Lecture #4 Prelims – Let's Talk About Combinatorics (February 22, 2023)

REMINDERS!

1) ALWAYS FOLLOW MY CLASSROOM PROCEDURES.

2) ACTIVITIES ARE ALWAYS UPDATED IN GOOGLE CLASSROOM, & MYOPENMATH

3) ALWAYS SUBMIT YOUR WORK ON TIME. NO MORE SUBMISSSION OF LATE WORK.

4) THE WEEKLY HOMEWORK IS ALWAYS DUE EVERY SATURDAY at 10:00 AM

5) CHECK THE UPDATES ON GOOGLE CLASSROOM AND DO YOUR MISSING ASSIGNMENTS as the GRADES are always UPDATED.

Note: **Activities that you failed to submit on-time will automatically disappear on the system.**

MyOpenMath

1) Bellwork # 4 (Prelims) - Any Topics Covered 02.22.23
2) Activity #4 (Prelims) - Combinatorics and Everything 02.22.23
(Due Date: February 23, 2023 11:59 PM)
3) Weekly Homework # 4 (Prelims) – Knowledge Retention is Essential 02.22-28.23 ***(Due Date: February 28, 2023, 11:59 PM)***

ANNOUNCEMENT!

PRELIMINARY EXAMINATIONS (MARCH 1 – 4, 2023)
TOPICS COVERED – LESSONS 1 -3.

EXAM TYPES
MATCHING TYPE 1-10
MULTIPLE CHOICE 11-25
TRUE OR FALSE 26-35
PROBLEM SOLVING 36-60

At the end of the lesson, students will be able to:
1) apply basic counting principles.
2) identify combinatorics.
3) perform different operations related to combinatorics.
4) perform probability with permutations and combinations

CHECK for UNDERSTANDING 1
True or False

1) Every W is N.

2) {-2, 3, 8} is equivalent to **{0, 4, 5}**

3) {8, 2, 5} is not equal to {1, 2, 5}

4) {} $\subseteq$ **{1, 2, 5}**

5) {1, 2, 5} $\subseteq$ **{1, 2, 5}**

6) $27 \notin \{Z\}$

7) Every irrational number is a whole number

8) Every decimal number is an integer.

9) Rational and irrational numbers are complex numbers

10) Natural numbers start from 1.

11) The union of W and Z is W.

QUICK REVIEW (RATIONAL or IRRATIONAL)

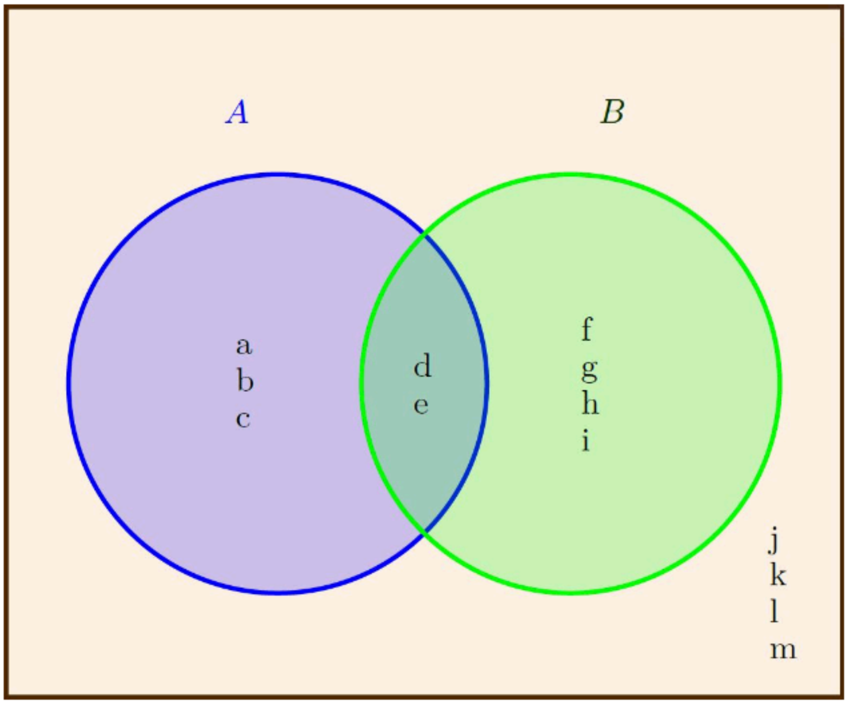
Determine for each number whether it is a rational or irrational number.
Select **Rational** or **Irrational** for each expression.

