Prelim: Lecture Notes #3 – Sets, Number System, Venn Diagram, & Propositional Calculus 02.15.23

Discrete Structures 1

REMINDER!

1) ALWAYS FOLLOW MY CLASSROOM PROCEDURES.

2) ACTIVITIES ARE ALWAYS UPDATED IN GOOGLE CLASSROOM & MYOPENMATH

3) ALWAYS SUBMIT YOUR WORK ON TIME. NO MORE SUBMISSSION OF LATE WORK.

4) THE WEEKLY HOMEWORK IS ALWAYS DUE EVERY SATURDAY.

5) CHECK THE UPDATES ON GOOGLE CLASSROOM AND DO YOUR MISSING ASSIGNMENTS as the GRADES are always UPDATED.

Note: **Activities that you failed to submit on-time will automatically disappear on the system.**

Activities for today

1) ***MyOpenMath***
   The course ID: ***175937***
   The enrollment key: ***faith2022***

   a) Bellwork #3 (Prelims) – Anything about Number, Sets, Venn Diagram, Propositional Calculus, & Truth Table 02.15.23
   b) Activity #3 (Prelims) – Everything You Know So Far 02.15.23
   c) Weekly Homework #3 (Prelims)- Retaining all the Knowledge You Know 02.15-22.23

2) ***Google Classroom: Class code:*** ciu2uyg
   Prelim: Lecture Notes #3 – Sets, Venn Diagram, & Propositional Calculus 02.01.23

QUICK REVIEW

TWO TYPES OF REAL NUMBERS (RATIONAL and IRRATIONAL)

RATIONAL VS. IRRATIONAL NUMBERS

RATIONAL NUMBERS **– numbers that can be written as the ratios or fractions of the two integers or decimals.**

As a fraction $\frac{a}{b}$, where *a* and *b* are integers (b $\neq 0$)

Examples: -12, $\frac{7}{8}$, $\frac{1}{2}$, 5

These numbers are natural numbers, whole numbers, integers, terminating decimals, repeating decimals, mixed fractions, perfect squares and perfect cubes.

SUBSETS OF RATIONAL NUMBERS

1) NATURAL NUMBERS OR COUNTING NUMBERS – the set of all positive counting numbers starting with 1.

Example: {1, 2, 3, 4, 5…}

Examples: 1, 300, 1000

2) WHOLE NUMBERS – the set of all positive counting numbers begin with zero

Example: {0, 1, 2, 3 4, 5 …}

Examples: 0, 50, 1230

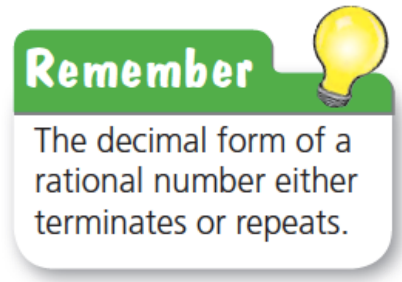
3) INTEGERS – the set of whole numbers and their opposites (positive, negative, zero). Numbers without decimal.

Example {…-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5…}

Examples: 0, -50, 100, 500

4) TERMINATING DECIMALS– decimal values that terminate

Examples: 0.6, $\frac{1}{4} = 0.25$, 0.75, $\frac{1}{2} = 0.5$



5) REPEATING DECIMALS– decimal values that

Examples: $-0.\overline{25}$ $\quad \frac{2}{3} = 0.\overline{66}$ $\quad \frac{1}{3} =$ 0.

6) MIXED FRACTIONS – fractions with whole number and fractional part

Examples: $5\frac{1}{2} = 5.5$, $3\frac{3}{4} = 3.75$

7) PERFECT SQUARES – numbers that when you get the square root the answer is a whole number

Examples: $\sqrt{4} = 2$, $\sqrt{25} = 5$, $\sqrt{100} = 10$,

8) PERFECT CUBES – numbers that when you get the cube root the answer is a whole number.

Examples: $\sqrt[3]{8} = 2$, $\sqrt[3]{64} = 4$

IRRATIONAL NUMBERS **–numbers that cannot be written as the ratios or fractions of the two integers or decimals**

Examples: π (The symbol for Pi)

e(Euler's number) = 2.718 281 828 459 045 235 360 287 471 352

SUBSETS OF IRRATIONAL NUMBERS

1) NON–TERMINATING DECIMALS/NON-REPEATING DECIMALS – decimal values that keep on going and non-repeating.

Examples: π (The symbol for Pi)= $\frac{22}{7} = 3.14285714\ldots$

e (Euler's number) = 2.718 281 828 459 045 235 360 287 471 352…

2) **NON-PERFECT SQUARE – numbers that when you get the square root the answer is not a whole number.**

Examples: $\sqrt{2} = 1.414213562\ldots$

$\sqrt{3} = 1.732050808\ldots$

3) **NON-PERFECT CUBE- numbers that when you get the cube root the answer is not a whole number.**

Examples: $\sqrt[3]{2} = 1.25992105\ldots$

> REMEMBER:
> *The decimal form of an irrational number neither terminates nor repeats.*

KEY IDEAS

✓To check if the fraction is rational or irrational number use your calculator and check if the decimal values are repeating or terminating (RATIONAL NUMBER) and non-terminating or non-repeating (IRRATIONAL NUMBER)

CHECK FOR UNDERSTANDING 2

YOU DO 1)

Which of the following would be an integer? (Answer all that apply.)

- ☐ $-\dfrac{8}{9}$
- ☐ $\dfrac{0}{9}$
- ☐ $\dfrac{9}{0}$
- ☐ $\dfrac{199}{25}$
- ☐ -8
- ☐ $\sqrt{-1}$
- ☐ $\sqrt{2}$
- ☐ π
- ☐ $-\dfrac{56}{7}$

Which of the following would a rational number? (Answer all that apply.)

- ☐ $-\frac{8}{9}$
- ☐ $\frac{0}{9}$
- ☐ $\frac{9}{0}$
- ☐ $\frac{199}{25}$
- ☐ -8
- ☐ $\sqrt{-1}$
- ☐ π
- ☐ $\sqrt{2}$
- ☐ $-\frac{56}{7}$

Which of the following would be considered an irrational number? (Answer all that apply.)

- ☐ $-\frac{8}{9}$
- ☐ $\frac{0}{9}$
- ☐ $\frac{9}{0}$
- ☐ $\frac{199}{25}$
- ☐ -8
- ☐ $\sqrt{-1}$
- ☐ π
- ☐ $\sqrt{2}$
- ☐ $-\frac{56}{7}$

QUICK REVIEW

Introduction of Sets

A set is defined as a collection of distinct objects of the same type or class of objects. The purposes of a set are called elements or members of the set. An object can be numbers, alphabets, names, etc.

