Prelim: Lecture Notes #1 – Sets & Venn Diagram 01.23.22
Discrete Structures 1

REMINDER!

1) ALWAYS FOLLOW MY CLASSROOM PROCEDURES.

2) ACTIVITIES ARE ALWAYS UPDATED IN GOOGLE CLASSROOM & MYOPENMATH

3) ALWAYS SUBMIT YOUR WORK ON TIME. NO MORE SUBMISSSION OF LATE WORK.

4) THE WEEKLY HOMEWORK IS ALWAYS DUE EVERY SATURDAY.

5) CHECK THE UPDATES ON GOOGLE CLASSROOM AND DO YOUR MISSING ASSIGNMENTS as the GRADES are always UPDATED.

Note: **Activities that you failed to submit on-time will automatically disappear on the system.**

Activities for today

1) ***MyOpenMath***
   The course ID: ***175937***
   The enrollment key: ***faith2022***

   a) Bellwork #1 (Prelims) – Math Related Topics 01.25.23
   b) Activity #1 (Prelims) - Introduction to Sets 01.25.23
   c) Weekly Homework #1 (Prelims)- All About Sets 01.25-01.28.23
2) ***Google Classroom: Class code:*** ciu2uyg
   a) Prelim Lecture Notes #1 01.25.23

Introduction to Sets & Venn Diagram

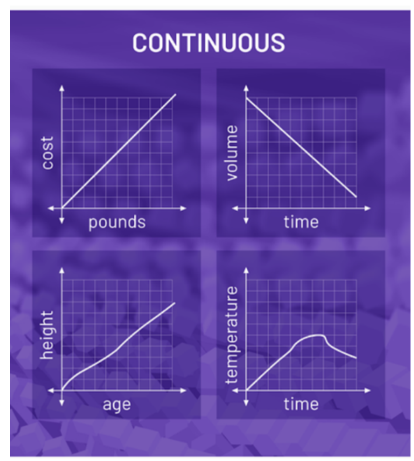
At the end of the lesson, students are expected to
1) identify what is discrete structure
1) identify what is set.
2) list different types of sets
3) perform set operations
4) identify Venn Diagrams

Discrete Structures Definition

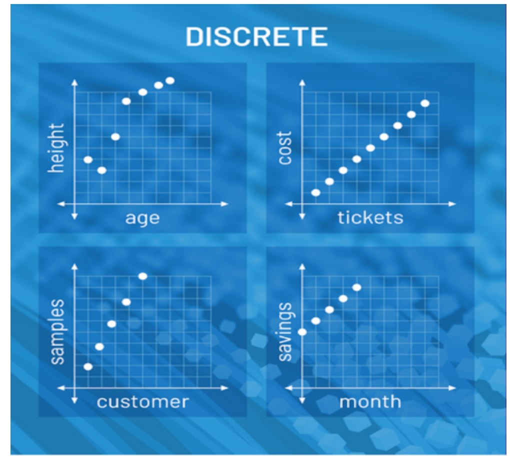
It is a branch of mathematics involving discrete elements that uses algebra and arithmetic. It is increasingly being applied in the practical fields of mathematics and computer science. It is a very good tool for improving reasoning and problem-solving capabilities

Two broad categories of mathematics

1. **Continuous Mathematics** – It is based upon continuous number line or the real numbers. It is characterized by the fact that between any two numbers, there are almost always an infinite set of numbers. For example, a function in continuous mathematics can be plotted in a smooth curve without breaks.



2. Discrete Mathematics – It involves distinct values; i.e. between any two points, there are a countable number of points. For example, if we have a finite set of objects, the function can be defined as a list of ordered pairs having these objects, and can be presented as a complete list of those pairs.



Discrete Data

- Discrete data is a count that involves integers. Only a limited number of values is possible. The discrete values cannot be subdivided into parts. For example, the number of children in a school is discrete data. You can count whole individuals. You can’ t count 1.5 kids.

Discrete Data Key Characteristics

- **You can count the data.** It is usually units counted in whole numbers.
- **The values cannot be divided** into smaller pieces and add additional meaning.
- **You cannot measure the data.** By nature, discrete data cannot be measured at all. For example, you can measure your weight with the help of a scale. So, your weight is not a discrete data.
- **It has a limited number of possible values** e.g. days of the month.
- Discrete data is **graphically displayed by a bar graph.**

Continuous Data

- Continuous data is information that could be meaningfully divided into finer levels. It can be measured on a scale or continuum and can have almost any numeric value. For example, you can measure your height at very precise scales — meters, centimeters, millimeters and etc.
- You can record continuous data at so many different measurements – width, temperature, time, and etc. This is where the key difference with discrete data lies.

Continuous Data Characteristics

- **In general, continuous variables are not counted.**
- **The values can be subdivided into smaller and smaller pieces and they have additional meaning.**
- **The continuous data is measurable.**
- **It has an infinite number of possible values within an interval.**
- **Continuous data is graphically displayed by histograms.**

Discrete mathematics in the real-world

It's often said that mathematics is useful in solving a very wide variety of practical problems. As time goes on, more and more mathematics that is done, both in academia and in industry, is discrete. But what are the actual applications people talk about when they say discrete mathematics can be applied? What problems are being solved?

Everyday applications of Discrete Mathematics

1. Computers run software and store files. The software and files are both stored as huge strings of 1s and 0s. Binary math is discrete mathematics.

2. Networks are, at base, discrete structures. The routers that run the internet are connected by long cables. People are connected to each other by social media ("following" on Twitter, "friending" on Facebook, etc.). The US highway system connects cities with roads.

3. Doing web searches in multiple languages at once, and returning a summary, uses linear algebra.

4. Google Maps uses discrete mathematics to determine fastest driving routes and times. There is a simpler version that works with small maps and technicalities involved in adapting to large maps.

5. **Data compression**, **reduction of noise in data**, and **automated recommendations of movies** all use the same tool from linear algebra.

6. Scaling COVID-19 testing by more efficiently using patient samples is assisted by linear algebra.

7. Scheduling problems---like deciding which nurses should work which shifts, or which airline pilots should be flying which routes, or scheduling rooms for an event, or deciding timeslots for committee meetings, or which chemicals can be stored in which parts of a warehouse---are solved either using graph coloring or using combinatorial optimization, both parts of discrete mathematics. One example is scheduling games for a professional sports league.

8. An **analog clock** has gears inside, and the sizes/teeth needed for correct timekeeping are determined using discrete math.

9. Wiring a computer network using the least amount of cable is a minimum-weight spanning tree problem.

10. Encryption and decryption are part of **cryptography**, which is part of discrete mathematics. For example, secure internet shopping uses public-key cryptography.

11. Discrete mathematics is **used in vaccine development**.

12. **Computer graphics** (such as in video games) use linear algebra in order to transform (move, scale, change perspective) objects. That's true for both applications like game development, and for operating systems.

13. **Digital image processing** uses discrete mathematics to merge images or apply filters.

A. Conventions in mathematics, some commonly used symbols, its meaning and an example

a) Sets and Logic

SYMBOL	NAME	MEANING	EXAMPLE
∪	Union	Union of set A and set B	A ∪ B
∩	Intersection	Intersection of set A and set B	A ∩ B
∈	Element	x is an element of A	x ∈ A
∉	Not an element of	x is not an element of set A	x ∉ A
{ }	A set of..	A set of an element	{a, b, c}
⊂	Subset	A is a subset of B	A ⊂ B
⊄	Not a subset of	A is not a subset of B	A ⊄ B
…	Ellipses	There are still other items to follow	a, b, c, … a + b + c + ….
∧	Conjunction	A and B	A ∧ B
∨	Disjunction	A or B	A ∨ B
~	Negation	Not A	~A
→	Implies (If-then statement)	If A, then B	A → B

$\leftrightarrow$	**If and only if**	**A if and only if B**	**A $\leftrightarrow$ B**
$\forall$	**For all**	**For all x**	$\forall$**x**
$\exists$	**There exist**	**There exist an x**	$\exists$
$\therefore$	**Therefore**	**Therefore C**	$\therefore C$