Number	Rational / Irrational
3.89	Rational ○ Irrational ○
$\sqrt{24}$	Rational ○ Irrational ○
$\frac{\sqrt{64}}{2}$	Rational ○ Irrational ○
π	Rational ○ Irrational ○
-4	Rational ○ Irrational ○

Question Help: Video

QUICK REVIEW (VENN DIAGRAM)

Use the following Venn diagram to find $A \backslash U$.

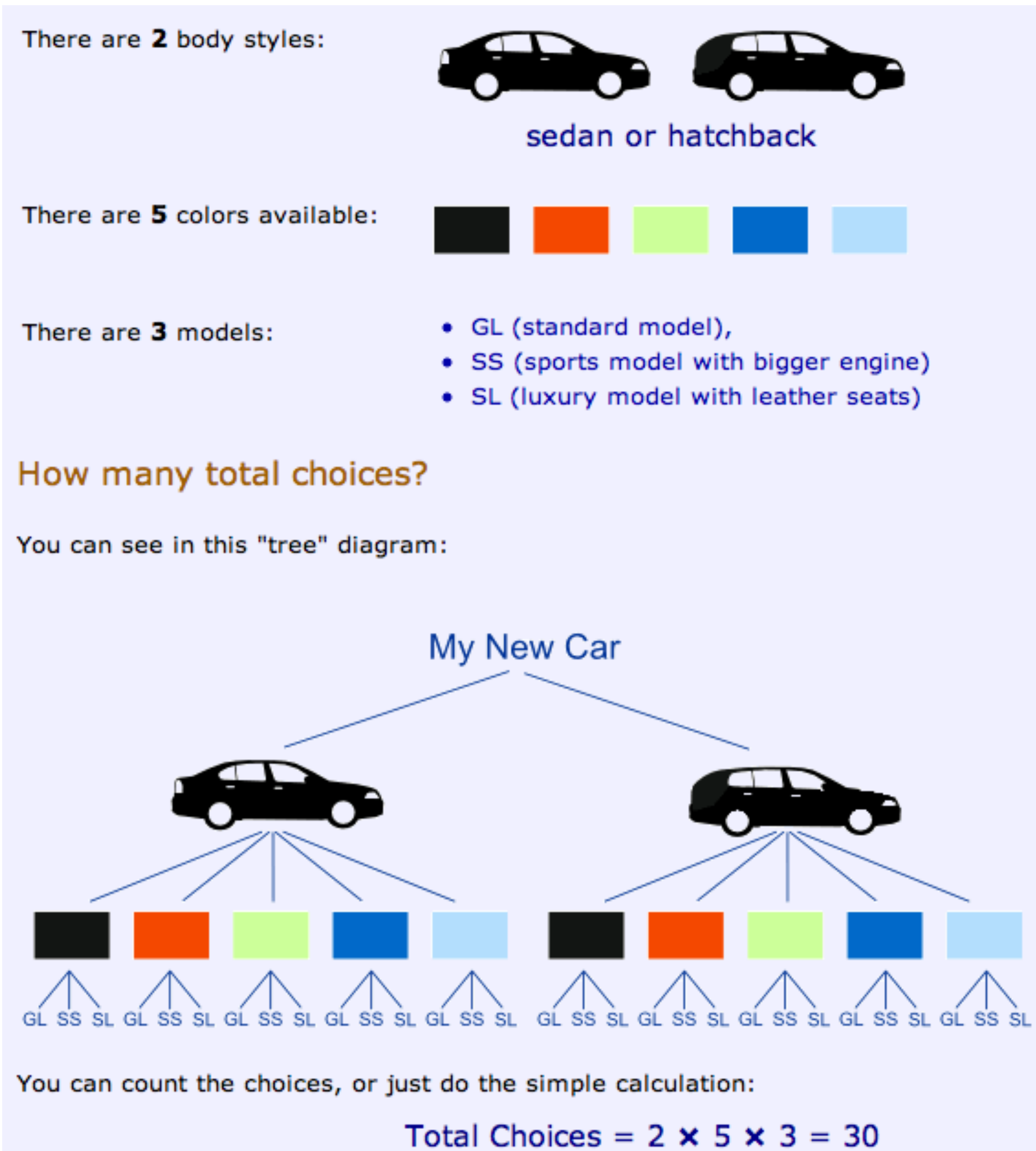


- {e,o,s,t,w}
- {h,k,p,r,y}
- {g,m,p,u,y}
- {p,q,s,v,y}
- {g,h,j,l,q}
- ∅

Basic Counting Principles

The Basic Counting Rule is used for scenarios that have multiple choices or actions to be determined.

The rule that states that when there are ***m*** ways to do one thing, and ***n*** ways to do another, then altogether there are ***m x n*** ways of doing both.



1) Sum Rule Principle: Assume some event *E* can occur in m ways and a second event *F* can occur in n ways and suppose both events cannot occur simultaneously. Then E ***or*** F can occur in m + n ways.

In general, if there are n events and no two events occurs in same time then the event can occur in $n_1 + n_2 \ldots\ldots\ldots n_j$ ways.

Example: If 8 male professors and 5 female professors teaching Discrete Structures, then the student can choose professor in ***8+5=13 ways.***

2) Product Rule Principle: Suppose there is an event E which can occur in m ways and, independent of this event, there is a second event F which can occur in n ways. Then combinations of E and F can occur in m x n ways.

In general, if there are n events occurring independently then all events can occur in the order indicated as $n_1 \times n_2 \times n_3 \ldots\ldots\ldots n_j$ ways.

Example: In class, there are 4 boys and 10 girls if a boy and a girl have to be chosen for the class monitor, the students can choose class monitor in 4 x 10 = 40 ways.

EXTRA EXAMPLE 1: If a boy owns 2 pairs of pants, 3 shirts, 8 ties, and 2 jackets. How many different outfits can he wear if he must wear one of each item?