Sets Representation:

Sets are represented in two forms:

a) Roster or tabular form: In this form of representation, we list all the elements of the set within braces { } and separate them by commas.

Example 1: If A=set of all odd numbers less than 10 then in the roster from it can be expressed as A= {1,3,5,7,9}.

Example 2: If A=set of all even numbers less than 10 then in the roster from it can be expressed as A= {2, 4, 6, 8}.

b) Set Builder or rule form: In this form of representation, we list the properties fulfilled by all the elements of the set. We note as {x: x satisfies properties P}. and read as 'the set of those entire x such that each x has properties P.'

It is also a method which makes use of the description {x | …}. This is read as *"x such that"*

Example 1: If B= {2, 4, 8, 16, 32}, then the set builder representation will be: $B = \{x: x=2^n$, where $n \in N$ and $1 \leq n \geq 5\}$

Example 2: Starting with all the real numbers, we can limit them to the interval between 1 and 6 inclusive. Hence, it will be represented as:

$S = \{x: x \geq 1 \text{ and } x \leq 6\}$

Set Builder Notation Symbols

The different symbols used to represent set builder notation are as follows:

- The symbol ∈ "is an element of".
- The symbol ∉ "is not an element of".
- The symbol W denotes the whole number.
- The symbol Z denotes integers.
- The symbol N denotes all natural numbers or all positive integers.
- The symbol R denotes real numbers or any numbers that are not imaginary.
- The symbol Q denotes rational numbers or any numbers that can be expressed as a fraction.

CHECK FOR UNDERSTANDING 2

Question #1) What does the rule method $\{x \in N \mid x > 5\}$ describe?

A) the set of positive integers
B) the set of all whole numbers greater than 5
C) the set of all natural numbers greater than 5
D) the set of all natural numbers

Question #2) Identify the elements of set *B*

$B = \{x: x=2^n, \text{ where } n \in N \text{ and } 1 \leq n \geq 3\}$

Cardinality of a Sets

The total number of unique elements in the set is called the *cardinality of the set.* The cardinality of the countably *infinite set is countably infinite.*

Examples:

1. Let P = {k, l, m, n}
The cardinality of the set P is 4.

2. Let A is the set of all non-negative even integers, i.e.
A = {0, 2, 4, 6, 8, 10......}.
As A is countably infinite set hence the cardinality.

Kinds of Sets

Sets can be classified into many categories. Some of which are ***finite, infinite, subset, universal, proper, power, singleton set, etc.***

Key Idea. A set which contains no element is called an empty set or null set. We denote the empty set { } or ∅. The set {∅} is not empty since it contains one element.

1. Finite Sets: A set is said to be finite if it contains exactly n distinct element where n is a non-negative integer. Here, n is said to be "cardinality of sets." The cardinality of sets is denoted by |A|, # A, card (A) or n (A).

Example:

1. Cardinality of empty set θ is 0 and is denoted by |θ| = 0
2. Sets of even positive integer is not a finite set.

A set is called a ***finite set*** if there is one to one correspondence between the elements in the set and the element in some set n, where n is a natural number and n is the cardinality of the set. Finite Sets are also called ***numerable sets***. n is termed as the cardinality of sets or a cardinal number of sets.

2. Infinite Sets: A set which is not finite is called as Infinite Sets.

Countable Infinite: If there is one to one correspondence between the elements in set and element in N. A countably infinite set is also known as ***Denumerable***. A set that is either finite or denumerable is known as ***countable***. A set which is not countable is known as ***Uncountable***. The set of a non-negative even integer is countable Infinite.

Uncountable Infinite: A set which is not countable is called Uncountable Infinite Set or non-denumerable set or simply Uncountable.

Example: Set R of all +ve real numbers less than 1 that can be represented by the decimal form 0. a_1, a_2, a_3..... Where a_1 is an integer such that $0 \leq a_i \leq 9$.

Key Idea. A set is infinite if the counting of elements has no end. The set of integers Z or positive integers N or Natural numbers), negative integers Z^-, and nonnegative integers (or whole numbers) are ***infinite sets.***

Z = {…, -2, -1, 0, 1, 2, …}

Z^- {-1, -2, -3, …}

N = {1, 2, 3, …}

W = {0, 1, 2, ...}

The sets of positive odd integers Z_0 = {1, 3, 5, …} and positive even integers Z_e = {2, 4, 6, …} are also infinite sets. An *odd integer* is denoted

by 2k + 1, where k is an integer while an *even integer* is denoted by 2k, where k is an integer.

3. Subsets: If every element in a set A is also an element of a set B, then A is called a subset of B. It can be denoted as $A \subseteq B$. Here B is called Superset of A.

Example: If A= {1, 2} and B= {4, 2, 1} the A is the subset of B or $A \subseteq B$.

4. Proper Subset: If A is a subset of B and $A \neq B$ then A is said to be a proper subset of B. If A is a proper subset of B then B is not a subset of A, i.e., there is at least one element in B which is not in A.

Example:

Let A = {2, 3, 4}

B = {2, 3, 4, 5}

$A \subset B$

A is a proper subset of B.

Key Idea: The null ∅ is a proper subset of every set.

5. Improper Subset: If A is a subset of B and A = B, then A is said to be an improper subset of B.

Example

A = {2, 3, 4}, B = {2, 3, 4}

$A \subseteq B$

A is an improper subset of B.

Key Idea: Every set is an improper subset of itself.

Proper subset (The symbol used for this is $\subset$)

Improper subset (The symbol used for this is $\subseteq$)

There are two subset symbols.

- ⊆, which is read as "is a subset or equal to" (or sometimes simply as "subset of")
- ⊂, which is read as "is a subset of" (means strictly subset of but NOT equal to)
-

6. Power Sets: The power of any given set A is the set of all subsets of A and is denoted by **P (A)**. If A has n elements, then **P (A)** has $\mathbf{2^n}$ elements.

Example:

A = {1, 2, 3}

P (A) = {∅, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3}}.

Properties of Subsets:

1. Every set is a subset of itself.
2. The Null Set i.e., ∅ is a subset of every set.
3. If A is a subset of B and B is a subset of C, then A will be the subset of C. If $A \subset B$ and $B \subset C \Longrightarrow A \subset C$
4. A finite set having n elements has 2^n subsets.

