1. **Member of the team:**

- Name: Tram Trinh
- NetID: htrinh

2. **Total time spent on the project:**

   ~3 hours

3. **Code development:**

a. Planning:

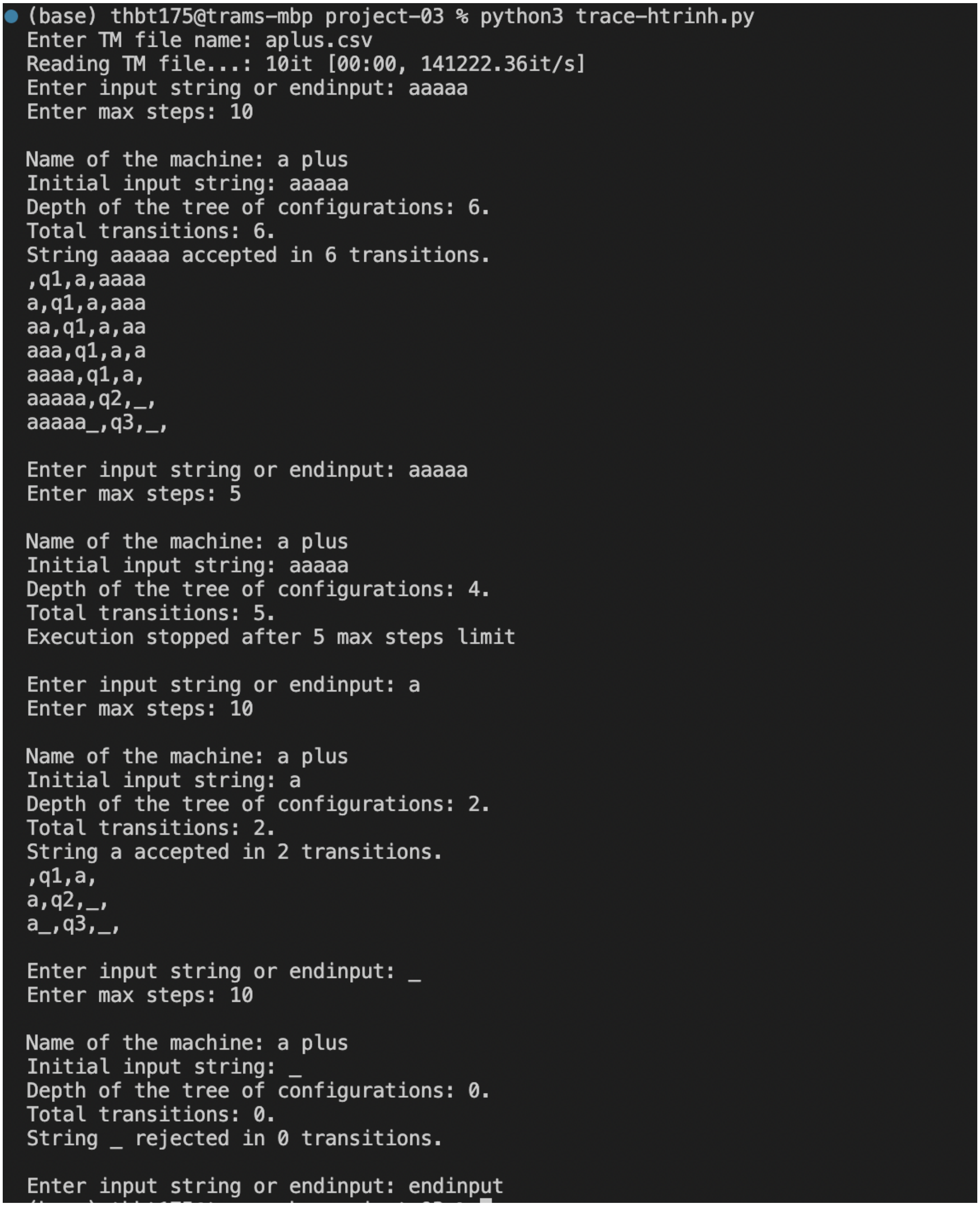
- Review lectures about the non-deterministic Turing Machine
- Understand the input and output format for the program
- Create the Turing Machine (TM) and Tape class objects
- Create the function to read the Turing Machine from the CSV input and store all needed data in the TM object
- Implement trace function
- Test the program

b. Overview of the program:

The program will ask the user to enter the file name of the Turing Machine. After reading the Turing Machine, it will continuously ask the user to input the string and the maximum steps (or maximum transition). Every time the program reads in an input string and a max step, it will print the output to the screen.