$\vert$	**Such that**	**x such that y**	**$x \mid y$**
■	**End of proof**		
$\equiv$	**Congruence / equivalent**	**A is equivalent to B** **a is congruent to b modulo n**	**$A \equiv B$** **$a \equiv b \bmod n$**
a, b, c, …, z **(lower case)**	**Variables** ***First part of English Alphabet uses as fixed variable*** ***Middle part of English alphabet use as subscript and superscript variable*** ***Last part of an English alphabet uses as unknown variable***	**$(ax_o)^p$**	**$(5x_2)^6$**

b) Basic Operations and Relational Symbols

SYMBOL	NAME	MEANING	EXAMPLE
+	Addition; Plus sign	a plus b a added by b a increased by b	3 + 2
-	Subtraction; minus sign	a subtracted by b a minus b a diminished by b	3 - 2
• ( )	Multiplication sign *we do not use x as a symbol for multiplication in our discussion since its use as a variable*	a multiply by b a times b	4 • 3 (4)(3)
÷ or |	Division sign; divides	a ÷ b b | a	10 ÷ 5 5 | 10

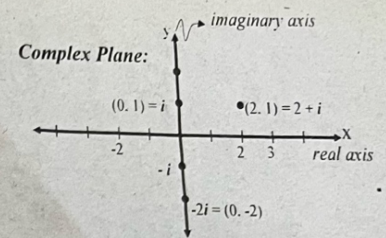
o	Composition of function	f of g of x	f o g(x)
=	Equal sign	a = a a + b = b + a	5 = 5 3 + 2 = 2 + 3
≠	Not equal to	a ≠ b	3 ≠ 4
>	Greater than	a > b	10 > 5
<	Less than	b < a	5 < 10
≥	Greater than or equal to	a ≥ b	10 ≥ 5
≤	Less than or equal to	b ≤ a	5 ≤ 10
*	Binary operation	a * b	a * b = a + 17b

c) Set of Numbers

SYMBOL	NAME	MEANING	EXAMPLE
$\mathbb{N}_0$	natural numbers / whole numbers set (with zero)	$\mathbb{N}_0 = \{0,1,2,3,4,...\}$	$0 \in \mathbb{N}_0$
$\mathbb{N}_1$	natural numbers / whole numbers set (without zero)	$\mathbb{N}_1 = \{1,2,3,4,5,...\}$	$6 \in \mathbb{N}_1$

$\mathbb{Z}$	integer numbers set	$= \{...-3, -2, -1,0,1,2,3,...\}$	$-6 \in$
$\mathbb{Q}$	rational numbers set	$= \{x \mid x=a/b,\ a,\ b\in\mathbb{Z}$ and $b\neq 0\}$	$2/6 \in$
$\mathbb{R}$	real numbers set	$= \{x \mid -\infty < x < \infty\}$	$6.343434 \in$
$\mathbb{C}$	complex numbers set	$= \{z \mid z=a+bi,\ -\infty<a<\infty,\ -\infty<b<\infty\}$	$6+2i \in$

Complex Plane:



- **COMPLEX NUMBERS = $\mathbb{C}$**
  $a + bi = (a, b)$, $i = \sqrt{-1}$, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$
  - **REAL NUMBERS = R**
    - **RATIONALS = Q**
      - **INTEGERS = Z**
        $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$
        - **NEGATIVE INTEGERS = Z^-**
          $\{\ldots, -3, -2, -1\}$
        - **NON-NEGATIVE INTEGERS = W or WHOLE NUMBER** $\{0, 1, 2, \ldots\}$
          - **ZERO**
            $\{0\}$
          - **POSITIVE INTEGERS = N (COUNTING or NATURAL NUMBERS)**
            $\{1, 2, 3, \ldots\}$
      - **DECIMALS (i.e. Fractions) = F**
        - **D_T = TERMINATING DECIMALS**
          Ex. $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \frac{1}{12}$
        - **D_{NR} = NON-TERMINATING REPEATING DECIMALS**
          Ex. $\frac{1}{3}, \frac{2}{3}, \frac{16}{9}, \frac{1}{9}$
    - **IRRATIONALS = Q′**
      NON-TERMINATING NON-REPEATING DECIMALS
      Ex. $\sqrt{2}, \sqrt[3]{3}, \sqrt[4]{4}, \sqrt{5}, \sqrt[3]{2}, \pi, e, \sqrt[5]{10}$

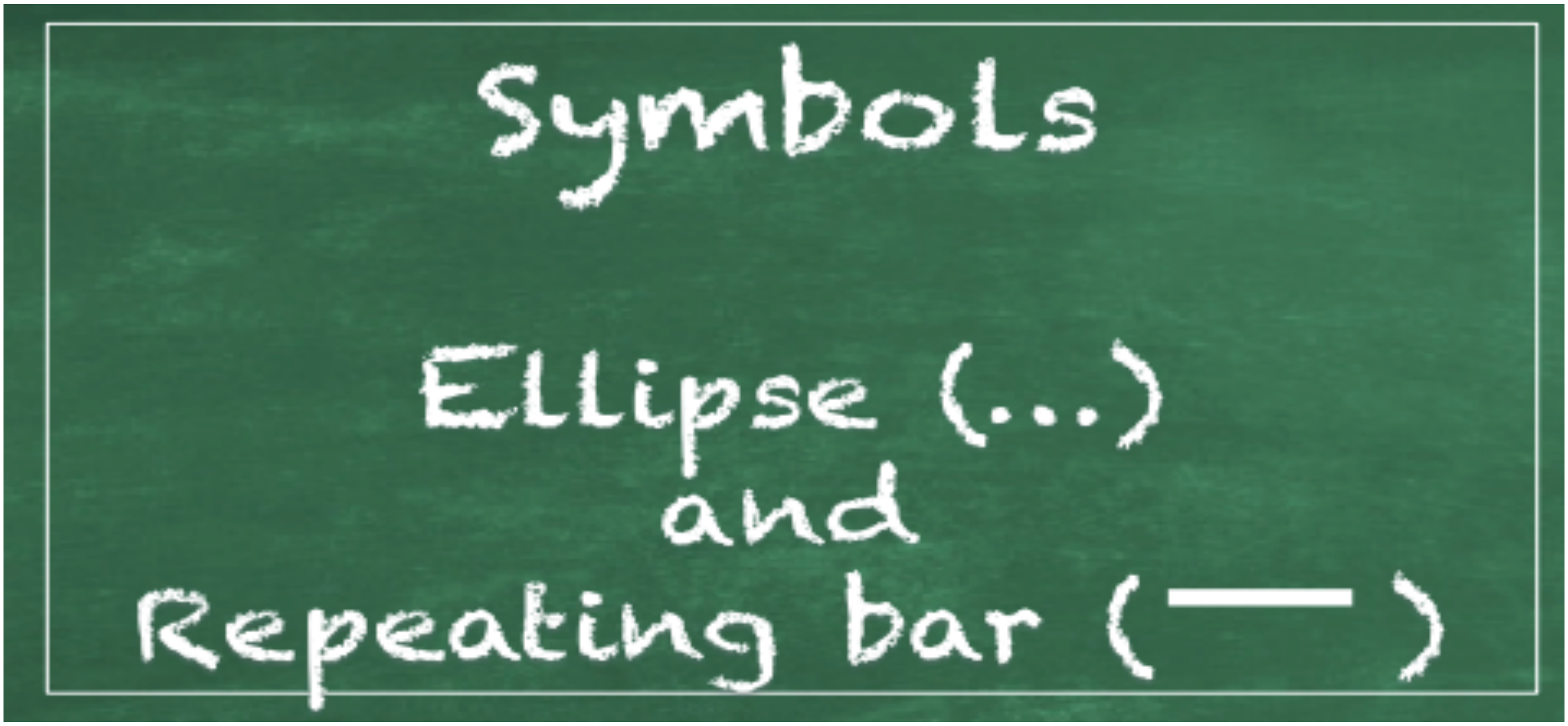
Note:
a. $N \subset W \subset Z \subset Q \subset R \subset \mathbb{C}$
b. $Z^- \cap W = \varnothing$; $Z^- \cup W = Z$
c. $D_T \cap D_{NR} = \varnothing$; $D_T \cup D_{NR} = F$
d. $Q \cap Q' = \varnothing$; $Q \cup Q' = R$

Fig. 1.3.1
SCHEMATIC DIAGRAM OF REAL NUMBER SYSTEM, ITS EXTENSION $\mathbb{C}$ and the SUBSETS

List of Mathematical Symbols

- $\mathbb{R}$ = real numbers, $\mathbb{Z}$ = integers, $\mathbb{N}$=natural numbers, $\mathbb{Q}$ = rational numbers, $\mathbb{P}$ = irrational numbers.
- $\subset$= proper subset (not the whole thing) $\subseteq$=subset
- $\exists$ = there exists
- $\forall$ = for every
- $\in$= element of
- $\bigcup$ = union (or)
- $\bigcap$ = intersection (and)
- s.t.= such that
- $\implies$ implies
- $\iff$ if and only if
- $\sum$ = sum
- $\setminus$ = set minus
- $\therefore$= therefore

Symbols to remember when working with fractions and decimals.