For example, 0 ϵ {0}, but ∅ ∉ { 0}. The elements 0 and ∅ are two different symbols. The set {∅} has one element designated by the symbol ∅, a symbol note considered as an empty set in this particular example.

Example: The empty set A = { } has only one subset, i.e., A.

Example: If A = {0, 1}, then the subsets are { }, {0}, {a}, A.

Take note that the number of subsets of a given set is 2^n where *n* denotes number of elements of the set. In the preceding example, $2^2 = 4$ subsets.

Example: The set B = {a, b, c} has 2^3 subsets. The power set has the elements with breakdown as follows

Improper subset with 3 elements: {a, b, c} or B

Proper subsets with 2 elements: {a, b}, {a, c}, {b, c}

Proper subsets with 1 element: {a}, {b}, {c}

Improper subset with no element: { }

Note: In any given set, there are always two improper subsets. The set itself and the null set or empty set.

P(B) = {B, ∅, {a, b}, {a, c}, {b, c}, {a}, {b}, {c}}

In the sets Z, W, and N we have the following subsets as follows:

$$N \subset W \subset Z \text{ i.e.}$$

$\{1, 2, 3, \ldots\} \subset \{0, 1, 2, 3 \ldots\} \subset \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

Also Z^- (the negative integers) ⊂ W (the non-positive integers) ⊂ Z (the integers

Note: **The expression to find the subsets of a given set is 2^n**, where 2 is constant and ***n*** is the number of elements of the given set.

7) Universal Set: If all the sets under investigations are subsets of a fixed set U, then the set U is called Universal Set.

Example: In the human population studies the universal set consists of all the people in the world.

Key Idea. Universal set is the totality of elements under consideration. If A = {1, 2, 3} and B = {3, 4, 5} then the universal set, considering no other set present is U = {1, 2, 3, 4, 5}

8. Null Set or Empty Set: A set having no elements is called a Null set or void set. It is denoted by $\emptyset$ or { }.

Example: A = { }, B = $\emptyset$

9. Singleton Set: It contains only one element. It is denoted by {s}.

Example: $S= \{x \mid x \in N, 7<x<9\} = \{8\}$

10. Equal Sets: Two sets A and B are said to be equal and written as A = B if both have the same elements. Therefore, every element which belongs to A is also an element of the set B and every element which belongs to the set B is also an element of the set A.

$A = B \Leftrightarrow \{x \in A \Leftrightarrow x \in B\}$.

Example: The set A = {1, 2, 3} and B {2, 1, 3) are equal sets.

Note: If there is some element in set A that does not belong to set B or vice versa then $A \neq B$, i.e., A is not equal to B.

11) **Equivalent Sets:** If the cardinalities of two sets are equal, they are called equivalent sets.

Example:
If A= {1, 2, 6} and B= {16, 17, 22}, they are equivalent as cardinality of A is equal to the cardinality of B. i.e. $|A|=|B|=3$

Sets A and B are equivalent, denoted by A ~ B, if they have the same number of elements. The sets C = {a, b, c} and D = {4, 5, 6} are equivalent sets. Also, {2} ~ {∅}, since they have the same number of elements.1

12. Disjoint Sets: Two sets A and B are said to be disjoint if no element of A is in B and no element of B is in A.

Example:

R = {a, b, c}
S = {k, p, m}
R and S are disjoint sets.

Key Idea. Two sets are disjoint if they have no common element.

The sets E = {a, b, c} and F = {e, f, g} are disjoints sets, since no element is common. The sets {0} and {∅} are also disjoint sets. The positive odd integers Z_0 = {1, 3, 5, ...} and the nonnegative even integers Z_e = {0, 2, 4, ...} are disjoint sets. Also, the negative integers Z^- = {-1, -2, ...} and the nonnegative integers W = {0, 1, 2, ...} are ***disjoint sets.***

13. Joint Sets: Sets that have common elements are ***joint sets*****.**
The sets A = {4, 5, 6} and B= {6, 10, 11} are joint sets since 6 is common to both A and B.

CHECK FOR UNDERSTANDING 3

Question 1) Identify the proper and improper subsets of the given set.

A = {0, 1, 2}

QUESTION 2

True or False

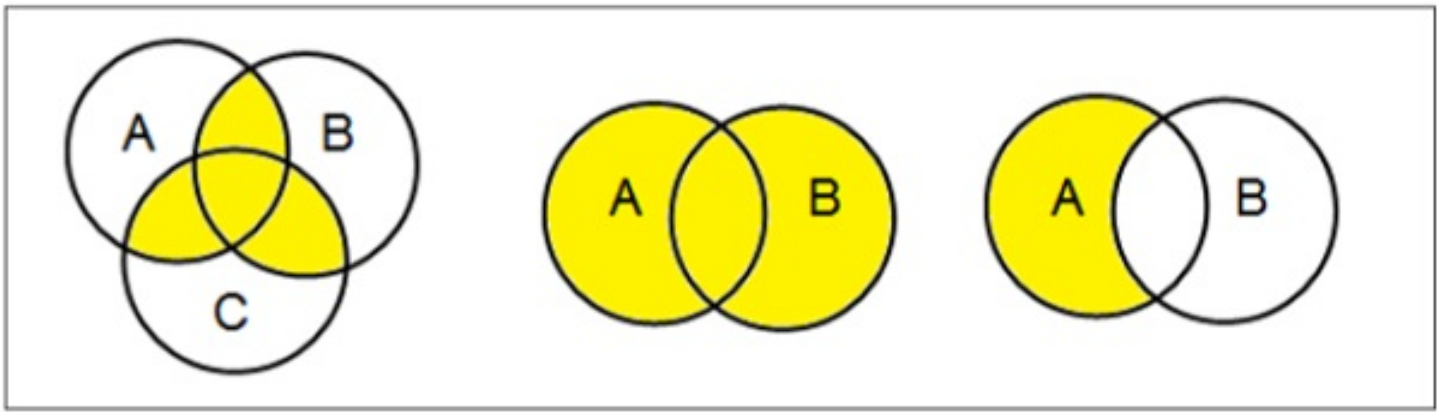
1) Every natural number is a whole number
2) $\{1, 4, 5\} \subseteq \{1, 4, 5, 8\}$
3) $\{1, 2, 5\} \subseteq \{1, 2, 5\}$
4) $2 \subseteq \{1, 2, 5\}$
5) $\{1\} \subseteq \{1, 2, 5\}$
6) $26 \notin \{2, 4, 6, 8, \ldots\}$
7) Every integer is a rational number.
8) Every natural number is an irrational number.
9) Some integers are not whole numbers.
10) All rational and irrational numbers are real numbers.
11) All equivalent sets have the same number of elements.
12) All equal sets have identical elements.