The program will terminate when the input string is “endinput”. The program will then record all the computed strings and their outputs in the text file with the name [name of machine]-output.txt

For example, given the a-plus Turing Machine and a list of input strings. The figure below shows all the outputs that were printed to the screen, and after reading “endinput”, the program stopped and wrote the file “a plus-output.txt”



```
(base) thbt175@trams-mbp project-03 % python3 trace-htrinh.py
Enter TM file name: aplus.csv
Reading TM file...: 10it [00:00, 141222.36it/s]
Enter input string or endinput: aaaaa
Enter max steps: 10

Name of the machine: a plus
Initial input string: aaaaa
Depth of the tree of configurations: 6.
Total transitions: 6.
String aaaaa accepted in 6 transitions.
,q1,a,aaaa
a,q1,a,aaa
aa,q1,a,aa
aaa,q1,a,a
aaaa,q1,a,
aaaaa,q2,_,
aaaaa_,q3,_,

Enter input string or endinput: aaaaa
Enter max steps: 5

Name of the machine: a plus
Initial input string: aaaaa
Depth of the tree of configurations: 4.
Total transitions: 5.
Execution stopped after 5 max steps limit

Enter input string or endinput: a
Enter max steps: 10

Name of the machine: a plus
Initial input string: a
Depth of the tree of configurations: 2.
Total transitions: 2.
String a accepted in 2 transitions.
,q1,a,
a,q2,_,
a_,q3,_,

Enter input string or endinput: _
Enter max steps: 10

Name of the machine: a plus
Initial input string: _
Depth of the tree of configurations: 0.
Total transitions: 0.
String _ rejected in 0 transitions.

Enter input string or endinput: endinput
```

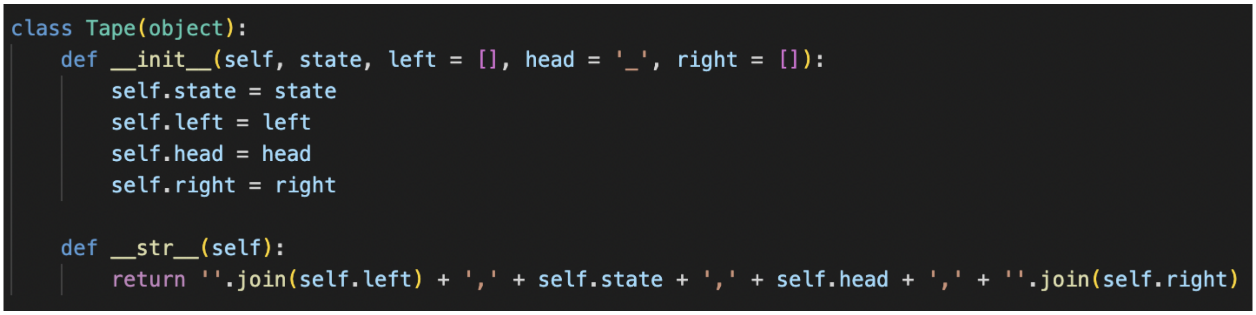
4. **Language and libraries:**

- Language: Python
- Libraries: csv, time, tqdm, sys, os, collections

5. Key data structure:

I applied BFS to trace all the nondeterministic paths of the tree configurations from the given Turing Machine and input string. I have deque to store the 3-tuple (tape, level, path).

The variable tape is an object that stores the current state, the current tape head, and characters on the left and right of the tape head.



```
class Tape(object):
    def __init__(self, state, left = [], head = '_', right = []):
        self.state = state
        self.left = left
        self.head = head
        self.right = right

    def __str__(self):
        return ''.join(self.left) + ',' + self.state + ',' + self.head + ',' + ''.join(self.right)
```

The level is the level of the tree that the tape is at, while the path stores all transitions from the start state to the final state (which can be either accept state, reject state, or neither if reaching the limit steps).

I used the BFS to get all the transitions at each level, and I used an unordered map (or dictionary in Python) visited to keep track of the depth of the tree.

