

Marching Prisms

Rapid section generation in 3D geological model

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Abstract—To efficiently construct the profile in 3D geological model expressed by triangular mesh and rapidly cut the model into separated parts, a new idea is presented in large scale compatible triangular mesh. It promotes the stratum surface meshes to volume model, and then treats the profile generation process as the isosurface extraction process in the model. A new method called **Marching Prisms (MP)** used to construct the isosurface. Using a divide-and-conquer approach to generate inter-slice connectivity, the MP algorithm uses a case table to define the inter-slice triangle topology in tri-prism cells just like the Marching Cubes algorithm in cube grids. After using linear interpolation to calculate the location of inter-slice triangle vertices, the MP algorithm uses the distance from every vertex of a prism to the isosurface to index the cases from the case table. According to the **practical application**, MP algorithm also can handle the ambiguous conditions in isosurface construction. To get the separated parts of the original meshes by the cutting, it is only need to modify the topology on top and bottom of the intersected prisms. Results from demo applications illustrate that the Marching Prisms is effective and efficient.

Keywords—surface intersection; analogical tri-prism; triangle mesh; geological modeling; marching cubes

I. INTRODUCTION

With the technology development, more and more 3D geological modeling use triangle mesh instead of rectangle mesh, it can better fitting the complex model surface and giving a more accurate description to the subsurface, at the same time it sharply reduces the amount of data. Driving by the great success achieved in industry application, there are so many people contribute to the method of triangulation and mesh generation. And now we already have some efficient method to get a better mesh. But in the practice, such as the implementation in geological modeling, not only need a good mesh, but also need serial operations on the mesh, including clipping, cutting, intersecting and so on. Unfortunately, the computational performance is not very well in triangle mesh, especially when do some intersection operation. So how to efficiently get the intersection line in triangle mesh becomes to one of the most important problem in 3D modeling. In this condition, this paper gives a new idea which is based on the isosurface extraction to generate the profile in triangle mesh expressed 3D model and the

performance is very well of this new method in a 3D modeling demo implementation.

II. ALGORITHM TO CUTTING TRIANGLE MESH

Cutting and clipping a stratum model can be generally described as five steps. (1) Modeling the geological model of a serial stratum and define the intersection profile. (2) Using the profile's location information to cutting and clipping all the stratum. (3) Get the separated parts of the cut stratum. (4) Get the attribute value on the profile of the intersection surface from the original stratum attribute. (5) Visualization and display all the results. The difficulty is how to construct the information on the profile surface and how to modify the original stratum to different slice. The whole workflow can be illustrated as the Fig1.

The traditional method to cutting the stratum model is projected the profile to every subsurface and solve the equation group to get the cutting points on every triangle of the stratum, then triangulate all the intersection points and the original points on every stratum to form the different surfaces of the separated results. To get the attribute on the profile, it must record all the points on every intersection line, then splicing them to construct the profile surface. To get all the intersection points in every triangle grid, it must do the time consuming equation group solving process. So the efficiency is not very well.

In this paper, we promote the triangle meshes to analogical tri-prisms (ATP), and treat the intersection profile to be the isosurface in the prism, using an efficient method which is called Marching Prisms to construct the profile which is represented by the triangle mesh as well. To form the separated parts of the result meshes, we make a tiny modify on the boundary cell's topology. This method greatly improves the computing efficiency.

A. Generated Compatible Triangle Mesh And Tri-Prisms

In 3D geological modeling, stratum is presented by triangle mesh, and in the adjacent two layers they usually have a very good consistency characteristic. So we can easily make the mesh to be compatible. Furthermore, the stratum has thickness. Then we connect the corresponding points in different layers to form the analogical tri-prisms grid. The algorithm to compatible triangulation is very well developed now. So in this paper, we use the method by Jinghan Zou, as in [3].