SET OPERATIONS

Venn Diagrams

Venn diagram, invented in 1880 by John Venn, is a schematic diagram that shows all possible logical relations between different mathematical sets.

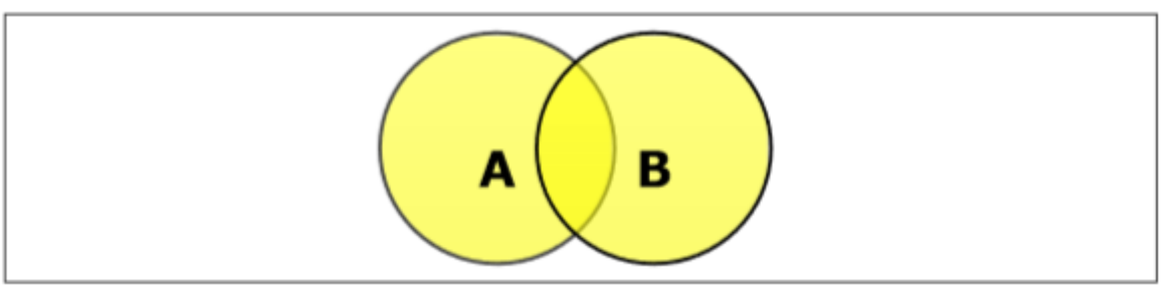
Examples



The basic set operations are:

1) Union of Sets: Union of Sets A and B is defined to be the set of all those elements which belong to A or B or both and is denoted by $A \cup B$.

Hence, $A \cup B = \{x \mid x \in A \text{ OR } x \in B\}$



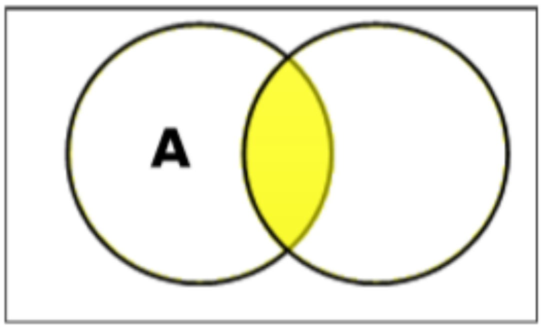
Example:

Let A = {1, 2, 3} and B = {3,4,5,6}

A∪B = {1, 2, 3, 4, 5, 6}.

2) Intersection of Sets: Intersection of two sets A and B is the set of all those elements which belong to both A and B and is denoted by A ∩ B.

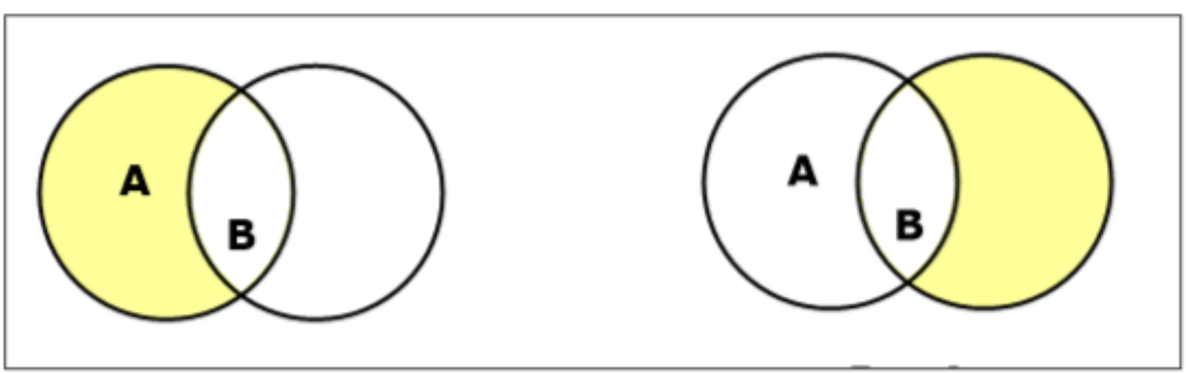
Hence, A ∩ B = {x | x ∈ A AND x ∈ B}



Example:
Let A= {11,12,13} and B= {13,14,15}

A∩B= {13}

3) Difference of Sets/ Relative Complement: The difference of two sets A and B is a set of all those elements which belongs to A but do not belong to B and is denoted by A - B. Hence, A−B or A/B = {x | x ∈ A AND x ∉ B}



Example:

If A= {10,11,12,13} and B= {13,14,15}

then (A−B) = {10,11,12} and (B−A) = {14,15} or

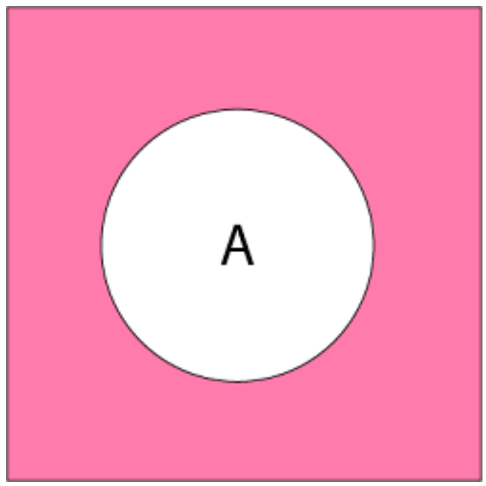
then (A/B) = {10,11,12} and (B/A) = {14,15} or

then (A\B) = {10,11,12} and (B\A) = {14,15} or

Here, we can see (A−B) ≠ (B−A)

4) Complement of a Set: The Complement of a Set A is a set of all those elements of the universal set which do not belong to A and is denoted by A^c.

Hence, A^c or A'= U - A = {x: x ∈ U and x ∉ A} = {x: x ∉ A}



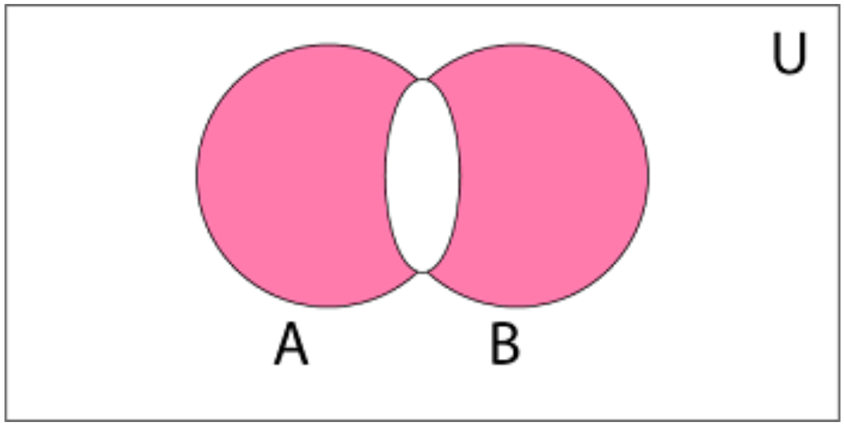
Example: Let U is the set of all natural numbers.