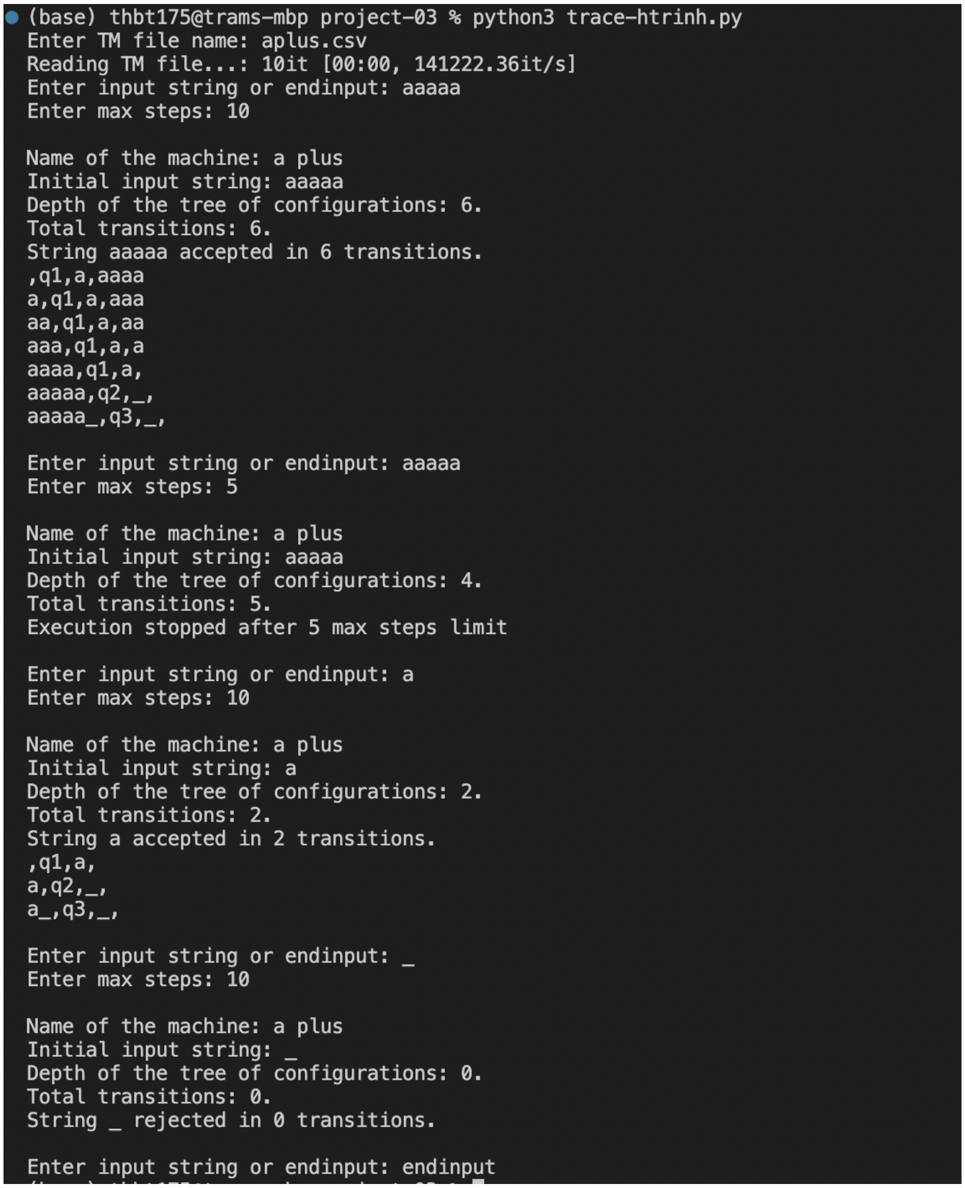
I started the queue with the initial tape at the start state, level 0, and a path that starts that includes the start state. Then I started looping through the queue with two conditions: the queue is not empty and the program has not reached the limit steps.

In every loop cycle, I pop an element out of the queue which gives the 3-tuple (tape, level, path). If the level is new, I added it as the new key to the visited dictionary. I also append the tape to its current level in the visited dictionary for the debugging purpose so that I