Note: If there is a bar ($\overline{3}$) at the top of the number, it means that the sequence of that number infinitely repeating.

Example: $15.2\overline{34}$ = 15.23434343434343434…

Note: If there is a period next to a decimal number (5…) it means that the sequence of number infinitely going.
Example: $13.12345\ldots$ = 13.12345678…
Did you know that?

The word "non-terminating" is the same as never stop or never end.
The word "terminating" means stop or end.

KEY IDEAS

Non-terminating decimal (...) means the decimal number infinitely going and never stops.
Example: 3.142857143...

Non-repeating decimal means the decimal number infinitely going and never repeats.
Example: 3.142857143...

Repeating decimal ($\overline{}$) means the decimal number infinitely repeating and never stops.
Example:

$15.2\overline{34}$ = **15.2343434343434**
1/3 = 0.3333333333.......

Terminating decimal means decimal that stops.
Example: ½= 0.5; ¼ = 0.25

GENERAL TYPES OF NUMBERS

1) NATURAL NUMBERS OR COUNTING NUMBERS (N) – the set of all positive counting numbers starting with 1.

Examples: {1, 2, 3, 4, 5…}

Examples: 1, 300, 1000

Natural Numbers or Counting Numbers

Natural Numbers	• Positive whole numbers - Not Including Zero - Examples: {1, 2, 3, 4, 5...} • Fractions that simplify to a positive whole number. - Examples: $\frac{28}{8} = 4$, $\frac{35}{7} = 5$ • Square roots that simplify to a positive whole number - Examples: $\sqrt{64} = 8$, $\sqrt{100} = 10$ • Cube roots that simplify to a positive whole number - Examples: $\sqrt[3]{27} = 3$, $\sqrt[3]{8} = 2$

2) WHOLE NUMBERS (W)– the set of all positive counting numbers begin with zero

Example: {0, 1, 2, 3 4, 5 …}

Examples: 0, 50, 1230

Whole Numbers

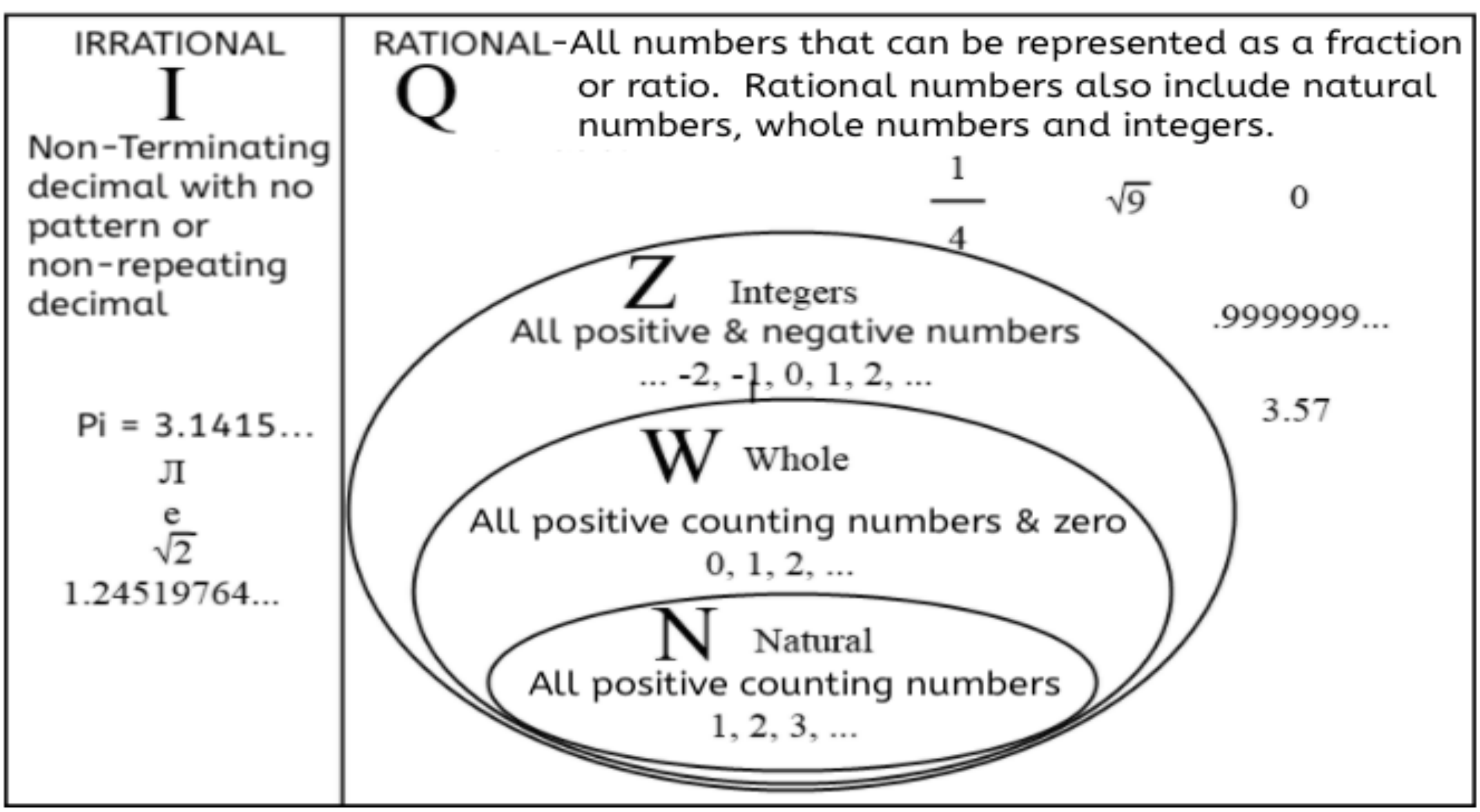
Whole Numbers Include Natural Numbers {1, 2, 3, 4, 5…}	• Positive whole numbers - Including Zero - Examples: {0, 1, 2, 3, 4, 5...} • Fractions that simplify to a positive whole number. - Examples: $\frac{28}{8} = 4$, $\frac{35}{7} = 5$ • Square roots that simplify to a positive whole number - Examples: $\sqrt{64} = 8$, $\sqrt{100} = 10$ • Cube roots that simplify to a positive whole number - Examples $\sqrt[3]{27} = 3$, $\sqrt[3]{8} = 2$

3) INTEGERS (Z)– the set of whole numbers and their opposites (positive, negative, zero). Numbers without decimal.
Example {…-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5…}
Examples: 0, -50, 100, 500

Integers Numbers

Integers Include Natural Numbers and Whole Numbers {0, 1, 2, 3, 4, 5…}	• Positive and negative whole numbers including zero - Examples: {…-3, -2, -1, 0, 1, 2, 3…} • Fractions that simplify to a positive or negative whole number. - Examples: $\frac{28}{7} = 4$, $\frac{35}{7} = 5$ • Square roots that simplify to a positive or negative whole number - Examples: $\sqrt{64} = 8$, $\sqrt{100} = 10$, $-\sqrt{36} = -6$, $-\sqrt{144} = -12$ • Cube roots that simplify to a positive whole number - Examples $\sqrt[3]{27} = 3$, $\sqrt[3]{8} = 2$, $\sqrt[3]{-27} = -3$, $-\sqrt[3]{8} = -2$

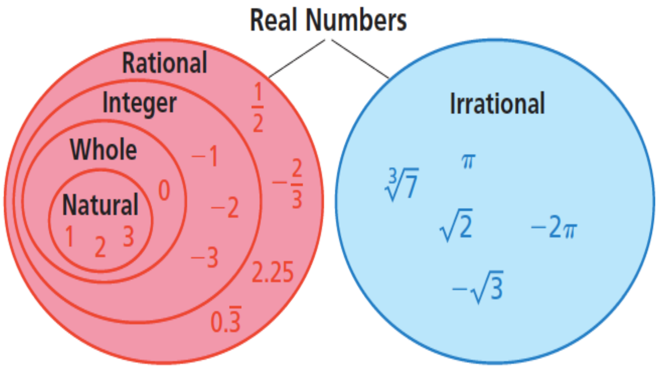
Real Numbers



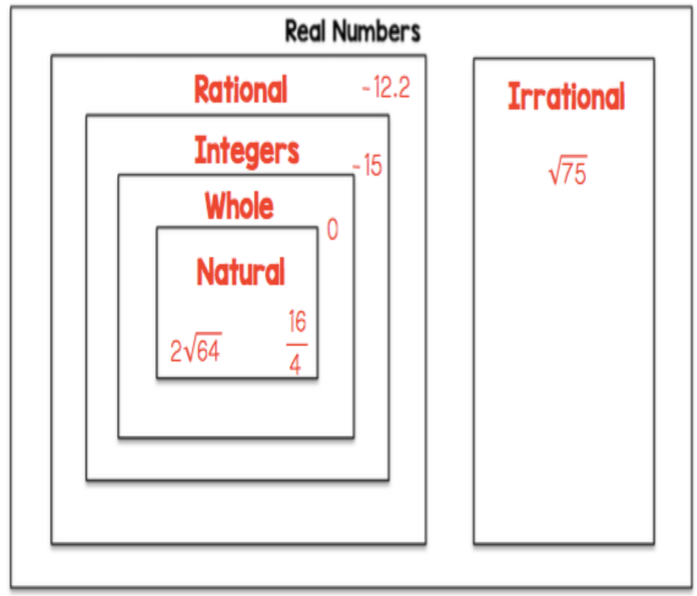
Key Idea

Real Numbers

Rational numbers and irrational numbers together form the set of **real numbers**.