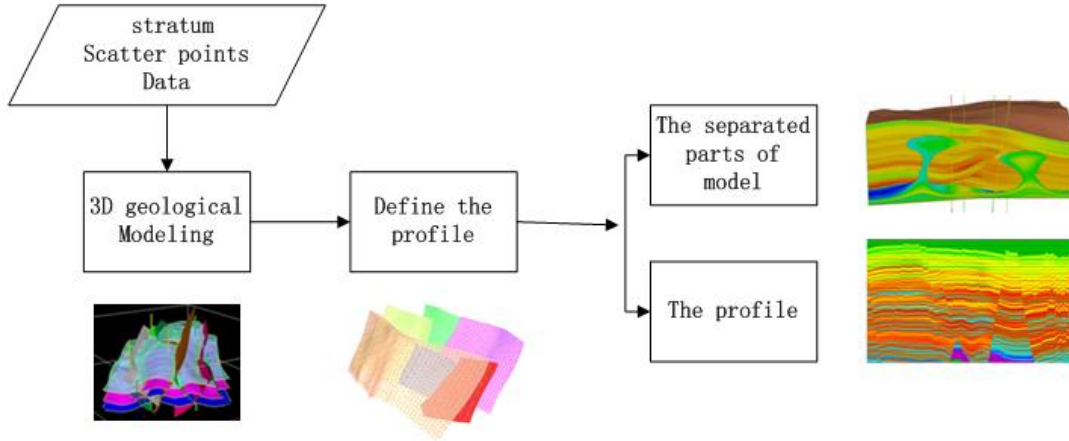


Figure 1. The workflow of cutting a geological model

B. Extract Isosurface In Prisms

1) Marching Cubes algorithm

The basic principle in Marching Cubes is processing the cubes one by one in all the dataset. Calculate the intersection points of the isosurface in every cube. Then look up to the case table for the connection information and connect all the intersect points to form triangle slices which is used to presented the isosurface. When all the cubes are marched, the whole isosurface is well done and it is a triangle mesh. To construct the connection information table, Lorensen list all the conditions of isosurface' topology in a cube, using the attribute value such as positive value means this point above the isosurface and negative value means it below the isosurface at every vertex of the cube to distinguish different conditions. All of the conditions in one cube are $2^8=256$.

2) Marching Tri-Prisms algorithm

Like the Marching cubes, in prisms grid, every cell has 6 vertexes. Give an ID number to every vertex and edge separately as Fig 2. And use the location information relative to the isosurface to set the attribute value in every vertex, use 0 or 1 to demonstrate the different value. All the condition in prisms grid is $2^6=64$. Fig 3 shows each condition that how the isosurface can locate and also it presents all the condition how an intersection profile can be constructed. The case table which is indexed by the attribute value of vertexes, records all the topology information. We use the edge ID of the prism to identify different vertexes of triangle slice of the isosurface. Then interpolation the exact value of isosurface intersection points in every edge of the prism. Finally, we record all the triangle slices in every prism and construct the isosurface and the profile in the whole model.

The interpolation can be easily implementation by use the linear interpolation formula as in (1), p_1 , p_2 is the adjacent vertexes in the prism, p is the intersection point of the isosurface and the prism's edge which between p_1 and p_2 , $isolevel$ is the attribute value on the isosurface. $valp_1$, $valp_2$ is the attribute value on p_1 and p_2 .