EXTRA EXAMPLE 2: If a girl owns 3 pairs of blouses, 6 shirts, 2 ties, 2 jackets, and 3 pairs of shoes. How many different outfits can she wear if she must wear one of each item?

I DO 1) A license plate is to have the following form: ***three letters followed by three numbers.*** An example of a license plate like this would be MTH 314. How many different license plates can be made, assuming that ***letters and numbers can be reused?***

YOU DO 1) License plates in the state of Maryland are in the following form: one number (which cannot be zero), two letters, four numbers (which includes zero). For instance, a valid plate is 4MD1234

WE DO 1) Standard automobile license plates in a country display 2 numbers, followed by 3 letters, followed by 2 numbers. How many different standard plates are possible in this system? (Assume repetitions of letters and numbers are allowed.)

Factorial

Definition

The factorial of a non–negative integer n, denoted by $n!$, is the product of all positive integers less than or equal to n.

Example:

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

Convention:

$$0! = 1$$

EXTRA EXAMPLE 1)

How many three-letter "words" can be made from 7 letters "FGHIJKL" if repetition of letters

(a) is allowed?

There are ______ possible three-letter "words".

(b) is not allowed?

There are ______ possible three-letter "words".

Question Help: Video Video

EXTRA EXAMPLE 2)

A true-false test contains 19 questions. In how many different ways can this test be completed. (Assume we don't care about our scores.)

Your answer is :

Question Help: Video

EXTRA EXAMPLE 3)

A child rolls 3 standard dice of different colors and records the numbers showing. Find the number of different outcomes.

EXTRA EXAMPLE 4)

Find the number of different finishes for a race with 7 runners. (Assume no ties occur.)

EXTRA EXAMPLE 5)

In how many ways can first, second, and third prizes be awarded in a contest with 615 contestants?