A = {1, 2, 3}

A^c = {all natural numbers except 1, 2, and 3}.

5) Symmetric Difference of Sets: The symmetric difference of two sets A and B is the set containing all the elements that are in A or B but not in both and is denoted by A ⊕ B i.e.

Hence, A ⊕ B = (A ∪ B) - (A ∩ B)



Example:
Let $A = \{a, b, c, d\}$
$B = \{a, b, l, m\}$
$A \oplus B = \{c, d, l, m\}$

6) Cartesian Product. Given sets A and B, the Cartesian product of A and B, denoted by A x B and read as "A cross B", is the set of all ordered pair (a,b) where a is in A and b is in B. Symbolically:
$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$ Note that A x B is not equal to B x A.

Example 1)
If $A = \{1, 2\}$ and $B = \{a, b\}$, what is A x B?
$A \times B = \{(1, a), (1, b), (2, a), (2, b)\}$. How many elements in a A x B?
Example 2: Let $A = \{1, 2, 3\}$ and $B = \{a, b\}$. Then
$A \times B = \{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)\}$.
Example 3: For the same A and B as in Example 1,
$B \times A = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)\}$.

Check for Understanding 4

Let U ={Z}
A = {N}
B = {W}

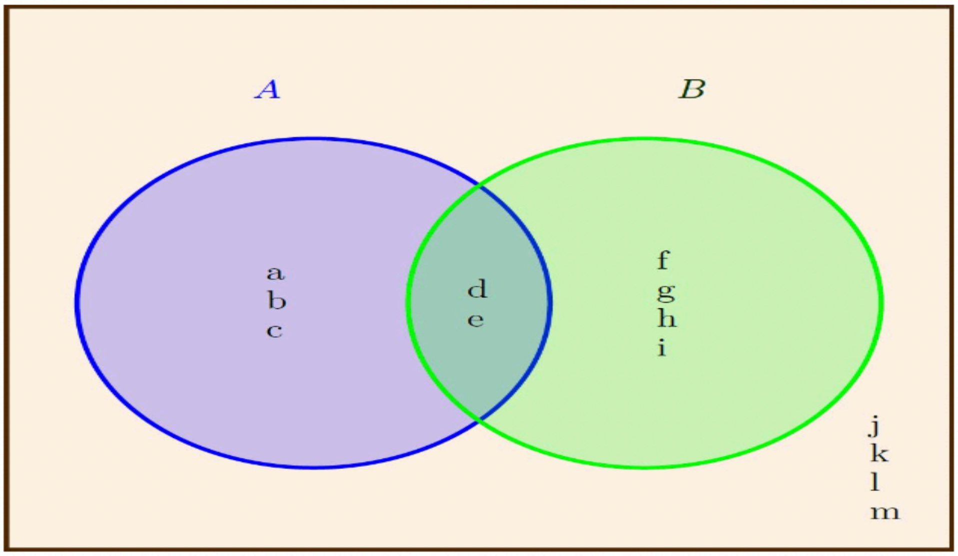
Perform the indicated set operations

QUESTION #1 $(A \cup B) \cap (B^c - A)$

QUESTION #2 $A \oplus B$

QUESTION #3

Use the following Venn diagram to find $A \setminus U$.



- ○ {c,g,q,t,z}
- ○ {e,j,m,v,z}
- ○ {b,d,o,t,v}
- ○ {j,q,u,v,w}
- ○ {l,m,p,v,w}
- ○ ∅

Steps in solving real-world problems involving Venn Diagram

Step 1: Read the problems/scenarios carefully.
Step 2: Identify the number of relationships given in the problem to identify how many circles you need to draw.
Step 3: Identify the point of intersections.
Step 4: Start solving or identifying the relationships from the bottom.

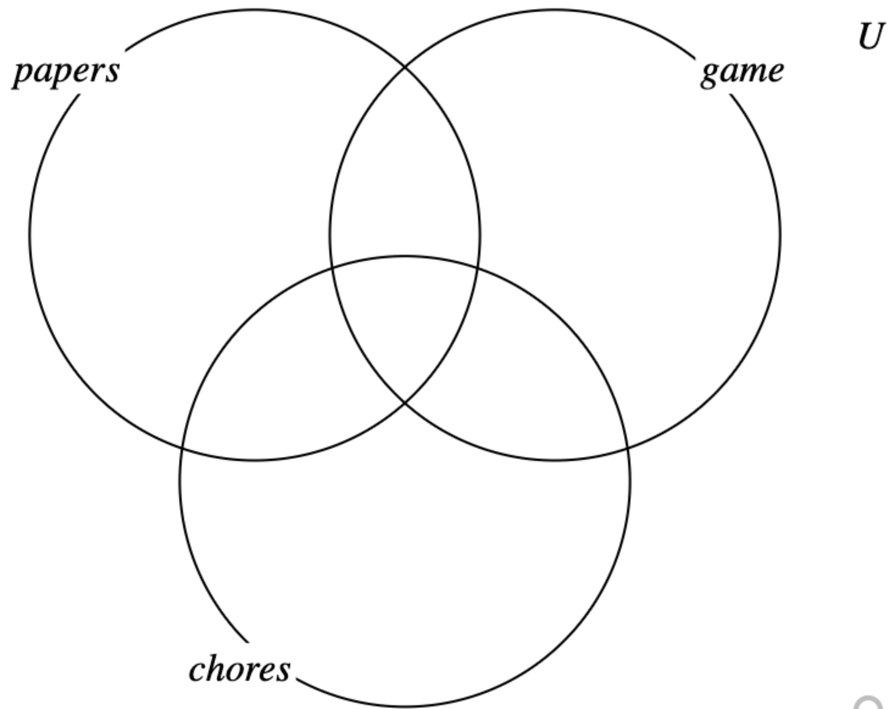
CHECK FOR UNDERSTANDING 5

Video: https://www.youtube.com/watch?v=XidkM5J3OQU

Two hundred twenty-five TCC faculty members were surveyed regarding their plans for the upcoming weekend. The results were as follows:

116	plan to grade papers
95	plan to attend a football game
87	plan to do household chores
19	plan to grade papers and attend a football game
53	plan to grade papers and do household chores
28	plan to attend a football game and do household chores
7	plan to grade papers, attend a football game, and do household chores

Sketch a Venn diagram and answer the questions below



1. How many planned to only grade papers?
2. How many planned to grade papers or do household chores?
3. How many did not plan to do any of these activities?
4. How many planned to do at lease two of these activities?
5. How many planned to attend a football game and do household chores, but did not have any plans to grade?
6. How many planned to attend a football game or grade papers, but did not plan to do household chores?
7. How many planned to do at most one of these activities?

