mathcal{A} \vdash_C \varphi_1 \Rightarrow^Q \varphi \quad \mathcal{S}, \mathcal{A} \vdash_C \varphi_2 \Rightarrow^Q \varphi \\ \hline \mathcal{S}, \mathcal{A} \vdash_C \varphi_1 \vee \varphi_2 \Rightarrow^Q \varphi \end{array}$$

ABSTRACTION :

$$\begin{array}{c} \mathcal{S}, \mathcal{A} \vdash_C \varphi \Rightarrow^Q \varphi' \quad X \cap \text{FreeVars}(\varphi') = \emptyset \\ \hline \mathcal{S}, \mathcal{A} \vdash_C \exists X \varphi \Rightarrow^Q \varphi' \end{array}$$

CIRCULARITY :

$$\begin{array}{c} \mathcal{S}, \mathcal{A} \vdash_{C \cup \{\varphi \Rightarrow^Q \varphi'\}} \varphi \Rightarrow^Q \varphi' \\ \hline \mathcal{S}, \mathcal{A} \vdash_C \varphi \Rightarrow^Q \varphi' \end{array}$$