am able to keep track of the tree of configurations. The program continues checking if the current state is an accept state. If so, it then halts and returns the output. If not, it continues searching for new transitions, appends them to the queue, and updates the level and path. If there is no transition, the program then moves on to another element in the queue.

6. Test cases:

a. aplus.csv: (Test case from the Drive)

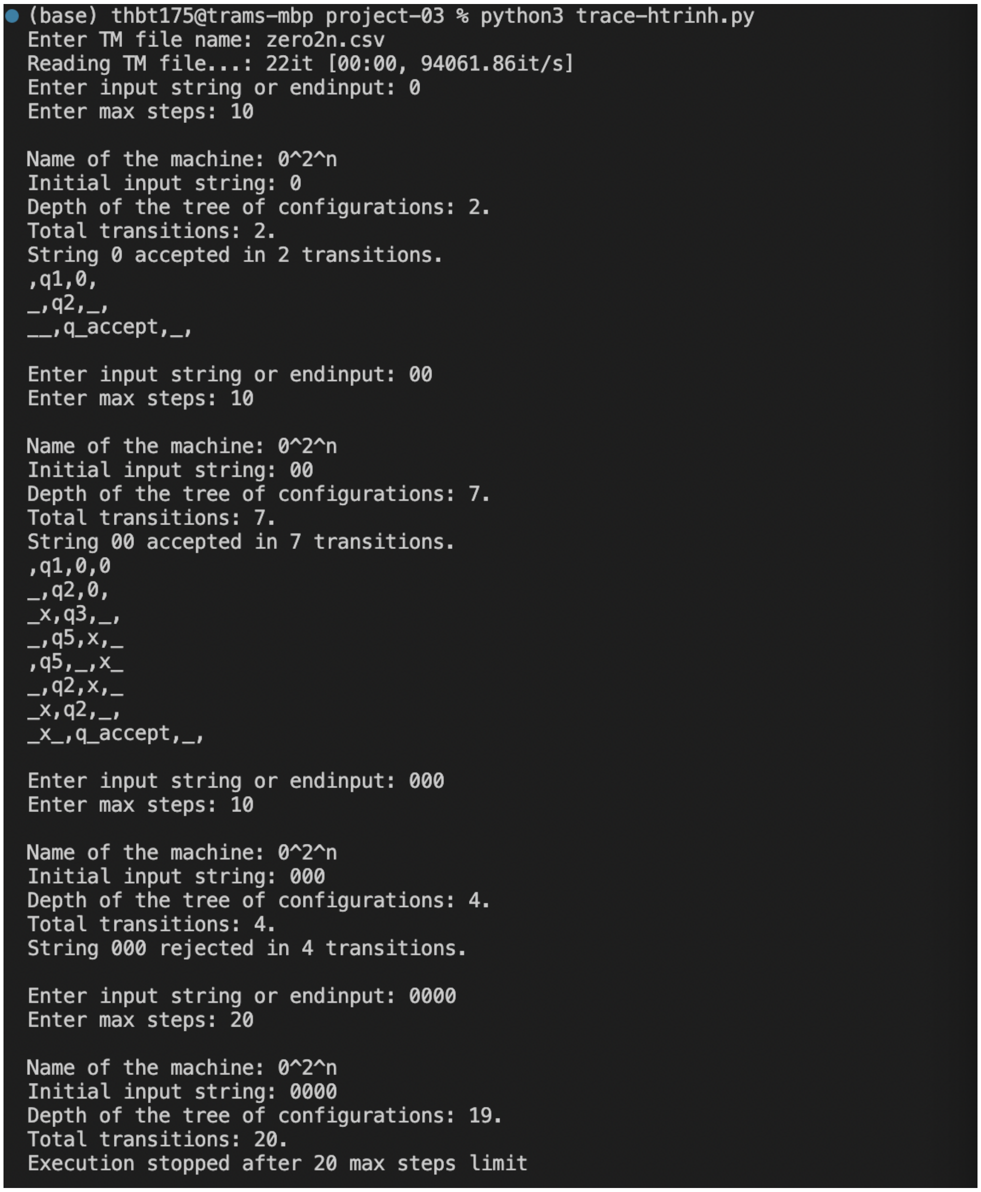
- The Turing Machine accepts the language a+
- The program outputs the correct answers for all the input strings.