Question Help: Video

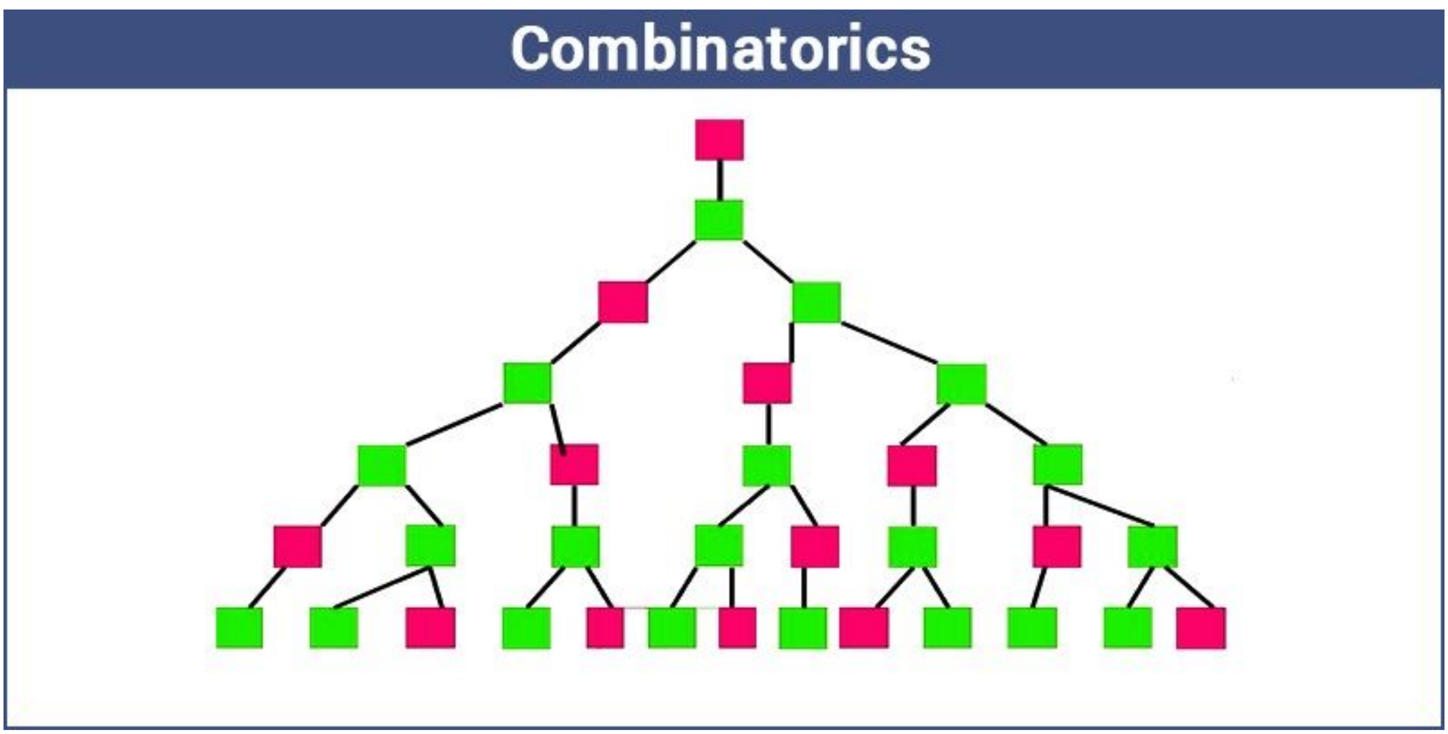
Source: ***https://byjus.com/maths/combinatorics/***

Combinatorics

Combinatorics is a stream of mathematics that concerns the study of finite discrete structures. It deals with the study of ***permutations and combinations,*** enumerations of the sets of elements. It characterizes Mathematical relations and their properties.

Mathematicians uses the term ***"Combinatorics"*** as it refers to the larger subset of Discrete Mathematics. It is frequently used in computer Science to derive the formulas and it is used for the estimation of the analysis of the algorithms. In this lesson, let us discuss what is combinatorics, its features, formulas, applications, and examples in details.

Features of combinatorics



Some of the important features of the combinatorics are as follows:

- Counting the structures of the provided kind and size.
- To decide when criteria can be fulfilled and analyzing elements of the criteria, such as combinatorial designs.
- To identify “greatest”, “smallest” or “optimal” elements, known as external combinatorics.