Check for Understanding 7(5 minutes)

Let U (Universal Set) = {1, 2, 3, 4, 5… 10}

A = {1, 2, 3, 4, 5, 6}

B = {2, 3, 4, 6, 8, 9, 10}

Perform the indicated set operations

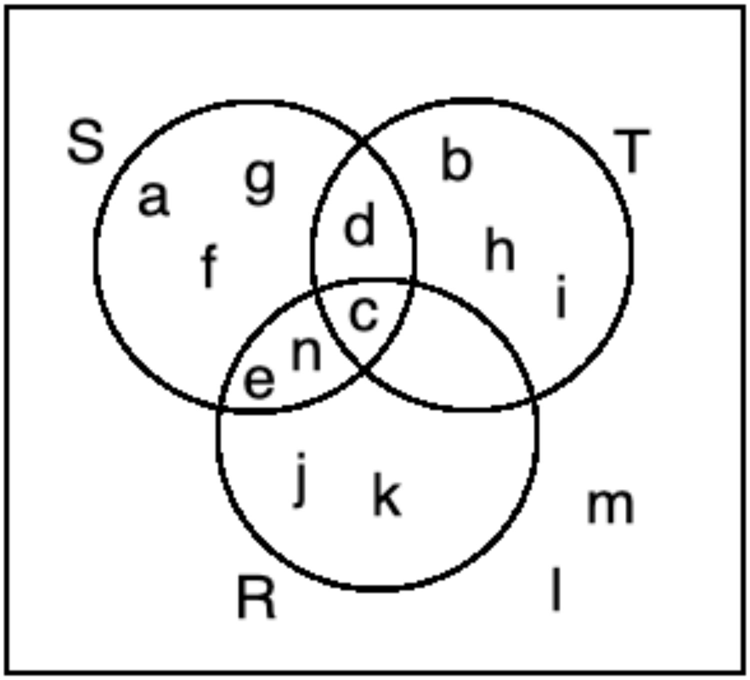
1) $(A \cup B) \cap B$

2) $A - B \cap B^c$

3) $A \oplus B$

4) The sets R, S, and T are displayed in the Venn diagram below where the universal set is the lowercase letters from a to n.

Find the cardinality of the sets.



$n(S \cup R) =$

$n(R \cap S) =$

$n(S^c) =$

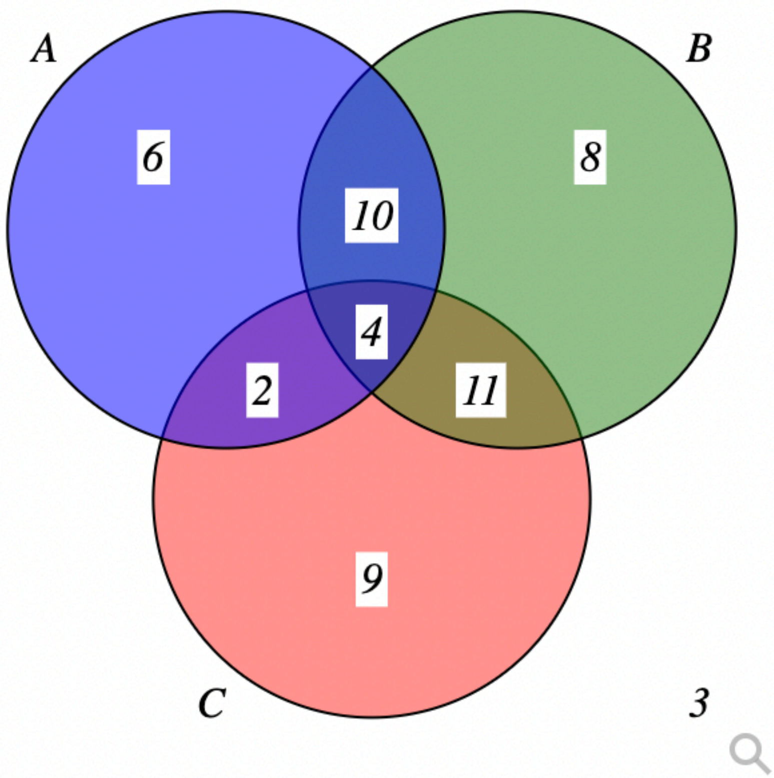
$n(R \cap T) =$

$n(S \cup T) =$

$n(T') =$

MORE PROBLEMS

Consider the Venn diagram shown below.



Determine the following cardinalities.

$n\left(\overline{A}\right) =$

$n(A \cup B) =$

$n\left(A \cap \overline{C}\right) =$

$n(A \cap B \cap C) =$

$n(A \cup B \cup C) =$

$n\left(\overline{A} \cap B \cap C\right) =$

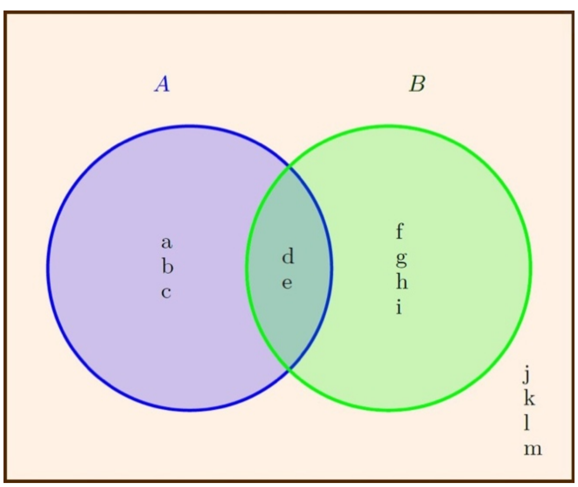
Question Help: Video

5) Let U= {1,2, 3,...,10}, A= {1,3,5,7}, B= {1,2,3,4}, and C={3,4,6,7,9}.