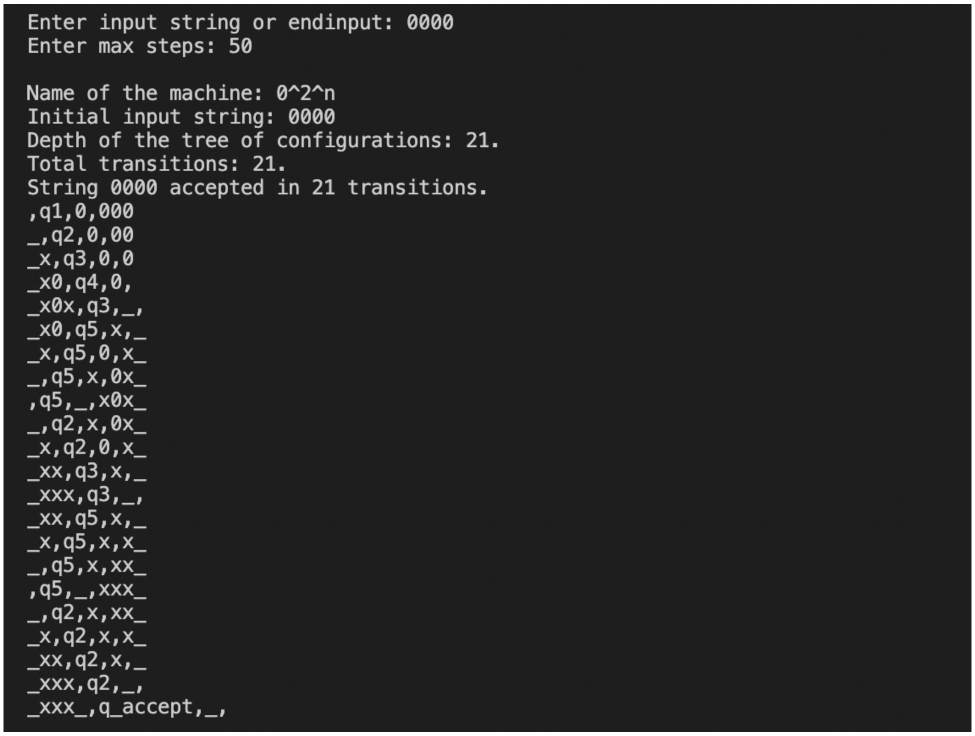
```
(base) thbt175@trams-mbp project-03 % python3 trace-htrinh.py
Enter TM file name: aplus.csv
Reading TM file...: 10it [00:00, 141222.36it/s]
Enter input string or endinput: aaaaa
Enter max steps: 10

Name of the machine: a plus
Initial input string: aaaaa
Depth of the tree of configurations: 6.
Total transitions: 6.
String aaaaa accepted in 6 transitions.
,q1,a,aaaa
a,q1,a,aaa
aa,q1,a,aa
aaa,q1,a,a
aaaa,q1,a,
aaaaa,q2,_,
aaaaa_,q3,_,

Enter input string or endinput: aaaaa
Enter max steps: 5

Name of the machine: a plus
Initial input string: aaaaa
Depth of the tree of configurations: 4.
Total transitions: 5.
Execution stopped after 5 max steps limit

Enter input string or endinput: a
Enter max steps: 10

Name of the machine: a plus
Initial input string: a
Depth of the tree of configurations: 2.
Total transitions: 2.
String a accepted in 2 transitions.
,q1,a,
a,q2,_,
a_,q3,_,

Enter input string or endinput: _
Enter max steps: 10

Name of the machine: a plus
Initial input string: _
Depth of the tree of configurations: 0.
Total transitions: 0.
String _ rejected in 0 transitions.

Enter input string or endinput: endinput
```

b. zero2n.csv: (My test case, but also in the drive)