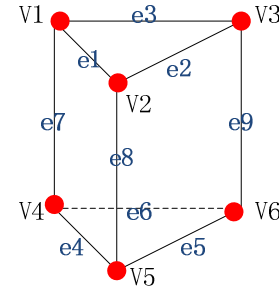


Figure 2. Define ID numbers in a prism grid

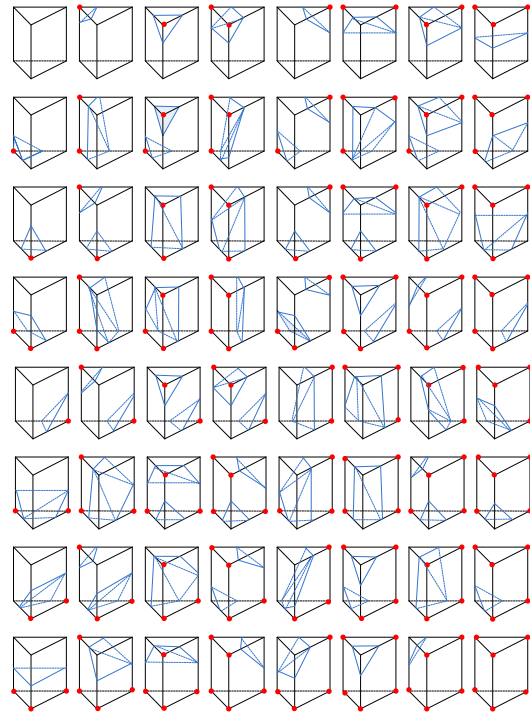


Figure 3. All the intersection conditions in a prism grid

$$\begin{aligned}
p.x &= p1.x + \frac{(isolevel - valp1)}{(valp2 - valp1)} * (p2.x - p1.x); \\
p.y &= p1.y + \frac{(isolevel - valp1)}{(valp2 - valp1)} * (p2.y - p1.y); \\
p.z &= p1.z + \frac{(isolevel - valp1)}{(valp2 - valp1)} * (p2.z - p1.z);
\end{aligned} \tag{1}$$

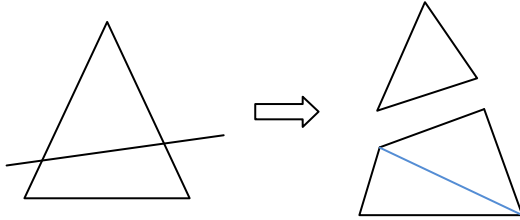


Figure 4. Cut one triangle to form the separated parts

As described above, the isosurface is presented by the triangle mesh, so there is no need to triangulate the intersection points and make it to be a triangle net as the other stratum expressed in the model.

C. Generate The Separate Parts of The Original Model

Because after the intersection, there is no change in the model which is relatively far from the intersection line, we could only focus on the nearby parts of the mesh. Actually, in the MP algorithm it is very easy to identify the boundary cells along the intersection line. In another word, we can easily figure out the no change cells which corresponding to the first and last condition in the case table indicated in the Fig 3. And to the boundary cells, because there could be only one intersection line in one triangle, we can easily modify the boundary triangles to separate them to different parts. The modify operation is show in Fig 4. We use the attribute value on the prism's vertices to distinguish the different sides of the profile, and then separate the triangle mesh to two parts.

D. Algorithm Description

In conclusion, as subscribed above, use the MP algorithm to cutting a stratum model can be shortly written in the following steps.

Input:

- A serial of triangle nets $G_1, G_2, G_3 \dots G_n$;
- The location information or formula Q of the profile.

Output:

- The intersection profile surface in triangle net model;
- The separated parts of $G_{11}, G_{12}, G_{21}, G_{22}, G_{31}, G_{32} \dots G_{n1}, G_{n2}$.

a) Generated the compatible triangle meshes from $G_1, G_2, G_3 \dots G_n$, get the new serial meshes of $G_1', G_2', G_3' \dots G_n'$, then construct the prism grids.

b) Calculate the distance from every vertex of a prism to the target surface Q . And use the distance value to initial the attribute value at every vertex in prisms.

c) Use the attribute value in vertex to index the case table. Index = $[v1][v2][v3][v4][v5][v6]$.

d) Use linear interpolation to get the exact intersection point in every edge of prism $Ve1, Ve2, Ve3, Ve4, Ve5, Ve6$.

e) Look up to the connection table, by the indication of connection information to construct the triangle slices and form the entire isosurface.

f) Modify the topology in original mesh cells which are cut by the intersection profile to construct the separated parts of it. Get the new serial meshes $G_{11}, G_{12}, G_{21}, G_{22}, G_{31}, G_{32} \dots G_{n1}, G_{n2}$.

III. ALGORITHM ANALYSIS

A. The Ambiguous In MP Algorithm

In the well know Marching Cubes, since 1988, people have found that there are ambiguous in this algorithm, which means the same vertex attribute value corresponded to different isosurface connection. This problem makes the isosurface have holes when using this method to construct it from a volume dataset.

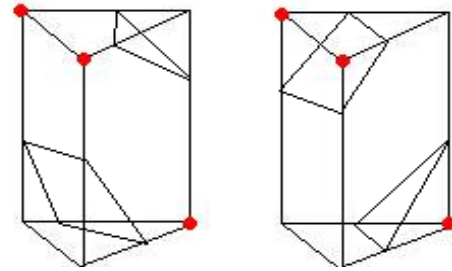


Figure 5. The ambiguous connection condition

In the MP algorithm, we still suffer from this problem, such as Fig 5. In this condition, we cannot decide which connection way should be use by the information in this prism merely. But the isosurface usually smooth and have the similar normal in geologic modeling. So we can use the normal in adjacent cell which is similar to this cell to determine how to form the surface in this prism. This way can efficiently solve this ambiguous. Moreover, usually we use only one surface to intersect the model in one time. So in this condition, we consider it as a very large curvature surface. And the MP algorithm can efficiently handle the complex condition such as show in Fig 5.