Key Idea. Combinatorial structures that rise in an algebraic concept, or applying algebraic techniques to combinatorial problems, known as algebraic combinatorics.

Applications of combinatorics

Combinatorics is applied in most of the areas such as:

- Communication networks, cryptography, and network security
- Computational molecular biology
- Computer architecture
- Scientific discovery
- Languages
- Pattern analysis
- Simulation
- Databases and data mining
- Homeland security
- Operations research

What are permutations and Combinations?

In English, we make use of the word "combination" without thinking if the order is important. Let's take a simple instance. The fruit salad is a combination of grapes, bananas, and apples. The order of fruits in the salad does not matter because it is the same fruit salad.

But let us assume that the combination of a key is 475. You need to take care of the order, since the other combinations like 457, 574, or others won't work. Only the combination of 4 – 7 – 5 can unlock.

Hence, to be precise;

- When the ***order does not have much impact***, it is said to be a ***combination.***
- When the ***order does have an impact***, it is said to be a ***permutation.***

Video: https://www.youtube.com/watch?time_continue=510&v=JyRKTesp6fQ&feature=emb_logo

Permutation: The act of an arranging all the members of a set into some order or sequence, or rearranging the ordered set, is called the process of permutation.

Definition

A permutation of r objects from a collection of n objects is any ordered arrangement of r distinct objects from the n objects.

- Notation: $(n)_r$ or ${}_nP_r$
- Formula: $(n)_r = \frac{n!}{(n-r)!}$

The special permutation rule states that anything permute itself is equivalent to itself factorial.

Example:

$$(3)_3 = \frac{3!}{(3-3)!} = \frac{3!}{0!} = 3! = 3 \times 2 \times 1 = 6$$

Note: Permutations of *n* items taken *r* at a time.

P (n, r) represents the number of permutations of *n* items *r* at a time.

$$P(n,r) = \frac{n!}{(n-r)!} = {}_{n}P_{r}$$

I DO 1) P (7, 3)

YOU DO 1) P (15, 5)

WE DO 1) If a class has 28 students, how many different arrangements can 5 students give a presentation to the class?

EXTRA PROBLEM 1) How many ways can the letter of the word PHOENIX be arranged?

SPECIAL CASE OF PERMUTATIONS

Permutations with indistinguishable items

The number of different permutations of ***n*** objects, where there are n_1 indistinguishable items, n_2 indistinguishable items, ..., and n_k indistinguishable items , is

$$\frac{n!}{n_1!n_2!\ldots n_k!}$$

I DO 1) How many ways can the letter of the word MATHEMATICS be arranged?

YOU DO 1) How many ways can the letter of the word COMMITTEE be arranged?

WE DO 1) How many ways can you order 3 blue marbles, 4 red marbles, and 5 green marbles? Marbles of the same color look identical.

EXTRA PROBLEM 1

A State's License plates consist of 3 letters followed by 4 digits. They will only allow you to choose from 18 letters and 9 numbers.

How many license plate options do they have if they CAN repeat letters and numbers?

How many license plate options do they have if they CANNOT repeat letters and numbers?

Question Help: Video

Video: https://www.youtube.com/watch?time_continue=6&v=SGn1913lOYM&feature=emb_logo

Combination: Selection of members of the set where the order is disregarded. ***The order does not MATTER!***

Combination

Definition

A combination of r objects from a collection of n objects is any unordered arrangement of r distinct objects from the n total objects.

Remark: The difference between a combination and a permutation is that order of the objects is not important for a combination.

- Notation: $\binom{n}{r}$ or ${}_nC_r$
- Formula: $\binom{n}{r} = \frac{n!}{(n-r)! \times r!}$

C (n, r) represents the number of permutations of n items r at a time.

$$C(n,r) = \frac{n!}{(n-r)!r!} = {}_n\mathrm{C}_r$$

I DO 1) C (7, 3)

YOU DO 1) C (15, 5)

WE DO 1) The soccer team has 20 players. There are always 11 players on the field at any one time. How many different groups of players can be on the field at any one time?