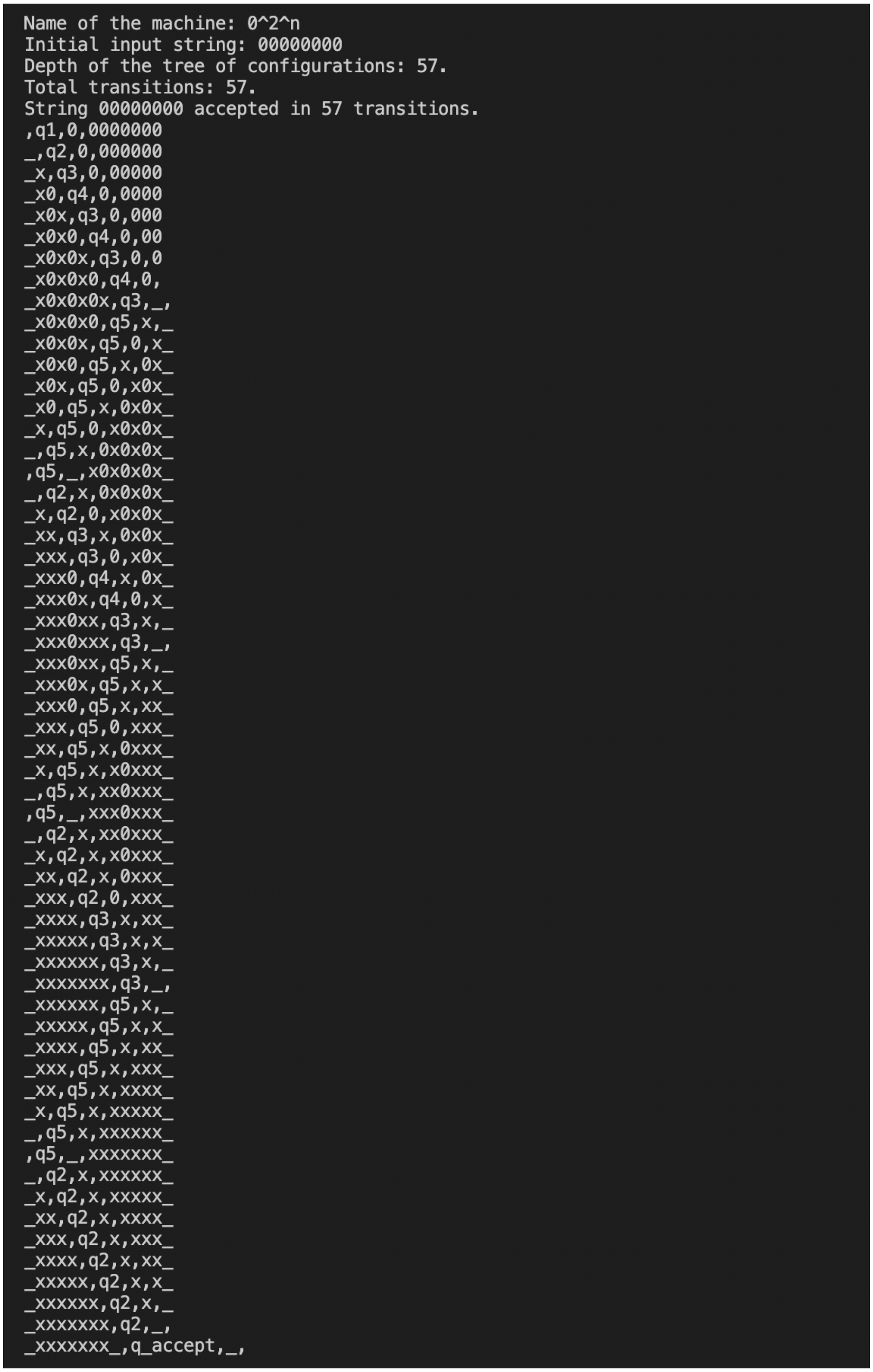
- The given Turing Machine accepts the language L = {0^(2^n) | n >= 0}. It means that the machine would accept 0, 00, 0000, 00000000, etc. and reject 000, 00000, 000000, etc.
- The program produces the right output for test cases.



```
● (base) thbt175@trams-mbp project-03 % python3 trace-htrinh.py
  Enter TM file name: zero2n.csv
  Reading TM file...: 22it [00:00, 94061.86it/s]
  Enter input string or endinput: 0
  Enter max steps: 10

  Name of the machine: 0^2^n
  Initial input string: 0
  Depth of the tree of configurations: 2.
  Total transitions: 2.
  String 0 accepted in 2 transitions.
  ,q1,0,
  _,q2,_,
  __,q_accept,_,

  Enter input string or endinput: 00
  Enter max steps: 10

  Name of the machine: 0^2^n
  Initial input string: 00
  Depth of the tree of configurations: 7.
  Total transitions: 7.
  String 00 accepted in 7 transitions.
  ,q1,0,0
  _,q2,0,
  _x,q3,_,
  _,q5,x,_
  ,q5,_,x_
  _,q2,x,_
  _x,q2,_,
  _x_,q_accept,_,

  Enter input string or endinput: 000
  Enter max steps: 10

  Name of the machine: 0^2^n
  Initial input string: 000
  Depth of the tree of configurations: 4.
  Total transitions: 4.
  String 000 rejected in 4 transitions.

  Enter input string or endinput: 0000
  Enter max steps: 20

  Name of the machine: 0^2^n
  Initial input string: 0000
  Depth of the tree of configurations: 19.
  Total transitions: 20.
  Execution stopped after 20 max steps limit
```