$\{2\sqrt{64}, -15, \frac{16}{4}, \sqrt{75}, 0, -12.2\}$



CLASSIFICATIONS OF REAL NUMBES

REAL NUMBERS	DEFINITION	EXAMPLES
Natural	• The set of all positive counting numbers (numbers on a number line); starting with _1_	$42,\ \frac{50}{2},\ 800$
Whole	• The set of all positive counting numbers starting with _0_	$\sqrt{100},\ 0,\ 4+\sqrt{4}$
Integers	• The set of whole numbers and their _opposites_	$-\frac{15}{3},\ 25,\ -\sqrt{81}$
Rational	• Numbers that can be written as fractions • Terminating decimals, repeating decimals and _perfect_ squares	$12\frac{5}{7},\ -6.8,\ 5$
Irrational	• Numbers that cannot be written as _fractions_ • Non-terminating decimals, non-repeating decimals and _non-perfect_ squares	$3\pi,\ \sqrt{10},$ 6.7234...

TWO TYPES OF REAL NUMBERS (RATIONAL and IRRATIONAL)

RATIONAL VS. IRRATIONAL NUMBERS

RATIONAL NUMBERS **– numbers that can be written as the ratios or fractions of the two integers or decimals.**

As a fraction $\frac{a}{b}$, where *a* and *b* are integers (b $\neq 0$)

Examples: -12, $\frac{7}{8}$, $\frac{1}{2}$, 5

These numbers are natural numbers, whole numbers, integers, terminating decimals, repeating decimals, mixed fractions, perfect squares and perfect cubes.

SUBSETS OF RATIONAL NUMBERS

1) NATURAL NUMBERS OR COUNTING NUMBERS – the set of all positive counting numbers starting with 1.

Example: {1, 2, 3, 4, 5…}

Examples: 1, 300, 1000

2) WHOLE NUMBERS – the set of all positive counting numbers begin with zero

Example: {0, 1, 2, 3 4, 5 …}

Examples: 0, 50, 1230

3) INTEGERS – the set of whole numbers and their opposites (positive, negative, zero). Numbers without decimal.

Example {…-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5…}

Examples: 0, -50, 100, 500

4) TERMINATING DECIMALS– decimal values that terminate

Examples: 0.6, $\frac{1}{4}$ = 0.25, 0.75, $\frac{1}{2}$ = 0.5

Remember

The decimal form of a rational number either terminates or repeats.

5) **REPEATING DECIMALS– decimal values that repeat**

Examples: $-0.\overline{25}$ $\frac{2}{3} = \mathbf{0.\overline{66}}$ $\frac{1}{3} =$ $0.\overline{33}$

6) **MIXED FRACTIONS – fractions with whole number and fractional part**

Examples: $5\frac{1}{2} = 5.5,\ 3\frac{3}{4} = 3.75$

7) **PERFECT SQUARES – numbers that when you get the square root the answer is a whole number**

Examples: $\sqrt{4} = 2, \sqrt{25} = 5, \sqrt{100} = 10,$

8) **PERFECT CUBES – numbers that when you get the cube root the answer is a whole number.**

Examples: $\sqrt[3]{8} = 2, \sqrt[3]{64} = 4$

IRRATIONAL NUMBERS **–numbers that cannot be written as the ratios or fractions of the two integers or decimals**

Examples: π (The symbol for Pi)

e(Euler's number) = 2.718 281 828 459 045 235 360 287 471 352

SUBSETS OF IRRATIONAL NUMBERS

1) **NON–TERMINATING DECIMALS/NON-REPEATING DECIMALS – decimal values that keep on going and non-repeating.**

Examples: π (The symbol for Pi)= $\frac{22}{7} = 3.14285714 \ldots$

e (Euler's number) = 2.718 281 828 459 045 235 360 287 471 352…

2) NON-PERFECT SQUARE – numbers that when you get the square root the answer is not a whole number.

Examples: $\sqrt{2} = 1.414213562\ldots$

$\sqrt{3} = 1.732050808\ldots$

3) NON-PERFECT CUBE- numbers that when you get the cube root the answer is not a whole number.

Examples: $\sqrt[3]{2} = 1.25992105\ldots$

REMEMBER:
The decimal form of an irrational number neither terminates nor repeats.

KEY IDEAS

✓To check if the fraction is rational or irrational number use your calculator and check if the decimal values are repeating or terminating (RATIONAL NUMBER) and non-terminating or non-repeating (IRRATIONAL NUMBER)

WE DO 1) CLASSIFY THE NUMBERS IN THE TABLE BY CHECKING ALL THAT APPLY.

VALUE	NATURAL	WHOLE	INTEGER	RATIONAL	IRRATIONAL	REAL
1) $\frac{1}{9}$						
2) π						
3) $\sqrt[3]{-64}$						
4) $-\frac{22}{7}$						
5) $\frac{1}{12}$						
6) $0.888\ldots$						
7) $-17.65\overline{8}$						

8) $\sqrt{5}$						
9) - 18.123456…						

YOU DO 1. CLASSIFY THE NUMBERS IN THE TABLE BY CHECKING ALL THAT APPLY.

VALUE	NATURAL	WHOLE	INTEGER	RATIONAL	IRRATIONAL	REAL
1) $\frac{3}{9}$						
2) $5\frac{1}{2}$						
3) $\sqrt[3]{-64}$						
4) $\frac{22}{7}$						
5) $\frac{1}{15}$						
6) $0.333\ldots$						
7) $-13.17\overline{98}$						
8) $\sqrt{15}$						
9) $\sqrt[3]{7}$						

QUICK REVIEW

1) Select two irrational numbers.

A) $\pi - \pi$

B) $-12.0\overline{76}$

C) $\sqrt{12}$

D) $3.\overline{14}$

E) $\frac{1.\overline{234}}{9}$

F) $\pi + \pi$

2) Which set includes ONLY rational numbers?

A) $\{0.06, \sqrt{36}, \sqrt{41}\}$

B) $\{-\frac{2}{3}, \frac{\sqrt{9}}{\sqrt{16}}, -0.00025$

C) $\{0.012345, \sqrt{30}, -2.23\}$

D) $\{\sqrt{2}, \sqrt{3}, \sqrt{4}\}$

3) Which inequality is correct?

A) $\frac{3}{4} > \frac{3}{\sqrt{2}} > \frac{8}{3}$

B) $\frac{3}{\sqrt{2}} > \frac{8}{3} > \frac{3}{4}$

C) $\frac{8}{3} > \frac{3}{4} > \frac{3}{\sqrt{2}}$

D) $\frac{3}{4} > \frac{3}{\sqrt{2}} > \frac{3}{4}$

4) What type of number is 19.8?

A) an irrational number

B) a rational number

C) a whole number

D) an integer

Introduction of Sets

Video: https://www.youtube.com/watch?v=l3-A0O42Lyo

- German mathematician **G. Cantor** introduced the concept of sets. He had defined a set as a collection of definite and distinguishable objects selected by the means of certain rules or description.
- **Set** theory forms the basis of several other fields of study like counting theory, relations, graph theory and finite state machines. In this chapter, we will cover the different aspects of **Set Theory**.
- A set is defined as a collection of distinct objects of the same type or class of objects. The purposes of a set are called elements or members of the set. An object can be numbers, alphabets, names, etc.

Examples of sets are:

a. A set of rivers of India.

b. A set of vowels.

We broadly denote a set by the capital letter A, B, C, etc. while the fundamentals of the set by small letter a, b, x, y, etc.