EXTRA PROBLEM 1) A student needs 8 more classes to complete her degree. If she has met the prerequisites for all the courses, how many ways can she take 4 classes next semester?

EXTRA PROBLEM 2) There are 4 men and 5 women in a small office. The customer wants a site visit from a group of 2 men and 2 women. How many different groups can be formed from the office?

EXTRA PROBLEM 3)

Lucy Furr must supply 3 different bags of chips for a party. She finds 18 varieties at her local grocer. How many different selections can she make?

Soruce: **https://www.mathplanet.com/education/pre-algebra/probability-and-statistics/probability-of-events**

Quick Lesson about Probability of Events

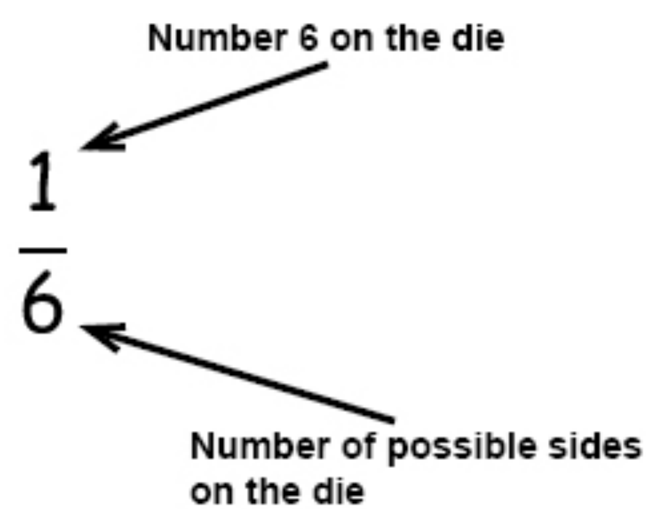
Probability is a type of ratio where we compare how many times an outcome can occur compared to all possible outcomes.

$$P(E) = \frac{favorable\ number\ of\ outcomes}{total\ number\ of\ outcomes}$$

Example 1:

What is the probability to get a 6 when you roll a die?

A die has 6 sides, 1 side contain the number 6 that give us 1 wanted outcome in 6 possible outcomes.



Independent events: Two events are independent when the outcome of the first event does not influence the outcome of the second event.

Note: ***When we determine the probability of two independent events, we multiply the probability of the first event by the probability of the second event.***

$$P(X \text{ } and \text{ } Y) = P(X) \cdot P(Y)$$

To find the probability of an independent event we are using this rule:

Example

If one has three dice what is the probability of getting three 4s?

The probability of getting a 4 on one die is 1/6

The probability of getting 3 4s is:

$$P\,(4 \text{ } and \text{ } 4 \text{ } and \text{ } 4) = \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{216}$$

Key Idea. When the outcome affects the second outcome, which is what we called dependent events.

Dependent events: Two events are dependent when the outcome of the first event influences the outcome of the second event. The probability of two dependent events is the product of the probability of X and the probability of Y **AFTER** X occurs.

$$P(X\ and\ Y) = P(X) \cdot P(Y\ after\ x)$$

Example. What is the probability for you to choose two red cards in a deck of cards?

A deck of cards has 26 black and 26 red cards. The probability of choosing a red card randomly is:

$$P\ (red) = \frac{26}{52} = \frac{1}{2}$$

The probability of choosing a second red card from the deck is now:

$$P\ (red) = \frac{25}{51}$$

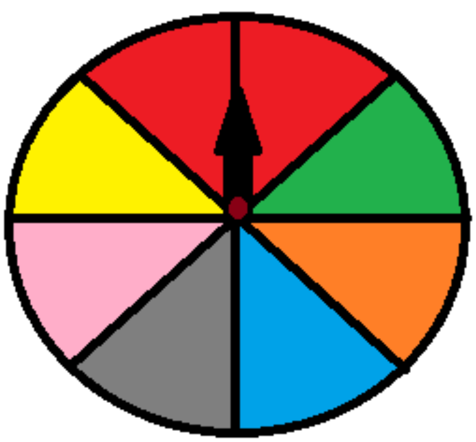
The probability:

$$P\ (2\ red) = \frac{1}{2} \cdot \frac{25}{51} = \frac{25}{102}$$

Key Idea. Two events are mutually exclusive when two events cannot happen at the same time. The probability that one of the mutually exclusive events occur is the sum of their individual probabilities.

$$P(X \text{ or } Y) = P(X) + P(Y)$$

Example: An example of two mutually exclusive events is a wheel of fortune. ***Let's say you win a bar of chocolate if you end up in a red or a pink field.***



What is the probability that the wheel stops at red or pink?