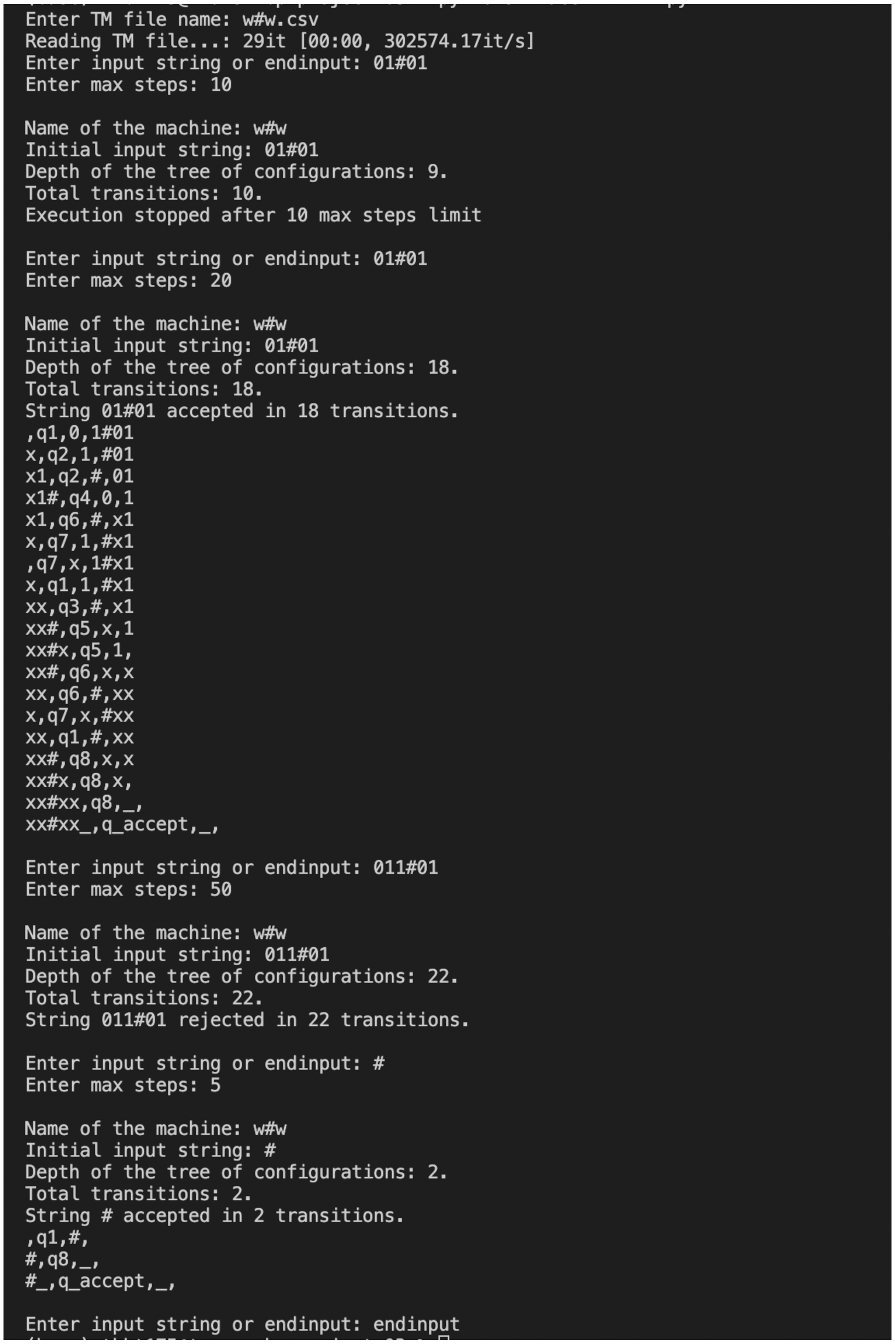
```
Enter input string or endinput: 0000
Enter max steps: 50

Name of the machine: 0^2^n
Initial input string: 0000
Depth of the tree of configurations: 21.
Total transitions: 21.
String 0000 accepted in 21 transitions.
,q1,0,000
_,q2,0,00
_x,q3,0,0
_x0,q4,0,
_x0x,q3,_,
_x0,q5,x,_
_x,q5,0,x_
_,q5,x,0x_
,q5,_,x0x_
_,q2,x,0x_
_x,q2,0,x_
_xx,q3,x,_
_xxx,q3,_,
_xx,q5,x,_
_x,q5,x,x_
_,q5,x,xx_
,q5,_,xxx_
_,q2,x,xx_
_x,q2,x,x_
_xx,q2,x,_
_xxx,q2,_,
_xxx_,q_accept,_,
```



```
Name of the machine: 0^2^n
Initial input string: 00000000
Depth of the tree of configurations: 57.
Total transitions: 57.
String 00000000 accepted in 57 transitions.
,q1,0,0000000
_,q2,0,000000
_x,q3,0,00000
_x0,q4,0,0000
_x0x,q3,0,000
_x0x0,q4,0,00
_x0x0x,q3,0,0
_x0x0x0,q4,0,
_x0x0x0x,q3,_,
_x0x0x0,q5,x,_
_x0x0x,q5,0,x_
_x0x0,q5,x,0x_
_x0x,q5,0,x0x_
_x0,q5,x,0x0x_
_x,q5,0,x0x0x_
_,q5,x,0x0x0x_
,q5,_,x0x0x0x_
_,q2,x,0x0x0x_
_x,q2,0,x0x0x_
_xx,q3,x,0x0x_
_xxx,q3,0,x0x_
_xxx0,q4,x,0x_
_xxx0x,q4,0,x_
_xxx0xx,q3,x,_
_xxx0xxx,q3,_,
_xxx0xx,q5,x,_
_xxx0x,q5,x,x_
_xxx0,q5,x,xx_
_xxx,q5,0,xxx_
_xx,q5,x,0xxx_
_x,q5,x,x0xxx_
_,q5,x,xx0xxx_
,q5,_,xxx0xxx_
_,q2,x,xx0xxx_
_x,q2,x,x0xxx_
_xx,q2,x,0xxx_
_xxx,q2,0,xxx_
_xxxx,q3,x,xx_
_xxxxx,q3,x,x_
_xxxxxx,q3,x,_
_xxxxxxx,q3,_,
_xxxxxx,q5,x,_
_xxxxx,q5,x,x_
_xxxx,q5,x,xx_
_xxx,q5,x,xxx_
_xx,q5,x,xxxx_
_x,q5,x,xxxxx_
_,q5,x,xxxxxx_
,q5,_,xxxxxxx_
_,q2,x,xxxxxx_
_x,q2,x,xxxxx_
_xx,q2,x,xxxx_
_xxx,q2,x,xxx_
_xxxx,q2,x,xx_
_xxxxx,q2,x,x_
_xxxxxx,q2,x,_
_xxxxxxx,q2,_,
_xxxxxxx_,q_accept,_,
```