If A is a set, and a is one of the elements of A, then we denote it as $a \in A$. Here the symbol $\in$ means - "Element of."

Sets Representation:

Sets are represented in two forms:

a) Roster or tabular form: In this form of representation, we list all the elements of the set within braces { } and separate them by commas.

Example 1: If A=set of all odd numbers less than 10 then in the roster from it can be expressed as $A= \{1,3,5,7,9\}$.

Example 2: If A=set of all even numbers less than 10 then in the roster from it can be expressed as A= {2, 4, 6, 8}.

b) Set Builder or rule form: In this form of representation, we list the properties fulfilled by all the elements of the set. We note as {x: x satisfies properties P}. and read as 'the set of those entire x such that each x has properties P.'

It is also a method which makes use of the description {x | …}. This is read as *"x such that"*

Example 1: If B= {2, 4, 8, 16, 32}, then the set builder representation will be: B = {x: x=2^n, where n ∈ N and 1≤ n ≥5}

Example 2: Starting with all the real numbers, we can limit them to the interval between 1 and 6 inclusive. Hence, it will be represented as:

S = {x: x ≥ 1 and x ≤ 6}

Set Builder Notation Symbols

The different symbols used to represent set builder notation are as follows:

- The symbol ∈ "is an element of".
- The symbol ∉ "is not an element of".
- The symbol W denotes the whole number.
- The symbol Z denotes integers.

- The symbol N denotes all natural numbers or all positive integers.
- The symbol R denotes real numbers or any numbers that are not imaginary.
- The symbol Q denotes rational numbers or any numbers that can be expressed as a fraction.

I DO. Write the following set builder or rule notation to a tabular or roster form

1) {x| x is N greater than 15}

2) $\{\frac{x}{4}$ | x is W less than 5}

3) {2x +4 | x is W between -1 and 6 inclusive}

YOU DO. Write the following set builder or rule notation to a tabular or roster form

1) {x| x is Z less than 4}

2) $\{\frac{x}{5}$ | x is a W less than 4}

3) $\{x \mid x$ is an even integer between 2 and 12$\}$

4) $\{k \mid k$ is a multiple of 4 between 4 and 20$\}$

Additional Example 1

Write the given set using the roster (listing) method. Write your answer in numerical order, without space.

$\{x \in \mathbb{N} \mid 2 \leq x < 8\}$

Additional Example 2

Write the set of letters in the word "messenger" using the roster (listing) method. Write your answer in alphabetical order with no space, example: {a,b,c}.

Cardinality of a Sets

The total number of unique elements in the set is called the *cardinality of the set.* The cardinality of the countably *infinite set is countably infinite.*

Examples:

1. Let P = {k, l, m, n}
The cardinality of the set P is 4.

2. Let A is the set of all non-negative even integers, i.e.
A = {0, 2, 4, 6, 8, 10......}.
As A is countably infinite set hence the cardinality.

CHECK FOR UNDERSTANDING 3

1. Let M = {p, q, r, s, t, u}
The cardinality of the set M is ___________.

2. Let Q = { 2, 4, 6…}
The cardinality of the set Q is ___________.

Kinds of Sets

Video: https://www.youtube.com/watch?v=VBzlvKP-2yI

Sets can be classified into many categories. Some of which are ***finite, infinite, subset, universal, proper, power, singleton set, etc.***

Key Idea. A set which contains no element is called an empty set or null set. We denote the empty set { } or ∅. The set {∅} is not empty since it contains one element.

1. Finite Sets: A set is said to be finite if it contains exactly *n* distinct element where *n* is a non-negative integer. Here, *n* is said to be "cardinality of sets." The cardinality of sets is denoted by |A|, # A, card (A) or n (A).

Example:

1. Cardinality of empty set θ is 0 and is denoted by |θ| = 0
2. Sets of even positive integer is not a finite set.

A set is called a ***finite set*** if there is one to one correspondence between the elements in the set and the element in some set *n*, where *n* is a natural number and *n* is the cardinality of the set. Finite Sets are also called ***numerable sets***. *n* is termed as the cardinality of sets or a cardinal number of sets.

2. Infinite Sets: A set which is not finite is called as Infinite Sets.

Countable Infinite: If there is one to one correspondence between the elements in set and element in N. A countably infinite set is also known as ***Denumerable***. A set that is either finite or denumerable is known as ***countable***. A set which is not countable is known as ***Uncountable***. The set of a non-negative even integer is countable Infinite.

Uncountable Infinite: A set which is not countable is called Uncountable Infinite Set or non-denumerable set or simply Uncountable.

Example: Set R of all +ve real numbers less than 1 that can be represented by the decimal form 0. a_1, a_2, a_3..... Where a_1 is an integer such that $0 \leq a_i \leq 9$.

Key Idea. A set is infinite if the counting of elements has no end. The set of integers Z or positive integers N or Natural numbers), negative integers Z^-, and nonnegative integers (or whole numbers) are ***infinite sets.***

Z = {…, -2, -1, 0, 1, 2, …}

Z^- {-1, -2, -3, …}

N = {1, 2, 3, …}

W = {0, 1, 2, ...}

The sets of positive odd integers Z_0 = {1, 3, 5, …} and positive even integers Z_e = {2, 4, 6, …} are also infinite sets. An *odd integer* is denoted by 2k + 1, where k is an integer while an *even integer* is denoted by 2k, where k is an integer.

3. Subsets: If every element in a set A is also an element of a set B, then A is called a subset of B. It can be denoted as $A \subseteq B$. Here B is called Superset of A.

Example: If A= {1, 2} and B= {4, 2, 1} the A is the subset of B or $A \subseteq B$.

4. Proper Subset: If A is a subset of B and $A \neq B$ then A is said to be a proper subset of B. If A is a proper subset of B then B is not a subset of A, i.e., there is at least one element in B which is not in A.

Example:
Let A = {2, 3, 4}
B = {2, 3, 4, 5}

A ⊂ B
A is a proper subset of B.

Key Idea: The null ∅ is a proper subset of every set.

5. Improper Subset: If A is a subset of B and A = B, then A is said to be an improper subset of B.

Example

A = {2, 3, 4}, B = {2, 3, 4}
A ⊆ B
A is an improper subset of B.

Key Idea: Every set is an improper subset of itself.

Proper subset (The symbol used for this is ⊂)
Improper subset (The symbol used for this is ⊆)

There are two subset symbols.

- ⊆, which is read as "is a subset or equal to" (or sometimes simply as "subset of")
- ⊂, which is read as "is a subset of" (means strictly subset of but NOT equal to)
-

6. Power Sets: The power of any given set A is the set of all subsets of A and is denoted by **P (A)**. If A has n elements, then **P (A)** has $\mathbf{2^n}$ elements.
Example:
A = {1, 2, 3}
P (A) = {∅, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3}}.

Properties of Subsets:

1. Every set is a subset of itself.
2. The Null Set i.e., ∅ is a subset of every set.
3. If A is a subset of B and B is a subset of C, then A will be the subset of C. If $A \subset B$ and $B \subset C \Longrightarrow A \subset C$
4. A finite set having n elements has 2^n subsets.

For example, $0 \in \{0\}$, but $\emptyset \notin \{0\}$. The elements 0 and ∅ are two different symbols. The set {∅} has one element designated by the symbol ∅, a symbol note considered as an empty set in this particular example.

Example: The empty set A = { } has only one subset, i.e., A.

Example: If A = {0, 1}, then the subsets are { }, {0}, {a}, A.

Take note that the number of subsets of a given set is 2^n where *n* denotes number of elements of the set. In the preceding example, $2^2 = 4$ subsets.

Example: The set B = {a, b, c} has 2^3 subsets. The power set has the elements with breakdown as follows

Improper subset with 3 elements: {a, b, c} or B

Proper subsets with 2 elements: {a, b}, {a, c}, {b, c}

Proper subsets with 1 element: {a}, {b}, {c}

Improper subset with no element: { }

KEY IDEA: In any given set, **there are always two improper subsets**. **The set itself and the null set or empty set.**

P(B) = {B, ∅, {a, b}, {a, c}, {b, c}, {a}, {b}, {c}}

In the sets Z, W, and N we have the following subsets as follows:

$$N \subset W \subset Z \text{ i.e.}$$

$\{1, 2, 3, \ldots\} \subset \{0, 1, 2, 3\ldots\} \subset \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

Also Z^- (the negative integers) $\subset$ W (the non-positive integers) $\subset$ Z (the integers

Note: **The expression to find the subsets of a given set is 2^n**, where 2 is constant and ***n*** is the number of elements of the given set.