P (red or pink) = P(red) + P(pink)

$$P\,(red) = \frac{2}{8} = \frac{1}{4}$$

$$P\,(pink) = \frac{1}{8}$$

$$P\,(red\ or\ pink) = \frac{1}{8} + \frac{2}{8} = \frac{3}{8}$$

Key Idea. Inclusive events are events that can happen at the same time. To find the probability of an inclusive event we first add the probabilities of the individual events and then subtract the probability of the two events happening at the same time.

$$P\,(X\ or\ Y) = P\,(X) + P\,(Y) - P\,(X\ and\ Y)$$

Example

What is the probability of drawing a black card or a ten in a deck of cards?

There are 4 tens in a deck of cards P(10) = 4/52

There are 26 black cards P(black) = 26/52

There are 2 black tens P(black and 10) = 2/52

$$P\,(black\ or\ ten) = \frac{4}{52} + \frac{26}{52} - \frac{2}{52} = \frac{30}{52} - \frac{2}{52} = \frac{28}{52} = \frac{7}{13}$$

KEYWORDS:

With replacement means the same item can be chosen more than once.

Without replacement means **the same item cannot be selected more than once**.

Probability with permutations

$$\boldsymbol{P(E) = \frac{favorable\ number\ of\ outcomes}{total\ number\ of\ outcomes}}$$

$$= \frac{number\ of\ outcomes\ in\ event\ space}{number\ of\ outcomes\ in\ sample\ space} * 100\%$$

I DO 1: **You pick 3 digits (0-9) at random without replacement and write them in the order picked. What is the probability that you have written the first 3 digits of your phone number? Assume there are no repeats of digits in your phone number. Give your answer as a fraction.**

YOU DO 1) You pick 2 digits (0-9) at random without replacement and write them in the order picked.

What is the probability that you have written the first 2 digits of your phone number? Assume there are no repeats of digits in your phone number.

Give your answer as a fraction.

EXTRA EXAMPLE 1

Kosumi has 13 books and he wants to read 5 over the summer. He has 7 fiction books and 6 nonfiction books.

If Kosumi randomly selects the 5 books, what is the probability that the first 2 books are fiction and the next 3 books are nonfiction?

%

Question Help: Written Example

EXTRA EXAMPLE 2

Three cards are chosen without replacement from a deck of 52 cards and placed in order from first to third.

What is the probability that all three cards are spades?

%

Question Help: Video Written Example

Probability with combinations

$$P(E) = \frac{\boldsymbol{favorable\ number\ of\ outcomes}}{\boldsymbol{total\ number\ of\ outcomes}}$$

$$= \frac{\boldsymbol{number\ of\ outcomes\ in\ event\ space}}{\boldsymbol{number\ of\ outcomes\ in\ sample\ space}} * 100\%$$

I DO 1) From a group of 12 people, you randomly select 4 of them. What is the **probability** that they are the 4 oldest people in the group?

YOU DO 1) From a group of 7 people, 2 are randomly selected. What is the probability the 2 oldest people in the group were selected? *Give your answer as a reduced fraction.*

EXTRA EXAMPLE 1

Combinations and Probability

An employee group requires 6 people be chosen for a committee from a group of 14 employees. Determine the following probabilities of randomly drawn committee of 6 employees.

Write your answers as percents rounded to 4 decimal places.

The employee group has 8 women and 6 men.

What is the probability that 4 of the people chosen for the committee are women and 2 people chosen for the committee are men?

%

The committee requires that exactly 2 people from IT serve on the committee. There are 5 people in IT.

What is the probability that exactly 2 of the people chosen for the committee are from IT?

%

Question Help: Video Written Example

Thank you for listening!

Do the assigned activities