Select $A^c \cap B^c$ from the choices below.
A) {1,3,7,8}
B) {2,5,6,9}
C) {1,4,5,10}
D) {6,8,9,10}
E) {1,2,5,8}
F) ∅

6) Let U= {1,2, 3,...,10}, A={1,3,5,7}, B= {1,2,3,4}, and C= {3,4,6,7,9}.
Select A∪∅ from the choices below.
A) {1,4,5,8}
B) {4,7,8,10}
C) {2,4,6,8}
D) {1,3,5,7}
E) {5,7,8,10}
F) ∅

7) Use the following Venn diagram to find $(A \cup B)^c$.



A) {e, p, v}
B) {g, l, m}
C) {l, u, y}
D) {g, t, v}
E) {j, k, l, m}
F) ∅

Sources: https://www.javatpoint.com/sets-operations

Important Notes on Set Operations

- Set operation formula for union of sets is $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ and set operation formula for intersection of sets is $n(A \cap B) = n(A)+n(B)-n(A \cup B)$.
- The union of any set with the universal set gives the universal set and the intersection of any set A with the universal set gives the set A.
- Union, intersection, difference, and complement are the various operations on sets.
- The complement of a universal set is an empty set $U' = \phi$. The complement of an empty set is a universal set $\phi' = U$.

WE DO 1) If A and B are two sets such that $n(A \cup B) = 70$, $n(A) = 42$, and $n(B) = 50$. How many elements does $A \cap B$ have?

YOU DO 1) If A and B are two sets such that $n(A \cup B) = 40$, $n(A) = 20$, and $n(B) = 10$. How many elements does $A \cap B$ have?

CHECK for UNDERSTANDING 1

True or False

1) Every Z is N.
2) {-2, 2, 8} is equivalent to **{3, 4, 5}**
3) {1, 2, 5} **is not equal to {1, 2, 5}**
4) 4 $\subseteq$ **{1, 2, 5}**
5) {1} $\subseteq$ **{1, 2, 5}**
6) 27 $\notin$ {W}
7) Every irrational number is a whole number
8) Every integer is a decimal number.
9) Rational and irrational numbers are not real numbers.
10) Natural numbers start from 0.
11) The union of N and Z is Z.

Properties of Set Operations

The properties of set operations are similar to the properties of fundamental operations on numbers. The important properties on set operations are stated below:

1) Commutative Law - For any two given sets A and B, the commutative property is defined as,

A ∪ B = B ∪ A

This means that the set operation of union of two sets is commutative.

Example 1.

A = {1, 2, 3, 4, 5, 6}

B = {2, 3, 4, 6, 8, 9, 10}

A ∩ B = B ∩ A

This means that the set operation of intersection of two sets is commutative.

Example 2.

A = {N}

B = {W}

2) Associative Law - For any three given sets A, B and C the associative property is defined as,

(A ∪ B) ∪ C = A ∪ (B ∪ C)
This means the set operation of union of sets is associative.

Example 1.
A = {1, 2, 3}
B = {2, 3,4}

(A ∩ B) ∩ C = A ∩ (B ∩ C)
This means the set operation of intersection of sets is associative.
Example 2.
A = {1, 2, 3}
B = {2, 3,4}

3) De-Morgan's Law - The De Morgan's law states that for any two sets A and B, we have **(A ∪ B)' = A' ∩ B'** and **(A ∩ B)' = A' ∪ B'**
Example 1.
U= {1, 2, 3, 4)
A = {1, 2, 3}
B = {2, 3,4}

Thus,
- A ∪ A = A
- A ∩ A = A
- A ∩ ∅ = ∅
- A ∪ ∅ = A
- A ∩ B ⊆ A
- A ⊆ A ∪ B

What is a proposition?

- A proposition is the basic building block of logic.
- It is defined as a declarative sentence that is either True or False, but not both.
- The **Truth Value** of a proposition is True (denoted as T) if it is a true statement, and False (denoted as F) if it is a false statement.

Note: Some sentences that do not have a truth value or may have more than one truth value are not propositions.

For Example,

1. What time is it?
2. Go out and play.
3. $x + 1 = 2$.

The above sentences are not propositions as the first two do not have a truth value, and the third one may be true or false.

Note: To represent propositions, **propositional variables** are used. By convention, these variables are represented by small alphabets such as p, q, r, s.

Key Idea 1. The area of logic which deals with propositions is called ***propositional calculus*** *or* ***propositional logic***. It also includes producing new propositions using existing ones.

Key Idea 2. Propositions constructed using one or more propositions are called ***compound propositions***.

The propositions are combined using ***Logical Connectives*** *or* ***Logical Operators***.

CHECK FOR UNDERSTANDING 6

QUESTION # 1)

Select all below which are propositions.

- [ ] 2+3=5
- [ ] Hawaii is in the Atlantic Ocean.
- [ ] Turn left at the corner.
- [ ] Today is Thursday.
- [ ] Hello!
- [ ] What time is it?

QUESTION # 2)

Select all below which are propositions.

- ☐ No chicken can swim.
- ☐ What time is it?
- ☐ Where is the orange juice?
- ☐ Look out!
- ☐ Today is Thursday.
- ☐ Turn left at the corner.

Truth Table

Since we need to know the truth value of a proposition in all possible scenarios, we consider all the possible combinations of the propositions which are joined together by Logical Connectives to form the given compound proposition. ***This compilation of all possible scenarios in a tabular format is called a*** *truth table*.

Most Common Logical Connectives-

1. Negation – If p is a proposition, then the negation of p is denoted by $\neg p$, which when translated to simple English means- "It is not the case that p" or simply "not p". The truth value of $\neg p$ is the opposite of the truth value of p. The truth table of $\neg p$ is-

p	$\neg p$
T	F
F	T

Example.

The negation of "It is raining today", is "It is not the case that is raining today" or simply "It is not raining today".

2. Conjunction – For any two propositions p and q, their conjunction is denoted by $p \wedge q$, which means "p and q". The conjunction $p \wedge q$ is True when both p and q are True, otherwise False. The truth table of $p \wedge q$ is-

p	q	p∧q
T	T	T
T	F	F
F	T	F
F	F	F

Example, The conjunction of the propositions p– "Today is Friday" and q– "It is raining today", $p \wedge q$ is "Today is Friday and it is raining today". This proposition is true only on rainy Fridays and is false on any other rainy day or on Fridays when it does not rain.