B. The Efficiency Improvement

Compare to the traditional algorithm, the Marching Prism method avoids the time consuming calculation to solve equation group to get the intersection point also it remove the triangulation to the intersection points with the original mesh to form the new separated parts of meshes. The MP algorithm use the efficient look up table process and a relatively rapid interpolation computation to construct the isosurface. It makes the cutting and clipping process in geological model more quickly. As we can see from the equation 1, the interpolation one point only need 1 addition, 3 subtractions, 2 multiplications and 1 division. But if we use

an equation group to calculate the intersection point, consider the simplest condition, the profile is a plane, and then we have the equations like (2).

$$\begin{cases} \frac{p-p1.x}{p2.x-p1.x} = \frac{p-p1.y}{p2.y-p1.y} = \frac{p-p1.z}{p2.z-p1.z} \\ Ax + By + Cz + D = 0 \end{cases} \quad (2)$$

To solve this equation group we need too much calculation than the interpolation. The compare can be seen in table 1. Well, on the other conditions, we use almost the same time to get the intersection points in MP, because we always use the same interpolation process to get the intersection points.

Table 1 compare of different target profile computation efficiency between MP and Traditional method to calculate one section point.

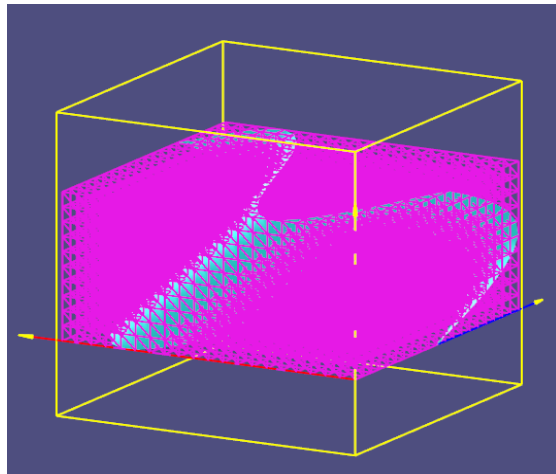
TABLE I. COMPARISON

	<i>MP</i>	<i>Traditional method</i>
Addition	1 step	8 steps
Subtraction	3 steps	6 steps
Multiplication	2 steps	9 steps
Division	1 step	1 step

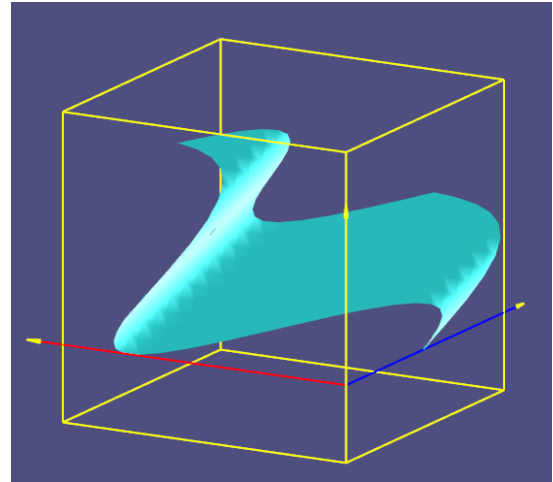
IV. IMPLEMENTATION

In our demo program, firstly, set the target profile which is presented by a formula such as a plane, a sin function or whatever you want. And in the simple condition of a plane, we also offer a convenience operation of mouse press to construct the plane. And in the implementation, use mouse drag to modify the parameter of the target surface's formula which realized the real time profile generation controlled by the user's will.

In Fig 6, it is a result in a mesh set which size is 80*160*80; and the isosurface is a sin function. (b) is the isosurface profile extracted from the prism grid in (a).



