Assignment 1

Saturday, January 30, 2021 03:54 PM

p 28 1a-d, 3, 4

1. absolute error & relative error

a) $p = \pi$, $p^* = 22/7$

abs = $|\pi - 22/7| = \boxed{0.00126}$

relative = $\frac{|\pi - 22/7|}{\pi} = \boxed{4.02 \cdot 10^{-4}}$

b) $p = \pi$, $p^* = 3.1416$

abs: $|\pi - 3.1416| = 7.34 \cdot 10^{-6}$

rel: $\frac{|\pi - 3.1416|}{\pi} = 2.34 \cdot 10^{-6}$

c) $p = e$, $p^* = 2.718$

abs: $|e - 2.718| = 2.82 \cdot 10^{-4}$

rel: $\frac{|e - 2.7181|}{e} = 1.04 \cdot 10^{-4}$

d) $p = \sqrt{2}$, $p^* = 1.414$

abs: $|\sqrt{2} - 1.414| = 2.14 \cdot 10^{-4}$

rel: $\frac{|\sqrt{2} - 1.4141|}{\sqrt{2}} = 1.51 \cdot 10^{-4}$

3. p^* must approx p with relative error at most 10^{-3}.
Largest interval where p^* must lie for each value p.

a) 150, max abs error = 0.15 $|150 - p^*|$

150

$p^* \in [149.85, 150.15]$

b) 900, Max abs error = 0.9

$= 10^{-3} p^* \in [899.1, 900.9]$

c) 1500

$p^* \in [1498.5, 1501.5]$

d) 90

$p^* \in [89.91, 90.09]$

4. exact (I), 3-digit chop (II), 3-digit round (III), rel error for II & III (IV)

a) $\frac{4}{5} + \frac{1}{3} = \boxed{\frac{17}{15}}$ (I) $= 1.1\overline{3}$

chop: $0.8 + 0.333 = \boxed{1.133}$

round: $0.8 + 0.333 = \boxed{1.133}$

chop rel error $\left|\frac{17}{15} - 1.133\right| = \boxed{3.\overline{3}E-4}$

round rel error $\left|\frac{17}{15} - 1.133\right| = \boxed{3.\overline{3}E-4}$

b) $\frac{4}{5} \cdot \frac{1}{3} = \boxed{\frac{4}{15}}$ (I) $= .2\overline{66}$

chop: $0.8 \cdot 0.333 = \boxed{0.2664}$

round: $0.8 \cdot 0.333 = \boxed{0.2664}$

chop rel error $\left|\frac{4}{15} - 0.2664\right| = \boxed{2.\overline{6}E-4}$

round rel error $\left|\frac{4}{15} - 0.2664\right| = \boxed{2.\overline{6}E-4}$

$\frac{400}{660} - \frac{97}{660} = \frac{303}{660}$

$\left(\frac{11}{33} + \frac{9}{33}\right)\frac{20}{20} - \frac{3.33}{660}$

c) $\left(\frac{1}{3} - \frac{3}{11}\right) + 3/20 = \frac{2}{33} + 3/20 = \boxed{\frac{139}{660}}$ (I) $= 0.2\overline{106}$

chop: $(0.333 - 0.272) + 0.15 = \boxed{0.211}$

round: $(0.333 - 0.273) + 0.15 \approx \boxed{0.210}$

chop rel error: $\left|\frac{\frac{139}{660} - .211}{\frac{139}{660}}\right| = \boxed{1.87E-3}$

round rel error: $\left|\frac{\frac{139}{660} - .21}{\frac{139}{660}}\right| = \boxed{2.88E-3}$

d) $\left(\frac{1}{3} + \frac{3}{11}\right) - 3/20 = \left(\frac{11}{33} + \frac{9}{33}\right) - \frac{99}{660} = \boxed{\frac{303}{660}}$ (I) $= 0.45\overline{90}$

chop: $(0.333 + 0.272) - 0.15 = \boxed{0.455}$

round: $(0.333 + 0.273) - 0.15 = \boxed{0.456}$

chop rel error: $\left|\frac{\frac{303}{660} - .455}{\frac{303}{660}}\right| = \boxed{8.91E-3}$

round rel error: $\left|\frac{\frac{303}{660} - 0.456}{\frac{303}{660}}\right| = \boxed{6.73E-3}$

$\left(\frac{11}{33} + \frac{9}{33}\right)\frac{20}{20} = \frac{66}{660} = \frac{303}{660}$