Check for Understanding 4

I DO. Identify the improper and proper subsets of the given set.

B = {a, b, c, d, e}

WE DO. Identify the improper and proper subsets of the given set.

B = {1, 2, 3, 4}

7) Universal Set: If all the sets under investigations are subsets of a fixed set U, then the set U is called Universal Set.

Example: In the human population studies the universal set consists of all the people in the world.

Key Idea. Universal set is the totality of elements under consideration. If A = {1, 2, 3} and B = {3, 4, 5} then the universal set, considering no other set present is U = {1, 2, 3, 4, 5}

8. Null Set or Empty Set: A set having no elements is called a Null set or void set. It is denoted by ∅ or { }.

Example: A = { }, B = ∅

9. Singleton Set: It contains only one element. It is denoted by {s}.

Example: S= {x | x ∈ N, 7<x<9} = {8}

10. Equal Sets: Two sets A and B are said to be equal and written as A = B if both have the same elements. Therefore, every element which belongs to A is also an element of the set B and every element which belongs to the set B is also an element of the set A.

$A = B \Leftrightarrow \{x \in A \Leftrightarrow x \in B\}$.

Example: The set A = {1, 2, 3} and B {2, 1, 3) are equal sets.

Note: If there is some element in set A that does not belong to set B or vice versa then $A \neq B$, i.e., A is not equal to B.

11) **Equivalent Sets:** If the cardinalities of two sets are equal, they are called equivalent sets.

Example:
If A= {1, 2, 6} and B= {16, 17, 22}, they are equivalent as cardinality of A is equal to the cardinality of B. i.e. |A|=|B|=3

Sets A and B are equivalent, denoted by A ~ B, if they have the same number of elements. The sets C = {a, b, c} and D = {4, 5, 6} are equivalent sets. Also, {2} ~ {∅}, since they have the same number of elements.1

12. Disjoint Sets: Two sets A and B are said to be disjoint if no element of A is in B and no element of B is in A.

Example:

R = {a, b, c}
S = {k, p, m}
R and S are disjoint sets.

Key Idea. Two sets are disjoint if they have no common element.

The sets E = {a, b, c} and F = {e, f, g} are disjoints sets, since no element is common. The sets {0} and {∅} are also disjoint sets. The positive odd integers Z_0 = {1, 3, 5, ...} and the nonnegative even integers Z_e = {0, 2, 4, ...} are disjoint sets. Also, the negative integers Z^- = {-1, -2, ...} and the nonnegative integers W = {0, 1, 2, ...} are ***disjoint sets.***

13. Joint Sets: Sets that have common elements are ***joint sets*.**
The sets A = {4, 5, 6} and B= {6, 10, 11} are joint sets since 6 is common to both A and B.

CHECK for UNDERSTANDING 5
1) Sets with common elements are called ?
A) Joint

B) Disjoint
C) Equal
D) Equivalent

2) Sets with common elements are called ?
A) Joint
B) Disjoint
C) Equal
D) Equivalent

3) Set A = {1, 2, 3} and Set B = {a, b, c} are?
A) Equivalent
B) Equal
C) Joint
D) Universal

4) Set A = {a, c, b, d} and Set B = {a, b, c, d} are?
A) Equivalent
B) Equal
C) Joint
D) Universal

CHECK for UNDERSTANDING 6
True or False
1) Every natural number is a whole number
2) $\{1, 2, 5\} \subseteq \{1, 2, 5, 8\}$
3) $\{1, 2, 5\} \subseteq \{1, 2, 5\}$
4) $1 \subseteq \{1, 2, 5\}$
5) $\{1\} \subseteq \{1, 2, 5\}$

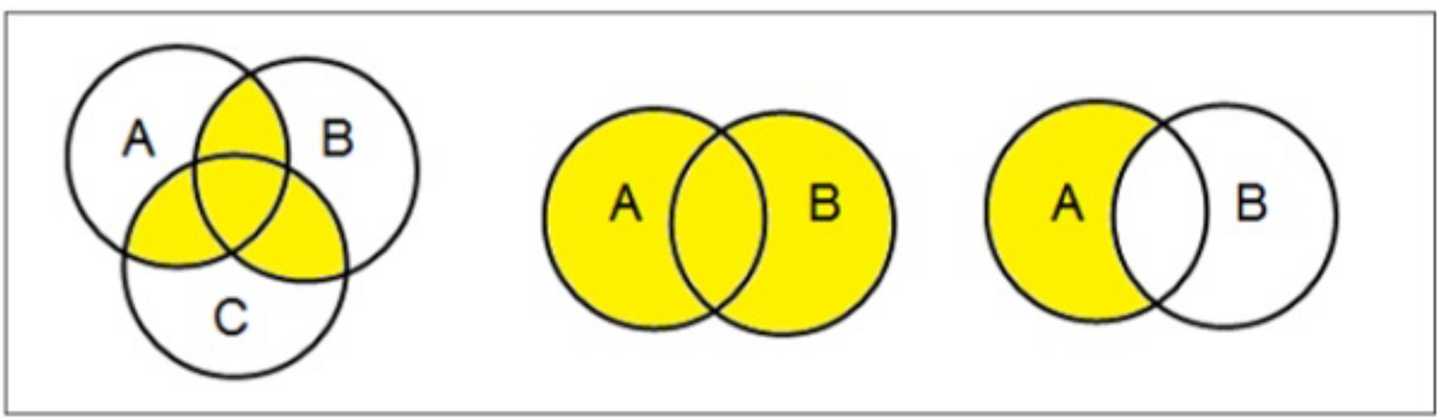
6) 26 $\notin$ {1, 2, 3, 4, ...}
7) Every rational number is an integer.
8) Every integer is a rational number.
9) Some whole numbers are not integers.
10) Some real numbers are not rational numbers.
11) All equivalent sets are equal sets.
12) All equal sets are equivalent sets.

SET OPERATIONS

Venn Diagrams

Venn diagram, invented in 1880 by John Venn, is a schematic diagram that shows all possible logical relations between different mathematical sets.

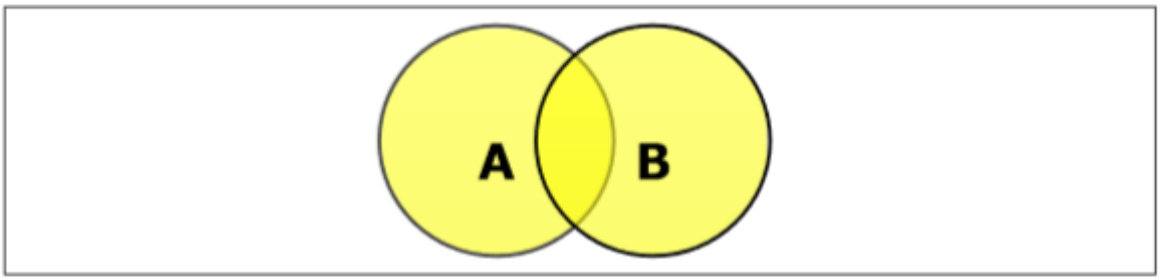
Examples



The basic set operations are:

1) Union of Sets: Union of Sets A and B is defined to be the set of all those elements which belong to A or B or both and is denoted by $A \cup B$.

Hence, $A \cup B = \{x \mid x \in A \text{ OR } x \in B\}$



Example:

Let $A = \{1, 2, 3\}$ and $B = \{3,4,5,6\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$.

I DO. Let $A = \{a, b, c, d, e\}$ and $B = \{a \ldots z\}$

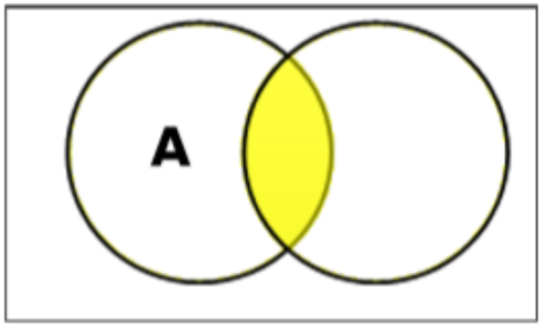
$A \cup B =$

YOU DO. Let $A = \{N\}$ and $B = \{Z\}$

$A \cup B =$

2) Intersection of Sets: Intersection of two sets A and B is the set of all those elements which belong to both A and B and is denoted by $A \cap B$.