c. w#w.csv: (My test case, but also in the Drive)

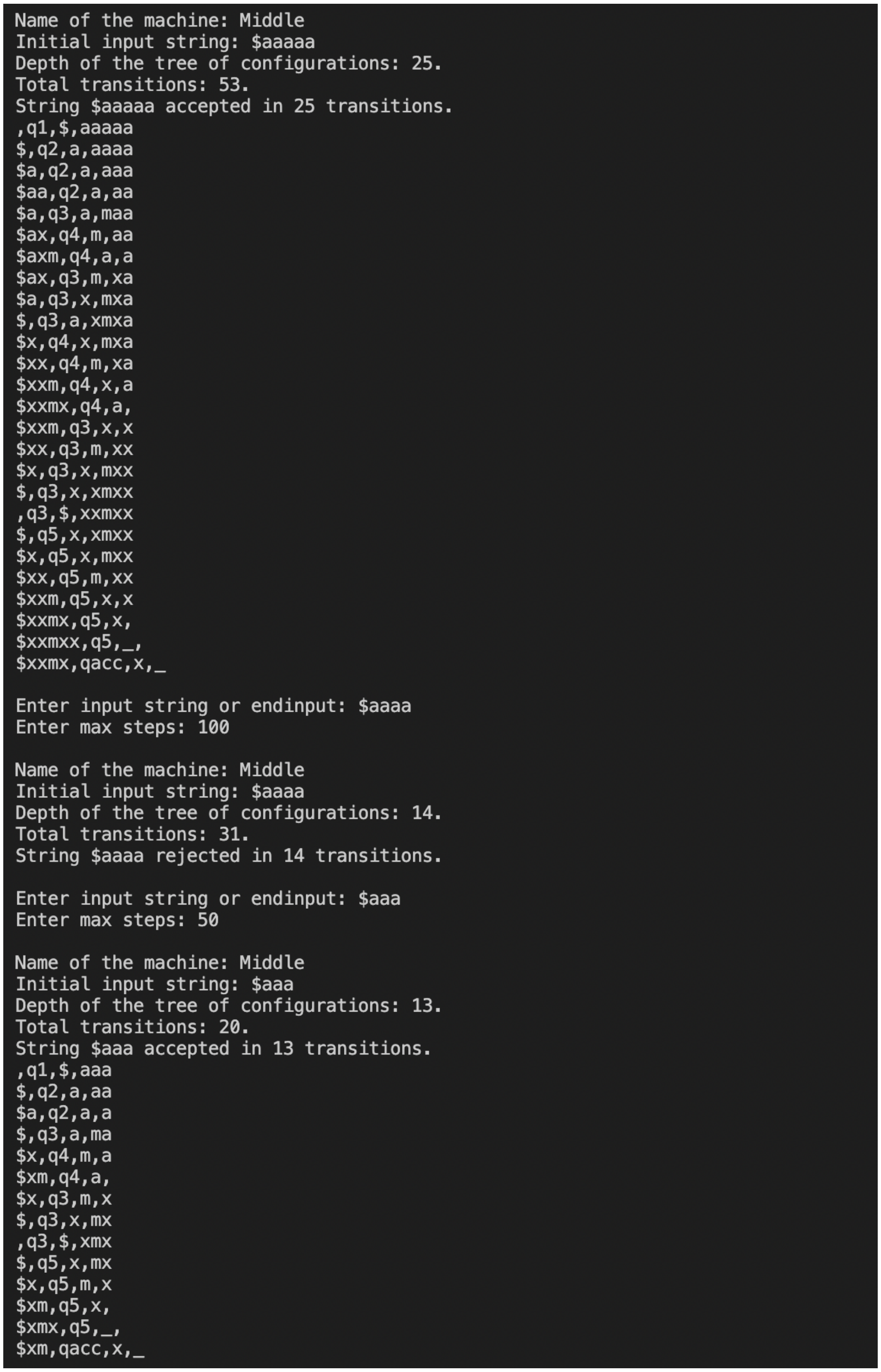
- The Turing Machine accepts the language L = {w#w | w ∈ {0,1}*}. It accepts the strings such as #, 01#01, 000#000, 101#101, etc., and rejects 011#01, #000, etc.
- The program produces the right output for test cases.



```
Enter TM file name: w#w.csv
Reading TM file...: 29it [00:00, 302574.17it/s]
Enter input string or endinput: 01#01
Enter max steps: 10

Name of the machine: w#w
Initial input string: 01#01
Depth of the tree of configurations: 9.
Total transitions: 10.
Execution stopped after 10 max steps limit

Enter input string or endinput: 01#01
Enter max steps: 20

Name of the machine: w#w
Initial input string: 01#01
Depth of the tree of configurations: 18.
Total transitions: 18.
String 01#01 accepted in 18 transitions.
,q1,0,1#01
x,q2,1,#01
x1,q2,#,01
x1#,q4,0,1
x1,q6,#,x1
x,q7,1,#x1
,q7,x,1#x1
x,q1,1,#x1
xx,q3,#,x1
xx#,q5,x,1
xx#x,q5,1,
xx#,q6,x,x
xx,q6,#,xx
x,q7,x,#xx
xx,q1,#,xx
xx#,q8,x,x
xx#x,q8,x,
xx#xx,q8,_,
xx#xx_,q_accept,_,

Enter input string or endinput: 011#01
Enter max steps: 50

Name of the machine: w#w
Initial input string: 011#01
Depth of the tree of configurations: 22.
Total transitions: 22.
String 011#01 rejected in 22 transitions.

Enter input string or endinput: #
Enter max steps: 5

Name of the machine: w#w
Initial input string: #
Depth of the tree of configurations: 2.
Total transitions: 2.
String # accepted in 2 transitions.
,q1,#,
#,q8,_,
#_,q_accept,_,

Enter input string or endinput: endinput
```

d. <u>Middle.csv: (Test case from the Drive)</u>

- The program produces the right output for input strings.



```
Name of the machine: Middle
Initial input string: $aaaaa
Depth of the tree of configurations: 25.
Total transitions: 53.
String $aaaaa accepted in 25 transitions.
,q1,$,aaaaa
$,q2,a,aaaa
$a,q2,a,aaa
$aa,q2,a,aa
$a,q3,a,maa
$ax,q4,m,aa
$axm,q4,a,a
$ax,q3,m,xa
$a,q3,x,mxa
$,q3,a,xmxa
$x,q4,x,mxa
$xx,q4,m,xa
$xxm,q4,x,a
$xxmx,q4,a,
$xxm,q3,x,x
$xx,q3,m,xx
$x,q3,x,mxx
$,q3,x,xmxx
,q3,$,xxmxx
$,q5,x,xmxx
$x,q5,x,mxx
$xx,q5,m,xx
$xxm,q5,x,x
$xxmx,q5,x,
$xxmxx,q5,_,
$xxmx,qacc,x,_

Enter input string or endinput: $aaaa
Enter max steps: 100

Name of the machine: Middle
Initial input string: $aaaa
Depth of the tree of configurations: 14.
Total transitions: 31.
String $aaaa rejected in 14 transitions.

Enter input string or endinput: $aaa
Enter max steps: 50

Name of the machine: Middle
Initial input string: $aaa
Depth of the tree of configurations: 13.
Total transitions: 20.
String $aaa accepted in 13 transitions.
,q1,$,aaa
$,q2,a,aa
$a,q2,a,a
$,q3,a,ma
$x,q4,m,a
$xm,q4,a,
$x,q3,m,x
$,q3,x,mx
,q3,$,xmx
$,q5,x,mx
$x,q5,m,x
$xm,q5,x,
$xmx,q5,_,
$xm,qacc,x,_
```