(a) The prisms and isosurface with a sin function



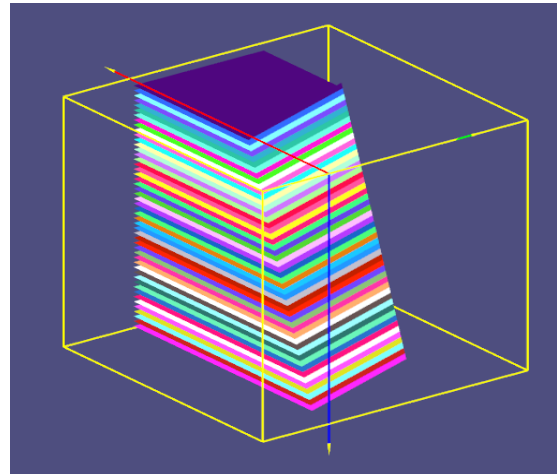
(b) The isosurface extracted from (a)

Figure 6. The sample of isosurface extraction from ATP

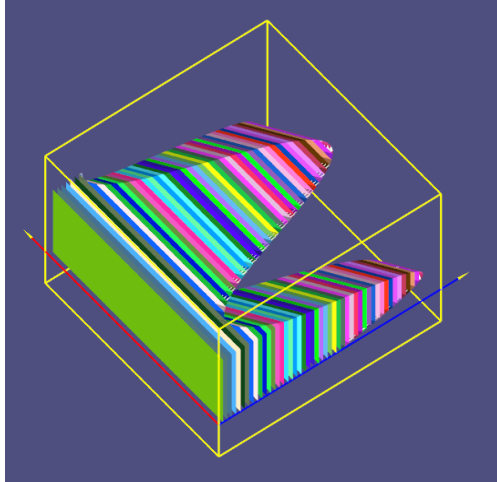
In Fig 7, there is another model which has 50 layers, we use a plane which is defined by the user's mouse click to cut the model and the result is show in (a). (b) is the model cutting by a sin function surface.

V. CONCLUSION

According to the application of the 3D geological modeling in the practice, this paper demonstrates the effectiveness and efficiency of the Marching Prisms method, furthermore, this method sharply improves the computational performance, reaching the expected goal. This paper gives a new idea on intersection profile construction and triangle mesh cutting. The MP algorithm also offers a new idea to construct the fault surface in geological modeling and the attributes in slice modeling. The MP algorithm is based on the ATP grid, which can be easily got from the 3D geological model and the special thin stratum in the petroleum prospecting area; it can also be widely used in ATP grid in the other field.



(a)The model is cutting by a plane



(b)The model cutting by a sin surface

Figure 7. A geological model after cutting by different profile

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REFERENCES

- [1] William E. Lorensen, Harvey E. Cline, "Marching Cubes: A high resolution 3D surface construction algorithm," *Computer Graphics*, Volume 21, Number 4, July 1987[SIGGRAPH '87, Anaheim, July 27-31, 1987].
- [2] Qin Yang, "Computer graphics," Tsinghua University Press, 2005.
- [3] Zou Jinghan, Yang Qin, "3D geologic modeling based on analogical tri-prism grid," *The paper collection of 5th Academic Forum for graduate students at Beihang University*. 2008, pp 275-281.
- [4] Hong Xiong, Zhang Jian-xun, Zeng Qing-sen, "Research on section of 3D geological model based on similar tri-prism," *Computer Engineering and Design*, vol. 29, No. 15, Aug. 2008, pp. 4097-4099.
- [5] Li Jiangwei, Li Jigang, "Rapid rectangular grid-based clipping algorithm of surface," *Microcomputer information*, vol 24, sept. 2008, pp 157-159.
- [6] Zaoli Li, "The Research on Visualization in the 3D Electromagnetic Field Calculation by the Finite Element Method," North China electronic university, Master thesis, 2002.
- [7] Zhang Yu, Bai Shi-wei, "An Approach of 3D Stratum Modeling Based on Tri-Prism Volume Elements," *Journal of Image and Graphics*, vol. 6(A), No. 3, Mar 2001, pp 285-290.
- [8] Simon W. Houlding, "3D geoscience modeling—computer techniques for geological characterization," Springer-Verlag, 1994.
- [9] Gregory M. Nielson, "On Marching Cubes," *IEEE Transactions on visualization and computer graphics*, vol. 9, No. 3, July-sept, 2003, pp 283-297.