Hence, $A \cap B = \{x \mid x \in A \text{ AND } x \in B\}$



Example:

Let A= {11,12,13} and B= {13,14,15}

$A \cap B = \{13\}$

I DO. Let A = {N} and B = {W}

$A \cap B$

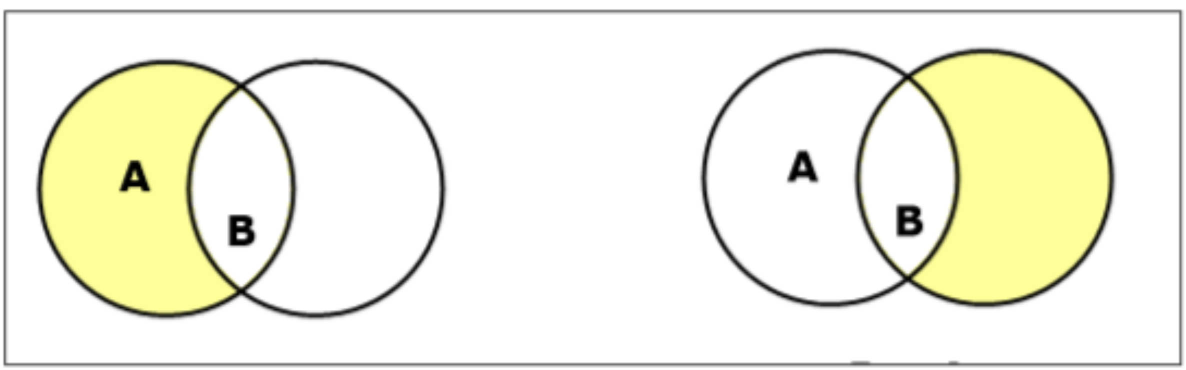
YOU DO. Let A = {Z} and B = {N}

$A \cap B$

WE DO) If A and B are two sets such that n $(A \cup B) = 60$, $n(A) = 32$, and $n(B) = 40$. How many elements does $A \cap B$ have?

3) Difference of Sets/ Relative Complement: The difference of two sets A and B is a set of all those elements which belongs to A but do not belong to B and is denoted by A - B.

Hence, $A-B$ or $A/B = \{x \mid x \in A \text{ AND } x \notin B\}$



Example:

If A= {10,11,12,13} and B= {13,14,15}

then $(A-B) = \{10,11,12\}$ and $(B-A) = \{14,15\}$

Here, we can see $(A-B) \neq (B-A)$

I DO. Let A = {1, 2, 3, 4, 5….20} and B = {1, 2, 3, 4, 5, 6…10}

a. (A−B) =

b. (B−A) =

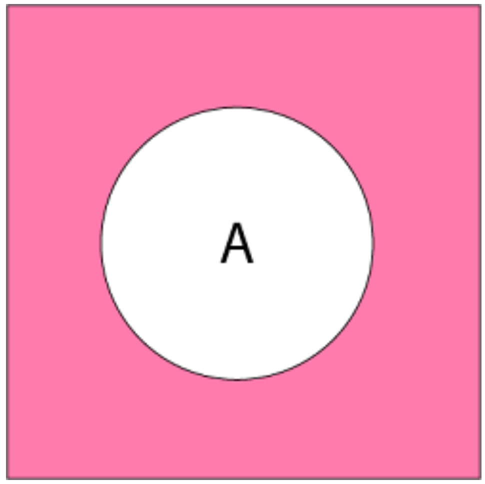
YOU DO. Let A = {a, b, c, d, e, f, g} and B = {a, b, c, d, e, l, m, x}

a. (A−B) =

b. (B−A) =

4) Complement of a Set: The Complement of a Set A is a set of all those elements of the universal set which do not belong to A and is denoted by A^c.

Hence, A^c or $A' = U - A = \{x: x \in U \text{ and } x \notin A\} = \{x: x \notin A\}$



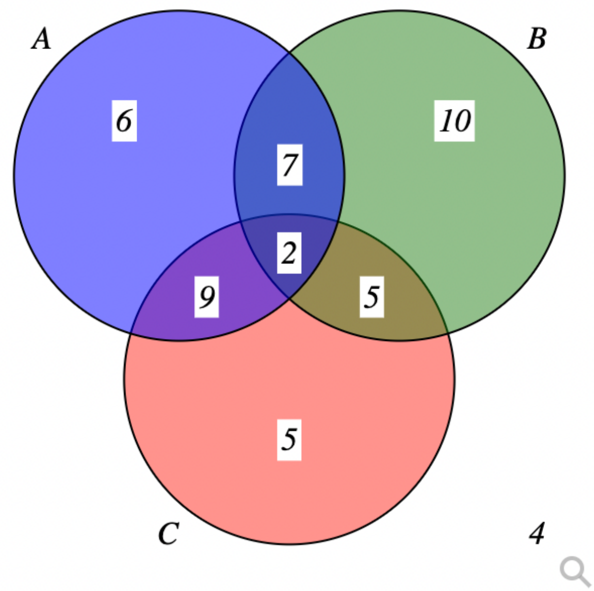
Example: Let U is the set of all natural numbers.
$A = \{1, 2, 3\}$
A^c = {all natural numbers except 1, 2, and 3}.

I DO. Let U is the set of all Integers and $A = \{1, 2, 3\ldots100\}$
$A^c =$

YOU DO. Let U is the set of all Natural Numbers and $B = \{1, 2, 3\ldots100\}$
$B^c =$

WE DO)

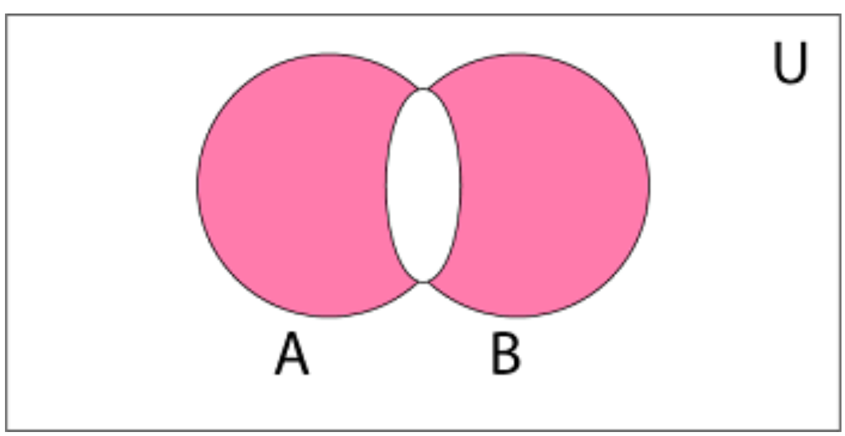
The Venn diagram here shows the cardinality of each set. Use this to find the cardinality of the given set.



$n\left(A^C \cap B^C\right) =$

5) Symmetric Difference of Sets: The symmetric difference of two sets A and B is the set containing all the elements that are in A or B but not in both and is denoted by A ⊕ B i.e.

Hence, A ⊕ B = (A ∪ B) - (A ∩ B)



Example:

Let A = {a, b, c, d}

B = {a, b, l, m}

$A \oplus B$ = {c, d, l, m}

I DO. Let A = {a, b, c, d, e} and B = {a, b, l, m, n}

$A \oplus B$ =

YOU DO. Let A = {1, 2, 3, 4, 5} and B = {1, 2, 3, 4,5}

$A \oplus B$ =

6) Cartesian Product. Given sets A and B, the Cartesian product of A and B, denoted by A x B and read as "A cross B", is the set of all ordered pair (a,b) where a is in A and b is in B. Symbolically:
$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$ Note that A x B is not equal to B x A.

Example 1)

If $A = \{1, 2\}$ and $B = \{a, b\}$, what is A x B?

$A \times B = \{(1, a), (1, b), (2, a), (2, b)\}$. How many elements in a A x B?

Example 2: Let $A = \{1, 2, 3\}$ and $B = \{a, b\}$. Then

$A \times B = \{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)\}$.

Example 3: For the same A and B as in Example 1,

$B \times A = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)\}$.