3. Disjunction – For any two propositions $p$and q, their disjunction is denoted by $p \lor q$, which means "$p$or q". The disjunction $p \lor q$is True when either $p$or $q$is True, otherwise False. The truth table of $p \lor q$is-

p	q	p$\lor q$
T	T	T
T	F	T
F	T	T
F	F	F

Example, The disjunction of the propositions p– "Today is Friday" and q– "It is raining today", $p \lor q$is "Today is Friday or it is raining today". This proposition is true on any day that is a Friday or a rainy day(including rainy Fridays) and is false on any day other than Friday when it also does not rain.

4. Exclusive Or – For any two propositions p and q, their exclusive or is denoted by $p \oplus q$, which means "either p or q but not both". The exclusive or $p \oplus q$ is True when either p or q is True, and False when both are true or both are false. The truth table of $p \oplus q$ is-

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Example, The exclusive or of the propositions p– "Today is Friday" and q– "It is raining today", $p \oplus q$ is "Either today is Friday or it is raining today, but not both". This proposition is true on any day that is a Friday or a rainy day(not including rainy Fridays) and is false on any day other than Friday when it does not rain or rainy Fridays.

5. Implication – For any two propositions $p$and q, the statement "if $p$then q" is called an implication and it is denoted by $p \to q$. In the implication $p \to q$, $p$is called the **hypothesis** or **antecedent** or **premise** and $q$is called the **conclusion** or **consequence**. The implication is $p \to q$is also called a **conditional statement**. The implication is false when $p$is true and $q$is false otherwise it is true. The truth table of $p \to q$is-

p	q	p$\to q$
T	T	T
T	F	F
F	T	T
F	F	T

You might wonder that why is $p \to q$true when $p$is false. This is because the implication guarantees that when $p$and $q$are true then the implication is true. But the implication does not guarantee anything when the premise $p$is false. There is no way of knowing whether or not the implication is false since $p$did not happen. This situation is similar to the "Innocent until proven Guilty" stance, which means that the implication $p \to q$is considered true until proven false. Since we cannot call the implication $p \to q$false when $p$is false, our only alternative is to call it true. This follows from the **Explosion Principle** which says- "A False statement implies anything" Conditional statements play a very important role in mathematical reasoning, thus a variety of terminology is used to express $p \to q$, some of which are listed below.

```
"if p, then q""p is sufficient for q""q when p""a necessary conditio
```

Example, "If it is Friday then it is raining today" is a proposition which is of the form $p \to q$. The above proposition is true if it is not Friday(premise is false) or if it is Friday and it is raining, and it is false when it is Friday but it is not raining.

6. Biconditional or Double Implication – For any two propositions $p$and q, the statement "$p$if and only if(iff) q" is called a biconditional and it is denoted by $p \leftrightarrow q$. The statement $p \leftrightarrow q$is also called a **bi-implication**. $p \leftrightarrow q$has the same truth value as $(p \rightarrow q) \land (q \rightarrow p)$ The implication is true when $p$and $q$have same truth values, and is false otherwise. The truth table of $p \leftrightarrow q$is-

p	q	p$\leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Some other common ways of expressing $p \leftrightarrow q$are-

```
"p is necessary and sufficient for q""if p then q, and conversely""p
```

Example, "It is raining today if and only if it is Friday today." is a proposition which is of the form $p \leftrightarrow q$. The above proposition is true if it is not Friday and it is not raining or if it is Friday and it is raining, and it is false when it is not Friday or it is not raining. **Exercise:**

ORDER OF OPERATIONS

Any expression inside the parenthesis must be evaluated first. Without the parenthesis, the order of evaluation is

$\neg, \land, \lor, \rightarrow, \leftrightarrow, \oplus$ from left to right.

Video: https://www.youtube.com/watch?v=hfz1gAoNd1w&t=2s

STEPS IN CREATING A TRUTH TABLE

Step 1: Identify the number of variables.

Step 2: Use the expression 2^n to identify the number of rows, where n represents the number of variables or propositions.

Step 3: Set the values of T and F on your table.

Step 4: Evaluate the relationships

WE DO 1) Perform the indicated operations.

$((p \rightarrow q \wedge p) \vee q) \leftrightarrow (\neg p \oplus q)$

YOU DO 1) Create the truth table for the given propositional expressions and then evaluate.

$(\neg p \vee \neg q) \wedge (p \rightarrow p) \leftrightarrow (p \oplus q)$

WE DO 2) Create the truth table for the given propositional expressions and then evaluate.

$\sim (\sim p \wedge \sim q) \vee (p \rightarrow \sim r) \leftrightarrow (p \oplus r)$

YOU DO 2) Create the truth table for the given propositional expressions and then evaluate.

$(\sim p \wedge \sim r) \vee \sim (p \rightarrow \sim q) \leftrightarrow (p \oplus r)$

CHECK FOR UNDERSTANDING 7
QUESTION #1

Fill in the truth table below for the statements shown.

(Note: you may need to work these out on paper, using additional columns.)

p	q	$\sim(p \vee q)$	$\sim p \vee \sim q$	$\sim(p \wedge q)$
T	T			
T	F			
F	T			
F	F			

Which pair of statements are equivalent?

- ○ $\sim p \vee \sim q$ and $\sim(p \wedge q)$
- ○ $\sim(p \vee q)$ and $\sim p \vee \sim q$
- ○ $\sim(p \vee q)$ and $\sim(p \wedge q)$
- ○ All three are equivalent
- ○ None are equivalent

QUESTION #2

Complete the truth table for the statements.

p	q	$p \vee q$	$p \wedge q$	$p \to q$	$p \leftrightarrow q$
T	T	?	?	?	?
T	F	?	?	?	?
F	T	?	?	?	?
F	F	?	?	?	?

Question Help: Video

QUESTION 3

Complete the truth table for the statement $s \wedge (p \wedge r)$.

s	p	r	$s \wedge (p \wedge r)$
T	T	T	?
T	T	F	?
T	F	T	?
T	F	F	?
F	T	T	?
F	T	F	?
F	F	T	?
F	F	F	?

ADDITIONAL PROBLEM 1

Complete the truth table for the following compound statement.

$$(\sim q \rightarrow p) \vee \sim r$$

p	q	r	$\sim q$	$\sim q \rightarrow p$	$\sim r$	$(\sim q \rightarrow p) \vee \sim r$
T	T	T	?	?	?	?
T	T	F	?	?	?	?
T	F	T	?	?	?	?
T	F	F	?	?	?	?
F	T	T	?	?	?	?
F	T	F	?	?	?	?
F	F	T	?	?	?	?
F	F	F	?	?	?	?

-End of the lesson-

Thank you so much for listening!

Do the assigned activities