YOU DO. If $A = \{1, 2, 3\}$ and $B = \{a, b, c\}$, what is A x B

Video: https://www.youtube.com/watch?v=XidkM5J3OQU

Check for Understanding 7(5 minutes)

Let U (Universal Set) = {1, 2, 3, 4, 5… 10}

A = {1, 2, 3, 4, 5, 6}

B = {2, 3, 4, 6, 8, 9, 10}

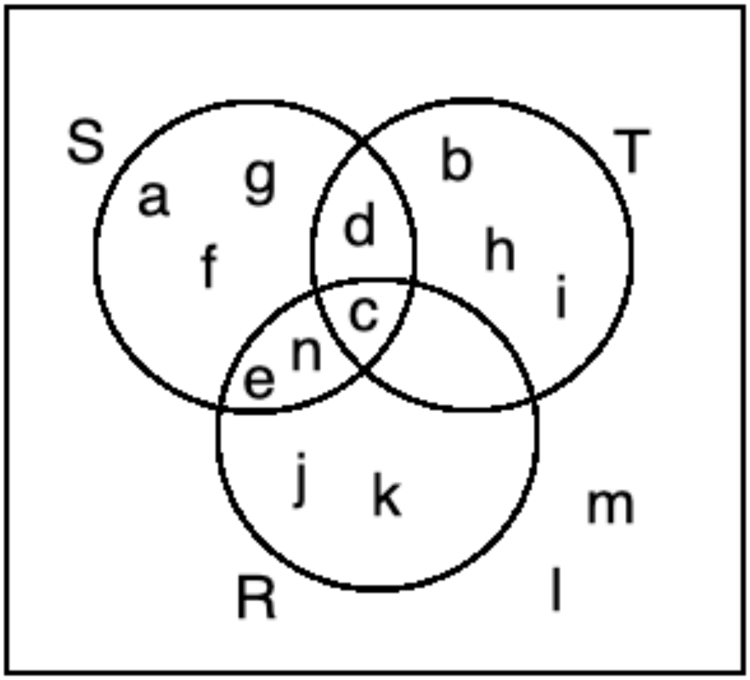
Perform the indicated set operations

1) $(A \cup B) \cap B$ 2) $A - B \cap B^c$

3) $A \oplus B$

4) The sets R, S, and T are displayed in the Venn diagram below where the universal set is the lowercase letters from a to n.

Find the cardinality of the sets.



$n(S \cup R) =$

$n(R \cap S) =$

$n(S^c) =$

$n(R \cap T) =$

$n(S \cup T) =$

$n(T') =$

5) Let U= {1,2, 3,...,10}, A= {1,3,5,7}, B= {1,2,3,4}, and C={3,4,6,7,9}.

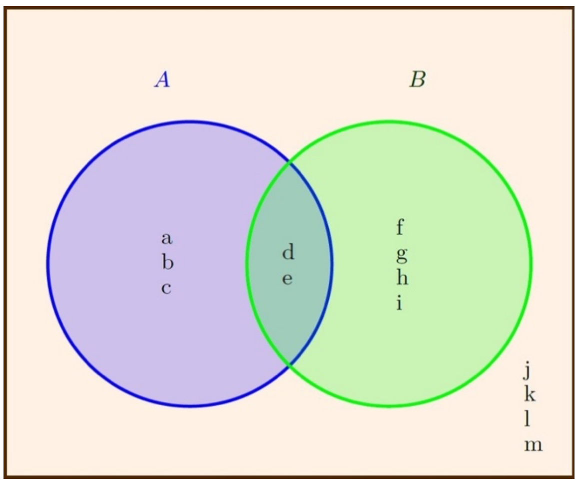
Select $A^c \cap B^c$ from the choices below.

A) {1,3,7,8}
B) {2,5,6,9}
C) {1,4,5,10}
D) {6,8,9,10}
E) {1,2,5,8}
F) Ø

6) Let U= {1,2, 3,...,10}, A={1,3,5,7}, B= {1,2,3,4}, and C= {3,4,6,7,9}. Select A∪∅ from the choices below.

A) {1,4,5,8}
B) {4,7,8,10}
C) {2,4,6,8}
D) {1,3,5,7}
E) {5,7,8,10}
F) ∅

7) Use the following Venn diagram to find $(A\cup B)^c$.



A) {e, p, v}
B) {g, l, m}
C) {l, u, y}
D) {g, t, v}
E) {j, k, l, m}
F) ∅

Do the Assigned Activity

Sources: https://www.javatpoint.com/sets-operations

Important Notes on Set Operations

- Set operation formula for union of sets is $n(A\cup B) = n(A) + n(B) - n(A\cap B)$ and set operation formula for intersection of sets is
  $n(A\cap B) = n(A)+n(B)-n(A\cup B)$.
- The union of any set with the universal set gives the universal set and the intersection of any set A with the universal set gives the set A.
- Union, intersection, difference, and complement are the various

operations on sets.

- The complement of a universal set is an empty set $U' = \phi$. The complement of an **empty set** is a universal set $\phi' = U$.

WE DO 1) If A and B are two sets such that $n(A \cup B) = 70$, $n(A) = 42$, and $n(B) = 50$. How many elements does $A \cap B$ have?

YOU DO 1) If A and B are two sets such that $n(A \cup B) = 40$, $n(A) = 20$, and $n(B) = 10$. How many elements does $A \cap B$ have?

CHECK for UNDERSTANDING 1

True or False

1) Every Z is N.
2) {-2, 2, 8} is equivalent to **{3, 4, 5}**
3) {1, 2, 5} **is not equal to {1, 2, 5}**
4) 4 ⊆ **{1, 2, 5}**
5) {1} ⊆ **{1, 2, 5}**
6) 27 ∉ {W}
7) Every irrational number is a whole number
8) Every integer is a decimal number.
9) Rational and irrational numbers are not real numbers.
10) Natural numbers start from 0.
11) The union of N and Z is Z.

Properties of Set Operations

The properties of set operations are similar to the properties of fundamental operations on numbers. The important properties on set operations are stated below:

1) Commutative Law - For any two given sets A and B, the commutative property is defined as,

A ∪ B = B ∪ A
This means that the set operation of union of two sets is commutative.

Example 1.
A = {1, 2, 3, 4, 5, 6}
B = {2, 3, 4, 6, 8, 9, 10}

A ∩ B = B ∩ A
This means that the set operation of intersection of two sets is commutative.

Example 2.
A = {N}
B = {W}

2) Associative Law - For any three given sets A, B and C the associative property is defined as,

$(A \cup B) \cup C = A \cup (B \cup C)$

This means the set operation of union of sets is associative.

Example 1.

A = {1, 2, 3}

B = {2, 3,4}

$(A \cap B) \cap C = A \cap (B \cap C)$

This means the set operation of intersection of sets is associative.

Example 2.

A = {1, 2, 3}

B = {2, 3,4}

3) De-Morgan's Law - The De Morgan's law states that for any two sets A and B, we have **$(A \cup B)' = A' \cap B'$** and **$(A \cap B)' = A' \cup B'$**

Example 1.

U= {1, 2, 3, 4)

A = {1, 2, 3}

B = {2, 3,4}

Thus,

- $A \cup A = A$
- $A \cap A = A$
- $A \cap \emptyset = \emptyset$
- $A \cup \emptyset = A$
- $A \cap B \subseteq A$
- $A \subseteq A \cup B$

Steps in solving real-world problems involving Venn Diagram

Step 1: **Read the problems/scenarios carefully.**

Step 2: **Identify the number of relationships given in the problem to identify how many circles you need to draw.**

Step 3: **Identify the point of intersections.**

Step 4: **Start solving or identifying the relationships from the bottom.**

WE DO 1) **In a survey involving 150 different factories, it was found out that**

70 purchased brand A
75 purchased brand B
95 purchased brand C
30 purchased brands A and B
45 purchased brands A and C
40 purchased brands B and C
10 purchased brands A, B, and C

YOU DO 1) In an excursion at Pagsanjan Falls, 80 students brought sandwiches, drinks, and cans as follows.

50 students brought sandwiches (S)
30 students brought drinks (D)
30 students brought cans (C)
18 students brought cans and drinks
15 students brought sandwiches and cans
8 students brought sandwiches and drinks
5 students brought sandwiches, drinks, and cans

WE DO 2) In a certain school, 50 students in a certain class were enrolled in three subjects: Algebra, Physics, and Chemistry.
30 enrolled in Algebra
25 enrolled in Physics
25 enrolled in Chemistry
14 enrolled in Chemistry and Algebra
12 enrolled in Chemistry and Physics
11 enrolled in Algebra and Physics
5 enrolled in three subjects

How many students were enrolled in
1)
________ a. exactly one subject
________ b. exactly two subjects

2)
________ a. at most two subjects
________ b. at most one subject

3)
________ a. Algebra or Physics
________ b. Algebra and Physics

4)
________ a. Algebra and Physics but not Chemistry
________ b. Physics and Chemistry but not Algebra

5)

_________ a. Algebra only
_________ b. Physics only

6)

_________ a. How many students did not enroll in any subjects
_________ b. How many students did not enroll in Algebra and Physics

-End of the lesson-
Thank you so much for listening!